UNIVERSITY OF GHANA

FACULTY OF ENGINEERING SCIENCE

SECOND SEMESTER EXAMINATION: 2013/2014

FAEN 302 : STATISTICS FOR ENGINEERS

ALLOWED TIME : 3 HOURS

INSTRUCTION : ANSWER ALL QUESTIONS IN SECTION A AND TWO IN SECTION B.

Statistical Tables are to be provided.

[1]Examiner : Mr. Louis Asiedu

SECTION A (50 Marks)
Answer All Questions

1. With reference to hypothesis testing, distinguish each of the following pairs of concepts:

   (a) Simple Hypothesis and Composite Hypothesis. (2 marks)

   (b) Significance Probability ($p - value$) and Level of Significance (3 marks)

   (c) Sample Statistic and Test Statistic. (2 marks)

2. State without proof the Total Probability Rule (TPR). (2 marks)

   (a) There is a $50 - 50$ chance that the queen carries the gene for hemophilia. If she is a carrier, then each prince has a $50 - 50$ chance of having hemophilia. If the queen has had three princes without the disease, what is the probability that the queen is a carrier? If there is a fourth prince, what is the probability that he will have hemophilia? (3 marks)

3. Distinguish between Type I error and Type II error. (2 marks)

   (a) Nielsen reported that young men in the United States watch 56.2 minutes of prime-time TV daily *(The Wall Street Journal Europe, November 18, 2003)*. A researcher believes that young men in Germany spend more time watching prime-time TV. A sample of German young men will be selected by the researcher and the time they spend watching TV in one day will be recorded. The sample results will be used to test the following null and alternative hypotheses.

$$H_o = 56.2$$
$$H_1 \neq 56.2$$

   i. What is the Type I error in this situation? What are the consequences of making this error? (2 marks)

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ii. What is the Type II error in this situation? What are the consequences of making this error? **(2 marks)**

4. Given that a random variable X follows the geometric distribution with parameter p, $0 < p < 1$, show that

   (a) $E[X] = \dfrac{1}{p}$ **(3 marks)**

   (b) $Var(X) = \dfrac{1-p}{p^2}$. **(5 marks)**

5. (a) If X is a binomial random variable with expected value 6 and variance 2.4, find $P(X = 5)$. **(3 marks)**

   (b) i. Find $Var(X)$ if, $P(X = a) = p = 1 - P(X = b)$ **(2 marks)**

   ii. Show that $\dfrac{X - b}{a - b}$ is a Bernoulli random variable. **(2 marks)**

6. An urn contains N white and M black balls. Balls are randomly selected, one at a time, until a black one is obtained. If we assume that each ball selected is replaced before the next one is drawn, what is the probability that

   (a) exactly n draws are needed? **(2 marks)**

   (b) at least k draws are needed? **(3 marks)**

7. Buses arrive at a specified stop at 15-minute intervals starting at 7 A.M. That is, they arrive at 7, 7:15, 7:30, 7:45, and so on. If a passenger arrives at the bus stop at a time that is uniformly distributed between 7 and 7:30, find the probability that he waits

   (a) less than 5 minutes for a bus. **(2 marks)**

   (b) more than 10 minutes for a bus. **(2 marks)**

8. (a) Suppose that X is a normal random variable with mean 5. If $P\{X > 9\} = 0.2$, approximately what is $Var(X)$? **(2 marks)**

   (b) Let Y be a normal random variable with mean 12 and variance 4. Find the value of c such that $P\{Y > c\} = 0.10$. **(2 marks)**

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(c) An automotive part must be machined to close tolerances to be acceptable to customers. Production specifications call for a maximum variance of 0.0004 in the lengths of the parts . Suppose the sample variance for 30 parts turns out to be $s^2 = 0.0005$. Use $\alpha = 0.05$ to test whether the population variance specification is being violated. **(4 marks)**

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SECTION B (50 Marks)

Attempt two questions

(All questions carry equal marks)

1. (a) The density function of X is given by

$$f(x) = \begin{cases} a + bx^2, & 0 \leq x \leq 1 \\ 0, & otherwise \end{cases}$$

