Min-max Fair Optimal Precoding and Decoding Matrix Design for Linear Video Coders

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Abstract

SoftCast, a linear Joint Source-Channel Video Coding (JSCVC) scheme, has proven that it can provide transmission fairness as it only uses linear operators, thus ensuring that a cliff effect does not appear when the Signal to Noise Ratio (SNR) of the canal is below the SNR for which the encoder is designed. In this paper, we aim at minimizing the maximum Mean Squared Error (MSE) of the estimated signal among all estimators by designing precoding and decoding matrices. We propose an optimal solution, and apply it to a simple case, and to real data. We show that our algorithm brings satisfying results, and could be implemented in a real data coding scheme.

Keywords: Semi-definite positive optimization, Video coding, Multicast

1. Introduction

The Quality of Service (QoS) specification for SoftCast coding that aims at minimizing the total MSE of the receiver under per-subchannel power constraints has been established in S. Zheng's thesis and in [3]. This thesis was supervised by M. Kieffer, and defended in October 2018 at the University Paris-Saclay. This paper adopts another approach, which is the one of minimizing the maximum MSE of the signal waveform estimation among receivers. One of the goals of this work is to promote the fairness of Linear Video Coding (LVC) schemes even more. We transmit a single message to multiple receivers through one transmitter, and we compute the MSE, which can be interpreted as the signal reconstruction error energy. We identify which user has the worse channel, that is to say the greatest MSE, and we aim at minimizing its energy subject to a power constraint at the transmitter. We use a precoding matrix $\mathbf{G}$ and decoding matrices $\mathbf{H}_i$ and iterate to minimize the greatest MSE at every step. An algorithm for the Min-Max optimization problem has already been proposed in [2] and we aim at applying it on SoftCast.

This paper is organized as follows. First, Section 2 describes a simple SoftCast scheme and the state of the art in JSCVC. Section 3 introduces a model of the considered system and explains the optimization problem and optimal algorithm. In Section 4, an application of our algorithm on a video sequence is studied, and the interpretation of the results as well as a comparison between different channels are provided. Section 5 compares our work with other state-of-the art optimization methods, and finally section 6 proposes possible developments for this project and conclusion on our work.

Appendices detail some of the mathematical background used in the report.

2. SoftCast, a Joint Source Channel Video Coding Scheme

SoftCast [citation] is a joint source-channel coding scheme that can resolve the unfairness problem encountered by conventional video codecs in the broadcast scenario. Firstly, we present the general structure of a Softcast video coder and we then present the specific coder we implemented.

2.1. Architecture of a Sofcast video coder

The architecture of SoftCast is shown in Figure 4. SoftCast is a video coder that only uses linear operators, so the video quality increases linearly with channel quality. In a broadcast scenario, the receiver who has a high SNR channel can receive a high PSNR video, and the receiver who has a low SNR channel receives a low PSNR video, thus it ensures transmission fairness. It is unlike conventional video coders, which would required a minimal bit rate for transmission, otherwise would appear a cliff effect, that is to say an abrupt decrease in SNR.

The input video signal undergoes a linear 3D-DCT, consisting of a full-frame 2D-DCT followed by a temporal 1D-DCT on a Group of Pictures (GoP) of n_F frames of $n_r \times n_c$ pixels. SoftCast works independently GoP by GoP. After a GoP has been transformed, the resulting coefficients are grouped into chunks. A chunk is a set of $n_r \times n_c$ spatial coefficients belonging to the same temporal sub-band (assuming they follow a similar distribution). The n_{Ck} chunks are sorted according to their variance i , where $i = 1, \ldots, n_{Ck}$ and only the first l of them may be sent, according to the bandwidth limitations and to the power constraint of the channel. The selected chunks are scaled by power allocation for error protection in order to minimize the reconstruction MSE at the decoder.

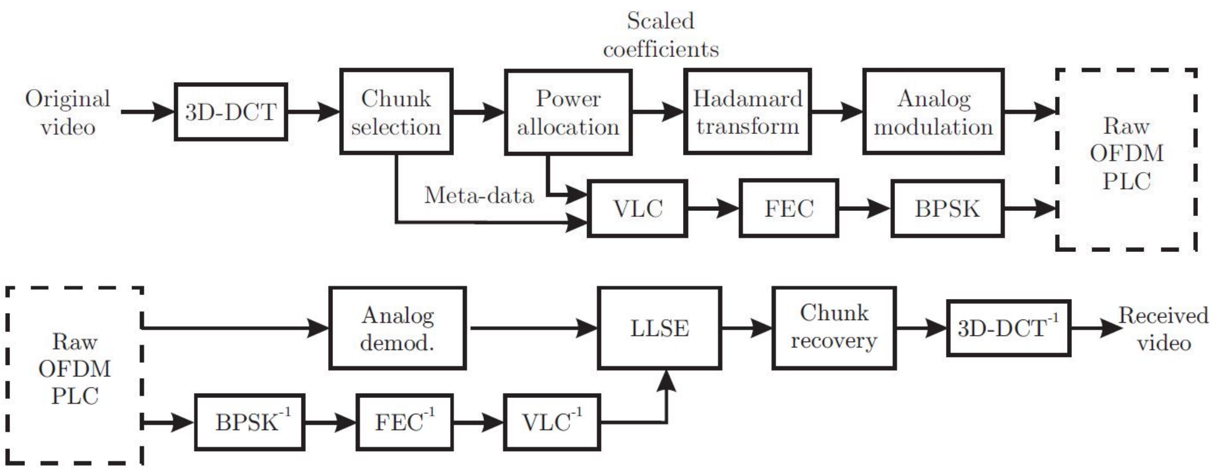


FIGURE 1: SoftCast transmitter (a) and receiver (b). Figures courtesy of Z. Shuo

2.2. Our Softcast video coder

In our scenario, we consider an image. This image is decomposed in the YCbCr color space and each of the three components undergoes a 2D-DCT. After the image has been transformed, a sequence of $n_r \times n_c$ vectors $\mathbf{t}_i$ of dimension n_{Ck} is formed by selecting one coefficient per chunk for each vector, see Figure 2. The variances λ_i of each vector $\mathbf{t}_i$ are then computed and the covariance matrix Λ is constructed based on the assumption that the vectors are not correlated. In this case, we have $\Lambda = diag(\lambda_1, \ldots, \lambda_{n_r \times n_c})$. Each vector is then linearly precoded by a precoding matrix and transmitted to every receiver. The receiver then linearly decodes the received vector $\mathbf{y}_i$ with a decoding matrix to obtain an estimation of the vector $\mathbf{t}_i$. This estimation $\hat{\mathbf{t}}_i$ then allows the construction of three images that need to undergo a reverse 2D-DCT which lead to estimations of the three components of the image at the transmitter in the YCbCr color space.

