Mathematical Modelling and Simulation of Pneumatic Systems

Dr. Djordje Dihovicni and Dr. Miroslav Medenica
Visoka tehnicka skola, Bulevar Zorana Djindjica 152 a, 11070
Serbia

1. Introduction

Program support, simulation and the animation of dual action pneumatic actuators controlled with proportional spool valves are developed. Various factors are involved, such as time delay in the pneumatic lines, leakage between chambers, and air compressibility in cylinder chambers as well as non-linear flow through the valve. Taking into account the complexity of the model, and the fact that it is described by partial different equations, it is important to develop the program support based on numerical methods for solving this kind of problems. Simulation and program support in Maple and Matlab programming languages are conducted, and it is shown the efficiency of the results, from engineering view of point.

These pneumatic systems have a lot of advantages if we compare them with the same hydraulic types; they are suitable for clean environments, and much safer. In accordance with project and space conditions, valves are positioned at relatively large distance from pneumatic cylinder.

Considering real pneumatic systems, it is crucial to describe them with time delay, nonlinearities, with attempt of not creating only academic model. Despite of these problems, development of fast algorithms and using the numerical methods for solving partial different equations, as well as enhanced simulation and animation techniques become the necessity. Various practical stability approaches, for solving complex partial equations, used similar algorithms, (Dihovicni, 2006).

In the third part it is described special group of distributed parameter systems, with distributed control, where control depends of one space and one time coordinate. It has been presented mathematical model of pneumatic cylinder system. The stability on finite space interval is analyzed and efficient program support is developed.

Solving problem of constructing knowledge database of a decision making in process safety is shown in fourth part. It is provided analyses of the requirements as well the analyses of the system incidents caused by specification, design and the implementation of the project. Main focus of this part is highlighted on practical stability problem and conditions for optimal performance of pneumatic systems. Algorithm of decision making in safety of pneumatic systems is developed, and the system has been realized taking into account C# approach in Windows environment.

2. Representation of pneumatic cylinder-valve system

Deatiled mathematical model of dual action pneumatic actuator system, controlled by proportional spool valves, is shown in paper (Richer, 2000), and it is carefully considered effects of non-linear flow through the valve, leakage between chambers, time delay, attenuation and other effects.

These pneumatic systems have a lot of advantages if we compare them with the same hydraulic types, they are suitable for clean environments, and much safer. In accordance with project and space conditions, valves are positioned at relatively large distance from pneumatic cylinder.

Typical pneumatic system includes pneumatic cylinder, command device, force, position and pressure sensors, and as well as connecting tubes.
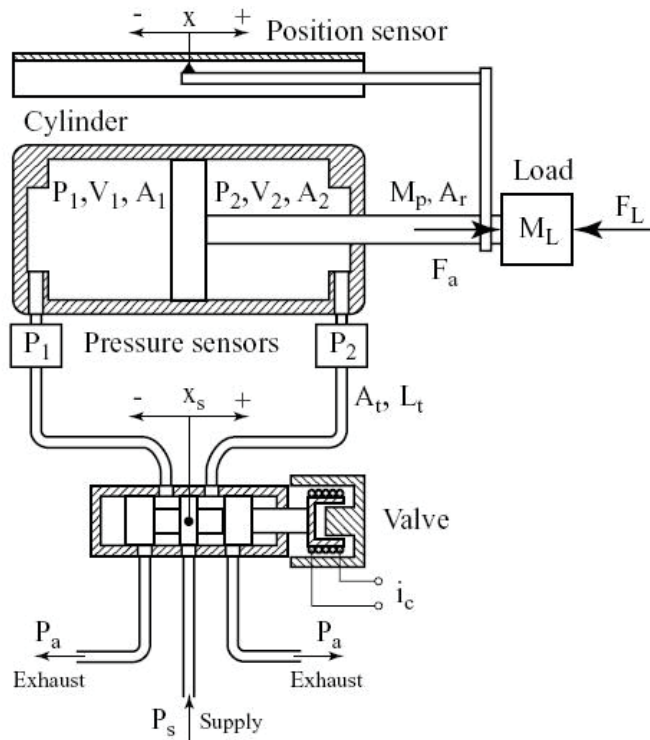

Fig. 1. Schematic representation of the pneumatic actuator system

The motion equation for the piston- road assembly is described as:

$$\left(M_L + M_p\right) \cdot \frac{d}{dt}\dot{x} + \beta \cdot \dot{x} + F_f + F_L = P_1 \bullet A_1 - P_2 \bullet A_2 - P_a \bullet A_r \tag{1}$$

where M_L is the external mass, M_p is the piston and rod assembly mass, x represents the piston position, β is the viscous friction coefficient, F_f is the Coulomb friction force, F_L is the external force, P_1 and P_2 are the absolute pressures in actuator's chambers, P_a is the absolute

ambient pressure, A_1 and A_2 are the piston effective areas, and A_r is the rod cross sectional area.

The general model for a volume of gas consists of state equation, the concervation of mass, and the energy equation. Using the assumptions that the gas is ideal, the pressures and temperature within the chambre are homogeneous, and kinetic and potential energy terms are negligible, it should be written the equation for each chamber. Taking into account that control volume V, density ρ, mass m, pressure P, and temperature T, equation for ideal gas is described as:

$$P = \rho \bullet R \bullet T \tag{2}$$

where, R is gas constant. Mass flow is given with:

$$\dot{m} = \frac{d}{dt} \bullet (\rho \bullet V) \tag{3}$$

and it can be expressed as:

$$\dot{m}_{ul} - \dot{m}_{iz} = \dot{\rho} \bullet V + \rho \bullet \dot{V} \tag{4}$$

where, $\dot{m}_{ul}, \dot{m}_{iz}$ are input and output mass flow. Energy equation is described with:

$$q_{ul} - q_{iz} + k \bullet C_v \bullet (\dot{m}_{ul} \bullet T_{in} - \dot{m}_{iz} \bullet T) - W = \dot{U} \tag{5}$$

where q_{ul} and q_{iz} are the heat transfer terms, k is the specific heat ratio, C_v is the specific heat at constant volume, T_{in} is the temperature of the incoming gas flow, W is the rate of change in the work, and $\dot{U}$ is the change of the internal energy. The total change of the internal is given:

$$\dot{U} = \frac{d}{dt}(C_v \bullet m \bullet T) = \frac{1}{k-1} \bullet \frac{d}{dt}(P \bullet V) = \frac{1}{k \cdot 1}(V \bullet \dot{P} + P \bullet \dot{V}) \tag{6}$$

and it is used the expression, $C_v = R / (k-1)$. If we use the term $\dot{U} = P \bullet \dot{V}$ and supstitute the equation (6) into equation (5), it yields:

$$q_{ul} - q_{iz} + \frac{k}{k-1} \cdot \frac{P}{\rho \bullet T}(\dot{m}_{ul} \bullet T_{ul} - \dot{m}_{iz} \bullet T) - \frac{k}{k-1} \bullet P \bullet \dot{V} = \frac{1}{k-1} \bullet V \bullet \dot{P} \tag{7}$$

If we assume that input flow is on the gas temperature in the chambre, which is considered for analyze, then we have:

$$\frac{k}{k-1} \bullet (q_{ul} - q_{iz}) + \frac{1}{\rho} \bullet (\dot{m}_{ul} - \dot{m}_{iz}) - \dot{V} = \frac{V}{k \bullet P} \bullet \dot{P} \tag{8}$$

