University of Ghana

BACHELOR OF SCIENCE IN ENGINEERING

FIRST SEMESTER EXAMINATIONS 2015/2016

DEPARTMENT OF COMPUTER ENGINEERING

CPEN 401: Control System Analysis and Design (3 Credits)

INSTRUCTION: Answer All Questions. Show all work for full marks

TIME ALLOWED: 3 Hours

SECTION A: Control System Concepts and Theory

A1. Name three reasons for using feedback control systems and at least one reason for not using them. [3 marks]

A2. What is the difference between open loop and closed loop control systems. Give three examples of open-loop systems. [3 marks]

A3. Name the three major design criteria for control systems. Describe a typical control system design task. [6 marks]

A4. Name three approaches used in the mathematical modeling of control systems. [3 marks]

A5. What is system modelling? Give two reasons for modelling systems using state space representation. [5 marks]

A6. Describe a typical control system analysis task. [5 marks]

A7. Where do system poles have to be to ensure that a system is not unstable? [2 marks]

A8. Briefly describe the design procedure for a controller. [4 marks]

A9. What is an observer? Under what conditions would you use an observer in your state-space design of a control system? [4 marks]

A10. Name two functions that the digital computer can perform when used with feedback control systems. [2 marks]

A11. Name three advantages of using digital computers in a closed loop control system. [3 marks]

SECTION B: System Analysis and Design Applications

B1. (a) Create a signal flow graph for the block diagram shown in Fig. 1, and using signal flow graph and Mason's gain formula, determine the transfer function $\frac{Y(s)}{R(s)}$.

[5 marks]

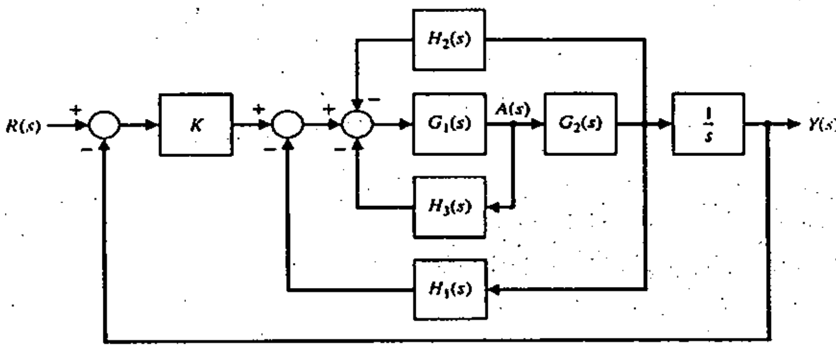

Fig. 1: Block diagram of control system

(b) A system has an open loop transfer function $G(s) = \frac{2}{s(s+1)(s+5)}$ and it is used in a closed loop with unity feed back.

i. Draw the block diagram arrangement and determine if the system is stable or not.

[5 marks]

ii. Determine the damping ratio and natural frequency of the approximated second order system and find the $\pm$ 2% settling time of a unit step response.

[5 marks]

B2. Consider the closed-loop system below, where

$$G(s) = \frac{K}{s+10} \text{ and } H(s) = \frac{14}{s+5}$$

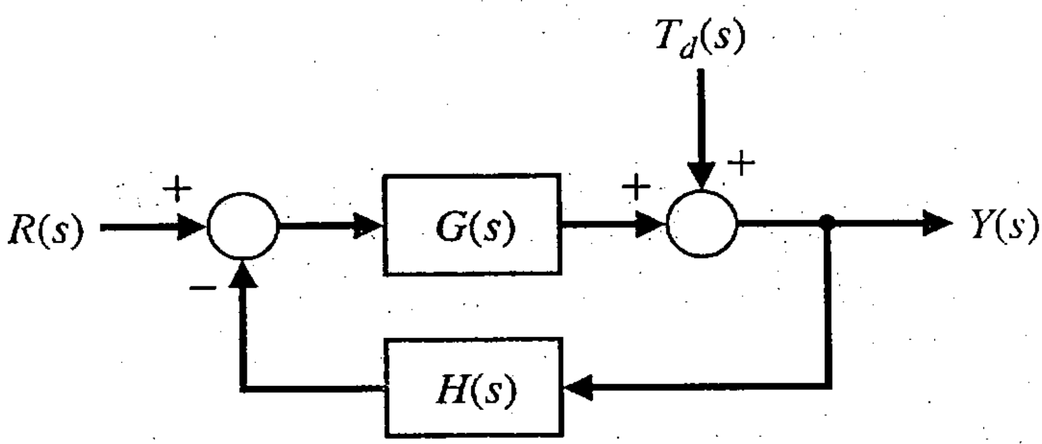

Fig. 2: Analog control system

(a) Compute the transfer function $T(s) = \frac{Y(s)}{R(s)}$ [2 marks]

(b) Define the tracking error to be $E(s) = R(s)Y(s)$. Compute $E(s)$ and determine the steady-state tracking error $e(t)$ due to a unit step input. [4 marks]

(c) Compute the transfer function $\frac{Y(s)}{T_d(s)}$ and determine the steady-state error $e(t)$ of the output due to a unit step disturbance input. [4 marks]

(d) A standard second order system has a damping ratio of 0.7 and a natural frequency of 40 rads/s. Calculate the damped resonant frequency, the rise time, the peak time, the peak value and the 5% settling time for a unit step

[5 marks]

B3. Find the control canonical state-space formulation (state equations) for a discrete time system described by the difference equation

$$y(k+2) - 0.3y(k+1) + 0.02y(k) = e(k+2) - 1.7e(k+1) + 0.72e(k)$$

where, y is the output and e is the input.

