University of Ghana

Second Semester Examinations 2016/2017

Bachelor of Science in Engineering

Level 400: CPEN 404: Computer Vision and Robotics

3 Credit Hours

Time Allowed: 3 Hours

INSTRUCTION: Answer All Questions

SECTION A: Computer Vision

A1. (a) What is Computer Vision ? Give five (5) application areas for computer vision with explanations. **[6 marks]**

(b) A fruit picking robot detects ripe fruit using a colour camera mounted at the end of its manipulator. The vision system then tracks the fruit's outline as the camera moves at constant speed towards the fruit. Describe how the change in the apparent area of the fruit can be used to guide the robot manipulator. **[5 marks]**

A2. Consider an input image $I[i,j]$ and an $m \times n$ filter $F[i,j]$. The 2D convolution $I * F$ is defined as

$$(I * F)[i,j] = \sum_{k,l} I[i-k, j-l]F[k,l] \tag{1}$$

Hint: The above operation is run for each pixel (i,j) of the result.

(a) Convolve the following I and F. Assume we use **zero-padding** where necessary.

$$I = \begin{bmatrix} 2 & 0 & 1 \\ 1 & -1 & 2 \end{bmatrix} \qquad F = \begin{bmatrix} 1 & -1 \\ 1 & -1 \end{bmatrix} \tag{2}$$

Show all necessary steps to receive full credit. **[5 marks]**

(b) Note that the F given in (2) is separable; that is, it can be written as a product of two 1D filters: $F = F_1F_2$. Here, we have

$$F_1 = \begin{bmatrix} 1 \\ 1 \end{bmatrix} \qquad F_2 = \begin{bmatrix} 1 & -1 \end{bmatrix} \tag{3}$$

Compute $(I * F_1)$ and $(I * F_1) * F_2$, i.e. first perform 1D convolution on each column, followed by another 1D convolution on each row. *Show all necessary steps to receive full credit.* [5 marks]

(c) Prove from the definition of the convolution expression given above that for any separable filter $F = F_1 F_2$,

$$I * F = (I * F_1) * F_2 \tag{4}$$

Hint: Expand equation (1) directly. [5 marks]

A3. (a) Name two different color representations. Are they linearly related? [3 marks]

(b) Give two reasons why a Gaussian filter is preferred to a box filter? [2 marks]

(c) What is the difference between the derivative of a Gaussian filter and the difference of Gaussian filters? [2 marks]

(d) List the five (5) main steps of the RANSAC algorithm as applied to line fitting with outliers. [5 marks]

(e) List the five (5) main steps of the Harris corner detector. [5 marks]

A4. There are two classic techniques for edge detection in computer vision namely gradient-based and Lapalcian-based edge detection methods.

(a) What is the motivation for the gradient-based and the Laplacian-based edge detection methods? [4 marks]

(b) Describe two ways to calculate the gradient magnitude for an image.

[4 marks]

(c) Write down the expression for the Laplacian for edge detection from first principles.

[4 marks]

A5. (a) Consider the following three filters $\mathcal{G}$, $\mathcal{E}$ and $\mathcal{M}$. $\mathcal{G}$ is a Gaussian smoothing kernel, $\mathcal{E}$ is one of the linear kernels used by the Sobel edge detector and $\mathcal{M}$ is a median filter. Is applying $\mathcal{G}$ to an image followed by $\mathcal{E}$ equivalent to applying $\mathcal{E}$ to an image followed by $\mathcal{G}$? Explain your answer.

[5 marks]

(b) In the continuous domain, a two dimensional Gaussian kernel $\mathcal{G}$ with standard deviation σ is given by

$$G(x,y) = \frac{1}{2\pi\sigma^2} \exp\left(-\frac{x^2+y^2}{2\sigma^2}\right).$$

Show that convolution with $\mathcal{G}$ is equivalent to convolving with $\mathcal{G}_x$ followed by $\mathcal{G}_y$ where $\mathcal{G}_x$ and $\mathcal{G}_y$ are $1-$ dimensional Gaussian kernels in the x and y directions respectively with standard deviation σ. From a computational efficiency perspective, explain which is better, convolving with $\mathcal{G}$ in a single step or the two step $\mathcal{G}_x$ -and-$\mathcal{G}_y$ approach.

