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SPECIAL RELATIVITY IN BEAM TRAJECTORY SIMULATION IN SMALL ACCELERATORS

Pramudita Anggraita, Budi Santosa, Taufik, Emy Mulyani, Frida Iswinning Diah
Center for Technology of Accelerators and Material Process,
National Nuclear Energy Agency, Jl. Babarsari, Yogyakarta, Indonesia 55281
Email : pramudita@batan.go.id

ABSTRACTS

Calculation for trajectory simulation of particle beam in small accelerators should account special relativity effect in the beam motion, which differs between parallel and perpendicular direction to the beam velocity. For small electron beam machine of 300 keV, the effect shows up as the rest mass of electron is only 511 keV. Neglecting the effect yields wrong kinetic energy after 300 kV of dc acceleration. For a 13 MeV PET (positron emission tomography) baby cyclotron accelerating proton beam, the effect increases the proton mass by about 1.4% at the final energy. To keep the beam isochronous with the accelerating radiofrequency, a radial increase of the average magnetic field must be designed accordingly.

Keywords: special relativity, beam trajectory, electron beam machine, proton cyclotron

INTRODUCTION

In 2003 a home made low energy (small) 300 keV dc (direct current) electron beam machine was commissioned at a research center of the National Nuclear Energy Agency in Yogyakarta. To study the electron beam trajectory in the machine, a simulation of the beam trajectory was carried out.[1] The simulation showed that appropriate relativistic beam dynamics should be applied to obtain the correct result. When this was not done and non-relativistic dynamics were applied, the simulation yielded kinetic energy which was less than it was supposed to be at the end of 300 kV dc acceleration. The necessity of the relativistic approach in this case could be easily understood as the electron mass is only 511 keV/c^2, which is comparable to the final low energy of the small electron beam machine.

In 2009 at the same research center a project to design and build a 13 MeV PET (positron emission tomography) was started. Even at a relatively small final energy of 13 MeV, which is much smaller than the proton rest mass energy of about 938 MeV, the proton mass increases by about 1.4% due to the special relativistic effect. Since the cyclotron frequency (f) is proportional to the magnetic field (B) and inversely proportional to the particle mass (m), isochronous condition for the proton beam at constant accelerating radio frequency is achieved only when the magnet design allows the increase of the magnetic field at the increase of the beam energy. If this condition is not satisfied the beam will be out of phase for the acceleration by the radio frequency.

RELATIVISTIC DYNAMICS

Trajectory calculation of an ion beam of charge q and velocity $\vec{v}$ is based on the Lorentz force $\vec{F}$ acting on the charged particle in electric field $\vec{E}$ and magnetic field $\vec{B}$

$$\vec{F} = q(\vec{E} + \vec{v} \times \vec{B}) \ . \qquad (1)$$

In a cyclotron the electric field $\vec{E}$ depends on position (x,y,z) and time (t)(time varying electric field), while the magnetic field $\vec{B}$ is position dependent only (static magnetic field). The magnitude of both fields are obtained from calculation or measurement (*mapping*).

The components of the Lorentz force in equation (1) can be written as

$$F_x = q(E_x + v_y B_z - v_z B_y),$$

$$F_y = q(E_y + v_z B_x - v_x B_z),$$

$$F_z = q(E_z + v_x B_y - v_y B_x).$$

In the theory of special relativity, the Lorentz force $\vec{F}$ affect different change of momentum and velocity at parallel and at perpendicular to the particle velocity $\vec{v}$. [2] The equation of motion in parallel to $\vec{v}$ is given by

$$\vec{F}_s = d\vec{p}_s / dt = m\gamma^3 d\vec{v}_s / dt, \qquad (2)$$

where the Lorentz force in parallel to $\vec{v}$ is given by

$$\vec{F}_s = s\vec{v},\ s = (\vec{F}.\vec{v})/v^2 \qquad (3)$$

and the relativistic factor $\gamma = 1/\sqrt{1 - v^2/c^2}$, c = speed of light in empty space.

