# UNIVERSITY OF GHANA



**SECOND SEMESTER EXAMINATIONS: 2014/2015**
**LEVEL 200: BACHELOR OF SCIENCE IN ENGINEERING**
**FAEN 202: DIFFERENTIAL EQUATIONS (4 Credits)**

**TIME ALLOWED: 3 HOURS**

**SECTION A [60 marks]**

Attempt **ALL FOUR (4)** questions in SECTION A.

**Question 1**

1.1. State Newton's law of cooling (heating), and find a general solution for its model in the case of:

(a) cooling
(b) heating

[6 marks]

1.2. Show that the total differential equation

$$(y+z)dx + \left(y - \frac{x-z}{2}\right)dz = \frac{x+z}{2}dy$$

is:

(a) homogeneous and integrable.
(b) use the double transformation $x = uz,\ y = vz$ to solve it.

[6 marks]

1.3. Show that a solution of $y' = 1 + 2xy$, subject to $y(1) = 0$ is $y = e^{x^2}\int_1^x e^{-t^2}dt$.

[3 marks]

**Question 2**

2.1. A mass on a spring undergoes a forced vibration given by

$$m\frac{d^2x}{dt^2} + \beta\frac{dx}{dt} + kx = F(t).$$

A

In the case where damping is neglected and $F(t) = \alpha\cos\omega t$,

(a) find a complementary solution for this model. [2 marks]
(b) determine a steady-state oscillation and show that there are two values (to be found) of $\omega$ at which resonance occurs. [4 marks]

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2.2. Given the differential equation $(D^2 - \cot x\, D - sin^2x)y = \cos x - cos^3x$, where $D = d/dx$, show that the introduction of $z = -\cos x$ as a new independent variable leads to the new equation $\frac{d^2y}{dz^2} - y = -z$. Hence obtain a general solution for the original equation. [6 marks]

2.3. Solve $\frac{d^4y}{dx^4} - 4\frac{d^3y}{dx^3} - 5\frac{d^2y}{dx^2} + 36\frac{dy}{dx} - 36y = 0$ if one solution is $xe^{2x}$.

[3 marks]

**Question 3**

3.1. Let $Y(t) = t^n$ so that $Y'(t) = nt^{n-1}$, $Y(0) = 0$. Then using Laplace transform of the derivative show that, in general,

$$\mathcal{L}\{t^n\} = \frac{n(n-1)\dots 2.1}{s^{n+1}} = \frac{n!}{s^{n+1}},$$

$n = 1,2,3,\dots,$ for $s > 0$. [5 marks]

3.2. (a) By the definition and an appropriate property of Laplace transform, evaluate $\mathcal{L}\left\{\frac{\sin 3x}{x}\right\}$. [3 marks]

(b) Find the inverse Laplace transform $\mathcal{L}^{-1}\left\{\frac{s+1}{s^2+s+1}\right\}$. [3 marks]

3.3. Use Laplace transform to solve the following initial-value problem. All unknowns are functions of $x$.

$$w' - w - 2y = 1$$
$$y' - 4w - 3y = -1$$
$$w(0) = 1, \; y(0) = 2.$$

[4 marks]

**Question 4**

Find the power series solution of the differential equation $y'' = y$, $y(0) = 1$, $y'(0) = 0$ near $x = 0$, showing that this solution converges to $\cosh x$. [15 marks]

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## SECTION B [40 marks]

Attempt **ONLY TWO (2)** questions.

### Question 5

**Velocity of Chemical Reactions and the Law of Mass Action. Chemical Mixtures.**

In the bimolecular reaction $A + B \rightarrow M, \alpha$ moles per liter of $A$ and $\beta$ moles per liter of $B$ are combined. If $x$ denotes the number of moles per liter which have reacted after time $t$, the rate of reaction is given by $\frac{dx}{dt} = k(\alpha - x)(\beta - x)$.

(a) Show that if $\alpha \neq \beta$, $x = \frac{\alpha\beta[1-e^{(\beta-\alpha)kt}]}{\alpha-\beta e^{(\beta-\alpha)kt}}$ and find $\lim_{t\to\infty} x$, considering the two cases $\alpha > \beta, \beta > \alpha$.

(b) If $\alpha = \beta$, show that $x = \alpha^2 kt/(1 + \alpha kt)$ and find $\lim_{t\to\infty} x$.

[20 marks]

### Question 6

**Absorption of Drugs in Organs or Cells**

A liquid carries a drug into an organ of volume 500 $cm^3$ at a rate of 10 $cm^3/s$ and leaves at the same rate. The concentration of the drug in the entering liquid is 0.08 $g/cm^3$. Assuming that the drug is not present in the organ initially, find (a) the concentration of the drug in the organ after 30 s and 120 s, respectively; (b) the steady-state concentration.

[20 marks]

### Question 7

**Electric Circuits**

One of the basic equations in electric circuits is $L\frac{di}{dt} + Ri = E(t)$, where $L$ is the inductance, $R$ is the resistance, $i$ is the current, and $E$ is the electromotive force or emf. (Here $R$ and $L$ are constants). (a) Solve the equation when $E(t) = E_0$ and the initial current is $i_0$. (b) Solve the equation when $L = 3$ H, $R = 15\ \Omega$, $E(t)$ is the 60 cycle sine wave of amplitude 110 V and $i = 0$ when $t = 0$.

[20 marks]

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