[10 marks]

SECTION B: Robotics

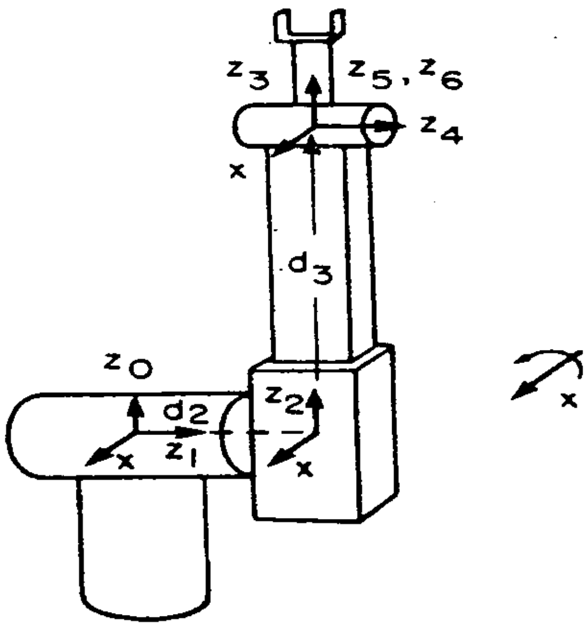
B1. (a) State the definition of robotics as outlined by the Robot Institute of America (RIA).

[3 marks]

(b) Give three (3) applications of robotic systems with explanations.

[9 marks]

B2. Given the Stanford arm as following figure, where $d_2 = 0.1m$



(a) Find the link parameters for the arm using the Denavit-Hartenberg convention. Note: d_3 is a prismatic joint variable, other joints are rotational joints; the link coordinate frames have been established as shown in the figure above. [4 marks]

(b) Find the forward kinematic model for the arm and represent it in homogeneous matrix form. [8 marks]

(c) Find the velocity kinematics model for the robotic arm. [8 marks]

B3. In modelling the dynamics of robotic manipulators, certain steps are undertaken namely:

1. Identify model mechanics by writing down the equations of motion for the robotic arm
2. For each link, calculate the mass, m_i, length, l_i, center of gravity, l_{Ci} and moment of Inertia, I_i

3. Formulate the Lagrangian function **L** defined as difference between kinetic energy (**T**) and potential energy (**V**) of the system: **L** is a function of the link parameters $\mathbf{L}(q, \dot{q})$

