# 18BTMT402: PROBABILITY AND QUEUING THEORY

Dr. Monika Vishnoi
Assistant Professor
Dept. of ASH (Mathematics)
MIT-SOE, MIT- ADT University, Pune

# UNIT I
# RANDOM VARIABLES

- Discrete and continuous random variables –
- Probability density function,
- Moments,
- Moments generating functions,
- Binomial, Poisson,
- Geometric,
- Negative binomial,
- Uniform, Exponential,
- Gamma and Normal distribution.

# Introduction: Random Variable

* Random variables are very important in statistics and probability and a must have if any one is looking forward to understand probability distributions. Random Variables many a times confused with traditional variables. In this lecture we will see what random variables are and why do we need them over traditional variables. Random Variables play a vital role in probability distributions and also serve as the base for Probability distributions.

- In probability and statistics, random variable, random quantity or stochastic variable is a variable whose possible values are the outcomes of a random phenomenon.

- Random variable is different from our traditional variable in terms of the value which it takes. It's a function which performs the mapping of the outcomes of a random process to a numeric value.

# WHY RANDOM VARIABLES?

If we flip 5 coins and want to answers questions like:

1. What is the probability of getting exactly 3 heads?

2. What is the probability of getting less than 4 heads?

3. What is the probability of getting more than 1 head?

Then our general way of writing would be:

· P(Probability of getting exactly 3 heads when we flip a coin 5 times)

· P(Probability of getting less than 4 heads when we flip a coin 5 times)

· P(Probability of getting more than 1 head when we flip a coin 5 times)

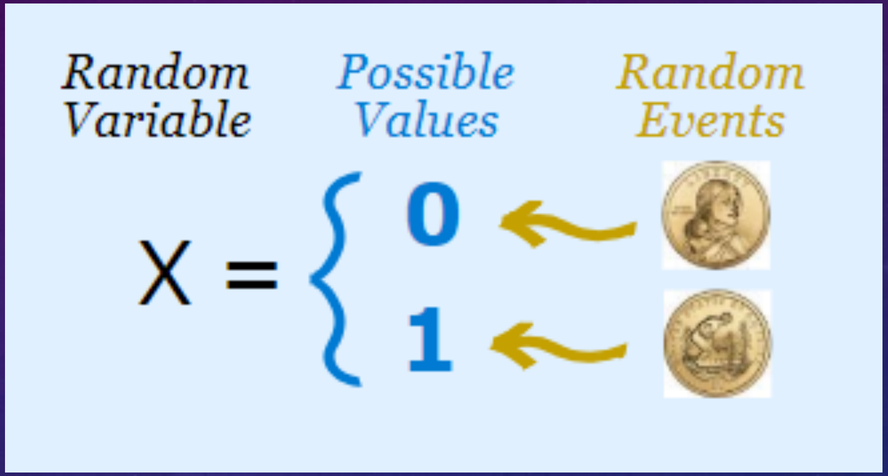
But if we use random variables to represent previous questions then we would write:

- 1. $P(X = 3)$
- 2. $P(X < 4)$
- 3. $P(X > 1)$

As we can see above random variables makes our task much easier to quantify results of any random process and apply math and perform further computation.

Suppose we have a random process/experiment of flipping a coin. One of the two possible outcomes could be either a head or a tail. So here we use X to denote random variable, which represents the outcomes of this random process.

Therefore we can write
X= 1, if the outcome is head
X= 0, if the outcome is tail




Here the random Variable X is mapping the outcomes of the random process(flipping a coin) to the numerical values (1 and 0).
The values assigned to denote head and tail can be anything it’s not necessarily be 1 and 0.
To make understanding simple we have used 1 and 0.

Other way of assigning numerical values to outcomes of a random process could be:
X= 100, if the outcome is head
X= 50, if the outcome is tail

# Summarizing In Three Points

- We have an experiment(tossing a coin)
- We give values to each event
- These set of values is a random variable.

# How Random Variables Are Different From Traditional Variables Used In Algebra?

Let's say variables used in algebra as x, y, z. Here x can be the number of cell phones, y = no of heads or z= no of students. A variable is nothing but an alphabetical character which represents an unknown number.

For Example:
$x + 5 = 10$
x is the variable whose value is unknown and we are trying to find its value.
After evaluating, $x=5$.

A Random Variable is different from the variable in algebra as it has whole set of values and it can take any of those randomly. Variable used in algebra cannot have more than a single value at a time.
If random variable $X=\{0,1,2,3\}$
Then X could be 0, 1, 2 or 3 randomly where each of them might have a different probability.
We use capital letter for random variables to avoid confusion with traditional variables.