The equation of motion perpendicular to $\vec{v}$ is given by

$$\vec{F}_t = d\vec{p}_t / dt = m\gamma d\vec{v}_t / dt, \qquad (4)$$

where the Lorentz force perpendicular to $\vec{v}$ is given by

$$\vec{F}_t = \vec{F} - \vec{F}_s. \qquad (5)$$

From equations (3) and (4) the components of $\vec{F}_s$ and $\vec{F}_t$ can be calculated as

$$F_{si} = sv_i,\ \ F_{ti} = F_i - F_{si},\ i = x, y, z. \qquad (6)$$

If all components of the Lorentz force are known then by using equations (2) and (4) the change of velocity $d\vec{v} = d\vec{v}_s + d\vec{v}_t$, new velocity $\vec{v} = \vec{v}_0 + d\vec{v}$, and new particle position $\vec{x} = \vec{x}_0 + \vec{v}dt$ can be determined. The time interval $dt = dx/v$ can be chosen such that the use of field data or the computation time can be optimized.

ELECTRON BEAM MACHINE

The 300 kV dc accelerator tube of the electron beam machine was modeled as a cylinder with 34 electrodes each at potential difference of 300 kV/35 ≈ 8571 volt, separation distance between electrodes about 31 mm, and the thickness of the electrodes 3 mm. Due to the symmetry, the accelerator tube can be divided into 17 cells, each of 68 mm in length divided into 68 axial mesh points containing a pair of electrodes. The effective radius of each electrode was 72 mm with accelerating hole of 30 mm radius at the center. For the simulation it was sufficiently done for each half cell, each radially divided into 74 mesh points. The result of the electrostatic potensial distribution is shown in Fig. 1, iterated using SOR (successive over relaxation) method at 0.1% accuracy. [1,3]

For the electron trajectory simulation radial and axial components of the electric fileds E_r and E_z was taken as the radial and the axial gradients of the potential. The result of the trajectory along the accelerator tube is shown in Fig. 2, at initial electron beam of 16 keV. The final energy at the end of the simulation is 299 keV, which is close to the given acceleration voltage of 300 kV. If the non-relativity dynamics were used, however, the final energy would be far less.

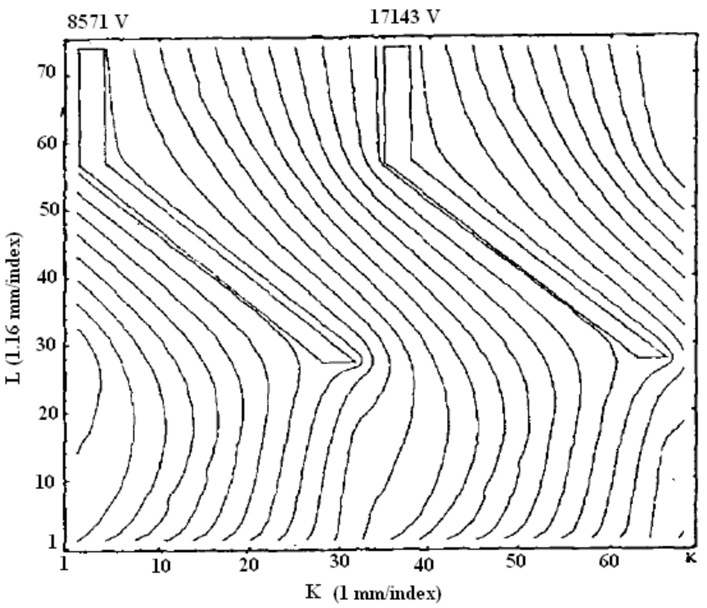

Fig. 1. Electrostatic potential distribution in each of a half cell. [1]

PET CYCLOTRON

PET (positron emission tomography) cyclotron is a small (low) energy cyclotron that accelerates proton or deuteron beam up to 5 to 18 MeV to hit a target producing positron emitting radionuclides. Because of its small size and low energy, it used to be called baby cyclotron.

