Introduction to Artificial Intelligence
Lab 3

Bayesian Inference

In this laboratory, we will analyse the use of the independence between random variables, Bayesian inference and Bayesian networks. The work should be deposited on moodle until **27 April 23:55**.

Part A.
Bayesian Networks have shaped complicated issues with few knowledge and resources. It is incorporated into the most cutting-edge technologies of the modern day, including Artificial Intelligence and Machine Learning.

Question 1.
Why independence between random variables is mandatory for Bayes Theorem?

Question2.
Give the difference between independence and conditional independence.

Question 3. You have three coins in your pocket:

1. Coin 1 is a fair coin that comes up heads with probability 1/2.
2. Coin 2 is a biased coin that comes up heads with probability 1/4.
3. Coin 3 is a biased coin that comes up heads with probability 3/4.

1. Suppose you pick one of the coins uniformly at random and flip it three times. If you observe the sequence HHT (where H stands for heads and T stands for tails), what is the probability that you chose Coin 3?

2. Suppose X and Y are independent random variables over the domain 1, 2, 3 with P (X = 3) = 1/6. Given the following partially specified joint distribution, what are the remaining values? Write your answers as simplified fractions in the blanks.

$$P(X=1,Y=1)=1/4 \quad P(X=2,Y=1)=1/6 \quad P(X=3,Y=1)=??$$
$$P(X=1,Y=2)=1/16 \quad P(X=2,Y=2)=1/24 \quad P(X=3,Y=2)=??$$
$$P(X=1,Y=3)=?? \quad P(X=2,Y=3)=?? \quad P(X=3,Y=3)=??$$

Part B.
Bayesian Statistics over Bayesian Networks and Inferencing them using **Pgmpy Python** library**:**
!pip install pgmpy

Theorem of Bayes

Bayes' theorem is a fundamental concept in Bayesian statistics, as it is used by Bayesian methods to update probabilities, or degrees of belief, following the acquisition of new data. The conditional probability of A occurring given that B occurs is represented as follows:

$$P(A \mid B) = \frac{P(B \mid A)P(A)}{P(B)}$$

The law of total probability can be used to calculate the probability of the evidence P(B). If {A1, A2, …, An} values is a partition of the sample space, which is the collection of all experimental results, then:

$$P(B) = P(B \mid A_1)P(A_1) + P(B \mid A_2)P(A_2) + \cdots + P(B \mid A_n)P(A_n) = \sum_i P(B \mid A_i)P(A_i)$$

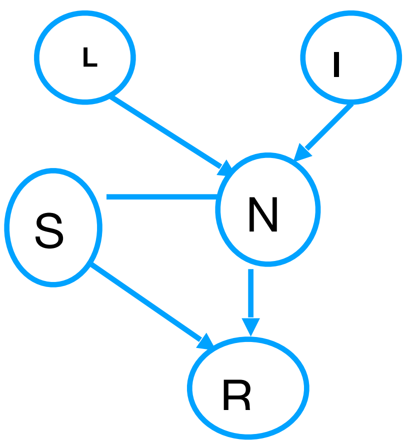
Bayesian network

A Bayesian network is a probabilistic graphical model (a kind of statistical model) that utilises a directed acyclic graph to describe a set of variables and their conditional relationships (DAG). Bayesian networks are perfect for analysing a past occurrence and estimating the probability that any of numerous known causes had a role.

We can use bayes rule and total probability theorem to infer probabilities in a bayes network.
Lets consider an example, where a the students should return (deposit) a Project for Introduction in IA and the Professor try to predict if students copied (plagiated) the content. For this the Professor would like to create a statistical method that can take the preemtive measure based on information given. Lets assume we have following information:

- **L** : A prediction from a NLP ML (Natural Language Processing Machine Learning) model that can read the content of the Project and give a score (signalled) (probability) that this content is copied/plagiated.
- **I** : Another student(colleague) marks the material as inappropriate/plagiarism.
- **S** : The Project was suspended before for any bad remarks.
- **N** : Score (Probability) that the Project should be not considered (not scored)
- **R** : Score (Probability) that Student should be restricted for this lecture (ratrappage)

We have the following Bayes net for this problem:



Following are the probability distribution tables:

I	T	F
P(I)	0.21	0.79

L	T	F
P(L)	0.08	0.92

S	T	F
P(S)	0.12	0.88

R	T	F
P(R|NS)	0.38	0.62
P(R|N!S)	0.08	0.92
P(R|!NS)	0.16	0.84
P(R|!N!S)	0.05	0.95

N	T	F
P(N|LSI)	0.92	0.08
P(N|LS!I)	0.88	0.12
P(N|L!SI)	0.79	0.21
P(N|L!S!I)	0.73	0.27
P(N|!LSI)	0.22	0.78
P(N|!LS!I)	0.08	0.92
P(N|!L!SI)	0.17	0.83
P(N|!L!S!I)	0.03	0.97

We will use the **pgmpy** python library to compute the probabilities.
You can follow the tutorial, and analyse the existed functions:
https://pgmpy.org/models/bayesiannetwork.html

Import the libraries :

```
from pgmpy.models import BayesianModel
from pgmpy.factors.discrete import TabularCPD
from pgmpy.inference import VariableElimination
import numpy as np
```

Question 1.

1. Create the Bayes nets using: bayesNet = BayesianModel()

- For all nodes:

```
bayesNet.add_node("Name_Node")
```

- For all anges:

```
bayesNet.add_edge("Name_Node1", "Name_Node2")
```

2. Add CPDs to each node, while adding probabilities, we have to give FALSE values first using the function TabularCPD() :
pgmpy.factors.discrete.CPD.TabularCPD(variable, variable_card, values, evidence=None, evidence_card=None, state_names={})

For more details, you can analyse: https://pgmpy.org/factors/discrete.html?highlight=tabularcpd#pgmpy.factors.discrete.CPD.TabularCPD

3. Check if model is correctly created using bayesNet.check_model()

4. Creating solver that uses variable elimination internally for inference using :
solver = VariableElimination(bayesNet)

Question 2.
Compute proability of « Project should be not considered (not scored)»**

1. Compute manually this probability

2. Compute this probability using pgmpy library:

result = ***solver.query()***

Question 3.
Compute the probability of « Project should be not considered (no scored) given the ML model si-gnalled it"

1. Compute manually this probability

2. Compute this probability using pgmpy library:

Question 4.
Find (in)dependencies between the variables using the function ***get_independencies()***.