UNIVERSITY OF GHANA

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BSc/BA, FIRST SEMESTER EXAMINATIONS: 2017/2018

SCHOOL OF ENGINEERING

DEPARTMENT OF BIOMEDICAL ENGINEERING

FAEN 201: CALCULUS II (4 credits)

INSTRUCTION:

ANSWER ANY 4 OUT OF THE FOLLOWING 6 QUESTIONS

TIME ALLOWED:

TWO HOURS AND THIRTY MINUTES $\left(2\frac{1}{2}\text{ hours}\right)$

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1. (a) Let $\sum_{n=1}^{\infty} a_n$ be an infinite series. When do we say that the series converge? Write down an expression for the sum of the series. [10 Marks]

   (b) Consider the infinite series: $\sum_{n=1}^{\infty} \frac{1}{(n+1)(n+2)}$

   (i) write down S_n, the n^{th} partial sum of the seires

   (ii) Find the sum of the series.

   [25 Marks]

   (c) Evaluate $\sum_{n=1}^{\infty} \frac{3^n + 2^{n+1}}{5^n}$.

   [15 Marks]

2. (a) Let $f(x, y) = x^2 \sin(x + y)$, find $\frac{\partial f}{\partial x}$ and $\frac{\partial f}{dy}$. [15 Marks]

   (b) Given that $f(x, y) = x^3 + y^3 - 6xy$ where $x = \cos t$ and $y = \sin t$, find

   (i) the value of $\frac{df}{dt}$ when $t = \frac{\pi}{2}$.

   (ii) find the critical point(s) of $f(x, y)$

   (iii) determine the nature of the critical point(s) of $f(x, y)$.

   [35 Marks]

3. (a) Let $\Gamma(x) = \int_0^{\infty} t^{x-1}e^{-t}dt$ and $B(x, y) = \int_0^1 t^{x-1}(1-t)^{y-1}dt$ be *Gamma* and *Beta* functions respectively. Use the above definitions to evaluate

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(i) $\int_0^\infty \sqrt{y}e^{-y^2}dy$ (ii) $B(2,3)$.

[20 Marks]

(b) By substituting $t = \sin^2\theta$ in the Beta function, show that

$$B(x,y) = 2\int_0^{\frac{\pi}{2}} (\sin\theta)^{(2x-1)}(\cos\theta)^{(2y-1)}d\theta.$$

Hence or otherwise, evaluate the integral $\int_0^{\frac{\pi}{2}} \sin^4\theta\cos^5\theta d\theta$.

[30 Marks]

4. (a) Let $F(x,y,z) = (2xz^2)\mathrm{i} + \mathrm{j} + (y^3xz)\mathrm{k}$ be a vector function. Given that $\phi(x,y) = x^2y$ is a scalar function, compute

(i) $\nabla\phi$ (ii) $F \times \nabla\phi$ (iii) $F\cdot(\nabla\phi)$ (iv) $\mathrm{Curl}(F)$

[30 Marks]

(b) Evaluate each of the following integrals

(i) $\iint\limits_R \sin(x+y)dxdy$ where $R = [0,1]\times[0,1]$

(ii) $\iint\limits_D e^{\frac{x}{y}}dA$ where $D = \{(x,y) : 1 \le y \le 2,\ y \le x \le y^3\}$

[20 Marks]

5. (a) Evaluate the improper integrals:

(i) $\int_0^2 (2-x)^{-\frac{1}{2}}dx$ (ii) $\int_{-\infty}^{\infty} xe^{-x^2}dx$. [15 Marks]

(b) For what interval of x does the power series $\sum_{n=1}^{\infty}\left(\frac{x^n}{3^{(n-1)}n^3}\right)$ converge absolutely?

[20 Marks]

(c) Test for convegence for the following:

(i) $\int_3^4 \frac{\ln x}{(x-3)^4}dx$ (ii) $\sum_{n=1}^{\infty}\frac{n}{n^4-3}$.

[15 Marks]

[Hint: Use the comparison test].

6. (a) Use Green's theorem to evaluate $\oint\limits_C xydx + x^3y^3dy$ where C is a triangle with vertices $(0,0)$, $(1,0)$ and $(1,2)$ with positive orientation. [30 Marks]

(b) Compute the directional derivative of the function

$$f(x,y,z) = xy^2 + y^2z^3 + z^3x$$

at the point $P(4,-2,-1)$ in the direction of $Q(1,3,2)$ [20 Marks]