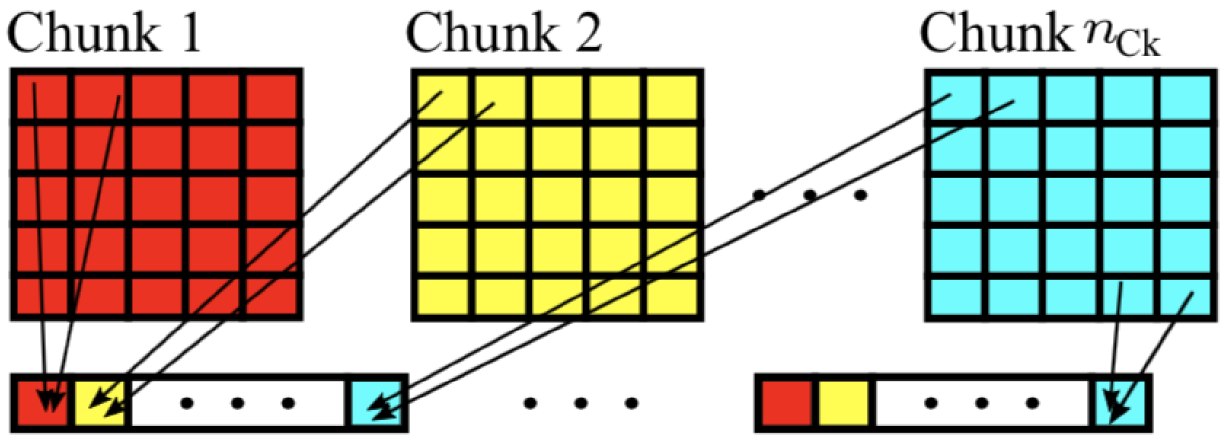


FIGURE 2: Vectorization of the chunks

3. Precoding and decoding matrix design algorithm

3.1. Min-Max optimization problem

System Model In this paper, we focus on a single-hop message transmission problem, in which a transmitter multicasts a message $\mathbf{t}$ to L receivers. That means that there is no relay between the transmitter and the receivers, the message goes through a single network before reaching the transmitter. It is the case in the WIFI protocol for instance, where a single router allows multiple machines to access internet. In order to transmit the message, it is transformed with a 3D-Discrete Cosinus Transform in order to reduce its entropy, and thus to be encoded within less bits. After the 3D-DCT transform, we have n_{Ck} chunks of size $n_r \times n_c$. A sequence of vectors $\mathbf{t}_i$ is constructed by taking an element of each chunk, we then have $n_r \times n_c$ realizations of a n_{Ck} vector, assumed to be independent and identically distributed (i.i.d.) Gaussian vectors with zero mean and a $\mathbf{\Lambda} = diag(\lambda_1, ..., \lambda_{n_{Ck}})$ covariance matrix. The index i of $\mathbf{t}_i$ is omitted since all vectors $\mathbf{t}_i$ have similar distribution and undergo the same processing.

Transmission The transmitter linearly precodes $\mathbf{t}$ by an $n_{Se} \times n_{Ck}$ precoding matrix $\mathbf{G}$

$$\mathbf{x} = \mathbf{G}\mathbf{t} \tag{1}$$

The precoded message $\mathbf{x}$ of size n_{Se} is then linearly transmitted to all L receivers

Reception Each of the L receivers welcome a $\mathbf{y}_i$ received message

$$\mathbf{y}_i = \mathbf{W}_i(\mathbf{G}\mathbf{t}) + \mathbf{v}_i, \quad i = 1, ..., L \tag{2}$$

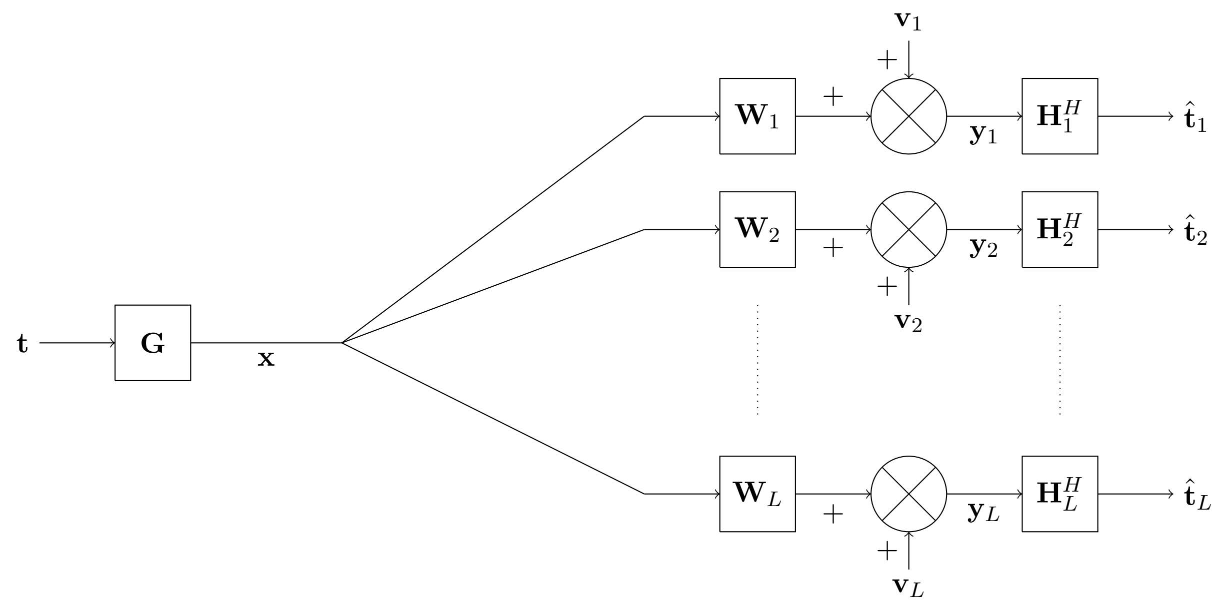


FIGURE 3: Block diagram of a one-hop SIMO system

with $\mathbf{W}_i$ the $n_{Sr} \times n_{Se}$ channel matrix between the transmitter and the i^{th} receiver. $\mathbf{v}_i$ is the received additive noise at the i^{th} receiver, with $E(\mathbf{v}_i) = \mathbf{0}_{n_{Sr}}$ and $\mathrm{E}[\mathbf{v}_i\mathbf{v}_i^H]=\mathbf{N}\ \forall i$, where $\mathrm{E}[\cdot]$ stands for the statistical expectation and $(\cdot)^H$ denotes the matrix Hermitian transpose.

