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Probability and Queueing Theory
UNIT-I
ASSIGNMENT-I

Q.1. Determine the discrete probability distribution, expectation, variance, S.D. of a discrete random variable (D.R.V.) X which denotes the minimum of the two numbers that appear when a pair of fair dice is thrown once.

Q.2. Prove that
(a) $E(kX)=kE(X)$
(b) E $(X+k)=E(X)$ +k
(c) E $(X+Y)=E(X)$ +E(Y)

Q.3. Suppose a continuous R.V. x has the probability density

$$f(x)=\begin{cases} k(1-x^2) \; for \; 0<x<1 \\ 0 \; Elsewhere \end{cases}$$

(a) Find k
(b) Find P(0.1 < x < 0.2)
(c) P(x > 0.5)
Using distribution function, determine the probabilities that
(d) X is less than 0.3
(e) Between 0.4 and 0.6
(f) Calculate mean and variance for the probability density function.

Q.4. The first four moments of a distribution about x = 2 are 1, 2.5, 5.5 and 16. Calculate the four moments about $\acute{X}$ and about zero.

Q.5. Calculate first four moments about the mean and also the value of β_1 and β_2 from the following data:

Marks	0-10	10-20	20-30	30-40	40-50	50-60	60-70
No. of students	8	12	20	30	15	10	5

Q.6. A player tosses 3 fair coins. He wins Rs. 500 if 3 heads occur, Rs. 300 if 2 heads occur, Rs. 100 if one head occurs. On the other hand, he losses Rs.1500 if 3 tails occur. Find the value of the game to the player. Is it favourable?

Q.7. A person wins Rs. 80 if 3 heads occur, Rs. 30 if 2 heads occur, Rs. 10 if only on head occurs in a single toss of 3 fair coins. If the game is to be fair, how much should he lose if no heads occur?

Q.8. Find the mean and variance of P.D.F.n

$$f(x)=\begin{cases}\frac{1}{4}e^{\frac{-x}{4}} & ,for\ x>0\\ 0\ elsewhere. & \end{cases}$$

Q.9. A random variable X has the following distribution

X	-2	-1	0	1	2	3
P (X)	0.1	k	0.2	2K	0.3	K

Determine (i) k (ii) Mean

Q.10. For the continuous probability function $f(x) = kx^2e^{-x}$ when $x \geq 0$. Find the value of k.

Q.11. A coin is tossed six times. What is the probability of obtaining four or more heads?

Q.12. Probability of Man aged 60 years will live for 70 years is $\frac{1}{10}$. Find the probability of 5 mean selected at random 2 will live for 70 years.

Q.13. Out of 800 families with 5 children each, how many would you expect to have

(i) 3 boys. (ii) 5 girls (iii) either 2 or 3 boys. Assume equal probabilities for boys and girls.

Q.14. The probability that a patient recovers from a rare blood disease is 0.4. If 15 people are known to have contracted this disease, what is the probability that (a) at least 10 survive (b) from 3 to 8 survive and (c) Exactly 5 survive?

Q.15. A large chain retailer purchases a certain kind of electronic device from a manufacturer. The manufacturer indicates that the defective rate of the device is 3%.

(a) The inspector randomly picks 20 items from a shipment. What is the probability that there will be at least one defective item among these 20?

(b) Suppose that the retailer receives 10 shipment in a month and the inspector randomly tests 20 devices per shipment. What is the probability that there will be exactly 3 shipments each containing at least one defective device among the 20 that are selected and tested from the shipment?

Q.16. It is conjectured that an impurity exists in 30% of all drinking wells in a certain rural community. In order to gain some insight into the true extent of the problem, it is determined that some testing is necessary. It is too expensive to test all of the wells in the area, so 10 are randomly selected for testing.

(a) Using the binomial distribution, what is the probability that exactly 3 wells have the impurity, assuming that the conjecture is correct?

(b) What is the probability that more than 3 wells are impure?

Q.17. In a binomial distribution mean and standard deviation are 12 and 2 respectively, find n and p.

Q.18. During a laboratory experiment, the average number of radioactive particles passing through a counter in 1 millisecond is 4. What is the probability that 6 particles enter the counter in a given millisecond?

