UNIVERSITY OF GHANA

BACHELOR OF SCIENCE IN ENGINEERING
FIRST SEMESTER EXAMINATIONS: 2018/2019
DEPARTMENT OF COMPUTER ENGINEERING
CPEN 301: SIGNALS AND SYSTEMS (3 Credits)

INSTRUCTION: *Answer any four (4) Questions of your choice*

TIME ALLOWED: *THREE (3) HOURS*

1. **(a)** Explain the difference between a signal and a system. Give two application areas of each. [4 marks]

(b) Using the concept of signals and systems, sketch a simple block diagram of a feedback system and briefly explain how the system can be used to control a gas cooker in a house. [5 marks]

(c) Sketch and carefully label the continuous time signal y(t):

$y(t) = u(t+2) - u(t-2)$ for $-4 \leq t \leq 4$. [2 marks]

(d) Suppose the sound from the engine of a car is modelled by the function

$x(t) = \cos[\omega(t+\tau)+\theta]$ where τ is a time delay, and θ is a phase.

Find the frequency (in Hz) and period (in seconds) of the signal $x(t)$ when:

(i) $[\omega, \tau, \theta] = [\pi/3, 1/2, \pi/2]$ [2 marks]

(ii) $[\omega, \tau, \theta] = [3\pi/4, 1/2, \pi/4]$. [2 marks]

(e) Suppose the signal from the braking activities of a car is modelled as:

$x(t) = 3e^{-2t}u(t)$.

Sketch this signal and find the energy and power of the signal. [5 marks]

(f) The relationship between the input and output of a discrete time system is given by the expression:

$y[n] + 2y[n-1] = x[n] + 2x[n-2]$, where $x[n]$ is the input and $y[n]$ is the output of the system.

(i) Sketch the block diagram representation of the system. [2 marks]

(ii) Find the output when the input $x[n] = \{2, 3, 2\}$ for $-1 \leq n \leq 1$ and $x[n] = 0$ for all other values of n. Assume that $y[n] = 0$ for $n \leq -2$. [3 marks]

2. **(a)** Briefly explain the following properties of a system and give one example of each type: [6 marks]

(i) *causal system*

(ii) *memory system*

(iii) *linear system*.

(b) A voltage signal $x(t)$ from an earth quake is captured by a sensor and passed through an active highpass filter system for processing.

(i) Sketch the circuit diagram for the process. [2 marks]

(ii) Derive an expression for the transfer function of the system. [5 marks]

(iii) Find the values of the circuit components required to produce an output of 5V if an input of 20mV is applied and the cut-off frequency is 2.5kHz. [6 marks]

(iv) If an input voltage of 2V is applied to the filter system, write an expression for this input in terms of unit step function and find the output $y(t)$ of the system. To minimize the calculation, assume all the circuit components in the transfer function equation have unity values. [6 marks]

3. **(a)** Suppose the heart beat or signal $x(t)$ captured from an Apple wrist watch is modelled by the function:

$$x(t) = 5cos(50\pi t) + 2cos(200\pi t) + 3sin(350\pi t).$$

The signal is captured every 2-ms for processing of the data.

(i) Sketch a simple diagram to illustrate how the sampling operation is done on the signal $x(t)$ to produce the discrete output signal. [3 marks]

(ii) Find the sampling frequency (Hz) and number of samples that could be obtained for processing 1-minute duration of the heart beats. [3 marks]

(iii) If the ADC on the wrist watch has 12-bits resolution, find the storage space (kB) that would be required to store 1-minute heart beat recording. [3 marks]

(b) What sampling period should the wrist watch use for sampling the signal to ensure that the signal can be adequately recovered from the samples. [4 marks]

(c) If the sampling rate used for the sampling is 100Hz, find the discrete output signal that would be obtained from the process in the simplest form. What is the angular frequency of this discrete signal? [8 marks]

(d) Suppose the signal $x[n]$ in Q3(c) is passed through an ideal filter to obtain the recovered signal $y(t)$, where $y(t) = y[n] = x[tF_s]$. Find the signal $y(t)$. Is the signal $y(t)$ the same as $x(t)$? Explain your answer. [4 marks]

