# Physics 111B
# Hall Effect with Semiconductor

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## Abstract

In this lab we investigated the physics of semiconductors and the Hall effect. The Hall effect was useful for discovering some properties of the semiconductor. The Hall effect is the name of the effect when you have a magnetic field perpendicular to an input current. This leads to a Lorentz force that deflects the charge carriers and develops a Hall voltage. The semiconductor is essentially a material with variable resistivity; which can act both like an insulator, at low temperatures, and a conductor, at high temperatures. You can dope a semiconductor to decrease its resistivity at low temperatures. We explored the following: determining the doping of the semiconductor from the Hall coefficient, investigating the disparity in the magnitude of the resistance and magnetoresistance, estimating the electron and hole concentrations, calculating the mobilities of the charged particles, determining the resistivity and conductivity as a function of temperature, and finding the temperature, 243.75 $\pm$ 3K that corresponds to the transition between the extrinsic and intrinsic region of the semiconductor.

## Introduction

The main goal of this lab is to gain insight into the properties of Germanium (Ge) semiconductor sample that we have in the apparatus. One goal is to measure the resistivity and conductivity as a function of temperature to help us attain the temperature at which the material transitions from the extrinsic to intrinsic region. In addition, we attain the rest of the properties from the Hall effect measurements. We have the mission to find the Hall voltages, which leads us to Hall coefficients. We can take this information to measure electron and hole concentrations. Also, we measure the difference between resistance and magnetoresistance, and we analyze the drift and Hall mobilities; where mobilities basically explain how easily a charge particle can move through the material. It is explained in the theory section in more details just exactly what a mobility is in regards to a constant of proportionality. We have the goal to derive the drift mobility from the resistivity, and derive the Hall mobilities from the Hall coefficient. Not only that but by observing the Hall coefficient we learn if this is a p- or n-doped Ge crystal.

This is the introduction to the physics that is used in this experiment. We are going to observe the Hall Effect that was first discovered by Edwin Hall in 1879. The basic physics lies in how a magnetic field acts on moving charged particles in a material. In fact, in 1881 we see Thomson publishing a paper that first gives the magnetic force on a charge particle by magnetic force.

$$\vec{\mathrm{F}}_{\mathrm{B}} = q\vec{\mathrm{v}} \times \vec{\mathrm{B}} \tag{1}$$

Later in 1895 we saw the first derivation of the total electromagnetic force exerted on a charged particle by both the electric and magnetic field. This is a crucial concept that is used in this experiment for observing the Hall effect.

$$\vec{\mathrm{F}}_{\mathrm{B}} = q\vec{\mathrm{v}} \times \vec{\mathrm{B}} + q\vec{\mathrm{E}} \tag{2}$$

The Lorentz equation is a quantitative way to understand the Hall effect. It puts in mathematical terms how the charged carrier particles are deflected in a current. It even gives the direction by using the physicists favorite arm exercise, the infamous right hand rule.
This is essentially what the hall effect is in a nutshell. It is a perpendicular magnetic field exerting force on charged particles which causes the charge to accumulate in specific places in the electrical conductor which causes a potential electric difference. That is we get what is referred to as Hall voltage because it is the voltage related to the Hall effect.

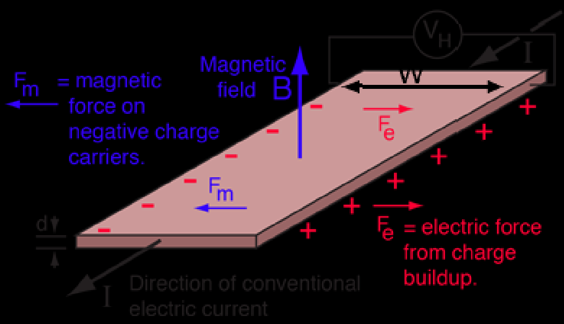



Figure 1: Diagram of the hall effect in an electrical conductor with a magnetic field perpendicular to the current. Notice the charge accumulated on the sides of the conductor because of the magnetic force exerted on the particles.

$$\mathrm{V}_{\mathrm{H}} = \mathrm{w}\mathrm{E}_{\mathrm{H}} \tag{3}$$

Where w represents the length perpendicular to the current of the conductor. It is the length along which you are measuring the Hall voltage in the conductor. The $\mathrm{E}_{\mathrm{H}}$ represents the Hall field, which is basically the electric field that develops between the interaction of two differently charged sides of the semiconductor. The physical idea is that at first we have the $F_m$ which is the magnetic force that is being exerted on the electrons. This curves the trajectory of the electrons and we attain a buildup of negative charge on the left side in figure 1. This buildup of electrons occurs rapidly after you introduce a magnetic field perpendicular to the current. This generates an electric field which gives an electric force that results from the charge buildup, $F_e$. The $F_m + F_e = 0$ overall force is zero as soon as the accumulated charge particles produces an electric force of equal magnitude to the magnetic force, and we are at equilibrium. At equilibrium the current just continues to travel in a straight line with a net force of zero acting upon the charged particles. Thus in reality the current is curved initially when we turn on the magnetic field. But very quickly it builds up the charges and the current travels in a straight line thereafter. The main point is that the Hall voltage is from the Hall effect.
Sweet, now we understand the basics interactions that cause this observed effect. What else could we possibly want to do with the Hall effect? In fact, in this experiment we implement the Hall effect in a semiconductor. Actually a p-doped semiconductor. So this means we should have a little

bit introduction to the physics and theory of semiconductors.

## Theory

### Resistivity and Semiconductors

What is a semiconductor? I am sure glad you asked because that is what the next paragraph is going to be about, else it could have been a little bit of a dull. Anyway to understand the physics of a semiconductor theoretically: quantum physics, Fermi-levels and the structure of the energy bands are discussed.

A semiconductor is a metal that is not exactly an electrical conductor in terms of resistance. In an electrical conductor the resistance to charged particles is extremely low. Let's begin with some of the basics of the concept of the connection between resistivity and semiconductors. Firstly, we can define a useful physics term instead of resistance. We can define the resistance of a specific material that is independent of the geometry of the material, which is called resistivity. Resistivity is a property of the metal itself and not the shape, nor size. A quick definition of resistivity is that it is quantitative representation of how strongly a material opposes an electric current. Take an example of a conductor, such as Copper (Cu), it has a resistivity of about $10^{-8}$. A semiconductor on the other hand, has a variable resistivity, where the resistivity is a function of temperature. At low temperatures it has the resistivity that matches an insulator, but at higher temperatures its resistivity decreases and it can become as low as a metal. Then you have the semiconductor conducting current just as well as a metal. Anyway, this is the connection between resistivity and semiconductors.

There is another important physical standpoint of semiconductors that is crucial for this lab. While many semiconductors have resistivity on par with an insulator at low temperatures. You can dope semiconductors to effectively decrease the resistivity and even at low temperatures it can conduct just like a metal. There are two types of semiconductors with regard to dopants: p-type, and n-type semiconductors. The basic idea is that in a p-type, it is essentially introducing particles or impurities into the crystal lattice structure of the semiconductor, which are electron acceptors. These impurities attract electrons from the valence band of the atoms in the crystal and this leaves behind a positive charge, that is referred to as a hole.

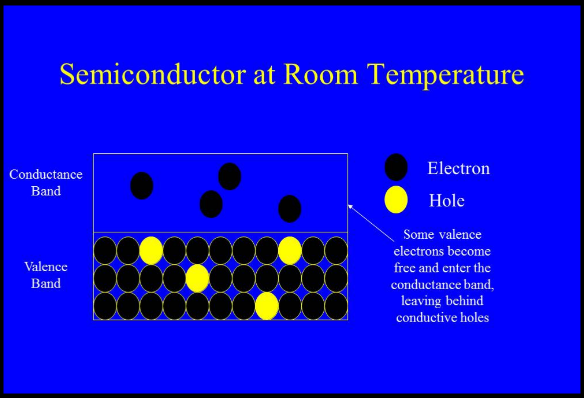



Figure 2: Diagram that depicts the concept of the holes in the valence band.

This brings me to another important consequence of semiconductors, when you move an electron out of the valence band you essentially form a spot that has an electron absent which takes on a positive charge effectively. This hole will move because electrons will repel each other to take over the holes spot and then it creates a hole in another place. If we have an applied electric field the holes will move in the same direction as the electric field, obviously electrons move in the opposite direction. This also relates to a bit of the physics that for an electron to be capable of being free there must be somewhere for it to go energetically, this will make sense once you read the band theory next.
We also have the existence of the n-type semiconductor, which is when you introduce an impurity that donates electrons to the atoms in the crystal lattice that are able to conduct those electrons.

Band Theory

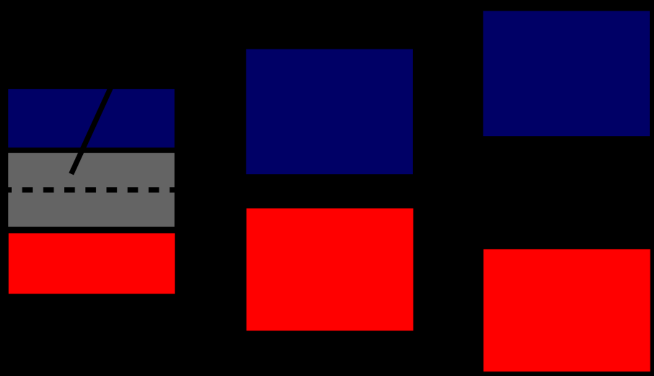


Figure 3: A diagram comparing the band gap theory for insulator, semiconductor, and conductor.

The theory of solids and their electrical conductivity depends on quantum physics. The quantum physics depends on treating each state as being discrete and that can be described by the quantum numbers. In the ground state, all the electrons are at the lowest energy states in the atom. Then consider if you were to bring N atoms together in a solid. It is thought that in terms of quantum mechanical wave functions, we are overlapping these wave functions. But the problem here is that we cannot have all of these N atoms be at the same energy level. The Pauli exclusion principle forbids this for fermions like electrons. Thus it makes sense that it would split the N states to be at different energy levels. This can be imagined as having more than N splittings, where all are at slightly different energy levels. This can be viewed graphically as a solid band to a good approximation. Furthermore, in an actual solid this N is on the order of Avogadro's number. All of the electrons at ground state, lowest energy, make up the valence band. The difference energy is so small we just represent it as a solid band. Then we have regions that are forbidden where it is called a bandgap. So we could have a conduction band separated by a bandgap from the valence band.
If we consider an insulator for the first example. It usually has a large bandgap energy between the

valence and conduction band. The fact that the valence band is full of electrons with no empty spots, means that when you apply an electric field, which usually is capable of imparting kinetic energy to an electron. It cannot move any electrons around, because imparting kinetic energy to an electron is essentially moving it to a different energy level. And only empty energy level for an electron to go to is in the conduction band which is separated by a very large energy gap. So you would need enough energy to move the electron through the bandgap. This leaves an insulator basically stuck without the ability for electrons to move around different energy levels even with an applied electric field.

