UNIVERSITY OF GHANA, LEGON

BSC. ENGINEERING FIRST SEMESTER EXAMINATIONS: 2015/2016

DEPARTMENT OF COMPUTER ENGINEERING

CPEN 205: DISCRETE MATHEMATICAL STRUCTURES (2 CREDITS)

INSTRUCTION: ***ANSWER ALL QUESTIONS.***

TIME ALLOWED: ***TWO (2) HOURS***

SECTION A

Answer all Questions in Section A. Show detailed work for full marks.

1. Let the function $f: \mathbf{Z} \rightarrow \mathbf{R}$ be given by $f(x) = x^2$ [10 marks]

a. Find :

i) Domain?

ii) Co-domain?

b. Compute the image of -3?

c. Find the pre-image of the following:

i) 3.

ii) 4.

d. What is the range $f(\mathbf{Z})$?

2. Compute the composite $f \circ g$ where [6 marks]

a. $f: \mathbf{Z} \rightarrow \mathbf{R}, f(x) = x$ and $g: \mathbf{R} \rightarrow \mathbf{R}, g(x) = x^3$

b. $f: \mathbf{Z} \rightarrow \mathbf{Z}, f(x) = x + 1$ and $g(x) = x - 1$

c. f: {people} → {people}, $f(x)$ = the father of x, and $g(x) = f(x)$

3. [6 marks]

a. Given $f: \mathbf{Z} \rightarrow \mathbf{Z}, f(x) = x^2$ find f^4

b. Given $g: \mathbf{Z} \rightarrow \mathbf{Z}, g(x) = x + 1$ find g^n

c. Given $h(x)$ = the mother of x, find h^n

4. Using the notation: floor$(x) = \lfloor x \rfloor$, ceiling$(x) = \lceil x \rceil$, compute the following: [4 marks]

a. $\lfloor 1.7 \rfloor$

b. $\lceil -2.5 \rceil$

5. Compute the cardinality of each of the following sets. [8 marks]
   a. {1, -13, 4, -13, 1}
   b. { }
   c. {3, {1, 2, 3, 4}, ∅}
   d. { {}, {{}}, {{{}}} }

6. Given S = {1, 2, 3}, A = {1,2}, B = {3,4} and, C = {5,6,7} [8 marks]
   a. Find the power set of S.
   b. Compute the Cartesian product $A \times B \times C$.

7. Compute the cardinality of each of the following sets. [8 marks]
   a. {1, -13, 4, -13, 1}
   b. { }
   c. {3, {1, 2, 3, 4}, ∅}
   d. { {}, {{}}, {{{}}} }

8. Given S = {1, 2, 3}, A = {1, 2}, B = {3, 4} and, C = {5, 6, 7} [6 marks]
   a. Find the power set of S.
   b. Compute the Cartesian product $A \times B \times C$.

9. Given two sets A and B, use Venn diagrams to show the following: [10 marks]
   a. $A \dot{\cup} B$ (disjoint union)
   b. $A - B$ (set difference)
   c. $A \oplus B$ (symmetric difference)
   d. $\overline{A \cup B} = \overline{A} \cap \overline{B}$ (De Morgan's first law)

10. Consider the universe U = {ant, beetle, cicada, dragonfly}, use bit-strings to represent the following sets: [6 marks]
    a. {beetle, cicada, dragonfly}.
    b. symmetric difference of {beetle} with {ant, beetle, dragonfly}.

11. Construct the truth table of the compound proposition $(p \vee \neg q) \rightarrow (p \wedge q)$. [4 marks]

12. How can big-O be used to estimate the logarithm of a factorial of a positive integer N? [4 marks]

SECTION B

Answer all questions in Section B. Show detailed work for full marks.

13. Write a C++ programme to implement a recursive function called *computeLucas*. *computeLucas* computes and returns the N^{th} Lucas number from the sequence of Lucas numbers defined by $L_0 = 2$, $L_1 = 1$, and $L_N = L_{N-1} + L_{N-2}$ for $N = 2, 3, 4, \ldots$ [10 marks]

14. Using diagrams, design a digital circuit that produces the output $(p \vee \neg r) \wedge (\neg p \vee (q \vee \neg r))$ when given input bits p, q, and r. [10 marks]