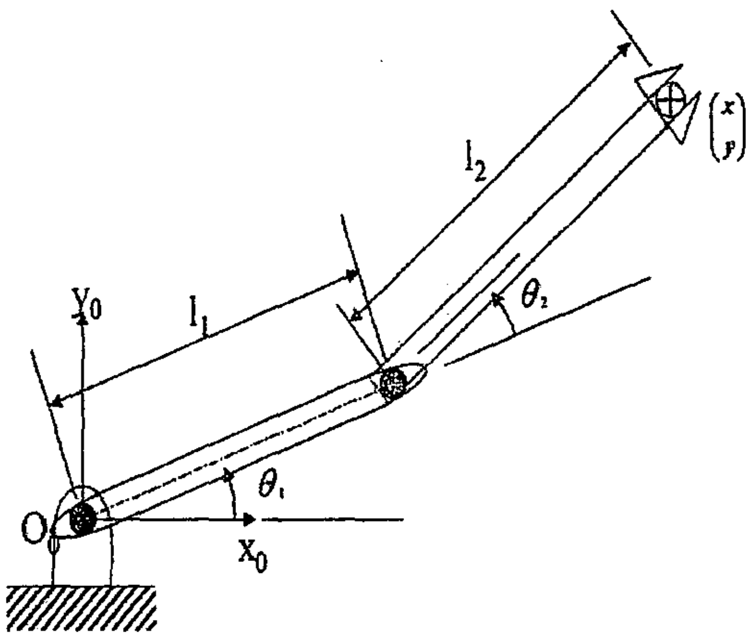


Figure 1: Robotic arm for 2-link manipulator

The robotic arm for a 2-link manipulator is shown above in figures 1 and 2, following the modelling dynamics outlined above; answer the following questions.

(a) Write down the expression for the kinetic energies for the two links taking into account both translational and rotational kinetic energies.

[4 marks]

(b) Write down the expression for the potential energies for the 2-link manipulator.

[4 marks]

(c) Calculate the Lagrangian function $\mathbf{L}(\mathrm{q}, \dot{\mathrm{q}})$.

[2 marks]

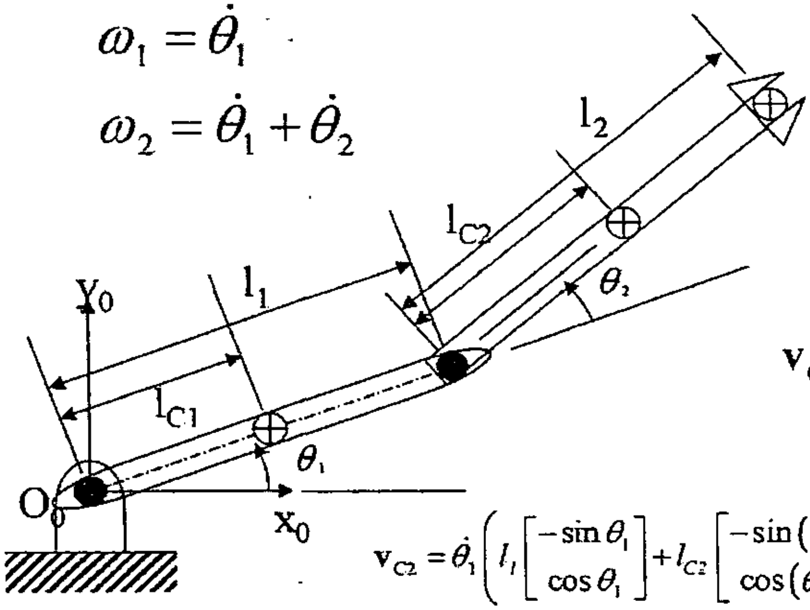
(d) Calculate the equations of motion using the expression

$$\frac{\mathrm{d}}{\mathrm{dt}}\left(\frac{\partial \mathrm{L}}{\partial \mathrm{q}'}\right) - \frac{\partial \mathrm{L}}{\partial \mathrm{q}} = \mathrm{Q}_{\mathrm{i}}' \text{ for } i = 1, 2, \ldots, n$$

[8 marks]

$$\omega_1 = \dot{\theta}_1$$

$$\omega_2 = \dot{\theta}_1 + \dot{\theta}_2$$



$$\mathbf{v}_{C1} = l_{C1}\dot{\theta}_1 \begin{bmatrix} -\sin\theta_1 \\ \cos\theta_1 \end{bmatrix}$$

$$\mathbf{v}_{C2} = \dot{\theta}_1 \left(l_1 \begin{bmatrix} -\sin\theta_1 \\ \cos\theta_1 \end{bmatrix} + l_{C2} \begin{bmatrix} -\sin(\theta_1+\theta_2) \\ \cos(\theta_1+\theta_2) \end{bmatrix} \right) + l_{C2}\dot{\theta}_2 \begin{bmatrix} -\sin(\theta_1+\theta_2) \\ \cos(\theta_1+\theta_2) \end{bmatrix}$$

Figure 2: Robotic arm for 2-link manipulator with equations