1 Definition

Declination: The angle between the direction of the compass and the true direction of the north.
Inclination: The vertical angle between the north and the direction the compass points when it is suspended in free fall.
Magnetic poles: Positions on Earth's Surface for where the horizontal component of the magnetic field is 0.
Magnetic Equator: An undulating curve near the Earth's geographical equator where the vertical component of the magnetic field is 0.
Cosmic Rays: High energy particles coming from cosmic. Solar particles count as one of them.
Before modeling the Earth as a magnetic dipole, it is best to define a few terminologies about magnetic dipole, which is the first order approximation of the Earth's magnetic field.

1. **Magnetic Dipole:** A pair of South and North magnetic poles kept closed together or viewed from a large distance. More formally, it is the limit of either a closed loop of electric current or a pair of poles as the dimensions of the source are reduced to zero while keeping the magnetic moment constant.

2. **Magnetic Dipole Moment:** A quantity that represents the magnetic field strength and orientation of a magnet or other objects that produces a magnetic field.

3. **Magnetic Vector Potential ($\vec{A}$):** It is a vector field that serves as the potential for magnetic field. The curl of the magnetic field potential is the magnetic field.

$$\vec{B} = \nabla \times \vec{A}$$

2 Magnetic field as Dipole

We are able to write the Earth's magnetic pole by modeling it according to a dipole - a first order approximation.

$$\mathbf{B(r)} = (B_r, B_\theta, B_\phi) \tag{1}$$

When we align the Earth's magnetic field along the z-axis:

$$\begin{aligned} V(\mathbf{r}) &= \frac{1}{4\pi r^3} m \cdot r \\ &= \frac{mr\cos(\theta)}{4\pi r^3} \\ &= \frac{m\cos(\theta)}{4\pi r^2} \end{aligned} \tag{2}$$

We can derive three equations separately for B_r, B_θ, and B_ϕ, stated as the following. We fine **m** as the dipole moment, and μ_o as the vacuum permeability.

$$\begin{aligned} B_r(r,\theta,\phi) &= \frac{-2\mu_o mcos(\theta)}{4\pi r^3} \\ B_\theta(r,\theta,\phi) &= \frac{-\mu_o msin(\theta)}{4\pi r^3} \\ B_\phi(r,\theta,\phi) &= 0 \end{aligned} \tag{3}$$

Thus the total field, which is just the vector sum of the three B functions.

$$B(r,\theta,\phi) = \sqrt{B_r^2 + B_\theta^2 + B_\phi^2} = \frac{\mu_o m}{4\pi r^3}\sqrt{1+3cos^2(\theta)} \tag{4}$$

At the **North Pole**

$$\begin{aligned} B_r(r,\theta,\phi) &= \frac{-\mu_o mcos(\theta)}{2\pi r^3} \\ B_\theta(r,\theta,\phi) &= 0 \end{aligned} \tag{5}$$

At the **Magnetic Equator**

$$\begin{aligned} B_r(r,\theta,\phi) &= 0 \\ B_\theta(r,\theta,\phi) &= \frac{-\mu_o msin(\theta)}{4\pi r^3} \end{aligned} \tag{6}$$

Let's define

$$k_o = \frac{\mu_o m}{4\pi}$$

and we know that at the Earth's Dipole $k_o = 8 * 10^{15}\ Tm^3$
At the dipole, $\theta = \pm 90°$, and we return the value of B as $B = \frac{2k_o}{R_E^3} \approx 60\mu T$
At the dipole, $\theta = \pm 0°$, and we return the value of B as $B = \frac{k_o}{R_E^3} \approx 30\mu T$

Conceptually, Earth's magnetic field does not lie directly on the north and south pole and is not necessarily antipodal. The best fit dipole currently is 11.5° from the geographical north pole.

3 Modeling the Earth's Magnetic Field - 2D

In this section, We strive to plot the magnetic field lines from the Earth first to see how might the magnetic field lines behave when particles from the Sun enters the field.
From the equations we are given above, we acquire:

$$\begin{aligned} B(r,\theta) &= \frac{-2\mu_o mcos(\theta)}{4\pi r^3} + \frac{-\mu_o mcos(\theta)}{4\pi r^3} \\ B(r,\theta) &= \frac{-\mu_o m}{4\pi r^3}(2cos(\theta) + sin(\theta)) \end{aligned} \tag{7}$$

Here, the m stands for the Earth's magnetic dipole moment, which from L03 we know the best value should be $m = 7.94 * 10^{22} Am^2$. The value of is $\mu_o = 4\pi * 10^{-7} Hm^{-1}$.
We define

$$M_E = \frac{-\mu_o m}{4\pi} = -7.94 * 10^{15} AHm \tag{8}$$

which, satisfactorily, is the highly as k_0 we acquire from other source we found. Thus we acquire the equation:

$$B(r,\theta) = \frac{-7.94 * 10^{15}}{r^3}(2cos(\theta) + sin(\theta)) \tag{9}$$

which gives us a beautiful plot of the shape of a pea, tilted at 11.5° to the left.

