Uncertainty and Methods

Uncertainty

- Agents may need to handle **uncertainty**, whether due to partial observability, nondeterminism, or a combination of the two.
- diagnosing a dental patient’s toothache. Diagnosis—whether for medicine, automobile repair, or whatever—almost always involves uncertainty.

$Toothache \Rightarrow Cavity$

$Toothache \Rightarrow Cavity \lor GumProblem \lor Abscess \ldots$

$Cavity \Rightarrow Toothache$

Reasons.....

- **Laziness:** It is too much work to list the complete set of antecedents or consequents needed to ensure an exceptionless rule and too hard to use such rules.
- **Theoretical ignorance:** Medical science has no complete theory for the domain.
- **Practical ignorance:** Even if we know all the rules, we might be uncertain about a particular patient because not all the necessary tests have been or can be run.

- The agent’s knowledge can at best provide only a degree of belief in the relevant sentences. Main tool for dealing with degrees of belief is probability theory.
- Probability provides a way of summarizing the uncertainty that comes from our laziness and ignorance.

An agent must first have preferences between the different possible outcomes of the various plans.
An outcome is a completely specified state.
We use utility theory to represent and reason with preferences.
(The term utility is used here in the sense of “the quality of being useful,” not in the sense of the electric company or water works.)
Utility theory says that every state has a degree of usefulness, or utility, to an agent and that the agent will prefer states with higher utility.

Decision theory = probability theory + utility theory

Bayesian Probability and belief network

- Product rule can be written in two forms:

  $P(a \wedge b) = P(a \mid b)P(b)$ and $P(a \wedge b) = P(b \mid a)P(a)$

- Equating the two right-hand sides and dividing by P(a), we get

  $P(b \mid a) = P(a \mid b)P(b)/P(a)$

- This equation is known as Bayes' rule
- This simple equation underlies most modern AI systems for probabilistic inference.

Applying Bayes' rule:

- Bayes' rule is useful in practice because there are many cases where we do have good probability estimates for these three numbers and need to compute the fourth.
- Often, we perceive as evidence the effect of some unknown cause and we would like to determine that cause. In that case, Bayes' rule becomes

$$P(cause \mid effect) = \frac{P(effect \mid cause)P(cause)}{P(effect)}$$

Example:

A doctor knows that the disease meningitis causes the patient to have a stiff neck, say, 70% of the time. The doctor also knows some unconditional facts: the prior probability that a patient has meningitis is 1/50,000, and the prior probability that any patient has a stiff neck is 1%. Letting s be the proposition that the patient has a stiff neck and m be the proposition that the patient has meningitis, we have

$$\begin{aligned} P(s \mid m) &= 0.7 \\ P(m) &= 1/50000 \\ P(s) &= 0.01 \\ P(m \mid s) &= \frac{P(s \mid m)P(m)}{P(s)} = \frac{0.7 \times 1/50000}{0.01} = 0.0014 \end{aligned}$$

- **Combining evidence**

when we have two or more pieces of evidence?

For example, what can a dentist conclude if her nasty steel probe catches in the aching tooth of a patient?

$$\mathbf{P}(Cavity \mid toothache \land catch) = \alpha \langle 0.108, 0.016 \rangle \approx \langle 0.871, 0.129 \rangle$$

where α is the normalization constant needed to make the entries in P(Y |X) sum to 1.

$$\begin{aligned} &\mathbf{P}(Cavity \mid toothache \land catch) \\ &\quad = \alpha\, \mathbf{P}(toothache \land catch \mid Cavity)\, \mathbf{P}(Cavity) \end{aligned}$$

$$\mathbf{P}(Toothache, Catch \mid Cavity) = \mathbf{P}(Toothache \mid Cavity)\mathbf{P}(Catch \mid Cavity)$$

$$\begin{aligned} &\mathbf{P}(Toothache, Catch, Cavity) \\ &= \mathbf{P}(Toothache, Catch \mid Cavity)\mathbf{P}(Cavity) \quad \text{(product rule)} \\ &= \mathbf{P}(Toothache \mid Cavity)\mathbf{P}(Catch \mid Cavity)\mathbf{P}(Cavity) \quad \text{(us} \end{aligned}$$