If $E[X] = \dfrac{3}{5}$, find

i. the cumulative density function, $F(X)$. **(5 marks)**

ii. $P(X < \dfrac{1}{2})$ **(2 marks)**

(b) The joint density function of X and Y is given by

$$f(x, y) = \begin{cases} \dfrac{1}{y} e^{-(y + x/y)} & x > 0,\, y > 0 \\ 0, & \text{otherwise} \end{cases}$$

i. Find $E[X]$, $E[Y]$ **(6 marks)**

ii. Show that $Cov(X, Y) = 1$. **(6 marks)**

(c) The width of a slot of a duralumin forging (in inches) is normally distributed with $\mu = 0.9000$ and $\sigma = 0.0030$. The specification limits were given as 0.9000 ± 0.0050.

i. What percentage of forgings will be defective? **(3 marks)**

ii. Assuming the standard deviation σ is unknown, what is the maximum allowable value of σ that will permit no more than 1 in 100 defectives when the widths are normally distributed with $\mu = 0.9000$ and σ? **(3 marks)**

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2. (a) Bank of America's Consumer Spending Survey collected data on annual credit card charges in seven different categories of expenditures: transportation, groceries, dining out, household expenses, home furnishings, apparel, and entertainment (US Airways Attaché, December 2003). Using data from a sample of 42 credit card accounts, assume that each account was used to identify the annual credit card charges for groceries (population 1) and the annual credit card charges for dining out (population 2). Using the difference data, the sample mean difference was $\bar{d} = 850$, and the sample standard deviation was $s_d = 1123$.

   i. Formulate the null and alternative hypotheses to test for no difference between the population mean credit card charges for groceries and the population mean credit card charges for dining out. **(3 marks)**

   ii. What is the $p - value$? **(5 marks)**

   iii. Use $\alpha = 0.05$ level of significance. Can you conclude that the population means differ? **(3 marks)**

   iv. Which category, groceries or dining out, has a higher population mean annual credit card charge? **(2 marks)**

   (b) The joint probability density function of X and Y is given by

$$f(x,y) = \begin{cases} \frac{6}{7}(x^2 + \frac{xy}{2}) & 0 < x < 1,\ 0 < y < 2 \\ 0, & \text{otherwise} \end{cases}$$

   i. Verify that this is indeed a joint density function. **(3 marks)**

   ii. Find $P\{X > Y\}$ **(4 marks)**

   iii. Find $P\{Y > \frac{1}{2} | X < \frac{1}{2}\}$ **(5 marks)**

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3. (a) State without proof the Bayes' theorem. **(3 marks)**

   (b) A departmental store may adopt a new credit policy to try to reduce the number of customers defaulting on payments. A suggestion is made to discontinue credit to any customer who has been one week or more late with his/her payment at least two times. Past records show 95% of defaults were of at least two times. Also, 3% of all customers default, and 30% of those who have not defaulted have at least two late payments.

      i. Find the probability that a customer with at least two late payment will default. **(5 marks)**

      ii. Based on part (i), should the policy be adopted? Explain. **(3 marks)**

   (c) The joint density function of X and Y is given by

$$f(x,y) = \begin{cases} \dfrac{e^{-x/y}e^{-y}}{y} & 0 < x < \infty, 0 < y < \infty \\ 0, & \text{otherwise} \end{cases}$$

   Compute $E[X^2|Y = y]$ **(6 marks)**

   (d) University of Ghana wishes to demonstrate that car ownership is detrimental to academic achievement. Random samples of 100 students are selected each from students who do not own cars and students who own cars. Their respective mean grade point averages (GPA) were 2.68 with standard deviation of 0.7 and 2.55 with standard deviation of 0.6. Assuming that the independence assumption holds. Take μ_1 = the mean GPA for all students who are not car owners and μ_2 = the mean GPA for all students who are car owners.

      i. Compute a 95% confidence interval for the difference in mean $(\mu_1 - \mu_2)$? **(3 marks)**

      ii. On the basis of the interval calculated in part (i) or otherwise, can the university statistically justify that car ownership is detrimental to academic achievement? Interpret your results. Take $\alpha = 0.05$ **(5 marks)**

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