Slopes of U-operators

Gebhard Böckle, Peter Mathias Gräf, Theresa Kaiser

March 4, 2025

This document contains further results of the computations explained in [BGK25]. Throughout, bold exponents denote multiplicities of the slopes. If for $\Gamma \in \{\mathrm{GL}_3(A), \Gamma_0^P, \Gamma_2^P\}$ the space of Γ-invariant harmonic cocycles is zero, the corresponding field in the table is empty. If for some k, the space of $\Gamma_0(t)$-invariant harmonic cocycles is zero, the smaller spaces are zero as well and the entire row is omitted.

List of Tables

k	T_1-Slopes	$U_1^{\Gamma_0^P}$-Slopes	$U_1^{\Gamma_2^P}$-Slopes	$U_1^{\Gamma_0(t)}$-Slopes
0				$0^{\mathbf{1}}$
1		$\infty^{\mathbf{1}}$	$0^{\mathbf{1}}$	$0^{\mathbf{1}}$, $\infty^{\mathbf{2}}$
2		$1^{\mathbf{1}}$, $\infty^{\mathbf{1}}$	$0^{\mathbf{1}}$, $1^{\mathbf{1}}$	$0^{\mathbf{1}}$, $1^{\mathbf{2}}$, $\infty^{\mathbf{3}}$
3		$\frac{3}{2}^{\mathbf{2}}$, $\infty^{\mathbf{1}}$	$0^{\mathbf{1}}$, $\frac{3}{2}^{\mathbf{2}}$	$0^{\mathbf{1}}$, $\frac{3}{2}^{\mathbf{4}}$, $2^{\mathbf{1}}$, $\infty^{\mathbf{4}}$
4	$1^{\mathbf{1}}$	$1^{\mathbf{1}}$, $2^{\mathbf{1}}$, $\infty^{\mathbf{3}}$	$0^{\mathbf{1}}$, $1^{\mathbf{2}}$, $2^{\mathbf{1}}$, $\infty^{\mathbf{1}}$	$0^{\mathbf{1}}$, $1^{\mathbf{2}}$, $2^{\mathbf{2}}$, $\frac{8}{3}^{\mathbf{3}}$, $\infty^{\mathbf{7}}$
5	$2^{\mathbf{1}}$	$2^{\mathbf{1}}$, $\frac{5}{2}^{\mathbf{2}}$, $\infty^{\mathbf{4}}$	$0^{\mathbf{1}}$, $2^{\mathbf{3}}$, $\frac{5}{2}^{\mathbf{2}}$, $\infty^{\mathbf{1}}$	$0^{\mathbf{1}}$, $2^{\mathbf{3}}$, $\frac{5}{2}^{\mathbf{4}}$, $\frac{10}{3}^{\mathbf{3}}$, $\infty^{\mathbf{10}}$
6	$1^{\mathbf{1}}$	$1^{\mathbf{1}}$, $3^{\mathbf{3}}$, $\infty^{\mathbf{5}}$	$0^{\mathbf{1}}$, $1^{\mathbf{2}}$, $3^{\mathbf{5}}$, $\infty^{\mathbf{1}}$	$0^{\mathbf{1}}$, $1^{\mathbf{2}}$, $3^{\mathbf{8}}$, $4^{\mathbf{4}}$, $\infty^{\mathbf{13}}$
