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DYNAMICS OF PNEUMATIC CYLINDER SYSTEMS

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ABSTRACT

To drive a pneumatic cylinder, meter-out circuit is used in many cases, since time to move the piston is constant even if the load is changed. That is, the meter-out restriction method realized velocity control. However this principle of velocity control has not been explained sufficiently. The purpose of this study is to make clear the velocity control mechanism.. In order to clarify this mechanism, nondimensional equations and their responses are presented. Consequently six parameters governing the cylinder response are obtained. As a result, by drawing the block diagram of cylinder system, it is found that the compliance of a cylinder makes the feedback loop in cylinder velocity and the flow characteristics of restriction in choking compensates the constant velocity for changing a load force.

KEY WORDS

Pneumatic Cylinder, Meter-out Circuit, Velocity Feedback, Nondimensional Equations

NOMENCLATURE

A : cylinder piston area [m^2]
C : viscous friction coefficient [Ns/m]
C_V : specific heat at constant volume [J/(kg K)]
F_q : coulomb friction [N], F_s : static friction [N]
G : mass flow rate [kg/s]
g : acceleration of gravity [m/s]
h : heat transfer coefficient [W/(m^2 K)]
L : cylinder stroke [m], M : load mass [kg]
P : pressure [kPa], R : gas constant [J/(kg K)]
S_e : restriction effective area [m^2]
S_h : heat transfer area [m^2]
t : time [s], u : piston velocity [m/s]
V : cylinder volume [m^3], x : cylinder position [m]
κ : specific heat ratio, θ : temperature [K]

Suffix

$_a$: atmosphere, $_d$: discharging side
$_s$: supply, $_u$: charging side
$_\infty$: terminal, * : nondimensional

1 INTRODUCTION

Pneumatic cylinders are widely utilized in factory automation[1] and are usually driven by the meter-out restriction method. By using only a restriction, which is normally installed in the speed control valve, the meter-out circuit realizes velocity control. This method has the advantage that a load change does not

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affect much the piston velocity.

Up to now Miyata et.al[2]. have proved in the meter-out restriction method that equilibrium velocity depends on the effective area of the meter-out restriction. However the velocity control mechanism and its principle have been unknown. On the other hand it is difficult to grasp the dynamic characteristics of pneumatic cylinder systems since they include many parameters: cylinder dimensions, a load mass, etc.; although Kawakami et.al.[3] have studied the cylinder response on the meter out circuit.

The purpose of this study is to find the principle of velocity feedback mechanism by deriving nondimensional equations and its governing parameters, and to make clear the influence of nondimensionalized parameters on the cylinder response.

2 NONDIMENSIONAL EQUATIONS

2.1 Standard Values for Nondimensionalization

A pneumatic cylinder is driven by the circuit shown in Fig.1. When $S_{eu} >> S_{ed}$ and the flow through the discharging restriction is choked, this circuit is called a meter-out circuit. It is known that terminal velocity of cylinder is a function of restriction area S_{ed} and its velocity is given by

$$u_\infty = \frac{S_{ed}}{A_d}\left(\frac{2}{\kappa+1}\right)^{\frac{1}{\kappa-1}}\sqrt{\frac{2\kappa}{\kappa+1}R\theta_a} \qquad (1)$$

The time that the piston takes to move from end to end with terminal velocity is:

$$T_P = \frac{L}{u_\infty} \qquad (2)$$

This time is also a time constant in a pneumatic RC circuit composed by stroke volume $(=A_d L)$ and restriction area of discharging side $(=S_{ed})$. Time is nondimensionalized by T_P. This transformation

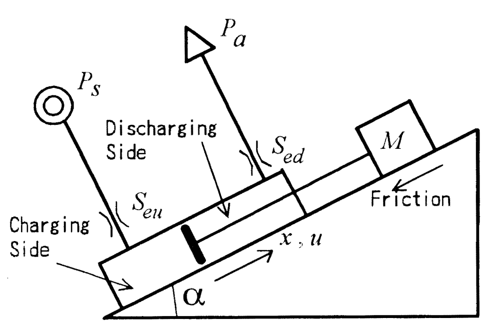

Fig.1 Pneumatic Cylinder System

accompanies the following nondimensionalization on cylinder velocity and position.

