UNIVERSITY OF GHANA

BACHELOR OF SCIENCE IN ENGINEERING
FIRST SEMESTER EXAMINATIONS: 2017/2018

DEPARTMENT OF COMPUTER ENGINEERING
CPEN 301: SIGNALS AND SYSTEMS (3 Credits)

INSTRUCTION: *Answer any five (5) Questions of your choice*

TIME ALLOWED: *THREE (3) HOURS*

1. **(a)** Explain the difference between a continuous time signal and a discrete time signal and give an expression that describes each signal type. List two areas of application of signals and two application areas of systems. [6 marks]

(b) Using the concept of signals and systems and simple block diagrams of a hybrid system with feedback, briefly describe how a water pump at the School of Engineering Sciences building can be controlled. [5 marks]

(c) The Ghana stock market wants to see whether the average stock level of a certain commodity is increasing or decreasing. As systems engineer, you have been consulted to develop a model for the computation. Based on your analysis of the information, you proposed the following:

average_stock[*today*] = *0.2*average_stock*[*yesterday*] + *0.4**(*stock*[*today*] + *0.5*stock*[*yesterday*]) + *0.2*stock*[*2 days ago*] – *0.8*stock*[*3 days ago*].

Write a difference equation model for the stock level estimation and sketch the block diagram for its implementation. [4 marks]

(d) A continuous time signal $x(t)$ from a pressure sensor at a plant is defined as $x(t) = e^{-t}u(t)$. Find the energy and power of this signal. [5 marks]

2. **(a)** What is the difference between a periodic signal and a causal signal? Give an expression that describes each signal type. [4 marks]

(b) A voltage signal $x(t)$ monitored from a pacemaker is defined by the function below. Sketch the signal and state whether it is periodic or aperiodic, even or odd, and causal or non-causal. [6 marks]

$$x[(t) = \begin{cases} 3e^{-2t}, & t \geq 0 \\ 0, & otherwise \end{cases}$$

(c) The impulse response h[n] of a certain LTI system is given as $h[n] = \{-1, 1, 2, 3\}$. If a signal $x[n] = \{-1, -2, 2, 3\}$ is applied to the system, find the output $y[n]$ of the system. [5 marks]

(d) A signal $x[n]$ is applied to a system that has an impulse response $h[n] = u[n]$, where $u[n]$ is a unit step. If the input $x[n] = \alpha^n u[n]$, where $-1 < \alpha < 1$, find the output $y[n]$ of the system. [5 marks]

3. **(a)** A 30 minutes interview with level 300 Computer Engineering student at the studios of Radio Univers was amplitude modulated and transmitted to receivers on campus. The transmitted signal $x(t)$ is described by the expression:

$x(t) = Bcos(16\pi t)$ where $B = 3cos(10\pi t)$.

(i) Write an expression for $x(t)$ as a sum of sinusoids and find the frequency components of the signal $x(t)$. [5 marks]

(ii) Find the rate at which the signal should be sampled in order for it to be adequately recovered at the receiver. [2 marks]

(iii) If the signal $x(t)$ is sampled at 10 Hz, find the discrete signal $x[n]$ that was transmitted in a reduced form and the angular frequency. [5 marks]

(iv) Find the number of samples obtained for the 30 minutes duration and the time that will be required to compute the DFT. [3 marks]

(v) Find the amount of storage space that will be required to record the 30 minutes interview. Assume a 16-bit ADC system is used. [2 marks]

(vi) At the receiver, the discrete signal $x[n]$ is passed through an ideal filter system to obtain the recovered signal y(t), where $y(t) = y[n] = x[tF_s]$. Find the signal $y(t)$. Is the signal $y(t)$ the same as $x(t)$? Explain your answer. [3 marks]

4. **(a)** A signal $x(t)$ monitored from the engine of a car is passed through a system that comprises a cascade of two passive RC lowpass filters to produce an output signal $y(t)$. Sketch the circuit diagram and derive the differential equation that describes the system. [7 marks]

(b) State whether the RC system in 4(a) above is: [5 marks]

(i) linear or non-linear system;
(ii) causal or non-causal system;
(iii) time variant or time-invariant system;
(iv) memory or memoryless system;
(v) invertible or non-invertible system.