Further simplification may be developed by analysing the terms of heat transfer in the equation (8). If we consider that the process is adiabatic ($q_{ul}-q_{iz}=0$), the derivation of the pressure in the chamber is:

$$\dot{P} = k \bullet \frac{P}{\rho \bullet V} \bullet (\dot{m}_{ul} - \dot{m}_{iz}) - k \bullet \frac{P}{V} \bullet \dot{V} \tag{9}$$

and if we substitute ρ from the equation (2), then it yields:

$$\dot{P} = k \bullet \frac{R \cdot T}{V} \bullet \left(\dot{m}_{ul} - \dot{m}_{iz} \right) - k \bullet \frac{P}{V} \bullet \dot{V} \qquad (10)$$

and if we consider that the process is isothermal (*T=constant*), then the change of the internal energy can be written as:

$$\dot{U} = C_v \bullet \dot{m} \bullet T \qquad (11)$$

and the equation (8) can be written as:

$$q_{in} - q_{out} = P \bullet \dot{V} - \frac{P}{\rho} \bullet \left(\dot{m}_{in} - \dot{m}_{out} \right) \qquad (12)$$

and then:

$$\dot{P} = \frac{R \cdot T}{V} \bullet \left(\dot{m}_{in} - \dot{m}_{out} \right) - \frac{P}{V} \bullet \dot{V} \qquad (13)$$

Comparing the equation (10) and the equation (13), it can be shown that the only difference is in heat transfer factor term *k*. Then both equations are given:

$$\dot{P} = \frac{R \cdot T}{V} \bullet \left(a_{ul} \bullet \dot{m}_{in} - a_{iz} \bullet \dot{m}_{out} \right) - a \bullet \frac{P}{V} \bullet \dot{V} \qquad (14)$$

where *a*, a_{ul}, and a_{iz} can take values between 1 i *k*, in accordance with heat transfer during the time of the process .
If we choose the origin of piston displacement at the middle of the stroke, the volume equation can be expressed as:

$$V_i = V_{0i} + A_i \bullet \left(\left((\frac{1}{2} L \pm x) \right) \right) \qquad (15)$$

where i=1,2 is cylinder chambers, index V_{0i} is non active volume at the end of the stroke, A_i is effective piston area, *L* is the piston stroke, and x is the position of the piston. If we change the equation (15) into equation (14), and pressure time derivation in pneumatic cylinder chambers, then it yields:

$$\dot{P}_i = \frac{R \cdot T}{V_{oi} + A_i \cdot \left(\frac{1}{2} \cdot L \pm x \right)} \cdot \bullet \left(a_{ul} \bullet \dot{m}_{ul} - a_{iz} \bullet \dot{m}_{iz} \right) - a \bullet \frac{P \bullet A_i}{V_{oi} + A_i \bullet \left((\frac{1}{2} \bullet L \pm x) \right)} \bullet \dot{x} \qquad (16)$$

2.1 Mathematical model valve-cylinder

In the literature, (Andersen, 1967), two basic equations which consider the flow change in pneumatic systems, are:

$$\frac{\partial P}{\partial s} = -R_i \bullet u - \rho \bullet \frac{\partial w}{\partial t} \qquad (17)$$

$$\frac{\partial u}{\partial s} = -\frac{1}{\rho \bullet c^2} \bullet \frac{\partial P}{\partial t} \tag{18}$$

where P is the pressure through the tube, u is the velocity, ρ is the air density, c is the sound speed, s is the tube axis coordinate, and R_t is the tube resistance. If we use mass flow through the tube, $\dot{m}_t = \rho \bullet A_t \bullet w$, where A_t is cross sectional area., finally it yields:

$$\frac{\partial P}{\partial s} = -\frac{1}{A_t} \bullet \frac{\partial \dot{m}_t}{\partial t} - \frac{R_t}{\rho \bullet A_t} \bullet \dot{m}_t \tag{19}$$

$$\frac{\partial \dot{m}_t}{\partial s} = -\frac{A_t}{c^2} \bullet \frac{\partial P}{\partial t} \tag{20}$$

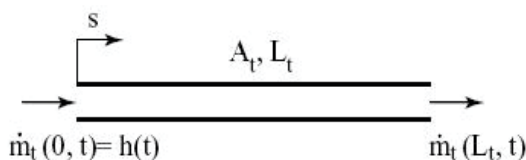The overall analysis is based on turbulent flow in the tube, which is presented in figure 2.

Fig. 2. Turbulent flow in the tube

Differentiating the equation (19), and the equation (20) with respect s, it is given the equation of mass flow through the tube:

$$\frac{\partial^2 \dot{m}_i}{\partial t^2} - c^2 \bullet \frac{\partial^2 \dot{m}_i}{\partial s^2} + \frac{R_i}{\rho} \bullet \frac{\partial \dot{m}_i}{\partial t} = 0 \tag{21}$$

The equation represents generalized wave equation, with new terms, and can be solved by using the form (Richer, 2000), like:

$$\dot{m}_t(s,t) = \varphi(t) \bullet \xi(s,t) \tag{22}$$

where $\xi(s,t)$ and $\varphi(t)$ are new functions. Supstituing equation (22) into equation (21) it yields:

$$\frac{\partial^2 \xi}{\partial t^2} - c^2 \bullet \varphi \bullet \frac{\partial^2 \xi}{\partial s^2} + \left((\varphi \bullet \frac{R_t}{\rho} + 2 \bullet \varphi') \right) \bullet \frac{\partial \xi}{\partial t} + \left((\varphi' \bullet \frac{R_t}{\rho} + \varphi'') \right) \bullet \xi = 0 \tag{23}$$

Simplifying the equation for ξ, it is determined $\varphi(t)$, so after the supstitution in equation (23), remaining of the equation ξ, doesn't contain the terms of firt order, so:

$$2 \bullet \varphi' + \varphi \bullet \frac{R_t}{\rho} = 0 \tag{24}$$

and that yields to:

$$\varphi(t) = e^{\frac{R_t}{2 \bullet \rho} \bullet t} \tag{25}$$

The result equation for ξ, will be in that case:

$$\frac{\partial^2 \xi}{\partial t^2} - c^2 \bullet \varphi \frac{\partial^2 \xi}{\partial s^2} + \frac{R_t^2}{4 \cdot \rho^2} \bullet \xi = 0 \qquad (26)$$

which represents dispersive hyperbolic equation. Tubes are usually not so long, so it might be assumpted that the dispersion is small, and it can't be neglected. Using that assumption it yields:

$$\frac{\partial^2 \xi}{\partial t^2} - c^2 \bullet \varphi \frac{\partial^2 w}{\partial s^2} = 0 \qquad (27)$$

which represents the classical one-dimension wave equation, which can be solved by using specific boundary and initial conditions:

$$\begin{cases} \xi(s,0) = 0 \\ \dfrac{\partial \xi}{\partial t}(s,0) = 0 \\ \xi(0,t) = h(t) \end{cases} \qquad (28)$$

The solution for the problem of boundary-initial values is given in the literature, (Richer, 2000), and can be expressed as:

$$\xi(s,t) = \begin{cases} 0 & \text{if } t < s/c \\ h \bullet \left(\left(t - \dfrac{s}{c} \right) \right) & \text{if } t > s/c \end{cases} \qquad (29)$$

