Conceptual approach to the link between motif composition and functioning

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A motif is any set of three species having a degree of at least one. For simplicity we focus on the case of single-link degrees, of which there are five, are these are the most commonly occuring.

We assume that all species have equal growth rate r and crowding factor q, so that (i) they can persist at an equilibrium density higher than 0 when alone, and (ii) the only differences in their equilibrium dynamics is triggered by interactions. For any species i in a motif, its dynamics is given by

$$\frac{dN_i}{dt} = N_i \left[r - qN_i + \sum_j \left(\alpha_{ij} - \alpha_{ji} \right) N_j \right] \tag{1}$$

To alternate between one of the five single-link motifs, one simply needs to switch the relevant α coefficients to 0. For example, in a linear food chain, all coefficients but α_{12} and α_{23} are turned off.

We derive the equilibrium density of this system with three species, yielding general expressions for N_1^*, N_2^* and N_3^*. We furthermore define the total equilibrium biomass of the system as $T^* = \sum_i N_i^*$. We investigate the function of T^* as a function of the values of all α coefficients.

The general expression of T^* is

$$T^* = r \times \frac{\alpha_{12} - \alpha_{13} - \alpha_{21} + \alpha_{23} + \alpha_{31} - \alpha_{32} + 3q}{(\alpha_{12} - \alpha_{21})(\alpha_{12} - \alpha_{21} - \alpha_{23} - \alpha_{32}) + q \times (q - \alpha_{13} + \alpha_{31})} \tag{2}$$

Using this general expression, we can derive the equilibrium biomass T_n^* in any motif n. To simplify this expression, we correct the total biomass by the species growth rate, so that $B_n^* = T_n^*/r$ In a linear food chain ($n = 1$), all coefficients but α_{12} and α_{23} are set to 0, yielding

$$B_1^* = \frac{\alpha_{12} + \alpha_{23} + 3q}{\alpha_{12} \times (\alpha_{12} - \alpha_{23})} \tag{3}$$

In an omnivory motif ($n = 2$), species 1 can consume species 3, so that α_{13} is restored. This yields

$$B_2^* = \frac{\alpha_{12} - \alpha_{13} + \alpha_{23} + 3q}{\alpha_{12} \times (\alpha_{12} - \alpha_{23}) + q \times (q - \alpha_{13})} \tag{4}$$

In a trophic loop motif ($n = 3$), species 3 becomes a consumer of species 1, so that α_{13} is set to 0, and α_{31} is restored. This yields

$$B_3^* = \frac{\alpha_{12} + \alpha_{23} + \alpha_{31} + 3q}{\alpha_{12} \times (\alpha_{12} - \alpha_{23}) + q \times (q + \alpha_{31})} \tag{5}$$

In an exploitative competition motif ($n = 4$), species 2 is consumed by both species 1 and 3, so that only α_{12} and α_{32} are kept. This yields

$$B_4^* = \frac{\alpha_{12} - \alpha_{32} + 3q}{\alpha_{12} \times (\alpha_{12} - \alpha_{32})} \tag{6}$$

Finally, in an apparent competition motif ($n = 5$), species 2 is consuming both species 1 and 3, so that only α_{21} and α_{23} are kept. This yields

$$B_5^* = \frac{\alpha_{23} - \alpha_{21} + 3q}{\alpha_{21} \times (\alpha_{21} + \alpha_{23})} \tag{7}$$