UNIVERSITY OF GHANA

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BSc/BA, SUPPLEMENTARY RE-SIT EXAMINATIONS: 2020/2021

SCHOOL OF ENGINEERING SCIENCES

FAEN 202: DIFFERENTIAL EQUATIONS (4 credits)

INSTRUCTION:

ANSWER ANY 3 OUT OF THE FOLLOWING 5 QUESTIONS

TIME ALLOWED:

TWO HOURS (2 hours)

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1. (a) Find the general solution of the following differential equations:

$$i.\ \frac{d^2y}{dx^2} + 12\frac{dy}{dx} + 36y = 0 \qquad ii.\ \frac{d^2y}{dx^2} - 4\frac{dy}{dx} - 12y = 3e^{5x}.$$

(30 Marks)

(b) Find the real constants p and q so that $y(t) = (t+3)e^{3t}$ is a solution of the initial value problem:

$$\frac{dy}{dt} = py + e^{3t}, \quad y(0) = q.$$

(20 Marks)

2. (a) Compute the Laplace transform of $f(t) = \begin{cases} 2, & 0 < t \leq 5 \\ e^{4t}, & t > 5. \end{cases}$ **(20 Marks)**

(b) Use the Lapace transform to solve the initial value problem:

$$\frac{d^2y}{dt^2} - 2\frac{dy}{dt} - 5y = -8e^{-t}, \quad y(0) = 2, \quad \frac{dy}{dt} = 12 \ \ at\ t = 0,$$

expressing your answer in the form $y(t) = \mathcal{L}^{-1}[F(s)]$, where $\mathcal{L}^{-1}$ is the inverse Laplace transform and $F(s)$ is a function of s.
Hint: $\mathcal{L}\left[e^{at}\right] = \frac{1}{s-a}$, $s > a$.

(30 Marks)

3. (a) Let $A = \begin{pmatrix} 1 & 2 \\ 3 & 2 \end{pmatrix}$ be a 2×2 real matrix. Find the eigenvalues and eigenvectors of A. Hence or otherwise, solve the differential equation $\frac{dx}{dt} = Ax(t)$. **(30 Marks)**

(b) Suppose the water in a hot tub is heated to $150^{o}C$. After the heater is turned off, the tub takes an hour to cool to $120^{o}C$. The tempereture of the surrounding air is $80^{o}C$. Use Newton's law of cooling to find the temperature of the tub after 3 hours. **(20 Marks)**

4. (a) Obtain the power series solution of $\frac{dy}{dx} - xy = 0$ about $x = 0$. **(30 Marks)**

(b) Use the substitution $y = zx$ to find the general solution of the differential equation

$$x^2 \frac{dy}{dx} = y(x + y).$$

(20 Marks)

5. (a) i. Solve the differential equation $\frac{dy}{dx} = xy \sin x$.

ii. Solve the initial value problem: $\frac{dy}{dx} = (y-1)^2, \;\; y(0) = 2.$ **(20 Marks)**

(b) Give a precise definition of Fourier series of a 2π-periodic function $f(x)$ on the interval $[-\pi, \pi]$.

Use your definition to find the Fourier series of $f(x) = \begin{cases} 0, & -\pi \leq x \leq 0 \\ 1, & 0 < x \leq \pi. \end{cases}$ **(30 Marks)**