

Rwanda Maths Olympiad Team Selection Test

Paper 2

November 30th 2022

Each question is worth 7 marks

1. Find all integer solutions to

$$(y^2 - y)(x + y) = 4(x - 1) - y(y^2 + x)$$

2. We have 2 circles, S_1 and S_2 . The circle S_2 is inside S_1 , and they touch at point P . The line p is tangent to both circles at P . Draw the line l which is tangent to S_2 at R , and perpendicular to p . It touches S_1 at N and M . p and l intersect at L

(a) Prove that $\angle LPR$ is 45°

(b) Prove that RP bisects angle MPN

3. Let n be a positive integer. Is it possible to have $12n - 119$ and $75n - 539$ both being perfect squares? If yes, find all values of n for which it is possible.
4. Let a, b, c, x and y be five real numbers such that $a^3 + ax + y = 0$, $b^3 + bx + y = 0$, $c^3 + cx + y = 0$. If a, b and c are all distinct real numbers, show that the sum $a + b + c$ is zero.
5. Given five distinct points P_1, P_2, P_3, P_4, P_5 in the plane having integer coordinates, prove that there is at least one pair (P_i, P_j) , with $i \neq j$, such that the line $P_i P_j$ contains a point Q having integer coordinates and lying strictly between P_i and P_j .