

ECE411 LAB1 Lab Report

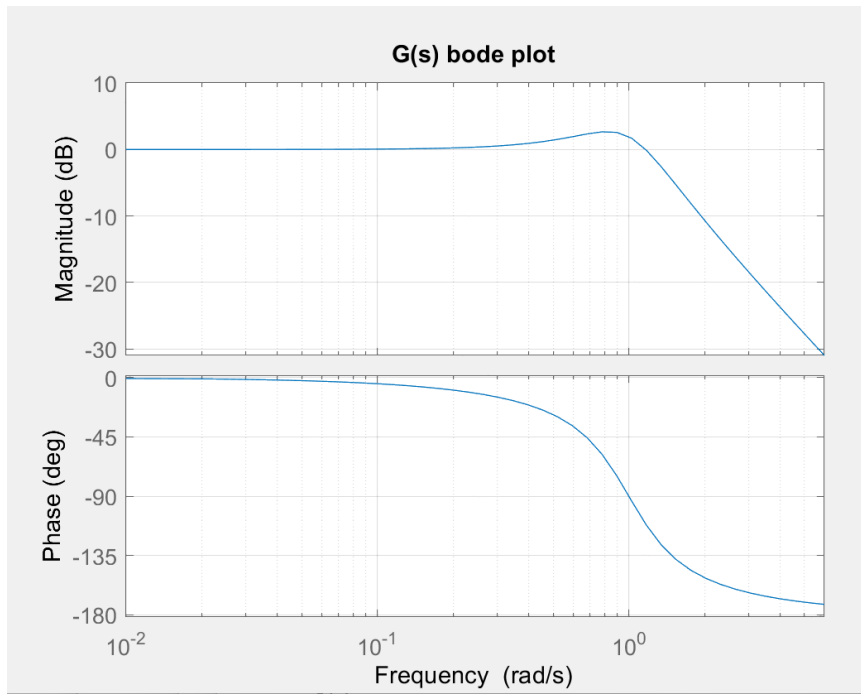
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3.1 Aliasing

1.

```
%===== Q1
% G = 1 / (s^2 + 0.8s + 1)
G = tf([1],[1 0.8 1]);
figure;
bode(G,{0,6});
title('G(s) bode plot');
grid on;
```



2.

(i)

```
%===== Q2
```

```
% i)
```

```
T = 1;
```

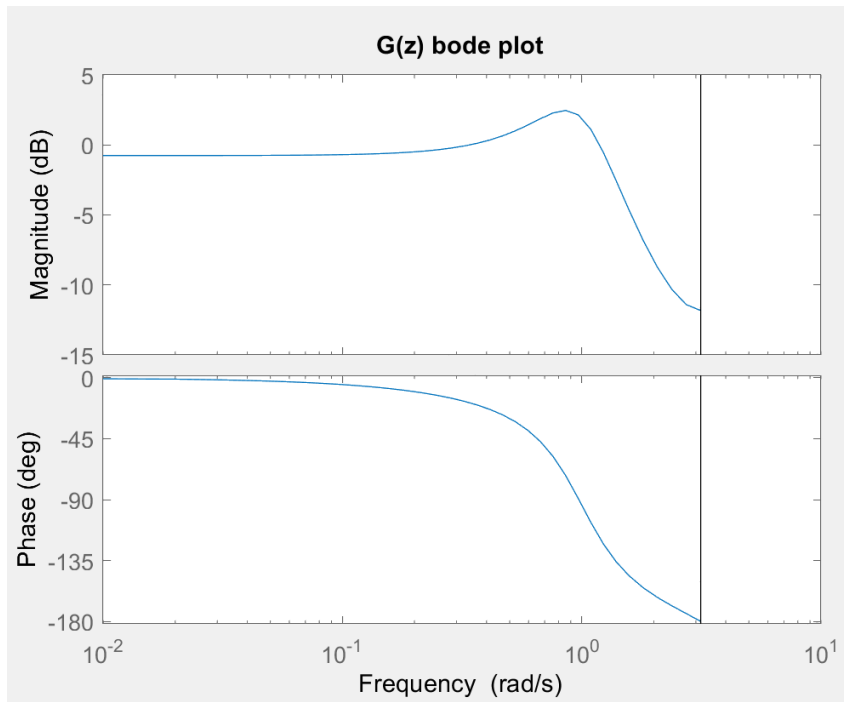
```
Gz = c2d(G,T,'impulse'); Sample time: 1 seconds
Discrete-time transfer function.
```

Gz =

$$\frac{0.5803 z}{z^2 - 0.8159 z + 0.4493}$$

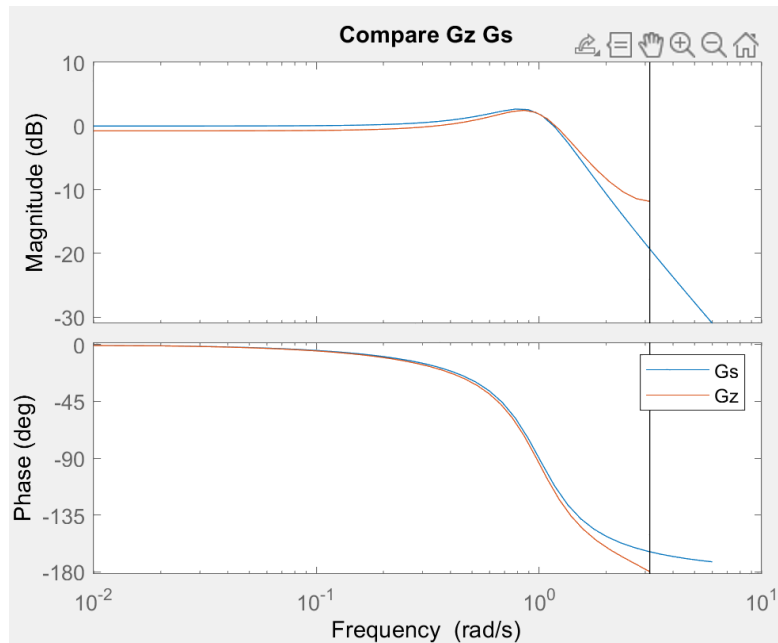
(ii)

```
% ii)
figure;
dbode(Gz,T,[0 10]);
title('G(z) bode plot');
```



(iii)

```
% iii)
figure;
bode(G,{0,6});
hold on;
dbode(Gz,T,[0 10]);
legend('Gs','Gz');
title('Compare Gz Gs');
```



From the figure the aliasing occurs at frequency = 1 rad/s

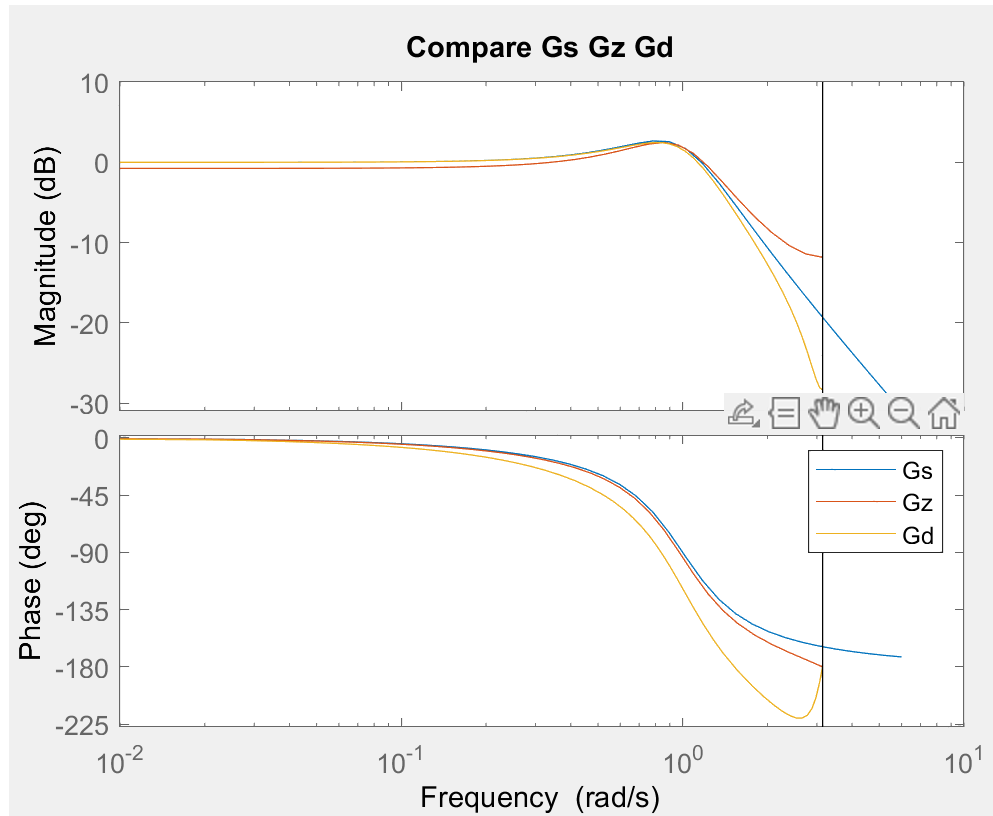
When the frequency > 1rad/s, there is large difference between Gs, Gz.