The input wave will reach the end of the tube in time interval $\tau = L_t/c$. Supstituing t with Lt/c in the equation (24), and ρ from the state equation, it is given:

$$\varphi = e^{\frac{R_T \cdot R \cdot T}{2 \cdot P} \bullet \frac{L_T}{c}} \qquad (30)$$

where P is the pressure. Mass flow at the end of the tube, when $s=L_t$ is given with:

$$\dot{m}_t(L_t,t) = \begin{cases} 0 & \text{if } t < \dfrac{L_t}{c} \\ e^{\frac{R_T \cdot R \cdot T}{2 \cdot P} \cdot \frac{L_T}{c}} \bullet h \left(t \cdot \dfrac{L_t}{c} \right) & \text{if } t > \dfrac{L_t}{c} \end{cases} \qquad (31)$$

The tube resistance R_t, can be calculated, (Richer, 2000/, and it is shown by following equation:

$$\Delta p = f \bullet \frac{L_t}{D} \bullet \frac{\rho \cdot w^2}{2} = R_t \bullet w \bullet L_t \qquad (32)$$

where f is the friction factor, and D is inner diameter of the tube. For laminar flow, $f=64/Re$, where Re, is Reynolds number. The resistance of the tube then becomes:

$$R_t = \frac{32 \bullet \mu}{D^2} \tag{33}$$

where μ is dynamic viscosity of the air. If the Blasius formula is used, then it yields:

$$f = \frac{0.316}{\text{Re}^{1/4}} \tag{34}$$

so the flow resistance for the turbulent flow then becomes:

$$R_t = 0.158 \bullet \frac{\mu}{D^2} \bullet \text{Re}^{3/4} \tag{35}$$

2.2 Program support
Program support is developed in Maple programing language.
```
#
# File for simulation
# the disperzive hyperbolic equation
# Definition the number of precision
Digits: =3:
# Definition of the disperse hyperbolic equation
pdeq : = diff(ξ, t)^2-c^2*φ*diff(ω,s)^2;
# Definition of initial conditions
init1:= ξ(s,0)=0:
init2:= diff(ξ, t)=0
init3:= ξ(0,t)=Heaviside(t)
# Creating the procedure for numeric solving
pdsol :=pdsolve(pdeq,init1, init2,init3):
#Activating the function for graphic display
# the solutions of the equation
plot (i(t), t=0.. 10, i=0.04…008, axes=boxed, ytickmarks=6);
```

2.3 Animation
The animation is created in Matlab programming language, (Dihovicni, 2008).
```
%      Solving the disperse equation
%
```
$$\% \qquad \frac{\partial^2 \xi}{\partial t^2} \bullet c^2 - \varphi \frac{\partial^2 w}{\partial s^2} = 0$$
```
%

%      Problem definition
g='squareg'; %
b='squareb3'; % 0 o
%            0
c=1;
a=0;
f=0;
d=1;
```

```
%       Mesh
[p,e,t]=initmesh('squareg');

%       Initial coditions:
%       ξ(s,0) = 0
```

$$\xi(s,0) = 0$$

```
%       ∂ξ/∂t (s,0) = 0
```

$$\frac{\partial \xi}{\partial t}(s,0) = 0$$

```
%       ξ(0,t) = h(t)
```

$$\xi(0,t) = h(t)$$

```
%       using higher program modes
%       time definition
x=p(1,:)';
y=p(2,:)';
u0=atan(cos(pi/2*x));
ut0=3*sin(pi*x).*exp(sin(pi/2*y));
%       Desired solution in 43 points between 0 and 10.
n=43;
tlist=linspace(0,10,n);
%       Solving of hyperbolic equation
uu=hyperbolic(u0,ut0,tlist,b,p,e,t,c,a,f,d);
%       Interpolation of rectangular mesh
delta=-1:0.1:1;
[uxy,tn,a2,a3]=tri2grid(p,t,uu(:,1),delta,delta);
gp=[tn;a2;a3];
%       Creating the animation
newplot;
M=moviein(n);
umax=max(max(uu));
umin=min(min(uu));
for i=1:n,...
  if rem(i,10)==0,...
  end,...
  pdeplot(p,e,t,'xydata',uu(:,i),'zdata',uu(:,i),'zstyle','continuous',...
      'mesh','off','xygrid','on','gridparam',gp,'colorbar','off');...
  axis([-1 1 -1 1 umin umax]); caxis([umin umax]);...
  M(:,i)=getframe;...
  if i==n,...
  if rem(i,10)==0,...
  end,...
  pdeplot(p,e,t,'xydata',uu(:,i),'zdata',uu(:,i),'zstyle','continuous',...
      'mesh','off','xygrid','on','gridparam',gp,'colorbar','off');...
  axis([-1 1 -1 1 umin umax]); caxis([umin umax]);...
  M(:,i)=getframe;...
  if i==n,...
  if rem(i,10)==0,...
  end,...
```
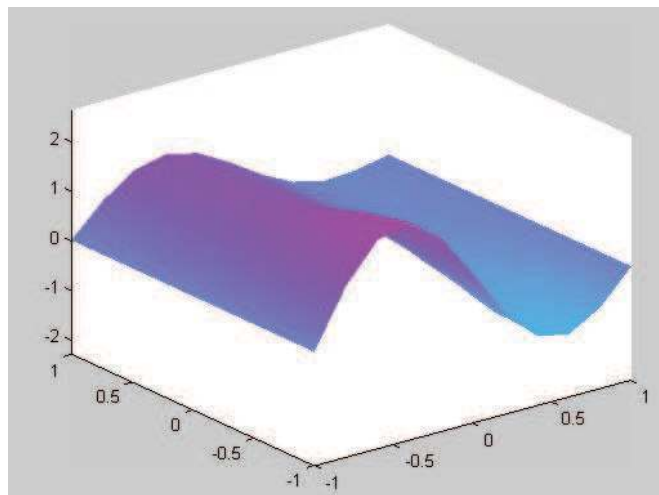
Fig. 3. Animation of the disperse equation

2.4 Conclusions

The complex mathematical model of dual action pneumatic actuators controlled with proportional spool valves (Richer, 2000) is presented. It is shown the adequate program support in *Maple* language, based on numerical methods. The simulation and animation is developed in *Matlab* programming language. The simplicity of solving the partial different equations, by using this approach and even the partial equations of higher order, is crucial in future development directions.

3. Program support for distributed control systems on finite space interval

Pneumatic cylinder systems significantly depend of behavior of pneumatic pipes, thus it is very important to analyze the characteristics of the pipes connected to a cylinder. Mathematical model of this system is described by partial different equations, and it is well known fact that it is distributed parameter system. These systems appear in various areas of engineering, and one of the special types is distributed parameter system with distributed control.

3.1 Mathematical model of pneumatic cylinder system

The Figure 4 shows a schematic diagram of pneumatic cylinder system. The system consists of cylinder, inlet and outlet pipes and two speed control valves at the charge and discharge sides. Detailed procedure of creating this mathematical model is described in, (Tokashiki, 1996).

For describing behavior of pneumatic cylinder, the basic equations that are used are: state equation of gases, energy equation and motion equation, (Al Ibrahim, 1992).

$$\frac{dP}{dt} = \frac{1}{V} \bullet \left(\frac{P \bullet V}{\theta} \frac{d\theta}{dt} + R \bullet \theta \bullet G - P \bullet \frac{dV_d}{dt} \right) \tag{36}$$

where P is pressure (kPa), V- is volume (m^3), θ - temperature (K), R- universal gas constant (J/kgK), and V_d- is dead volume (m^3).

