UNIVERSITY OF GHANA

BSC. ENGINEERING
SECOND SEMESTER EXAMINATIONS: 2015/2016

DEPARTMENT OF BIOMEDICAL ENGINEERING
BMEN 404: BIOMEDICAL ENGINEERING SYSTEMS (3 CREDITS)

INSTRUCTIONS:
ATTEMPT ALL QUESTIONS.
ALL QUESTIONS SHOULD BE ANSWERED IN THE BOOKLET PROVIDED.
EACH MAJOR QUESTION SHOULD START ON A NEW PAGE.
CALCULATIONS SHOULD BE DETAILED AND SYSTEMATIC. MARKS ARE ALLOCATED TO STEPS.
RELEVANT FORMULAE ARE PROVIDED AT THE END OF THE QUESTION SETS.

TIME ALLOWED: THREE (3) HOURS

1. Control systems play an important role both in nature and in the engineered world. Due to this, we tend to derive representations of these systems for different purposes, each with specific knowledge requirements.

   a. Briefly explain any four (4) differences between engineering control systems and physiological control systems.

   (12 marks)

   b. What are the knowledge requirements for:
      i. A model to be used for prediction?

      (4 marks)

      ii. A model to be used for control?

      (4 marks)

   c. What type of problem are you trying to solve when you use a model for:
      i. Prediction?

      (2 marks)

      ii. Control?

      (2 marks)

2. The knee jerk is a routine method for assessing the state of the nervous system. A sharp tap to the patellar tendon in the knee leads to an abrupt stretching of the extensor muscle in the thigh to which the tendon is attached. This activates the muscle spindles, which are stretch receptors. Neural impulses, which encode information about the magnitude of the stretch, are sent along afferent nerve fibres to the spinal cord. Since each afferent nerve is synaptically connected with one motor neuron in the spinal cord, the motor

neurons get activated and, in turn, send efferent neural impulses back to the same thigh muscle. These produce a contraction of the muscle, which acts to straighten the lower leg [Khoo (1999)].

a. With the above description of the muscle stretch reflex
   i. Identify the controller, plant, feedback sensor and disturbance.

   (4 marks)

   ii. Draw the block diagram representation indicating the controlling, controlled and feedback variables.

   (8 marks)

b. Assume a model of the above system has an input $x(t)$ and output $y(t)$ with a disturbance (δx) producing a proportional change in the output (δy). The controller, the plant and feedback sensor have a gain of G_c, G_p and G_s respectively. Derive an expression for the following:
   i. The open loop gain (OLG)

   (3 marks)

   ii. The closed loop gain (CLG)

   (4 marks)

   iii. The loop gain (LG)

   (3 marks)

c. Differentiate between a feedback control system and a feedforward control system.

   (4 marks)

3. A linear model of lung mechanics is given by the mathematical equation expressed below which relates the input: pressure at the airway entry ($p(t)$) to the output: airflow into the lungs ($q(t)$). This can be transformed into a state-space model.

$$\frac{d^2q(t)}{dt^2} + 620\frac{dq(t)}{dt} + 4000q(t) = 420p(t)$$

a. Determine if the system is:
   i. Linear or non-linear.

   (3 marks)

   ii. Causal or non-causal.

   (2 marks)

b. Derive the state-space model by:
   i. Determining the state-space equations.

   (6 marks)

   ii. Determining the output equation.

   (4 marks)

c. Draw the block diagram representation of the state-space model.

   (10 marks)

4. Figure 1 shows the electrical analogue of the 3-element Windkessel model that has been used to approximate the haemodynamic properties of the arterial tree. R_P represents the peripheral resistance, C_P is the peripheral compliance and R_A is the resistance of the aortic wall.

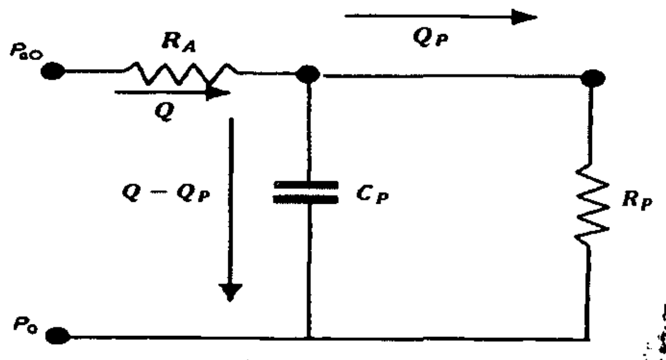

Figure 1: 3-element Windkessel model of cardiovascular function

a. Derive a mathematical expression relating the aortic pressure (P_{ao}) to aortic flow (Q). Assume venous pressure (P_0) is 0.

(10 marks)

b. Derive the transfer function of the model. Assume initial conditions to be 0.

(3 marks)

c. The second order differential equation below represents a model of some physiological control system,

$$\frac{d^2y}{dt^2} + \frac{\lambda}{LC}y(t) = \frac{1}{LC}x(t)$$

i. Determine the transient response under an inpulse test signal (in the complex s domain)

(7 marks)

ii. Is the system undamped, underdamped, critically damped or overdamped. Justify your answer.

(5 marks)

Relevant information (all letters have their usual contextual meanings)

System	Expression
Fluid storage	$\Delta P = \frac{1}{C}\int Q\,dt$
Fluid inertance	$\Delta P = L\frac{dQ}{dt}$
Fluid resistance	$\Delta P = RQ$

$f(t)$	$F(s)$
$\delta(t)$	1
$u(t)$	$\frac{1}{s}$
$tu(t)$	$\frac{1}{s^2}$
$t^n u(t)$	$\frac{n!}{s^n+1}$
$e^{-at}u(t)$	$\frac{1}{s+a}$
$\sin \omega t u(t)$	$\frac{\omega}{s^2+\omega^2}$
$\cos \omega t u(t)$	$\frac{s}{s^2+\omega^2}$

Theorem	
$\mathscr{L}[f(t)] = F(s)$	$= \int_{0-}^{\infty} f(t)e^{-st}dt$
$\mathscr{L}[kf(t)]$	$= kF(s)$
$\mathscr{L}[f_1(t) + f_2(t)]$	$= F_1(s) + F_2(s)$
$\mathscr{L}[e^{-at}f(t)]$	$= F(s+a)$
$\mathscr{L}[f(t-T)]$	$= e^{-sT}F(s)$
$\mathscr{L}[f(at)]$	$= \frac{1}{a}F\left(\frac{s}{a}\right)$
$\mathscr{L}\left[\frac{df}{dt}\right]$	$= sF(s) - f(0-)$
$\mathscr{L}\left[\frac{d^2f}{dt^2}\right]$	$= s^2F(s) - sf(0-) - f'(0-)$
$\mathscr{L}\left[\frac{d^nf}{dt^n}\right]$	$= s^nF(s) - \sum_{k=1}^{n} s^{n-k}f^{k-1}(0-)$
$\mathscr{L}\left[\int_{0-}^{t} f(\tau)d\tau\right]$	$= \frac{F(s)}{s}$
$f(\infty)$	$= \lim_{s\to 0} sF(s)$
$f(0+)$	$= \lim_{s\to\infty} sF(s)$

Reference

Khoo, M.C.K. Physiological Control Systems: Analysis, Simulation, and Estimation. Wiley; 1999.

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