

UNIVERSITY OF GHANA

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BSc/BA, SECOND SEMESTER EXAMINATIONS: 2015/2016

SCHOOL OF ENGINEERING

DEPARTMENT OF BIOMEDICAL ENGINEERING

FAEN 202: DIFFERENTIAL EQUATIONS (4 credits)

INSTRUCTION:

ANSWER ANY FIVE OUT OF THE FOLLOWING SEVEN QUESTIONS TIME ALLOWED:

THREE (3) HOURS

1. (a) Find the eigenvalues and eigenvectors of

$$A = \left(\begin{array}{rrr} 1 & 2 & -1 \\ 1 & 0 & 1 \\ 4 & -4 & 5 \end{array}\right).$$

Hence or otherwise, find a general solution to $\frac{dx}{dt} = Ax$, given that x is a function of t.

(30 marks)

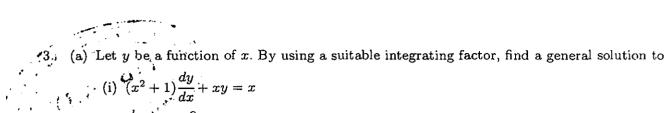
- (b) Let y be a function of x. Find a general solution to $\frac{d^2y}{dx^2} 4\frac{dy}{dx} + 5y = 0$. (10 marks)
- 2. (a) Given that y is a function of x, verify that the following differential equations are exact and solve them.

(i)
$$(3x^2 + 2y^2) + (4xy + 6y^2)\frac{dy}{dx} = 0$$
 (10 marks)

(ii)
$$\left(2x - \frac{\ln y}{x^2}\right) + \frac{1}{xy}\frac{dy}{dx} = 0, \ x > 0, y > 0.$$
 (15 marks)

(b) A cold drink at $36^{\circ}F$ is placed in a sweltering conference room at $90^{\circ}F$. After 15 minutes, its temperature is $54^{\circ}F$. Find the temperature T(t) of the drink after t minutes, assuming it obeys Newton's law of cooling.

(15 marks)



(i)
$$(x^2 + 1)\frac{dy}{dx} + xy = x$$

(ii)
$$\frac{dy}{dx} - y = \frac{8}{9}e^{-\frac{x}{3}},$$

(iii)
$$\frac{dy}{dx} + y \tan x = \sec x, \ x > 0.$$
 (25 marks)

- (b) The rate of change of the volume Vm^3 of water in a draining tank is proportional to the square root of the depth y metres of water in the tank.
 - (i) write down a differential equation connecting V and y.
 - (ii) If the tank is a cylinder with vertical sides and cross sectional area Am^2 , show that $\frac{dy}{dt} + h\sqrt{y} = 0$, where h is a constant and related to A.
 - (iii) solve the differential equation in (ii) above. (15 marks)
- (a) (i) Let f(t) be a real valued function defined on $(0,\infty)$. Define the Laplace transform F(s) of f stating the values of s for which F(s) is defined.
 - (ii) Use your definition in (i) above to determine the Laplace transform of

$$f(t) = \begin{cases} 2, & 0 < t < 5 \\ 0, & 5 < t < 10 \\ e^{4t}, & t > 10. \end{cases}$$

(15 marks)

(b) Given that y is a function of x, solve the initial value problem:

(i)
$$\frac{dy}{dx} = 2\sqrt{y+1}\cos x, \quad y(\pi) = 0,$$

(ii)
$$\frac{dy}{dx} + y \tan x = 2x \cos x, \ y\left(\frac{\pi}{4}\right) = -\frac{15\sqrt{2}}{32}\pi^2.$$

(25 marks)

- (a) (i) Show that the Laplace transform of the identity function f(x) = x is $F(s) = \frac{1}{s^2}$, s > 0
 - (ii) Let y be a function of x. Use Laplace transforms to solve $\frac{d^2y}{dx^2} + y = x$, where y(0) = 0and $\frac{dy}{dx} = 2$ at x = 0,

Hint: The Laplace transform of $\sin x$ is $\frac{1}{s^2+1}$. (20 marks)

(b) (b) By first resolving $\frac{4s-2}{s^3-s}$ into partial fractions, find the inverse Laplace transform of $F(s) = \frac{4s-2}{s^3-s}$.

(20 marks)

- 6. (a) (i) Give the exact definitions of the Fourier series and the Fourier transform of a 2L-periodic function defined on (-L, L).
 - (ii) Expand

$$f(x) = \begin{cases} 0, & -5 < x < 0 \\ 3, & 0 < x < 5 \end{cases}$$

in a Fourier series if the period of f is 10.

(25 Marks)

(b) Determine the Fourier transform of

$$f(x) = \begin{cases} e^{2x}, & x < 0 \\ e^x, & x > 0 \end{cases}$$

(15 marks)

7. (a) Use the method of undetermined coefficients to find a particular solution of

(i)
$$3\frac{d^2y}{dx^2} + \frac{dy}{dx} - 2y = 2\cos x;$$

(ii)
$$\frac{d^2y}{dx^2} - 4y = 2e^{2x}$$
.

(20 marks)

(b) Given that $\frac{dy}{dx} - 2xy = 0$, show that $c_0 \sum_{n=0}^{\infty} \left(\frac{x^{2n}}{n!}\right)$ is a power series solution to the differential equation about x = 0.

(20 marks)

