



UNIVERSITY OF GHANA

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BSC. ENGINEERING

FIRST SEMESTER EXAMINATIONS: 2016/2017

SCHOOL OF ENGINEERING SCIENCES

FAEN 205: THERMODYNAMICS (3 CREDITS)

INSTRUCTIONS: ANSWER QUESTION ONE AND ANY OTHER THREE (3) QUESTIONS

TIME ALLOWED: THREE (3) HOURS

Some of the Equations and Constants Needed are Provided on Page 5

1.

- a. No single constant Equation of State is accurate over a wide range of states. However, a number are useful over a limited range of states. One of such equations is:

$$V = \frac{RT}{P} - \frac{b}{T^3}, \quad b = \text{constant}, \quad V = V(T, P)$$

- i. Determine an expression for the exact differential, dV . [8 Marks]
 - ii. Prove that the answer in part (i) is an exact differential (make use of the Maxwell's relation). [5 Marks]
 - iii. Determine a thermodynamic relationship between C_p and C_v for this equation of state. HINT: $C_p - C_v = \frac{TV\alpha^2}{\beta}$ [7 Marks]
- b. For a system constrained to be at a constant T and P, the Gibbs free energy function decreases for every spontaneous change. Assume that the system under consideration is experimentally controlled so that, whatever processes occur, its temperature and pressure are maintained to be constant. Consider two states "a" and "b", along the *isothermal* and *isobaric* path traversed by the system. The Gibbs free energy for each state is given as:

$$G^a = U^a + P^a V^a - T^a S^a \text{ and } G^b = U^b + P^b V^b - T^b S^b$$

- i. Impose the mechanical and thermal conditions of equilibrium and prove that

$$\Delta G = [Q + W + W'] - Q_{rev} - W_{rev}$$

Where $[Q + W + W'] = \Delta U$, all the symbols have their usual meaning.

[14 Marks]

- ii. Using undergraduate thermodynamic arguments, show that the ΔG expression (in b(i)) above reduces to $\Delta G < 0$ for an *irreversible* spontaneous change in an *isothermal* and *isobaric* system. [6 Marks]

2.

- a. Steam flows at steady state through a converging, insulated nozzle, 25 cm long and with an inlet diameter of 5 cm. At the nozzle entrance (state 1), the temperature and pressure are 598.15 K and 700 KPa, and the velocity is 30 m/s. At the nozzle exit (state 2), the steam temperature and pressure are 513.15 K and 350 KPa.

- i. What is the velocity of the steam at the nozzle exit?
- ii. What is the exit diameter?
- iii. Calculate the change in entropy and comment on the answer.

[Use the steam tables provided]

[10 Marks]

- b. Given that H is a function of T and V , that is $H(T, V)$, derive the following expression:

$$dH = \left[C_v + V \left(\frac{\partial P}{\partial T} \right)_v \right] dT + \left[T \left(\frac{\partial P}{\partial T} \right)_v - \frac{1}{\beta} \right] dV$$

Where H is the enthalpy of the system, and all other variables have the usual meaning. Use the above expression to calculate the enthalpy change for 1 mol of an ideal gas when it is compressed from $T_1 = 273 \text{ K}$, $P_1 = 1 \text{ atm}$ and $V_1 = 12 \text{ L}$ to $P_2 = 10 \text{ atm}$ and $V_2 = 6 \text{ L}$. For an ideal gas, $C_v = 1.5 R$. [10 Marks]

3.

- a. The following heat engine produces power at 95,000 KW. Determine in each case the rate at which heat is absorbed from the heat reservoir and discarded to the cold reservoir.

- i. A Carnot engine operates between heat reservoirs 750 K and 300 K.

- ii. A practical engine operates between the same heat reservoirs but with a thermal efficiency of 0.35.

[8 Marks]

- b. Iron with face centered cubic (FCC) structure is brought without phase transformation from a temperature of 1250 K and a pressure of 1 atm to 1600 K and 1,000 atm. The following data are provided:

$$V_{Fe}^0 = 7.31 \text{ cm}^3/\text{mol} \text{ (volume at 1600 K)}$$

$$\alpha = 0.63 \times 10^{-4} \text{ K}^{-1}, \beta = 1.10 \times 10^{-6} \text{ atm}^{-1}$$

$$C_p^0 = 5.80 + 1.98 \times 10^{-3} T \text{ Cal/K mol}$$

- i. Derive the relation between the enthalpy of the above system as a function of Temperature and Pressure; $H = H(T, P)$ [6 Marks]
- ii. Calculate the enthalpy change. *Ignore the effect of pressure on volume* in your calculation. [6 Marks]

4.

- a. Reported values for the virial coefficient of isopropanol vapor at 473.15 K are

$$B = -0.388 \text{ m}^3 \text{ kmol}^{-1}$$

$$C = -26 \times 10^{-3} \text{ m}^6 \text{ kmol}^{-2}$$

Calculate V and Z for isopropanol vapor at 473.15 K and 10 bar by:

- i. The ideal gas equation [2 Marks]
- ii. $Z = \frac{PV}{RT} = 1 + \frac{B}{V} + \frac{C}{V^2}$ (solve by iteration) [8 Marks]
- b. Derive the expression for U (internal energy) as a function of P and V; that is $U = U(P, V)$ and use it to compute the change in internal energy when 12 L of argon gas at 273 K and 1 atm is compressed to 6 L with the final pressure equal to 10 atm. Assume that argon is an ideal gas with $C_v = \left(\frac{3}{2}\right) R$. [10 Marks]

