Bayesian System Identification and Predictive Algorithms in Wavefront Sensing and Control for Exoplanet Imaging

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Abstract

In direct high-contrast imaging, the residual electric field from the host star is predominant in the face of the electric field from the planet we are interested in. Wavefront Sensing and Control (WFS&C) are methods using both estimation and control algorithms to correct errors in the telescope's optics and allow to observe planets much fainter than their host stars in the search for life. **Until now, no space telescope has integrated WFS&C algorithms** but they could greatly enhance the image processing contrast. In current designs, previous information is discarded at each step of estimation and control, and the telescope use is frozen to allow estimation. We demonstrate that it is beneficial to account for past history of photons on the science camera and propose a prediction framework that significantly reduces the instrument observation time. The algorithm has been validated in both simulation and in experimentation on the laboratory testbed. We show that it **greatly reduces the estimation time**, and it has been added to the **open-source package 'catkit'** that will be deployed on other testbeds.

Context

The great challenge in future NASA flagship missions is the **direct imaging and characterization of exoplanets in the search for life**. The difficulty is that those exoplanets orbit around a star that is much brighter than them. There remains an important residual electric field (speckles) from the star on the science camera, which prevents us from achieving sufficient contrast to observe planets that are 10^{10} times fainter than their host star.

Model of the System

We assume that the actuation of the deformable mirrors (DMs) is *small* and that the coronagraph has a *linear* contribution on the system.

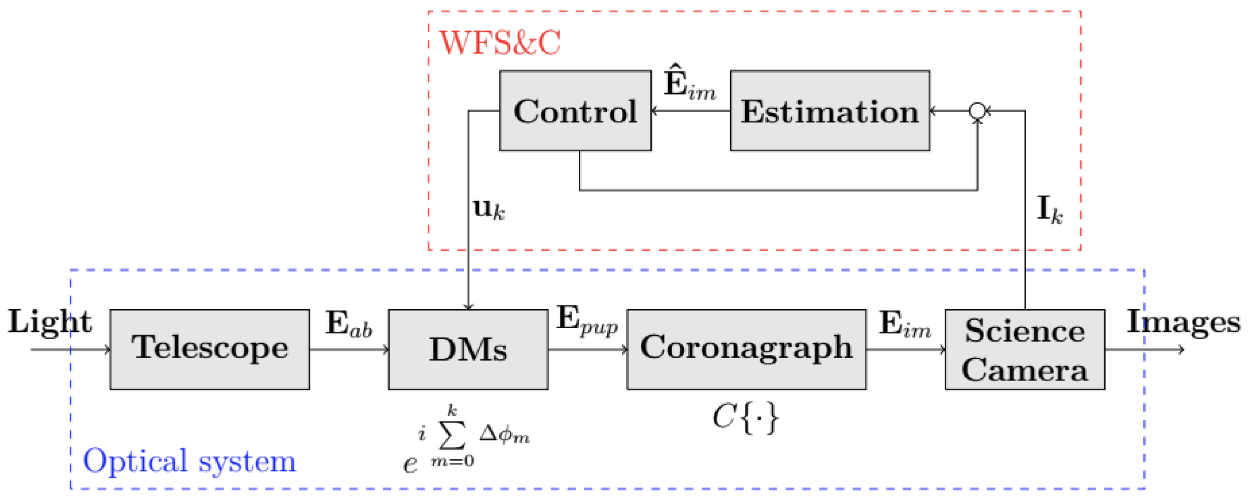

Figure 1: Block diagram of the estimation and control closed-loop.

$$\mathbf{E}_{im,k} = \mathbf{E}_{im,k-1} + \mathbf{G}_k \mathbf{u}_k \tag{1}$$

With $\mathbf{E}_{im,k} \in \mathbb{R}^{N_{pix}}$ the electric field vector on the science camera, $\mathbf{u}_k \in \mathbb{R}^{N_{act}}$ the vector of DMs actuations at step $k+1$ and $\mathbf{G}_k \in \mathbb{R}^{N_{pix} \times N_{act}}$ the Jacobian matrix of the DMs influence.