Fig. 4. Schematic diagram of pneumatic cylinder system

The temperature change of the air in each cylinder chamber, from the first law of thermodynamics, can be written as:

$$\frac{d\theta_d}{dt} = \frac{1}{C_v \bullet m_d} \cdot$$
$$\left\{ S_{hd} \bullet h_d \left(\theta_a - \theta_d \right) + R \bullet \dot{m}_d \bullet \theta_d - P_d \bullet \frac{dV_d}{dt} \right\} \tag{37}$$

$$\frac{d\theta_u}{dt} = \frac{1}{C_v \bullet m_u} \cdot$$
$$\left\{ S_{hu} \bullet h_u \left(\theta_a - \theta_u \right) + C_p \bullet \dot{m}_u \bullet T_1 - C_v \bullet \dot{m}_u \bullet \theta_u - P_u \bullet \frac{dV_u}{dt} \right\} \tag{38}$$

where C_v- is specific heat at constant volume (J/kgK), m- mass of the air (kg), S_h –heat transfer area (m^2), $\dot{m}$ - mass flow rate (kg/s), and subscript d denotes downstream side, and subscript u denotes upstream side.

Taking into account that thermal conductivity and the heat capacity of the cylinder are sufficiently large compared with them of the air, the wall temperature is considered to be constant.

In equation of motion, the friction force is considered as sum of the Coulomb and viscous friction, and force of viscous friction is considered as linear function of piston velocity, and other parameters have constant effect to friction force of cylinder. Then, equation of motion may be presented in following form:

$$M \bullet \frac{dw_p}{dt} = P_u \bullet S_u - P_d \bullet S_d + P_a \bullet \left(S_d - S_u \right) - M \bullet g \bullet \sin a - c \bullet w_p \bullet F_q \tag{39}$$

where S- cylinder piston area (m^2), w_p- piston velocity (m/s), M- load mass (kg), c-cylinder viscous friction (Ns/m), P_a- atmospheric pressure (kPa), F_q- Coulomb friction (N), g-acceleration of gravity (m/s^2).

By using the finite difference method, it can be possible to calculate the airflow through the pneumatic pipe. The pipe is divided into n partitions.

Applying the continuity equation, and using relation for mass of the air $m = \rho \bullet A \bullet \partial z$ and mass flow $\dot{m} = \rho \bullet A \bullet w$, it can be obtained:

$$\frac{\partial m_i}{\partial t} = \dot{m}_{i\text{-}1} - \dot{m}_i \tag{40}$$

Starting from the gas equation, and assuming that the volume of each part is constant, deriving the state equation it follows, (Kagawa, 1994):

$$\frac{dP_i}{dt} = \frac{R \bullet \theta_i}{V}\left(\dot{m}_{i\text{-}1} - \dot{m}_i\right) + \frac{R \bullet m_i}{V} \bullet \frac{d\theta_i}{dt} \tag{41}$$

The motion equation of the air, is derived from Newton's second law of motion and is described as:

$$\frac{\partial w}{\partial t} = \frac{P_i - P_{i+q}}{\rho_i \bullet \delta z} \bullet \frac{\lambda}{2d} \bullet w_i \bullet |w_i| - |w_i| \bullet \frac{\partial w_i}{\partial z} \tag{42}$$

where λ is pipe viscous friction coefficient and is calculated as a function of the Reynolds number:

$$\lambda = \frac{64}{\text{Re}}, \quad \text{Re} < 2.5x10^3 \tag{43}$$

$$\lambda = 0.3164 \, \text{Re}^{0.25}, \quad \text{Re} \geq 2.5x10^3 \tag{44}$$

The respective energy can be written as:

$$\Delta E_{st} = E_{1i} - E_{2i} + L_{1i} - L_{2i} + Q_i \tag{45}$$

where E_{1i} is input energy, E_{2i} is output energy, L_{1i} is cylinder stroke in downstream side, and L_{2i} is cylinder stroke in upstream side of pipe model, and the total energy is calculated as sum of kinematic and potential energy.

Deriving the total energy ΔE_{st} , it is obtained the energy change ΔE_{st}:

$$\Delta E_{st} = \frac{d}{dt}\left\{\left((C_v \bullet m_i \bullet \theta_i + \frac{1}{2} \bullet m_i \bullet \left(\left(\frac{w_{i-1} + w_i}{2}\right)^2\right)\right)\right\} \tag{46}$$

In equation (45), the inflow and outflow energy as well as the work made by the inflow and outflow air can be presented :

From the following equation the heat energy Q can be calculated:

$$Q = h_i \bullet S_h \bullet \left(\theta_a - \theta_i\right) \tag{47}$$

where h is is heat transfer coefficient which can be easily calculated from the Nusselt number Nu, and thermal conductivity k:

$$h_i = \frac{2Nu_i \bullet k_i}{d_p} \tag{48}$$

where d_p is pipe diameter.

Nusselt number can be calculated from Ditus and Boelter formula for smooth tubes, and for fully developed turbulent flow:

$$Nu_i = 0.023 \bullet \mathrm{Re}_i^{0.8} \bullet \mathrm{Pr}^{0.4} \tag{49}$$

and thermal conductivity k can be calculated as a linear function of temperature:

$$k_i = 7.95 \bullet 10^5 \bullet \theta_i + 2.0465 \bullet 10^3 \tag{50}$$

3.2 Distributed parameter systems

During analyzes and synthesis of control systems, fundamental question which arises is determination of stability. In accordance with engineer needs, we can roughly divide stability definitions into: Ljapunov and non-Ljapunov concept. The most useful approach of control systems is Ljapunov approach, when we observe system on infinite interval, and that in real circumstances has only academic significance.

Let us consider n- dimensional non-linear vector equation:

$$\frac{dx}{dt} = f\left(\underline{x}\right) \tag{51}$$

for $\frac{dx}{dt} = 0$ solution of this equation is $\underline{x}_s = 0$ and we can denote it as equilibrium state.

Equilibrium state $\underline{x}_r = 0$ is stable in sense of Ljapunov if and only if for every constant and real number ε, exists $\delta(\varepsilon) > 0$ and the following equation is fulfilled, (Gilles, 1973):

$$\left\| \underline{x}_0 \right\| = \left\| \underline{x} \right\|_{t=0} \le \delta \tag{52}$$

for every t ≥ 0

$$\left\| \underline{x} \right\| < \varepsilon \tag{53}$$

If following equation exists:

$$\left\| \underline{x}_0 \right\| \to 0 \ \text{ for } \ t \to \infty \tag{54}$$

then system equilibrium state is asymptotic stable.

System equilibrium state is stable, if and only if exists scalar, real function V($\underline{x}$), Ljapunov function, which for $\left\| \underline{x} \right\| < r, r = const > 0$, has following features:

a. $V\left(\underline{x}\right)$ is positively defined

b. $\frac{dV\left(\underline{x}\right)}{dt}$ is negatively semi defined for t$\ge$0

System equilibrium state is asymptotic stable, if and only if exists:

$\dfrac{dV(\underline{x})}{dt}$ is negatively defined for t≥0

Derivation of function V(x) , $\dfrac{dV(\underline{x})}{dt}$ can be expressed:

$$\frac{dV(x)}{dt} = \nabla_x{}^T \bullet V(\underline{x}) \bullet \frac{d\underline{x}}{dt} = \nabla_x{}^T V(\underline{x}) \bullet \underline{f}(\underline{x}) \tag{55}$$

and

$$\nabla_x = \left[\begin{bmatrix} \dfrac{\partial}{\partial x_1} \\ . \\ . \\ . \\ . \\ \dfrac{\partial}{\partial x_n} \end{bmatrix} \right] \tag{56}$$

By using Ljapunov function successfully is solved problem of asymptotic stability of system equilibrium state on infinite interval.