Q.19. Ten is the average number of oil tankers arriving each day at a certain port. The facilities at the port can handle at most 15 tankers per day. What is the probability that on a given day tankers have to be turned away?

Q.20. Suppose the average number of telephone calls coming into a telephone exchange between 10 AM to 11 AM is 2, while between 11 AM to 12 noon is 6, Determine the probability that more than five calls come in between 10 AM to 12 noon, assuming that calls are independent.

Q.21. Fit the Poisson distribution to the following data

X	0	1	2	3	4	5	6	7	8
Observed frequency f_i	56	156	132	92	37	22	4	0	1

Q.22. Lots of 40 components each are deemed unacceptable if they contain 3 or more defectives. The procedure for sampling a lot is to select 5 components at random and to reject the lot if a defective is found. What is the probability that exactly 1 defectives is found in the sample if there are 3 defectives in the entire lot? Find the mean and variance of the random variable.

Q.23. Determine the probability that exactly one defective is found in a sample of 5 from a lot of 40 components containing 3 defectives (in the entire lot). Also find mean and variance of the random X.

Q.24. In an NBA (National Basketball Association) championship series, the team that wins four games out of seven is the winner. Suppose that teams A and B face that other in the championship games and that team A has probability 0.55 of winning a game over team B.
(a) What is the probability that team A will win the series in 6 games?
(b) What is the probability that team A will win the series?
(c) If teams A and B were facing each other in a reginal playoff series, which is decided by winning three out of five games, what is the probability that team A would win the series?

Q.25. For a certain manufacturing process it is known that on the average, 1 in every 100 items is defective. What is the probability that the fifth item inspected is the first defective item found?

Q.26. At a "busy time", a telephone exchange is very near capacity, so callers have difficulty placing their calls. It may be of interest to know the number of attempts necessary in order to make a connection. Suppose that we let p = 0.05 be the probability of a connection during a busy time. We are interested in knowing the probability that 5 attempts are necessary for a successful call.

Q.27. A certain type of storage battery lasts, on average, 3.0 years with a standard deviation of 0.5 year. Assuming that battery life is normally distributed, Find the probability that a given battery will last less than 2.3 years.

Q.28. An electrical firm manufactures light bulbs that have a life, before burn-out, that is normally distributed with mean equal to 800 hours and a standard deviations of 40 hours. Find the probability that a bulb burns between 778 and 834 hours.

Q.29. In an industrial process, the diameter of a ball bearing is an important measurement. The buyer sets specifications for the diameter to be $3.0 \pm 0.01\,cm$. The implication is that no part falling outside these specifications will be accepted. It is known that in the process the diameter of a ball bearing has a normal distribution with mean $\mu = 3.0$ and standard deviation $\sigma = 0.005$. On an average, how many manufactured ball bearings will be scrapped?

Q.30. A certain machine makes electrical resistors having a mean resistance of 40 ohms and a standard deviation of 2 ohms. Assuming that the resistance follows a normal distribution and can be measured to any degree of accuracy, what percentage of resistors will have a resistance exceeding 43 ohms?

Q.31. The probability that a patient recovers from a rare blood disease is 0.4. If 100 people are known to have contracted this disease, what is the probability that fewer than 30 survive?

Q.32. A multiple-choice quiz has 200 questions, each with 4 possible answers of which only 1 is correct. What is the probability that sheer guesswork yields from 25 to 30 correct answers for the 80 of the 200 problems about which the student has no knowledge?

Q.33. Assume that the average life span of computers produced by a company is 2040 hours with s.d. of 60 hours. Find the expected number of computers whose life span is

(a) more than 2150 hours

(b) less than 1950 hours

(c) more than 1920 hours and less than 2160 hours

From a pool of 2000 computers assuming that the life span X is normally distributed.

Q.34. The mean height of 500 students is 151 Cm and the standard deviation is 15 Cm. Assuming that the heights are normally distributed, find how many students have heights between 120 and 155 Cm.

Q.35. If a random variable X has the exponential distribution with mean $\mu=\frac{1}{\lambda}=\frac{1}{2}$ calculate the probabilities that

(a) X will lie between 1 and 3

(b) X is greater than 0.5

(c) X is at most 4.

Q.36. Suppose that reaction time X has a standard gamma distribution with r = 2. Find

(a) $P(3 \leq X \leq 5)$ (b) $P(X > 4)$