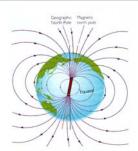


# Describing the Earth's field the best fit dipole

This first order simple model/description of the field allows use of paleomagnetic observations to determine past plate motions



#### **Magnetic potential**

$$V(\mathbf{r}) = \frac{1}{4\pi r^3} \mathbf{m} \cdot \mathbf{r}$$

The Earth's best fit dipole moment,  $m = 7.94x10^{22} Am^2$ 

Magnetic field is the derivative of the potential

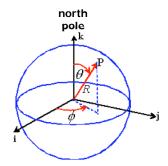
$$\mathbf{B}(\mathbf{r}) = -\mu_0 \nabla V(\mathbf{r})$$

given the magnetic permeability of free space,  $\mu_0 = 4\pi x 10^{-7} \ kg \ m \ A^{-2} \, s^{-2}$ 

dot or scalar product

 $\mathbf{m} \cdot \mathbf{r} = mr \cos \theta$ 

### Spherical polar coordinates



- R radius
- **6** colatitude, 0 to pi (degrees from north pole)
- Φ longitude, 0 to 2pi

## Conversion from/to Cartesian coordinates

$$R = \sqrt{x^2 + y^2 + z^2} \qquad x = R\sin\theta\cos\phi$$
  

$$\theta = \cos^{-1}z/R \qquad y = R\sin\theta\sin\phi$$
  

$$\phi = \tan^{-1}y/x \qquad z = R\cos\theta$$

#### **Gradient operator**

$$\nabla f = \left(\frac{\partial f}{\partial r}, \frac{1}{r} \frac{\partial f}{\partial \theta}, \frac{1}{r \sin \theta} \frac{\partial f}{\partial \phi}\right)$$

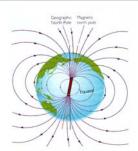
EPS 122: Lecture 5 – Earth's magnetic field

## Describing the Earth's field the best fit dipole

Magnetic field in spherical polar coordinates

$$\mathbf{B}(\mathbf{r}) = (B_r, B_\theta, B_\phi)$$

the radial, southerly, easterly components



If the Earth's magnetic dipole moment is aligned along the z-axis:

$$V(\mathbf{r}) = \frac{1}{4\pi r^3} \mathbf{m} \cdot \mathbf{r}$$
$$= \frac{mr \cos \theta}{4\pi r^3} = \frac{m \cos \theta}{4\pi r^2}$$

we can calculate the magnetic field at any point....

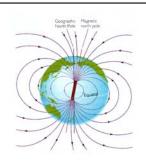
# Describing the Earth's field the best fit dipole

Three components

$$B_r(r,\theta,\phi) = -\frac{2\mu_0 m \cos \theta}{4\pi r^3}$$

$$B_{\theta}(r,\theta,\phi) = -\frac{\mu_0 m \sin \theta}{4\pi r^3}$$

$$B_{\phi}(r,\theta,\phi) = 0$$



tal field
$$B(r,\theta,\phi) = \sqrt{B_r^2 + B_{\theta}^2 + B_{\phi}^2 + \frac{\mu_0 m}{4\pi r^3}} \sqrt{1 + 3\cos^2 \theta}$$

At the north pole

$$B_r(r,0,\phi) = -\frac{\mu_0 m}{2\pi r^3}$$

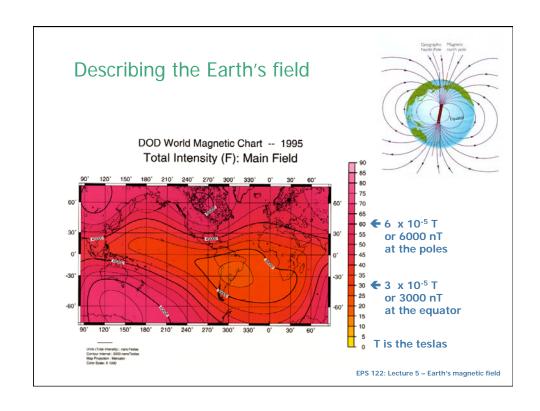
$$B_{\theta}(r,0,\phi) = 0$$

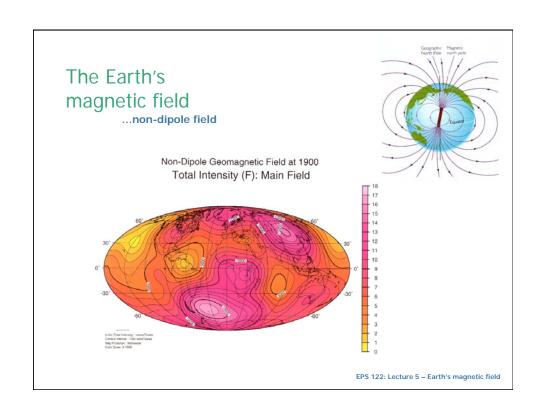
At the equator

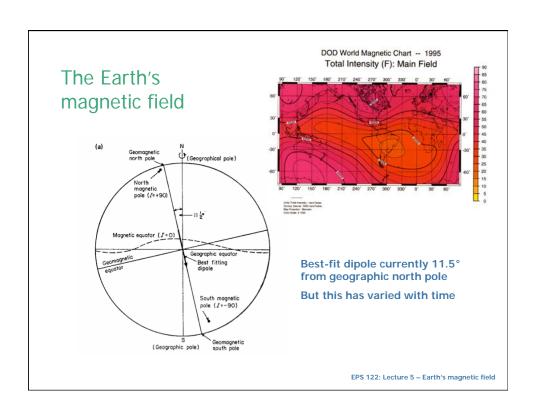
$$B_r(r,90,\phi)=0$$

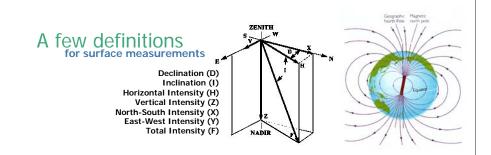
$$B_{\theta}(r,90,\phi) = -\frac{\mu_0 m}{4\pi r^3}$$