From strictly engineering point of view it is very important to know the boundaries where system trajectory comes during there's motion in state space. The practice technical needs are responsible for non- Ljapunov definitions, and among them is extremely important behaving on finite time interval- practical stability. Taking into account that system can be stabile in classic way but also can posses not appropriate quality of dynamic behavior, and because that it is not applicable, it is important to take system in consideration in relation with sets of permitted states in phase space which are defined for such a problem. In theory of control systems there are demands for stability on finite time interval that for strictly engineering view of point has tremendous importance. The basic difference between Ljapunov and practical stability is set of initial states of system (S_α) and set of permitted disturbance (S_ε). Ljapunov concept of stability, demands existence of sets S_α and S_ε in state space, for every opened set S_β permitted states and it is supplied that equilibrium point of that system will be totally stable, instead of principle of practical stability where are sets (S_α and S_ε) and set S_β which is closed, determined and known in advance.

Taking into account principle of practical stability, the following conditions must be satisfied:

- determine set S_β - find the borders for system motion
- determine set S_ε - find maximum amplitudes of possible disturbance
- determine set S_α of all initial state values

In case that this conditions are regularly determined it is possible to analyze system stability from practical stability view of point.

3.2 Practical stability

Problem of asymptotic stability of system equilibrium state can be solved for distributed parameter systems, which are described by equation:

$$\frac{\partial \underline{x}}{\partial z} = \underline{f}\left((t,\underline{x},\frac{\partial \underline{x}}{\partial t},\frac{\partial^2 x}{\partial t^2}...) \right) \quad t \in (0,T) \tag{57}$$

with following initial conditions:

$$\underline{x}(t,0) = \underline{x}_0(t) \tag{58}$$

To satisfy equation (57), space coordinate z cannot be explicitly defined. The solution of equation (23) is $\dfrac{\partial \underline{x}}{\partial z} = \underline{0}$, and let the following equation exists:

$$\frac{\partial \underline{x}}{\partial z} = \underline{x}(t,z) - \underline{x}_r(t) \tag{59}$$

Assumption 1: Space coordinate z on time interval t∈ (0,T) is constant.

Accepting previous assumption, and equation (58), we have equation for equilibrium state for system described by equation (57):

$$\underline{x}_r(t) = 0 \tag{60}$$

For defining asymptotic stability of equilibrium state the functional V is used:

$$V(x) = \int_0^l W(\underline{x})\,dt \tag{61}$$

where W is scalar of vector $\underline{x}$.

We choose functional V like:

$$V(x) = \frac{1}{2} \bullet \int_0^T \underline{x}^T \underline{x} \bullet dt \tag{62}$$

when it is used expression for norm:

$$\|\underline{x}\| = \sqrt{\int_0^T \underline{x}^T \bullet \underline{x}\,dt} \tag{63}$$

For asymptotic stability of distributed parameter systems described by equation (7), we use Ljapunov theorems, applied for functional V :

$$\frac{dV(\underline{x})}{dz} = \int_0^l \nabla^T{}_x \bullet W(\underline{x}) \bullet f\left((t,\underline{x},\frac{\partial \underline{x}}{\partial t},\frac{\partial^2 x}{\partial t^2}...) \right) \bullet dt \tag{64}$$

where W is scalar function of x.

Let consider distributed parameter system described by following equation:

$$\frac{\partial^3 x}{\partial t^3} = \frac{\partial x}{\partial t} \tag{65}$$

and initial conditions:

$$x(0,z) = \frac{K}{2} \bullet x(T,z) \tag{66}$$

We use the assumption that equation (61) and initial conditions (62) are not explicit functions of space coordinate z, so stationary state of system (61) with appropriate border conditions is represented by following equation:

$$x_r(z) = 0, \ \text{with} \frac{\partial^3 x}{\partial z^3} = 0 \tag{67}$$

For determination of asymptotic stability of equilibrium system state, we use functional V which is expressed:

$$V(x) = \int_0^l W(\underline{x}) dt \tag{69}$$

where W is scalar function of $\underline{x}$.

Functional V is described by :

$$V = \frac{1}{4} \bullet \int \left[x(t,z) \right]^4 \bullet dt \tag{70}$$

and the following condition V(x)>0 is fulfilled.

Derivation of functional V is given by following equation:

$$\frac{dV(x)}{dz} = \int_0^L x^3 \bullet \frac{\partial^3 x}{\partial z^3} \bullet dt = \int_0^L x^3 \bullet \frac{\partial x}{\partial t} \bullet dt =$$
$$\frac{1}{4} \left(\left[x(T,z) \right]^4 \bullet \left[x(0,z) \right]^4 \right) \tag{71}$$

By using equation (65) and by including it in equation (71) it is obtained:

$$\frac{dV(x)}{dz} = \left(1 - \frac{K^4}{4} \right) \bullet \left(\left[x(T,z) \right]^4 \right) \tag{72}$$

and it yields:

$$\frac{dV(x)}{dt} < 0 \ \text{when K}^4 < 1/4, \ |K| < 0.7 \tag{73}$$

which is necessary and sufficient condition for asymptotic stability of equilibrium state for system described by equations (61) and (62).

3.3 Distributed control

Control of distributed parameter systems, which depends of time and space coordinate is called distributed control. If we choose control *U*, for pressure difference in two close parts of pneumatic pipe, and for state *X*, if we choose air velocity through the pneumatic pipe, with assumptions that are shown during derivation of mathematical model of pneumatic pipe, finally it is obtained:

$$\frac{\partial X}{\partial t} + |X| \bullet \frac{\partial X}{\partial z} + a \bullet X \bullet |X| = b \bullet U, z \in [0, L] \tag{74}$$

where $a = \dfrac{\lambda}{2 \bullet d}$, $b = \dfrac{1}{\rho \bullet \delta z}$.

Nominal distributed control can be solved by using procedure which is described in (Novakovic, 1989), and result of that control is nominal state $w_N(t,z)$ of chosen system. In that case it yields:

$$L\big(X_N(t,z)\big) = \frac{1}{b} \bullet \frac{\partial X_N}{\partial t} + \frac{1}{b} \bullet |X| \bullet \frac{\partial X}{\partial z}$$
$$+ \frac{1}{b} \bullet a \bullet X \bullet |X| = U(t,z) \tag{75}$$

where *L* is appropriate operator.

