UNIVERSITY OF GHANA

SCHOOL OF ENGINEERING SCIENCES

COLLEGE OF BASIC AND APPLIED SCIENCES

SUPPLEMENTARY RESIT EXAMINATIONS 2020/2021

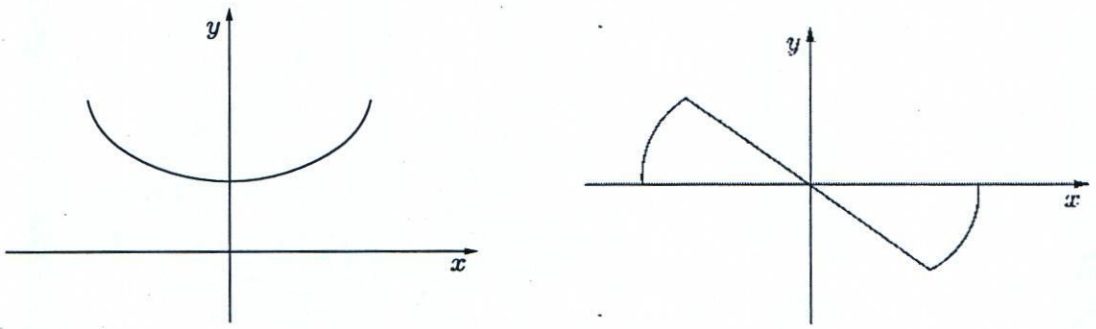
FAEN102 - CALCULUS I (4 CREDITS)

TIME ALLOWED: TWO HOURS [2 hrs]

INSTRUCTION: Answer any 3 of the following 5 questions.

Question 1

[50 marks] The graphs of two functions $y = f(x)$ are shown in Figure 1 below. For each of them, sketch a qualitatively accurate graphs of the first derivative $f'(x)$ and the second derivative $f''(x)$ from the graphs of $f(x)$.



(a) (b)

Figure 1: The graph of $y = f(x)$.

Question 2

(a) **[20 marks]** Evaluate the limit $\lim_{x\to-\infty}\left(x+\sqrt{x^2-4x+1}\right)$.

(b) **[30 marks]** Decide whether or not the real function

$$f(x)=\begin{cases}\frac{1-\cos x}{e^{x^2}-1}, & x\neq 0,\\ -1, & x=0,\end{cases}$$

is continuous at $x=0$.

Question 3

(a) **[30 marks]** If $f(x)=\ln(\cos(2x)+1)$, then show that

$$\frac{df}{dx}=-2\tan x.$$

(b) **[20 marks]** Find the derivative of the function

$$f(x)=|x|e^{-x^2}.$$

Question 4

(a) **[20 marks]** Without solving for $y(x)$, find the value of $y''(0)$ in the initial value problem

$$(x+2)y'+y=1,\quad x>0,\quad y(0)=2.$$

(b) **[30 marks]** If $f(x)=\cos x$ for all $x\in(0,\frac{\pi}{2})$, prove that f is invertible and find $(f^{-1})'(\frac{1}{2})$.

Question 5

(a) **[20 marks]** Find an equation of the curve $y=f(x)$ if $y''=3x^2$ and the tangent line to the curve at $(2,-2)$ is horizontal.

(b) **[30 marks]** Compute the following indefinite integral

$$\int x^3\cos(x^2)\,dx$$

---------- Good Luck ----------

Examiner: Vincent Teyekpiti (PhD)