# AIMS Maths Competition 2021 Round 3

Note: Please explain your working on all solutions. There are no marks for answer only. Calculators are not allowed. Do not use the same paper for multiple questions. Write your name on every piece of paper.

1. (4 marks) A circle has centre O. OF is perpendicular to chord DC at F and is perpendicular to chord AB at E. If $AB = 8$, $DC = 6$ and $EF = 1$, determine the radius of the circle.
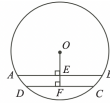
2. (4 marks) Find the number of solutions (x,y) to $x^2 - y^2 = 24$ where x and y are both integers.

3. (4 marks) I choose 3 random integers from 1 to 6 with replacement. What is the probability that the sum of the 3 integers is the square of an integer?

4. (7 marks) A positive integer divisor of a number X is any positive integer that can divide X with no remainder. E.g. 3 is a positive integer divisor of 9. The product of all of the positive integer divisors of 81^8 equals 3^k for some integer k. Determine the value of k.

5. (7 marks) What is the largest three-digit number with the property that the number is equal to the sum of its hundreds digit, the square of its tens digit and the cube of its units digit? E.g. 123 is not such a number because $1 + 2^2 + 3^3$ is not equal to 123.

6. (7 marks) Triangle ABC has $AC = 450$ and $BC = 300$. Points K and L are located on $\overline{AC}$ and $\overline{AB}$ respectively so that $AK = CK$, and $\overline{CL}$ is the angle bisector of angle C. Let P be the point of intersection of $\overline{BK}$ and $\overline{CL}$, and let M be the point on line BK for which K is the midpoint of $\overline{PM}$. If $AM = CP = 180$, find LP.
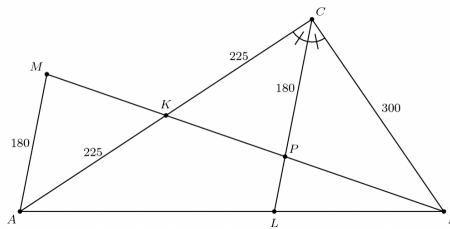
7. (7 marks) As shown in the diagram, you can create a 3x4 rectangle using exactly 31 matches. (Note that a 3x4 rectangle is the same as a 4x3 rectangle). List all rectangles that can be constructed using:
a) exactly 22 matches.
b) exactly 337 matches