We suppose that all channels are quasi-static, *i.e.*, $\mathbf{W}_i$, $i = 1, ..., L$ are constant during a block of transmission.

Estimation We use a linear estimator to retrieve the transmitted signal $\mathbf{t}$ at the each receiver, see Figure 3.

$$\widehat{\mathbf{t}}_i = \mathbf{H}_i^H \mathbf{y}_i, \quad i = 1, ..., L \tag{3}$$

with $\widehat{\mathbf{t}}_i$ the estimation of the message $\mathbf{t}$ and $\mathbf{H}_i$ the $n_{Sr} \times n_{Ck}$ the decoding matrix at the i^{th} receiver.

We aim at minimizing the maximum MSE signal wave form estimation among all receivers. Its expression, at the i^{th} receiver, is given by

$$\begin{aligned} E_i &= E[(\hat{\mathbf{t}}_i - \mathbf{t})^H(\hat{\mathbf{t}}_i - \mathbf{t})] \\ &= tr(E[(\hat{\mathbf{t}}_i - \mathbf{t})(\hat{\mathbf{t}}_i - \mathbf{t})^H]) \\ &= tr(E[(\mathbf{H}_i^H(\mathbf{W}_i\mathbf{G}\mathbf{t} + \mathbf{v}_i) - \mathbf{t})(\mathbf{H}_i^H(\mathbf{W}_i\mathbf{G}\mathbf{t} + \mathbf{v}_i) - \mathbf{t})^H]) \\ &= tr(\mathbf{H}_i^H\mathbf{W}_i\mathbf{G}\boldsymbol{\Lambda}\mathbf{G}^H\mathbf{W}_i^H\mathbf{H}_i - \mathbf{H}_i^H\mathbf{W}_i\mathbf{G}\boldsymbol{\Lambda} - \boldsymbol{\Lambda}\mathbf{G}^H\mathbf{W}_i^H\mathbf{H}_i + \boldsymbol{\Lambda} + \mathbf{H}_i^H\mathbf{N}\mathbf{H}_i) \\ &= tr((\mathbf{H}_i^H\mathbf{W}_i\mathbf{G}\boldsymbol{\Lambda} - \boldsymbol{\Lambda})(\mathbf{G}^H\mathbf{W}_i^H\mathbf{H}_i - \mathbf{I}_{n_{Ck}}) + \mathbf{H}_i^H\mathbf{N}\mathbf{H}_i) \\ &= tr((\mathbf{H}_i^H\mathbf{W}_i\mathbf{G} - \mathbf{I}_{n_{Ck}})\boldsymbol{\Lambda}(\mathbf{H}_i^H\mathbf{W}_i\mathbf{G} - \mathbf{I}_{n_{Ck}})^H + \mathbf{H}_i^H\mathbf{N}\mathbf{H}_i) \end{aligned} \tag{4}$$

Assuming all $\mathbf{t}$ and $\mathbf{v}_i$ are independant, $\mathrm{E}[\mathbf{t}\mathbf{v}_i^H]$=$\mathrm{E}[\mathbf{v}_i\mathbf{t}^H]$= $\mathbf{0}$. Here, $\mathbf{I}_k$ is the $k \times k$ identity matrix.

Given a power constraint at the transmitter, the matrix optimization problem can be rewritten as

$$\begin{array}{ccc} \underset{\mathbf{G},\{\mathbf{H}_i\}}{min} & \underset{i}{max} & E_i \\ & s.t. & tr\left(\mathbf{G\Lambda G}^H\right) \leqslant P_e \end{array} \tag{5}$$

where $\{\mathbf{H}_i\} \triangleq \{\mathbf{H}_i,\ i = 1, ..., L\}$, and $P_e > 0$ is the power budget at the transmitter.

3.2. Iterative Approach

The goal of this optimization would be that at each iteration, we minimize the greatest MSE among all canals by the design of a precoding matrix $\mathbf{G}$ and decoding matrices $\{\mathbf{H}_i\}$. Our two-step algorithm will first determine the $\{\mathbf{H}_i\}$ with a given $\mathbf{G}$ that respects the transmitter power constraint thanks to an explicit formulation of $\{\mathbf{H}_i\}$. It will then use these $\{\mathbf{H}_i\}$ to compute $\mathbf{G}$ with the Semi-Definite Programming (SDP) solver CVX. At every step, we check which MSE is the greatest before minimizing it.

1. With $\mathbf{G}$ given, determine $\mathbf{H}_i$ $\forall i$.

Using the orthogonality principle of the unbiased Minimum Mean Squared Error estimator(MMSE), and considering the estimation error and the signal are orthogonal, we have

$$\begin{array}{cl} & E[(\hat{\mathbf{t}}_i - \mathbf{t})(\mathbf{y}_i - \mathbf{m}_{y_i})^H] = 0 \\ \Longleftrightarrow & E[(\mathbf{H}_i^H\mathbf{W}_i\mathbf{G}\mathbf{t} + \mathbf{H}_i^H\mathbf{v} - \mathbf{t})(\mathbf{W}_i\mathbf{G}\mathbf{t} - \mathbf{W}_i\mathbf{G}\mathbf{m}_t)^H] = 0 \\ \Longleftrightarrow & E[(\mathbf{H}_i^H\mathbf{W}_i\mathbf{G} - \mathbb{I}_{n_{ck}})\mathbf{t} + \mathbf{H}_i^H\mathbf{v})(\mathbf{W}_i\mathbf{G}\mathbf{t} + \mathbf{v})^H] = 0 \\ \Longleftrightarrow & (\mathbf{H}_i^H\mathbf{W}_i\mathbf{G} - \mathbb{I}_{n_{ck}})E[\mathbf{t}\mathbf{t}^H]\mathbf{G}^H\mathbf{W}_i^H + \mathbf{H}_i^H E[\mathbf{v}\mathbf{v}^H] = 0 \\ \Longleftrightarrow & (\mathbf{H}_i^H\mathbf{W}_i\mathbf{G} - \mathbb{I}_{n_{ck}})\mathbf{\Lambda}\mathbf{G}^H\mathbf{W}_i^H + \mathbf{H}_i^H\mathbf{N} = 0 \\ \Longleftrightarrow & \mathbf{H}_i^H(\mathbf{W}_i\mathbf{G}\mathbf{\Lambda}\mathbf{G}^H\mathbf{W}_i^H + \mathbf{N}) = \mathbf{\Lambda}\mathbf{G}^H\mathbf{W}_i^H \\ \Longleftrightarrow & \mathbf{H}_i^H = \mathbf{\Lambda}\mathbf{G}^H\mathbf{W}_i^H(\mathbf{W}_i\mathbf{G}\mathbf{\Lambda}\mathbf{G}^H\mathbf{W}_i^H + \mathbf{N})^{-1} \\ \Longleftrightarrow & \mathbf{H}_i = (\mathbf{W}_i\mathbf{G}\mathbf{\Lambda}\mathbf{G}^H\mathbf{W}_i^H + \mathbf{N})^{-1}\mathbf{W_i}\mathbf{G}\mathbf{\Lambda}, \end{array} \tag{6}$$

where $(\ \cdot\ )^{-1}$ denotes matrix inversion.

