NOTE:
1. Please read the instructions on the front cover of this booklet.
2. Write all answers in the answer booklet provided.
3. For questions marked (lightbulb), place your answer in the appropriate box in the answer booklet and **show your work**.
4. For questions marked (pencil), provide a well-organized solution in the answer booklet. Use mathematical statements and words to explain all of the steps of your solution. Work out some details in rough on a separate piece of paper before writing your finished solution.
5. Diagrams are *not* drawn to scale. They are intended as aids only.
6. While calculators may be used for numerical calculations, other mathematical steps must be shown and justified in your written solutions, and specific marks may be allocated for these steps. For example, while your calculator might be able to find the $x$-intercepts of the graph of an equation like $y = x^3 - x$, you should show the algebraic steps that you used to find these numbers, rather than simply writing these numbers down.

**A Note about Bubbling**

Please make sure that you have correctly coded your name, date of birth and grade on the Student Information Form, and that you have answered the question about eligibility.

1. (a) If $x = 11$, what is the value of $\dfrac{3x+6}{x+2}$ ?

   (b) What is the $y$-intercept of the line that passes through $A(-1, 5)$ and $B(1, 7)$?

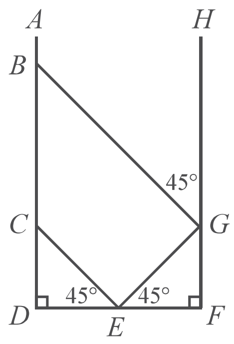
   (c) The lines with equations $y = 3x + 7$, $y = x + 9$, and $y = mx + 17$ intersect at a single point. Determine the value of $m$.

2. (a) The three-digit positive integer $m$ is odd and has three distinct digits. If the hundreds digit of $m$ equals the product of the tens digit and ones (units) digit of $m$, what is $m$?

   (b) Eleanor has 100 marbles, each of which is black or gold. The ratio of the number of black marbles to the number of gold marbles is $1 : 4$. How many gold marbles should she add to change this ratio to $1 : 6$?

   (c) Suppose that $n$ is a positive integer and that the value of $\dfrac{n^2+n+15}{n}$ is an integer. Determine all possible values of $n$.

3. (a) Donna has a laser at $C$. She points the laser beam at the point $E$. The beam reflects off of $DF$ at $E$ and then off of $FH$ at $G$, as shown, arriving at point $B$ on $AD$. If $DE = EF = 1$ m, what is the length of $BD$, in metres?




(b) Ada starts with $x = 10$ and $y = 2$, and applies the following process:

Step 1: Add $x$ and $y$. Let $x$ equal the result. The value of $y$ does not change.

Step 2: Multiply $x$ and $y$. Let $x$ equal the result. The value of $y$ does not change.

Step 3: Add $y$ and 1. Let $y$ equal the result. The value of $x$ does not change.

Ada keeps track of the values of $x$ and $y$:

| | $x$ | $y$ |
|---|---|---|
| Before Step 1 | 10 | 2 |
| After Step 1 | 12 | 2 |
| After Step 2 | 24 | 2 |
| After Step 3 | 24 | 3 |

Continuing now with $x = 24$ and $y = 3$, Ada applies the process two more times. What is the final value of $x$?

(c) Determine all integers $k$, with $k \neq 0$, for which the parabola with equation $y = kx^2 + 6x + k$ has two distinct $x$-intercepts.

4. (a) The positive integers $a$ and $b$ have no common divisor larger than 1. If the difference between $b$ and $a$ is 15 and $\frac{5}{9} < \frac{a}{b} < \frac{4}{7}$, what is the value of $\frac{a}{b}$?

(b) A geometric sequence has first term 10 and common ratio $\frac{1}{2}$.
An arithmetic sequence has first term 10 and common difference $d$.
The ratio of the 6th term in the geometric sequence to the 4th term in the geometric sequence equals the ratio of the 6th term in the arithmetic sequence to the 4th term in the arithmetic sequence.
Determine all possible values of $d$.

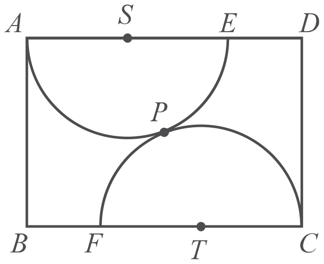
(An *arithmetic sequence* is a sequence in which each term after the first is obtained from the previous term by adding a constant, called the common difference. For example, $3, 5, 7, 9$ are the first four terms of an arithmetic sequence.
A *geometric sequence* is a sequence in which each term after the first is obtained from the previous term by multiplying it by a non-zero constant, called the common ratio. For example, 3, 6, 12 is a geometric sequence with three terms.)

5. (a) For each positive real number $x$, define $f(x)$ to be the number of prime numbers $p$ that satisfy $x \leq p \leq x + 10$. What is the value of $f(f(20))$?