System (73) is exposed to many disturbances, so the real dynamic must be different from nominal. It is applied deviation from nominal system state, then the nominal system state can be realized as:

$$x(t,z) = X(t,z) - X_N(t,z), 0 < z \cdot L . \tag{76}$$

Time derivation of deviation from nominal system state, can be presented by following equation:

$$\frac{\partial x(t,z)}{\partial t} = \frac{\partial X(t,z)}{\partial t} - \frac{\partial X_N(t,z)}{\partial t} \tag{77}$$

and from equations (74), it yields:

$$\frac{\partial x(t,z)}{\partial t} = r(t,z) + |X| \bullet \frac{\partial X}{\partial z} + a \bullet X \bullet |X| - b \bullet U \tag{78}$$

where $r = \dfrac{\partial X_N}{\partial t}$

3.4 Application

Using the concept of extern linearization, which is described in, (Meyer, 1983), we can include distributed control in the following form:

$$U(t,z) = \left[(a-k) \bullet X \bullet |X| + k \bullet X_N \bullet |X| + |X| \bullet \frac{\partial X}{\partial z} + r \right] / b,$$
$$0 \le z \le L \tag{79}$$

Including the equation (79) in the equation (78), it yields:

$$\frac{\partial x(t,z)}{\partial t} = -k \bullet x(t,z), \, 0 \le z \le L \tag{80}$$

Functional V is chosen in the form:

$$V(x) = \frac{1}{2} \bullet \int_0^L \left[x(t,z) \right]^2 \bullet dz = \frac{1}{2} \bullet \left\| x(t,z) \right\|^2 \tag{81}$$

Derivation of functional V is given as:

$$\frac{dV(x)}{dt} = \int_0^L x \bullet \frac{\partial x}{\partial t} \bullet dz$$
$$= k \bullet \int_0^L \left[x(t,z) \right]^2 \bullet dz = 2 \bullet k \bullet V(x) \tag{82}$$

Taking into account that $V(x)$ is positive defined functional, time derivation of functional given by equation (82) will be negative defined function for $k>0$, and in that way all necessary conditions from Ljapunov theorem applied to functional V, are fulfilled.

$$\nabla_x = \begin{bmatrix} \frac{\partial}{\partial x_1} \\ . \\ . \\ . \\ . \\ \frac{\partial}{\partial x_n} \end{bmatrix} \tag{83}$$

3.5 Program support

For this kind of symbolically presented problems, the most elegant solution can be achieved by using program language Maple.
#
Program support for determination of stability of
distributed parameter systems on finite space interval
described by equation

$\dfrac{\partial x}{\partial z^3} = f \bullet \left(\dfrac{\partial x}{\partial t} \right)$

#

Definition of procedure dpst
dpst:=proc(ulaz3)
Read input values
read ulaz3;
Determination of functional V
V:=1/4*int[x(t,z)∧2,z]=norm[x(t,z)];
Determination of functional V derivation
 dV/dz:=int[x(t,z)*diff(x,t),t]
# Applying partial integration on equation dV/dt
with(student)
intpart[Int(x*diff(x∧2,z∧2), x)]
Presentation of equation dV/dt
dV/dt;
Calculation of values for parameter K for which the system is stable
result:= solve(dV/dt,K)
#
If the procedure dpst would be operational for determination the values of parameter K for which the system is stable, it is necessary to create files with input parameters for current procedure. For case that is analytically calculated, it is created file *input3* with following input data:
dx/dt=diff(x∧3,z∧3)
 x(0,z)= =K/2*x(T,z)

Program support for distributed parameter systems with distributed control
#
Definition of procedure dpsdc
dpsdc:=proc(input 1)
Reading of input parameter values
read input1;
Determination of functional V
V:=1/2*int[x(t,z)^2,z]=norm[x(t,z)];
# Determinatation of time derivation of functional V
dV/dt:=int[x(t,z)*diff(x,t),z];
# Calculation of time derivation of functional V
derivationfunctional:=solve(dV/dt, z=0..1);
# Calculation the values of parameter K for which the system is stable
solution:=solve(derivationfunctional,K);
For using the procedure *dpsdc* for determination the values of parameter K it is necessary to create files that contain input parameters for given procedure. In case, which is calculated, the file *input 1* with input data:
dx/dt=-k*(x,z);
By using task *read* it yields to reading procedure *dpsdc*. Specification of input1 as argument of the procedure *dpsdc* starts the process of calculation the values of parameter K for which this distributed parameter system is stable on finite space interval.
It is developed, program support for other types of distributed parameter systems.

```
#
# Program support for determination of stability of
#  distributed parameter systems on finite space interval
# described by equation
```
$$\# \quad \frac{\partial \underline{x}}{\partial t} = f\left(\bullet \frac{\partial \underline{x}}{\partial z} \right)$$
```
#
# Definition of  procedure srppr
srppr:=proc(ulaz1)
# Read input values
read ulaz1;
# Determination of functional  V
V:=1/2*int[x(t,z)^2,z]=norm[x(t,z)];
#Determinatation of time derivation of functional V
dV/dt:=int[x(t,z)*diff(x,t),z];
#Solving the functional V
derivationfunctional:=solve(dV/dt, z=0..1);
# Calculation the solution for parameter K for which is
# the system stable
solution:=solve(derfunctional,K);
```

If the procedure *srppr* would be operational for determination of value of parameter K for which the system is stable, it is necessary to create files with input parameters for current procedure. It is created file *input1* with following data :
```
dx/dt=-vz*diff(x,z);
x(t,0)=-K*x(t,l)
# Program support for determination of stability of
# distributed parameter diffusion systems on finite space
# interval
# Definition of  procedure srppr
srpdp:=proc(ulaz2)
# Read input values
read input1;
# Determination of functional  V
V:=1/2*int[exp(2*γ*z)*x(t,z)^2, z=0..1];
# Determinatation of time derivation of functional V
dV/dt:=diff(V,t);
# Applying partial integration on equation dV/dt
intpart[Int(exp(2*γ*z)*diff(x,z)^2,z=0..1)]=( γ^2+π^2/l^2)*int[exp(2* γ*z)*x^2, z=0..1);
# Calculation of values for parameter K for which the system is stable
resenje:=solve(dV/dt,K);
```
If the procedure *spdsr* would be operational for determination of value of parameter K for which the system is stable, it is necessary to create files with input parameters for current procedure. It is created file *ula2* with following data:
```
dx/dt=diff(x^2,z^2)+2*a*diff(x,z)-b*x
x(t,0)=x(t,l)=0
```

3.6 Conclusion

Special class of control systems is focus of our scientific paper. Our main idea is to present a practical stability solution for this type of systems with distributed control. From practical view of point, it is crucial to find intervals on which the system is stable, and it is achieved by using this unique approach. Concerning on one-dimensional problem, where mathematical model of distributed parameter system is presented by equations which are dependable of time and only one space coordinate, successfully is applied new method for determination of stability on finite space interval for distributed parameter systems of higher order. The program support proved the efficiency of the theory, and it is developed for various types of distributed parameter systems.

4. Decision making in safety of pneumatic systems

One of the most important tasks in the safety engineering lays in the construction of a knowledge database of decision support for the pneumatic systems, and on that way to ensure optimal conditions, improve quality and boost efficiency. Methods of analysis of control systems and simulation methods, which are used for observing dynamic behavior of linear systems with time delay, and distributed parameter systems, based on linear algebra, operation calculus, functional analyse, integral differential equations and linear matrix non-equations has shown long ago that modern electronic components can be used to achieve more consistent quality at lower cost in safety engineering. The main idea to do so is that the quality service is maintained and controlled. Applying the Fuzzy theory in decision making has given very good results, and provided a flexible framework and over the years numerous mathematical models have been developed.