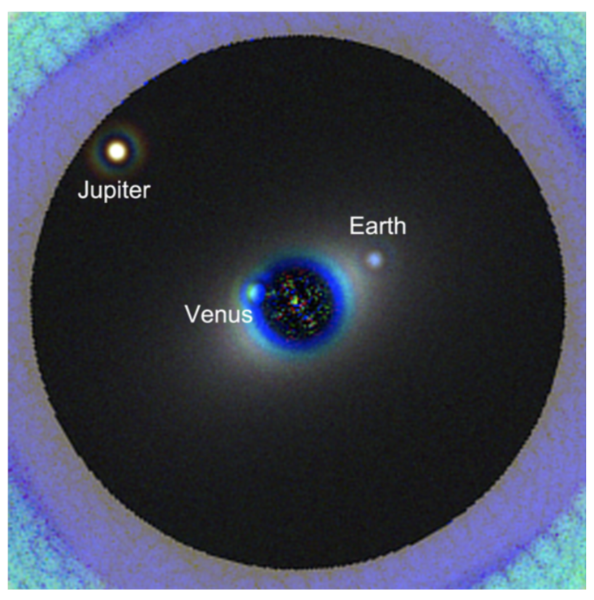

Figure 2: A simulated image of a solar system twin as seen with the proposed high-contrast optical system.

Contrast This complicated optical system has been designed to create a **dark hole**, a zone of great contrast that would potentially allow to see very dim exoplanets.

Estimation Problem

Problem How to estimate the electric field $\mathbf{E}$ given its intensity $\mathbf{I} = |\mathbf{E}|^2$?

State of the art Currently, we estimate $\mathbf{E}$ from $\mathbf{I}$ by choosing a set of probe functions u_j^p that are applied to DMs and we measure resulting intensity patterns :

$$\begin{aligned} \mathbf{I}_j^+ &= |\mathbf{E}_{im,k-1} + \mathbf{G}_k \mathbf{u}_j^p|^2 \\ \mathbf{I}_j^- &= |\mathbf{E}_{im,k-1} - \mathbf{G}_k \mathbf{u}_j^p|^2 \end{aligned} \tag{2}$$

This method is called **Pairwise estimation**, and measured intensities along with forward model allow to construct a system of equations and solve for real and imaginary part of field at each pixel in dark zone:

$$\begin{pmatrix} [\mathbf{I}_0^+ - \mathbf{I}_0^-]_m \\ \vdots \\ [\mathbf{I}_{s-1}^+ - \mathbf{I}_{s-1}^-]_m \end{pmatrix} = \mathbf{P} \begin{pmatrix} [\mathrm{Re}(\mathbf{E}_{im,k-1})]_m \\ \\ [\mathrm{Im}(\mathbf{E}_{im,k-1})]_m \end{pmatrix}, \forall\, m \tag{3}$$

Need for new algorithms The problem is that probing creates a huge loss in observation time because while we are tweaking the DMs actuation and sending light through the telescope, we can't use the image plane for science.

Main objectives of the internship

1. Develop statistical WFS&C methods that could preserve the contrast while reducing the observation time on the telescope.
2. Validate those algorithms and implement them on the testbed of the laboratory.

Empirical Orthogonal Functions

Presentation of the algorithm We want to train a linear filter $\mathbf{F}$ such that the **predicted value** of the wavefront w at an ulterior time is a **linear sum of past measurements** at the n previous times:

$$\hat{\mathrm{w}}(t+\delta t) = \mathbf{F}\mathbf{h}(t) \tag{4}$$

We learn the filter based on previous measurements contained in the $n \times m$ by l data matrix

$$\mathbf{D} = \begin{bmatrix} | & | & & | \\ \mathbf{h}(t) & \mathbf{h}(t-dt) & \cdots & \mathbf{h}(t-(l-1)dt) \\ | & | & & | \end{bmatrix} \tag{5}$$

which is a horizontal concatenation of the last l history vectors.

Multivariate prediction We want a filter $\mathbf{F}$ that minimizes the Euclidian distance between prediction and actual wavefront, temporally averaged over a sufficiently large number of measurements

$$\min_{\mathbf{F}} \langle\, || \underbrace{\hat{\mathrm{w}}(t+\delta t)}_{\mathbf{F}\mathbf{h}(t)} - \tilde{\mathrm{w}}(t+\delta t)||_2^2 \,\rangle_T \tag{6}$$

which writes in matrix form as the classical least squares solution, and can be regularized:

$$\min_{\mathbf{F}} ||\mathbf{D}^T\mathbf{F}^T - \tilde{\mathbf{P}}^T||_2^2 + \lambda\, ||\mathbf{F}||_2^2 \tag{7}$$

With $\mathbf{F}$ the m by $n \times m$ filter matrix and $\mathbf{P}$ the m by l a posteriori matrices for complete estimation, respectively.