Before we analyze the metals and where it fits in the band theory. We can take a look at the concept of Fermi energy and Fermi level. If you suppose that you have a metal at absolute zero temperature. Then all the electrons are at the lowest possible energy states in the metal. We define the highest energy level that is occupied by an electron in this configuration the Fermi level, and setting E=0 at the bottom of the valence band, we can say that this is the Fermi energy. The crucial part for a conductor is that the band structure is so that the Fermi energy is in the middle of a band of energies. Therefore when we have a temperature above the absolute zero, you already have enough energy to move the electrons at the Fermi level to the unoccupied states next to it. Electrons below Fermi level are still stuck because they are surrounded by electrons nearby. This means when you apply an electric field it can actually create a drift velocity, that is a directed current by imparting kinetic energy to the electrons near the Fermi level, moving them to higher energy states.

Lastly let's take a look at the band theory to explain a semiconductors behavior. The main idea is that the energy gap between the valence band that is fully occupied at absolute zero temperature and the conduction band that is empty is much smaller than that in an insulator. It is to the point that with enough thermal energy at a high enough temperature, charge carriers from the valence band can be excited to energy levels in the conduction band. Once this begins to happen, an electric field can now impart kinetic energy on the electrons in the conduction band because there are so many energy levels available, they are not stuck in traffic. In fact, even in the valence band the fact that we left a hole, now allows electrons to move in the valence band as well because there are energy levels that are unoccupied. So we get this idea of positive charges or holes moving in the direction of the electric field in the valence band.

## Hall Effect and Drift Mobility

Let's discuss a little bit of the theory that gives relevant equations for this experiment. So we have the Lorentz force again which is equation (2) that reaches equilibrium. At that point the net force is zero and electrons are travelling in a straight trajectory.

The following equation can represent the Hall field in terms of the magnetic field and the drift velocity of the charge carriers.

$$\mathrm{E_H = v_x B_z [V/m]} \tag{4}$$

We know that the Hall voltage $\mathrm{V_H}$ depends on which of the charger carriers is more prevalent in the material because $I \propto qv_x$. So let's use the Hall voltage to signify if we are dealing with an n-type or a p-type semiconductor. Equation (5) is for the n-type and equation (6) is for the p-type.

$$V_H = \frac{-I_x B_z}{end} \tag{5}$$

$$V_H = \frac{I_x B_z}{epd} \tag{6}$$

These are the relevant equations for the Hall coefficient for one charge carrier.

$$R_H = \frac{E_H}{J_x B_z} = \frac{1}{en} \tag{7}$$

$$R_H = \frac{-1}{ep} \tag{8}$$

$$R_H = \frac{V_H d}{I_x B_z} \tag{9}$$

Hall Coefficient for when you have two charge carriers.

$$R_H = \frac{\mu_p^2 p - \mu_n^2 n}{e(\mu_p p + \mu_n n)^2} \tag{10}$$

Where n represents the electron concentration and p is the hole concentration.
If we assume that our sample is p-doped we know that in the extrinsic region equation (8) is applicable. However, mobilities depend on temperature and in general $\mu_n > \mu_p$, then we know that a change in sign signifies a substantial increase in the n value (electron concentration). Where this change in sign happens is when $R_H = 0$ which is called an inversion in Mellisinos, but will only happen if $p > n$. thus in Mellisinos they conclude that "Hall coefficient inversion" is only possible for a p-type semiconductor. The magnetic field is not important in this case, and so the direction doesn't matter in figuring out which type it is.
We also can get the drift mobility via the following reasoning. But firstly let's discuss a little bit about resistivity and drift velocity because they are related to the physics of mobility. Mobility is a bit like a quantitative relationship of temperature and the ability of moving charges in the material. We already know that if there is no electric field the charged particles will move randomly. But consider an applied electric field $\vec{\mathrm{E}}$. This electric field will cause a small drift of charged particles in the direction of the field. We know that drift velocity is proportional to an applied electric field by working with Ohm's law we have.

$$\mathrm{v_d} = \mu \mathrm{E} \tag{11}$$

The proportionality constant $\mu$ is our mobility, so a higher mobility means that with some constant applied electric field we can achieve greater drift velocity that is charge particles are moving more quickly in the direction of the electric field, but a lower mobility causes them to move more slowly. This obviously relates to resistivity as well. Cause a greater resistivity will signify a smaller mobility. Let's express the current density in one dimension.

$$\mathrm{J} = \mathrm{epv_d} = \mathrm{en}\mu\mathrm{E} = \sigma\vec{\mathrm{E}} \tag{12}$$

e is the fundamental charge constant, and p is the hole concentration. For the next equation without deriving we can get the following result for the conductivity, but it is useful to understand some of the dependencies of conductivity on temperature.

$$\sigma = \mathrm{en}\mu = \frac{\mathrm{ne}^2\lambda}{2\sqrt{3\mathrm{kTm}}} \tag{13}$$

Where the $\lambda$ is the mean free path, which is essentially the average distance a particle must travel before a collision. If we make the next two assumptions. 1) the number of carriers is constant, 2) the mean free path is constant. Then we could state that the conductivity is decreasing by a factor of $\mathrm{T}^{-1/2}$. In Mellisinos they take $\lambda = 1/\mathrm{kT}$ to get the following equation for conductivity.

$$\sigma = \mathrm{ne}\mu = C\frac{ne^2}{m}\mathrm{T}^{-3/2} \tag{14}$$

Hall Mobility

There is another way to get mobilities. While the method above relied on some approximations with mean free path, and resistivity. We can also derive the mobility from the Hall effect with the following equation. Notice that this gives us theoretically the following $\mu_{\mathrm{d}} \propto \mathrm{T}^{-3/2}$.

$$\mu_{\mathrm{H}} = \mathrm{R}_{\mathrm{H}}\sigma \tag{15}$$

## Apparatus and Procedure

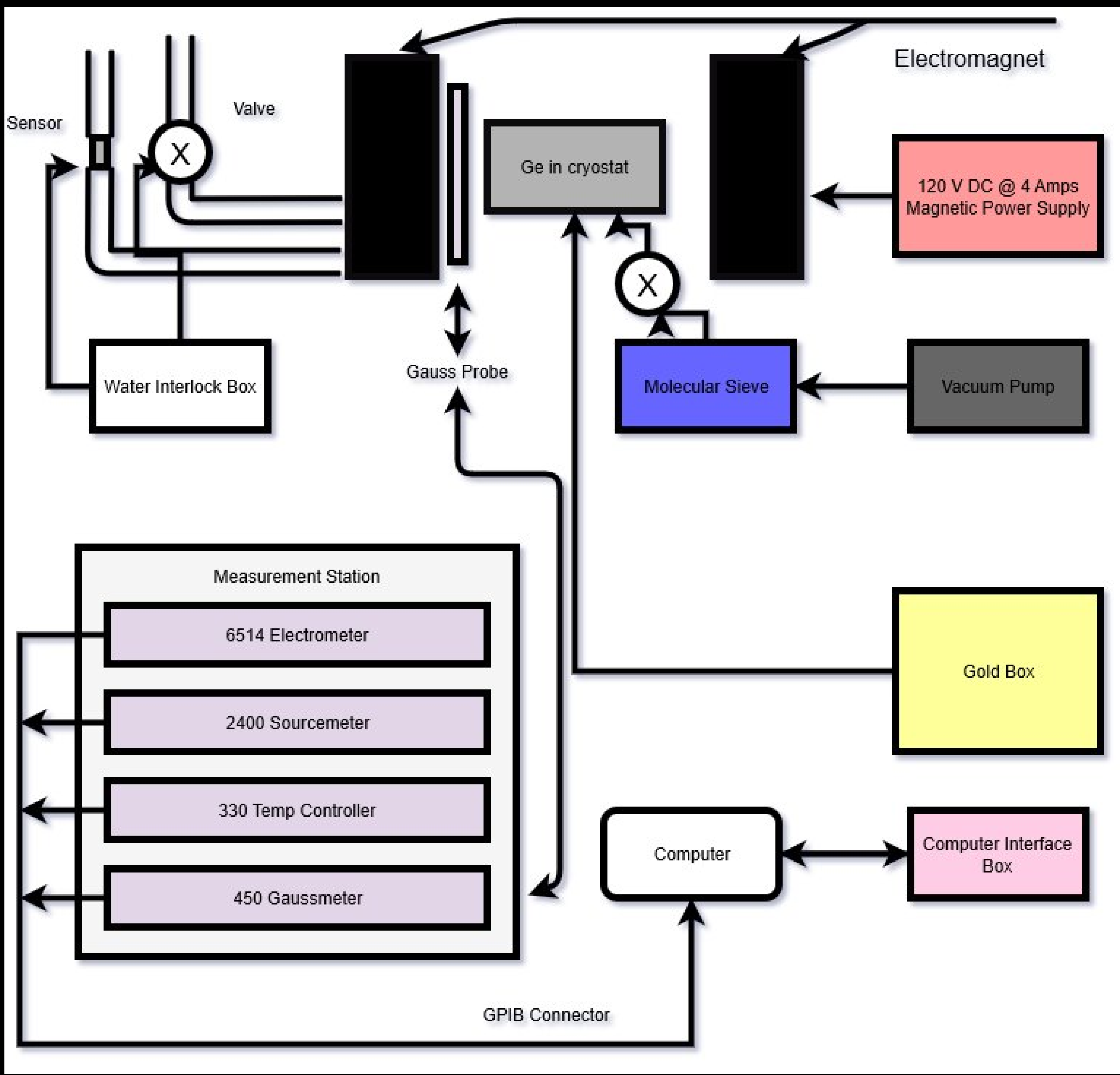



Figure 4: A block diagram of the experiment.

In this experiment we are measuring properties of a semiconductor as it relates to temperature. We are taking a temperature range from about 95 K to 350 K, where the program automatically collects data in the range. We have four measuring devices in our measurement section. These are connected to the computer in order to record the relevant data. The following four measuring devices are at the measurement station: 240 SourceMeter, to generate our input current set on the LabView program on the computer; 6514 Electrometer, to measure the voltage across any two contacts of the four contacts of the Germanium sheet. LakeShore 450 Gaussmeter, to measure the magnetic field through a gauss probe. 330 Temperature Controller, to measure and automatically control the temperature via the diode temperature sensor on the back of the sample. Lastly we have all of these meters connected to the computer via a GPIB connector. Also the Gold Box is connected to the computer via an interface box that connects to the computer with a DAC. The Gold Box is crucial for the experiment because it is responsible for triggering the SourceMeter to provide the input current provided by the user to the sample. The Gold box is controlled by the LabView program, and it can also change the current in the electromagnet to generate the magnetic field.