7	$\frac{3}{2}^{\mathbf{2}}$	$\frac{3}{2}^{\mathbf{2}}$, $\frac{7}{2}^{\mathbf{2}}$, $4^{\mathbf{1}}$, $\infty^{\mathbf{7}}$	$0^{\mathbf{1}}$, $\frac{3}{2}^{\mathbf{4}}$, $2^{\mathbf{1}}$, $\frac{7}{2}^{\mathbf{2}}$, $4^{\mathbf{2}}$, $\infty^{\mathbf{2}}$	$0^{\mathbf{1}}$, $\frac{3}{2}^{\mathbf{4}}$, $2^{\mathbf{1}}$, $\frac{7}{2}^{\mathbf{4}}$, $4^{\mathbf{3}}$, $\frac{14}{3}^{\mathbf{6}}$, $\infty^{\mathbf{17}}$
8	$1^{\mathbf{1}}$, $2^{\mathbf{1}}$	$1^{\mathbf{1}}$, $2^{\mathbf{1}}$, $4^{\mathbf{3}}$, $5^{\mathbf{1}}$, $\infty^{\mathbf{9}}$	$0^{\mathbf{1}}$, $1^{\mathbf{2}}$, $2^{\mathbf{2}}$, $\frac{8}{3}^{\mathbf{3}}$, $4^{\mathbf{3}}$, $5^{\mathbf{1}}$, $8^{\mathbf{1}}$, $\infty^{\mathbf{2}}$	$0^{\mathbf{1}}$, $1^{\mathbf{2}}$, $2^{\mathbf{2}}$, $\frac{8}{3}^{\mathbf{3}}$, $4^{\mathbf{6}}$, $5^{\mathbf{2}}$, $\frac{16}{3}^{\mathbf{6}}$, $8^{\mathbf{1}}$, $\infty^{\mathbf{22}}$
9	$2^{\mathbf{1}}$, $4^{\mathbf{1}}$	$2^{\mathbf{1}}$, $4^{\mathbf{1}}$, $\frac{9}{2}^{\mathbf{4}}$, $5^{\mathbf{2}}$, $\infty^{\mathbf{10}}$	$0^{\mathbf{1}}$, $2^{\mathbf{3}}$, $\frac{10}{3}^{\mathbf{3}}$, $4^{\mathbf{2}}$, $\frac{9}{2}^{\mathbf{4}}$, $5^{\mathbf{2}}$, $8^{\mathbf{1}}$, $\infty^{\mathbf{2}}$	$0^{\mathbf{1}}$, $2^{\mathbf{3}}$, $\frac{10}{3}^{\mathbf{3}}$, $4^{\mathbf{2}}$, $\frac{9}{2}^{\mathbf{8}}$, $5^{\mathbf{4}}$, $6^{\mathbf{7}}$, $8^{\mathbf{1}}$, $\infty^{\mathbf{26}}$
10	$1^{\mathbf{1}}$, $3^{\mathbf{1}}$, $4^{\mathbf{1}}$	$1^{\mathbf{1}}$, $3^{\mathbf{1}}$, $4^{\mathbf{1}}$, $5^{\mathbf{3}}$, $\frac{23}{4}^{\mathbf{4}}$, $\infty^{\mathbf{12}}$	$0^{\mathbf{1}}$, $1^{\mathbf{2}}$, $3^{\mathbf{4}}$, $4^{\mathbf{3}}$, $5^{\mathbf{5}}$, $\frac{23}{4}^{\mathbf{4}}$, $\infty^{\mathbf{3}}$	$0^{\mathbf{1}}$, $1^{\mathbf{2}}$, $3^{\mathbf{4}}$, $4^{\mathbf{3}}$, $5^{\mathbf{8}}$, $\frac{23}{4}^{\mathbf{8}}$, $\frac{20}{3}^{\mathbf{9}}$, $\infty^{\mathbf{31}}$
11	$\frac{3}{2}^{\mathbf{2}}$, $4^{\mathbf{2}}$	$\frac{3}{2}^{\mathbf{2}}$, $4^{\mathbf{2}}$, $\frac{11}{2}^{\mathbf{4}}$, $6^{\mathbf{1}}$, $\frac{13}{2}^{\mathbf{2}}$, $\infty^{\mathbf{15}}$	$0^{\mathbf{1}}$, $\frac{3}{2}^{\mathbf{4}}$, $2^{\mathbf{1}}$, $4^{\mathbf{5}}$, $\frac{11}{2}^{\mathbf{8}}$, $6^{\mathbf{1}}$, $\frac{13}{2}^{\mathbf{2}}$, $\infty^{\mathbf{4}}$	$0^{\mathbf{1}}$, $\frac{3}{2}^{\mathbf{4}}$, $2^{\mathbf{1}}$, $4^{\mathbf{5}}$, $\frac{11}{2}^{\mathbf{12}}$, $6^{\mathbf{2}}$, $\frac{13}{2}^{\mathbf{4}}$, $\frac{22}{3}^{\mathbf{12}}$, $\infty^{\mathbf{37}}$