(a) Calculate the transfer function $\frac{Y(z)}{E(z)}$ [3 marks]

(b) Draw a simulation diagram of this system in the control canonical form. [2 marks]

(c) Defining the output of the first delay element as a state x_2 and the output of the second delay element as a state x_1 (bottom up order), write a set of state equations, and an output equation. [5 marks]

(d) List the poles and zeros of this system [2 marks]

B4. For the system shown in the figure below, the digital filter solves the difference equation

$$m(k) = 0.8m(k-1) + 0.2e(k)$$

The plant transfer function $G_p(s)$ is given by

$$G_p(s) = \frac{1}{(s+1)(s+2)}$$

The sampling rate is 10Hz or $T = 0.1$ sec. The Digital to Analog converter (D/A) is a Zero Order Hold (ZOH).

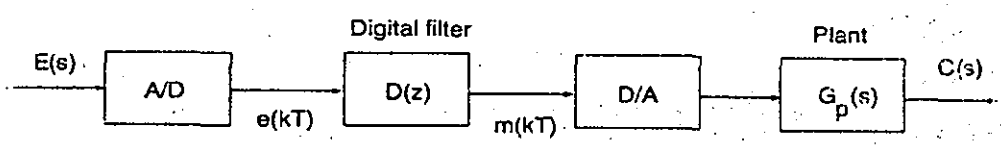

Fig. 3: Digital Control system block diagram

(a) Write the transfer function of the digital filter, $D(z)$

[3 marks]

(b) Find the system transfer function $\frac{C(z)}{E(z)}$. You must show the entire mathematical process to calculate the starred transform, then to convert to the z-transform.

[10 marks]

B5. An analog system with the transfer function,

$$G(s) = \frac{1}{s^2 + s + 1}$$

has state feedback to form a feedback control system as shown in Figure 4. The state variables are x_1 and x_2. The sampling rate is 10 samples per second.

(a) Using the state variables defined by $\begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} y \\ \dot{y} \end{bmatrix}$, obtain the analog state equation of $G(s)$ which takes the form of

$$\mathbf{x}'(t) = \mathbf{A}\mathbf{x}(t) + \mathbf{b}u(t)$$

and the output equation which gives $y(t)$. [8 marks]

(b) Obtain the state transition matrix $\phi(t)$ from $\mathbf{A}$ [5 marks]

(c) Is the system given above controllable and observable? Explain. [5 marks]

(d) Obtain the discrete time state equation expressed in the form of

$$\mathbf{x}(k+1) = \mathbf{P}\mathbf{x}(k) + \mathbf{q}u(k)$$

using exact method derived from first principles. [5 marks]

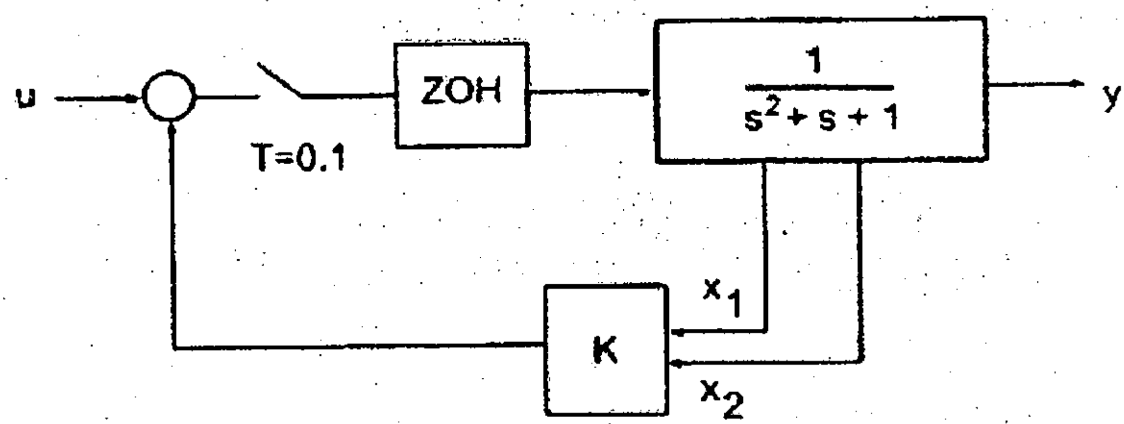

Figure 4: A state feedback control system