$$t^* = \frac{t}{T_P} \quad (3) \qquad u^* = \frac{u}{u_\infty} \quad (4) \qquad x^* = \frac{x}{L} \quad (5)$$

On the condition there is not a load force, pressure balances at supply pressure and temperature recovers to atmospheric temperature. Then discharging flow rate $G_{d\infty}$ is constant. Therefore these values are used by the normalization respectively.

$$P^* = \frac{P}{P_s} \quad (6) \qquad \theta^* = \frac{\theta}{\theta_a} \quad (7) \qquad G^* = \frac{G}{G_{d\infty}} \quad (8)$$

2.2 Nondimensionalization of governing equations

Based on these standard values nondimensionalized equations are derived from governing equations on a cylinder system which consider state change of the air[4]. The downstream flow rate is expressed as follows: When $P_a^*/P_d^* \geq 0.528$,

$$G_d^* = -P_d^*\left(\frac{\kappa+1}{2}\right)^{\frac{1}{\kappa-1}}\sqrt{\frac{\kappa+1}{(\kappa-1)\theta_d^*}\left\{\left(\frac{P_a^*}{P_d^*}\right)^{\frac{2}{\kappa}}-\left(\frac{P_a^*}{P_d^*}\right)^{\frac{\kappa+1}{\kappa}}\right\}} \qquad (9)$$

and when $P_a^*/P_d^* < 0.528$,

$$G_d^* = -\frac{P_d^*}{\sqrt{\theta_d^*}} \qquad (10)$$

Likewise upstream flow rate is expressed as follows: In the case $P_u^* \geq 0.528$,

$$G_u^* = S_{eu}^*\left(\frac{\kappa+1}{2}\right)^{\frac{1}{\kappa-1}}\sqrt{\frac{\kappa+1}{\kappa-1}\left\{P_u^{*\frac{2}{\kappa}}-P_u^{*\frac{\kappa+1}{\kappa}}\right\}} \qquad (11)$$

and in the case $P_u^* < 0.528$,

$$G_u^* = S_{eu}^* \qquad (12)$$

In discharging side the state equation and the energy equation for the air can be written as

$$V_d^*\frac{dP_d^*}{dt^*} = \theta_d^* G_d^* + \frac{P_d^* V_d^*}{\theta_d^*}\frac{d\theta_d^*}{dt^*} + P_d^* u^* \qquad (13)$$

$$V_d^*\frac{d\theta_d^*}{dt^*} = (\kappa-\theta_d^*)\frac{\theta_d^* G_d^*}{P_d^*} + \frac{S_{hd}^*}{T_{hd}^*}(1-\theta_d^*) + (\kappa-1)\theta_d^* u^* \qquad (14)$$

While in charging side the following equations are obtained.

$$V_u^*\frac{dP_u^*}{dt^*} = \theta_u^* G_u^* + \frac{P_u^* V_u^*}{\theta_u^*}\frac{d\theta_u^*}{dt^*} - A_u^* P_u^* u^* \qquad (15)$$

$$V_u^* \frac{d\theta_u^*}{dt^*} = \left(\kappa - \theta_u^*\right)\frac{\theta_u^* G_u^*}{P_u^*} + \frac{S_{hu}^* h_u^*}{T_{hu}^*}\left(1 - \theta_u^*\right) - A_u^*\left(\kappa - 1\right)\theta_u^* u^* \quad (16)$$

Here the cylinder volume and the heat transfer area are nondimensionalized by stroke volume $A_d L$ and its surface area $2\left(A_d + L\sqrt{\pi A_d}\right)$ respectively. In these equations values of downstream cylinder chamber are introduced as standard values; i.e. cylinder piston area, restriction effective area, heat transfer coefficient, taking into account the cylinder is driven by meter-out restriction method.

Supposing a packing friction as the sum of a coulomb and a viscous friction, the motion equation of cylinder is transformed to the next equation.

$$\frac{du^*}{dt^*} = \left(\frac{2\pi}{T_f}\right)^2 \left(A_u^* P_u^* - P_d^* - K^* u - F_r^* - F^*\right) \quad (17)$$

2.3 Nondimensional parameters

Through nondimesionalization all cylinder parameters are arranged into six nondimensional parameters defined as follows.