# TYPES OF RANDOM VARIABLES

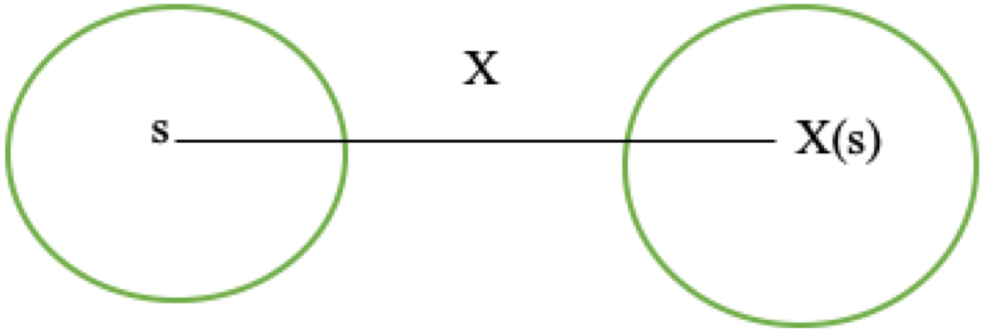
RANDOM VARIABLES

DISCRETE

CONTINUOUS

RANDOM VARIABLES CAN BE EITHER **DISCRETE OR CONTINUOUS**

A **random variable** X on a sample space S is a function X: S→R from S to the set of real numbers R, which assigns a real number X(s) to each sample point s of S.




S: Sample Space  $R_x$: Possible Values of x

Range space $R_x$: is the set of all possible values of X is a subset of real numbers R.

Although X is called a random "variable" note that it is infact a "single-valued function".

**Notation:** If R.V. is denoted by X, then x (corresponding small letter) denoted one of its values.

**Discrete:** A R.V. X is said to be discrete R.V. if its set of possible outcomes, the sample space S, is countable (finite or an unending sequence with as many elements as there are whole numbers).

**Continuous:** A R.V. is said to be continuous R.V. if S contains infinite numbers equal to the number of points on a line segment.

# DISCRETE RANDOM VARIABLE:

- If a variable can take countable number of distinct values then it's a **discrete random variable**.

For example:

In an experiment of tossing 2 coins, we need to find out the possible number of heads.
In this case, X is the random variable and the possible values taken by it is 0, 1 and 2 which is discrete.
Therefore X= {0, 1, 2}

*Discrete probability distribution,* probability function or probability mass function of a discrete R.V. X is the function f(x) satisfying the following conditions:

- $f(x) \geq 0$
- $\sum f(x) = 1$
- $P(X = x) = f(x)$

Thus probability distribution is the set of ordered pairs $(x, f(x))$ ie, outcome x and its probability (chance) $f(x)$

Example: Let X denote the discrete R.V. which denotes the minimum of the two numbers that appear in a single throw of a pair of fair dice. Then X is a function from the sample space S consisting of 36 ordered pair $\{(1,1), (1,2), \dots \dots (6,6)\}$ to a subset of real numbers $\{1,2,3,4,5,6\}$.

The event minimum 5 can appear in the following cases (occurrences) (5,5), (5,6),(6,5). Thus R.V. X assigns to this event of the sample space a real number 3. The probability of such an event happening is $\frac{3}{36}$ since there are 36 exhaustive cases. This is represented as

$$P(X = x_i) = \mathrm{p}_i = f(x_i) = P(X = 5) = f(5) = \frac{3}{36}$$

Calculating in a similar way the other probabilities, the distribution of probabilities of this discrete R.V. is denoted by the discrete probability distribution as follows:

| $X = x_i$ | 1 | 2 | 3 | 4 | 5 | 6 |
|---|---|---|---|---|---|---|
| $P(X = x_i)$ $= f(x_i)$ $= \mathrm{p}_i$ | $\frac{11}{36}$ | $\frac{9}{36}$ | $\frac{7}{36}$ | $\frac{5}{36}$ | $\frac{3}{36}$ | $\frac{1}{36}$ |

For example, consider a six-sided die, pictured below.

We could let $X$ be the random value that gives the value observed on the upper face of the six-sided die after a single roll. Then if $x$ denotes a particular value of the upper face, the expression $X = x$ becomes well-defined. Specifically, the notation $X = x$ signifies the event that the *random variable* $X$ assumes the *particular value* $x$. For the six-sided die example, $x$ can be any integer from 1 to 6. So the expression $X = 4$ would express the event that a random roll of the die would result in observing the value 4 on the upper face of the die.