2. With $\mathbf{H}_i$ given $\forall i$, determine $\mathbf{G}$.

Starting with our initial min-max optimization problem (5), we have

$$\begin{array}{ccc} \underset{\mathbf{G}}{min} & \underset{i}{max} & tr(\mathbf{H}_i^H\mathbf{W}_i\mathbf{G\Lambda G}^H\mathbf{W}_i^H\mathbf{H}_i - \mathbf{H}_i^H\mathbf{W}_i\mathbf{G\Lambda} - \mathbf{\Lambda G}^H\mathbf{W}_i^H\mathbf{H}_i + \mathbf{\Lambda} + \mathbf{H}_i^H\mathbf{N}\mathbf{H}_i) \\ & s.t. & tr\left(\mathbf{G\Lambda G}^H\right) \leqslant P_e \end{array}$$

Let $\mathbf{L}_i$ be $\mathbf{H}_i^H\mathbf{W}_i\mathbf{G}$, then we can rewrite (5) as

$$\underset{\mathbf{G}}{min} \quad \underset{i}{max} \quad tr(\mathbf{L}_i\mathbf{\Lambda}\mathbf{L}_i^H - \mathbf{L}_i\mathbf{\Lambda} \ - \mathbf{\Lambda}\mathbf{L}_i^H) + tr(\mathbf{\Lambda} + \mathbf{H}_i^H\mathbf{N}\mathbf{H}_i)$$

then let's introduce :

$$\begin{array}{cc} \mathbf{\Pi}_i \succcurlyeq \mathbf{L\Lambda L}_i^H - \mathbf{L}_i\mathbf{\Lambda} - \mathbf{\Lambda L}_i^H & i = 1, ..., L \\ \mathbf{\Sigma} \succcurlyeq \mathbf{G\Lambda G}^H & \end{array}$$

where $\mathbf{A} \succcurlyeq \mathbf{B}$ means that $\mathbf{A} - \mathbf{B} \succcurlyeq 0$, or $\mathbf{A} - \mathbf{B}$ is positive semi-definite.

With a real slack variable t_e, the problem (5) can be equivalently transformed to :

$$\begin{aligned}
\underset{\mathbf{t_e},\mathbf{G},\{\mathbf{\Pi_i}\},\mathbf{\Sigma}}{min} \quad & t_e \\
s.t. \quad & tr(\mathbf{\Pi}_i) + tr(\mathbf{\Lambda} + \mathbf{H}_i^H\mathbf{N}\mathbf{H}_i) \leq t_e \qquad i = 1, ..., L \\
& \begin{pmatrix} \mathbf{\Pi}_i + \mathbf{L}_i\mathbf{\Lambda} + \mathbf{\Lambda}\mathbf{L}_i^H & \mathbf{L}_i \\ \mathbf{L}_i^H & \mathbf{\Lambda}^{-1} \end{pmatrix} \succcurlyeq 0 \\
& tr(\mathbf{\Sigma}) \leq P_e \\
& \begin{pmatrix} \mathbf{\Sigma} & \mathbf{G} \\ \mathbf{G}^H & \mathbf{\Lambda}^{-1} \end{pmatrix} \succcurlyeq 0
\end{aligned} \tag{7}$$

If we look at our first problem (8), we have

$$\begin{aligned}
\underset{\mathbf{G}}{min}\ \underset{i}{max}\ & tr(\mathbf{H}_i^H\mathbf{W}_i\mathbf{G}\mathbf{\Lambda}\mathbf{G}^H\mathbf{W}_i^H\mathbf{H}_i - \mathbf{H}_i^H\mathbf{W}_i\mathbf{G}\mathbf{\Lambda} - \mathbf{\Lambda}\mathbf{G}^H\mathbf{W}_i^H\mathbf{H}_i) \\
& + tr\left(\mathbf{\Lambda} + \mathbf{H}_i^H\mathbf{N}\mathbf{H}_i\right) \\
s.t. \quad & tr\left(\mathbf{G}\mathbf{\Lambda}\mathbf{G}^H\right) \leqslant P_{\mathrm{e}}
\end{aligned} \tag{8}$$

If we impose $\mathbf{\Sigma}$ and $\{\mathbf{\Pi}_i\}$ such that

$$\begin{aligned}
\mathbf{\Sigma} & \succcurlyeq \mathbf{G}\mathbf{\Lambda}\mathbf{G}^H \\
\mathbf{\Pi}_i & \succcurlyeq \mathbf{H}_i^H\mathbf{W}_i\mathbf{G}\mathbf{\Lambda}\mathbf{G}^H\mathbf{W}_i^H\mathbf{H}_i - \mathbf{H}_i^H\mathbf{W}_i\mathbf{G}\mathbf{\Lambda} - \mathbf{\Lambda}\mathbf{G}^H\mathbf{W}_i^H\mathbf{H}_i,\ i = 1, \ldots, L
\end{aligned}$$

