UNIVERSITY OF GHANA

BSC. ENGINEERING
SECOND SEMESTER EXAMINATIONS: 2016/2017

DEPARTMENT OF BIOMEDICAL ENGINEERING
BMEN 404: BIOMEDICAL ENGINEERING SYSTEMS (3 CREDITS)

INSTRUCTIONS:
ATTEMPT ALL QUESTIONS.
ALL QUESTIONS SHOULD BE ANSWERED IN THE BOOKLET PROVIDED.
EACH MAJOR QUESTION SHOULD START ON A NEW PAGE.
CALCULATIONS SHOULD BE DETAILED AND SYSTEMATIC. MARKS ARE ALLOCATED TO STEPS.
RELEVANT FORMULAE ARE PROVIDED AT THE END OF THE QUESTION SETS.

TIME ALLOWED: THREE (3) HOURS

1. Control systems play an important role both in nature and in the engineered world. Due to this, we tend to derive representations of different systems for different purposes, each with specific requirements.

   a. Briefly explain any two (2) reasons why systems require control.
   (6 marks)

   b. Briefly explain the three (3) main purposes for which we model these systems.
   (9 marks)

   c. State three (3) differences between the analog and systems approaches of analysing physiological control systems.
   (6 marks)

   d. A system can be said to be causal and instantaneous. Are all causal systems also instantaneous? Briefly explain.
   (4 marks)

2. Blood glucose levels are naturally regulated by the glucose regulatory mechanism. B-cells in the pancreas respond to eating and an attendant increase in blood glucose levels by secreting insulin, which increases the rate of transport of glucose for metabolism or storage. In liver cells, especially, the glucose is stored as glycogen. When blood sugar levels fall below the proper level through exercise, this causes α-cells to increase the production of glucagon. The increased levels of glucagon cause the liver to increase the rate of conversion of glycogen to glucose which is then released to the blood. In a Type I diabetic patient, insulin production is low and an external insulin pump is used in

addition. The pump works by continuously measuring blood glucose levels and delivering a continuous but varying dose of insulin.

a. With the above description,
   i. Draw a block diagram representation of this diabetic patient's blood glucose regulation.
   (7 marks)
   ii. Identify the controller(s), plant(s), sensor(s) and disturbance(s) if present/applicable.
   (4 marks)

b. A model of a given system has an input $x(t)$ and output $y(t)$ with a disturbance (δx) producing a change in the output (δy). The system employs feedforwards and feedbacks to improve its performance. In steady state, the system components attain the gains indicated in Figure 1 below. Derive an expression for the following:
   i. The closed loop gain (CLG)
   (10 marks)
   ii. The loop gain (LG)
   (2 marks)

c. State one example of a physiological control system that employs a feedforward mechanism.
   (2 marks)

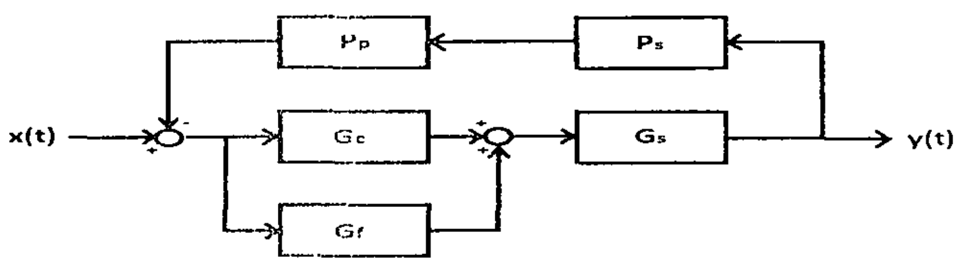

Figure 1. Block diagram of a hypothetical control system in steady-state..