The block diagram depicts that we have a cryostat that holds the Ge crystal. We pour liquid nitrogen into the cryostat. It is in contact with Copper which then thermally conducts and becomes colder because of the liquid nitrogen. Basically the heat is moved from the Cu into the liquid nitrogen. The Cu is connected to the Ge crystal. But this is not the entire picture of the cryostat, because if this were the case the Ge crystal would just keep getting colder as it lost its heat to the Cu that was transferring its thermal energy to the liquid nitrogen, until it boils off. The actual experiment introduces a brass part between the Cu and the Ge crystal, it acts as a thermal resistance because we don't want to cool the sample down too quickly. Once you get the temperature down to 95K we want to start heating the sample up in a controlled manner. We have the heating coils wrapped around the Cu that comes after the brass. This allows us to heat the sample up because we heat up the Cu that is attached to the Ge crystal, and then this heating is not lost instantly because the brass acts as a thermal resistor that prevents the thermal energy from instantly transferring to the liquid nitrogen. We are then able to continue to heat the Cu portion after the brass component. We also use the temperature controller to keep track of the temperature via the sensor diode on the back of the Ge crystal. The reason for the vacuum is simply to prevent Germanium from oxidizing at high temperatures.

Then we have two large electromagnets surrounding the Ge in the cryostat. In general the physics of an electromagnet is that it has coils wound around a magnetic core, that is normally ferromagnetic like Iron. You can control the magnetic field based on the amount of current that you generate in the coils. The basic physics is that by generating a current in the coils you are creating a magnetic field that then interacts with the random magnetic domains of the magnet core, and causes them to align so they are magnetized. B = $\mu$nI, where n is the number of turns of coil around the magnetic core, and I is the current, $\mu$ is a constant of proportional that has to do with magnetic permeability. There is one more complication we need to consider for the electromagnets. If we allow the magnet to overheat it could become demagnetized. The solution is by having a water system to cool the electromagnet.

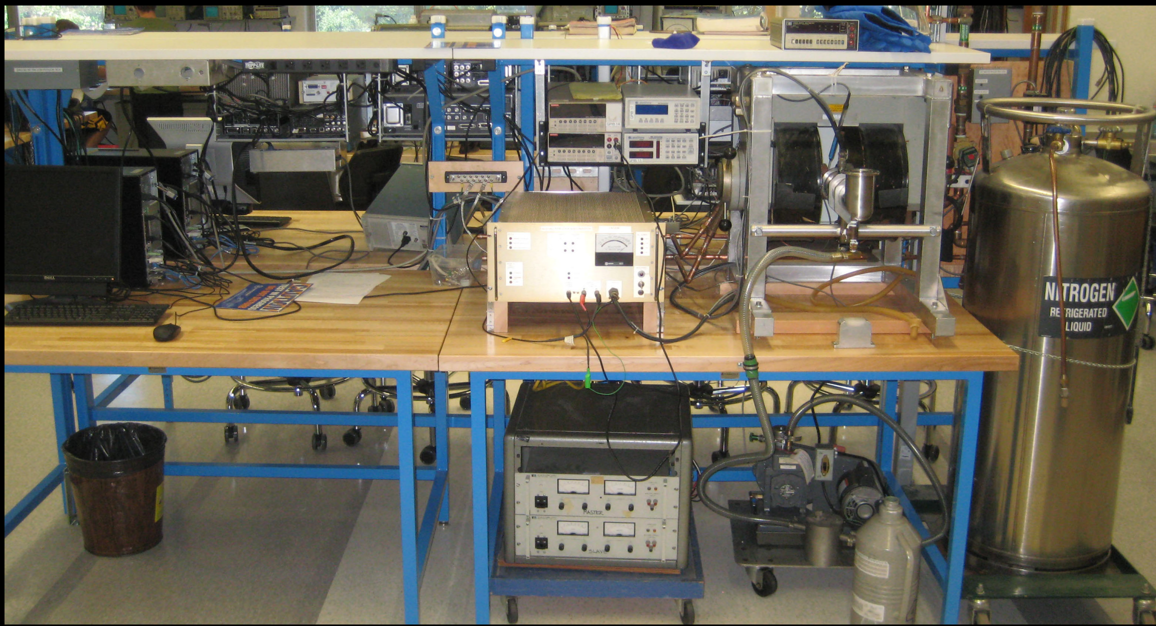


Figure 5: A picture of the general setup of the experiment.


Figure 6: A picture of the front panel of the Gold Box.

## Van Der Pauw method

For this experiment we collect the crucial data by the Van Der Pauw method. At each temperature we take a total of 8 measurements, four of them are by reversing the current. We measure current and voltage across the contacts. The four contacts are on the edges of the square sheet of Ge sample.

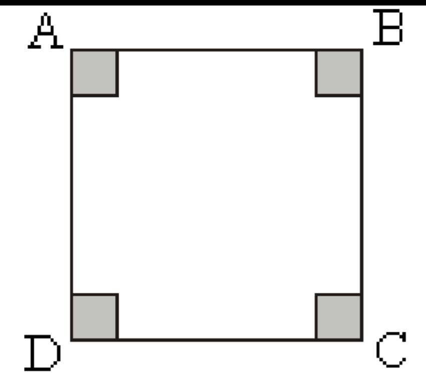



Figure 13: Van der Pauw configuration

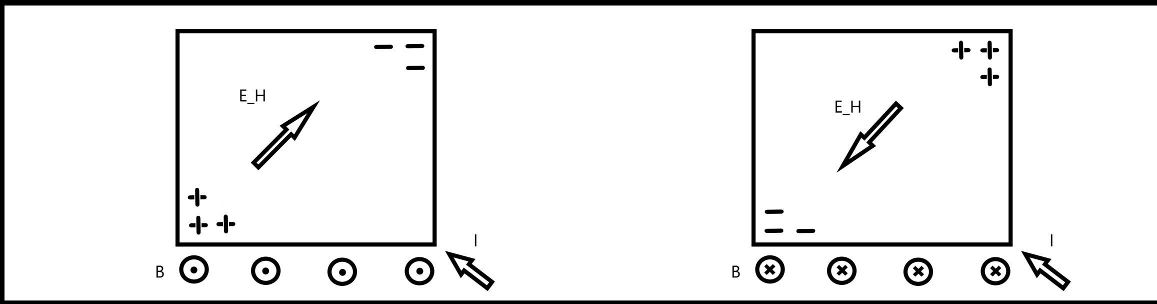
Figure 7: A diagram of the Ge crystal with the four peripheral and symmetrical contacts on the edges of the square sheet.

We took the following measurements, and the same with the current reversed.

| Current | Voltage | Trans-resistance |
|---|---|---|
| $I_{AB}$ | $V_{CD}$ | $R_{AB,CD}$ |
| $I_{AD}$ | $V_{CB}$ | $R_{AD,CB}$ |
| $I_{AC}$ | $V_{BD}$ | $R_{AC,BD}$ |
| $I_{BD}$ | $V_{CA}$ | $R_{BD,CA}$ |

Table 1: Trans-resistance results from Van Der Pauw Technique

The first two rows represent input current and voltage measurements of adjacent contact points, which are used for the resistivity measurements. We also take the same measurements for the reverse polarity input current in order to improve our data by canceling any constant voltage offsets. We took the last two rows as a measurement of the hall voltage directly but because of an offset we again take the reverse polarity. In addition, We also improve the accuracy of our results by taking measurements with the B-field in suppose the +z direction and the -z direction. So we are just averaging between the positive and negative magnetic field to improve accuracy. I show an example of what I mean when getting the Hall voltage.




(a) Diagram of the hall effect with the magnetic field into the page.

(b) Diagram of the hall effect with the magnetic field out of the page.

Figure 8: Hall voltage by the Van Der Pauw method.

These figures show that when we have a current being sent through the nonadjacent points the charge carriers will be deflected into the two other nonadjacent corners. Measuring the voltage between the nonadjacent points where the charge carriers accumulated gives the Hall voltage with some voltage offset caused by the dissimilarities between the semiconductor and the contact metal. One possible dissimilarity is that the metals can be at a different temperatures because of differences in thermal resistance which causes the formation of a potential difference generated over a temperature gradient at the contacts, called the thermoelectric effect. There is a good chance this will happen in this experiment. The Ge is heated up by the transfer of thermal energy from the Cu that is heated, but then the Ge is going to transfer its thermal energy to the metal at the contacts. Since these two metals have a different thermal resistance, one of them will be at a different temperature with respect to the other one. There is also another potential candidate for a voltage offset, which is the difference in electrical resistance between the two metals as well that can generate a voltage as well at the contact point. Mathematically it looks like the following. This is mathematically how the apparatus measures voltage. $\tilde{\mathrm{V}}_{\mathrm{CD}} = \mathrm{V}_{\mathrm{C}} - \mathrm{V}_{\mathrm{D}}$. Then we have the following facts.

$$\tilde{\mathrm{V}}_{\mathrm{CD}} = \mathrm{V}_{\rho} + \mathrm{V}_{\mathrm{offset}}$$

$$\tilde{\mathrm{V}}_{\text{-CD}} = \mathrm{V}_{-\rho} + \mathrm{V}_{\mathrm{offset}}$$

$$\mathrm{V}_{\mathrm{CD}} = \frac{1}{2}(\tilde{\mathrm{V}}_{\mathrm{CD}} - \tilde{\mathrm{V}}_{\text{-CD}})$$

Where $\tilde{\mathrm{V}}_{\mathrm{CD}}$ is a measurement of the voltage with input current, and the $\tilde{\mathrm{V}}_{\text{-CD}}$ is for the reverse current. Then $\mathrm{V}_{\rho}$ is the voltage drop due to the resistivity. Notice that when we average the difference between these two measurements we are canceling the offset voltage and left with the voltage drop due to the resistivity. Then to increase accuracy we take the measurement between anti-parallel magnetic fields at the same magnitude.

$$\mathrm{V}_{\mathrm{CD,H}} = \frac{1}{2}(\mathrm{V}_{\mathrm{CD,\ +B}} - \mathrm{V}_{\mathrm{CD,\text{-}B}})$$

Another important part of this procedure to consider is the reason we start the temperature at 95K. The reason is that at this temperature we are already at the point of saturation of the impurities, which physically means we are not going to see a drastic change in the concentration of holes, since this is p-doped, in the extrinsic region.
This is partly the reason to choose our temperatures, in addition we do not want to go too high,

that is well above 350K because at some temperature we could melt the soldered contacts.
The reason for the temperature range is to allow us to see the influence of temperature on the p-doped Germanium sample. We get to see the extrinsic and intrinsic region. We can find the hole concentration from the saturation at 95K, and the relatively constant concentration in the extrinsic region. By increasing the temperature we are eventually able to overcome the bandgap energy $E_g$, where essentially the kT value is of same magnitude as the $E_g$. This allows the thermal excitations of electrons from the valence to conduction band to occur. Now that electrons can be freed into the conduction band this leads to the ability for the electrons to move around, because the conduction band is empty with many energy levels all smeared very close to each other. So the electron easily moves between the energy levels and this is as shown in theory the idea of electrical conduction of charged particles. That is in other words, the electrons have energy levels to travel at their disposal. At this point, we are in the intrinsic region and this will be explored in the analysis. But that is the reason for the temeprature range.

