**UNIVERSITY OF GHANA**

**SECOND SEMESTER EXAMINATION 2013/2014**

**LEVEL 300: BACHELOR OF SCIENCE IN ENGINEERING**

**CPEN 304: DIGITAL SIGNAL PROCESSING (3 Credits)**

**TIME: THREE (3) HOURS**

**INSTRUCTIONS**

*Answer Question 1 and any other three (3) questions of your choice.*

**Q1.** **(a)** Assume you have been tasked to design a digital system for measuring the heart rate signals of a patient. Briefly describe the various stages you will go through from the signal capture through to the output. Support your answer with a block diagram. [6 marks]

**(b)** Give four (4) reasons why digital signal processing is preferred over analog processing and state two (2) limitations of digital signal processing. [6 marks]

**(c)** If the resolution of your device in Q1 (a) above is 0.25Hz and you need to capture is at least 90 harmonics, find the sampling rate that will be required and the number of points needed if the heart rate to be measured is between 60 and 200 beats per minute. [4 marks]

**(d)** To process the signals in Q1 (a), assume an ideal ADC is used to convert the captured signal $x(t)$ to the discrete time signal y[n] that is converted to the continuous time signal y(t) by an ideal reconstruction, i.e., $y(t) = y[n] = y[t/T_s] = y(tF_s)$. If the captured continuous time input signal $x(t)$ is given as below, determine the following:

$x(t) = 3cos(60\pi t) + 3sin(300\pi t) + 2cos(340\pi t) + 4cos(500\pi t) + 10sin(660\pi t)$,

(i) sampling rate at which the signal $x(t)$ should be sampled in order to avoid the problem of aliasing. Show all your calculations. [3 marks]

(ii) discrete time signal $y[n]$ in a reduced form and the angular frequency if the signal is sampled at 200Hz. [7 marks]

(iii) reconstructed continuous time signal $y(t)$. [2 marks]

**(e)** If the impulse response of the filter system of your device in Q1 (a) above is given by, $h[n] = \delta[n] - \delta[n-1]$, find the output of the filter system {y(n) = h(n)*x(n)} if the input is $x[n] = 0.5\delta[n] + 0.5\delta[n-1] + 0.5[n-2]$. [4 marks]

**(f)** Suppose you need to design an active highpass first order RC filter for your

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device in Q1 (a). Give a sketch of the circuit diagram and derive an expression for the transfer function of the filter system. If the cutoff frequency is 250Hz, and C is 30nF, design the filter circuit for a gain of 200. [8 marks]

**Q2.** (**a**) Show that if a causal discrete time sequence $x[n]$ is shifted by a variable k, then the z-transform of the shifted sequence will be given as $z^{-1}X(z)$. [3 marks]

(**ii**) If the sequence, $x$[n] in Q2 (a) is defined by $x[n] = \lambda^n u[n]$, where $u[n]$ is the unit step function, find the $z$-transform of the sequence $x$[n]. [4 marks]

(**b**) Describe the sequential steps you will follow to design an IIR digital filter and an FIR digital filter. Give two (2) reasons why the FIR digital filter is usually preferred to the IIR digital filter. [7 marks]

(**c**) Design a simple first order IIR highpass digital filter to achieve a 3-dB cutoff frequency $\omega_c$ of $0.4\pi$. [Hint: find H(z) of the filter]. [6 marks]

**Q3.** The input $x$[n] and output $y$[n] of an LTI filter system are related by the linear difference equation:

$$2y[n] + 3y[n-1] + y[n-2] = 3x[n-1] + 2x[n].$$

(**a**) Sketch the filter structure. [1 mark]

(**b**) Find the transfer function, $H$(z) and draw the zero-pole map. [7 marks]

(**c**) Explain whether the system is recursive or non-recursive and whether such a system will be stable or unstable. [3 marks]

(**d**) Find the impulse response, $h$[n], of the filter system using the partial fraction expansion method. [9 marks]