In a cyclotron, a static magnetic field (usually electromagnet) is applied to keep charged particle beam in successive circular trajectories for multiple acceleration when an alternating voltage is applied to the same electrodes (called *dees* in cyclotron). The cyclotron frequency of the alternating voltage (f), the magnetic field (B), and the mass of the accelerated particle (m) of charge e in a cyclotron are related by

$$2\pi f = eB/\gamma(r)m, \tag{7}$$

where $\gamma = 1/\sqrt{1 - v^2/c^2}$ is the relativistic factor which is higher when the radius or the beam energy is getting larger. In a baby cyclotron with a final energy of 13 MeV, this factor is about 1.14. To keep the beam isochronous at constant accelerating radiofrequency, a radial increase of the average magnetic field must be designed accordingly.

We are developing a 13 MeV PET cyclotron (DECY-13, Development of Experimental Cyclotron in Yogyakarta) based on the design of KIRAMS-13 (Korean Institute of Radiation and Medical Sciences) [4]. The magnetic field B_x, B_y, B_z was computed using Opera3D and Tosca module for the whole region of the cyclotron, covering size of 960 mm × 960 mm × 40 mm at 1 mm resolution. The model used for the computation is shown in Fig. 3 and due to the symmetry chosen it is sufficient to take only 1/8 of the whole data for $0 \geq x, y, z \geq$ 480mm and the rest are available using

$$B_x = \mathrm{sign}(x)\mathrm{sign}(z)B_x,\ B_y = \mathrm{sign}(y)\mathrm{sign}(z)B_y,\ B_z = B_z. \tag{8}$$

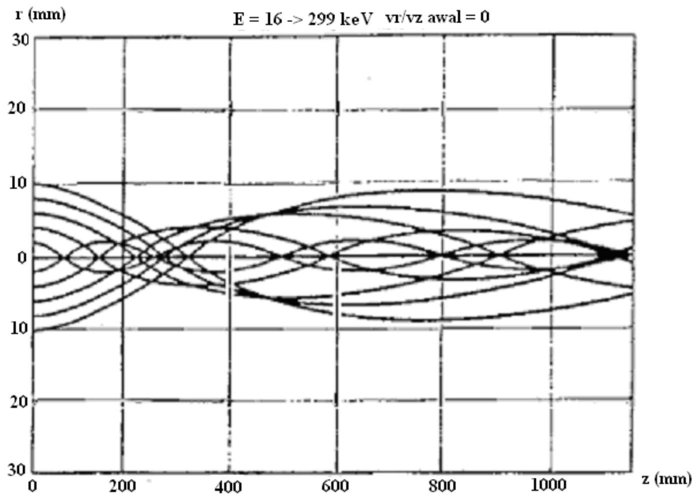

Fig. 2. Electron trajectory simulation at various initial position at axial direction and initial beam energy of 16 keV.

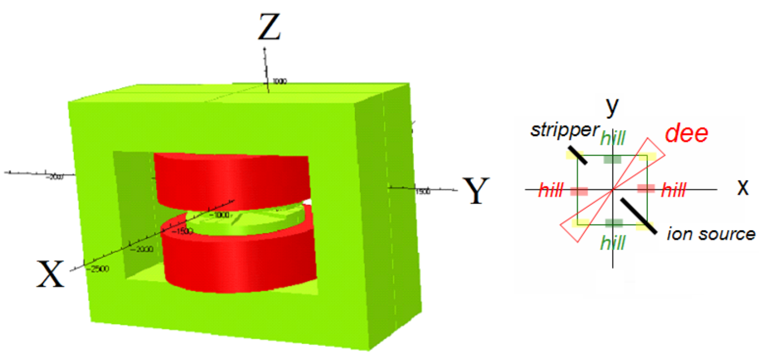

Fig. 3. Model of yokes, poles, hills, and valleys coordinates for the computation of the magnetic field using Opera3D and Tosca.