4. (a) The impulse response h[n] of a certain discrete LTI system is given as $h[n] = \{-1, 1, 2, -3, 4\}$. If a signal $x[n] = \{-1, -2, 5, 3, 4\}$ is applied to the system, find the output $y[n]$ of the system. [5 marks]

(b) A signal $x[n]$ is applied to a system that has an impulse response $h[n] = u[n]$, where $u[n]$ is a unit step. If the input $x[n] = \alpha^n u[n]$, where $-1 < \alpha < 1$, find the output $y[n]$ of the system. [5 marks]

(c) A pulse train $x(t) = 0.5\delta(t+3) + \delta(t) + 0.5\delta(t-3)$ is applied to a system to produce an output $y(t)$. Sketch the signal $x(t)$ and find its Fourier transform. Sketch the frequency response for $\omega = 0, \pi/3, 2\pi/3, \pi$. [7 marks]

(d) An LED on an *i*-phone blinks repeatedly at a constant rate of 5s. The duration for the ON is 2s while that of the OFF is 3s. The amplitude of the voltage signal is 5V.

(i) Write a mathematical expression for the LED blinking operation and sketch the signal for 15s operation. [3 marks]

(ii) Find the Fourier coefficients (spectral components) of the signal. [5 marks]

5. (a) Suppose the input $x(t)$ and output y(t) of an RC system is described by the linear differential equation defined below.

$$\frac{d^2y(t)}{dt^2} + 3\frac{dy(t)}{dt} + 2y(t) = 3\frac{dx(t)}{dt} + 2x(t).$$

(i) Find the transfer function and frequency response of the system. Plot the zero-pole locations of the system. [6 marks]

(ii) Find the output $y(t)$ of the system in Q5(a) if an impulse $x(t) = \delta(t)$ is applied to the system. [8 marks]

(b) Suppose the decay of the vibration of a car engine is modeled as a function

$x(t) = e^{-bt}u(t)$ for $t \geq 0$ and $x(t) = 0$ for $t < 0$ where $b > 0$ is a constant.

Sketch the signal $x(t)$ and find its Fourier transform. [5 marks]

(c) Suppose the frequency domain signal $X(\omega)$ obtained from a time domain signal $x(t)$ is given by the expression:

$X(\omega) = 1/(5 + j\omega)$.

(i) Find the real and imaginary components of $X(\omega)$. [3 marks]

(ii) Find the signal $x(t)$ that produced the signal $X(\omega)$. [3 marks]

Useful Formulae

[1] Laplace transform :$X(s) = \int_{-\infty}^{\infty} x(t)e^{-st}dt$

[2] Complex Fourier Series: $x[t] = \sum_{k=\infty}^{\infty} a_k e^{jk\omega_0 n}$ where $\omega_0 = 2\pi/N$ with N of samples

[3] Fourier coefficient:$a_k = \frac{1}{T}\int_{-T/2}^{T/2} x(t)\, e^{-jn\omega_0 t}dt$ with period T.

[4] Inverse Fourier Transform $x(t) = \frac{1}{2\pi}\int_{-\pi}^{\pi} X(j\omega)e^{j\omega n}\, d\omega$

[5] Signal Power $E = \frac{1}{T}\int_{-T/2}^{T/2} |x(t)|^2 dt$, where $T = 2\pi/\omega_0$

[6] Signal Energy $W = \int_{-\infty}^{\infty} |X(t)|^2 dt = \frac{1}{2\pi}\int_{-\infty}^{\infty} |X(j\omega)|^2 d\omega$

[7] Discrete function for $\frac{d^2y(t)}{dt^2}\Big|_{t=nT} = \frac{y[nT+2T]-2y[nT+T]+y[nT]}{T^2}$

[8] Sum of series $S(p) = \sum_{p=0}^{N-1} r^p = \frac{1-r^N}{1-r}$, $|r| \leq 1$