1) T_f^* **parameter** has been regarded as nondimensionalized inertia. However it is not true. We have gotten obvious physical meaning that T_f^* is the natural period determined by the cylinder compliance and the inertia mass.

$$T_f^* = 2\pi\sqrt{\frac{M u_\infty^2}{A_d P_S L}} = 2\pi\sqrt{\frac{ML}{A_d P_S}} \Big/ T_P \quad (18)$$

2) K^* **parameter** is the viscous friction divided by the maximum driving force of the cylinder.

$$K^* = \frac{C u_\infty}{A_d P_S} \quad (19)$$

3) F^* **parameter** expresses a load force and includes the coulomb friction and the unbalance force caused by the difference in piston areas.

$$F^* = \frac{Mg\sin\alpha + F_c - P_a\left(A_d - A_u\right)}{A_d P_S} \quad (20)$$

4) F_r^* **parameter** represents the difference between the maximum static friction and the coulomb friction.

$$F_r^* = \frac{F_s - F_c}{A_d P_S} \quad (21)$$

5) S_{eu}^* **parameter** is the ratio of restriction area in charging to that in discharging.

$$S_{eu}^* = \frac{S_{eu}}{S_{ed}} \quad (22)$$

6) T_h^* **parameter** is a nondimensionalized time constant on heat transfer in the cylinder chamber.

$$T_h^* = \frac{C_V W_{d0}}{S_{hd} h_d} \Big/ T_P \quad \left(W_{d0} = \frac{R\theta_a}{P_s A_d L}\right) \quad (23)$$

3 FEEDBACK MECHANISM

3.1 Characteristics of Meter-out circuit

For simplicity we assume that state change of air is isothermal. Substituting the state equation, Eq.(13), into flow rate equation, Eq.(10):

$$V_d^* \frac{dP_d^*}{dt^*} = P_d^*\left(u^* - 1\right) \quad (24)$$

This equation has the characteristic that when the piston velocity becomes higher than the terminal velocity, which nondimensionalized value is 1, the cylinder pressure in discharging side rises up and the piston velocity decreases, the converse is true. This means that the cylinder dynamics has a feedback loop.

3.2 Block Diagram

Let us investigate this mechanism in more detail through the block diagram of pneumatic cylinder system. Fig.2 results of linearizing at equilibrium point. In Fig.2 ξ^* is

$$\xi_d^* = S_{ed}^*\left(\frac{\partial G_d^*}{\partial P_d^*}\right)_\infty - A_d^* = \left(\frac{\partial G_d^*}{\partial P_d^*}\right)_\infty - 1 \quad (25)$$

Linearizing equations relating to charging side, the same block diagram is drawn at all. In discharging $\xi_d^* = 0$ always holds because $\left(\partial G_d^* / \partial P_d^*\right)_\infty = 1$. Therefore there is not the loop drawing by the broken line and pressure in the cylinder chamber only depends on the

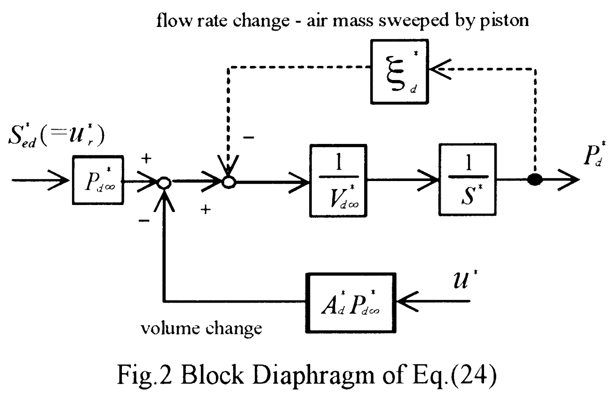

Fig.2 Block Diaphragm of Eq.(24)

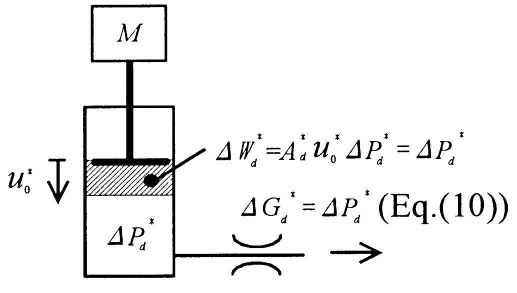

Fig.3 Relation between ΔW and ΔG

cylinder velocity. Refering Fig.3 this mean that when the cylinder pressure in discharging side rise, the increase of sweeped air mass by a piston ΔW is just canceled by the increase in flow rate ΔG. On the other hand in charging $\xi^* = 0$ does not hold. This cancellation brings the feature of meter out drive. The complete block diagram is presented in Fig.4.