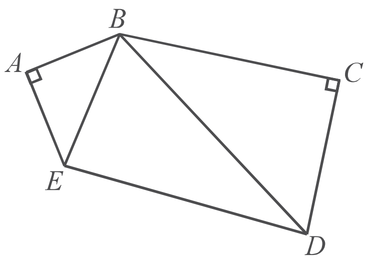
(b) Determine all triples $(x, y, z)$ of real numbers that satisfy the following system of equations:

$$\begin{aligned}(x-1)(y-2) &= 0 \\ (x-3)(z+2) &= 0 \\ x + yz &= 9\end{aligned}$$

6. (a) Rectangle $ABCD$ has $AB = 4$ and $BC = 6$. The semi-circles with diameters $AE$ and $FC$ each have radius $r$, have centres $S$ and $T$, and touch at a single point $P$, as shown. What is the value of $r$?




(b) In the diagram, $\triangle ABE$ is right-angled at $A$, $\triangle BCD$ is right-angled at $C$, $\angle ABC = 135°$, and $AB = AE = 7\sqrt{2}$. If $DC = 4x$, $DB = 8x$ and $DE = 8x - 6$ for some real number $x$, determine all possible values of $x$.




7. (a) Suppose that the function $g$ satisfies $g(x) = 2x - 4$ for all real numbers $x$ and that $g^{-1}$ is the inverse function of $g$. Suppose that the function $f$ satisfies $g(f(g^{-1}(x))) = 2x^2 + 16x + 26$ for all real numbers $x$. What is the value of $f(\pi)$?

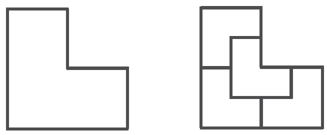
(b) Determine all pairs of angles $(x, y)$ with $0° \leq x < 180°$ and $0° \leq y < 180°$ that satisfy the following system of equations:

$$\begin{aligned}\log_2(\sin x \cos y) &= -\frac{3}{2} \\ \log_2\left(\frac{\sin x}{\cos y}\right) &= \frac{1}{2}\end{aligned}$$

8. (a) Four tennis players Alain, Bianca, Chen, and Dave take part in a tournament in which a total of three matches are played. First, two players are chosen randomly to play each other. The other two players also play each other. The winners of the two matches then play to decide the tournament champion. Alain, Bianca and Chen are equally matched (that is, when a match is played between any two of them, the probability that each player wins is $\frac{1}{2}$). When Dave plays each of Alain, Bianca and Chen, the probability that Dave wins is $p$, for some real number $p$. Determine the probability that Bianca wins the tournament, expressing your answer in the form $\dfrac{ap^2 + bp + c}{d}$ where $a$, $b$, $c$, and $d$ are integers.

(b) Three microphones $A$, $B$ and $C$ are placed on a line such that $A$ is 1 km west of $B$ and $C$ is 2 km east of $B$. A large explosion occurs at a point $P$ not on this line. Each of the three microphones receives the sound. The sound travels at $\frac{1}{3}$ km/s. Microphone $B$ receives the sound first, microphone $A$ receives the sound $\frac{1}{2}$ s later, and microphone $C$ receives it 1 s after microphone $A$. Determine the distance from microphone $B$ to the explosion at $P$.

9. (a) An L shape is made by adjoining three congruent squares. The L is subdivided into four smaller L shapes, as shown. Each of the resulting L's is subdivided in this same way. After the third round of subdivisions, how many L's of the smallest size are there?


Original L    After 1 round

(b) After the third round of subdivisions, how many L's of the smallest size are in the same orientation as the original L?

(c) Starting with the original L shape, 2020 rounds of subdivisions are made. Determine the number of L's of the smallest size that are in the same orientation as the original L.

10. Kerry has a list of $n$ integers $a_1, a_2, \ldots, a_n$ satisfying $a_1 \le a_2 \le \ldots \le a_n$. Kerry calculates the pairwise sums of all $m = \frac{1}{2}n(n-1)$ possible pairs of integers in her list and orders these pairwise sums as $s_1 \le s_2 \le \ldots \le s_m$. For example, if Kerry's list consists of the three integers $1, 2, 4$, the three pairwise sums are $3, 5, 6$.

(a) Suppose that $n = 4$ and that the 6 pairwise sums are $s_1 = 8$, $s_2 = 104$, $s_3 = 106$, $s_4 = 110$, $s_5 = 112$, and $s_6 = 208$. Determine two possible lists $a_1, a_2, a_3, a_4$ that Kerry could have.

(b) Suppose that $n = 5$ and that the 10 pairwise sums are $s_1, s_2, \ldots, s_{10}$. Prove that there is only one possibility for Kerry's list $a_1, a_2, a_3, a_4, a_5$.

(c) Suppose that $n = 16$. Prove that there are two different lists $a_1, a_2, \ldots, a_{16}$ and $b_1, b_2, \ldots, b_{16}$ that produce the same list of sums $s_1, s_2, \ldots, s_{120}$.