UNIVERSITY OF GHANA

B.Sc. ENGINEERING/FIRST SEMESTER EXAMINATIONS: 2018/2019

DEPARTMENT OF COMPUTER ENGINEERING

CPEN 205: DISCRETE MATHEMATICAL STRUCTURES (2 CREDITS)

INSTRUCTIONS:

ANSWER ALL QUESTIONS

EACH QUESTION CARRIES 25 MARKS

TIME ALLOWED: TWO (2) HOURS

Q1.

a) How many license plates can be made using either *two* or *three letters* followed by either *two* or *three digits* and contain no letter or digit twice? [4 marks]

b) The *sixth* permutation of the lexicographic permutations of the 24 elements of the set {1, 2, 3, 4} is *1432*. Find the next **ten** permutations in *lexicographic order* after *1432*. [5marks]

c) Suppose that a department contains 10 men and 15 women. How many ways are there to select a committee with *six* members if it must have

i. At most three women? [3 marks]

ii. At least 1 woman and at least 1 man? [4 marks]

d) Use the binomial theorem to expand $(x + y)^5$. Hence, find the value of $\frac{32}{243}(1+\frac{1}{2})^5$ using the expression obtained from the expansion of $(x + y)^5$.

[3 marks]

e) Prove that

$$\binom{n}{2} + \binom{n+1}{2} = n^2$$, where n is a positive integer. [4 marks]

f) A test contains 100 true/false questions. How many different ways can a student answer the questions on the test, if answers may be left blank? [2 marks]

Q2.

a) Given the sets A = {1, 2, 3, 4, 5} and B = {2, 4, 6}, calculate the following sets:

i.	$A \cup (B \cap A)$	[1 mark]
ii.	$(A \cap B) \cup B$	[1 mark]
iii.	$A - B$	[1 mark]
iv.	$(B - A) \cap B$	[1 mark]
v.	$A \cup (B - A)$	[1 mark]

b) Prove the following using Venn diagrams:

i. $A \cap (B \cup C) = (A \cap B) \cup (A \cap C)$ [4 marks]

ii. $(A \cap B)' = A' \cup B'$ [3 marks]

c) In propositional logic, a ***contradiction*** is a compound proposition that is always ***false***, no matter what the truth values of the propositional variables that occur in it are. Show by means of a truth table that $p \wedge \neg (q \vee p)$ is a contradiction.

[6 marks]

d)

i. Define a *relation* and hence a *function.* [2 marks]

ii. Consider the functions

$f = \{(1,3), (2,5), (3,3), (4,1), (5,2)\}$

$g = \{(1,4), (2,1), (3,1), (4,2), (5,3)\}$

from $X = \{1, 2, 3, 4, 5\}$ into X.

State the ranges of f and g and also find the composite function fg.

[5 marks]

Q3.

a)

i. Define a *recurrence relation.* [1 mark]

ii. What is the solution of the linear homogeneous recurrence relation?

$a_n = -2a_{n-1} + 15a_{n-2}$

with $a_0 = 0$, and $a_1 = 1$ [4 marks]

iii. What is the solution of the linear homogeneous recurrence relation?

$a_n = 8a_{n-1} - 16a_{n-2}$

with $a_0 = 1$ and $a_1 = 7$ [4 marks]

b) The **Lucas numbers** satisfy the linear homogeneous recurrence relation

$L_n = L_{n-1} + L_{n-2}$ with initial conditions $L_0 = 2$ and $L_1 = 1$.

Prove that the solution (explicit formula) to the Lucas numbers is

$$L_n = \left(\frac{1+\sqrt{5}}{2}\right)^n + \left(\frac{1-\sqrt{5}}{2}\right)^n$$

[5 marks]

c) A bank pays 6 % interest annually on savings, compounding the interest yearly. If we deposit $1000, how much will this deposit be worth 12 years later? [4 marks]

d) A model for the number of lobsters caught per year is based on the assumption that the number of lobsters caught in a year is the average of the number caught in the two previous years.

i. Find a recurrence relation for $\{Ln\}$, where Ln is the number of lobsters caught in year n, under the assumption for this model. [2 marks]

ii. Find Ln if 100,000 lobsters were caught in year 1 and 300,000 were caught in year 2. [5 marks]

Q4.

a) Let $Q(x, y)$ be the statement "Student x has taken class y," where the domain for x consists of *all students in your class* and for y consists of *all computer science courses at your school*. Express each of these quantifications in English.

i. $\exists x \exists y Q(x, y)$ [1 mark]
ii. $\exists x \forall y Q(x, y)$ [1 mark]
iii. $\forall x \exists y Q(x, y)$ [1 mark]
iv. $\exists y \forall x Q(x, y)$ [1 mark]
v. $\forall y \exists x Q(x, y)$ [1 mark]
vi. $\forall x \forall y Q(x, y)$ [1 mark]

b) Let $W(x, y)$ mean that student x has visited website y, where the domain for x consists of all students in your school and the domain for y consists of all websites. Express each of these statements by a simple English sentence.

i. W(Sarah Smith, www.att.com) [1 mark]
ii. $\exists x W(x,$ www.imdb.org$)$ [1 mark]
iii. $\exists y W$(José Orez, y) [1 mark]
iv. $\exists y(W$(Ashok Puri, $y) \wedge W$(CindyYoon, y)) [1 mark]

c) Let $F(x, y)$ be the statement "*x can fool y*," where the domain consists of all people in the world. Use quantifiers to express each of these statements.

i.	Everybody can fool Kanga.	[1 mark]
ii.	Amadou can fool everybody.	[1 mark]
iii.	Everybody can fool somebody.	[1 mark]
iv.	Everyone can be fooled by somebody.	[1 mark]

d) Let $P(x)$, $Q(x)$, and $R(x)$ be the statements "x is a professor," "x is ignorant," and "x is vain," respectively. Express each of these statements in symbolic form using quantifiers; logical connectives; and $P(x)$, $Q(x)$, and $R(x)$, where the domain consists of *all people*.

i.	No professors are ignorant.	[1 mark]
ii.	All ignorant people are vain.	[1 mark]
iii.	No professors are vain.	[1 mark]
iv.	There is a professor who is vain	[1 mark]
v.	Some ignorant people are not vain	[1 mark]
vi.	Prof. Agbotui is both ignorant and vain	[2 marks]

e) Let $A(x)$, $C(x)$, $S(x)$ and $GP(x)$ be the statements "x is an animal," "x is a cat," "x is small," and "x is a good pet," respectively. Express each of these quantifications in English where the universe consists of *all animals*.

i.	$\forall x\ (C(x) \rightarrow A(x))$	[1 mark]
ii.	$-\exists x(C(x) \land \neg S(x))$	[1 mark]
iii.	$\forall x\ (C(x) \rightarrow S(x) \land A(x))$	[1 mark]
iv.	$\forall x\ (S(x) \land A(x) \rightarrow GP(x))$	[1 mark]