## Analysis

In all of this analysis except for the last section, I used the data from our run with an input current of 10 $\mu$A because it generated the most consistent data. Unfortunately during the 500 $\mu$A run the input current fluctuated and caused our data to be inconsistent. Due to problems with the experiment we had only two good runs at the end of the alloted time, 10 $\mu$A and 1 $\mu$A run.

### Resistivity and Conductivity by Van Der Pauw

In this part of analysis we used the Van Der Pauw method in order to solve for the resistivity of the sample as a function of temperature. This allowed us to realize that this is a doped semiconductor and we were able to find the extrinsic and intrinsic regions. Following the procedure section, we took the current and voltage measurements from adjacent points. I realized that one of the voltages is horizontal and the other one vertical from figure 7. So I took $V_{CD} = V_{vertical}$ and the other one is the horizontal voltage. From Ohm's law I can compute $R_{horizontal}$ and $R_{vertical}$. We used the following equation to get the Resistivity corresponding to temperatures.

$$exp(\frac{-\pi d}{\rho} R_{AB,CD}) + exp(\frac{-\pi d}{\rho} R_{AD,CB}) = 1 \tag{16}$$

This is a useless equation that is difficult to solve, but there is a mathematical technique by using the ratio $x = \frac{R_{AB,CD}}{R_{AD,CB}}$ and $f(x) = \frac{1}{cosh(ln(x)/2.403)}$ and a function that exists we get the following simplification.

$$\rho = \frac{\pi \mathrm{d}}{\log(2)} \cdot \frac{\mathrm{R_{horizontal}} + \mathrm{R_{vertical}}}{2} \cdot f(\frac{\mathrm{R_{horizontal}}}{\mathrm{R_{vertical}}})$$

Where d represents the sample depth and $\rho$ represents the resisitivity.

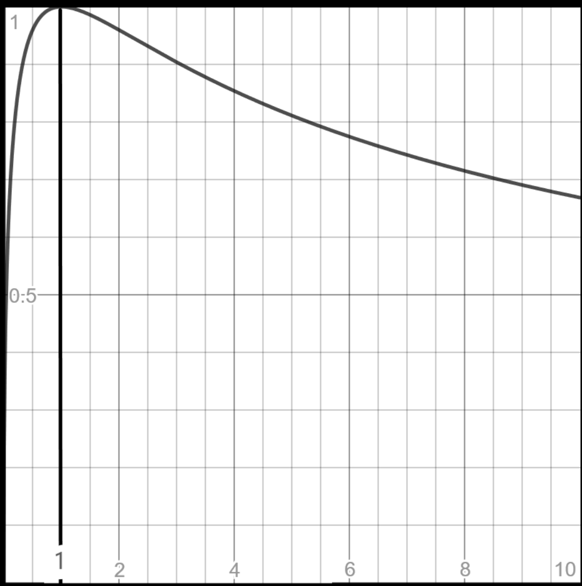



Figure 9: The graph of the function f(x) which represents the value of the ratio of the trans-resistances which is proportional to the resistivity.

I performed analysis to find the maximum or critical temperature. This represents the temperature at which the resistivity achieves its highest value with our data collected. I found that $T_{critical} = 243.75 \pm 3K$. This represents the temperature at which the material switches from the extrinsic to intrinsic region. Also the approximate bin size for our temperature measurements for one magnetic field strength was 4.5, that is why we introduced the uncertainty because while I can tell that at temperature 4.5 below and above the critical temperature that the resistivity is lower. It is any where between this regime that the resistivity is the highest. I found it to be between 237.55 and 247.1. Thus from this plot and our experiment we discovered for our configuration that we get the following for the temperature ranges for the regions.

$$\begin{cases} \text{Extrinsic} & \text{T} < 243.75 \pm 3K \\ \text{Intrinsic} & \text{T} > 243.75 \pm 3K \end{cases}$$

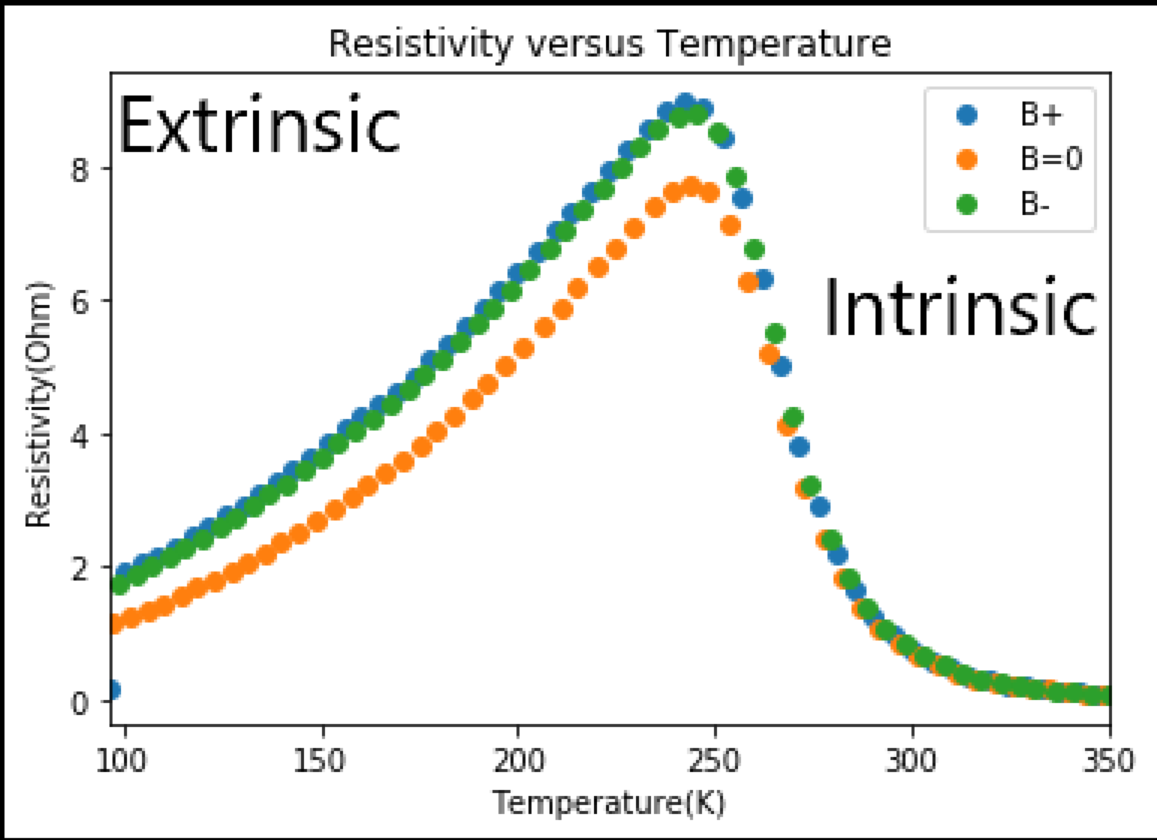



Figure 10: Plot of the resistivity versus the temperature. Error in units, it should be Ohm*m for resistivity.

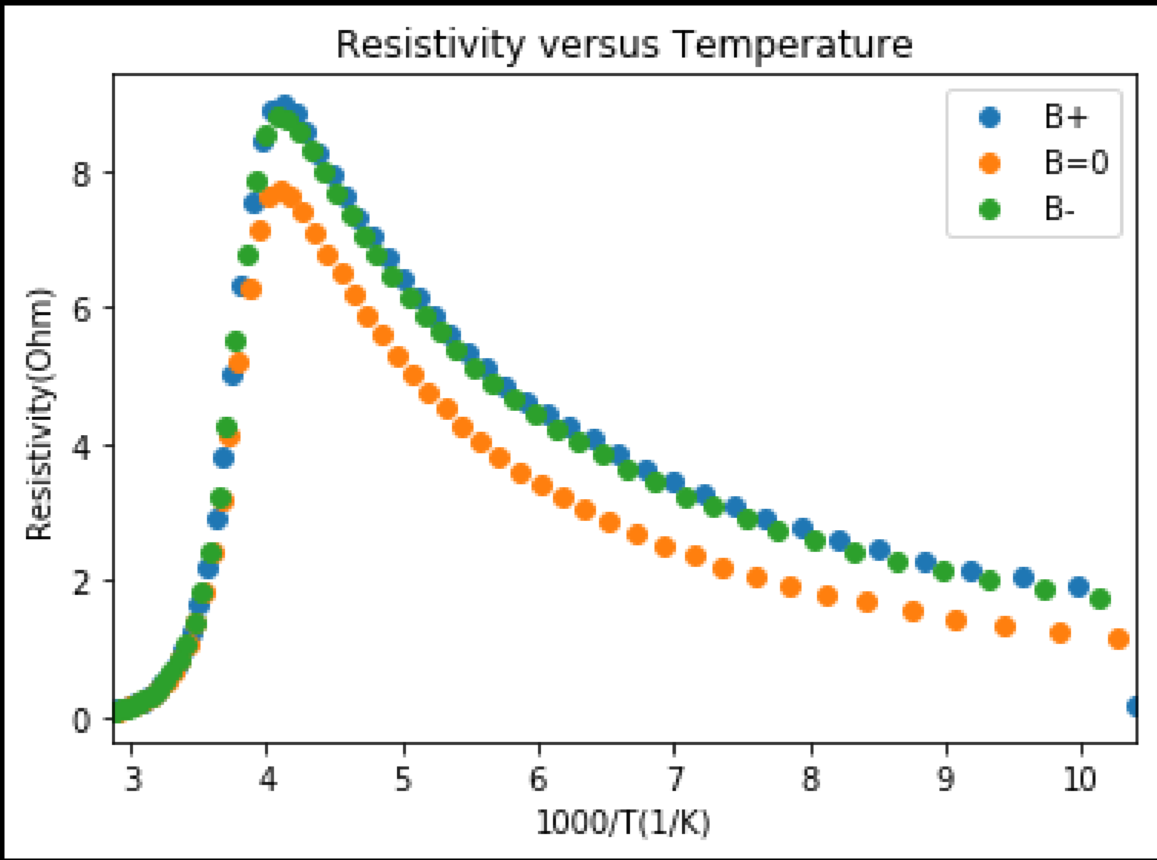



Figure 11: A plot of the resistivity versus the inverse temperature. Error in units, it should be Ohm*m for resistivity.