The computation of the magnetic field is based on B_z = 1.275 T at r = 0 (below soft iron magnetic saturation at about 2 T, but strong enough to keep 13 MeV proton radius at a reasonable radius of about 42 cm) or proton resonant frequency of 77.88 MHz at the fourth harmonics, with 4 pairs of hills and valleys to provide Thomas focusing of the proton beam along the circular trajectory. Plot of B_z at z = 0 for $0 \geq x,\ y \geq 480$ mm showing hill and valley is shown in Fig. 4a. Fig. 4b shows the ideal (blue) and computed (green and red) increase of the isochronous average B_z to compensate the increase of proton relativistic mass, which is about 1.4% higher at the final energy of 13 MeV. The ideal isochronous average B_z follows equation (7) and was averaged using Bcalc code [5], and the computed ones were achieved by widening the hills' angle at larger radius. From two models of hills' angle widening, one model (red) is used in computation of the beam trajectory.

The electric fields E_x, E_y, E_z were also computed by using Opera3D and Tosca module, using the model of KIRAMS-13 central region developed at the Korean Institute for Radiation and Medical Sciences [4]. The DECY-13 PET cyclotron is designed to operate with negative proton beam (for easier beam extraction using a stripper foil), 40 kV peak dee voltage at fourth harmonic accelerating rf frequency of 77.88 MHz. The central region covers fields of 240 mm × 240 mm × 30 mm size at 1mm resolution. Mapping of E_x, E_y at $z = 0$ is shown in Fig. 5a.

The trajectory simulation was carried out using Scilab 5.3.3 code (a freeware) by applying equations (1) to (6). The simulations in the central region show up to 7 turns of orbital trajectories (Fig. 5b), reaching about 1 MeV of beam energy (Fig. 5c), which is still too small to see the relativistic effect. Work on simulations to reach the intended final energy of 13 MeV is still going on.

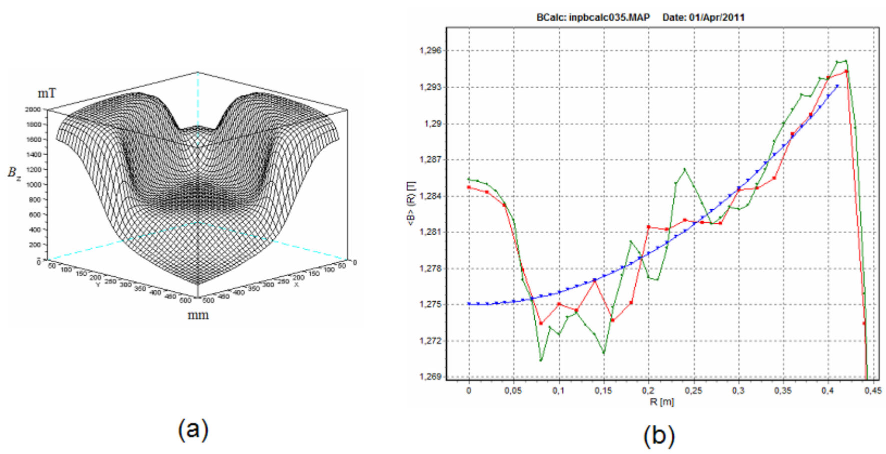

Fig. 4. (a) Plot of B_z at $z = 0$ for $0 \geq x,\ y \geq 480$mm showing hill and valley. (b) Plot of isochronous average B_z: ideal (blue), model I (green), model II (red). Model II is used in computation of the beam trajectory.

