UNIVERSITY OF GHANA

Second Semester Examination : 2012/2013

Level 200: Bachelor of Science in Engineering

FAEN 202; Differential Equations (4 Credits)

Time Allowed :Three (3) Hours

Instructions: Answer Any Four Questions in All

Question 1

(a) Solve the differential equation $\frac{dy}{dx} + 5y = \sin x$

[5 Marks]

(b) Solve the differential equation $\left(6x^5y^3 + 4x^3y^3\right) dx + \left(3x^6y^2 + 3x^4y^2\right) dy = 0$

[5 Marks]

(c) Solve the differential equation $(x^3 + y^3)\, dx - 3xy^2 dy = 0$

[5 Marks]

(d) According to Newton's law of cooling, the rate at which a substance cools in moving air is proportional to the difference between the temperature of the substance and that of air. If the temperature of air 300K and the substance cools from 370K to 340K in 15 minutes, find when the temperature will be 310K.

[10 Marks]

A

Examinar: John Afrim

Question 2

(a) Solve the differential equation $\frac{d^3y}{dx^3} + 2\frac{d^2y}{dx^2} + 25\frac{dy}{dx} + 50y = 3e^x\sin x$

[15 Marks]

(b) The equation of motion for the spring-mass system is given by

$m\frac{d^2x}{dt^2} + \delta\frac{dx}{dt} + kx = f(t)$ with their usual meaning.

The spring mass system has an attached mass of $10g$ and the spring constant is $100g/s^2$. A dashpot mechanism is attached, which has damping coefficient of $20g/s$. The mass is pulled down and released. At time $t = 0$, the mass is 3cm below the rest position and moving upward at $5cm/s$.Determine and solve the equation of motion if a force of $30\cos 4t$ acts on the particle.

[10 Marks]

Question 3

(a) Solve the differential equation $\frac{d^3y}{dx^3} + 2\frac{d^2y}{dx^2} - 5\frac{dy}{dx} - 6y = 3x^2e^x$

[15 Marks]

(b) Recall that the equation governing an L, R, C circuit is given by $L\frac{d^2q}{dt^2} + R\frac{dq}{dt} + \frac{1}{C}q = e(t)$ with their usual meaning.

An RLC circuit has a voltage source of $e(t) = 5cos2t$ volts. Values for the components are $R = 2\Omega$, $L = 1H$ and $C = \frac{1}{17}F$. Initially, the charge on the capacitor and the current in the resistor are zero. Find the charge on the capacitor and the current as a function of time.

[10 Marks]

Question 4

(a) Determine the number of arbitrary constants that the system of equations below should have.

[5 Marks]

$$2\frac{dx}{dt} + \frac{dy}{dt} - 4x - y = 4e^{2t} + t \quad \text{and} \quad \frac{dx}{dt} + 3x + y = e^{t} - 1$$

(b) Solve the equations

[15 Marks]

(c) Find a particular solution when $x = 1$, $y = 0$ and $t = 0$

[5 Marks]

Examiner: John Afrim

Question 5

(a) You are given that

$$(\alpha)\ \ \mathcal{L}\left(\cos bt\right) = \frac{s}{s^2+b^2} \qquad (\beta)\ \ \mathcal{L}\left(\sin bt\right) = \frac{b}{s^2+b^2}\ \text{ and } \mathcal{L}\left(e^{at}\right) = \frac{1}{s-a}$$

Show that

(i) $\mathcal{L}\left(e^{at}\cos bt\right) = \frac{s-a}{(s-a)^2+b^2}$ (ii) $\mathcal{L}\left(e^{at}\sin bt\right) = \frac{b}{(s-a)^2+b^2}$

(iii) $\mathcal{L}\left(\sinh at\right) = \frac{a}{s^2-a^2}$ (iv) $\mathcal{L}\left(\cosh at\right) = \frac{s}{s^2-a^2}$

[9 Marks]

Question 5(b)

Obtain the Laplace Transform of the following.

(i) $f(t) = 4e^{3t} - 5\sin 6t$

(ii) $f(t) = 5\sinh 3t + 3\cos 5t + 6$

(iii) $f(t) = e^{8t}\cos 9t$

[6 Marks]

Question 5(c)

Obtain the inverse Laplace transform for the following

(i) $\frac{4s+7}{s^2-25}$ (ii) $\frac{8}{s^2+8s+41}$

(iii) $\frac{s+3}{s^2+6s+19}$ (iv) $\frac{3s-17}{s^2-6s+13}$

[10 Marks]