UNIVERSITY OF GHANA

BACHELOR OF SCIENCE IN ENGINEERING
SECOND SEMESTER EXAMINATIONS: 2014/2015

CPEN 304: DIGITAL SIGNAL PROCESSING (3 Credits)

INSTRUCTION: *Answer any five (5) Questions of your choice*

TIME ALLOWED: *THREE (3) HOURS*

Q1. **(a)** Using block diagrams, explain the difference between analog signal processing and digital signal processing. Give four reasons why analog signal processing is not popular compared with its digital counterpart. [8 marks]

(b) Suppose heartbeat signals from biosensors are to be processed and display for visualization. If 90 harmonics with spectral resolution of 0.4Hz are captured, find the sampling rate that will be used if the heartbeat being monitored ranges between 65 and 190 beats per minute. [4 marks]

(c) State four (4) application areas of digital signal processing and for each area, give one (1) sample product. [4 marks]

(d) The relationship between the input and output of a system is defined by:

$$y[n] = a_0 x[n] + a_1 x[n-1] + a_2 x[n-2] + a_3 x[n-3] + a_4 x[n-4].$$

Write a mathematical expression to represent the above function. Indicate whether this filter is recursive or non-recursive and give reason(s) for your answer. Sketch the block diagram necessary to implement the filter structure. [4 marks]

Q2. **(a)** A continuous-time speech signal $x(t)$ defined by the expression below is passed through an ideal analog-to-digital converter (ADC) for processing by a system with transfer function H(z) and the output is passed through a DAC for reconstruction.

$$x(t) = 6\cos(120\pi t) + 2\cos(680\pi t) + 4\cos(1000\pi t).$$

Sketch the block diagram of the process and find the:

(i) frequency at which the speech signal should be sampled in order for the signal

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to be fully recovered at reconstruction. [2 marks]

(ii) discrete signal that is passed to the system for processing and its corresponding angular frequency if the signal is sampled at 400 samples per second. [5 marks]

(iii) reconstructed signal y(t) from the output of the DAC where y(t) = y[n] = x[tFs]. Is the reconstructed signal y(t) the same as x(t)?. Explain your answer. [8 marks]

(b) Suppose the system used to process the signal in **Q2(a)** above is a lowpass first order passive filter circuit which is defined by the differential equation below. Find the linear difference equivalent the filter system. Sketch the circuit diagram of the passive filter system. [5 marks]

$$\frac{dy}{dt} + ay(t) = bx(t), \qquad \text{where } a = 1/\mathrm{RC} \text{ and } b = 1/\mathrm{C}.$$

Q3. A certain causal system is described by the linear difference equation below:

$$y[n] = \frac{3}{4}y[n-1] - \frac{1}{8}y[n-2] + x[n].$$

(i) Sketch the filter implementation structure using direct form 1. [2 marks]

(ii) Find the transfer function of the system in the z-domain and plot the zero-pole map of the system. Do you consider the system stable? Explain. [6 marks]

(iii) Find the impulse response h[n] of the system. [4 marks]

(iv) Find the frequency response of the system. [3 marks]

(v) If the input to the system is a unit step function, find the output response y[n] of the system. [4 marks]

Q4. **(a)** You have been tasked to design a digital filter for implementation at the TOR plant. Give two (2) major reasons why you will select FIR type of digital filter over an IIR digital filter for your implementation. [2 marks]

(b) Outline the steps you will follow for the design of the FIR digital filter above. How different are these steps from that of the IIR digital filter design? [6 marks]

(c) Show that if a causal discrete time sequence $x[n]$ is shifted by a variable k, then the z-transform of the shifted sequence will be given as $z^{-k}X(z)$. [2 marks]

(ii) A causal LTI sequence x[n] is defined by $x[n] = \lambda^n u[n]$, where $u[n]$ is the unit step function. Find the discrete time Fourier transform of the sequence $x[n]$. [5 marks]

(d) Design a single pole lowpass active filter with cutoff frequency of 5 kHz and gain A_V of 20 to filter interference noise in a mobile phone. Sketch the filter circuit diagram and show all calculations. Use C = 250pF. [5 marks]

Q5. **(a)** Write a mathematical expression for the moving average FIR lowpass filter with length M = 3. Find the transfer function of this filter in the z-domain and the frequency response of the filter. [4 marks]

(b) Suppose you need a linear phase lowpass FIR digital filter to process signals from seismic activities for early warning system. If the specifications for such a filter system are as defined below, use the properties of the fixed window types in Table 1 to design the filter (find the ripples, choice of window type, filter order, and filter h[n]). Show all steps in your design calculation and sketch the magnitude response of the filter. [13 marks]

Passband edge frequency = 2.1 kHz
Stopband edge frequency = 8 kHz
Peak passband = 0.5dB
Minimum stopband attenuation = 45dB
Sampling rate = 20 kHz.

(ii) Is the filter order you obtained from the fixed window method the same as the filter order from the Kaiser approximation? [3 marks]

Q6. **(a)** Find the transfer function for the IIR first order lowpass digital filter necessary to achieve a 3-dB cutoff frequency ω_c of 0.6π. [6 marks]

(b) Suppose the digital filter in **Q5(b)** were to be designed using IIR digital filter with flat magnitude response characteristics. Show the steps you will follow to design the IIR filter $G(z)$. Use bilinear transformation method for the filter design. Assume the transfer function $H(s)$ for your analog filter resulting from your design is given by the expression below. [14 marks]

$$H(s) = \frac{(s+1)}{(s+1)(s+3)}.$$

Table 1

Type of Window	Mainlobe width	Mainlobe/ sidelobe	Minimum stopband attenuation
Rectangular	$4\pi/M$	13dB	21 dB
Hanning	$8\pi/M$	32dB	44 dB
Hamming	$8\pi/M$	43dB	53 dB
Blackman	$12\pi/M$	58dB	74 dB

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Useful Formulae

1. $(1-\delta_P) = \dfrac{1}{\sqrt{1+\varepsilon^2}}$

2. $H_{LP}(z) = \dfrac{(1-\alpha)}{2}\dfrac{[1+z^{-1}]}{[1-\alpha z^{-1}]}$

3. $\cos\omega_c = \dfrac{2\alpha}{1+\alpha^2}$

4. $\omega_a = \tan(\omega_D/2)$

5. $N \geq \dfrac{1}{2}\dfrac{\log_{10}\left(\dfrac{A^2-1}{\varepsilon^2}\right)}{\log_{10}\left(\dfrac{\omega_S}{\omega_P}\right)}$

6. $N = \dfrac{-20\log_{10}(\delta_P\delta_S)^{1/2} - 13}{14.6(\omega_S - \omega_P)/2\pi}$

7. $|H(\omega)|^2 = \dfrac{1}{[1+(\omega_P/\omega_C)^{2N}]} = \dfrac{1}{1+\varepsilon^2} = \dfrac{1}{A^2}$

8. $H(s) = \dfrac{\omega_C^N}{\prod_{k=0}^{N-1}(s-s_k)}$

9.

$$Hanning: w[n] = 0.5 + 0.5\cos\left(2\pi\frac{n}{2M \div 1}\right), \quad -M \leq n \leq M$$

$$Hamming: w[n] = 0.54 + 0.46\cos\left(2\pi\frac{n}{2M+1}\right), \quad -M \leq n \leq M$$

$$Blackman: w[n] = 0.42 + 0.5\cos\left(2\pi\frac{n}{2M+1}\right) \div 0.08\cos\left(4\pi\frac{n}{2M+1}\right) \quad -M \leq n \leq M$$

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