When the frequency <1 rad/s, the difference between Gs, Gz is small.

3.

```
%===== Q3
%convert TF - SS for G
A = [0 1; -1 -0.8];
B = [0 ; 1];
C = [1 0];
D = 0;
I = [1 0; 0 1];
sys_c = ss(A,B,C,D);
sys_d = c2d(sys_c,T);
[Ad,Bd,Cd,Dd] = ssdata(sys_d)
z = tf('z',T);
Gd = Cd * inv(z*I - Ad)*Bd + Dd;
```

```
figure;
bode(G,{0,6});
hold on;
dbode(Gz,T,[0 10]);
hold on;
dbode(Gd,T,[0 10]);
legend('Gs','Gz','Gd');
title('Compare Gs Gz Gd');
```



The G_s , G_z and G_d are close when frequency $< 1 \text{ rads/s}$

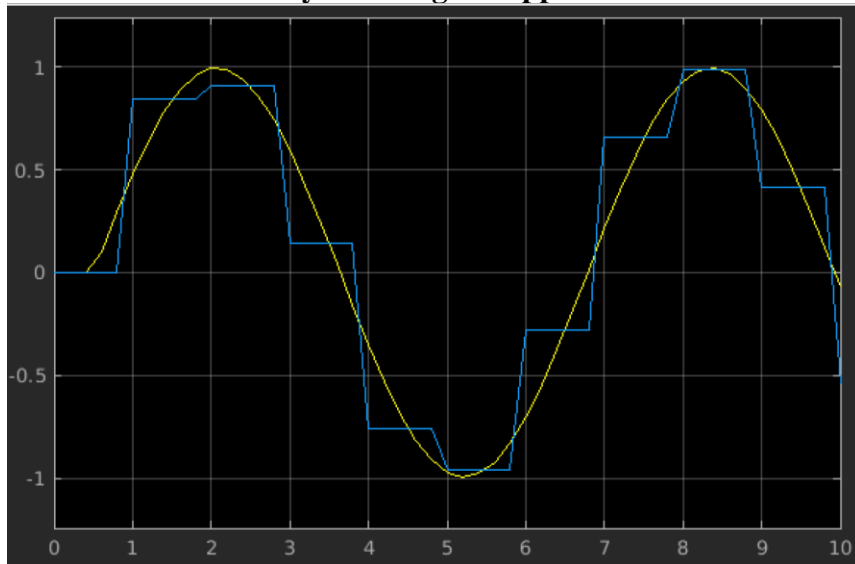
And diverges when frequency $> 1 \text{ rad/s}$

When frequency $> 1 \text{ rad/s}$, the magnitude in the bode plot $G_z > G_s > G_d$.

3.2 Effects of Sample and Hold on Control Design

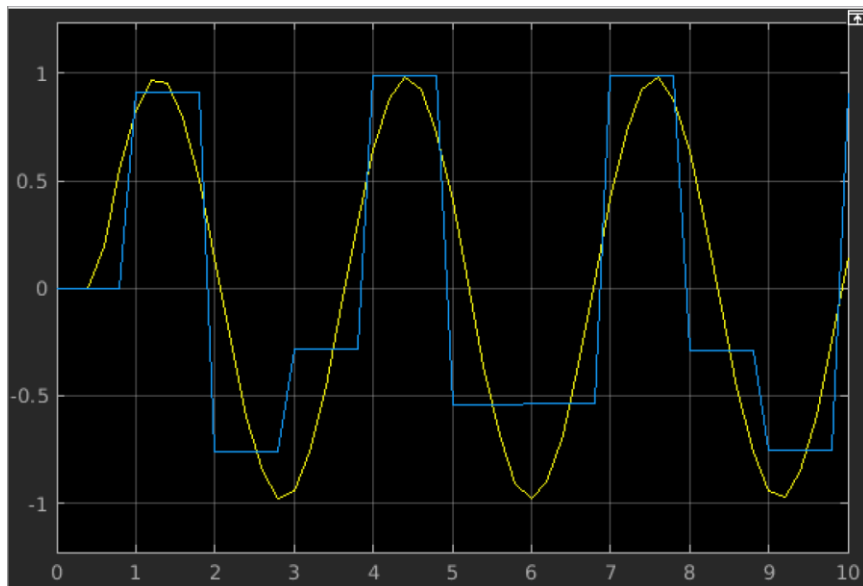
1.

The output figure of scope(sampling time $T = 1$, transport delay $T/2 = 0.5$) is shown below: the yellow line corresponds to Transport Delay and the blue line corresponds to Zero-Order Hold. The delay is not a good approximation.

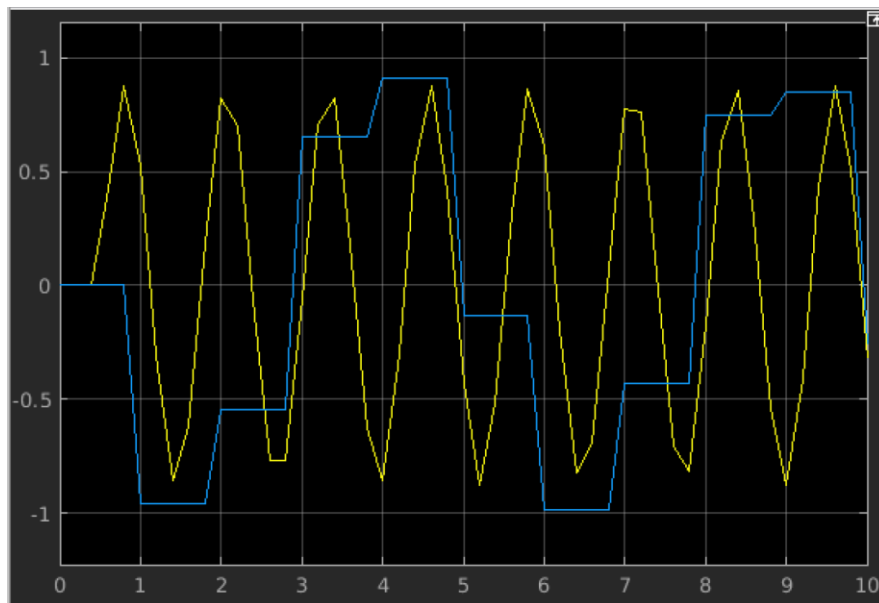


Varying the frequencies to see the change in approximation:

