**Probability and Queueing Theory**

**Unit-I**

**Discrete and Continuous random variables-Moments-Moments generating function and their properties, Binomial, Poisson, Geometric, Negative Binomial, Uniform, Exponential, Gamma, Normal and Weibull Distribution.**

**Probability:** In random phenomena past information no matter how voluminous, will not allow to formulate a rule to determine precisely (uniquely) what will happen in future. The theory of probability is the study of such random phenomena which are not deterministic. In analyzing and interpreting data that involves an element of ‘chance’ or uncertainty, probability theory plays a vital role in the theory and application of statistics. Blaise Pascal in the middle of 17th century was the first to use probability in problems of gambling. Laplace, De Moivre, Gauss, Poisson and Kolmogorov greatly contributed to the development of probability theory which finds application in engineering, biology, economics, computer science, politics, traffic control, medicine, meteorology, psychology, agriculture, geography and management of natural resources. Almost every human activity involves some kind of chance element and the role for the theory of probability to play.

‘The subject of statistics originated much earlier than probability and dealt mainly with the collection and organization of data. With the advent of probability and dealt mainly with the collection and organization of data. With the advent of probability, it was realized that statistics could be used in drawing valid conclusions and making reasonable decisions on the basis of analysis of data, such as in sampling theory and prediction of forecasting. As time progressed, probability theory found its way into many applications not only in engineering and science but also in the fields like actuarial science, agriculture, commerce, medicine and psychology. We consider the Baye’s theorem (also known as theorem of inverse probability) which determines the probability ‘of causes.

Probability is a concept which numerically measure the degree of uncertainty and therefore of certainty of the occurrence of events.

**Terminology-** Before we take-up the subject matter, we shall define and explain certain terms which are encountered so very often.

1. **Die:** It is a small cube. Dots are . marked on its faces. Plural of the die is dice. On throwing a die, the outcome is the number of dots on its upper face.
2. **Cards:** A pack of cards consists of four suits i.e., Spades, Hearts, Diamonds and Clubs. Each suit consists of 13 cards, nine cards numbered 2,3,4,….10 and Ace, a King, a Queen and a Jack or knave. Color of spades and clubs is black and that of Hearts and Diamonds is red. Aces, Kings, Queens and Jacks are known as face cards.
3. **Exhaustive Events or Sample space:** The set of all possible outcomes of a single performance of an experiment is exhaustive events or Sample space. Each outcome is called a sample point. In case of tossing a coin once S = (H,T) is the sample space. Two outcomes head and tail constitute an exhaustive event because no other is possible.
4. **Experiments:** Students of science and engineering are familiar with experiments which when performed repeatedly under the same conditions given identical results. In theory of probability, our interest is centered around the kind of experiment, which though repeated under essentially identical conditions, does not give unique results but may result in any one of the several possible outcomes, such an experiment is also called a trial and the outcome an event or a case. For example, the throw of a coin is an experiment or a trial which can result in one of the two outcomes a Head or Tail. Drawing a card from a well shuffled pack is a trial which may result in any one of 52 outcomes.
5. **Random Experiment:** There are experiments, in which results may be altogether different, even though they are performed under identical conditions. They are known as random experiments. Tossing a coin or throwing a die is random experiment.
6. **Trial and Events:** Performing a random experiment is called a trial and outcome is termed as event. Tossing of a coin is a trial and the turning up of head or tail is an event.
7. **Equally Likely Events:** Two events are said to be ‘equally likely’ if one of them cannot be expected in preference to the other. For instance, if we draw a card from well shuffled pack, we may get any card, then the 52 different cases are equally likely.
8. **Independent Events:** Two, Events may be independent when the actual happening of one does not influence in any way the probability of the happening of the other.

Example: The Event of getting head on first coin and the event of getting tail on the second coin in a simultaneous throw of two coins are independent.

1. **Mutually Exclusive Events:** Two Events are known as mutually exclusive, when the occurrence of one of them excludes the occurrence of the other. For example, on tossing of a coin, either we get head or tail, but not both.
2. **Compound Events:** When two or more events occur in composition with each other the simultaneous occurrence is called a compound event. When two dice are thrown, getting a 5 or 6 is a compound event.
3. **Favorable Events:** The Events which ensure the required happening are said to be favorable events. For Example, in throwing a die , to have the even numbers 2, 4 and 6 are favorable cases.
4. **Conditional Probability:** The probability of happening an event A, such that event B has already happened, is called the conditional probability of happening of A. on the condition that B has already happened. It is usually denoted by P(A/B) or P(A/B)
5. **Odds in Favor of an Event and Odds against an Event:**

If number of Favorable ways = m

number of not favorable ways/Events = n

1. Odds in favor of the event =
2. Odds in against of the event =
3. **Classical Definition of Probability:** If there are N equally likely, mutually exclusive and exhaustive events of an experiment and m of these are favorable, then the probability of the happening of the events is defined as .
4. **Expected value:** If P1, P2, P3,…..Pn of the probabilities of the events x1, x2, x3,….xn respectively.

E(x) = P1 x1 + P2 x2 + P3 x3 +…..Pn xn

=

**Q.1.** Find the probability of throwing

(a) 5 (b) An even number with an ordinary six faced die

Sol. (a) There are 6 possible ways in which the die can fall and there is only one way of throwing 5.

(b) Total number of ways of throwing a die = 6

Number of ways falling 2,4,6 = 3

The required probability =

**Q.2.** Find the probability of throwing 9 with two dice.

Sol. Total number of possible ways of throwing two dice =

Number of ways getting 9 i e. (3+6), (4+5), (5+4),(6+3) = 4

The required probability =

**Q.3.** From a pack of 52 cards, one is drawn at random. Find the probability of getting a king.

Sol. A king can be chosen in 4 ways. But a card can be drawn in 52 ways.

The required probability =

Addition Law of Probability: If be separate probabilities of mutually exclusive events, then the probability P, that any of these events will happen is given by

Note Mutually Exclusive Events: Consider the case where two evets A and B are not mutually exclusive. The probability of the event that either A or B or both occur is given as

B) = B)

**Q.4.** An bag contains 10 black and 10 white balls. Find the probability of drawing two balls of the same color.

Sol. Probability of drawing two black balls

Probability of drawing two white balls

Probability of drawing two balls of the same color

= + = Ans.

**Q.5.** A bag contains four white and two black balls and a second bag contains three of each color. A bag is selected at random, and a ball is then drawn at random from the bag chosen. What is the probability that the ball drawn is white?