I find the conductivity plots by just taking $\sigma = \frac{1}{\rho}$

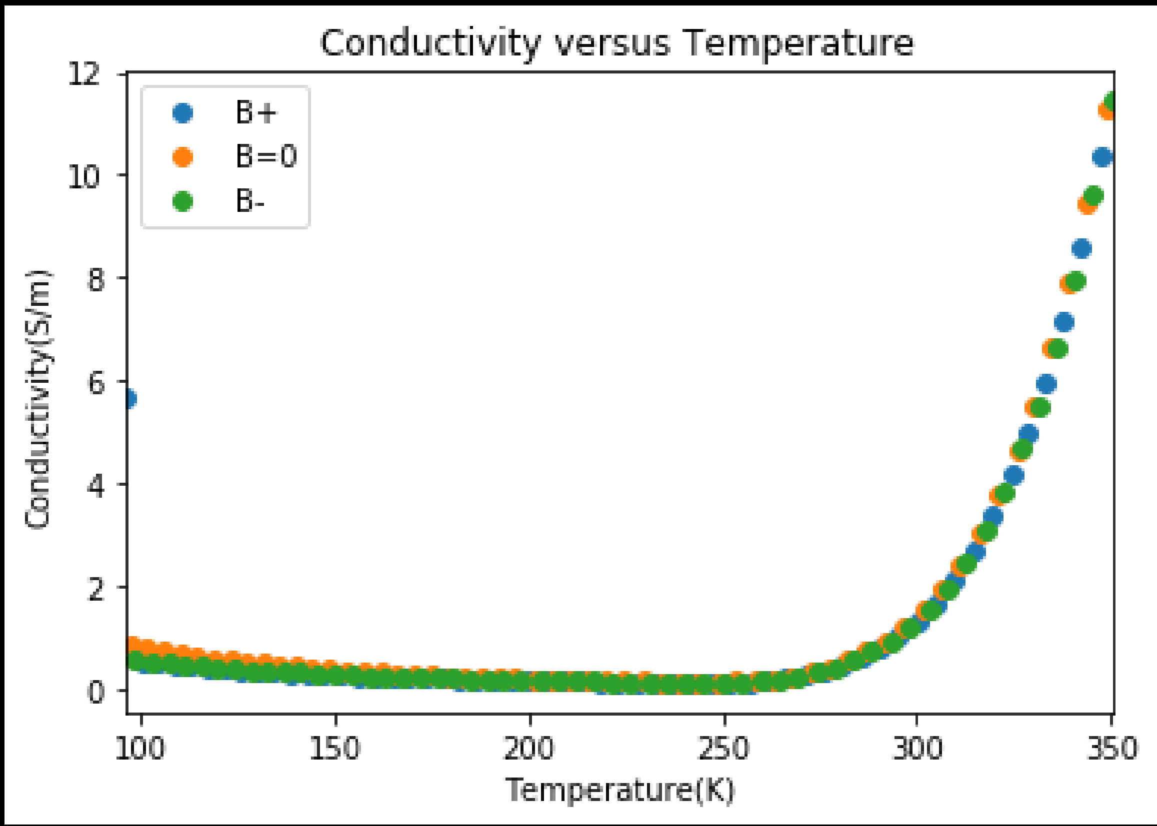



Figure 12: A plot of the conductivity versus the temperature.

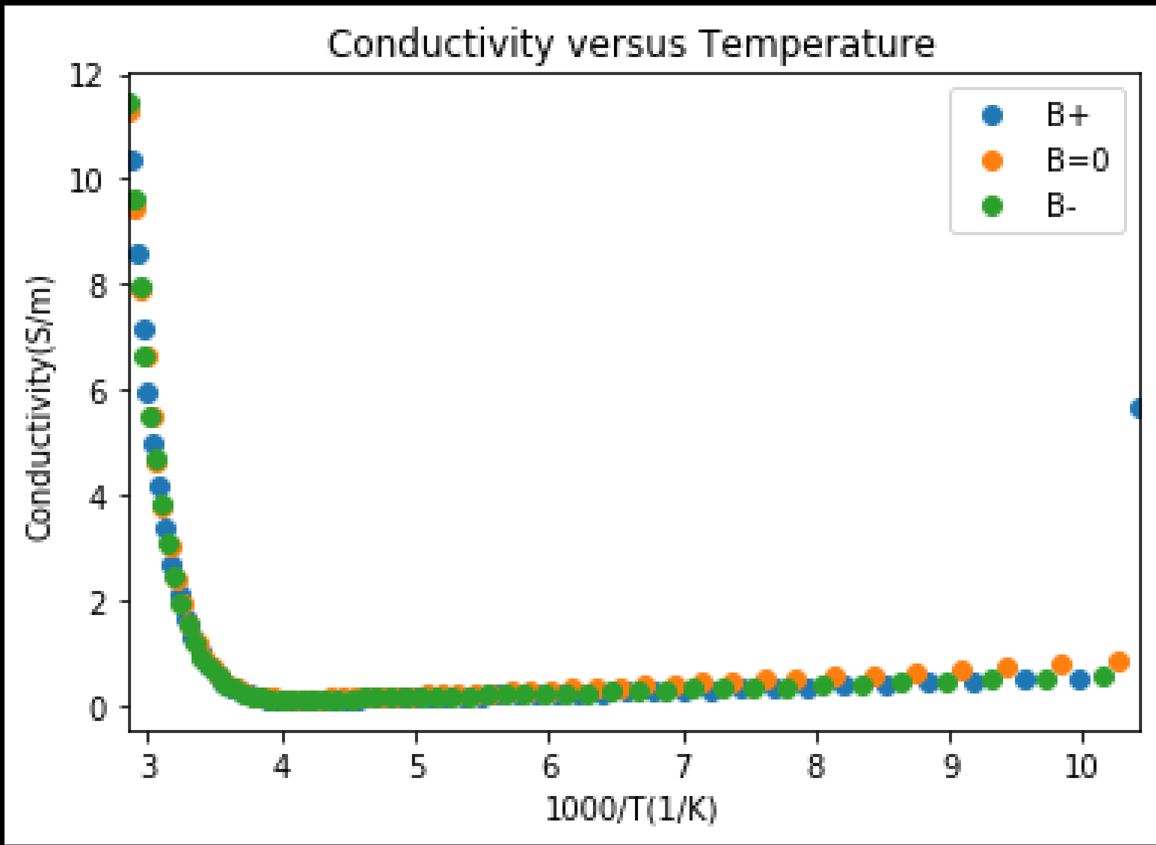



Figure 13: A plot of the conductivity versus the inverse temperature.

From the plots above, we see that the resistivity slowly increased but at higher temperatures we are in the intrinsic region, which means the electrical conductivity is a result of the thermal excitations from the valence to conduction band. As temperature increases the resistivity decreases in this region as it becomes more probable to excite electrons from valence to conduction band. This results in a surprising result though, which is extreme conductivity. Also the resistivity increased in the extrinsic region, the explanation for that is because we were already at the saturation point of the impurity, so as the temperature increased the hole concentration remained constant. However, the collisions between charged carriers increased with the greater kinetic energy of random motion. This results in lowering the drift velocity, which is another way to represent an increase in resistivity.

## Hall Coefficient

The Hall coefficient which measures the magnitude of the Hall effect in the sample. It does this by the fact that $\mathrm{R_H} \propto \mathrm{E_H}$, where $\mathrm{E_H}$ is the magnitude of the electric field produced by the Hall effect. We first found the Hall voltages by using the method described in the procedure section. The following are the voltages that I considered as Hall voltages.

$$\mathrm{V_{BD}} = \frac{1}{2}(\tilde{\mathrm{V}}_{\mathrm{BD}} - \tilde{\mathrm{V}}_{-\mathrm{BD}})$$

$$\mathrm{V_{CA}} = \frac{1}{2}(\tilde{\mathrm{V}}_{\mathrm{CA}} - \tilde{\mathrm{V}}_{-\mathrm{CA}})$$

And I use this equation to get the Hall coefficient.

$$\mathrm{R_H} = \frac{\mathrm{V_H d}}{\mathrm{I}B}$$

Where B is perpendicular to the current I. In the experiment we applied a constant magnetic field of 5000 gauss. It turned out to be the case that the measured magnetic field was not exactly 5000. Instead we saw the following, 5309 gauss for positive fields, -5157 gauss for negative fields. I took the average magnitude of the magnetic field between these two to make my representative magnetic field $5233 \pm 5$ gauss.

