

Rwanda Maths Olympiad Team Selection Test

Paper 1,

November 30th 2022

Each part (a) is worth 2 marks. Each part (b) is worth 4 marks.

- (a) Three consecutive positive integers are written on a board. One of them is erased. The sum of the remaining 2 numbers is 40. What is the erased number?

(b) Seven consecutive positive integers are written on a board. One of them is erased. The sum of the remaining six numbers is 197. What is the erased number?
- Consider x and y as real numbers.

(a) What is the minimum possible value of: $(x+3)^2 + (x-1)^2$

(b) What is the minimum value of: $(x+3)^2 + 2(y-2)^2 + 4(x-7)^2 + (y+4)^2$
- (a) Consider a circle with 6 equally spaced points on the circumference. Label the points 1 to 6, in increasing order. From each point draw a line through the centre of the circle to the other side. Each pair of numbers connected by a line is called "diametrically opposite". Then, what is diametrically opposite to 3?

(b) On a new circle we have n equally spaced points, labeled from 1 to n in increasing order. 19 and 59 are diametrically opposite. What is the value of n ?
- (a) If a and b are positive integers such that $\frac{1}{a} + \frac{1}{2a} + \frac{1}{3a} = \frac{1}{b^2 - 2b}$ then a must be divisible by:
(A)3 (B)5 (C)7 (D)11 (E)13

(b) Then, what is smallest possible value of $a + b$?
- (a) There is a group of 5 persons A, B, C, D and E. In the group, there is one badminton player, one chess player and one tennis player. A and D are unmarried ladies and they do not play any games. No lady is a chess player or a badminton player. There is a married couple of which E is a husband. B is the brother of C and is neither a chess player nor a tennis player. Who is the tennis player?

(b) A girl meets a lion and unicorn in the forest. The lion lies every Monday, Tuesday and Wednesday and the other days he speaks the truth. The unicorn lies on Thursdays, Fridays and Saturdays, and the other days of the week he speaks the truth. "Yesterday I was lying," the lion told the girl. "So was I," said the unicorn. What day is it?
- (a) Given that a_1, a_2 and a_3 is a permutation of 1, 2, and 3. Find the maximum possible value of $|a_1 - 1| + |a_2 - 2| + |a_3 - 3|$

(b) Given that $a_1, a_2, a_3, \dots, a_{97}$ is a permutation of 1, 2, 3, ..., 97 Find the maximum possible value of $|a_1 - 1| + |a_2 - 2| + |a_3 - 3| + \dots + |a_{97} - 97|$
- (a) A circle with radius 6 is inscribed in a square. Find the radius of the circle that is inscribed between two sides of the square and the original circle.

(b) A circle with radius 6 is inscribed in an equilateral triangle. Find the radius of the circle that is inscribed between two sides of the triangle and the original circle.
- Consider the x-axis on the coordinate plane. Point A is at (0,0). Point B at $(2a, 0)$ and Point C at $(2a + 2b, 0)$. Equilateral triangles ABD, BCE, CAF are constructed with D, E above the x-axis and F below the x-axis.

(a) Find the distance between the centroids of ABD and BCE in terms of x and y . You may use the fact that $\tan(30) = \frac{1}{\sqrt{3}}$

(b) Find the distance between the orthocenters of BCE and CAF in terms of x and y

Kinyarwanda translations:

3 (a) Kumuzenguruko w'uruziga hashushanyijeho utudomo 6 kuburyo harimo intera zingana hagati y'utudomo dukurikiranye. Utwo tudomo twite kuva kuri 1 ukazamuka kugera ku 6 uko dukurikiranye kuruziga. ca umurongo uva hagati na hagati muruziga (Center of the circle) ukagera kuri buri kado. Buri mibare ibiri ibiri ihujwe n'umurongo umwe unaca muri Center irateganye ikitwa (diametrically Opposite). Nuwuhwe mubare uteganye (diametrically opposite) na 3?

(b) Kurundi ruziga hashushanyijeho utudomo n muburyo bumwe n'ubwakoreshajwe muri (a). Nazo zitwa amazina kuva kuri 1 ugenda uzamuka kugera kuri n uko utudomo dukurikiranye. Niba 19 na 59 ziteganye (diametrically opposite), ubwo n ingana gute?

5 (a) Ufashe groupe irimo abantu batanu bitwa aya mazina "A", "B", "C", "D" hamwe na "E". Umwe muribo akina umukino witwa "badminton", Umwe muribo akina umukino witwa "Chess", Umwe muribo akina umukino witwa "Tennis". A na D ni abakobwa badafite abagabo, kandi nta mukino n'umwe bakina. Nta mukobwa cyangwa umugore ukina Chess cyangwa Badminton. Hari abantu babiri bashakanye E akaba umugabo. B ni musaza wa C akaba adakina Chess cyangwa Tennis. Ninde ukina Tennis?

5 (b) Umukobwa ari kugenda mwishyamba ahura n'intare anahura na "Unicorn" (akanyamanswa k'ihembe rimwe). Intare ibeshya Kuwambere, Kuwakabiri no Kuwagatatu hanyuma indi minsi yose ikavugisha ukuri. Naho akanyamanswa k'ihembe rimwe kabeshya kuwakane, kuwagatanu no kuwagatandatu hanyuma indi minsi yose kakavushisha ukuri. Intare ibwira umukobwa iti: "Ejo hashize narabeshyaga". Akanyamanswa k'ihembe rimwe nako kati: "nange ejo hashize narabeshyaga". Ubwo uwo munsu biri kuba ni kuwakangahe?