



UNIVERSITY OF GHANA  
FACULTY OF ENGINEERING SCIENCES  
FIRST SEMESTER EXAMINATIONS, 2011/2012

LEVEL 100: BACHELOR OF SCIENCE IN ENGINEERING

FAEN 101: ALGEBRA

DURATION: THREE (3) HOURS

Instructions:

1. Answer all questions from Section A and any two questions from Section B
2. Calculators NOT allowed
3. Graph paper recommended

1. Solve the following equations:

(i)  $\log_{10} x = 3$

(ii)  $\log_3 x + \log_3 2 = \log_3 8$

(iii)  $\frac{2}{1 - e^{-2s}} = 4$

(iv)  $\frac{3}{1+e^{-t}} = \frac{1}{1-e^{-t}}$

(v)  $\frac{3e^{2x}}{e^{2x} + 1} = e$

2. A radio-active element decays as time passes. The amount of the element present in a sample is given by the equation:

$$A = 6e^{-0.333t}$$

Where A is the amount of the element (grams) in the sample t years after the measurements started.

- (i) Find the amount, correct to two significant figures, of the element present 3 years after the start of the measurements.
- (ii) Find, to the nearest tenth of a year, the time taken before the amount in the sample is 2 grams.
- (iii) Find the 'half life' (time taken for the original element to decay to half its original amount) of the element, correct to the nearest tenth of a year.

3. Study the following function f: determining its domain, range, various asymptotes, the x- and y-intercepts and sketch its graph:  $f = \frac{x^2 + x - 2}{x^2 - x - 2}$



7.

4. (a) If  $z = \cos\theta + i\sin\theta$  show that

$$\frac{1}{z} = \cos\theta - i\sin\theta, \quad z^5 = \cos 5\theta + i\sin 5\theta, \quad \frac{1}{z^5} = \cos 5\theta - i\sin 5\theta$$

Show further that

$$\left(z + \frac{1}{z}\right)^5 = 32\cos^5\theta \text{ and by expanding } \left(z + \frac{1}{z}\right)^5, \text{ prove that}$$

$$\cos 5\theta + 5\cos 3\theta + 10\cos\theta = 16\cos^5\theta$$

(b) Find all the 6 roots of  $-\frac{1}{64}$  i.e. find  $x$  such that  $x^6 = -\frac{1}{64}$ .

## Section B

5. Find the eigenvalues and eigenvectors of the following:

a.  $A = \begin{bmatrix} -1 & -4 \\ 3 & -2 \end{bmatrix}$

b.  $A = \begin{bmatrix} 1 & -4 & -2 \\ 0 & 3 & 1 \\ 1 & 2 & 4 \end{bmatrix}$

6. (a) Find all the other solutions of  $f(x)$  for  $f(x) = x^4 + x^3 - 2x^2 - 6x - 4 = 0$  if  $-1+i$  is one of the solutions of  $f(x)$ . Sketch its graph.(b) Find the exact value of  $\tan 105^\circ$ .7. Given the vectors  $\mathbf{a} = (2, 1, 0)$ ,  $\mathbf{b} = (2, 1, -1)$  and  $\mathbf{c} = (0, 1, 1)$  evaluate

a)  $\mathbf{a} \times \mathbf{b}$

b)  $(\mathbf{a} \times \mathbf{b}) \times \mathbf{c}$

c)  $(\mathbf{a} \cdot \mathbf{c})\mathbf{b} - (\mathbf{b} \cdot \mathbf{c})\mathbf{a}$