Then the solution of the following problem

$$\begin{aligned}
\underset{t_{\mathrm{e}},\mathbf{G},\mathbf{\Sigma},\mathbf{\Pi}_i}{min}\ & t_{\mathrm{e}} \\
s.t.\ \ & tr\left(\mathbf{\Pi}_i\right) + tr\left(\mathbf{\Lambda} + \mathbf{H}_i^H\mathbf{N}\mathbf{H}_i\right) \leqslant t_{\mathrm{e}},\ i = 1, \ldots, N \\
& tr\left(\mathbf{\Sigma}\right) \leqslant P_{\mathrm{e}} \\
& \mathbf{\Sigma} \succcurlyeq \mathbf{G}\mathbf{\Lambda}\mathbf{G}^H \\
& \mathbf{\Pi}_i \succcurlyeq \mathbf{H}_i^H\mathbf{W}_i\mathbf{G}\mathbf{\Lambda}\mathbf{G}^H\mathbf{W}_i^H\mathbf{H}_i - \mathbf{H}_i^H\mathbf{W}_i\mathbf{G}\mathbf{\Lambda} - \mathbf{\Lambda}\mathbf{G}^H\mathbf{W}_i^H\mathbf{H}_i,\ i = 1, \ldots, L
\end{aligned} \tag{9}$$

guaranties that

$$tr\left(\mathbf{G}\mathbf{\Lambda}\mathbf{G}^H\right) \leqslant P_{\mathrm{e}}$$

because $\mathbf{\Sigma} \succcurlyeq \mathbf{G}\mathbf{\Lambda}\mathbf{G}^H$ and we have

$$tr\left(\mathbf{H}_i^H\mathbf{W}_i\mathbf{G}\mathbf{\Lambda}\mathbf{G}^H\mathbf{W}_i^H\mathbf{H}_i - \mathbf{H}_i^H\mathbf{W}_i\mathbf{G}\mathbf{\Lambda} - \mathbf{\Lambda}\mathbf{G}^H\mathbf{W}_i^H\mathbf{H}_i\right) + tr\left(\mathbf{H}_i^H\mathbf{N}\mathbf{H}_i + \mathbf{\Lambda}\right) \leqslant t_{\mathrm{e}},\ i = 1, \ldots, L$$

We then have

$$\underset{i}{max}\ tr\left(\mathbf{H}_i^H\mathbf{W}_i\mathbf{G}\mathbf{\Lambda}\mathbf{G}^H\mathbf{W}_i^H\mathbf{H}_i - \mathbf{H}_i^H\mathbf{W}_i\mathbf{G}\mathbf{\Lambda} - \mathbf{\Lambda}\mathbf{G}^H\mathbf{W}_i^H\mathbf{H}_i\right) + tr\left(\mathbf{H}_i^H\mathbf{N}\mathbf{H}_i + \mathbf{\Lambda}\right) \leqslant t_{\mathrm{r}}$$

because $\mathbf{\Pi}_i \succcurlyeq \mathbf{H}_i^H\mathbf{W}_i\mathbf{G}\mathbf{\Lambda}\mathbf{G}^H\mathbf{W}_i^H\mathbf{H}_i - \mathbf{H}_i^H\mathbf{W}_i\mathbf{G}\mathbf{\Lambda} - \mathbf{\Lambda}\mathbf{G}^H\mathbf{W}_i^H\mathbf{H}_i,\ i = 1, \ldots, L.$

Consequently, the solution of (9) is a solution of (8).

4. Precoding and decoding matrix min-max optimization for a single-hop message transmission problem

Algorithm Min-Max Optimization for Precoding and Decoding Matrix Design

1 : Initialize : $\mathbf{G}^{(0)}$ and $\{\mathbf{H}_i\}^{(0)}$ respecting the power constraints at the transmitter, k=0
2 : **while** $||\mathbf{G}^{(k+1)} - \mathbf{G}^{(k)}||_1 \leq \varepsilon$ and $k < k_{max}$ **do**
3 :   Given $\mathbf{G}^{(k)}$, update $\{\mathbf{H}_i\}^{(k)}$
4 :   Given $\{\mathbf{H}_i\}^{(k)}$, update $\mathbf{G}^{(k)}$
5 :   k=k+1
6 : **end**

We can now iterate the algorithm shown above to find the optimal $\mathbf{G}$ and $\{\mathbf{H}_i\}$. Here $||\ \cdot\ ||_1$ denotes the matrix maximum absolute column sum norm, ε is a small positive number close to zero and the superscript (k) denotes the number of iterations. To compute the $\{\mathbf{H}_i\}$, CVX proposes two solvers : SeDuMi and SDPT3 4.0. Preemptive tests show no major differences between them as they both support SDP optimization. All tests are consequently run using SDPT3 4.0.

4.1. Application of the algorithm to a simple example

In this subsection, we describe the use of our algorithm with matrices of small sizes. We study a case where a single message is transmitted through a single hop to two receivers, but it could easily be extended to a greater number. Here, we take a simple case where $n_{Se} = n_{Ck} = n_{Sr} = 10$. The covariance matrices of the noises of each channel $\mathbf{N}_i$ are chosen diagonal, we set $N_1 = diag([1, 1, 2, 2, 3, 3, 4, 4, 5, 5]),\ N_2 = diag([1, 2, 3, 4, 5, 1, 2, 3, 4, 5])$. The covariance matrix of the chunks $\mathbf{\Lambda}$ and the transmission channel matrices $\mathbf{W}_i$ are identity matrices.

We initialize the precoding matrix $\mathbf{G}$ so that it respects the power constraint. We use

$$\mathbf{G} = \sqrt{P_e} \times \mathbf{I} \times \sqrt{tr(\mathbf{\Lambda})}^{-1}$$

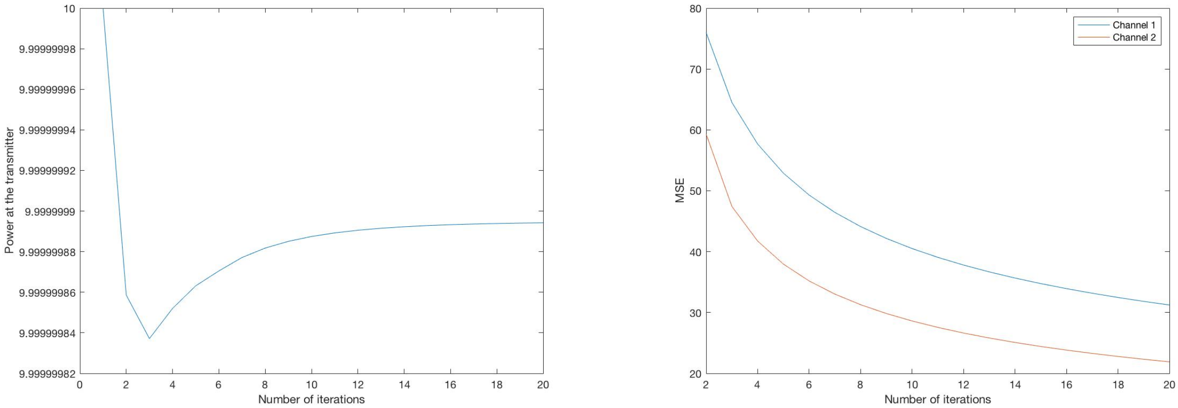
Finally, we set the power constraint to $P_e = 1$.

We show that the transmitter's power matches the power constraint P_e, Figure 4a. That saturation of the power constraint means that the algorithm uses all the resources given to solve the problem. We also see that the greatest MSE dimishes at each iteration as expected.