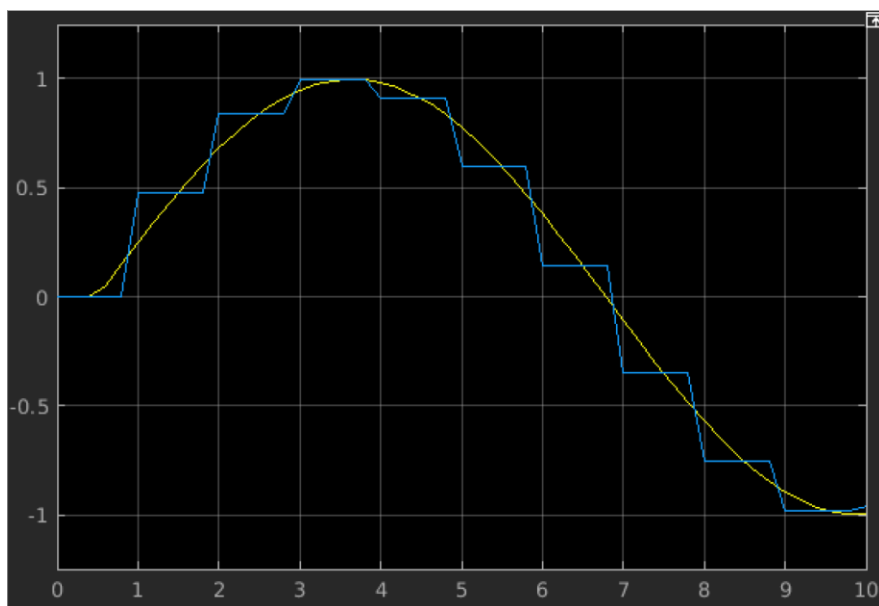
Set frequency = 2:



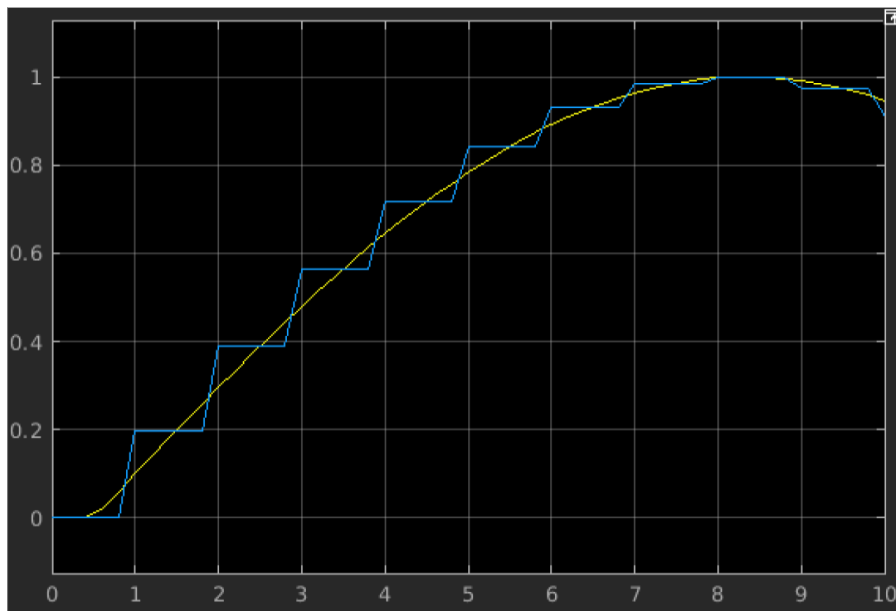
Set frequency = 5:



Set frequency = 0.5:



Set frequency = 0.2:

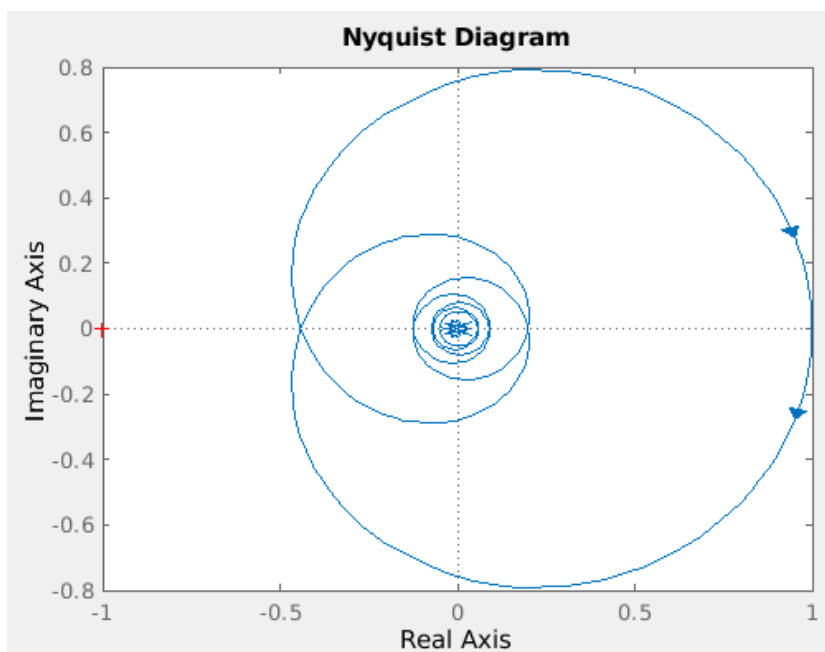


In conclusion, as frequency decreases(becomes smaller), the proximation becomes better and closer.

2.

(i)

Nyquist plot of $G(s)$ is shown below:

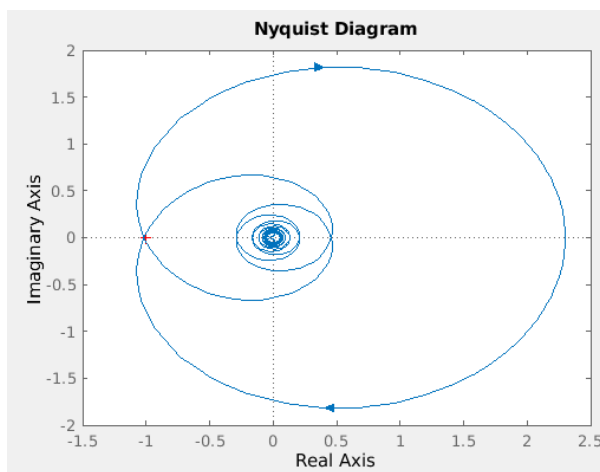


From the Nyquist plot, we learn that $G(s)$ is stable as the Nyquist plot does not encircle -1 and also $G(s)$ has a pole in OLHP.

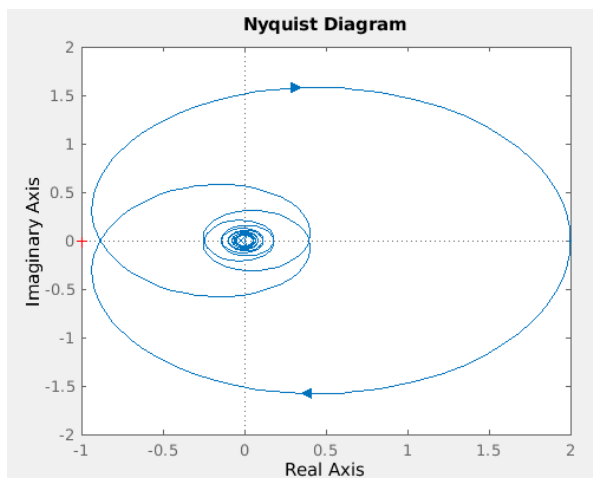
If we apply the same control law $u = k(r - y)$, the closed loop system will not be stable for all $k > 0$. From the Nyquist plot, we estimated that the closed loop system will be stable for $k < 2.3$ and the evidence is shown in below.

From Nyquist stability criterion, we know that stability is determined by the number of encirclements of point $(-1, 0)$. And also from Cauchy's argument principle we know that $N = P - Z$.

