UNIVERSITY OF GHANA

BACHELOR OF SCIENCE IN ENGINEERING

FIRST SEMESTER EXAMINATIONS: 2015/2016

DEPARTMENT OF COMPUTER ENGINEERING

FAEN 301: NUMERICAL METHODS (3 CREDITS)

INSTRUCTION: *ANSWER ALL QUESTIONS IN SECTION A and TWO QUESTIONS FROM SECTION B.*

TIME ALLOWED: *THREE (3) HOURS*

Section A (50 marks)
Answer all questions

1. Distinguish between the following;
   (a) Well-conditioned system and Ill-conditioned system. [3 marks]
   (b) Consistent system and Inconsistent system. [3 marks]
   (c) Bisection and Newton Raphson method of solving non-linear equations. [4 marks]

2. Define absolute relative approximate error and explain why it is preferred over an approximate error. [3 marks]

3. The strain in an axial member of a square cross-section is given by,

$$\varphi = \frac{F}{h^2 E}$$

where,
F = axial force in the member, N,
h = length of the cross-section, m
E = Young's modules, Pa.

Given, $F = 90 \pm 0.5\ N$, $h = 6 \pm 0.2\ mm$ and $E = 80 \pm 2.0\ GPa$,
Find the maximum possible error in the measured strain. [5 marks]

4. The Taylor polynomial of order n of a function $f(x)$ with $(n+1)$ continuous derivatives in the domain $[x, x+h]$ is given by;

$$f(x+h) = f(x) + f'(x)h + f''(x)\frac{h^2}{2!} + \cdots + f^n(x)\frac{h^n}{n!} + R_n(x).$$

The remainder is given by,

$$R_n(x) = \frac{(x-h)^{n+1}}{(n+1)!} f^{(n+1)}(c),$$

where, $x < c < x + h$.

(a) What is the truncation (true) error in the representation of e^1, if only five terms of the series at a point $x = 0$ are used. **[3 marks]**

(b) Use the remainder theorem to find the bound of the truncation error. **[3 marks]**

(c) How many terms would it require to get an approximation of e^1 within a magnitude of true error less than 10^{-6}. **[3 marks]**

5. Find the roots of the cubic equation,

$$x^3 - 0.03x^2 + 2.4 \times 10^{-6} = 0.$$

[7 marks]

6. A machine stores floating-point numbers in a hypothetical 10-bit binary word. It employs the first bit for the sign of the number, the second one for the sign of the exponent, the next four for the exponent, and the last four for the magnitude of the mantissa.

(a) Find how 0.02832 will be represented in the floating-point 10-bit word. **[4 marks]**

(b) Confirm that the magnitude of the relative true error that results from approximate representation of 0.02832 in the 10-bit format is less than the machine epsilon. **[3 marks]**

7. The upward velocity of a rocket is given as a function of time in the table below.

t(s)	0	10	15	20	22.5	30
$v(t), m/s$	0	227.04	362.78	517.35	602.97	901.67

Determine the value of the velocity at $t = 16s$ from second order polynomial interpolation using Lagrangian polynomial interpolant.

[4 marks]

8. Find the largest eigen value of

$$\mathbf{A} = \begin{bmatrix} 25 & 1 & 2 \\ 1 & 3 & 0 \\ 2 & 0 & -4 \end{bmatrix}$$

by power method and its corresponding eigen vector. Start with an initial eigen vector $\begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix}$. [5 marks]

Section B (50 marks)
Answer any <u>two</u> questions from this section.
All questions carry equal marks.

1. (a) Fisheries Science reported on a study of the variables that affects the endogenous nitrogen excretion (ENE) in carp raised in Ghana. Carp were divided into groups of 10 fish each according to the body weights. That is in each group, the body weight was approximately the same for ten fish. Each group was placed in a separate tank. The Carp were then fed a protein-free diet three times daily for a period of 20 days. On one day, the amount of ENE in each tank was measured after terminating the feeding experiment. The table below gives the mean body weight(in grams) and ENE amount (in milligrams of body weight per day) for each carp group.

Tank	Body weight, x	ENE, y
1	11.7	15.3
2	25.3	9.3
3	90.2	6.5
4	213.0	6.0
5	10.2	15.7
6	17.6	10.0
7	32.6	8.6
8	81.3	6.4
9	141.5	5.6
10	285.7	6.0

i. Form a system of linear equations from least squares estimation of the quadratic regression coefficients, $\beta_0, \beta_1, \beta_2$. [11 marks]

ii. Is the system consistent? [3 marks]

(b) Use Gaussian elimination method with partial pivoting to find the least squares estimate of the regression coefficients $\beta_0, \beta_1, \beta_2$. [6 marks]

(c) Calculate the coefficient of determination, r^2, for the linear and quadratic models. Which of these model seems adequate? Explain. [5 marks]

2. (a) Given the system of equations

$$3x_1 + 7x_2 + 13x_3 = 76,$$
$$x_1 + 5x_2 + 3x_3 = 28,$$
$$12x_1 + 3x_2 - 5x_3 = 1.$$

i. Is the system diagonally dominant? Justify. **[2 marks]**

ii. Is the system well-conditioned or ill-conditioned? Justify. **[3 marks]**

iii. Use LU decomposition to find the solution of the system. **[6 marks]**

iv. How many significant digits can be trusted in the solution of the system of equations? Assume single precision with 24 bits is used in the mantissa for real numbers. **[3 marks]**

(b) Given a non-singular square matrix A, and a solution vector X, show that,

$$\frac{||\Delta X||}{||X||} \leq ||A||||A^{-1}|| \frac{||\Delta A||}{||A||}.$$

[4 marks]

(c) The function $f(x)$ is defined as

$$f(x) = \frac{300x}{1 + e^x}$$

from $x = 0$ to $x = 10$.

i. Use Romberg's rule to find the area under the curve. Use the 1, 2, 4, and 8-segment trapezoidal rule and show these increasingly correct values in a tree graph. **[4 marks]**

ii. Find the true error, E_t from trapezoidal rule. **[3 marks]**

3. (a) State three uses and three abuses of regression analysis. [3 marks]

(b) Many patients get concerned when a test involves injection of a radioactive material. For example, when scanning a gallbladder, a few drops of Technetium-99m isotope is used. It however, takes about 24 hours for the radiation levels to reach what we are exposed to in day-to-day activities. The table below shows the relative intensity of radiation as a function of time.

Relative intensity of radiation as a function of time

t(hrs)	0	1	3	5	7	9
γ	1.000	0.891	0.708	0.562	0.447	0.355

If the level of the relative intensity of radiation is related to time via an exponential formula $\gamma = Ae^{\lambda t}$. Find

i. the value of the regression constants A and λ. [7 marks]

ii. the half-life of Technetium-99m. [3 marks]

iii. the relative radiation intensity after 24 hours. [3 marks]

(c) The velocity of a rocket is given by,

$$v(t) = 2000\ln\left[\frac{14 \times 10^4}{14 \times 10^4 - 2100t}\right] - 9.8t$$

i. Deduce the central difference approximation of the first derivative from forward and backward difference approximation.

[6 marks]

ii. Use the central difference approximation of the first derivative of $v(t)$ to calculate the acceleration at $t = 16s$ using a step size of $\Delta t = 2s$.

[3 marks]