12	$1^{\mathbf{1}}$, $2^{\mathbf{1}}$, $5^{\mathbf{2}}$	$1^{\mathbf{1}}$, $2^{\mathbf{1}}$, $5^{\mathbf{2}}$, $6^{\mathbf{5}}$, $\frac{20}{3}^{\mathbf{3}}$, $9^{\mathbf{1}}$, $\infty^{\mathbf{17}}$	$0^{\mathbf{1}}$, $1^{\mathbf{2}}$, $2^{\mathbf{2}}$, $\frac{8}{3}^{\mathbf{3}}$, $5^{\mathbf{6}}$, $6^{\mathbf{7}}$, $\frac{20}{3}^{\mathbf{3}}$, $9^{\mathbf{1}}$, $16^{\mathbf{1}}$, $\infty^{\mathbf{4}}$	$0^{\mathbf{1}}$, $1^{\mathbf{2}}$, $2^{\mathbf{2}}$, $\frac{8}{3}^{\mathbf{3}}$, $5^{\mathbf{6}}$, $6^{\mathbf{12}}$, $\frac{20}{3}^{\mathbf{6}}$, $8^{\mathbf{13}}$, $9^{\mathbf{2}}$, $16^{\mathbf{1}}$, $\infty^{\mathbf{43}}$
13	$2^{\mathbf{1}}$, $\frac{5}{2}^{\mathbf{2}}$, $6^{\mathbf{2}}$	$2^{\mathbf{1}}$, $\frac{5}{2}^{\mathbf{2}}$, $6^{\mathbf{2}}$, $\frac{13}{2}^{\mathbf{4}}$, $7^{\mathbf{2}}$, $\frac{22}{3}^{\mathbf{3}}$, $10^{\mathbf{1}}$, $\infty^{\mathbf{20}}$	$0^{\mathbf{1}}$, $2^{\mathbf{3}}$, $\frac{5}{2}^{\mathbf{4}}$, $\frac{10}{3}^{\mathbf{3}}$, $6^{\mathbf{7}}$, $\frac{13}{2}^{\mathbf{4}}$, $7^{\mathbf{2}}$, $\frac{22}{3}^{\mathbf{3}}$, $8^{\mathbf{1}}$, $10^{\mathbf{1}}$, $16^{\mathbf{1}}$, $\infty^{\mathbf{5}}$	$0^{\mathbf{1}}$, $2^{\mathbf{3}}$, $\frac{5}{2}^{\mathbf{4}}$, $\frac{10}{3}^{\mathbf{3}}$, $6^{\mathbf{7}}$, $\frac{13}{2}^{\mathbf{8}}$, $7^{\mathbf{4}}$, $\frac{22}{3}^{\mathbf{6}}$, $8^{\mathbf{1}}$, $\frac{26}{3}^{\mathbf{15}}$, $10^{\mathbf{2}}$, $16^{\mathbf{1}}$, $\infty^{\mathbf{50}}$
14	$1^{\mathbf{1}}$, $3^{\mathbf{3}}$, $7^{\mathbf{2}}$	$1^{\mathbf{1}}$, $3^{\mathbf{3}}$, $7^{\mathbf{7}}$, $\frac{31}{4}^{\mathbf{4}}$, $8^{\mathbf{1}}$, $9^{\mathbf{1}}$, $\infty^{\mathbf{23}}$	$0^{\mathbf{1}}$, $1^{\mathbf{2}}$, $3^{\mathbf{8}}$, $4^{\mathbf{4}}$, $7^{\mathbf{11}}$, $\frac{31}{4}^{\mathbf{4}}$, $8^{\mathbf{2}}$, $9^{\mathbf{1}}$, $16^{\mathbf{1}}$, $\infty^{\mathbf{6}}$	$0^{\mathbf{1}}$, $1^{\mathbf{2}}$, $3^{\mathbf{8}}$, $4^{\mathbf{4}}$, $7^{\mathbf{16}}$, $\frac{31}{4}^{\mathbf{8}}$, $8^{\mathbf{3}}$, $9^{\mathbf{2}}$, $\frac{28}{3}^{\mathbf{18}}$, $16^{\mathbf{1}}$, $\infty^{\mathbf{57}}$