Magnitude of the total field at the pole is twice as strong as at the equator







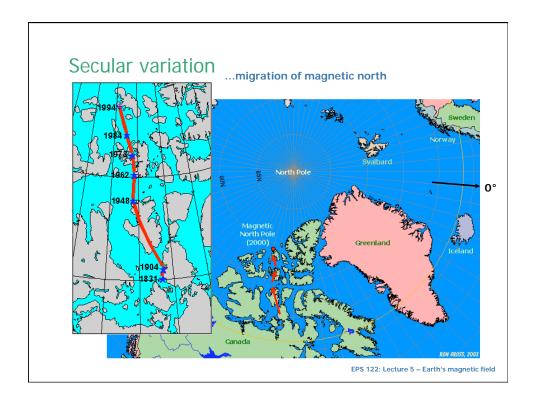


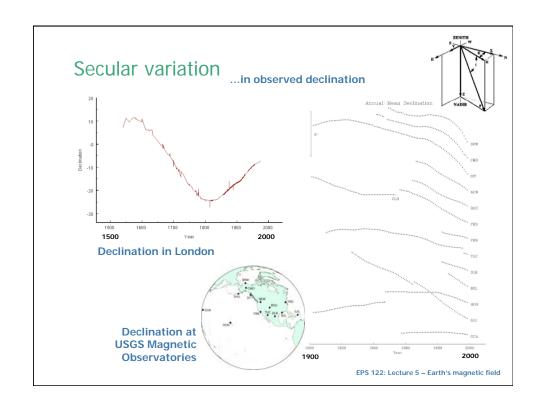
$$Z(R, \theta, \phi) = -B_r(R, \theta, \phi)$$

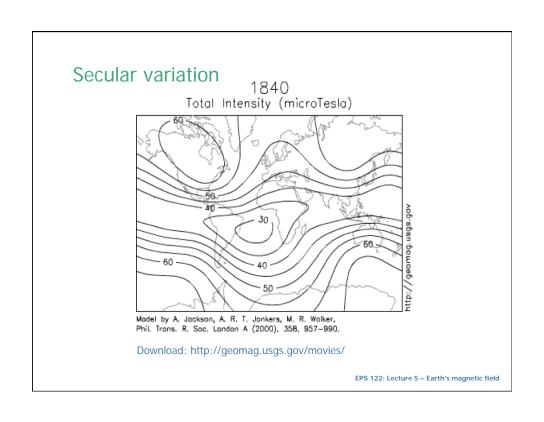
$$H(R,\theta,\phi) = |B_{\theta}(R,\theta,\phi)|$$

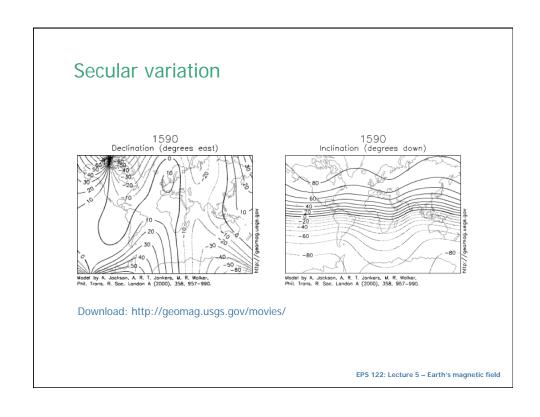
magnetic inclination 
$$\tan I = \frac{Z}{H} = \frac{2\cos\theta}{\sin\theta} = 2\cot\theta = 2\tan\lambda$$

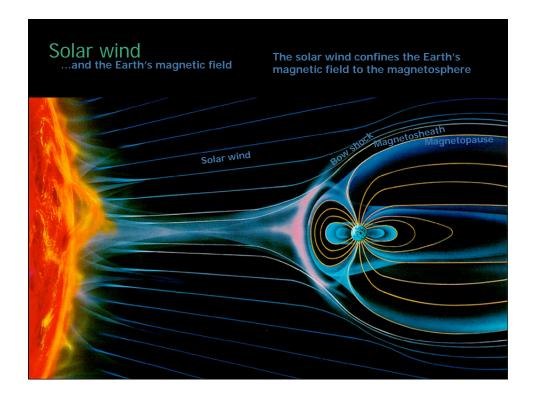
where  $\lambda$  is the magnetic latitude ( $\lambda = 90 - \theta$ )











#### The sunspot cycle and flips in the Sun's magnetic field X-ray image of the Sun · 11 year sunspot cycle Cycle 23 Sunspot Number Prediction (February 2001) Sunspots are intense magnetic loops which poke out of the photosphere · Sun's dipole flips at peak in sunspot activity · Last peak/flip: February 2001 · The magnetic south pole is now at the 1996 geographical north NASA - February 2001 pole • On Earth the field flips at intervals of ~200,000 years (5,000 to 50 mill) • The last reversal on Earth happened 740,000 years ago

