UNIVERSITY OF GHANA

BACHELOR OF SCIENCE IN ENGINEERING
FIRST SEMESTER EXAMINATIONS: 2016/2017

DEPARTMENT OF COMPUTER ENGINEERING
CPEN 301: SIGNALS AND SYSTEMS (3 Credits)

INSTRUCTION: *Answer any five (5) Questions of your choice*

TIME ALLOWED: *THREE (3) HOURS*

1. **(a)** Explain the difference between a continuous time signal and a discrete time signal. Give two areas of applications of signals and two application areas of systems. [4 marks]

(b) Sketch a simple diagram for a two-system feedback arrangement and explain how this system can be used in an industrial plant. [4 marks]

(c) Using the concept of signals and systems and block diagrams, describe how radio discussion at the studios of Radio Univers is conveyed in frequency to receivers and how the receivers are able to extract the information. [6 marks]

(d) The input-output relationship of a certain discrete time system is given by the expression: $y[n]+2y[n-1]=x[n]+2x[n-1]$ where $x[n]$ is the input and $y[n]$ is the output. Find the output of the system if the input $x[n]$ is defined as below. Sketch the input and output signals. Assume $y[n] = 0$ for $n < -2$. [6 marks]

$$x[n]=\begin{cases}2, & n=-1\\ 3, & n=0\\ 2, & n=1\\ 2, & n=2\\ 0, & otherwise\end{cases}$$

2. **(a)** The impulse response of a certain system is given as $h[n] = \{-1, 1, 2, 3\}$.

Find the output $y[n]$ of the system if an input signal $x[n] = \{-1, -2, 2, 3\}$ is presented to the system. (Use convolution concept $y[n] = x[n]*h[n]$). [5 marks]

(b) An amplitude modulated signal $x(t)$ is described by the expression:

$x(t) = Bcos(340\pi t)$ where $B = 3cos(60\pi t)$.

(i) Write an expression for $x(t)$ as a sum of two sinusoids. [3 marks]

(ii) At what sampling rate should the signal be sampled in order for the samples to be adequately recovered? [2 marks]

(iii) Write an expression (in a reduced form) for the discrete equivalent $x[n]$ of $x(t)$ if it is sampled at 200Hz. Find the angular frequency of the resulting discrete signal. [6 marks]

(iv) Suppose the discrete sequence $x[n]$ in **2b**(iii) above is passed through an ideal system to obtain the reconstructed signal $y(t)$. Write an expression for the reconstructed signal $y(t)$, where $y(t) = y[n] = x[tF_s]$. Is the reconstructed signal $y(t)$ the same as the input signal $x(t)$? Explain your answer. [4 marks]

3. **(a)** A rectangular input voltage signal $x(t)$ is applied to a system that is made up of a cascade of two passive RC lowpass filters to produce an output signal $y(t)$. Sketch the circuit diagram of the system and answer the following:

(i) Indicate whether the RC filter system described in **3(a)** is causal or non-causal, time invariant or time-variant, and memory or memoryless. Explain your answer for each case. [6 marks]

(ii) Derive an expression for the differential equation that describes the RC filter system in **3(a)** above. [5 marks]

(iii) Derive an expression for the discrete time equivalent of the continuous time RC filter system in **3a(ii)** above. [4 marks]

(b) Suppose an input signal $x[n]$ is presented to a system that has an impulse response $h[n] = u[n]$, where $u[n]$ is a unit step, to produce an output $y[n]$. If the input $x[n] = \alpha^n u[n]$, where $-1 < \alpha < 1$, find the output $y[n]$ of the system. Use the convolution sum. [5 marks]

4. **(a)** The input $x(t)$ of a causal LTI system is related to the output $y(t)$ by the linear differential equation :

$$\frac{d^2y(t)}{dt^2} + 3\frac{dy(t)}{dt} + 2y(t) = 3\frac{dx(t)}{dt} + 2x(t).$$

(i) Find the transfer function H(s) of the system. [2 marks]

(ii) Find the zero-pole locations of the system. [3 marks]