15	$\frac{3}{2}^{\mathbf{2}}$, $\frac{7}{2}^{\mathbf{2}}$, $4^{\mathbf{1}}$, $8^{\mathbf{1}}$	$\frac{3}{2}^{\mathbf{2}}$, $\frac{7}{2}^{\mathbf{2}}$, $4^{\mathbf{1}}$, $\frac{15}{2}^{\mathbf{6}}$, $8^{\mathbf{4}}$, $\frac{17}{2}^{\mathbf{2}}$, $\frac{19}{2}^{\mathbf{2}}$, $\infty^{\mathbf{26}}$	$0^{\mathbf{1}}$, $\frac{3}{2}^{\mathbf{4}}$, $2^{\mathbf{1}}$, $\frac{7}{2}^{\mathbf{4}}$, $4^{\mathbf{3}}$, $\frac{14}{3}^{\mathbf{6}}$, $\frac{15}{2}^{\mathbf{6}}$, $8^{\mathbf{8}}$, $\frac{17}{2}^{\mathbf{2}}$, $\frac{19}{2}^{\mathbf{2}}$, $16^{\mathbf{2}}$, $\infty^{\mathbf{6}}$	$0^{\mathbf{1}}$, $\frac{3}{2}^{\mathbf{4}}$, $2^{\mathbf{1}}$, $\frac{7}{2}^{\mathbf{4}}$, $4^{\mathbf{3}}$, $\frac{14}{3}^{\mathbf{6}}$, $\frac{15}{2}^{\mathbf{12}}$, $8^{\mathbf{11}}$, $\frac{17}{2}^{\mathbf{4}}$, $\frac{19}{2}^{\mathbf{4}}$, $10^{\mathbf{19}}$, $16^{\mathbf{2}}$, $\infty^{\mathbf{65}}$
16	$1^{\mathbf{1}}$, $2^{\mathbf{1}}$, $4^{\mathbf{3}}$, $5^{\mathbf{1}}$, $9^{\mathbf{1}}$	$1^{\mathbf{1}}$, $2^{\mathbf{1}}$, $4^{\mathbf{3}}$, $5^{\mathbf{1}}$, $8^{\mathbf{5}}$, $\frac{26}{3}^{\mathbf{6}}$, $9^{\mathbf{2}}$, $10^{\mathbf{3}}$, $\infty^{\mathbf{29}}$	$0^{\mathbf{1}}$, $1^{\mathbf{2}}$, $2^{\mathbf{2}}$, $\frac{8}{3}^{\mathbf{3}}$, $4^{\mathbf{6}}$, $5^{\mathbf{2}}$, $\frac{16}{3}^{\mathbf{6}}$, $8^{\mathbf{6}}$, $\frac{26}{3}^{\mathbf{6}}$, $9^{\mathbf{5}}$, $10^{\mathbf{3}}$, $16^{\mathbf{2}}$, $\infty^{\mathbf{7}}$	$0^{\mathbf{1}}$, $1^{\mathbf{2}}$, $2^{\mathbf{2}}$, $\frac{8}{3}^{\mathbf{3}}$, $4^{\mathbf{6}}$, $5^{\mathbf{2}}$, $\frac{16}{3}^{\mathbf{6}}$, $8^{\mathbf{11}}$, $\frac{26}{3}^{\mathbf{12}}$, $9^{\mathbf{6}}$, $10^{\mathbf{6}}$, $\frac{32}{3}^{\mathbf{21}}$, $16^{\mathbf{2}}$, $\infty^{\mathbf{73}}$

Table 1: Slopes for $q = 2$, $i = 1$. Slopes of the form $\frac{2k}{3}$ are marked in blue.