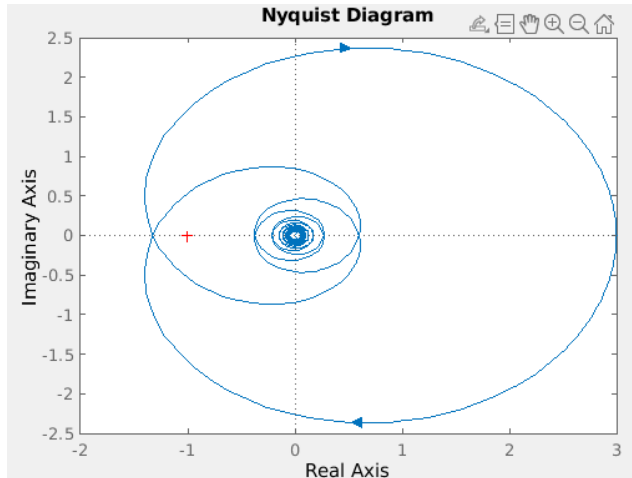
As shown in below, when k is set to 2.3, the Nyquist plot cross $(-1, 0)$.



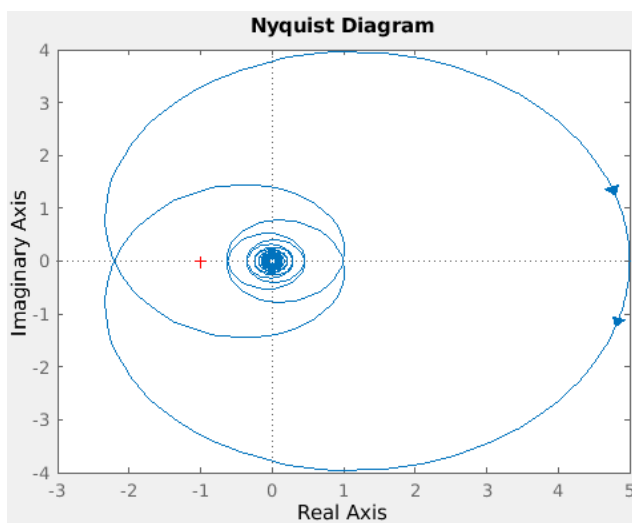
When k is set to 2, the Nyquist plot does not encircle $(-1, 0)$.



When k is set to 3, the Nyquist plot encircle $(-1, 0)$.

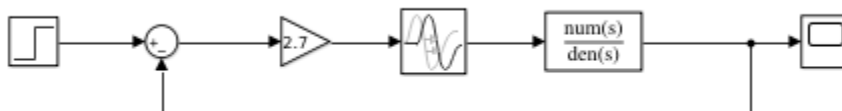


When k is set to 5, the Nyquist plot encircle $(-1, 0)$.



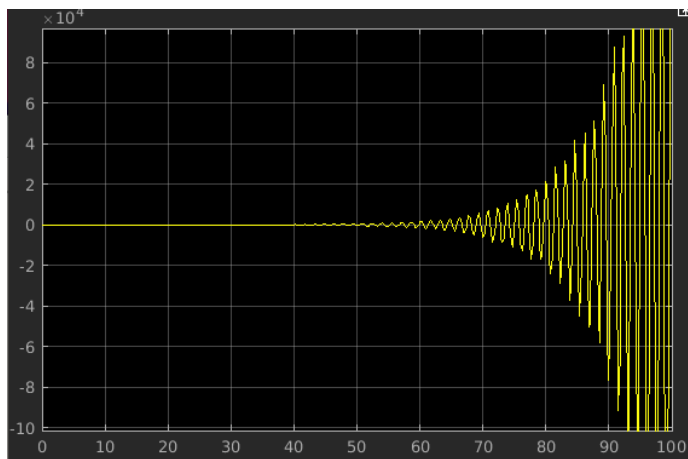
(ii)

The Simulink model is constructed as shown below.

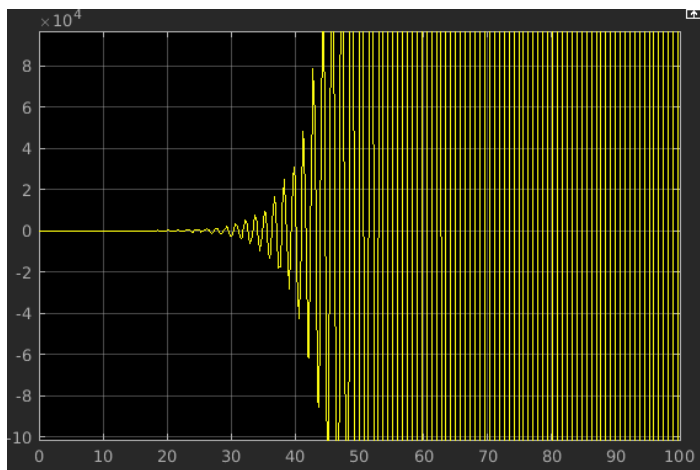


From experiments, the largest value of k for which the closed loop system becomes unbounded is 2.7. For $k \geq 2.7$, the closed loop system becomes unbounded as shown below.

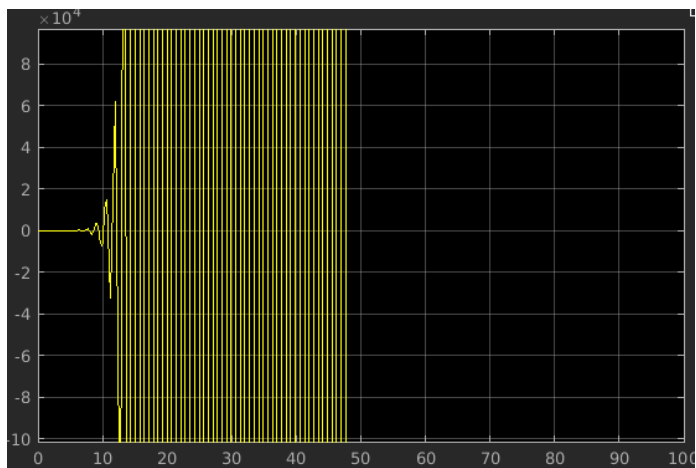
When $k = 2.7$:



When $k = 3$:

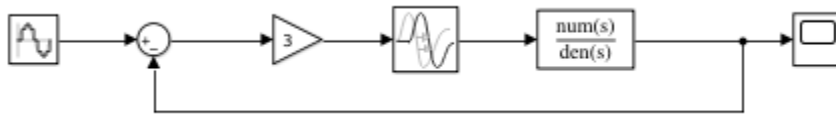


When $k = 5$:

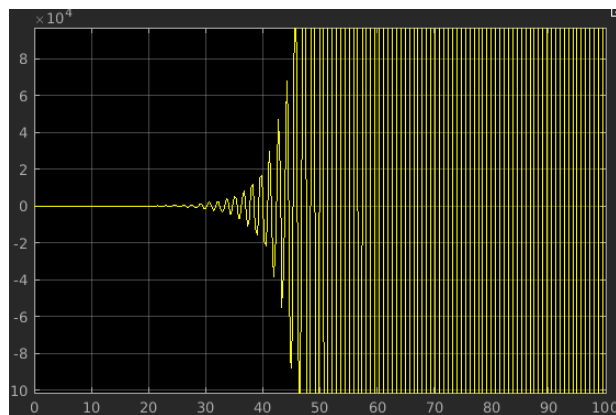


This value is close to the previous estimation from the Nyquist plot.