5:

- a. Iron with face centered cubic (FCC) structure is brought without phase transformation from a temperature of 1250 K and a pressure of 1 atm to 1600 K and 1,000 atm. The following data are provided:

$$V_{Fe}^0 = 7.31 \text{ cm}^3/\text{mol} \text{ (volume at 1600 K)}$$

$$\alpha = 0.63 \times 10^{-4} \text{ K}^{-1}, \beta = 1.10 \times 10^{-6} \text{ atm}^{-1}$$

$$C_p^0 = 5.80 + 1.98 \times 10^{-3} T \text{ Cal/K mol}$$

i. Derive the relation between the entropy of the above system as a function of Temperature and Pressure; $S = S(T, P)$ [6 Marks]

ii. Calculate the entropy change. *Ignore the effect of pressure on volume* in your calculation. [6 Marks]

b. A steady flow adiabatic turbine accepts gas at condition $T_1 = 450\text{ K}$, $P_1 = 5\text{ bar}$ and delivers it at $T_2 = 376\text{ K}$, $P_2 = 2\text{ bar}$. Assuming ideal gas behavior, determine;

i. W_s

ii. W_{ideal}

iii. S_G

Take $T_O = 300\text{ K}$ and $\frac{C_P}{R} = 4$

[8 Marks]

THE FOLLOWING EQUATIONS AND CONSTANTS ARE PROVIDED

$dS = \frac{C_p}{T} dT - V\alpha dP$ $dH = C_p dT + V(1 - T\alpha)dP$ $dU = (C_p - PV\alpha)dT + V(P\beta - T\alpha)dP$ $dV = V\alpha dT - V\beta dP$ $dG = VdP - SdT$ $C_p - C_v = R; \text{ for ideal gases}$ $R = 8.314 \text{ J/mol K or } 0.08206 \text{ atm L/mol K}$ $1 \text{ atm} \cdot \text{L} = 101.3 \text{ J}$ $1 \text{ atm} \cdot \text{cm}^3 = 0.1013 \text{ J}$ $C_v = 1.5R$ $\alpha = \frac{1}{v} \left(\frac{dv}{dT} \right)_P$ $\beta = -\frac{1}{v} \left(\frac{dv}{dP} \right)_T$ $\Delta \left(H + \frac{1}{2}u^2 + gz \right) \dot{m} = \dot{Q} + \dot{W}_s$ $\Delta \left(H + \frac{1}{2}u^2 + gz \right) = T_o \Delta(S) + W_{\text{ideal}}$ $\Delta(Sm)_{fs} - \sum \frac{\dot{Q}_s}{T_{o,j}} = \dot{S}_G$ $\Delta S - \sum_j \frac{\dot{Q}_s}{T_{o,j}} = \dot{S}_G$ $W_{\text{lost}} = W_s - W_{\text{ideal}}$	$\frac{\Delta S}{R} = \int \frac{C_p}{R} \frac{dT}{T} - \int \frac{dP}{P}$ $dU = TdS - PdV$ <p>for $dZ = Mdx + Ndy$, the Maxwell Relation,</p> $\left(\frac{\partial M}{\partial y} \right)_x = \left(\frac{\partial N}{\partial x} \right)_y$ $ W = QH \cdot QC $ $\Delta S_M = \frac{- QH }{T_H} + \frac{ QC }{T_C}$ $\Delta S_M = Q_H \left(\frac{T_H - T_C}{T_H T_C} \right)$ $\eta = 1 - \frac{ QC }{QH}$ $\frac{ QH }{ QC } = \frac{T_H}{T_C}$ $\frac{U_1 A_1}{V_1} = \frac{U_2 A_2}{V_2}$ $dF = -(S + PV\alpha)dT + PV\beta dP$ $dG = VdP - SdT$ $1 \text{ atm} \cdot \text{cm}^3 = 0.024 \text{ cal}$ <p>For an ideal gas; $\alpha = \frac{1}{T}$ and $\beta = \frac{1}{P}$</p>
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Table A.2 Values of the Universal Gas Constant

$R = 8.314 \text{ J mol}^{-1} \text{ K}^{-1} = 8.314 \text{ m}^3 \text{ Pa mol}^{-1} \text{ K}^{-1}$
 $= 83.14 \text{ cm}^3 \text{ bar mol}^{-1} \text{ K}^{-1} = 8314 \text{ cm}^3 \text{ kPa mol}^{-1} \text{ K}^{-1}$
 $= 82.06 \text{ cm}^3 \text{ atm mol}^{-1} \text{ K}^{-1} = 82\,363.95 \text{ cm}^3 \text{ torr mol}^{-1} \text{ K}^{-1} = 0.082\,06 \text{ m}^3 \text{ atm kmol}^{-1} \text{ K}^{-1}$
 $= 1.9872 \text{ (cal) mol}^{-1} \text{ K}^{-1} = 1.986 \text{ (Btu) (lb mole)}^{-1} \text{ (R)}^{-1}$
 $= 0.7302 \text{ (ft)}^3 \text{ (atm) (lb mole)}^{-1} \text{ (R)}^{-1} = 10.73 \text{ (ft)}^3 \text{ (psia) (lb mole)}^{-1} \text{ (R)}^{-1}$
 $= 1545 \text{ (ft) (lb}_f \text{) (lb mole)}^{-1} \text{ (R)}^{-1}$