k	T_2-Slopes	$U_2^{\Gamma_0^P}$-Slopes	$U_2^{\Gamma_2^P}$-Slopes	$U_2^{\Gamma_0(t)}$-Slopes
0				$0^{\mathbf{1}}$
1		$0^{\mathbf{1}}$	$0^{\mathbf{1}}$	$0^{\mathbf{2}}$, $\infty^{\mathbf{1}}$
2		$0^{\mathbf{2}}$	$0^{\mathbf{1}}$, $\infty^{\mathbf{1}}$	$0^{\mathbf{3}}$, $\infty^{\mathbf{3}}$
3		$0^{\mathbf{3}}$	$0^{\mathbf{1}}$, $\infty^{\mathbf{2}}$	$0^{\mathbf{4}}$, $1^{\mathbf{1}}$, $\infty^{\mathbf{5}}$
4	$0^{\mathbf{1}}$	$0^{\mathbf{4}}$, $\infty^{\mathbf{1}}$	$0^{\mathbf{2}}$, $\infty^{\mathbf{3}}$	$0^{\mathbf{5}}$, $\frac{4}{3}^{\mathbf{3}}$, $\infty^{\mathbf{7}}$
5	$0^{\mathbf{1}}$	$0^{\mathbf{5}}$, $1^{\mathbf{1}}$, $\infty^{\mathbf{1}}$	$0^{\mathbf{2}}$, $1^{\mathbf{1}}$, $\infty^{\mathbf{4}}$	$0^{\mathbf{6}}$, $1^{\mathbf{2}}$, $\frac{5}{3}^{\mathbf{3}}$, $\infty^{\mathbf{10}}$
6	$0^{\mathbf{1}}$	$0^{\mathbf{6}}$, $\frac{3}{2}^{\mathbf{2}}$, $\infty^{\mathbf{1}}$	$0^{\mathbf{2}}$, $\frac{3}{2}^{\mathbf{2}}$, $\infty^{\mathbf{5}}$	$0^{\mathbf{7}}$, $\frac{3}{2}^{\mathbf{4}}$, $2^{\mathbf{4}}$, $\infty^{\mathbf{13}}$
7	$0^{\mathbf{2}}$	$0^{\mathbf{7}}$, $1^{\mathbf{2}}$, $2^{\mathbf{1}}$, $\infty^{\mathbf{2}}$	$0^{\mathbf{3}}$, $1^{\mathbf{1}}$, $2^{\mathbf{1}}$, $\infty^{\mathbf{7}}$	$0^{\mathbf{8}}$, $1^{\mathbf{3}}$, $2^{\mathbf{2}}$, $\frac{7}{3}^{\mathbf{6}}$, $\infty^{\mathbf{17}}$
8	$0^{\mathbf{2}}$	$0^{\mathbf{8}}$, $\frac{4}{3}^{\mathbf{3}}$, $2^{\mathbf{1}}$, $4^{\mathbf{1}}$, $\infty^{\mathbf{2}}$	$0^{\mathbf{3}}$, $\frac{4}{3}^{\mathbf{3}}$, $4^{\mathbf{1}}$, $\infty^{\mathbf{8}}$	$0^{\mathbf{9}}$, $\frac{4}{3}^{\mathbf{6}}$, $2^{\mathbf{1}}$, $\frac{8}{3}^{\mathbf{6}}$, $4^{\mathbf{2}}$, $\infty^{\mathbf{21}}$
9	$0^{\mathbf{2}}$	$0^{\mathbf{9}}$, $1^{\mathbf{3}}$, $\frac{5}{3}^{\mathbf{3}}$, $4^{\mathbf{1}}$, $\infty^{\mathbf{2}}$	$0^{\mathbf{3}}$, $1^{\mathbf{1}}$, $\frac{5}{3}^{\mathbf{3}}$, $4^{\mathbf{1}}$, $\infty^{\mathbf{10}}$	$0^{\mathbf{10}}$, $1^{\mathbf{4}}$, $\frac{5}{3}^{\mathbf{6}}$, $3^{\mathbf{7}}$, $4^{\mathbf{2}}$, $\infty^{\mathbf{26}}$
