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Solutions for Assignment 3.

Sol. 1 → $\mu = 50$.

The formula for the Mean for the random variable defined as number of failure until first success is $\mu = \frac{1}{p} = \frac{1}{0.02} = 50$

The formula for the variance is σ^2

$$= \left(\frac{1}{p}\right)\left(\frac{1}{p} - 1\right) = \left(\frac{1}{0.02}\right)\left(\frac{1}{0.02} - 1\right) = 2,450$$

The standard deviation is σ

$$= \sqrt{\left(\frac{1}{p}\right)\left(\frac{1}{p} - 1\right)} = \sqrt{\left(\frac{1}{0.02}\right)\left(\frac{1}{0.02} - 1\right)} = 49.5$$

Sol. 3 → $P(x=3) = (1-0.32)^{3-1} \times .32 = 0.1480$.

In this case, the sequence is failure, failure success.

Sol. 4 → $P(x=3) = (1-0.80)^3 \times 0.80 = 0.0064$.

Sol. 5 → We have $A = 2$, $\mu_1' = 1$, $\mu_2' = 2.5$, $\mu_3' = 5.5$, $\mu_4' = 16$.

<u>Moment about Mean</u>

$\mu_1 = 0$

$\mu_2 = \mu_2' - (\mu_1')^2 = 2.5 - (1)^2 = 1.5$

$\mu_3 = \mu_3' - 3\mu_2'\mu_1' + 2(\mu_1')^3 = 5.5 - 3(2.5)(1) + 2(1)^3 = 0$

$\mu_4 = \mu_4' - 4\mu_3'\mu_1' + 6\mu_2'\mu_1'^2 - 3\mu_1'^4 = 16 - 4(5.5)(1) + 6(2.5)(1)^2 - 3(1)^4 = 6$.

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Moment about origin

$v_1 = \bar{x} = A + \mu_1'$, $\quad v_2 = \mu_2 + \bar{x}^2$

$v_3 = \mu_3 + 3\mu_2\bar{x} + \bar{x}^3$ $\quad v_4 = \mu_4 + 4\mu_3\bar{x} + 6\mu_2\bar{x}^2 + \bar{x}^4$

$\therefore$ $v_1 = \bar{x} = 2 + 1 = \underline{3}$ $\quad v_2 = 1.5 + (3)^2 = \underline{10.5}$

$v_3 = 0 + 3(1.5)(3) + (3)^3$
$= \underline{40.5}$

$v_4 = 6 + 4(0)(3) + 6(1.5)(3)^2 + (3)^4$
$= \underline{168}$.

Sol. 6:→ We have $A = 4$, $\mu_1' = -1.5$, $\mu_2' = 17$,
$\mu_3' = -30$, $\mu_4' = 308$

Moment about Mean

$\mu_1 = 0$

$\mu_2 = \mu_2' - \mu_1'^2 = 17 - (-1.5)^2 = \underline{14.75}$

$\mu_3 = \mu_3' - 3\mu_2'\mu_1' + 2\mu_1'^3 = -30 - 3(17)(-1.5) + 2(-1.5)^3$
$= \underline{39.75}$

$\mu_4 = \mu_4' - 4\mu_3'\mu_1' + 6\mu_2'\mu_1'^2 - 3\mu_1'^4$

$= 308 - 4(-30)(-1.5) + 6(17)(-1.5)^2 - 3(-1.5)^4$
$= \underline{342.3125}$

Moments about origin

$v_1 = \bar{x} = \mu_1' + A = -1.5 + 4 = \underline{2.5}$

$v_2 = \mu_2 + \bar{x}^2 = 14.75 + (2.5)^2 = \underline{21}$

$v_3 = \mu_3 + 3\mu_2\bar{x} + \bar{x}^3 = \underline{166}$

$v_4 = \mu_4 + 4\mu_3\bar{x} + 6\mu_2\bar{x}^2 + \bar{x}^4 = \underline{1332}$.

Calculation of β_1 and β_2

$\beta_1 = \dfrac{\mu_3^2}{\mu_2^3} = \underline{0.492377}$, $\quad \beta_2 = \dfrac{\mu_4}{\mu_2^2} = \underline{1.573398}$

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Sol. 7 → We have $A = 4$, $\mu'_1 = -1.5$, $\mu'_2 = 17$, $\mu'_3 = -30$
$\mu_4 = 108$.

Moment about Mean

$\mu_1 = 0$

$\mu_2 = \mu'_2 - \mu'^2_1 = 14.75$

$\mu_3 = \mu'_3 - 3\mu'_2\mu'_1 + 2\mu'^3_1 = 39.75$

$\mu_4 = \mu'_4 - 4\mu'_3\mu'_1 + 6\mu'_2\mu'^2_1 - 3\mu'^4_1 = 142.3125$.

Also, $\bar{x} = \mu'_1 + A = -1.5 + 4 = 2.5$

Moments about origin

$v_1 = \bar{x} = 2.5$

$v_2 = \mu_2 + \bar{x}^2 = 14.75 + (2.5)^2 = 21$

$v_3 = \mu_3 + 3\mu_2\bar{x} + \bar{x}^3 = 166$

$v_4 = \mu_4 + 4\mu_3\bar{x} + 6\mu_2\bar{x}^2 + \bar{x}^4 = 1132$.

Calculation of β_1 and β_2

$\beta_1 = \frac{\mu_3^2}{\mu_2^3} = 0.492377$, $\beta_2 = \frac{\mu_4}{\mu_2^2} = 0.654122$.

Moment about the point $x = 2$.

