ECE421 - Winter 2022
Homework Problems - Tutorial #11

Theme: PAC Learning and VC Dimension

Due: April 10, 2022 11:59 PM

Question 1 (Examples 2.2 from LFD)

Consider the following learning models.

1. $\mathcal{H}$ is the set of positive rays consisting of all hypothesis $h : \mathbb{R} \to \{-1, +1\}$ of the form $h(x) = \text{sign}(x - a)$.
2. $\mathcal{H}$ is the set of positive intervals consisting of all hypothesis $h : \mathbb{R} \to \{-1, +1\}$, such that $h(x) = +1$ if x is *within* some predefined interval and -1 otherwise.

For each of the above learning models, compute

- the maximum number of dichotomies $m_{\mathcal{H}}(N)$,
- the smallest breakpoint k, and
- the VC dimension d_{vc}.

Reason clearly how you arrive at the answers. The final expression for $m_{\mathcal{H}}(N)$ is provided on page 44 of LFD to help you to verify the answer.

Question 2 (Problem 2.3 from LFD)

Compute the maximum number of dichotomies $m_{\mathcal{H}}(N)$ for the following learning models, and compute the VC dimension d_{vc}.

1. Positive *or* negative ray: $\mathcal{H}$ contains the functions that are $+1$ on $[a, \infty)$ (for some a) and -1 otherwise, together with functions that are $+1$ on $(-\infty, a]$ (for some a) and -1 otherwise.
2. Two concentric spheres in $\mathbb{R}^d$: $\mathcal{H}$ contains the functions which are $+1$ for $a \leq \sqrt{x_1^2 + \ldots + x_d^2} \leq b$, and -1 otherwise.

Question 3

Consider an arbitrary hypothesis space $\mathcal{H}$, where $h \in \mathcal{H}$ is a binary linear classifier. Suppose that $m_{\mathcal{H}}(1) = 2$ and $m_{\mathcal{H}}(2) = 3$. Find the largest possible value for $m_{\mathcal{H}}(3)$. (Hint: proceed by induction, for instance, what does it mean for $m_{\mathcal{H}}(1) = 2$ and $m_{\mathcal{H}}(2) = 3$? A table could be helpful.).