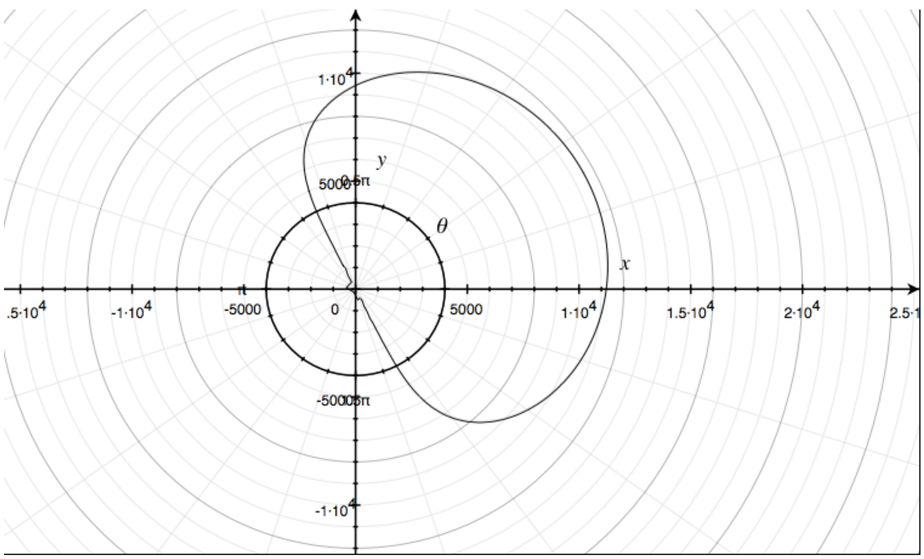


Figure 1: Polar Coordinate Graph of the Earth's Magnetic Field

However, if we want to plot it in Python, we will need to change the coordinates into Cartesian. For which is calculated as the following:

$$\begin{aligned}(x^2+y^2)^{\frac{5}{2}} &= \frac{-M_E(y+2x)}{2} \\ (x^2+y^2)^{\frac{5}{2}} &= \frac{-7.94 * 10^{15}(y+2x)}{2}\end{aligned} \tag{10}$$

which gives us almost the same graph (the origin of the Polar graph of Figure 1 is slight twisted).

If we add a negative sign to the graph, we acquire the other half of the pea. The next step will be to plot multiple lines of the magnetic field, as the

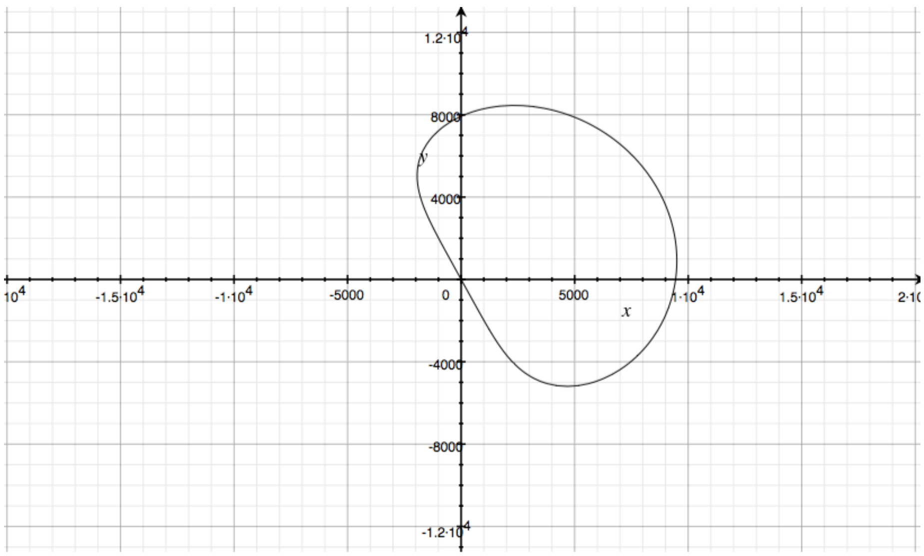


Figure 2: Cartesian Coordinate Graph of the Earth's Magnetic Field

Earth does. However, the equations should yield multiple lines according to r instead of just one! Something must be wrong. We need to draw the contour of B.

We complete this using the help of Wolfram Mathematica, which we can plot using the command Contour.

```
Y[r_, f_] = -7.94*10^15 (2 Cos[f] + Sin[f])/r^3
ContourPlot[Y[Sqrt[x^2 + y^2], ArcTan[x, y]], {x, -8*10^6, 8*10^6},
     {y, -8*10^6, 8 *10^6}]
```

which generates the following graph:

