MIT Arts, Design and Technology University, Rajbaug, Pune MIT

School of Engineering

B.TECH. (Computer Science and Engineering), T.Y.SEM-V

Probability and Queueing Theory

UNIT-III ASSIGNMENT-III

Q.1. Write the properties of Markov chain. (Any four)

Q.2. Define following terms with an example

(i) Markov Chain (ii) Transition Probability Matrix

(iii)one-step transition matrix (iv) n-step transition matrix

(v) Steady state (equilibrium) condition

Q.3. What do you understand by Markov process? In what areas of management can it be applied successfully?

Q.4. Define following terms with an example:

(i) Transition Probability matrix (ii) Transition diagram

Q.5. Define Chapman Kolmogorov Equations and Limiting distribution.

Q.6. The school of international studies for population found out by its survey that the mobility of the population of a state to the village, town and city is in the following percentage:

From	To		
	Village	Town	City
Village	50%	30%	20%
Town	10%	70%	20%
City	10%	40%	50%

What will be the proportion of population in village, town and city after two tears, given that present population has proportion of 0.7, and 0.2 and 0.1 in the village, town and city respectively? What will be the respective proportions in long run?

Q.7.The number of units of an item that are withdrawn from inventory on a day-to-day basis is a Markov chain process in which requirements for tomorrow depend on today's requirements. A one day transition matrix is given below:

Number of units withdrawn from inventory

$$\begin{array}{cc} & \text{Tomorrow} \\ & \begin{array}{ccc} 5 & 10 & 12 \end{array} \\ \text{Today} \begin{array}{c} 5 \\ 10 \\ 12 \end{array} & \begin{bmatrix} 0{\cdot}6 & 0{\cdot}4 & 0{\cdot}0 \\ 0{\cdot}3 & 0{\cdot}3 & 0{\cdot}4 \\ 0{\cdot}1 & 0{\cdot}3 & 0{\cdot}6 \end{bmatrix} \end{array}$$

(i) Construct a tree diagram showing inventory requirements on two consecutive days

(ii) Develop a two-day transition matrix.

(iii) Comment on how a two-day transition matrix might be helpful to a manage who is responsible for inventory management.

Q.8. Suppose there are three diaries in a town, say A, B and C. They supply all the milk consumed in the town. It is known by all the diaries that consumers switch from dairy to dairy overtime because of advertising, dissatisfaction with service and other reasons. All these diaries maintain records of the number of their customers and the daily from which they obtained each new customers over an observation period of one month, say June

Dairy	June 1 (Customers)	Gains from			Losses to			July 1 (Customers)
		A	B	C	A	B	C	
A	200	0	35	25	0	20	20	220
B	500	20	0	20	35	0	15	490
C	300	20	15	0	25	20	0	290

We assume that the matrix of transition probabilities remain fairly stable and that the July market shares are A = 22 %, B = 49 %, C = 29 %. Managers of these diaries are willing to know.

(i) Market share of their diaries on 1st August and 1st September.

(ii) Their market shares in steady state.

Q.9. Define Chapman Kolmogorov equation and Limiting distribution.

and consider the Markov chain shown in Figure.

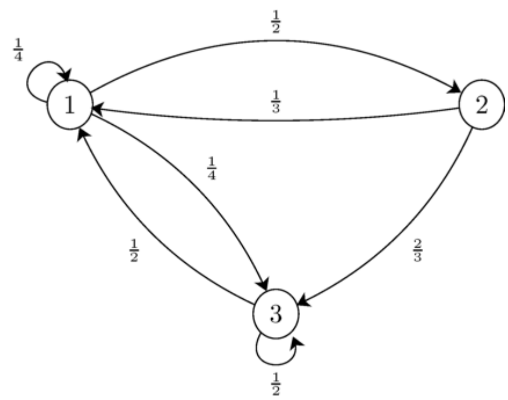

Figure- A state transition diagram.

Find (i) Is this chain irreducible?
(ii) Is this chain aperiodic?

(iii)The stationary distribution for this chain.

(iv)Is the stationary distribution a limiting distribution for the chain?

Q.10. The number of units of an item that are withdrawn from inventory on a day to day basis is a Markov chain process in which requirements for tomorrow depend on today's requirements, a one day transition matrix is given below:

Number of units withdrawn from inventory

		Tomorrow		
		5	10	12
Today	5	0.6	0.4	0.0
	10	0.3	0.3	0.4
	12	0.1	0.3	0.6

(i) Develop a two-day transition matrix.
(ii) Comments on how a two-day transition matrix might be helpful to a manager who is responsible for inventory management. Also construct a tree diagram showing inventory requirements on two consecutive days.

Q.11. On January 1 (this year) bakery A had 40% of its local market share which the other two bakeries B and C had 40% and 20% respectively of the market share. Based upon a study by a marketing research firm, the following facts were complied. Bakery A retains 90% of its customers while gaining 5 % of competitor B's customers and 10% of C's customers. Bakery B retains 85% of its customers while gaining 5 % of A's customer and 7% of C's customers. Bakery C retains 83% of its customers and gains 5% of A's and 10% of B's customers. What will each firms share be on January 1, next year, and what will each firm's market share be at equilibrium?

Q.11. What do you understand by Markov chains? Explain how it can be used for predicting sales force needs.

Q.13. The state transition matrix for retentions, gains and losses of firms A, B and C is given below .Using this matrix determine the steady state equilibrium conditions:

To			
From	A	B	C
A	0.700	0.100	0.200
B	0.100	0.800	0.100
C	0.200	0.100	0.700

Q.14. Suppose there are two market products of brands A and B respectively. Let each of these two brands have exactly 50% of the total market in same period and let the market be of fixed size .The transition matrix is given below:

		To	
From		A	B
	A	0.9	0.1
	B	0.5	0.5

If the initial market share breakdown is 50% for each brand, then determine their market shares in the steady-state.

Q.15. On January 1 (this year) bakery A had 40% of its local market share which the other two bakeries B and C had 40% and 20% respectively of the market share. Based upon a study by a marketing research firm the following facts were complied. Bakery A retains 90% of its customer while gaining 5 % of competitor B's customer and 10% of C's customers while gaining 5 % of A's customer and 7% of C's customer Bakery C retains 83% of its customers and gains 5% of A's and 10% of B's customers. What will each firms share be on January 1 Next year and what will each firm's market share be at equilibrium?

Q.16. A housewife buys three kind of cereals A, B and C. she never buys the same cereal on successive weeks. If she buys cereal A then the next week she buys cereal B. However, if she buys either B or C, then the next week she is three times as likely to buy A as the other brand. Obtain the transition probability matrix and determine how often he would but each of the cereals in the long run.

Q.17. Two manufacturers A and B are competing with each other in a restricted market. Over the years, A's customers have exhibited a high degree of loyalty as measured by the fact that customers are using A's product 80 percent of time. Also former customers purchasing the product from B have switched back to A's product 60 percent of the time. Construct and interpret the state transition matrix in

(i) Retention and loss and

(ii) Retention and gain.

Q.18. Let $X_i = 0$ if it rains on day i; otherwise, $X_i = 1$. Suppose $P_{00} = 0.7$ and $P_{10} = 0.4$. Then

$$P = \begin{pmatrix} 0.7 & 0.3 \\ 0.4 & 0.6 \end{pmatrix}$$

Suppose it rains on Monday. Then what is the probability that it rains on Friday? And also suppose $\alpha_0 = 0.4$ and $\alpha_1 = 0.6$. Find the probability that it will not rain on the 4th day after we start keeping records (assuming nothing about the first day)?