4.2. Application of the algorithm to image encoding

In this subsection, we describe the use of our algorithm with matrices of bigger sizes to put it in a image coding context, using the `Foreman.qcif` video sequence which has been sampled. We study a case with two receivers but it could easily be extended to a greater number. Here, we have $n_{se} = n_{ck} = n_{sr} = 24$. We use the covariance $\mathbf{\Lambda}$ of the chunks as seen in Section 3.1 and for the covariance matrices of the noises of each channel $\mathbf{N}_i$, we use real data that was provided by Z. Shuo. In a first time, we consider that the channel transmission matrices are identity matrices. The transmitter power is set to $P_e = 2$, and we verify that the initialization respects the transmitter power constraint.

Influence of the noise For the first example, we take a very small additive noise, with a diagonal covariance matrix, which coefficients are close to 10^{-5}. Both MSE then decrease and we

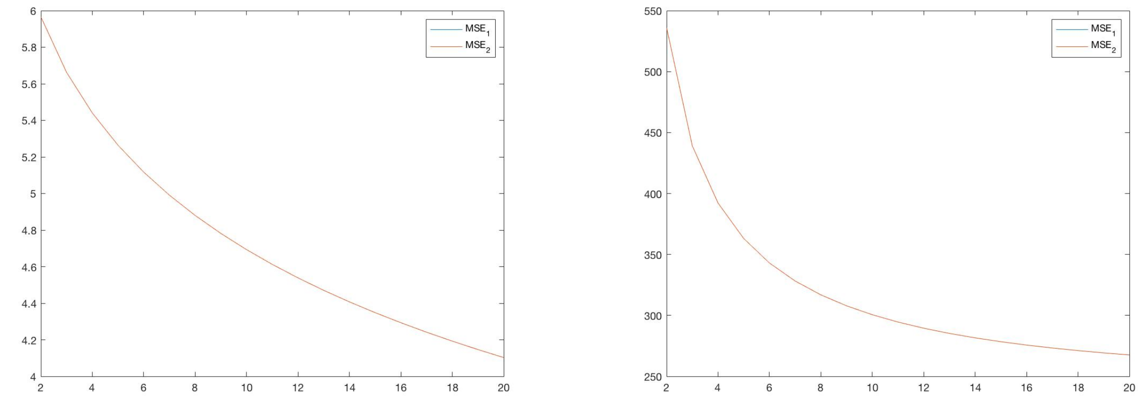


(a) Power constraint at the transmitter

(b) Minimization MSE

FIGURE 4: First tests of the algorithm on a simple case

can't see a notable difference between the channels' image reconstructions. If covariance matrices' coefficients are close to 10^{-3} on both channels, we can observe the same results, except that the MSE are higher of course, Figure 5.



(a) Weak noise on both channels

(b) Higher noise on both channels

FIGURE 5: Example 1 : Minimization of the MSE versus the number of iterations, same noise on both channels, $P_e = 2$

For the second example, the noises are different on both channels, for instance with a covariance close to 10^{-5} on one channel and the other close to 10^{-3}, we can see that we still minimize the highest MSE, see Figure 6. Our algorithm leads a $4dB$ increase in the PSNR of the image at both receptors, see Figure 7. The first channel being subject to a higher noise, its PSNR is lower than the second channel. If we compare the image received by the receptors with the matrices chosen for the initialization and with the matrices obtained with the algorithm, we can also see that the PSNR is higher in the second case.

If we take an extreme case where a channel which is noisier than the other by a factor 1000,

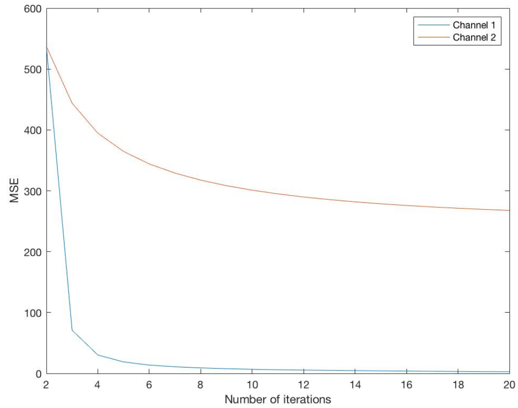


FIGURE 6: Minimization of the MSE versus the number of iterations, different noises on both channels, $P_e = 2$

we see that the PSNR of the noisier receptor decreases by $12dB$, and the one of the noisy receptor increases by $1,5dB$, showcasing the limits of fair video coding, where it benefits one user a little and detriments another a lot, Figure 8. Nevertheless, this has to be put in perspective because a PSNR of $53dB$ is huge and the quality of the reconstruction at the first receptor is still very good.

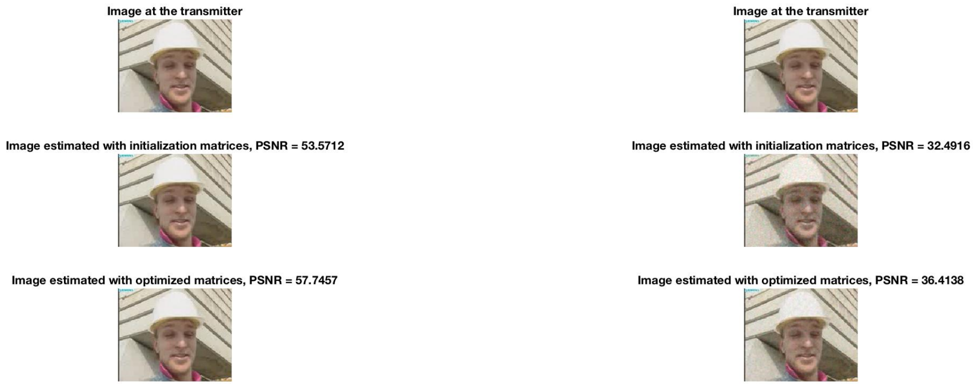
Influence of the transmitter power constraint Here, we study the case seen in the example 3, but we take different P_e. If we relax the power constraint at the transmitter, the space of precoding and decoding matrices becomes greater and we will more easily converge to the desired MSE. The solver does not encounter any problem and the MSE is monotonically decreasing. We obtain much better results than with $P_e = 2$ because we pre-amplified the signal with bigger coeffients before sending it, thus reducing the impact of the additive noise on the channels, Figure 9. Here, we can see that the PSNR of the second image rises above $33dB$. It is not showcased here, but the reader would have guessed that the opposite effect occurs when we decrease P_e.