3.3 Transfer Function

From Fig.4 it is found that if a charging restriction affects the cylinder response, the cylinder dynamics is expressed as a third order time lag system. Except in such case the transfer function becomes the following second order time lag system.

$$\frac{\hat{u}^*}{\hat{u}_r^*} = \frac{\omega_n^{*2}}{s^2 + 2\varsigma^* \omega_n^* s + \omega_n^{*2}} \tag{26}$$

Where natural frequency and damping ratio are given by the following equations respectively.

$$\omega_n^* = \frac{2\pi}{T_f^*}\sqrt{\frac{P_{d\infty}^*}{V_{d\infty}^*}} \quad (27) \qquad \varsigma^* = \frac{K^*}{2\omega_n^*} \quad (28)$$

T_f^* parameter determines the natural frequency of the system and the damping in the velocity response only depends on K^* parameter expressing the packing friction. It is found that friction assumes an important role.

Table 1 specifications of cylinder

diameter of cylinder	40 [mm]
stroke of cylinder	400 [mm]
supply pressure	500 [kPa]
load mass	10 [kg]
static friction	20 [N]
viscous friction coefficient	80 [Ns/m]
effective area of restriction	1.5 [mm^2]

The transfer function from external force to cylinder velocity is

$$\frac{\hat{u}^*}{\hat{F}^*} = \frac{V_{d\infty}^*}{P_{d\infty}^*} \frac{\omega_n^{*2} s}{s^2 + 2\varsigma^* \omega_n^* s + \omega_n^{*2}} \tag{29}$$

This is a 1 type transfer function which equilibrates at zero for a constant input. In other words a meter-out circuit has compensation for the cylinder velocity. So if packing friction or a handling load changes, the cylinder velocity always reaches the terminal velocity. The above 'cancellation' ($\xi_d^* = 0$) in flow rate, appearing in the integrate element in the block diaphragm of Fig.4, actualized this result. In meter-out circuit ($\xi_u^* \neq 0$) the same velocity is never gotten changing external force although feedback loop exists. As a result in the meter-out the cylinder velocity is robust for load change.

In the next section the nondimensiolized step response is examined using block diaphragm and transfer function.

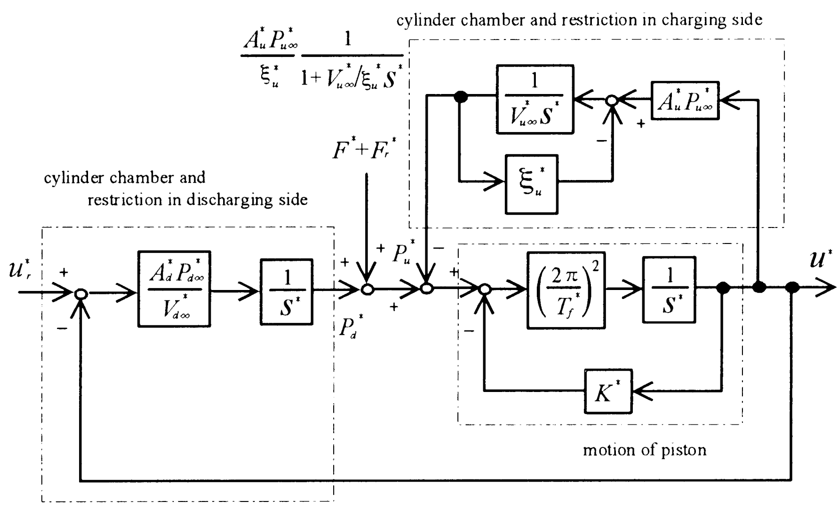

Fig.4 Block diaphragm of pneumatic cylinder driven by meter-out circuit

4 NONDIMENSIOLIZED RESPONSES

4.1 T_f^* parameter

Simulation is carried out using dimensions based on an actual cylinder which specifications are presented in Table 1. Fig.5 shows responses for various T_f^* values. T_f^* is the natural frequency of the system so the vibration period of response is equal to T_f^* value. For example doubling T_f^*, the cycle time increases twice. However the calculated average velocity of the cylinder will be nearly the terminal velocity because the velocity oscillates around the terminal velocity. Therefore stroke time of cylinder is almost the same as Fig.5 shows.