There are two basic problems to solve in decision making situations: obtaining alternative, and achieving consensus about solution from group of experts. First problem takes into account individual information which existed in collective information units. The later usually means an agreement of all individual opinions. Usually it is considered two approaches for developing a choice process in solving of decision making problems: a direct approach where solution is derived on the basis of the individual relations and as well indirect approach where solution is based on a collective preference relation. In safe engineering technical and economic benefits over hard-wired, discrete components has shown PLC. Main problem in process engineering is practical stability of the system. Chosen system should be stable in required period of time, and this important task is obtained by using practical stability theory for distributed parameter systems. Most pneumatic systems for instance, are described by partial different equations and they belong to group of distributed parameter systems.

4.1 Definitions and conditions of practical stability

Let us consider first order hyperbolic distributed parameter system, which is decribed by the following state- space equation:

$$\frac{\partial \underline{x}(t,z)}{\partial t} = A_0 \bullet \underline{x}(t,z) + A_1 \frac{\partial \underline{x}}{\partial z} \tag{84}$$

with appropriate function of initial state

$$\underline{x}_0(t,z) = \underline{\psi}_x(t,z)$$
$$0 \le t \le \tau, 0 \le z \le \zeta$$
(85)

where $\underline{x}(t,z)$ is n-component real vector of system state, A is matrix appropriate dimension, t is time and z is space coordinate.

Definition 1: Distributed parameter system described by equation (84) that satisfies initial condition (85) is stable on finite time interval in relation to [ξ(t,z), β, T, Z] if and only if:

$$\underline{\psi}_x^T(t,z) \bullet \underline{\psi}_x(t,z) < \xi(t,z)$$
$$\forall t \in [0,\tau], \forall z \in [0,\varsigma]$$
(86)

then it follows

$$\underline{x}^T(t,z)) \bullet \underline{x}(t,z) < \beta,$$
$$\forall t \in [0,T] \forall z \in [0,Z]$$
(87)

where ξ(t,z) is scalar function with feature $0 < \xi(t,z) \le a, 0 \le t \le \tau, 0 \le z \le \zeta$ where α is real number, β ∈ R and β > α.

Let calculate the fundamental matrix for this class of system:

$$\frac{d\Phi(s,\sigma)}{d\sigma} = A_1 \bullet (sI - A) \bullet \Phi(s,\sigma)$$
(88)

where after double Laplace transformation, and necessary approximation finally it is obtained, (Dihovicni, 2007):

$$\Phi(t,z) = \exp(A \cdot t \cdot z)$$
(89)

where $A = \dfrac{I - A_0 \cdot A_1}{A_1}$.

Theorem1: Distributed parameter system described by equation (84) that satisfies internal condition (85) is stable on finite time interval in relation to [ξ(t,z), β, T, Z] if it is satisfied following condition:

$$e^{2\mu(A) \bullet t \bullet z} < \frac{\beta}{a}$$
(90)

Proof: Solution of equation (84) with initial condition (85) is possible to describe as:

$$\underline{x}(t,z) = \Phi(t,z) \cdot \underline{\psi}(0,0)$$
(91)

By using upper equation it follows:

$$\underline{x}^T(t,z) \bullet \underline{x}^T(t,z) = \left[\underline{\psi}_x^T(0,0) \bullet \Phi(t,z) \right] \cdot$$
$$\left[\underline{\psi}_x^T(0,0) \bullet \Phi(t,z) \right]$$
(92)

By using well-known ineqality

$$\left\|\Phi(t,z)\right\| = \left\|\exp[A \bullet t \bullet z]\right\| \le \exp\{\mu(A) \bullet t \bullet z\} \tag{93}$$

and taking into account that:

$$\begin{aligned} \underline{\psi}_x^T(0,0) \bullet \underline{\psi}_x(0,0) < a \\ \left(\left\|\underline{\psi}_x^T(0,0)\right\| = \left\|\underline{\psi}_x^T(0,0)\right\| < a\right) \end{aligned} \tag{94}$$

then it follows:

$$\underline{x}^T(t,z) \bullet \underline{x}(t,z) \le e^{2\mu(A \bullet t \bullet z)} \bullet a \tag{95}$$

Applying the basic condition of theorem 1 by using equation (91) to further inequality it is obtained, (Dihovicni, 2007):

$$\underline{x}^T(t,z) \bullet \underline{x}(t,z) < \left(\frac{\beta}{a}\right) \bullet a < \beta \tag{96}$$

Theorem 2: Distributed parameter system described by equation (84) that satisfied initial condition (85) is stable on finite time interval in relation to [ξ(t,z), β, T, Z] if it is satisfied following condition:

$$e^{\mu(A) \bullet t \bullet z} < \frac{\sqrt{\beta/a}}{1 + \tau \bullet \zeta \|A\|} \tag{97}$$

$$\forall t \in [0,\tau] \forall z \in [0,\varsigma]$$

The proof of this theorem is given in (Dihovicni, 2006).

Let $|\underline{x}|_{(.)}$ is any vector norm and any matrix norm $\|\cdot\|_2$ which originated from this vector. Following expresions are used:

$$|\underline{x}|_2 = \left(\underline{x}^T \bullet \underline{x}\right)^{1/2} \text{ and } \|\cdot\|_2 = \lambda^{1/2}\left(A^* \bullet A\right)$$

where * and T are transpose-conjugate and transport matrixes.
It is important to define matrix measure as:

$$\mu(A) = \lim_{\varepsilon \to 0} \frac{\|1 + \varepsilon \bullet A\| - 1}{\varepsilon} \tag{98}$$

The matrix measure μ may be defined in three different forms according to the norm which is used:

$$\begin{aligned} \mu_1(A) &= \max\left(\operatorname{Re}(a_{kk}) + \sum_{i=1,i\neq k}^{n} |a_{ik}|\right) \\ \mu_2(A) &= \frac{1}{2}\max\lambda_i\left(A^T + A\right) \\ \mu_\infty(A) &= \max\left(\operatorname{Re}(a_{ii}) + \sum_{k=1}^{n} |a_{ki}|\right) \end{aligned} \tag{99}$$

Definition 2: Distributed parameter system described by equation (84) that satisfies initial condition (85) is stable on finite time interval in relation to [ξ(t,z), β, T, Z] if and only if, (Dihovicni, 2007):

$$\left| \underline{\psi}_x (t,z) \right|_2 < \xi(t,z) \tag{100}$$

then follows

$$\left| \underline{x}(t) \right|_2 < \beta \tag{101}$$

where ξ(t,z) is scalar function with feature $0 < \xi(t,z) \le a, \, 0 \le t \le \tau, \, 0 \le z \le \zeta)$ α is real number, β ∈ R and β > α.