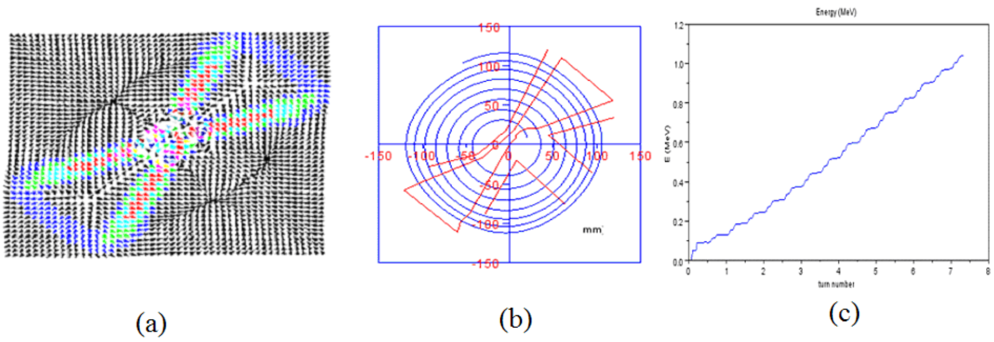

Fig. 5. (a) Mapping of E_x, E_y at $z = 0$, the red and green colors are the accelerating fields (b) simulated proton trajectory in the central region (c) increasing proton kinetic energy.

CONCLUSION

The relativistic effect showed up and should be taken into account in the trajectory simulation in 300 keV electron beam machine and in the design of the magnetic field in 13 MeV PET proton cyclotron. The relativistic effect in the trajectory simulation in the cyclotron is remain to be seen for higher energy.

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REFERENCES

1. **PRAMUDITA ANGGRAITA, DJOKO SUJONO, SANGADJI**, "Simulation of Electron Beam Trajectory in the Accelerator Tube of an Electron Beam Machine," Proceeding of Scientific Meeting and Presentation PPNY-BATAN, Yogyakarta 26-27 May 1998, page 450-458 (in Indonesian).
2. **H. WIEDEMANN**, Particle Accelerator Physics, Basic Principles and Linear Beam Dynamics, Springer-Verlag, Berlin 1993.
3. **M.Y. BERNARD**, Particles and Fields: Fundamental Equations, in Focussing of Charged Particles, Vol. I, ed. A. Septier, Academic Press Inc., New York 1967.
4. **DONG HYUN AN, ET AL.**, "Design of the Central Region in the KIRAMS-13 Cyclotron," APAC 2004, Gyoengju, March 2004.
5. **JOZEF FRANKO**, "Bcalc – A Computer Program for Processing The Cyclotron Magnetic Structure Measurement Data," Flerof Laboratory of Nuclear Reaction (FLNR), Dubna, Rusia, in Journal of Electrical Engineering, Vol.56, No. 1-2, 2005, 26-30.

TANYA JAWAB

Edi Trijono

- Mohon penjelasan perihal perbedaan trajectory partikel dalam percepatan yang mengalami efek relativitas dan tidak mengalami efek relativitas!

Pramudita Anggraita

- ✓ *Untuk lintasan elektron jika tidak dipakai efek relativistik tenaga akhir lebih rendah dan seharusnya hanya sekitar 278 keV dan jika dipakai efek relativistik diperoleh tenaga akhir 299 keV yang lebih mendekati nilai seharusnya (300 keV). Untuk lintasan proton pada siklotron 13 MeV, efek pada lintasan maupun tenaga geraknya tidak terlalu terlihat karena efek relativistiknya lebih kecil.*

Suprapto

- Berkas elektron pada simulasi pemercepatan untuk MBE terjadi perpotongan berkas dengan sumbu, bagaimana pemfokusannya agar berkas elektron tidak menumbuk elektrode tabung akselerator.

Pramudita Anggraita

- ✓ *Dari simulasi lintasan berkas sebenarnya sudah tampak adanya pemfokusan karena ukuran berkas akhir lebih kecil daripada ukuran berkas waktu masuk tabung pemercepat.*