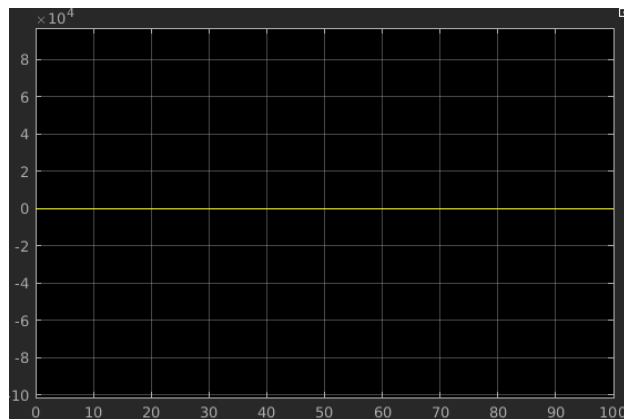
When we change the reference input r to a sinusoid, as shown below:



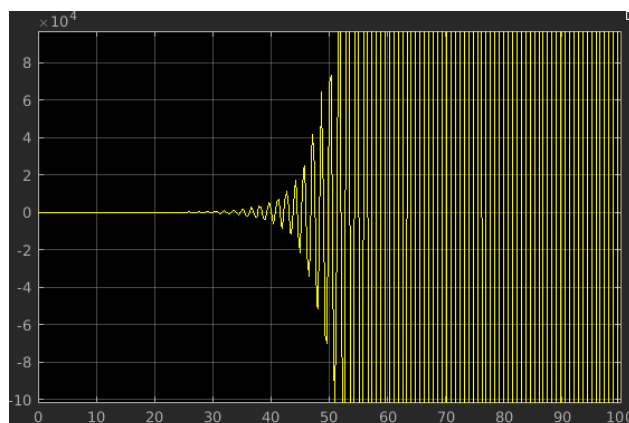
When frequency = 1, $k = 3$: unstable



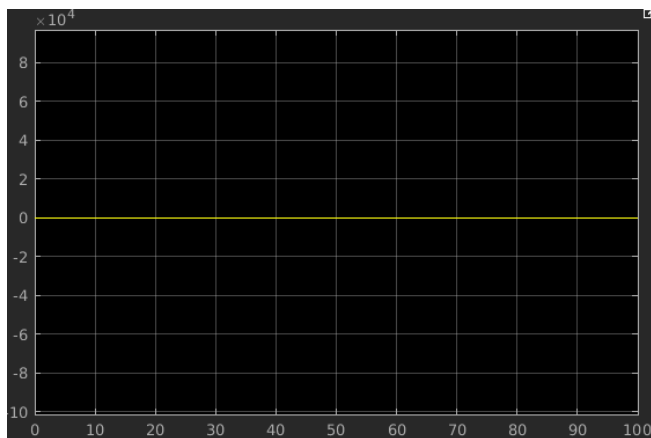
When frequency = 1, $k = 2.3$: stable



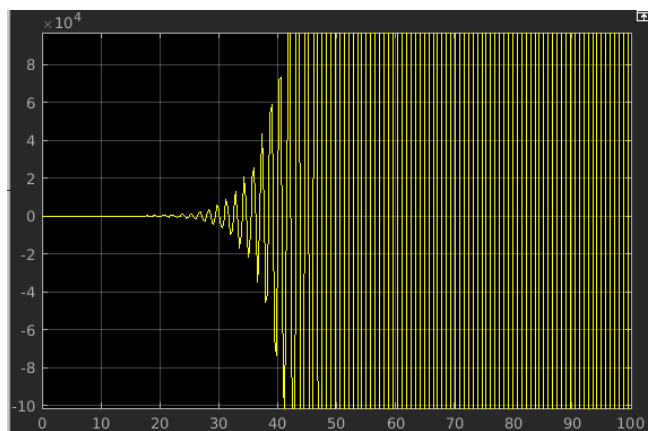
When frequency = 0.2, $k = 3$: unstable



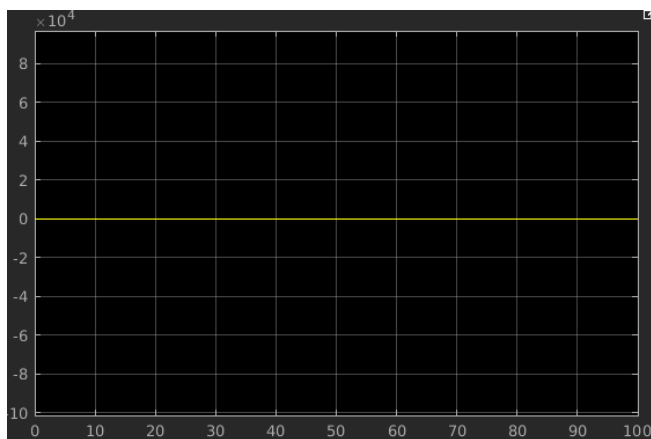
When frequency = 0.2, k = 2.3: stable



When frequency = 5, k = 3: unstable



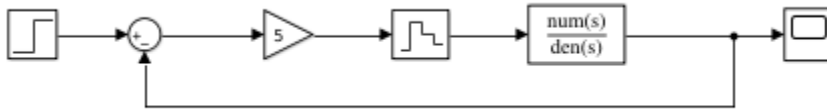
When frequency = 5, k = 2.3: stable



From experiments above, we found that regardless the frequency, the gain k for the closed loop system to become unbounded is similar and also is similar to the values from the Nyquist plot.

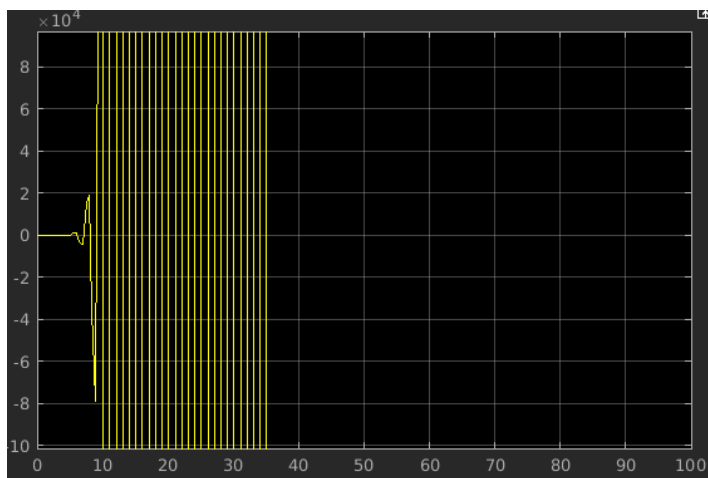
(iii)

Replace the transport delay block with a zero-order hold block.

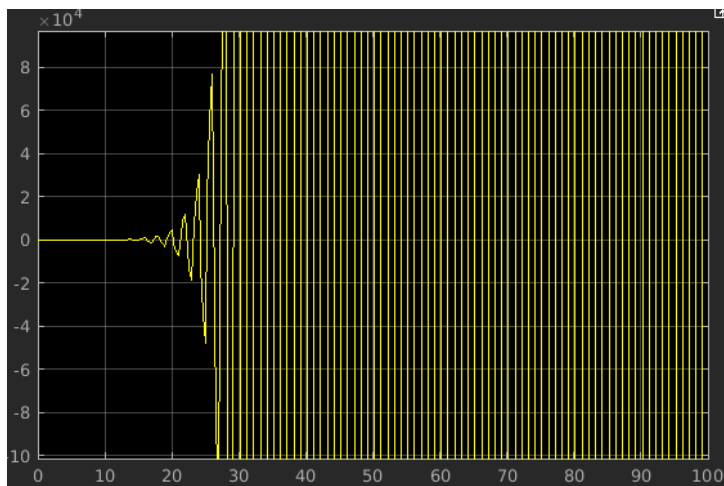


When reference signal r is the unit step, we found that for $k > 1.4$, the closed loop system becomes unstable and the experiments are shown below.