3. The Poynting-Thomson analogue model of muscle mechanics is given by Figure 2 below.

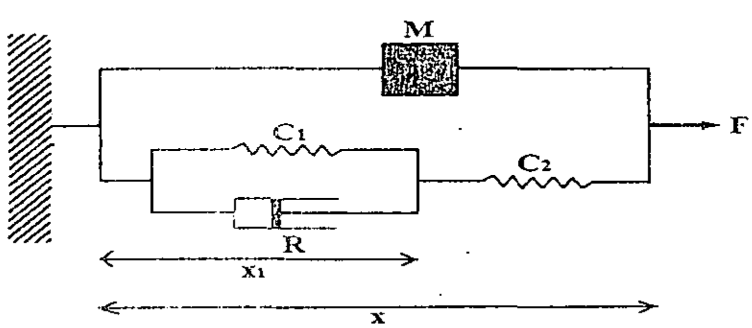

Figure 2. The Poynting-Thomas viscoelastic model of muscle.

a. Derive a mathematical expression relating the length, x, (input) to the force, F, generated (output) of the muscle.

(10 marks)

b. Determine if the above system is:

i. Linear or non-linear.

(6 marks)

ii. Determine the system response under a step test signal (leave answer in the complex s domain). [C_1=8.9, C_2=4, R=2 and M=0.1]

(8 marks)

iii. Determine the behaviour/nature of the response in *b ii)*.

(6 marks)

4. A physiological control system with input, $u(t)$, and output, $y(t)$, has been represented as a state space model as shown by the equations below:

$$\begin{bmatrix} \dot{x_1} \\ \dot{x_2} \\ \dot{x_3} \end{bmatrix} = \begin{bmatrix} 2 & -5 & 3 \\ -6 & -2 & 2 \\ 1 & -3 & -4 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} + \begin{bmatrix} 2 \\ 5 \\ 7 \end{bmatrix} u(t)$$

$$y(t) = \begin{bmatrix} -4 & 6 & 9 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} + \boldsymbol{D}u(t)$$

Where x_1, x_2 and x_3 are state variables and the vector $\boldsymbol{D} = 0$.

a. What is meant by the "state of the system" as applied to the state-space model?

(4 marks)

b. From the above model, derive a block diagram representation of the system.

(10 marks)

c. The second order differential equation below represents a model of some physiological control system,

$$\frac{d^2y}{dt^2} + \frac{\lambda}{LC} y(t) = \frac{1}{LC} x(t)$$

i. Express this model in state-space form if $y(t)$ and $x(t)$ are the output and input, respectively.

(6 marks)

Relevant information (all letters have their usual contextual meanings)

$x = FC_m$

$\Delta V = \Delta P C_f$

$q = \Delta\theta C_t$

$F = R_m v$

$\Delta P = R_f Q$

$\Delta\theta = R_t Q$

$\Delta\phi = R_c Q$

$\Delta P = \frac{1}{C}\int Q\,dt$

$\Delta P = L\frac{dQ}{dt}$

$\Delta P = RQ$

The Laplace Transform Table

$f(t)$	$F(s)$
$f'(t)$	$sF(s) - f(0)$
$f''(t)$	$s^2F(s) - sf(0) - f'(0)$
$\frac{d^n f(t)}{dt^n}$	$s^nF(s) - \sum_{i=0}^{n-1} s^{n-1-i} f^{(i)}(0)$
$f(t-t_0)u(t-t_0)$	$F(s)e^{-st_0}$
$\int_0^t f(\tau)d\tau$	$\frac{1}{s}F(s)$
$f(t-\tau)$	$e^{-s\tau}F(s)$
$e^{-at}f(t)$	$F(s+a)$
$-t\cdot f(t)$	$\frac{d}{ds}F(s)$
$\delta(t)$	1
1	$\frac{1}{s}$
e^{-at}	$\frac{1}{s+a}$
t	$\frac{1}{s^2}$
$\sin(at)$	$\frac{a}{s^2+a^2}$
$\cos(at)$	$\frac{s}{s^2+a^2}$
$f(0) = \lim_{t\to 0} f(t) = \lim_{s\to\infty} sF(s)$	
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