Theorem 3: Distributed parameter system described by equation (84) that satisfies initial condition (85) is stable on finite time interval in relation to [α, β, T, Z] if it is satisfied following condition:

$$e^{\mu_2(A)\cdot t \cdot z} < \frac{\sqrt{\beta / a}}{1 + \mu^{-1}{}_2(A)} \tag{102}$$

$$\forall t \in [0,T] \forall z \in [0,Z]$$

Proof: Solution of equation (1) with initial condition (2) is possible to describe by using fundamental matrix as:

$$\underline{x}(t,z) = \Phi(t,z) \bullet \underline{\psi}_x(0,0) \tag{103}$$

By using the norms of left and right side of the equation (103) it follows:

$$\underline{x}^T(t,z) \cdot \underline{x}(t,z) \le e^{2\mu(A \cdot t \cdot z)} \cdot a \tag{104}$$

and by using well-known inequality

$$\left\| \exp(A \bullet t \bullet z) \right\|_2 \le \exp\{\mu(A \bullet t \bullet z)\} \tag{105}$$

$$t \ge 0, \quad z \ge 0$$

it follows:

$$\left| \underline{x}(t,z) \right|_2 \le e^{\mu_2(A)\cdot t \cdot z} \bullet \left| \underline{\psi}_x(0,0) \right|_2 \tag{106}$$

and by using equation (100) it is obtained:

$$\left| \underline{x}(t,z) \right|_2 \le a \bullet e^{\mu_2(A) \bullet t \bullet z} \tag{107}$$

so finally it is obtained:

$$\left| \underline{x}(t,z) \right|_2 \le a \cdot e^{\mu_2(A) t \bullet z} \left\{ 1 + \mu_2^{-1}(A) \right\} \tag{108}$$

Applying the basic condition of theorem 3 by using equation (19) it is obtained:

$$\left| \underline{x}(t) \right|_2 < \beta$$
$$\forall t \in [0,T], \forall z \in [0,Z] \tag{109}$$

Theorem 4: Distributed parameter system described by equation (85) that satisfies initial condition (86) is stable on finite time interval in relation to [α, β, T, Z], if it is satisfied following condition, (Dihovicni, 2007):

$$e^{\mu(A \bullet t \bullet z)} < \frac{\beta}{a}$$
$$\forall t \in [0,T], \forall z \in [0,Z] \tag{110}$$

Theorem 5: Distributed parameter system described by equation (84) that satisfies initial condition (85) is stable on finite time interval in relation to [t_0, J, α, β, Z], if it is satisfied following condition:

$$\left[1 + (t - t_0) \bullet \sigma_{max} \right]^2 \bullet e^{2(t \cdot t_0) \cdot z \bullet \sigma_{max}} < \frac{\beta}{a},$$
$$\forall t \in [0,J] \; \forall z \in [0,Z] \tag{111}$$

where σ_{max} represents maximum singular value of matrix. The proof of this theorem is given in (Dihovicni, 2007*).

4.2 Architecture
There are few well known stages in developing computer decision support systems based on knowledge which include choosing suitable mathematical tools, formalization of the subject area, and development of the corresponding software. In the first phase the problem lays in making right diagnosis and in analyses of the requirements and as well the analyses of the system incidents caused by specification, design and the implementation of the project, (Bergmans, 1996). The problem of diagnostics may be stated such as finite number of subsets, or it should be applied classical investigation methods, (Thayse, 1996).
System architecture consists of the following modules:
- Stability checking module. This module is designed as program for checking the practical stability of the system. If the system passes this check it goes further to other modules.
- Analysis module of safe fault-tolerant controllers, I/O, engineering and pressure transmitters.
- Diagnostics module
- Knowledge Module of all possible situations and impacts to pneumatic systems
- Optimal solution- decision making module
- Presentation module

For system realization an object oriented programming approach has been used, and the program has been developed using the C# language. Each module has a supportive library, and the logical structure is based on the classes, which are described down below for ilustrating purpose.

- Main classes are:
- Analyses group which has a primary task of collecting necessary facts about system.
- Practical stability group which determines if the system is stable or not. If the system is unstable in view of practical stability, then it is automaticly rejected.
- **Diagnosis group** describes all possible casualities for not required results, or potencial casualities for not optimal costs.
- Performance group is used for the optimal performance.
- Cost group is used for the optimal cost effect.
- Decision making algorithm for optimal performance and cost consists of two phases:
- Phase 1 is used for input Analyses class, Practical stability class and diagnosis class.
- Phase 2 is used for output Performance and Cost group.

4.3 Conclusion

By analysing process systems from safety and optimal cost perspective, it is important to recognize which systems are not stable in real conditions. From engineering state of view we are interested in such a systems which are stable in finite periods of time, so our first concern should be to maintain stable and safe systems. Our knowledge database is created in DB2, and it involved all possible reasons for non adequate performanse. Key modules for obtaining best performance, safety and the low cost are a good base for the program support in C# programming language and the UML representation.

5. References

Al-Ibrahim, A.M., and Otis, D.R., *Transient Air Temperature and Pressure Measurements During the Charging and Discharging Processes of an Actuating Pneumatic Cylinder*, Proceedings of the 45th National Conference on Fluid Power, 1992

Andersen, B..; Wiley, J. (1967), *The Analysis and Design of Pneumatic Systems*, INC. New York-London-Sydney, 1967

Bergmans J., Gutnikov S., Krasnoproshin V., Popok S. and Vissia H., *Computer-Base Support to Decision-Making in Orthopaedics*, International Conference on Intelligent Technologies in Human related Sciences, vol. 2: Leon, Spain, (1996) pp 217-223

Dihovicni, Dj.; Nedic, N. (2006), *Stability of Distributed Parameter Systems on Finite Space Interval,* 32-end Yupiter Conference, Zlatibor, September 2006, pp 306-312

Dihovicni, Dj.; Nedic, N. (2007), *Practical Stability of Linear Systems with Delay in State,* AMSE, Association for the Advancement of Modelling & Simulation Techniques in Enterprises, Tassin La-Demi-Lune, France, Vol 62 n^0 2 (2007) pp 98-104

Dihovicni, Dj.; Nedic, N., "*Simulation, Animation and Program Support for a High Performance Pneumatic Force Actuator System*", Mathematical and Computer Modelling 48 (2008), Elsevier, Washington, pp. 761–768

Gilles, E; *Systeme mit verteilten Parametern*, Wien 1973

Kagawa T Fujita T, Takeuchi M. *Dynamic Response and Simulation Model of Pneumatic Pipe Systems*, Proc 7th Bath International Fluid Power Workshop 1994

Novakovic, B.; *Metode vodjenja tehnickih sistema,* Skolska knjiga, Zagreb 1989

Richer, E.; Hurmuzlu, Y., *A High Performance Pneumatic Force Actuator System,* ASME Journal of Dynamic Systems Measurement and Control, September 2000, pp 416-425

Thayse A., Gribont P, *Approche Logique de LIInteligence Artificielle l De la Logique*, Bordas, (1997).

Tokashiki, L.; Fujita, T. ; Kogawa T., Pan W., *Dynamic, Characteristics of Pneumatic Cylinders Including pipes*, 9th Bath International Fluid Power Workshop, September 1996, pp 1-14
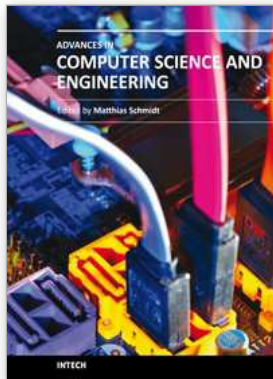
Advances in Computer Science and Engineering
Edited by Dr. Matthias Schmidt

The book Advances in Computer Science and Engineering constitutes the revised selection of 23 chapters written by scientists and researchers from all over the world. The chapters cover topics in the scientific fields of Applied Computing Techniques, Innovations in Mechanical Engineering, Electrical Engineering and Applications and Advances in Applied Modeling.

How to reference
In order to correctly reference this scholarly work, feel free to copy and paste the following:

Djordje Dihovicni and Miroslav Medenica (2011). Mathematical Modelling and Simulation of Pneumatic Systems, Advances in Computer Science and Engineering, Dr. Matthias Schmidt (Ed.), ISBN: 978-953-307-173-2, InTech, Available from: http://www.intechopen.com/books/advances-in-computer-science-and-engineering/mathematical-modelling-and-simulation-of-pneumatic-systems

INTECH
open science | open minds