Influence of the number of chunks Instead of taking $n_{Ck} = 24$, we take $n_{Ck} = 48$. As expected, the PSNR of the resulting images is overall better with twice more chunks, at the cost of a slower bit rate (we need to transmit twice more data, so we need one more bit), and computation time to optimize the matrices $\mathbf{G}$ and $\{\mathbf{H}_i\}$. We get a $2dB$ PSNR increase at each channel with twice more chunks, and a MSE three times lower Figure 10.

4.3. Application of the algorithm to a video sequence

In this section we study the overall performances obtained with the optimization algorithm on a video sequence. Here the additive noise on both receptors are comparable, with a diagonal covariance matrix close to $10^{-3} * \mathbf{I}_{n_{Ck}}$. The noise covariance matrix of the second channel is taken a little greater than the one of the first channel for the sake of the example. We set $P_e = 2$, $n_{Ck} = 24$, and channel matrices equal to the identity matrices.

The mean PSNR gained between the PSNR of the optimized reconstruction and the initial reconstruction is given in Figure 11. We observe that in the case of a 20-image video sequence, we gain an average of $3,7dB$ on both channels.



(a) First receptor

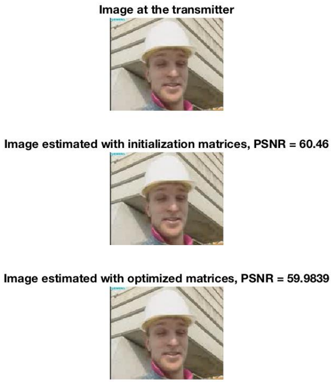
(b) Second receptor

FIGURE 7: Example 2 : Original image and reconstruction at the two receptors, using the initialization matrices and the optimized matrices, different noises on both channels, $P_e = 2$

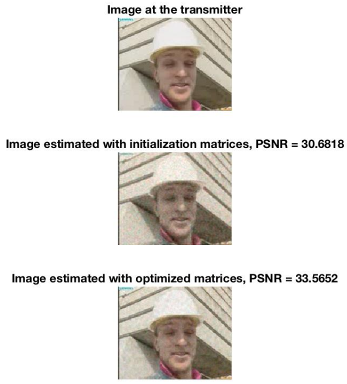
(a) First receptor, low noise

(b) Second receptor, much noisier

FIGURE 8: Example 3 : Original image and reconstruction at the two receptors, using the initialization matrices and the optimized matrices, very different noises on both channels, $P_e = 2$

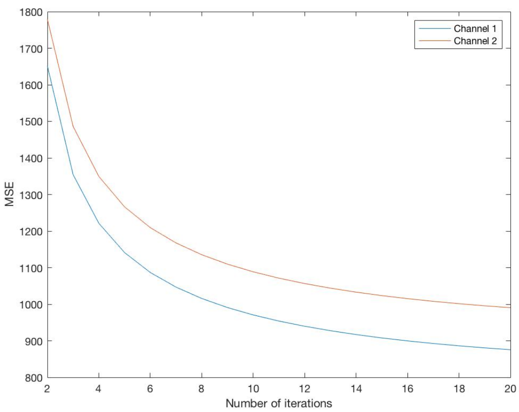


(a) First receptor, low noise

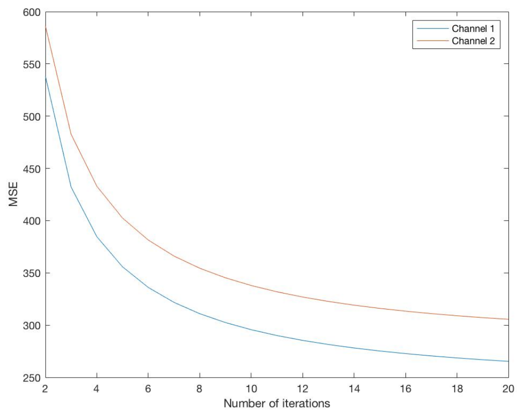


(b) Second receptor, much noisier

FIGURE 9: Example 4 : Original image and reconstruction at the two receptors, using the initialization matrices and the optimized matrices, $P_e = 10$



(a) MSE optimization, 24 chunks



(b) MSE optimization, 48 chunks

FIGURE 10: Example 5 : MSE optimization with 24 (a) and 48 (b) chunks, similar noises on both channels, $P_e = 2$

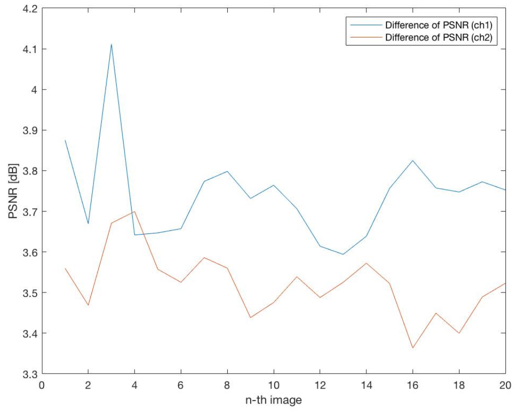


FIGURE 11: Average PSNR gained between the reconstruction with initial matrices and the reconstruction with optimized matrices, $P_e = 2$

5. Comparison of our method with state-of-the art video coding methods

As seen in [3], another method to estimate the MSE at the L^{th} receiver is to consider per-subchannel power constraints to compute $\mathbf{G}$ with the solution of [1], then $\mathbf{G}$ with the explicit solution of the MMSE estimator.

In our work, we focused on a min-max optimisation problem with a single power constraint at the receiver, with the motivation to create the fairest video coding algorithm possible. Another scheme that could have been explored is the min-mean algorithm, but a low mean doesn't necessarily make for a fair JSCVC. Let's say one wish to broadcast a single message to $L = 100$ receivers, then the mean could be very low but some users could nonetheless achieve a bad video PSNR. If we have 95 low MSE reconstruction of the signal waveform and 5 high ones, the min-mean algorithm shall not make for a fair coding, because some users could be sacrificed for the sake of the others.

In this section, we will initialize $\mathbf{G}$ with the optimal solution of [1] for one channel, and run the min-max algorithm. We expect the MSE of this channel to decrease while the MSE of the other channel increases. The symmetric case will also be studied.

— Initialize channel 1 with Total Power Constraint optimal solution and run the min-max algorithm

— Initialize channel 2 with Total Power Constraint optimal solution and run the min-max algorithm

6. Conclusion and possible developments for the project

We considered the case of a one-hop SIMO transmission scheme where a single message is transmitted to multiple receptors subjecting to QoS constraints. In order to guaranty transmission fairness among receptors, we used a mix-max optimization scheme for precoding and decoding matrix design. Our experiments on a simple case, then on a video sequence with real noises and chunks' covariances were able to showcase that it helps greatly reducing the MSE at the receptors.