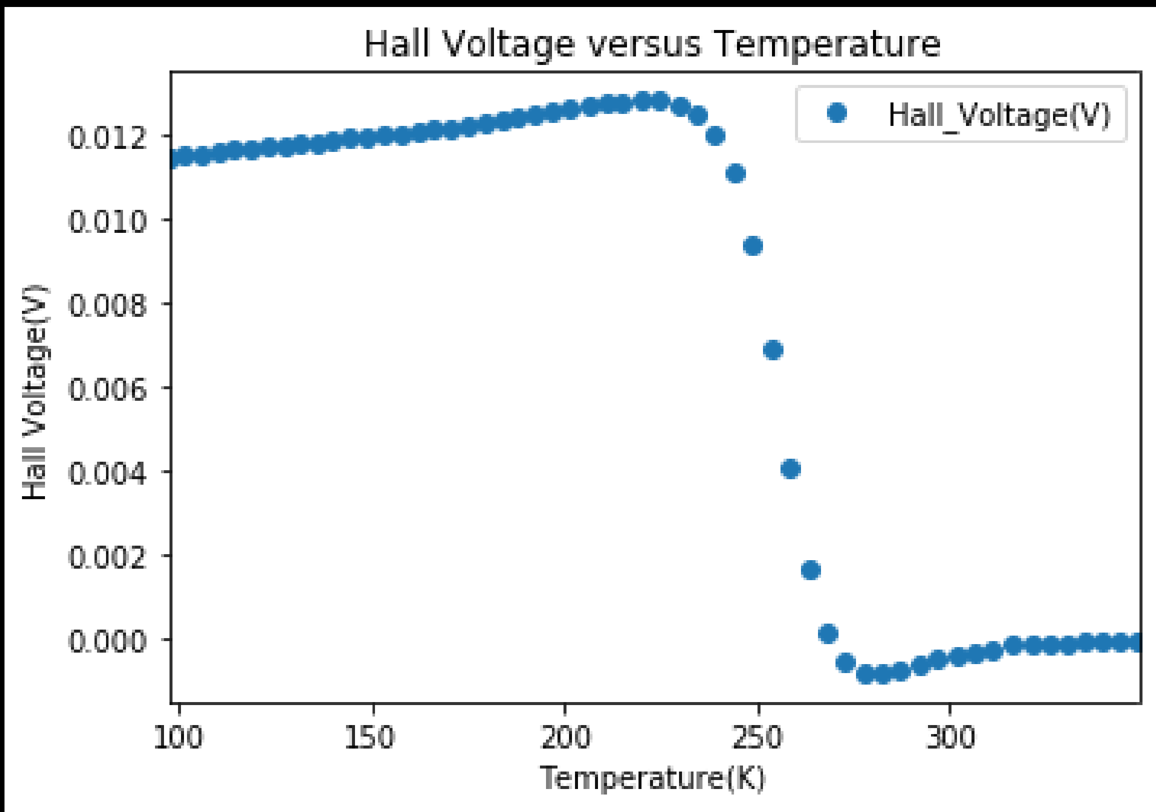



Figure 14: A plot of the hall voltage versus the Temperature, shows how the hall voltage changes sign at some point.

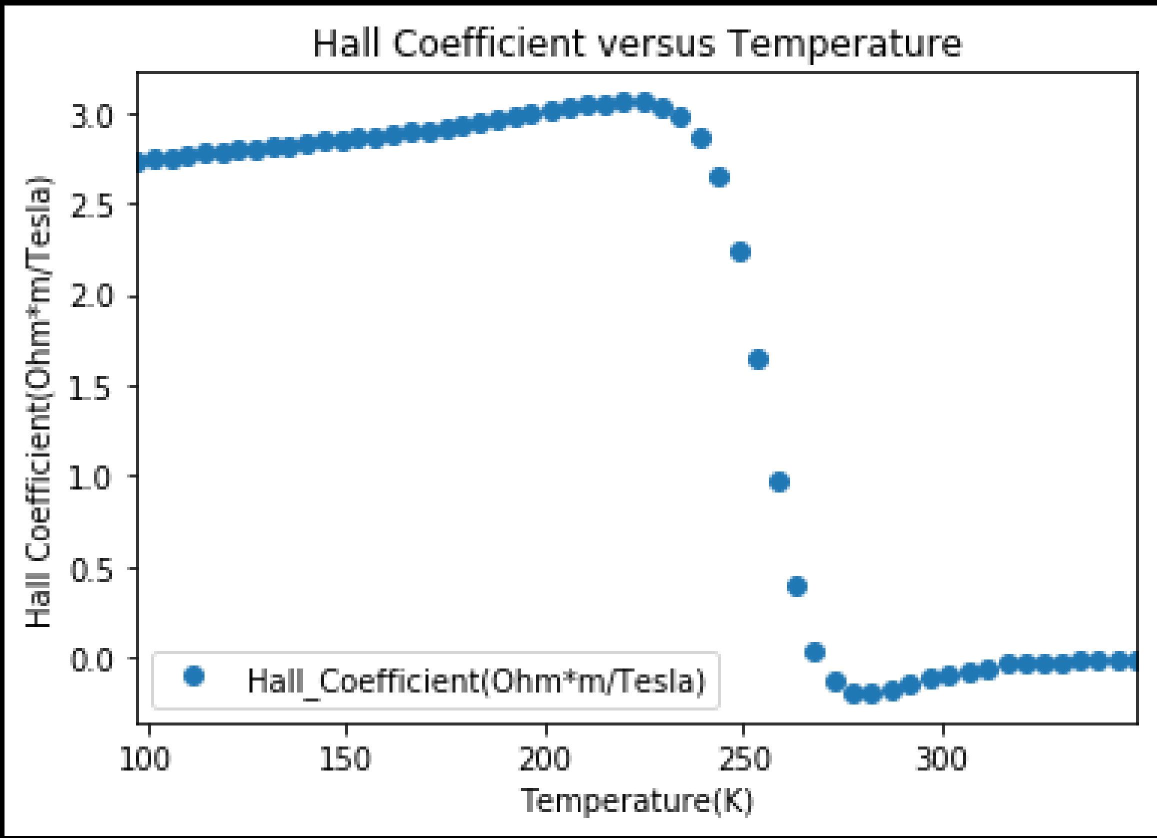



Figure 15: A plot of the hall coefficient versus temperature. Since it changes sign that indicates that we have a p-type semiconductor.

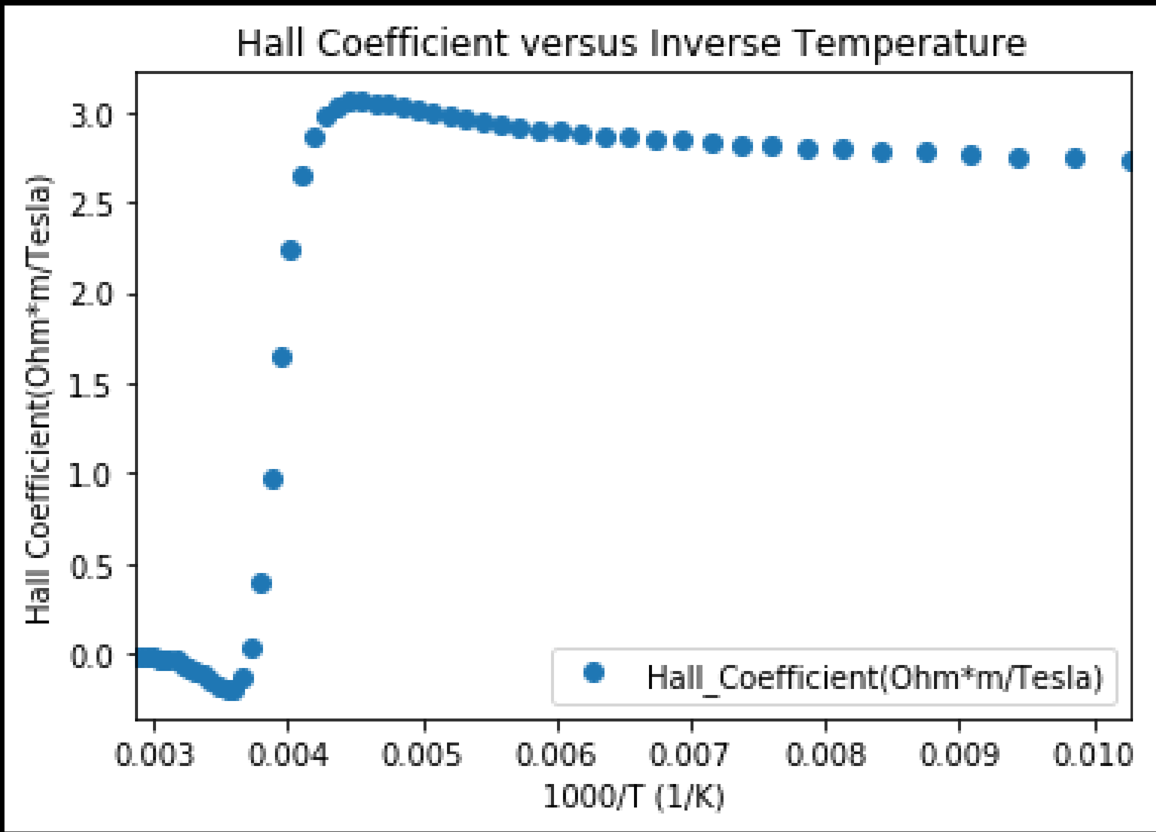



Figure 16: A plot of the hall coefficient in terms of the inverse temperature.

These figures above make sense in the context of the fact that for a two charge carrier system we can express the Hall coefficient in terms of equation (10).

$$R_H = \frac{\mu_p^2 p - \mu_n^2 n}{e(\mu_p p + \mu_n n)^2}$$

Since it changes sign that indicates that our Germanium was p-doped as explained in the theory section.

### Concentration and Drift Mobility in Extrinsic region

In our data collection it appears that the hole concentration is decreasing as temperatures increases in the extrinsic region. This implies to me that there is something slightly wrong with the theory, because I was expecting the hole concentration to be constant in the extrinsic region. I took the largest value for the hole concentration and set it as my constant concentration in the extrinsic region. This is just an assumption, but it does make sense because the variation is very small in the extrinsic region so it is not going to create too much error by simply assuming it is constant. I chose the largest value because as temperature increases there could have been more scattering of the holes, thus decreasing the concentration slightly.

$$p = 2.283 \cdot 10^{18} \pm 0.000001 \text{m}^{-3}$$

$$n = 0$$

The $\mu_d \propto T^{-3/2}$ is not always observed for the drift mobility experimentally.

$$\rho_{\text{ext}} \propto \mathrm{T}^{\alpha} \tag{17}$$

By equation above we know that the inverse of mobility is equal to resistivity. Aso cause when resistivity increases, mobility decreases. So we suppose

$$\mu_{\mathrm{d}} \propto \mathrm{T}^{-\alpha} \tag{18}$$

I can now find the hole mobility from the resistivity. Mobility is just a way to quantified how easily the charged carriers can move through the material. For this I am going to use the drift mobility, in contrast to a later section, where I find the Hall mobility. It makes sense that the resistivity of the material which dictates how much a material resists an electric current can influence the mobility. A higher resistivity would cause a lower mobility. In a sense, this is just mobility inferred from resistivity.