(c) Suppose the input $x(t)$ and output y(t) of the RC system described in 4(a) is characterized by the linear differential equation defined below. Find the output $y(t)$ of the system if an impulse $x(t) = \delta(t)$ is applied to the system. [8 marks]

$$\frac{d^2y(t)}{dt^2} + 3\frac{dy(t)}{dt} + 2y(t) = 3\frac{dx(t)}{dt} + 2x(t).$$

5. **(a)** You have been consulted by a company to design an active lowpass filter system for a pacemaker device. Sketch the circuit diagram of filter and find the frequency response of the system. Find the real part and imaginary parts of the system. If the monitored voltage signal from the heart $x(t) = e^{-t}u(t)$ is passed through the filter system, find the output $y(t)$ of the system. [10 marks]

(b) The ON and OFF blinking operation of an LED light on a mobile phone is controlled by a switching function where the LED is switched ON for 1 second at a time repeatedly and it is put OFF for 3 seconds in between the ON times. Write an expression for the blinking operation and sketch the signal. Find the Fourier coefficient (frequency spectra) of the signal. [5 marks]

(c) A voltage signal with unity amplitude and width from a temperature sensor has the following characteristics: at time $t = -1$, amplitude = -1V, at $t = -0.5$, amplitude = 1V, at $t = 0.5$, amplitude = 1V, and at $t = 1$, amplitude = -1. Sketch the waveform and find the Fourier coefficients of the signal. [5 marks]

6. **(a)** The relationship between the input $x[n]$ and output $y[n]$ of an LTI system at a manufacturing plant is defined by the linear difference equation below. Find the frequency response of the system and the impulse response. [10 marks]

$$y[n] + 0.4y[n-1] - 0.25y[n-2] = x[n] - x[n-1]$$

(b) A discrete time signal $x[n]$ from a device is given as: $x[n] = 6\cos^2(\pi n/4)$. Find the 4-point DFT of the sequence. [5 marks]

(c) The DTFT of a lowpass filter system is given by the expression below. Find the inverse DTFT of the system to obtain the impulse response $h[n]$. [5 marks]

$$H(e^{j\omega}) = \begin{cases} 1, & 0 \le |\omega| \le \omega_C \\ 0, & \omega_C < |\omega| < \pi \end{cases}$$

Useful Formulae

[1] Laplace transform : $X(s) = \int_{-\infty}^{\infty} x(t) e^{-st} dt$

[2] Complex Fourier Series: $x[t] = \sum_{k=\infty}^{\infty} a_k e^{jk\omega_0 n}$ where $\omega_0 = 2\pi/N$ with N of samples

[3] Fourier coefficient: $a_k = \frac{1}{T} \int_{-T/2}^{T/2} x(t) e^{-jn\omega_0 t} dt$ with period T.

[4] DFT $X(w) = \sum_{n=-\infty}^{\infty} x[n] e^{-j\phi n}$

[5] IDFT $x[n] = \frac{1}{2\pi} \int_{-\pi}^{\pi} X(e^{-j\omega}) e^{j\omega\omega}$

[6] DTFT $X(e^{j\omega}) = \sum_{n=-\infty}^{\infty} x[n] e^{-j\omega\omega}$

[7] IDTFT $x[n] = \frac{1}{2\pi} \int_{-\pi}^{\pi} X(e^{-j\omega}) e^{j\omega\omega}$

[8] Signal Power $E = \frac{1}{T} \int_{-T/2}^{T/2} |x(t)|^2 dt$, where $T = 2\pi/\omega_0$

[9] Signal Energy $W = \int_{-\infty}^{\infty} |X(t)|^2 dt = \frac{1}{2\pi} \int_{-\infty}^{\infty} |X(j\omega)|^2 d\omega$

[10] Discrete function for $\frac{d^2 y(t)}{dt^2}\Big|_{t=nT} = \frac{y[nT + 2T] - 2y[nT + T] + y[nT]}{T^2}$

[11] Sum of series $\sum_{p=0}^{N-1} r^p = \frac{1 - r^N}{1 - r}$, $|r| \le 1$