During the tests of our algorithm, we noticed that it was taking too long to solve the optimal problem. In the case of a 20×20 $\mathbf{\Lambda}$ matrix, we have 2463 variables and 1031 equality constraints, and in the case of a 50×50 $\mathbf{\Lambda}$ matrix we have 15153 variables and 6326 equality constraints. It appears that we have a computational problem due to the complexity of the problem, and that we can't generalize the code to signals of higher dimensions. We could seek a simplified algorithm with the hypothesis of a high SNR level to upper-bound our equations and approximate our problem.

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Appendices

Minimum Mean Squared Error (MMSE) estimator

The MMSE is a statistical estimator used when the value we seek to estimate is random and not deterministic. His general form is $\hat{\mathbf{x}} = \mathbf{H}\mathbf{y} + \mathbf{c}$, with $\mathbf{H}$ an $n \times m$ matrix and $\mathbf{c}$ an $n \times 1$ vector. This estimator needs statistical characteristics of the 1^{st} and 2^{nd} order.

Unbiased estimator

Intuitively, we can sense that an estimator is better if the estimated value $\hat{\mathbf{x}}$ is close to the real value $\mathbf{x}$. We define the bias of an estimator as the expectancy of the error, which the difference between the estimated value and the real value, that is to say

$$E[\tilde{\mathbf{x}}] = E[\hat{\mathbf{x}} - \mathbf{x}]$$

If the bias is null, we say that an estimator is unbiased, which can be interpreted as the expectation of the value given by the estimator to be the same as the real value.

We define the MSE of the estimator as $E[\tilde{\mathbf{x}}^2] = E[\tilde{\mathbf{x}}^T\tilde{\mathbf{x}}] = E[(\mathbf{H}\mathbf{y} + \mathbf{c} - \mathbf{x})^T(\mathbf{H}\mathbf{y} + \mathbf{c} - \mathbf{x})]$. To find the optimal $\mathbf{H}$ and $\mathbf{c}$ parameters of the minimum Mean Squared Error estimator for a given signal, we derive with respect to them, and set those derivatives to zero.

By derivating with respect to $\mathbf{c}$, we obtain $2E[\mathbf{H}\mathbf{y} + \mathbf{c} - \mathbf{x}] = 2E[\tilde{\mathbf{x}}] = \mathbf{0}$. Thus the MMSE is an unbiased estimator.

Orthogonality principle

By derivative with respect to $\mathbf{H}$, we obtain $2E[(\mathbf{Hy}+\mathbf{c}-\mathbf{x})\mathbf{y}^T] = 2E[\tilde{\mathbf{x}}\mathbf{y}^T] = \mathbf{0}$. Thus $\tilde{\mathbf{x}}$ and $\mathbf{y}$ are uncorrelated when the MSE is minimal, $\tilde{\mathbf{x}} \perp \mathbf{y}$.

Schur complement

Definition

In linear algebra and the theory of matrices, the Schur complement of a block matrix is defined as follows.

Suppose $\mathbf{A}$, $\mathbf{B}$, $\mathbf{C}$, $\mathbf{D}$ are respectively $p \times p$, $p \times q$, $q \times p$, and $q \times q$ matrices, and $\mathbf{D}$ is invertible.

Let

$$\mathbf{M} = \begin{bmatrix} \mathbf{A} & \mathbf{B} \\ \mathbf{C} & \mathbf{D} \end{bmatrix}$$

so that $\mathbf{M}$ is a $(p+q) \times (p+q)$ matrix.

Then the Schur complement of the block $\mathbf{D}$ of the matrix $\mathbf{M}$ is the $p \times p$ matrix defined by

$$\mathbf{M}/\mathbf{D} := \mathbf{A} - \mathbf{B}\mathbf{D}^{-1}\mathbf{C}$$

and, if $\mathbf{A}$ is invertible, the Schur complement of the block $\mathbf{A}$ of the matrix $\mathbf{M}$ is the $q \times q$ matrix defined by

$$\mathbf{M}/\mathbf{A} := \mathbf{D} - \mathbf{C}\mathbf{A}^{-1}\mathbf{B}$$

Schur complement condition for positive definiteness and positive semi-definiteness

Let $\mathbf{X}$ be a symmetric matrix given by

$$\mathbf{X} = \begin{bmatrix} \mathbf{A} & \mathbf{B} \\ \mathbf{B}^T & \mathbf{C} \end{bmatrix}$$

Let $\mathbf{X}/\mathbf{A}$ be the Schur complement of $\mathbf{A}$ in $\mathbf{X}$; i.e.,

$$\mathbf{X}/\mathbf{A} := \mathbf{C} - \mathbf{B}^T\mathbf{A}^{-1}\mathbf{B}$$

and $\mathbf{X}/\mathbf{C}$ be the Schur complement of $\mathbf{C}$ in $\mathbf{X}$; i.e

$$\mathbf{X}/\mathbf{C} := \mathbf{A} - \mathbf{B}\mathbf{C}^{-1}\mathbf{B}^T$$

Then

— $\mathbf{X}$ is positive definite if and only if $\mathbf{A}$ and $\mathbf{X}/\mathbf{A}$ are both positive definite :

$$\mathbf{X} \succ \mathbf{0} \;\Leftrightarrow\; \mathbf{A} \succ 0,\; \mathbf{X}/\mathbf{A} = \mathbf{C} - \mathbf{B}^T\mathbf{A}^{-1}\mathbf{B} \succ \mathbf{0}$$

— $\mathbf{X}$ is positive semi-definite if and only if $\mathbf{C}$ and $\mathbf{X}/\mathbf{C}$ are both positive semi-definite :

$$\mathbf{X} \succeq \mathbf{0} \;\Leftrightarrow\; \mathbf{C} \succeq 0,\; \mathbf{X}/\mathbf{C} = \mathbf{A} - \mathbf{B}\mathbf{C}^{-1}\mathbf{B}^T \succeq \mathbf{0}$$

— If $\mathbf{A}$ is positive definite, then $\mathbf{X}$ is positive semi-definite if and only if $\mathbf{X}/\mathbf{A}$ is positive semi-definite :

$$\text{If } \mathbf{A} \succ \mathbf{0}, \text{ then } \mathbf{X} \succeq 0 \;\Leftrightarrow\; \mathbf{X}/\mathbf{A} = \mathbf{C}\ \mathbf{B}^T\mathbf{A}^{-1}\mathbf{B} \succeq \mathbf{0}$$

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