I will see if I get something like this from my resistivity curve. I performed a power fit for this type of equation $\rho \propto \mathrm{T}^{\alpha}$ I found that $\alpha = 2.3082632 \pm 0.02$. Thus we have that $\rho \propto T^{2.3082632 \pm 0.02} \Omega \cdot \mathrm{m}$. It follows that $\mu_{\mathrm{d}} \propto T^{-2.3082632 \pm 0.02} \mathrm{m}^2/(\mathrm{Vs})$. It is reasonably close to the -2 that was found in Mellisinos. Anyway this is the answer for the drift mobility in this experiment. It does not exactly agree with the theoretical estimate for the dependence on temperature. This indicates that either the theory is only an approximation and some assumptions made in it are leading to a different result or the experiment was erroneous. Note in the derivation, they just choose a temperature dependence for $\lambda$, so that guess is maybe not exactly correct. Or there could be error in the experimental and the data analysis tools used. Note the uncertainties were given by the least squares fit done with Scipy by choosing a simulation error of 1% for the resistivity values. I chose a small percent because I know that most error has been removed via the method we are using, the Van Der Pauw method. In addition, from equation

$$\rho = \frac{\pi \mathrm{d}}{\log(2)} \cdot \frac{\mathrm{R}_{\text{horizontal}} + \mathrm{R}_{\text{vertical}}}{2} \cdot f(\mathrm{x})$$

We know from the lab manual that that we can take this $f(x)$ to a good approximation to have error around 0.1% for $x < 2.2$ and better than 1% for $x < 4.3$ I just took the upper bound for all values to simplify my error estimate. I took the 1%. Provided with this error estimate of 0.02 approximately it doesn't seem our error is large enough to explain why it is about 0.3 different from the theoretical value. This would lead me to conclude that the theoretical value could potentially be a little bit off.

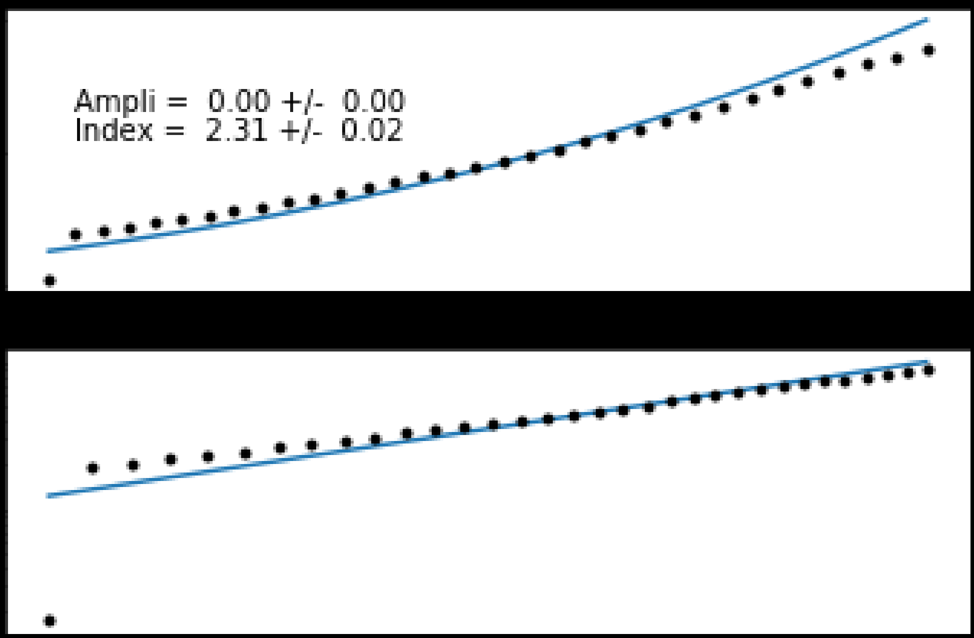



Figure 17: Shows the power fit, and the least squares approximate when we take the log to get a straight line. It shows that the straight line is a decent estimate to the points. This shows that a power fit is a good assumption. The error bars are so small as to not be visible.

Hall Mobility

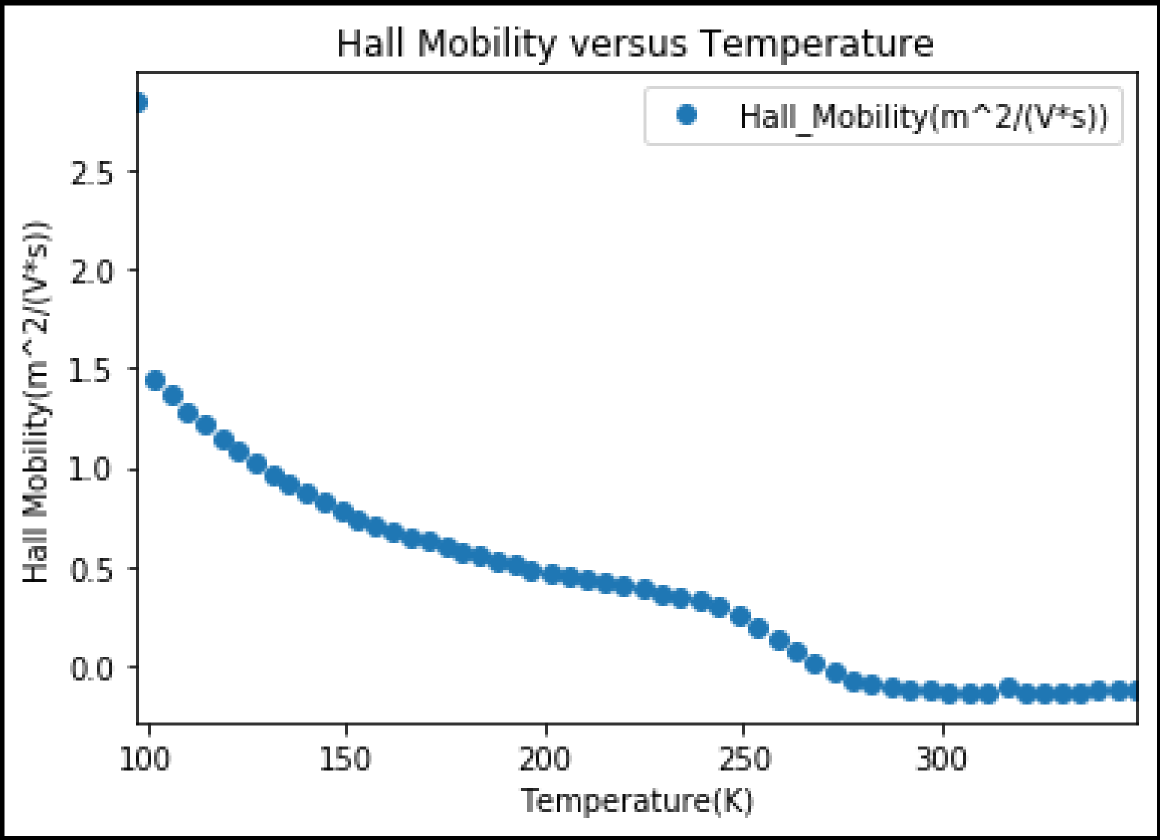



Figure 18: Plot of the Hall mobility versus temperature.

$$\mu_{\mathrm{H}} = \mathrm{R_H}\sigma \tag{19}$$

Of course it goes to zero at around the same temperature as the hall coefficient by equation (20), that is between temperatures 268.05 and 272.8. In fact, I will say my inversion temperature as it is called in Mellisinos is at $270 \pm 1K$. The inversion temperature tells us that this is a p-type semiconductor because from the equations for $\mathrm{R_H}$ seen above that is the only way that you can get a switch in sign for $R_H$. There is another important part about this inversion temperature. We can use it to find the ratio of mobilities $b = \mu_n/\mu_p$. We get the following when $R_H = 0$.

$$nb^2 - p = 0 \tag{20}$$

$$n = \frac{N_a}{b^2 - 1} \tag{21}$$

Basically $N_a = p$ in the extrinsic region. By performing some mathematics we can solve for the b with the equation seen above. So we have

$$b = \frac{R_e(T = T_0}{R_e(T = T0) - R_0} \tag{22}$$

We computed

$$\mathrm{b} = 1.45 \pm 0.01 \tag{23}$$

Now I can find the T dependence of the Hall mobility to compare to the drift mobility. The Hall mobility is just the concept of mobility inferred from the Hall effect.

$$\frac{1}{\rho} = e \cdot (n \cdot \mu_n + p \cdot \mu_p)$$

this is interesting but imagine if you rewrite this a little bit for the extrinsic region. $\Rightarrow \sigma = e(p\mu_p) \Rightarrow \frac{\sigma}{ep} = \mu_p$, wait a second $1/ep = R_H$ so this is the same thing as $\mu_p = R_H\sigma$, so this is actually the Hall mobility as it is derived from the Hall coefficient. Using the graph for the hall mobility from above we get the following results by performing a fit. $\mu_H \propto \mathrm{T}^{-1.84\pm0.02}$, So we found the following with a little bit of uncertainty at 0.02, this is still not the same as the value that was found in Melissinos to be -3/2. And the uncertainty doesn't fit it within the value as well.

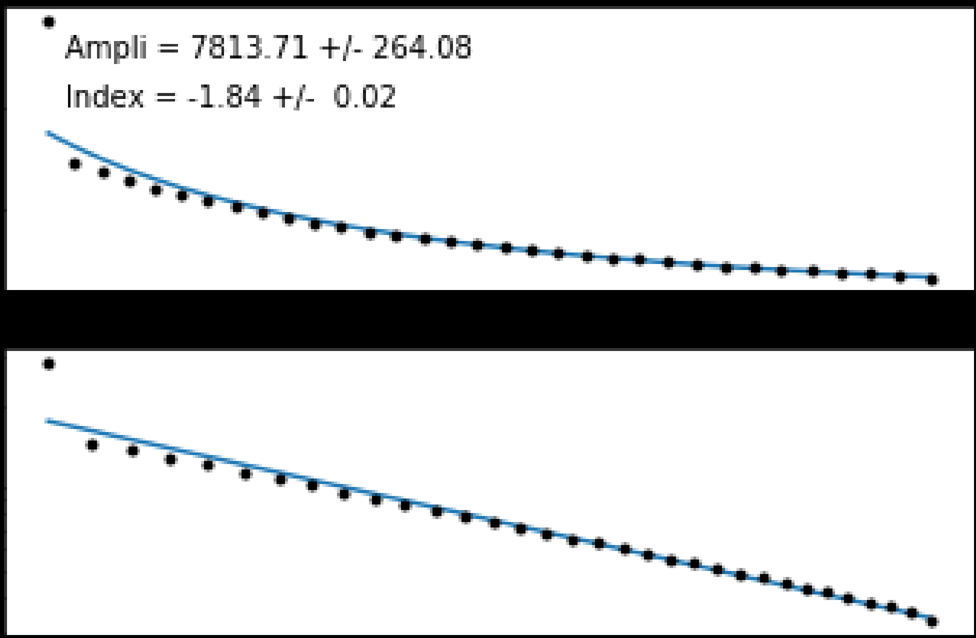



Figure 19: Shows the power fit, and the least squares approximate when we take the log to get a straight line. It shows that the straight line is a decent estimate to the points. This shows that a power fit is a good assumption. The error bars are so small as to not be visible.

