UNIVERSITY OF GHANA

BSC ENGINEERING/FIRST SEMESTER EXAMINATIONS: 2017/2018

DEPARTMENT OF COMPUTER ENGINEERING

CPEN 205: DISCRETE MATHEMATICAL STRUCTURES (2 CREDITS)

INSTRUCTIONS:

ANSWER ALL QUESTIONS

EACH QUESTION CARRIES 25 MARKS

TIME ALLOWED: TWO (2) HOURS

Q1.

a)

i. Define a *recurrence relation.* [1 mark]

ii. What is the solution of the linear homogeneous recurrence relation?
$a_n = a_{n-1} + 2a_{n-2}$
with $a_0 = 2$ and $a_1 = 7$ [4 marks]

iii. What is the solution of the linear homogeneous recurrence relation?
$a_n = 4a_{n-1} - 4a_{n-2}$
with $a_0 = 1$ and $a_1 = 2$ [4 marks]

b) The Fibonacci numbers satisfy the linear homogeneous recurrence relation

$f_n = f_{n-1} + f_{n-2}$ with initial conditions $f_0 = 0$ and $f_1 = 1$.

i. Prove that the solution (explicit formula) to the Fibonacci recurrence is

$$f_n = \frac{1}{\sqrt{5}}\left(\frac{1+\sqrt{5}}{2}\right)^n - \frac{1}{\sqrt{5}}\left(\frac{1-\sqrt{5}}{2}\right)^n$$

[5 marks]

ii. Hence find the first three Fibonacci numbers (i.e., f_0, f_1, and f_2) using the explicit formula derived in (i) above. [3 marks]

c) A deposit of \$100,000 is made to an investment fund at the beginning of a year. On the last day of each year two dividends are awarded. The first dividend is 20% of the amount in the account during that year. The second dividend is 45% of the amount in the account in the previous year.

  i. Find a recurrence relation for $\{P_n\}$, where P_n is the amount in the account at the end of n years if no money is ever withdrawn. [3 marks]
  ii. How much is in the account after n years if no money has been withdrawn? [5 marks]

Q2.

a) Show by means of **truth tables** that each of these conditional statements is a **tautology**.

  i. $(p \wedge q) \rightarrow (p \rightarrow q)$ [4 marks]
  ii. $\neg (p \rightarrow q) \rightarrow p$ [4 marks]

b) Prove that

$$\binom{2n}{2} - 2\binom{n}{2} = n^2,$$ where n is a positive integer. [4 marks]

c) Draw Venn diagrams to discover whether or not the following are true?
  i. $A \cap (B \cup C) = (A \cap B) \cup (A \cap C)$ [4 marks]
  ii. $(A \cap B)' = A' \cup B'$ [3 marks]

d)
  i. Define a *relation* and hence a *function*. [2 marks]
  ii. Consider the functions

$f = \{(1,3), (2,5), (3,3), (4,1), (5,2)\}$

$g = \{(1,4), (2,1), (3,1), (4,2), (5,3)\}$

from $X = \{1, 2, 3, 4, 5\}$ into X.

Determine the ranges of f and g and also find the composite function fg. [4 marks]

Q3.

a) A laboratory cage contains eight white mice and six brown mice. Find the number of ways of choosing five mice from the cage if
  i. They can be of either colour, [1 mark]
  ii. At least one of each colour must be chosen, [4 marks]
  iii. It must have at most two white mice? [3 marks]

b) The English alphabet contains 21 consonants and five vowels. How many strings of six lowercase letters of the English alphabet contain
  i. exactly one vowel? [2 marks]
  ii. at most three vowels? [3 marks]

iii. at least three vowels? [3 marks]

c) The name of a file in a computer directory consists of two uppercase letters followed by a digit, where each letter is either A, or B, and each digit is either 1 or 2. List the name of these files in *lexicographic* order, where we order letters using the usual alphabetic order of letters.

[4 marks]

d) How many different strings can be made from the letters in *MISSISSIPPI*, using all the letters?

[2 marks]

e) Suppose that there are eight runners in a race. The winner receives a gold medal, the second place finisher receives a silver medal, and the third-place finisher receives a bronze medal. How many different ways are there to award these medals, if all possible outcomes of the race can occur and there are no ties? [3 marks]

Q4.

a) Write each of these statements in the form "**if p, then q**" in English. [**Hint: Refer to the list of common ways to express conditional statements.**]

i. It is necessary to wash the boss's car to get promoted. [1 mark]
ii. Winds from the south imply a spring thaw. [1 mark]
iii. A sufficient condition for the warranty to be good is that you bought the computer less than a year ago. [1 mark]
iv. Willy gets caught whenever he cheats. [1 mark]

b) Let p be "Bonsu is rich" and let q be "Bonsu is happy". Write each of the following in symbolic form using logical operators or connectives.

i. Bonsu is poor but happy; [1 mark]
ii. Bonsu is neither rich nor happy; [1 mark]
iii. Bonsu is either rich or unhappy; and [1 mark]
iv. Bonsu is poor or else he is both rich and unhappy. [2 marks]

c) Let $P(x, y)$ be the statement "Student x has taken class y," where the domain for x consists of all students in your class and for y consists of all computer engineering courses at your school. Express each of these quantifications in English.

i. $\exists x \exists y P(x, y)$ [1 mark]
ii. $\exists x \forall y P(x, y)$ [1 mark]
iii. $\forall x \exists y P(x, y)$ [1 mark]
iv. $\exists y \forall x P(x, y)$ [1 mark]
v. $\forall y \exists x P(x, y)$ [1 mark]
vi. $\forall x \forall y P(x, y)$ [1 mark]

d) Let $W(x, y)$ mean that student x has visited website y, where the domain for x consists of all students in your school and the domain for y consists of all websites. Express each of these statements by a simple English sentence.

i. W(Sarah Smith, www.att.com) [1 mark]
ii. $\exists x W(x,$ www.imdb.org$)$ [1 mark]
iii. $\exists y W($José Orez$, y)$ [1 mark]
iv. $\exists y(W($Ashok Puri$, y) \wedge W($CindyYoon$, y))$ [1 mark]

e) Let $T(x, y)$ be the statement "x *trusts* y," where the domain consists of all people in the world. Use quantifiers to express each of these statements.

i. Everybody trusts Bob. [1 mark]
ii. Bob trusts somebody. [1 mark]
iii. Alice trusts herself. [1 mark]
iv. Everyone trusts somebody. [1 mark]
v. Someone trusts everybody. [1 mark]
vi. Somebody is trusted by everybody. [1 mark]