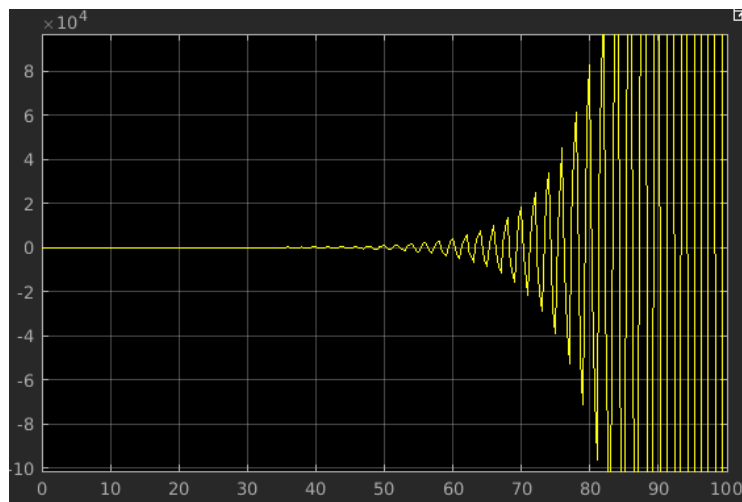
Set $k = 5$:



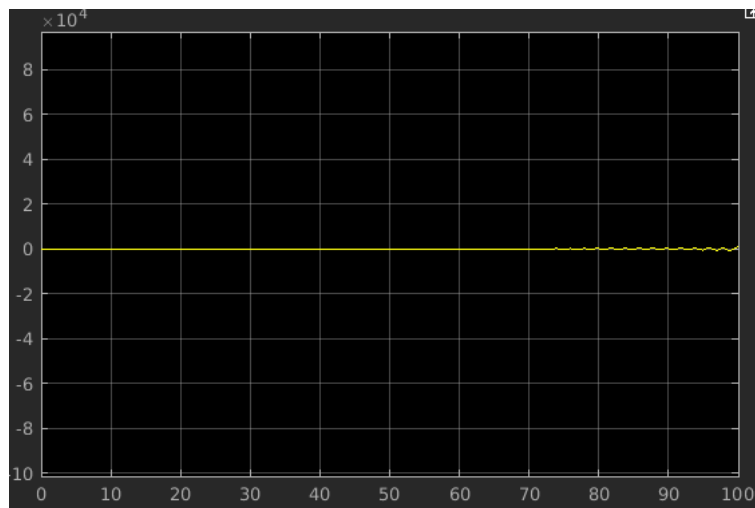
Set $k = 2$:



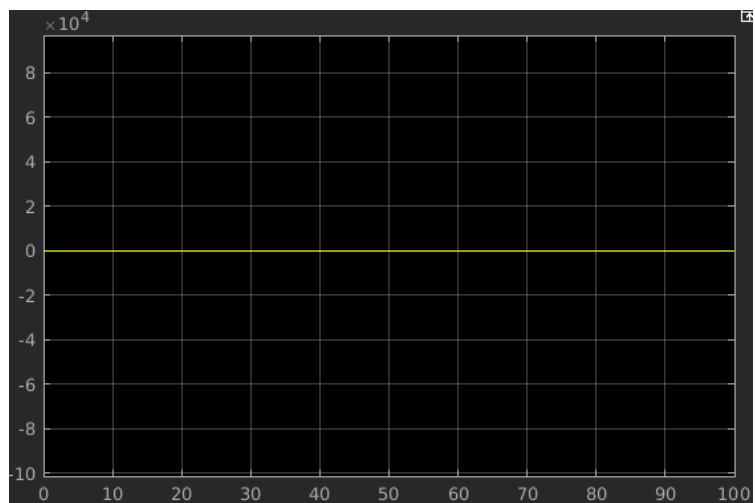
Set $k = 1.5$:



Set $k = 1.4$:

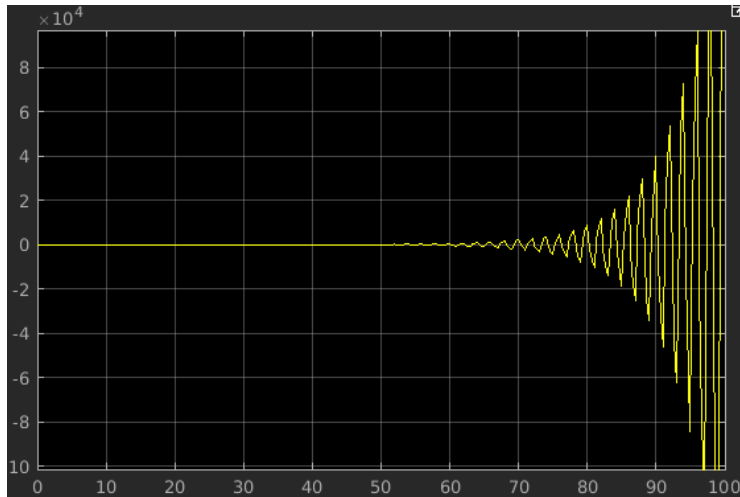


Set $k = 1.3$:

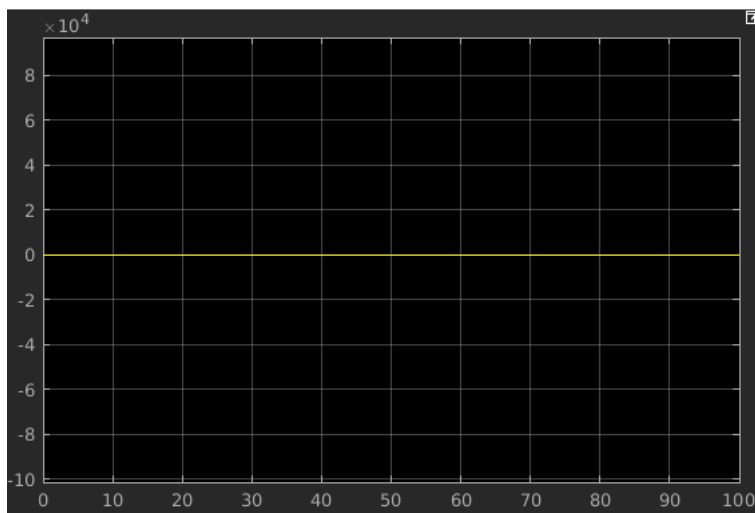


When reference signal r is the sinusoid, we found that for $k > 1.4$, the closed loop system becomes unstable and the experiments are shown below. Also we noticed that as frequency increases, it will become unstable/oscillate sooner.

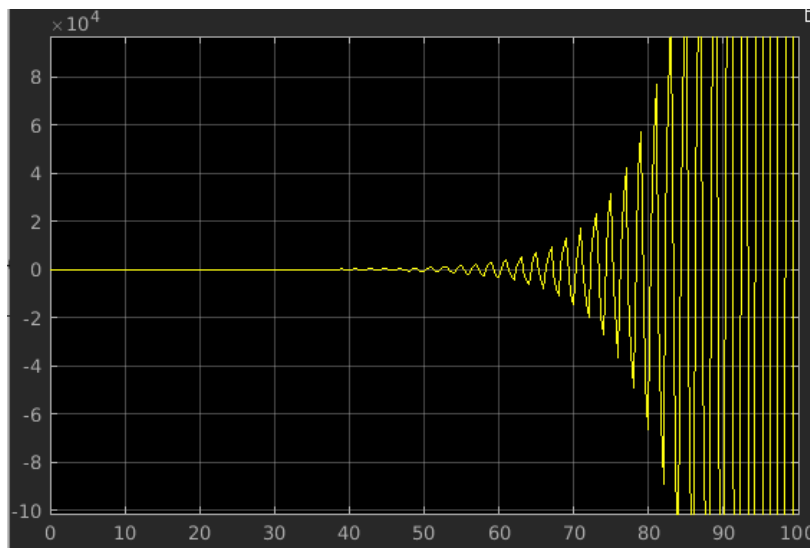
When frequency = 0.2, $k = 1.5$: unstable