By using the equation 3.3a and 3.3b from Mellisinos and then taking the drift velocity and the current density from equation (12) you can derive the relation for resistivity.

$$\rho = \frac{1}{\mu}\mathrm{T}^{-3/2}exp(\frac{\mathrm{E_g}}{2k_\mathrm{B}\mathrm{T}} \tag{24}$$

In the intrinsic region the exponential is going to be much more important. If we look at the plots we can qualitatively verify that it does seem to fit an exponential decline equation.

Magnetoresistance

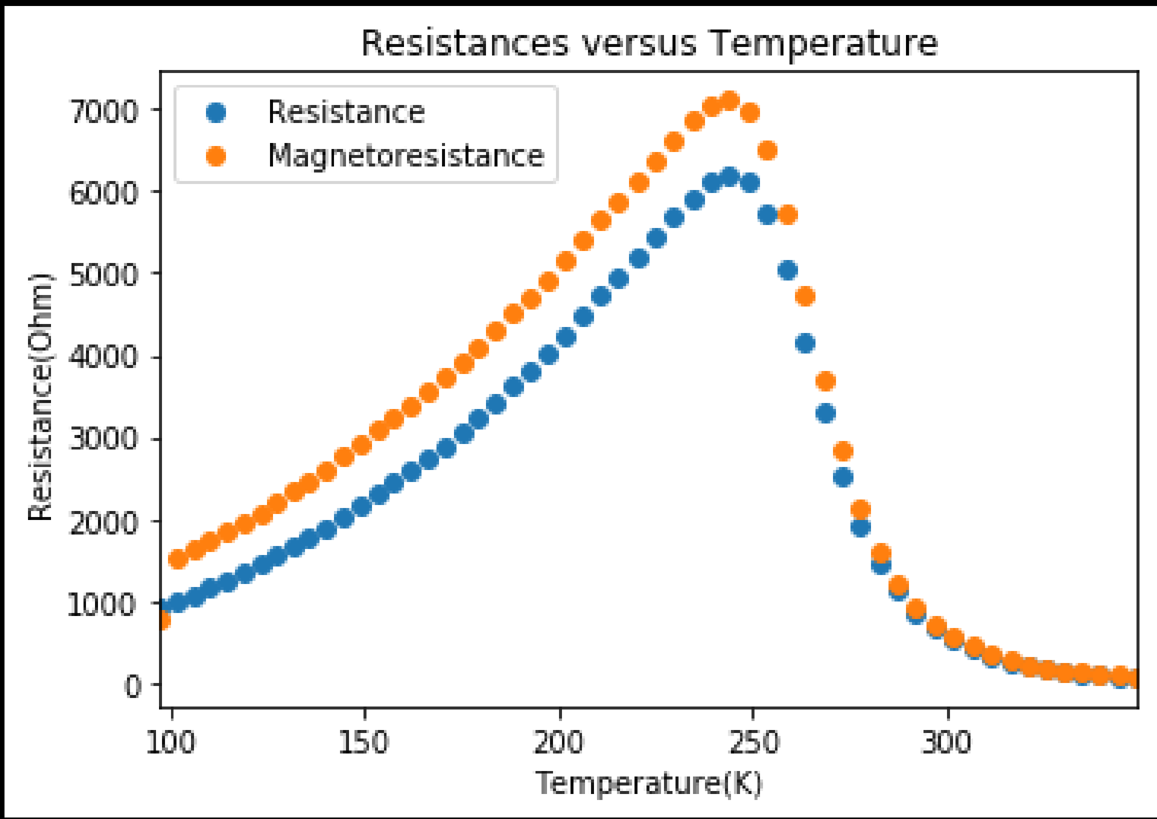



Figure 20: These plots show the relationship between the resistance and the magnetoresistance.

I also noticed a pattern between the resistance when the magnetic field was on versus when it was off. The first thing that I saw was that even when the magnet was supposedly off we picked up a small magnetic field of about 0.01 gauss. I believe the simplest explanation is that the current to the electromagnet was zero, but because of hysteresis some of the magnetic domains were still aligned in the magnetic core. Hysteresis says that when you apply a current to a magnet and generate a magnetic field, when you remove the current through the coils. There still exist some remnant magnetic field left over. In fact, in order to get zero magnetic field you need to drive the current a little bit in the reverse direction. But the crucial point here is that hysteresis, or magnetic memory of the magnetic core is a good explanation for the physical reason why our data did not show zero magnetic field when we eliminated the current going to the electromagnet.
From figure ? we see that resistance when the magnetic field was on was consistently higher than the resistance at almost zero magnetic field in the extrinsic region. The possible explanation for this is related the drift velocity, which again is the average velocity of the charge carriers in the direction of an applied electric field to a material.

$$F_B = qV \times B \tag{25}$$

This represents the magnetic force felt by the charge particles. Imagine we have electrons with a drift velocity in a certain direction, that is an average velocity moving down a conductive wire. If we applied a magnetic field out of the page it will exert a force on electrons towards the bottom of the wire. But wouldn't particles with more velocity be deflected greater than charge particles with slower velocity. This is true via the fact that the magnetic force is proportional to the velocity. This would cause more collisions of the charge carriers and thus slow down the drift velocity. This would directly lead to greater resistance. This is the effect that the magnetic field can have on the resistance of a conducting material. [3]

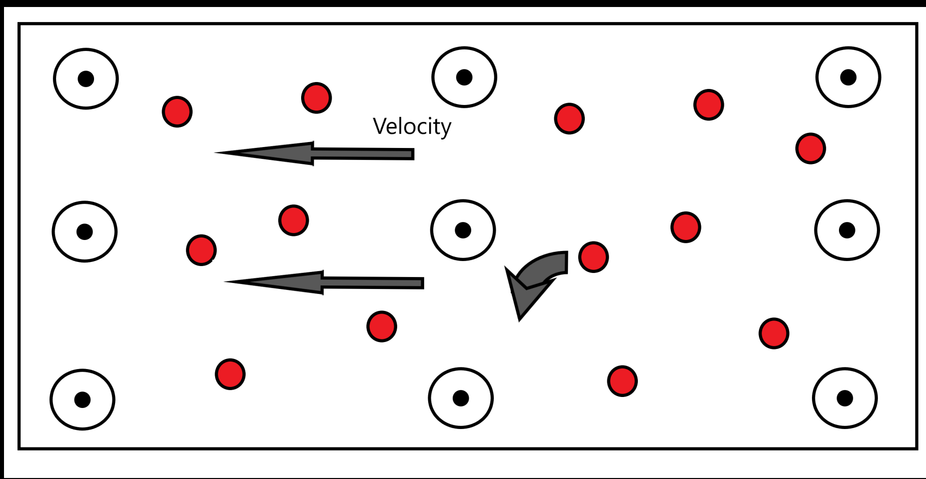



Figure 21: Shows the magnetic field along a wire deflecting electrons towards the bottom of the wire. The red dots are the electrons. The magnetic field is pointing out of the page.

## Room Temperature Measurements

I took room temperature to be my measurements at 296.825K with current of 10$\mu$Amp.

$$\rho = 0.8366 \pm 0.00001 \Omega \cdot \mathrm{m}$$

$$\mathrm{R_H} = -0.117 \pm 0.001 \mathrm{m}^2/(\mathrm{Vs})$$

Since $\mathrm{R_H} < 0$ it is in the intrinsic region at room temperature.
I also took a measurement to find at what magnetic field strength would the Hall field be of equal value as the Electric field caused by the resistivity of the material at room temperature. Note that I am doing this when the input current was 10$\mu$Amp.

$$\mathrm{E_H} = \frac{\mathrm{IBR_H}}{\mathrm{sd}}$$

We know that the electric field due to resistance is $\mathrm{V}_\rho = \mathrm{E_H s}$, after algebra we just need to plug into this equation.

$$\mathrm{B} = \frac{\mathrm{V}_\rho \mathrm{d}}{\mathrm{IR_H}}$$

We have the following values d=0.00125 m $\pm$0.01 mm, I=10$\mu$Amps, $\mathrm{V}_\rho = 0.0015705$ V. I get the following value, B = 16,780 $\pm$ 0.01 gauss. It is a rather large magnetic field required, in this experiment we do not provide a magnetic field in order to get the Hall field to be similar in value to the electric field caused by resistance. There is one thing that we could not do which was the plot of hall voltage with respect to current, because we lacked a suitable number of consistent runs to be able to find a pattern in the plot. With just two runs it is just going to be two dots. Although when I plotted it looked like as current increased the hall voltage decreased and became more negative in the intrinsic region. Also as magnetic field increased hall voltage increased. That is about all I could discern from two points and it is not statistically significant.

## Conclusion

In this experiment we tested many properties of the semiconductor from the Hall effect. We were able to find out that our semiconductor was p-doped because the Hall coefficient and the Hall voltage switch sign at the inversion temperature $\mathrm{T}_0 = 270 \pm 1\mathrm{K}$. An inversion temperature is a property of only a p-doped semiconductor by the equations in the Hall effect. Specifically equation (10), the Hall coefficient for two charge carriers. There were a couple of simplifying approximations we were able to make in the extrinsic region thanks to the physics. Such as the saturation of impurities allowed us to assume in the extrinsic region a constant hole concentration, which we found to be $p = 2.283 \cdot 10^{18} \pm 0.000001\mathrm{m}^{-3}$.
We plotted the resistivity and conductivity to show graphically the behavior of the semiconductor with respect to temperature. These graphs showed the characteristic increasing resistivity in the extrinsic region, then once it peaks it begins decreasing at an exponential rate in the intrinsic region. The conductivity shows the exponential increase in the intrinsic region. Basically where the semiconductor approximates a conductor. We also took a very crucial analysis of the mobilities. At the end of the day, the mobility is this constant of proportionality between the electric field and the drift velocity. So it just represents well the charge carriers will move with an average velocity in the direction of the electric field, or in other words it is how responsive they are to the electric field. For the extrinsic region I was able to perform a power fit of the resistitivity curve and the Hall mobility curve. I found two different results that do not agree with theory. This means some of the approximations made in the theory are potentially wrong. I suppose that the assumption that $\lambda = 1/\mathrm{kT}$ could be wrong, after all it is just a guess to make the math easier. The $\mu_\mathrm{d} \propto \mathrm{T}^{-2.308}$ and the $\mu_\mathrm{H} \propto \mathrm{T}^{-1.84}$ is what I found to be my results.

# 1 References

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2. D. Halliday, R. Resnick, J. Walker. "Chapter 46: Conduction of Electricity in Solids Fundamental of Physics extended. Excellent introduction to conductivity in solids. (1993).
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