**Q4.** (**a**) An ideal lowpass filter with frequency response $H(e^{j\omega})$ is defined by :

$$H_{LP}(e^{j\omega}) = \begin{cases} 1, & 0 \leq |\omega| \leq \omega_c \\ 0, & \omega_c \leq |\omega| \leq \pi \end{cases}.$$

Sketch the frequency response and find the inverse DTFT, $h$[n]. [4 marks]

(**b**) Suppose you have been tasked to design a linear phase lowpass FIR digital filter based on the windowing method to meet the following specifications: passband edge frequency of 1.8 kHz, stopband edge frequency of 2 kHz, sampling rate of 12 kHz, peak passband and stopband ripple values of 0.012 and 0.018 respectively. Sketch the magnitude response of the filter and determine from the given specifications the window type to use, filter length, and the filter h[n]. Find the filter coefficient for h[0] and h[1]. Use the properties of the fixed window types in *Table 1* below for your design. Is your estimated filter order from the fixed window approach the same as that of the Kaiser approximation? [16 marks]

**Q5.** (**a**) The simplest first order FIR digital filter is the moving average with length M = 2. Write a mathematical expression for this moving average filter and find the transfer function for the first order highpass FIR filter. Sketch the zero-pole map of this filter. [6 marks]

(**b**) Suppose you have been tasked to design a lowpass IIR digital filter with flat magnitude characteristics to remove interference noise in a line for control system. The filter is required to meet the following specifications: passband edge frequency of 800 Hz, stopband edge frequency of 1000 Hz, sampling rate of 4 kHz, peak passband ripple of 0.5dB, and minimum stopband attenuation of 40dB. Show all the calculation steps required to obtain the IIR digital filter G(z). Use the bilinear transformation method with coefficient of unity for your filter design. To determine your filter transfer function *G*(z), assume that your analog filter transfer function $H_a(s)$ from the specification is given by the following expression: [14 marks]

$$H_a(s) = \frac{(s+2)}{(s+3)(2s+5)}$$

*Table 1*

| Type of Window | Mainlobe width | Mainlobe/ sidelobe | Minimum stopband attenuation |
|---|---|---|---|
| Rectangular | $4\pi/M$ | -13dB | -20.9 dB |
| Hanning | $8\pi/M$ | -32dB | -43.9 dB |
| Hamming | $8\pi/M$ | -43dB | -54.5 dB |
| Blackman | $12\pi/M$ | -58dB | -75.3 dB |

 

## Useful Formulae

1. $(1-\delta_P) = \dfrac{1}{\sqrt{1+\varepsilon^2}}$

2. $H(z) = \dfrac{(1-\alpha)}{2}\dfrac{[1+z^{-1}]}{[1-\alpha z^{-1}]}$

3. $\cos\omega_c = \dfrac{2\alpha}{1+\alpha^2}$

4. $\omega_a = \tan(\omega_D/2)$

5. $N \geq \dfrac{1}{2}\dfrac{\log_{10}\left(\dfrac{A^2-1}{\varepsilon^2}\right)}{\log_{10}\left(\dfrac{\omega_S}{\omega_P}\right)}$

6. $N = \dfrac{-20\log_{10}(\delta_P\delta_S)^{1/2}-13}{14.6(\omega_S-\omega_P)/2\pi}$

7. $|H(\omega)|^2 = \dfrac{1}{[1+(\omega_P/\omega_C)^{2N}]} = \dfrac{1}{1+\varepsilon^2} = \dfrac{1}{A^2}$

8. $H(s) = \dfrac{\omega_C^N}{\prod_{k=0}^{N-1}(s-s_k)}$

9.

$$Hanning: w[n] = 0.5 + 0.5\cos\left(2\pi\frac{n}{2M+1}\right), \quad -M \leq n \leq M$$

$$Hamming: w[n] = 0.54 + 0.46\cos\left(2\pi\frac{n}{2M+1}\right), \quad -M \leq n \leq M$$

$$Blackman: w[n] = 0.42 + 0.5\cos\left(2\pi\frac{n}{2M+1}\right) + 0.08\cos\left(4\pi\frac{n}{2M+1}\right) \quad -M \leq n \leq M$$