Rwanda Mathematics Competition 2019

Date: March 14$^{\text{th}}$, 2019　　　　　　　　　　　　　　　　**Time limit: 3 hours**

Please enter the following information in **PRINT**.

NAME:	**GENDER:** M ☐ F ☐

SCHOOL:	**CLASS:**

INSTRUCTIONS AND REGULATIONS (please read it):

- **In each question in PART ONE and PART TWO cross ($\times$) for the alternative you find is true**. Only one of the alternatives A, B, C, D and E is true.

- **Write full solutions to ALL questions in PART THREE**.

- **No calculators, cellphones nor other electronic devices are allowed.** Only paper, pen, eraser, straightedge and compass.

- **Please do not make any calculation on the problem pages.** Those are only for answers. For your own calculations use scratch papers.

- **If you need additional paper or go to the toilet** please raise your hand wait until one of the invigilators will approach you.

- Students communicating with each other during the test will be removed from the classroom.

- **There are four questions in each category:** Easy (3 points each), Medium (5 points each) and Open problems (8 points each).

- **When the invigilators announce "TIME'S UP"** please collect your papers (with this page on top) and hand them over immediately. **Do NOT include scratch papers**.

Wishing you Good Luck
Celestin Kurujyibwami, Japhet Niyobuhungiro, Layla Sorkatti and Paul Vaderlind.

DO NOT write in the boxes below please

Problems	Easy	Medium	Open	Total Score
Score	/12	/20	/32	/64

PART ONE: EASY. Each correct answer will be awarded 3 points.

1. Consider the sentence *WE ALL LOVE RWANDAN MATH COMPETITION*. Every second the first letter in **each** word is moved to the other end of this word. In how many seconds the original sentence will appear back again?

   ☐ (A) 25 ☐ (B) 31 ☐ (C) 924 ☐ (D) 1848 ☐ (E) 2019

2. How many natural number n satisfy the inequality $\dfrac{1}{5} < \dfrac{n}{n+96} < \dfrac{1}{4}$?

   ☐ (A) 0 ☐ (B) 3 ☐ (C) 7 ☐ (D) 9 ☐(E) Infinitely many

3. Sixteen discs are arranged in four rows of four, the top row being number one. The discs have a number on one side and either red or green on the other. A number on a disc shows how many discs that touch that disc have green on the other side.
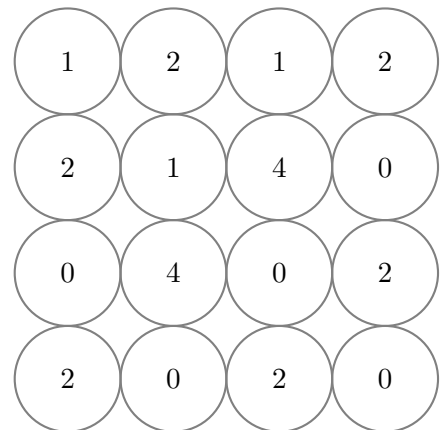
   Which of the following statements is true?
   ☐ (A) All of the rows have the same number of green discs
   ☐ (B) Row one has more green discs than any other row
   ☐ (C) Row two has more green discs than any other row
   ☐ (D) Row three has fewer green discs than any other row
   ☐ (E) Row four has fewer green discs than any other row

4. A piece of paper in the shape of an equilateral triangle has one corner folded over as shown. In the figure one of the angles is $40°$. What is the size of the angle marked by $x°$?
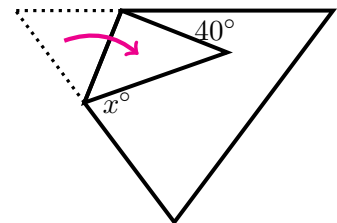
   ☐ (A) $40°$ ☐ (B) $45°$ ☐ (C) $60°$ ☐ (D) $80°$
   ☐ (E) $90°$

PART TWO: MEDIUM. Each correct answer will be awarded 5 points.

5. Vincent and Jeninah throw two dice (one each). How big is the chance that Vincent throws a number higher than Jeninah's?

   □ (A) $1/2$ □ (B) $1/3$ □ (C) $1/4$ □ (D) $1/6$ □ (E) $5/12$

6. The height of a building is 60 meters. At a certain moment during daytime, it casts a shadow of length 40 meters. If a vertical pole of length 2 meters is erected on the roof of the building, the length of the shadow of the pole at the same moment is?

   □ (A) $3/2\,m$ □ (B) $2/3\,m$ □ (C) $4/2\,m$ □ (D) $4/3\,m$ □ (E) None of the previous numbers

7. Let ABC be a triangle with AB and AC of the same length. A circle which passes through vertices B and C cuts the sides AB and AC in points D and E respectively. Assume that line segments BC and CD have the same length and that line segments BD and DE have the same length. Find the angles of the triangle ABC.

   □ (A) $60°, 60°, 60°$ □ (B) $72°, 72°, 36°$ □ (C) $75°, 60°, 45°$
   □ (D) $75°, 75°, 30°$ □ (E) $80°, 70°, 30°$

8. How many different planes are there, which each pass through three or more vertices of a given cube?

   □ (A) 6 □ (B) 12 □ (C) 14 □ (D) 20 □ (E) 26

PART THREE: OPEN PROBLEMS. Each correct solution will be awarded 8 points. **For each of the four problems you have one page to supply a solution. Please do not attach scratch papers.**

9. To a given integer T one may in a single step add 2 or 3, or one can multiply T by 2 or 3. If you start with an odd number T and perform all possible sequences of three steps as described, how many times will you end with an even number?

10. Three equal (congruent) squares form a figure below. Find the angle AMT.?
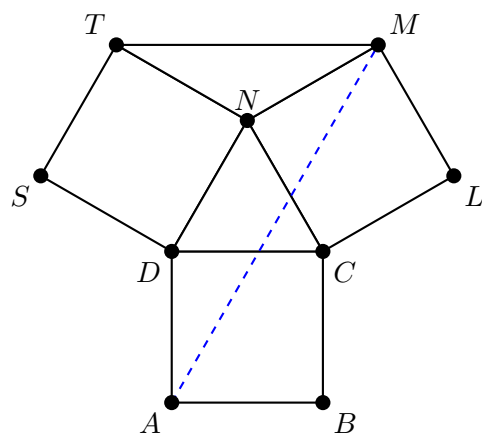
11. Find all pairs of real numvers (x, y) which are solutions to the system of equations

$$\begin{cases} x + y - \lfloor y \rfloor = 3.14 \\ x + \lfloor x \rfloor + \lfloor y \rfloor = 20.19 \end{cases},$$

where $\lfloor a \rfloor$ means the largest integer which is less or equal to a. For example $\lfloor 7.21 \rfloor = 7$, $\lfloor -7.21 \rfloor = -8$ and $\lfloor 13 \rfloor = 13$.

12. In each cell of a 4×4-grid we place one of the numbers $-1, 0$ or 1. Is it possible to do it so that the sums of numbers in each row, in each column and on both diagonals are all different?

************ T H E   E N D ************