Rwanda Mathematics Competition 2020/21
Final Stage

Date: May 7th, 2021 **Time limit: 3 hours**

Please enter the following information in **PRINT**.

NAME: **GENDER:** M ☐ F ☐

SCHOOL: **CLASS:**

INSTRUCTIONS AND REGULATIONS (please read it):

- **Write full solutions to ALL questions**.
- **No calculators, cellphones nor other electronic devices are allowed.** Only paper, pen, eraser, straightedge and compass.
- For your own calculations use scratch papers. **For the final solution use the page under each problem.**
- **If you need additional paper or go to the toilet** please raise your hand wait until one of the invigilators will approach you.
- Students communicating with each other during the test will be removed from the classroom.
- **When the invigilators announce "TIME'S UP"** please collect your papers (with this page on top) and hand them over immediately. **Do NOT include scratch papers**.
- Each answer will be awarded with up to 7 points.

Wishing you Good Luck

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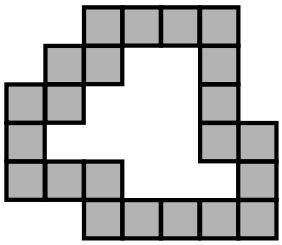
DO NOT write in the boxes below please

Problems	ONE	TWO	THREE	FOUR	FIVE	SIX	Total Score
Score	/7	/7	/7	/7	/7	/7	/42

PROBLEMS.

1. There are 660 cats and dogs in Huye, some of them are really crazy. 25% of dogs consider themselves as cats, and 20% of cats consider themselves as dogs. The remaining dogs and cats consider themselves as they are. After en inquiry among all cats and dogs, one found out that 40% of population said they are cats. How many cats and how many dogs there are in Huye?

2. In the figure below is an example of a ”chain” consisting of 22 identical squares. In such chain each of the squares has exactly two of its sides common with two other squares. Is it possible to construct a chain consisting of 123 squares?



3. Find all **integer** solutions x and y to the equation

$$(x + y^2)(x^2 + y) = (x + y)^3 - 6xy.$$

4. Let $ABCDE$ be a convex pentagon with five equal sides and right angles at C and D. Let P denote the intersection point of the diagonals AC and BD. Prove that the lengths of PA and PD are equal. (A convex polygon has all its internal angles smaller than 180 degrees.)

5. Suppose that A, B, C and D are integers. Show that the product

$$(A-B)(A-C)(A-D)(B-C)(B-D)(C-D)$$

is divisible by 12.

6. (a) Find which of the following six statements are true and which are false:

   (1) Exactly one of these six statements is false.

   (2) Exactly two of these six statements are false.

   (3) Exactly three of these six statements are false.

   (4) Exactly four of these six statements are false.

   (5) Exactly five of these six statements are false.

   (6) Exactly six of these six statements are false.

   (b) Remove the word ”Exactly” from each of the six statements above and then find out which statements are true and which are false.

********** THE END **********