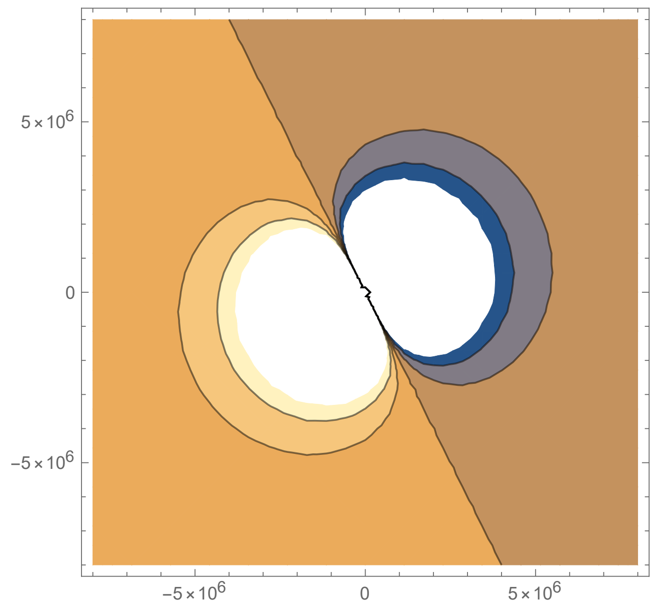


Figure 3: Earth's Magnetic Field Line contour Map using Wolfram Mathematica

I will try to use python to plot it later and include the code for it.

4 Modeling Earth's Magnetic Field - 3D

We therefore try to plot the 2D version of Magnetic field into 3D version by also using Mathematica. By using the formula

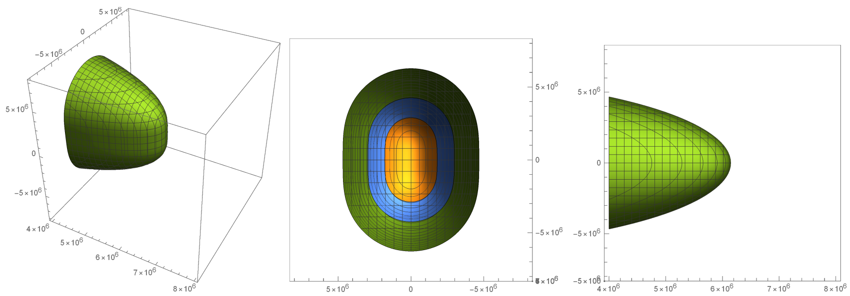
$$B(r,\theta,\phi) = \frac{\mu_o m}{4\pi r^3}\sqrt{1+3cos^2(\theta)} \tag{11}$$

The code should be the following:

```
Y[r_, f_, p_] = -7.94*10^15 (Sqrt[1 + 3 (Cos[f])^2])/r^3
ContourPlot3D[
Y[Sqrt[x^2 + y^2 + z^2], ArcCos[z/ Sqrt[x^2 + y^2 + z^2]],
ArcTan [y/ x]], {x, 5*10^6, 8*10^6}, {y, -8*10^6,
8*10^6}, {z, -8*10^6, 8*10^6}]
```

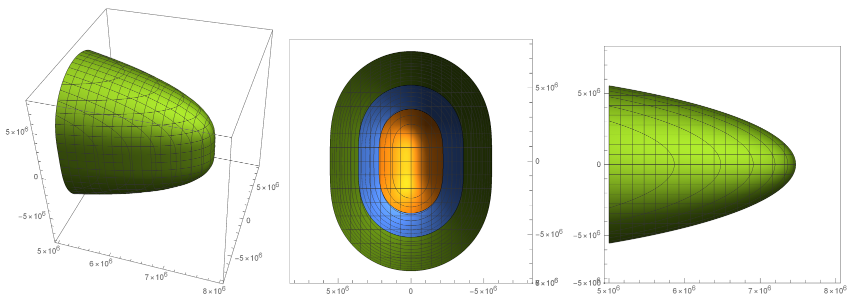
However, I realize that if we put the range of x, y, z, all as $\{x, -8*10^6, 8*10^6\}, \{y, -8*10^6, 8*10^6\}, \{z, -8*10^6, 8*10^6\}$. However, this will results in the command of $\frac{1}{0}$, and the command will not run.

Thus, I try to set the value of x axis into $\{x, 5*10^6, 8*10^6\}$ and $\{x, 4*10^6, 8*10^6\}$. Which gives us the following graphs.



(a) Ranging from $4.0*10^6$ (b) Side view of $4.0*10^6$ (c) Top view of $4.0*10^6$

We now try to plot the graph for the range $\{x, 5*10^6, 8*10^6\}$.



(a) Ranging from $5.0*10^6$ (b) Side view of $5.0*10^6$ (c) Top view of $5.0*10^6$

I'm not entirely sure about why it is not displaying lines like magnetic field. However, from the side view we can clearly see the layers of the contour map for which we can model that the magnetic field of the Earth decreases as it goes outwards.

5 Solar wind

When solar particles enter Earth's magnetic field, they are highly energized. Thus, we can view that particles are accelerated through sun as if it were passing through an electric or magnetic field. A spectra of particles of energy around 10^{20} is found.

The types of particles which are emitted from the solar flame varies but are mostly three types: electrons, protons and alpha particles (which includes two protons and two neutrons). In the following model, we will be most likely using electron as the first example we modeled

The velocity from the solar wind of the particle is approximately 400,000 m/s. While the position of which the solar wind enters the Earth magnetic field can be of our choice in a 3-D model (possibly any point 1 Earth diameter away).