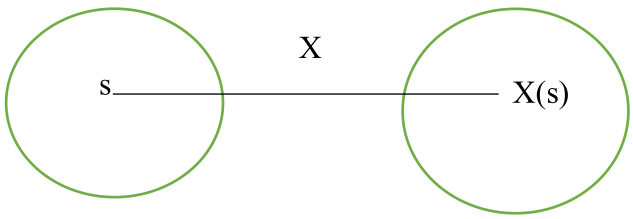
Probability Distribution: Probability distribution is the theoretical counterpart of frequency distribution and plays an important role in theoretical study of populations. A probability model can be developed for a given idealized conditions in a game of chance by incorporating all the factors that have a bearing on this game. In building such model, the empirical data of frequency distribution, A.M., Variance etc. are to be taken into account. In the discrete case we consider discrete uniform distributions. The continuous probability distributions we study are uniform distribution, normal distribution, exponential, gamma, Weibull distributions which are of great practical importance.

Recall that in a random experiment, the outcomes (or results) are governed by chance mechanism and the sample space S of such a random experiment consists of all outcomes of the experiment. When the elements (outcomes/events) of the sample space are non-numeric, they can be quantified by assigning a real number to every event of the sample space. The assignment rule, known as the random variable (R.V.) provides the power of observation and thus discards unimportant finest-grain description of the sample space.

A **random variable** X on a sample space S is a function $X: S \to R$ from S to the set of real numbers R, which assigns a real number X(s) to each sample point s of S.



S: Sample Space $\quad$ R_x: Possible Values of x

Range space R_x: is the set of all possible values of X is a subset of real numbers R.

Although X is called a random "variable" note that it is infact a "single-valued function".

Notation: If R.V. is denoted by X, then x (corresponding small letter) denoted one of its values.

Discrete: A R.V. X is said to be discrete R.V. if its set of possible outcomes, the sample space S, is countable (finite or an unending sequence with as many elements as there are whole numbers).

Continuous: A R.V. is said to be continuous R.V. if S contains infinite numbers equal to the number of points on a line segment.

Probability Distributions

Discrete Probability Distributions:

Each event in a sample space has certain probability (or chance) of occurrence (or happening). A formula representing all these probabilities which a discrete R.V. assumes, is known as the discrete probability distribution.

Example: Let X denote the discrete R.V. which denotes the minimum of the two numbers that appear in a single throw of a pair of fair dice. Then X is a function from the sample space S consisting of 36 ordered pair $\{(1,1),(1,2),\ldots\ldots(6,6)\}$ to a subset of real numbers $\{1,2,3,4,5,6\}$.

The event minimum 5 can appear in the following cases (occurrences) (5,5), (5,6),(6,5). Thus R.V. X assigns to this event of the sample space a real number 3. The probability of such an event happening is $\frac{3}{36}$ since there are 36 exhaustive cases. This is represented as

$$P(X = x_i) = \mathrm{p}_i = f(x_i) = P(X = 5) = f(5) = \frac{3}{36}$$

Calculating in a similar way the other probabilities, the distribution of probabilities of this discrete R.V. is denoted by the discrete probability distribution as follows:

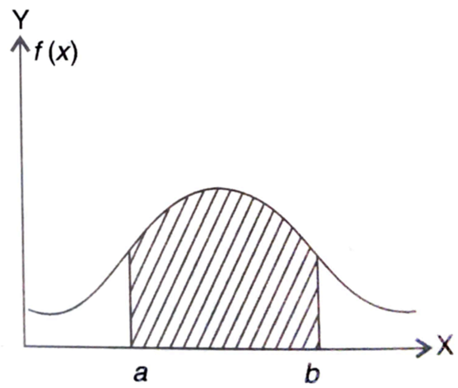
$X = x_i$	1	2	3	4	5	6
$P(X = x_i) = f(x_i) = \mathrm{p}_i$	$\frac{11}{36}$	$\frac{9}{36}$	$\frac{7}{36}$	$\frac{5}{36}$	$\frac{3}{36}$	$\frac{1}{36}$

Discrete Probability distribution, probability function or probability mass function of a discrete R.V. X is the function f(x) satisfying the following conditions:

(i) $f(x) \geq 0$
(ii) $\sum f(x) = 1$
(iii) $P(X = x) = f(x)$

Thus probability distribution is the set of ordered pairs $(x, f(x))$ ie, outcome x and its probability (chance) $f(x)$

Continuous Probability Distribution: For a continuous R.V. X, the function $f(x)$ satisfying the following is known as the probability density function (P.D.F.) or simply density function.



(i) $f(x) \geq 0$
(ii) $(ii) \int_{-\infty}^{\infty} f(x)\, dx = 1$

(iii) $P(a < X < b) = \int_a^b f(x)\, dx =$ area under $f(x)$ between ordinates $x = a$ and $x = b$

Note 1: $P(a < X < b) = P(a \leq X < b)$

$P(a < X \leq b) = P(a \leq X \leq b)$

i.e. inclusion or non-inclusion of end points, does not change the probability, which is not the case in the discrete distribution.

Note 2: Probability at a point

$$P(X = a) = \int_{a-\Delta x}^{a+\Delta x} f(x)dx$$