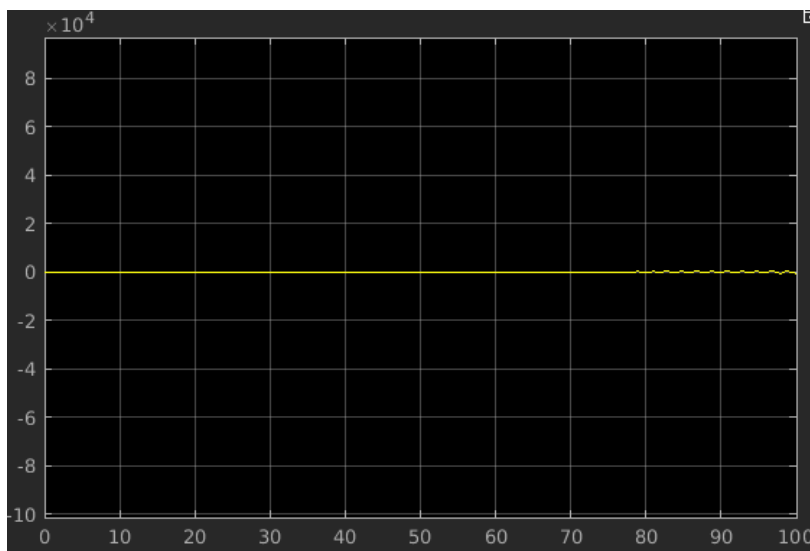
When frequency = 0.2, $k = 1.4$: stable



When frequency = 5, k = 1.5: unstable



When frequency = 5, k = 1.4: stable

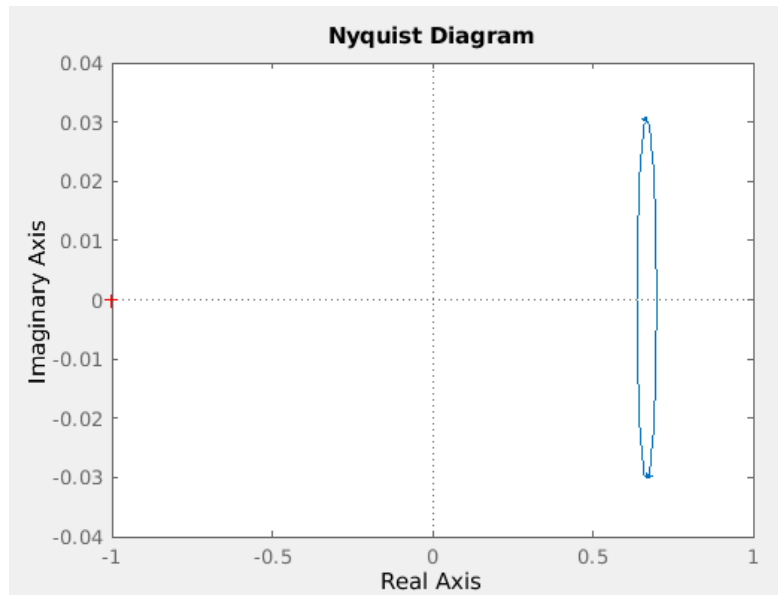


(iv)

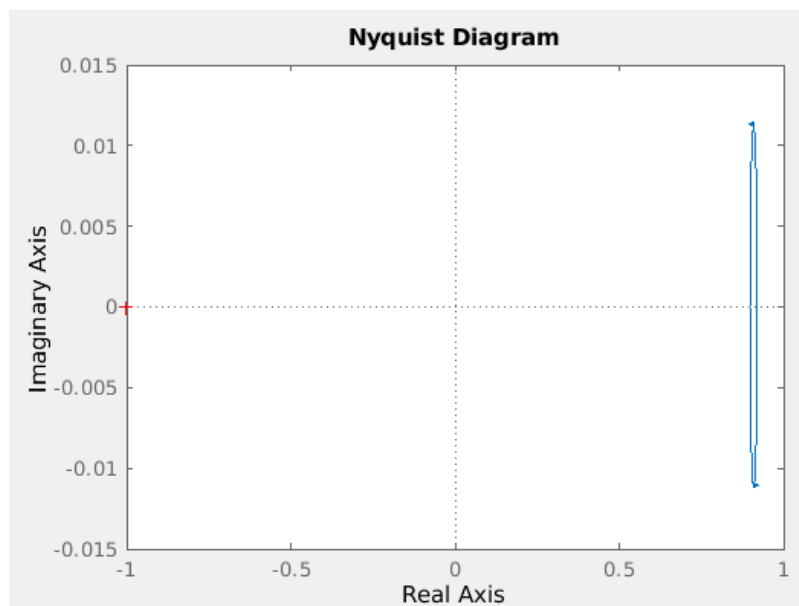
```
G0 = tf([1], [0.5, 1]);  
T = 1;  
G0_d = c2d(G0, T, 'impulse');  
k = 1;  
Gd = k * G0_d / (1 + k * G0_d);  
nyquist(Gd);
```

From the Nyquist plot, we determined the range of value of k for closed loop stability is $k > 0$. And the evidence is shown below.

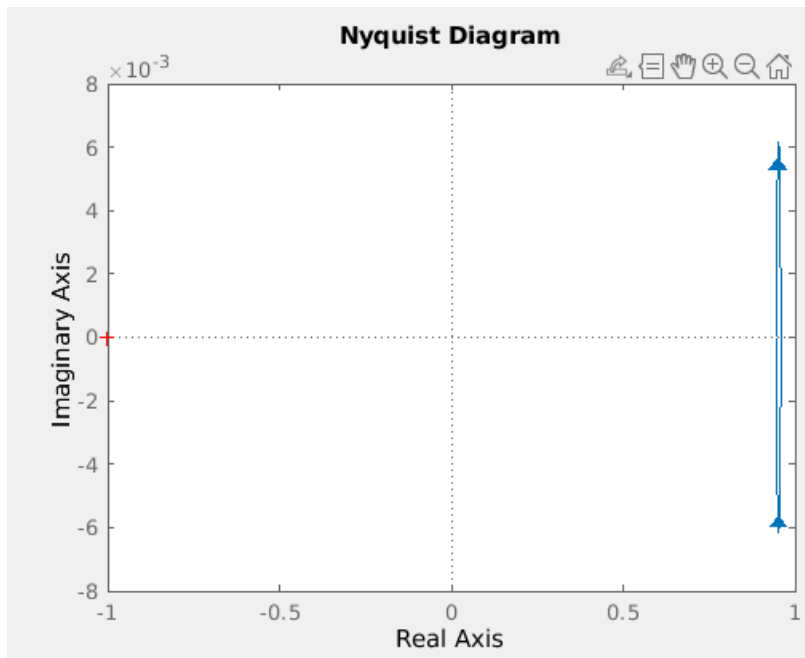
When $k = 1$:



When $k = 5$:



When $k = 10$:



3.3 Intersample Ripple

1.

```
P = tf([0.1], [1, 0.1, 0]);  
T = 1;  
P_d = c2d(P, T, 'impulse');  
C_d = tf([9, -7.2], [1, 0.8], T);  
G1 = (C_d * P_d) / (1 + C_d * P_d);  
G2 = feedback(C_d * P_d, T);  
step(G2);
```

The output $y(kT)$ when input $r(kT)$ is the discrete time unit step is:

G1 is the closed loop transfer function calculated and conducted from block diagram.

G1 =

$$\frac{0.8565 z^5 - 1.631 z^4 + 0.2268 z^3 + 1.044 z^2 - 0.496 z}{z^6 - 1.353 z^5 - 1.649 z^4 + 3.042 z^3 - 0.1722 z^2 - 1.392 z + 0.524}$$

Sample time: 1 seconds

Discrete-time transfer function.

G2 is the closed loop transfer function calculated using MATLAB built-in function.

G2 =

$$\frac{0.8565 z^2 - 0.6852 z}{z^3 - 0.2484 z^2 - 1.304 z + 0.7239}$$

Sample time: 1 seconds

Discrete-time transfer function.

Y1 is y(kT) when input r(kT) is the discrete time unit step and the closed loop transfer function is G1.

Y1 =

$$\frac{0.8565 z^6 - 1.631 z^5 + 0.2268 z^4 + 1.044 z^3 - 0.496 z^2}{z^7 - 2.353 z^6 - 0.2956 z^5 + 4.691 z^4 - 3.215 z^3 - 1.22 z^2 + 1.916 z - 0.524}$$

Sample time: 1 seconds

Discrete-time transfer function.

Y2 is y(kT) when input r(kT) is the discrete time unit step and the closed loop transfer function is G2.

