UNIVERSITY OF GHANA

FACULTY OF ENGINEERING SCIENCES
FIRST SEMESTER EXAMINATIONS: 2013/2014
LEVEL 300: BACHELOR OF SCIENCE IN ENGINEERING
**CPEN 301: SIGNALS AND SYSTEMS [*3 Credits*]**

**TIME:** 3 HOURS

**Instructions:**

1. Answer *ALL* questions in the answer booklet provided.
2. All symbols have their usual meaning.

## SECTION A [20 MARKS]

*Provide **concise** answers to the questions in this section.*

A1. What are the main parameters of a sinusoidal signal? **[3 marks]**

A2. An FM radio receiver can receive signals from different radio stations operating on different frequencies around 100MHz. However, a typical voice signal is limited between 300 Hz and 3.3 kHz. Explain briefly how the FM reception is made possible. **[4 marks]**

A3. How is a digital signal obtained from an analog signal? **[3 marks]**

A4. What is the relationship between discrete-time Fourier transform and Z transform? State condition(s) if any. **[3 marks]**

A5. What is the main condition for a signal to have a Fourier series expression? How does one obtain frequency domain components of signals that do not meet the condition? **[4 marks]**

A6. State the difference between BIBO stability and marginal stability. What are their implications on real life practical systems? **[3 marks]**

## SECTION B [44 MARKS]

B1.

Consider an amplitude modulation (AM) radio system using a carrier signal $c(t) = \cos(\omega_1 t)u(t)$. A message signal $m(t) = \sin(\omega_0 t)u(t)$ is transmitted over this system.

a. Derive the Fourier transform of the modulated signal $s(t) = [\cos(\omega_1 t)\sin(\omega_0 t)]u(t)$ with $\omega_1 >> \omega_0$. (All necessary steps must be clear). **[6 marks]**

b. Draw the spectrum of $s(t)$ **[3 marks]**



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B2.

An LTI system responds as follows:

if input $x(t) = u(t)$, then output $y(t) = 2(1 - e^{-t})u(t)$ ;

if input $x(t) = \cos(t)$, then output $y(t) = 1.414\cos(t - \frac{\pi}{4})$ .

Find $y(t)$ for the following inputs:

a. $x(t) = 5u(t) + 10\cos(2t)$

b. $x(t) = tu(t)$

**[6 marks]**

B3.

a. Compute $v[n] * x[n]$ given $v[n] = (0.25)^n u[n]$ and $x[n] = u[n]$.

b. Find the Z transform of the discrete-time signal defined as $x[n] = \{1, 2, 3, 4\}$.

**[8 marks]**

B4.

a. Evaluate $V(\omega) = \int_{-4}^{7} \sin(\omega t)(t-3)^2 \delta(3t+4)dt$

b. Evaluate the stability and the causality of the system defined by the input response $h(t) = e^{-4t}u(t+2)$.

c. Is the signal $x(t) = 2\cos(10t+1) - \sin(4t-1)$ periodic? If yes, find the period. If no, justify your answer.

**[10 marks]**

B5.

Consider an audio signal with frequency components limited to the frequency band 300Hz to 3.3 kHz. What is the maximum time interval between samples of this signal, if aliasing is to be avoided? Explain your answer.

**[5 marks]**

B6.

Consider the pulse defined for an integer $q > 0$ as

$$p[n] = \begin{cases} 1 & , n = 1, 2, \cdots q \\ 0 & elsewhere \end{cases}$$

Derive the expression of the Z transform of $p[n]$.

**[6 marks]**



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## SECTION C [36 MARKS]

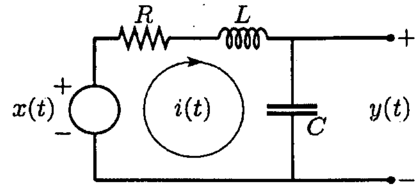


Figure 3

C1. Consider the voltage divider RLC circuit in Figure 3 to be a system where $x(t)$ and $y(t)$ are the input and the output, respectively.

a. Write the differential equation connecting the output to the input.

b. Assuming a zero-state, find the transfer function $H(s)$ of the system.

c. Find the expression of the poles of $H(s)$.

d. Assuming that the value of L and C are fixed and that the resistance R is variable, investigate the stability of the system as R varies from zero to infinity.

**[18 marks]**

C2. Consider the causal LTI system described by the following differential equation:

$$\frac{d^2y(t)}{dt^2}+6\frac{dy(t)}{dt}+8y(t)=2x(t),\ \ y(0^-)=1 \text{ and } \dot{y}(0^-)=2$$

a. Find the relationship between the Laplace transform $Y(s)$ of $y(t)$ and the Laplace transform $X(s)$ of $x(t)$.

b. Deduce the transfer function $H(s)$ of the system and the part due to initial conditions

c. Find the solution to the differential equation using the inverse Laplace transform.

**[18 marks]**

**Laplace Transform Table**

| Time Signal | Laplace Transform |
|---|---|
| $u(t)$ | $1/s$ |
| $u(t)-u(t-c),\ \ c>0$ | $(1-e^{-cs})/s,\ \ c>0$ |
| $t^N u(t),\ \ N=1,2,3,\ldots$ | $\frac{N!}{s^{N+1}},\ \ N=1,2,3,\ldots$ |
| $\delta(t)$ | 1 |
| $\delta(t-c),\ \ c$ real | $e^{-cs},\ \ c$ real |
| $e^{-bt}u(t),\ \ b$ real or complex | $\frac{1}{s+b},\ \ b$ real or complex |
| $t^N e^{-bt}u(t),\ \ N=1,2,3,\ldots$ | $\frac{N!}{(s+b)^{N+1}},\ \ N=1,2,3,\ldots$ |
| $\cos(\omega_o t)u(t)$ | $\frac{s}{s^2+\omega_o^2}$ |
| $\sin(\omega_o t)u(t)$ | $\frac{\omega_o}{s^2+\omega_o^2}$ |



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