UNIVERSITY OF GHANA
SCHOOL OF ENGINEERING SCIENCES

FIRST SEMESTER EXAMINATIONS, 2014/2015

LEVEL 100: BACHELOR OF SCIENCE IN ENGINEERING

FAEN 101: ALGEBRA

DURATION: THREE (3) HOURS

Instructions:

1. Answer <u>all questions from Section A</u> and any TWO questions from Section B
2. Calculators <u>NOT</u> allowed
3. Graph paper recommended

Section A

1. Study and sketch the graph of the function $f(x) = \frac{1}{x^2 - 4}$, giving relevant information such as but not limited to domain, range, x- and y- intercept.

2. Find the term in x^{10} in the expression of $(5+2x^2)^7$.

3. Part of the graph of $y = q + p \cos x$ is shown below. The graph passes through the points $(0, 3)$ and $(\pi, -1)$. Find the values of
   a. p
   b. q

xc: 3.14159   yc: -1
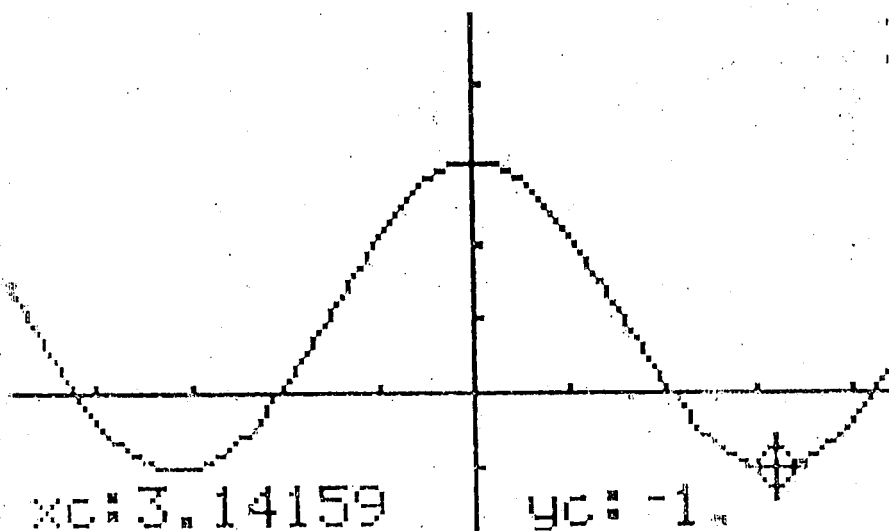
4. a) Find all quadratic equations with real coefficients having 1+2i as a root.

   b) Find the exact values of a and b if $(\sqrt{2} - i)$ is a root of $x^2 + ax + b = 0$ a, b ∈ R

5. i. Use the laws of logarithms to write the following as a single logarithm

      a. $\ln 5 + \ln 4$

      b. $\ln 24 - \ln 6$

      c. $\ln 5 - 2$

      d. $2\ln 7 - 5\ln 2$

      e. $2\ln 3 - 1$

   ii. Write the following equations without logarithms

       $\ln A = 2\ln c + 2$

       Find the value of A if c = 10.

       What is the approximate value of A to 2 significant figures?

Section B

5. i) Solve using determinants:
$$\begin{cases} \dfrac{1}{x} + \dfrac{2}{y} + \dfrac{1}{z} = 2 \\ \dfrac{3}{x} - \dfrac{4}{y} - \dfrac{2}{z} = 1 \\ \dfrac{2}{x} + \dfrac{5}{y} - \dfrac{2}{z} = 3 \end{cases}$$

   ii) Find the eigenvalues and eigenvectors of the following:
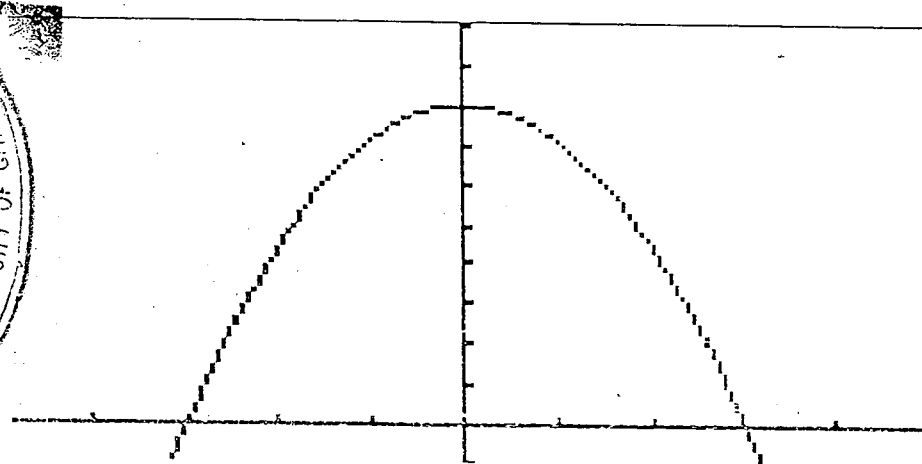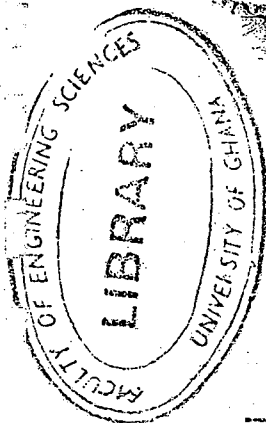$$\begin{pmatrix} 2 & 1 & 1 \\ 1 & 3 & 2 \\ -1 & 1 & 2 \end{pmatrix}$$

7. i) If A and B are measures of acute angles find A+B if

a. $\tan A = \dfrac{1}{4}$ and $\tan B = \dfrac{3}{5}$

b. $\tan A = \dfrac{5}{3}$ and $\tan B = 4$

ii) A tunnel is parabolic in shape with dimensions as follows: height 8m and width 6m see the sketch below.

A truck carrying a wide load is 4.8m high and 3.5m wide and needs to pass through the tunnel. Your task is to determine if the truck will fit through the tunnel. Show your calculations step by step.

8. i) Find all the 8 roots of 1 in the complex set

ii) Determine whether U and V are orthogonal, parallel or neither.

1. $U = \dfrac{1}{4}(3i-j)$ V= 5i+6j

2.   U=2i-2j   V=-i-j

3. $U=< \cos\theta, \sin\theta >$ $V=< \sin\theta, -\cos\theta >$