$\mu'_1 = \bar{x} - A = 2.5 - 2 = 0.5$

$\mu'_2 = \mu_2 + \mu'^2_1 = 14.75 + (.5)^2 = 15$

$\mu'_3 = \mu_3 + 3\mu'_2\mu'_1 = 2\mu'^3_1 = 39.75 + 3(15)(.5)$
$- 2(.5)^3 = 62$

$\mu_4 = \mu_4 + 4\mu'_3\mu'_1 - 6\mu'_2\mu'^2_1 \cancel{+} 3\mu'^4_1 = 244$.

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Sol. 8 → Moment generating function of the origin is given by

$$M_x(t) = \Sigma e^{tx} \cdot e^{-\lambda} \cdot \frac{\lambda^x}{x!} = e^{-\lambda} \Sigma \frac{(\lambda e^t)^x}{x!} = e^{-\lambda} \cdot e^{\lambda e^t}$$
$$= e^{\lambda(e^t-1)}$$

$$\nu_1 = \left[\frac{d}{dt} M_x(t)\right]_{t=0} = [e^{\lambda(e^t-1)} \lambda e^t]_{t=0} = \lambda$$

$$\nu_2 = \left[\frac{d^2}{dt^2} M_x(t)\right]_{t=0} = [\lambda \{e^t \cdot e^{\lambda(e^t-1)} \cdot \lambda e^t + e^{\lambda(e^t-1)} e^t\}]_{t=0}$$

$$= [\lambda e^{\lambda(e^t-1)} e^t (\lambda e^t + 1)]_{t=0} = \lambda(\lambda+1).$$

Hence, first and second moment about the Mean are given by

Since $\mu_1 = 0$
$\nu_1 = \bar{x} = \lambda$

$\therefore$ $$\mu_2 = \nu_2 - \bar{x}^2 = \nu_2 - \nu_1^2 = \lambda(\lambda+1) - \lambda^2 = \underline{\lambda}.$$

Sol. 9 → Moment generating function about the origin is defined ~~by~~ as

$$M_x(t) = E(e^{tx}) = \int_{-\infty}^{\infty} e^{tx} \cdot \frac{1}{\sigma\sqrt{2\pi}} e^{-\frac{1}{2}\left(\frac{x-\mu}{\sigma}\right)^2} dx$$

$$= \frac{e^{\mu t}}{\sqrt{2\pi}} \int_{-\infty}^{\infty} e^{-\frac{1}{2}z^2} \cdot e^{-t\sigma z} dz \qquad \boxed{\text{where } z = \frac{x-\mu}{\sigma}}$$

$$= \frac{1}{\sqrt{2\pi}} e^{(\mu t + \frac{1}{2}t^2\sigma^2)} \int_{-\infty}^{\infty} e^{-\frac{1}{2}(z-t\sigma)^2} dz)$$

$$= e^{\mu t + \frac{1}{2}t^2\sigma^2} \cdot 1 = e^{\mu t + \frac{1}{2}t^2\sigma^2}.$$

Sol. 10 →

$$p = \frac{1}{13}$$

$$p(20) = \binom{y-1}{r-1} p^r q^{y-r}$$

$$= \binom{19}{3}\left(\frac{1}{13}\right)^4\left(\frac{12}{13}\right)^{16} = \binom{19}{3}\left(\frac{12^{16}}{13^{20}}\right).$$

Sol. 11 →

$$p = \frac{1}{10}$$

$$q = \frac{9}{10}$$

$$r = 5$$

$$\mu = \frac{r}{p} = \boxed{50 \text{ years}}$$

$$\sigma^2 = \frac{rq}{p^2} = \frac{5\frac{9}{10}}{\frac{1}{10^2}} = 450$$

$$\sigma = \sqrt{450} = 15\sqrt{2} \approx \boxed{21.21 \text{ years}}$$

Sol. 12 → (a) This is the negative binomial distribution with

$$p = \frac{1}{10}, \quad q = \frac{9}{10}, \quad r = 3$$

$$p(10) = \binom{y-1}{r-1} p^r q^{y-r}$$

$$= \binom{9}{2}\left(\frac{1}{10}\right)^3\left(\frac{9}{10}\right)^7 = 36\left(\frac{9^7}{10^{10}}\right).$$

(b) Same as 'a'.

Sol. 13 → The Time is $T = 3(Y-3)+15 = 3Y+6$. The Mean is $E(T) = 3E(Y)+6 = 3\left(\frac{r}{p}\right)+6 = 3\left(\frac{3}{\frac{1}{10}}\right)+6$.

$\boxed{96 \text{ hours}}$.

The variance $(V(aY+b) = a^2V(Y))$ is $V(T)$
$= 9V(Y) = 9\frac{rq}{p^2} = 9\left(\frac{\frac{27}{10}}{\frac{1}{100}}\right)$

2430 hours^2.

The standard deviation is $\sqrt{2430} = 9\sqrt{30} \approx$

$\boxed{49.295 \text{ hours}}$

Sol. 14 → This is the negetive binomial distribut with $p = \frac{1}{6}$, $r = 4$.

$$\mu = \frac{r}{p} = \boxed{24 \text{ rolls}}$$

$$\sigma^2 = \frac{rq}{p^2} = \frac{4 \cdot \frac{5}{6}}{\frac{1}{36}}$$

$$= 120$$

$$\sigma = \sqrt{120} = \boxed{2\sqrt{30} \approx 10.95 \text{ rolls}}$$

Sol. 2 → Mean → $\mu = \frac{(1-p)}{p} = \frac{(1-0.0128)}{0.0128} = 77.12$

(ii) Standard deviation →

$$\sigma = \sqrt{\frac{1-p}{p^2}} = \sqrt{\frac{1-0.0128}{0.0128^2}} = 77.62.$$