(iii) Find the output $y(t)$ of the system if an impulse signal $x(t) = \delta(t)$ is applied to the input of the system. [7 marks]

(b) An LTI system produced an output $y(t) = e^{-bt}u(t)$ when an input signal $x(t) = e^{-at}u(t)$ is presented, where a and b are real numbers and $u(t)$ is a unit step function. Find the frequency response of the system and write an expression for the magnitude spectrum. [5 marks]

(c) A voltage signal x(t) with amplitude of 2V is delayed by a time of t_0 units after passing through a system. Sketch the output signal of the system. Show that the Laplace transform of the output signal is a multiplication by the factor $\exp(-st_0)$ in the s-domain. [3 marks]

5. **(a)** A continuous time signal $x(t)$ is defined as below. Write an expression for the complex exponential equivalent of $x(t)$. [3 marks]

$$x(t) = 10 + 8cos(5t + 1) + 6cos(7t + 1) + 4cos(9t + 3) .$$

(b) A signal $x[n]$ is defined by the expression $x[n] = \alpha^n u(n)$ where $|\alpha| < 1$. Find Fourier transform of the sequence using the DTFT. [5 marks]

(c) A discrete time signal $x[n]$ has a period N and Fourier series coefficients a_k. Find the Fourier coefficients for the signal $y[n] = x[n] - x[n - 1]$. [7 marks]

(d) A certain periodic signal $x(t) = B\cos(\omega_0 t + \varphi)$ is passed through a system for processing. Find the power of this signal. [5 marks]

6. **(a)** A rectangular periodic voltage signal that is symmetric about the origin is passed through an RC filter circuit. The signal has amplitude of 2 and width of 1. The characteristics of the signal are as follows: at $t = -1$, the amplitude is -2, at $t = -0.5$, the amplitude is +2, at $t = +0.5$, the amplitude is 2, and at $t = +1$, the amplitude is -2. Compute the Fourier series coefficients C_n of this signal. What is the fundamental frequency? Assume the period T = 2. [8 marks]

(b) A signal $x(t)$ from a temperature sensor is passed through an active lowpass RC filter circuit to produce an output $y(t)$. Sketch the diagram of the filter circuit and derive and expression for the frequency response of the system. [6 marks]

(c) The DTFT of a lowpass filter system is given by the expression below. Find the inverse DTFT of the system to obtain the impulse response $h[n]$. [6 marks]

$$H(e^{j\omega}) = \begin{cases} 1, & 0 \le |\omega| \le \omega_C \\ 0, & \omega_C < |\omega| < \pi \end{cases}$$

EXAMINER: DR. G. A. MILLS

Useful Formulae

[1] Laplace transform $X(s) = \int_{-\infty}^{\infty} x(t)e^{-st}\,dt$

[2] Complex Fourier Series $x[t] = \sum_{k=\infty}^{\infty} a_k e^{jk\omega_0 n}$ where $\omega_0 = 2\pi/N$ with N of samples

[3] Fourier Coefficient $a_k = \frac{1}{T}\int_{-T/2}^{T/2} x(t)e^{-jn\omega_0 t}\,dt$ with period T.

[4] DTFT $X(e^{j\omega}) = \sum_{n=-\infty}^{\infty} x[n]e^{-j\omega\omega}$

[5] IDFT $x[n] = \frac{1}{2\pi}\int_{-\pi}^{\pi} X(e^{-j\omega})e^{j\omega\omega}$

[6] Signal Power $E = \frac{1}{T}\int_{-T/2}^{T/2} |x(t)|^2\,dt$, where $T = 2\pi/\omega_0$

[7] Signal Energy $W = \int_{-\infty}^{\infty} |X(t)|^2\,dt = \frac{1}{2\pi}\int_{-\infty}^{\infty} |X(j\omega)|^2\,d\omega$

[8] Discrete function for $\frac{d^2y(t)}{dt^2}\Big|_{t=nT} = \frac{y[nT+2T] - 2y[nT+T] + y[nT]}{T^2}$

[9] Sum of series $\sum_{p=0}^{N-1} r^p = \frac{1-r^N}{1-r}$, $|r| \le 1$