



UNIVERSITY OF GHANA  
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UNIVERSITY OF GHANA  
FACULTY OF ENGINEERING SCIENCES

Second SEMESTER EXAMINATIONS, 2012/2013

LEVEL 100: BACHELOR OF SCIENCE IN ENGINEERING

FAEN 102: CALCULUS 1

TIME ALLOWED: THREE (3) HOURS

**Instructions:**

1. Answer all questions from Section A and any two questions from Section B
2. Calculators allowed
3. Graph paper provided

**Section A**

1. Find the derivative of the following

- i. using the limit process:

$$F(x) = 4x^6$$

- ii. using various theorems

1.  $F(x) = \sin^2 x$

2.  $F(x) = x^{x+1} \ln(x^2 + x + 1)$

3.  $F(x) = x + \cos 2x$

4.  $F(x) = \tan 2\frac{\pi}{3} + x$

2. Find the integral of the following

i.  $F(x) = \cos^7 x$

ii.  $F(x) = \frac{4x}{(x^2-1)(x^2+1)}$

iii.  $F(x) = \frac{1}{x} + \ln x$

iv.  $F(x) = x^3 \ln x$

v.  $F(x) = x^2 \sin x$

3. Two functions  $f(x) = e^x$  and  $g(x) = 1 + \ln 9x$  are given

- a. Draw each and determine their points of intersection if any.
- b. Draw the graph given by  $k(x) = f(x) - g(x)$  on the same graph paper as in (a).

- c. Determine the x and y intercepts of the function  $k(x)$  if any.
  - d. Determine its asymptotes if any.
  - e. The graph of  $k(x)$  cuts the x axis at a point between 0 and 0.5. Find the value by the method of Newton-Raphson to 5 decimal places.
  - f. Find the value for which the graph has a horizontal tangent. Is it a minimum or a Maximum?
  - g. Find  $\int k(x) dx$
  - h. Find the area between the graph of  $k$  and the x axis for  $0.5 \leq x \leq 1$ .
4. Evaluate the area  $\int_0^1 \sqrt[2]{x+2} dx$  using , for  $n=8$  and the answer to 5d.p,
- i. The trapezium method and
  - ii. The Simpson method.
  - iii. Which method gives the best approximation compared to the definite integral?

### Section B

5. A rectangular open-topped box is to be constructed out of 20cm-square sheets of thin cardboard by cutting x-cm squares out of each corner and bending the sides up.
- i. Sketch the operations described above
  - ii. Express each of the following quantities as a polynomial in both factored and expanded form
    - 1. The area of the cardboard after the corners have been removed
    - 2. The volume of the box
    - 3. The value of x that will lead to maximum volume.
6. Find the area between  $f(x) = x^3 e^x$  for x varying between 0 and 1 for  $n = 8$  using
- i. the trapezoidal rule and
  - ii. the Simpson's rule.
  - iii. Compare the two areas from (a) and (b) with  $\int_0^1 f(x) dx$  and conclude which method gives the best approximation to 5 decimal places.
7. i. Find  $y^{(6)}$  or  $\left(\frac{d^6 y}{dx^6}\right)$  using Leibnitz's theorem for  $y=x^3 \sin 2x$ .
- ii. Prove that  $\frac{d^2 y}{dx^2} + 4y = 0$  for  $y = a \sin 2x$  for  $a \in \mathbb{R}$ .