4.2 K^* parameter

Simulation results in Fig.6 indicate that K^* parameter governs damping in the velocity response. The time period in the oscillation keeps constant for any K^* value. However from Eq.(28) ζ^* is not only a function of K^* but also a function of T_f^* so that the response does not damp as T_f^* increases. It is known that in meter-out drive, packing friction is significant for controllability of velocity control.

4.3 F^* parameter

Fig.7 is the result of changing F^* parameter. The piston does not move until the force generated by cylinder pressures becomes bigger than the load force. Also stopping time becomes longer when increasing F^* values.Therefore F^* parameter affects moving time the most among all parameters; although equilibrium velocity is also terminal velocity because of the above compensation; furthermore each response can be fitted transforming the time scale.

4.4 F_r^* parameter

From the phase plane of responses, it is known that F_r^* parameter determines the initial condition. Fig.8 shows that the degree of overshoot is greater. And it is interesting that the stroke time is the same and its settling time is not longer with increment of F_r^* value.

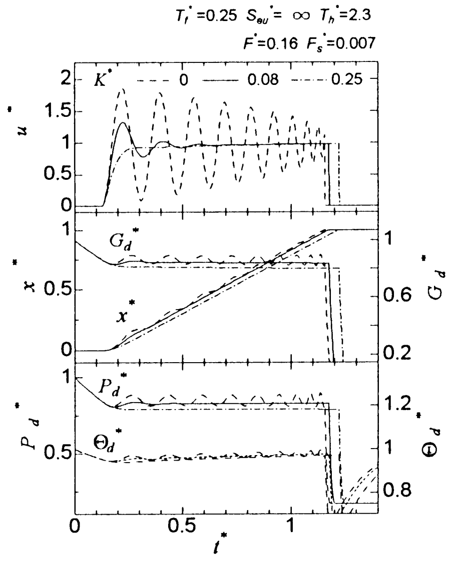

Fig.6 Step responses changing K^* parameter

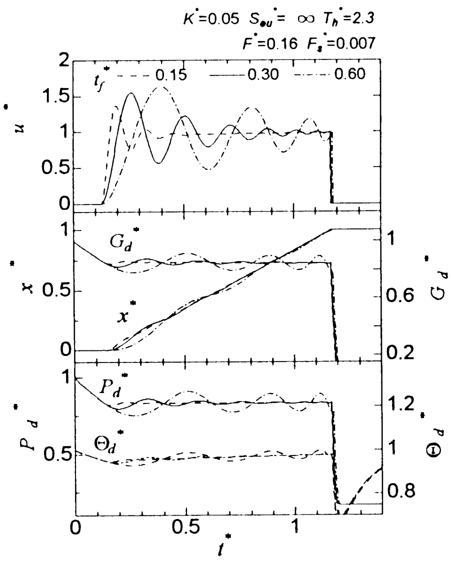

Fig.5 Step responses changing T_f^* parameter

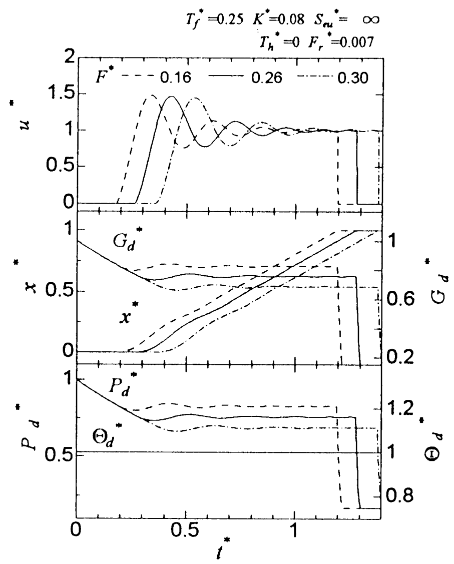

Fig.7 Step responses changing F^* parameter

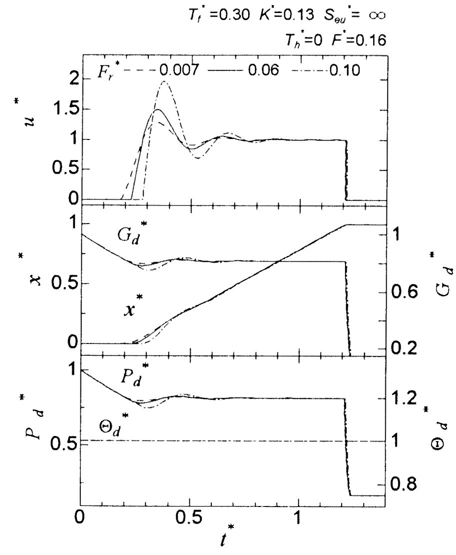

Fig.8 Step responses changing F_r^* parameter

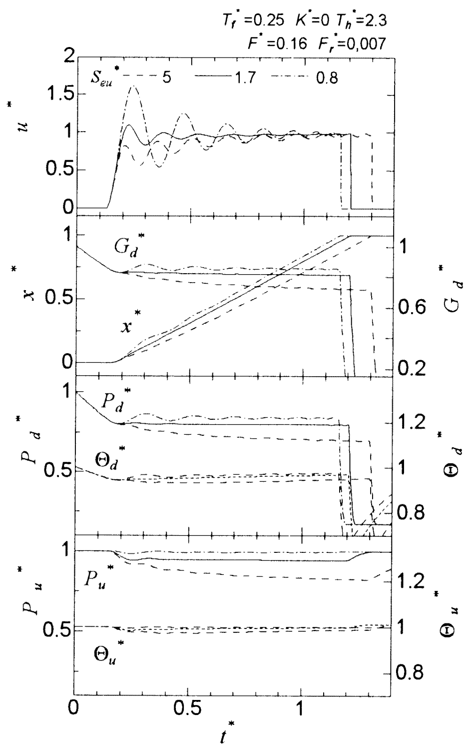

Fig.9 Step responses changing S_{eu} parameter

F_r^* parameter has been confused with F^* parameter. However this result proves that the effect on the response of F_r^* parameter is different from that of F^* parameter .

4.5 S_{eu}^* parameter

