
Online Training Exam Aug 2020

- Total marks = 30 (20 marks on section A and 10 marks on section B)
- You must show your work to get full marks on each question. (i.e. answer only is not enough)
- Time: 10am - 12:30pm Monday 17th August.
- All work submitted should be your own. You are not allowed to use the internet or ask your friends for help.

Section A

1. **(1 mark)** Prove that having 100 whole numbers, one can choose 15 of them so that the difference of any two is divisible by 7
2. **(2 marks)** Prove that in any collection of 31 distinct integers between 1 and 60, you can find two integers such that one divides the other.
3. **(2 marks)** Prove that among any ten points located on a circle with diameter 5, there exist at least two at a distance less than 2 from each other.
4. **(2 marks)** Consider a cyclic quadrilateral $ABCD$ with $|AB| = |BC| = |CD|$. Draw in the diagonals AC and BD . Let $\angle DAC = a$. Mark in all the angles in terms of a .
5. **(3 marks)** Let ABC be an isosceles triangle with $|AB| = |AC|$. The bisector of the angle BAC meets the side BC at the point D . Prove that if the triangle ACD is isosceles, the triangle ABD must also be isosceles
6. Consider the number $x = 2 \times 4 \times 6 \times 8 \times 12 \times 14 \times 16 \times 18 \times 22 \dots \times 98$.
 - (a) **(1 mark)** What is the last digit of x ?
 - (b) **(1 mark)** What is the remainder of x on division by 13?
 - (c) **(1 mark)** What is the remainder of x on division by 5?
7. **(2 marks)** Gilbert is turning 15 on Tuesday 18th August 2020. What day of the week was he born? (Note that 2020, 2016, 2012, 2008 etc are leap years). Show your working.
8. **(2 marks)** What is the domain, co-domain and range of the following functions?
 - (a) $f : \mathbb{Z} \rightarrow \mathbb{R}$ with $f(x) = x + 1$
 - (b) $f : \mathbb{R} \rightarrow \mathbb{R}$ with $f(x) = x^4$
9. **(3 marks)** State whether the following functions are injective, surjective, bijective or None.
 - (a) **(1 mark)** $f : \mathbb{Z} \rightarrow \mathbb{R}$ with $f(x) = 5x + 3$
 - (b) **(1 mark)** $f : \mathbb{R} \rightarrow \mathbb{R}^+$ with $f(x) = x^2 - 6x + 9$
 - (c) **(1 mark)** $f : \mathbb{R} \rightarrow \mathbb{R}$ with $f(x) = x^3$