Exponential Distribution

Exercises

1. The time intervals between successive barges passing a certain point on a busy waterway have an exponential distribution with mean 8 minutes.

   (a) Find the probability that the time interval between two successive barges is less than 5 minutes. 0.4647

   (b) Find a time interval t such that we can be 95% sure that the time interval between two successive barges will be greater than t. 0.4103

2. It is believed that the time X for a worker to complete a certain task has probability density function $f_X(x)$ where

$$f_X(x) = \begin{cases} 0 & (x \leq 0) \\ kx^2 e^{-\lambda x} & (x > 0) \end{cases}$$

where λ is a parameter, the value of which is unknown, and k is a constant which depends on λ.

   (a) Show that if $\quad I_n = \displaystyle\int_0^\infty x^n e^{-\lambda x}\, dx \quad$ then $\quad I_n = \dfrac{n}{\lambda} I_{n-1}, \quad$ where $n > 0$ and $\lambda > 0$.

   Evaluate $\quad I_0 = \displaystyle\int_0^\infty e^{-\lambda x}\, dx \quad$ and hence find a general expression for I_n.

   This result can be used in the rest of this question. $I_n = \dfrac{n!}{\lambda^{n+1}}$

   (b) Find, in terms of λ, the value of k. $\lambda^3/2$

   (c) Find, in terms of λ, the expected value of X. $3/\lambda$

   (d) Find, in terms of λ, the variance of X. $3/\lambda^2$

   (e) Write down the expected value and variance of the sample mean of a sample of n independent observations on X. $\dfrac{3}{\lambda}, \dfrac{3}{n\lambda^2}$

   (f) Find, in terms of λ, the expected value of X^{-1}. $\lambda/2$

Uniform Distribution

Exercises

1. In the manufacture of petroleum the distilling temperature (T°C) is crucial in determining the quality of the final product. T can be considered as a random variable uniformly distributed over 150°C to 300°C. It costs £C_1 to produce 1 gallon of petroleum. If the oil distills at temperatures less than 200°C the product sells for £C_2 per gallon. If it distills at a temperature greater than 200°C it sells for £C_3 per gallon. Find the expected net profit per gallon. $\frac{C_2 - 3C_1 + 2C_3}{3}$

2. Packages have a nominal net weight of 1 kg. However their actual net weights have a uniform distribution over the interval 980 g to 1030 g.

   (a) Find the probability that the net weight of a package is less than 1 kg. 0.4

   (b) Find the probability that the net weight of a package is less than w g, where $980 < w < 1030$. $\frac{w-980}{50}$

   (c) If the net weights of packages are independent, find the probability that, in a sample of five packages, all five net weights are less than wg and hence find the probability density function of the weight of the heaviest of the packages. (Hint: all five packages weigh less than w g if and only if the heaviest weighs less that w g). $0.1\left(\frac{w-980}{50}\right)^4$ ~~980~~ $w \leq 1030$