Results

The algorithm has been understood, validated in simulation and implemented on the testbed. It allows **dark hole maintenance** and a **reduction in computation time**.

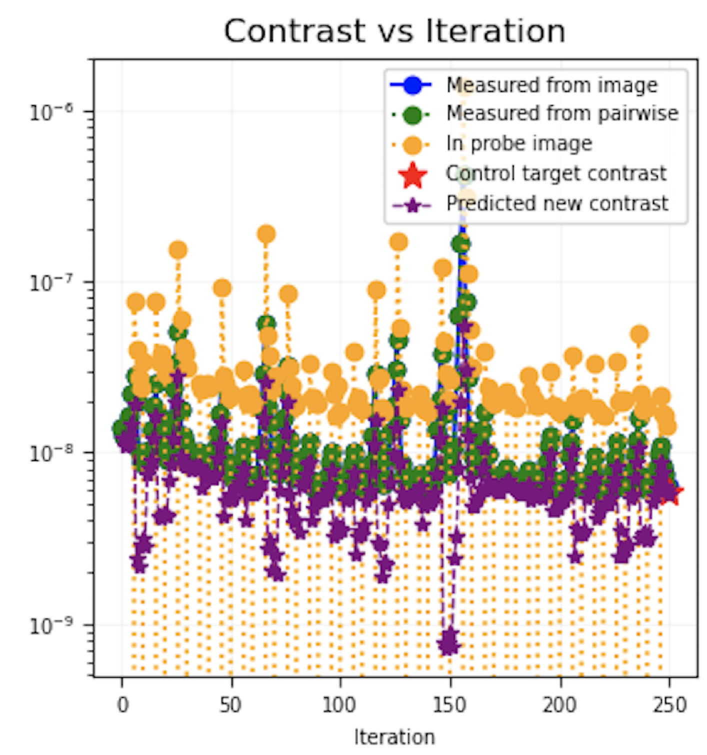

Figure 3: Dark hole maintenance run in simulation. The algorithm keeps the contrast below $1e-7$ which is comparable to state of the art methods. The regularization parameter λ is $1e-3$, the number of past measurements in each history vector is 5 and the number of vectors in the training set is 80. Every five iterations, we alternate between pairwise estimation and filter prediction, keeping all pairwise estimates for the next computation of the filter. The module for prediction has been implemented in Python and deployed on the testbed using Git.

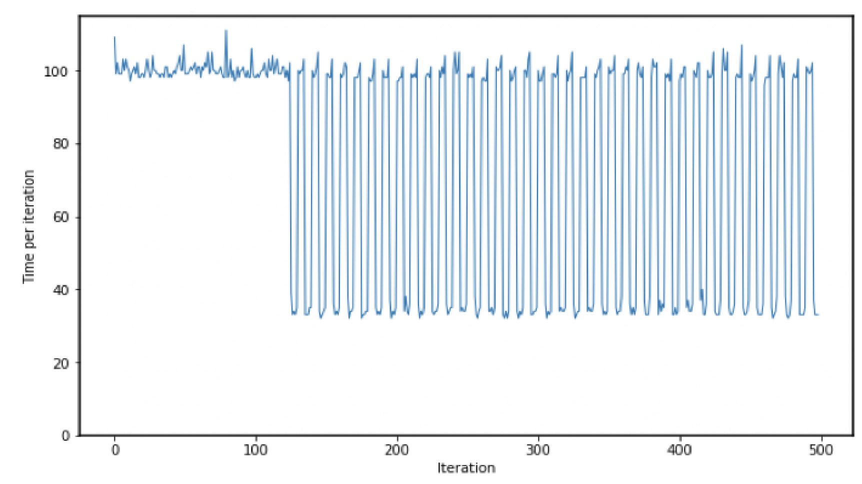

Figure 4: Dark hole maintenance run on the testbed, time in seconds is shown on the Y-axis. The algorithm demonstrates significant reduction in computation time. Furthermore, on a real telescope, the gain in time will be much greater because we are not using a laser like on the testbed, meaning probing techniques will take much longer to estimate the electric field. Every time we simulate probes on the optical system, we can't use it for observation because the DMs are not correcting the image plane anymore, which is another reason why this method is very promising. The filter is computed using the last 100 history vectors of 20 past measurements each.

Next Steps

- Fine-tune the algorithm parameters such as the regularization and the size of the training set.
- Investigate how the algorithm behaves when we take into account photon noise and speckle drift.
- Expand this work onto other testbeds to compare their performances with ours.

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