UNIVERSITY OF GHANA

SCHOOL OF ENGINEERING SCIENCES

FIRST SEMESTER EXAMINATIONS: 2016/2017

FAEN 101: ALGEBRA (4 credits)

INSTRUCTION:

ANSWER QUESTION ONE(1) AND ANY OTHER FOUR(4) OUT OF THE FOLLOWING SEVEN(7) QUESTIONS

EACH QUESTION CARRIES 40 MARKS

TIME ALLOWED:

THREE HOURS (3 hours)

(a) If vectors a and b are inclined to one another at an angle of $\frac{\pi}{4}$ and $|\underline{a}| = 5$ and $|\underline{b}| = 8\sqrt{2}$.

(i) Find $\underline{a} \cdot \underline{b}$. (ii) Hence, find $|\underline{a} - \underline{b}|$.

(b) If $\underline{a} = -3\underline{i} + 2\underline{j} + 2\underline{k}$ and $\underline{b} = 6\underline{i} + 3\underline{j} + \underline{k}$, find $\underline{a} \times \underline{b}$ and hence or otherwise give two unit vectors which are perpendicular to both $\underline{a}$ and $\underline{b}$.

(c) Show that the vectors $\underline{u} = 2\underline{i} + 3\underline{j} - 6\underline{k}$, $\underline{v} = 6\underline{i} + 2\underline{j} + 3\underline{k}$ and $\underline{w} = 3\underline{i} - 6\underline{j} - 2\underline{k}$ are mutually perpendicular and find the direction cosines of the vector $\underline{u}$.

(a) Solve for the values of x and y, if $\log_2(xy^2) = 0$ and $\log_2(x^2y) = 6$.

(b) Write as a quadratic in e^x and solve for all values of x, if $\dfrac{e^x + 5e^{-x}}{2} = 3$.

(c) Let $f(x) = \dfrac{x+3}{x+2}$, $x \neq -2$, by choosing a suitable range for f, show that f is bijective and hence find the inverse of f.

(a) Express $f(x) = \dfrac{x^2+2}{x(x+2)^2}$ as a partial fraction.

(b) Find all values of x which satisfy $\left|\dfrac{3x-2}{2x-3}\right| \leq 2$, solve for x.

(c) Find the range of values of x for which $\dfrac{x-1}{x(x+2)} > \dfrac{1}{x+2}$.

Let $f(x) = \dfrac{3x(x-1)}{x^2-x-2}$.

(i) Find the domain of f.

(ii) Find the intercepts of f, if they exist.

(iii) Find the horizontal and vertical asymptotes if it exists.

(iv) Find the range and turning points of f, if they exist.

(v) Sketch the graph of f.

5. (a) Write the following complex numbers in the form $a + ib$, where $a, b \in \mathbb{R}$.

(i) $\dfrac{(3-i)}{1+i}$ (ii) $(1+i)^5$ [Hint: Change into polar form and use De Moivre's theorem].

(b) If the complex number $z = \left[\cos\dfrac{\pi}{24} + i\sin\dfrac{\pi}{24}\right]^6$, find the conjugate of z and hence or otherwise find the modulus of z.

(c) Find the fourth root of $z = (1 + i)$.

6. (a) Find the value of r if the coefficients of x^r and x^{r+1} are equal in the binomial expansion of $(1 + 2x)^5$.

(b)

(i) Write out the first 5 terms of the expansion of $(1 + 2x)^{-2}$.

(ii) Use the above expansion to evaluate $\dfrac{1}{(1.22)^2}$, leave your answer in 4 decimal places.

(c) If p is a real number and the term corresponding to p^4 is 1120, find the value of p in the expansion of $\left(\dfrac{p}{2} + 2\right)^8$.

7. (a) The points A, B and C have position vectors $\underline{a} = -\underline{i} + 2\underline{j} + 3\underline{k}$, $\underline{b} = 8\underline{i} + 7\underline{j} - 9\underline{k}$ and $\underline{c} = 2\underline{i} - 3\underline{j} - \underline{k}$ respectively.
Show that angle ACB is $\frac{\pi}{2}$ and find the area of the triangle ABC.

(b) Using the Pythagorean identity, prove each of the following identities

(i) $\dfrac{\text{cosec}x}{\sin x} - \dfrac{\cot x}{\tan x} = 1$ (ii) $\dfrac{\cos x}{1-\sin x} - \tan x = \sec x$

(c) Solve for the value(s) of x in the interval $0 \leq x < 2\pi$, for the following equations

(i) $\sin^2 x \cos x = 4\cos x$ (ii) $(1 - \sin x) = \sqrt{3}\cos x$ [Hint: Square each side.]