Fig.9 tells that in the cylinder velocity response of 3rd order time lag system is realized. It is recognized that S_{eu}^* parameter has the effect to dump the response similar to K^* parameter except that S_{eu}^* parameter works with the first order time lag of V_u^*/ξ_u^* as seeing in the block diaphragm of Fig.4. Due to this time lag, it is difficult to realize response without overshoot when initial volume is very large. Even in the case of $S_{eu}^* = 5$, which is nearly the ratio often used to drive cylinder in velocity control, the effect of damping is obvious comparing broken line in Fig.6.

4.6 T_h^* parameter

Step responses changing T_h^* parameter can not be shown here for lack of space. Previously one of the authors had been reported that drop of temperature of the air makes the cylinder velocity to be slow. In this study it is newly found that T_h^* parameter decides the degree of this velocity drop.

All nondimensional response are not able to be presented since there is many combination of nondimensiolized parameters. However it is easy to predict the cylinder response in various conditions.

5 CONCLUSIONS

1. Nondimensional equations and its responses have been presented. By nondimensiolization it has been found that six parameter determine the cylinder response. It has made clear the influence of each parameter on the cylinder dynamics.
2. We have clarified the feedback mechanism: The air compressibility makes feedback loop and the flow characteristics of restriction in choking compensates the constant cylinder velocity for a changing external force.

REFERENCES

1. J.F.Blackburn, C.Reethof and J.L.Shearer, Fluid Power Control, Wiley, MIT press, 1960.
2. K.Miyata et.al., Studies on a Constant Speed Drive of a Pneumatic Cylinder, Journal of JHPS, 1988, **20**-3, pp.240-246 (in Japanese).
3. K.Kawakami et.al., Some Considerations on Dynamic characteristics of Pneumatic Cylinder, Journal of Fluid Control, 1988, pp.22-36.
4. T.Kagawa and Y.Ishii, Air Temperature Change of Pneumatic Cylinder with Meter-out Control and Its Effect on the Velocity, Proceedings of 3rd, 1991, pp.549-554.