## Six-Sided Die Example

Using our six-sided die example above, we have the random variable $X$ which represents the value we observe on the upper face of the six-sided die after a single roll. Then the probability that $X$ is equal to 5 can be written as:

Using our identities for complimentary events and for disjoint events, we find that the probability that $X$ is equal to 1, 2, 3 or 4 can be computed as:

$$\Pr(X = 5) = \frac{1}{6}$$

$$\begin{aligned}
\Pr(1 \leq X \leq 4) &= \Pr(X = 1, \text{ or } X = 2, \text{ or } X = 3, \text{ or } X = 4) \\
&= 1 - \Pr(X = 5, \text{ or } X = 6) \\
&= 1 - [\Pr(X = 5) + \Pr(X = 6)] \\
&= 1 - (\frac{1}{6} + \frac{1}{6}) \\
&= \frac{2}{3}
\end{aligned}$$

Notice that $X \sim$ Uniform(6); i.e. $X$ has a uniform distribution on the integers from 1 to 6. Indeed, the probability of observing any one of these integer values (the value on the upper face of the rolled die) is the same for any value. Thus, $X$ must be a uniform random variable.

# CONTINUOUS RANDOM VARIABLE:

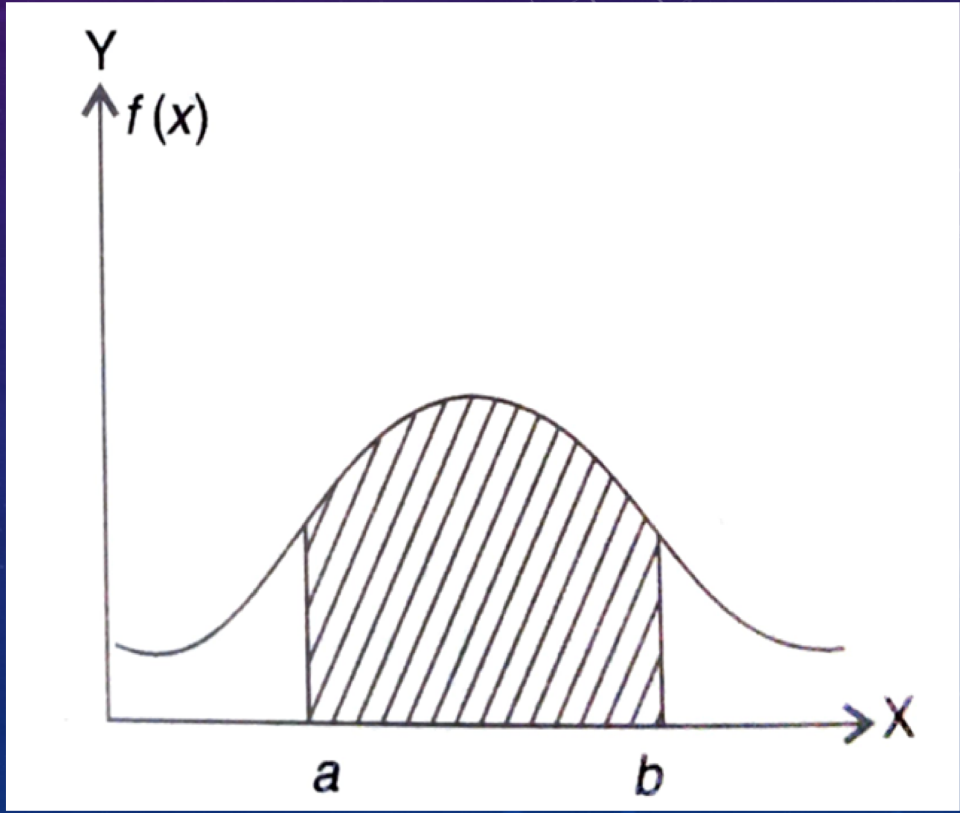
A random variable is said to be **continuous** if it takes infinite number of values in an interval.

For example: Suppose the temperature in a city lies between $30^{o}$ and $45^{o}$ centigrade. The temperature can take any value in the interval $30^{o}$ to $45^{o}$. So the temperature can be either $30.13^{o}$ or $40.15^{o}$ or it may be in $30.13^{o}$ and $40.15^{o}$. When we say temperature is $38^{o}$, it means it lies somewhere between 37.5 and 38.5. So there is nothing exact or discrete observation in continuous random variable.

So this was brief introduction of random variables.

**Continuous probability distribution:** for a continuous R.V. X, the function $f(x)$ satisfying the following is known as the probability density function (P.D.F.) Or simply density function.

- (i) $f(x) \geq 0$
- $(ii) \int_{-\infty}^{\infty} f(x)\,dx = 1$
- $(iii) \quad P(a < X < b) = \int_a^b f(x)\,dx =$ area under $f(x)$ between ordinates $x = a$ and $x = b$




**Note 1:** $P(a < X < b) = P(a \leq X < b)$

$P(a < X \leq b) = P(a \leq X \leq b)$

i.e. inclusion or non-inclusion of end points, does not change the probability, which is not the case in the discrete distribution.

**Note 2:** Probability at a point

$$P(X = a) = \int_{a-\Delta x}^{a+\Delta x} f(x)dx$$

# Thank You