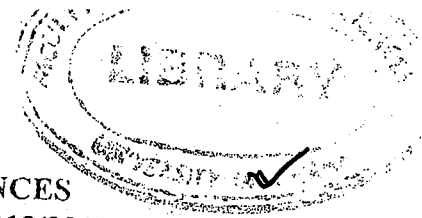




UNIVERSITY OF GHANA
FACULTY OF ENGINEERING SCIENCES
FIRST SEMESTER EXAMINATIONS, 2012/2013



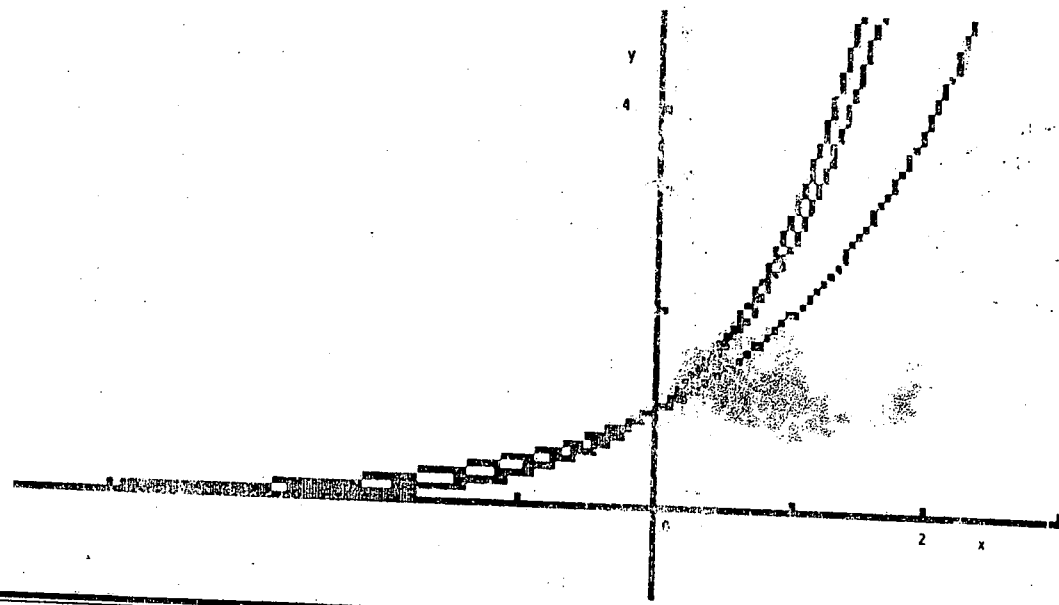
LEVEL 100: BACHELOR OF SCIENCE IN ENGINEERING
FAEN 101: ALGEBRA
DURATION: THREE (3) HOURS

Instructions:

1. Answer all questions from Section A and any two questions from Section B
2. Calculators NOT allowed
3. Graph paper provided

SECTION A

1. Study the following function $f(x) = \frac{x^2}{(x-1)(x-3)}$ indicating clearly the domain, range, y and x-intercepts, the asymptotes. Sketch the graph of the function.
2. A graphing utility was used to graph the functions $f(x) = 3^x$, $g(x) = 2^x$ and $h(x) = e^x$ in the figure below. Where do the graphs intersect? Which graph lies between the others? Which graph is above the others when $x > 0$? When $x < 0$? Discuss the behaviour of the three functions as $x \rightarrow \infty$ and as $x \rightarrow -\infty$.



3. Solve for x without using a calculator or table

a. $\log_2(x+3) = 2\log_2 3$

b. $\log_3 x + \log_3(x+2) = \frac{1}{2}\log_3 9$

c. If $\ln 3 = 1.10$, $\ln 5 = 1.609$, $\ln 7 = 1.95$ and $\ln 2 = 0.693$ find the following

i. $\ln \sqrt[4]{21}$

ii. $\ln \frac{5^5}{8}$

iii. $\ln \sqrt[4]{\frac{8^2}{5}}$

4. Find the quotient $\frac{z_1}{z_2}$ of the complex numbers

$z_1 = 24(\cos 330^\circ + i\sin 330^\circ)$ $z_2 = 8(\cos 105^\circ + i\sin 105^\circ)$

(a) Using the trigonometric form of a complex number

(b) Using the standard form $a+ib$ of a complex number

5. Evaluate without using a calculator or tables the following. If it is impossible to give an exact answer, leave surds in the answers where appropriate

a. $1 - 2\sin^2 67\frac{1}{2}^\circ$

b. $\frac{1 - \tan^2 75^\circ}{\tan 75^\circ}$

c. $\sec \frac{\pi}{8} \operatorname{cosec} \frac{\pi}{8}$

d. $2\cos^2 105^\circ - 1$

SECTION B

6. Find the eigenvalues and eigenvectors of the following

a. $A = \begin{pmatrix} 1 & -1 \\ 3 & 2 \end{pmatrix}$

b. $A = \begin{pmatrix} 2 & 1 & 1 \\ 1 & 3 & 2 \\ -1 & 1 & 2 \end{pmatrix}$

7. (a). Find all the roots of $x^8 - 256 = 0$ for $x \in \mathbb{C}$.

(b). Find all the solutions of $x^4 + 2x^3 + 3x^2 + 2x - 2 = 0$ for $x \in \mathbb{C}$.

8. Given the vectors $a = (2, 1, 1)$ $b = (2, -1, 1)$ and $c = (0, 1, 1)$ evaluate

a) $a \times b$

b) $(a \times b) \times c$

c) $(a \cdot c)b - (b \cdot c)a$