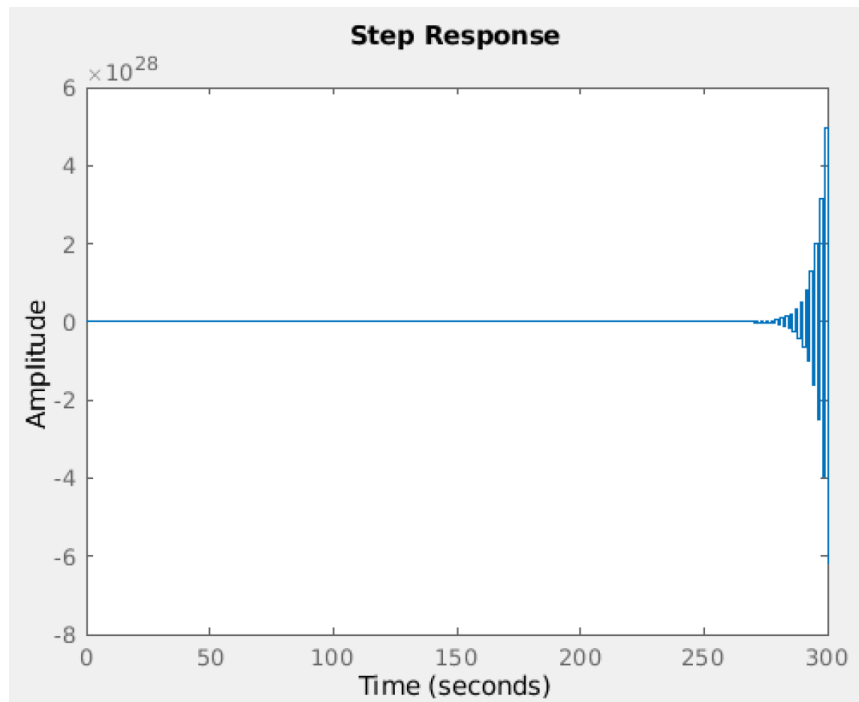
Y2 =

$$\frac{0.8565 z^3 - 0.6852 z^2}{z^4 - 1.248 z^3 - 1.056 z^2 + 2.028 z - 0.7239}$$

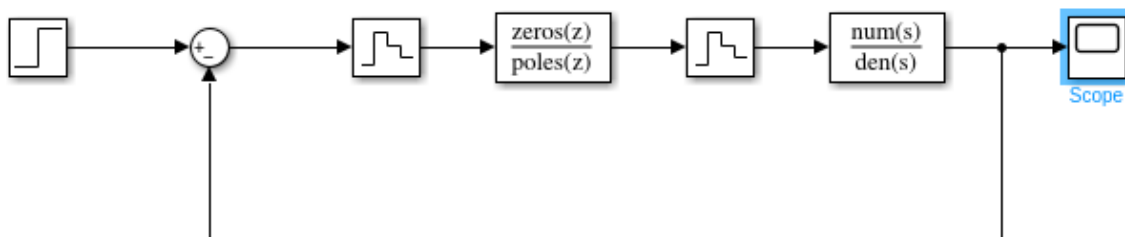
Sample time: 1 seconds

Discrete-time transfer function.

The figure of $y(kT)$ is shown as below:

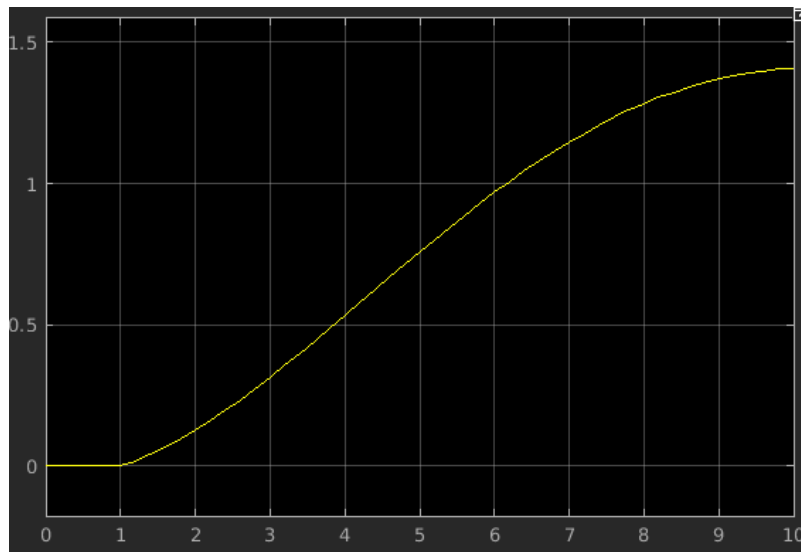


2.

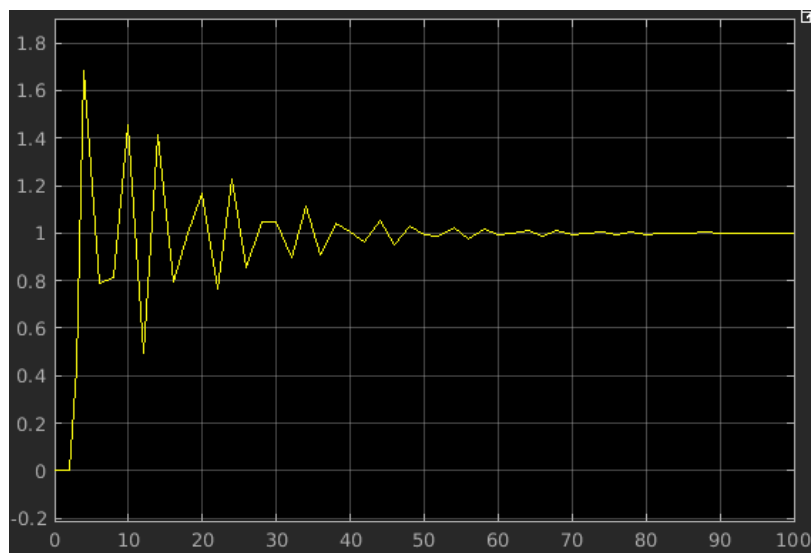


Simulink is constructed as the lab handout shows.

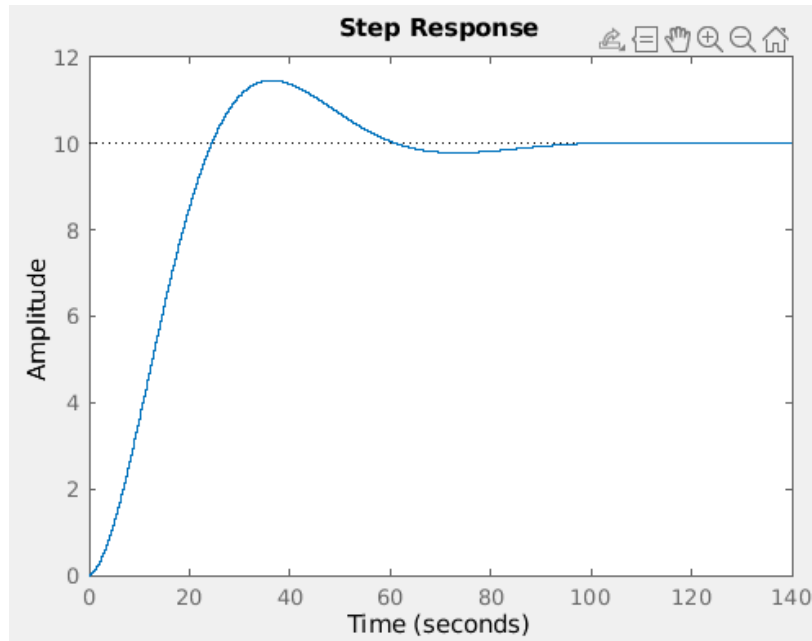
The output of scope when stop time is 10 is shown below:



The output of scope when stop time is 100 is shown below:



We noticed that the behavior obtained from problem 2 is different compared to the behavior obtained from the discrete time closed loop system from problem 1. This difference is caused by the choice of sampling time. This is intersample ripple. We also found that when we set the sample time $T = 0.1$ in problem 1, then the behavior obtained from the discrete time closed loop system will be similar to behavior obtained from problem 2, as shown below.



We also noticed that with different sampling time, $P_d(z)$ are different.