10	$0^{\mathbf{3}}$	$0^{\mathbf{10}}$, $\frac{3}{2}^{\mathbf{6}}$, $2^{\mathbf{1}}$, $\frac{5}{2}^{\mathbf{2}}$, $\infty^{\mathbf{3}}$	$0^{\mathbf{4}}$, $\frac{3}{2}^{\mathbf{2}}$, $2^{\mathbf{1}}$, $\frac{5}{2}^{\mathbf{2}}$, $\infty^{\mathbf{13}}$	$0^{\mathbf{11}}$, $\frac{3}{2}^{\mathbf{8}}$, $2^{\mathbf{2}}$, $\frac{5}{2}^{\mathbf{4}}$, $\frac{10}{3}^{\mathbf{9}}$, $\infty^{\mathbf{32}}$
11	$0^{\mathbf{3}}$, $1^{\mathbf{1}}$	$0^{\mathbf{11}}$, $1^{\mathbf{4}}$, $2^{\mathbf{3}}$, $\frac{11}{4}^{\mathbf{4}}$, $\infty^{\mathbf{4}}$	$0^{\mathbf{4}}$, $1^{\mathbf{2}}$, $2^{\mathbf{1}}$, $\frac{11}{4}^{\mathbf{4}}$, $\infty^{\mathbf{15}}$	$0^{\mathbf{12}}$, $1^{\mathbf{5}}$, $2^{\mathbf{4}}$, $\frac{11}{4}^{\mathbf{8}}$, $\frac{11}{3}^{\mathbf{12}}$, $\infty^{\mathbf{37}}$
12	$0^{\mathbf{3}}$, $2^{\mathbf{1}}$	$0^{\mathbf{12}}$, $\frac{4}{3}^{\mathbf{6}}$, $2^{\mathbf{2}}$, $\frac{5}{2}^{\mathbf{2}}$, $3^{\mathbf{2}}$, $6^{\mathbf{1}}$, $8^{\mathbf{1}}$, $\infty^{\mathbf{4}}$	$0^{\mathbf{4}}$, $\frac{4}{3}^{\mathbf{3}}$, $2^{\mathbf{1}}$, $\frac{5}{2}^{\mathbf{2}}$, $3^{\mathbf{2}}$, $8^{\mathbf{1}}$, $\infty^{\mathbf{17}}$	$0^{\mathbf{13}}$, $\frac{4}{3}^{\mathbf{9}}$, $2^{\mathbf{2}}$, $\frac{5}{2}^{\mathbf{4}}$, $3^{\mathbf{4}}$, $4^{\mathbf{13}}$, $6^{\mathbf{1}}$, $8^{\mathbf{2}}$, $\infty^{\mathbf{43}}$
13	$0^{\mathbf{4}}$, $1^{\mathbf{1}}$	$0^{\mathbf{13}}$, $1^{\mathbf{5}}$, $\frac{5}{3}^{\mathbf{6}}$, $3^{\mathbf{3}}$, $4^{\mathbf{1}}$, $7^{\mathbf{1}}$, $8^{\mathbf{1}}$, $\infty^{\mathbf{5}}$	$0^{\mathbf{5}}$, $1^{\mathbf{2}}$, $\frac{5}{3}^{\mathbf{3}}$, $3^{\mathbf{3}}$, $4^{\mathbf{1}}$, $8^{\mathbf{1}}$, $\infty^{\mathbf{20}}$	$0^{\mathbf{14}}$, $1^{\mathbf{6}}$, $\frac{5}{3}^{\mathbf{9}}$, $3^{\mathbf{6}}$, $4^{\mathbf{2}}$, $\frac{13}{3}^{\mathbf{15}}$, $7^{\mathbf{1}}$, $8^{\mathbf{2}}$, $\infty^{\mathbf{50}}$
14	$0^{\mathbf{4}}$, $\frac{3}{2}^{\mathbf{2}}$	$0^{\mathbf{14}}$, $\frac{3}{2}^{\mathbf{10}}$, $2^{\mathbf{5}}$, $\frac{7}{2}^{\mathbf{2}}$, $4^{\mathbf{2}}$, $8^{\mathbf{1}}$, $\infty^{\mathbf{6}}$	$0^{\mathbf{5}}$, $\frac{3}{2}^{\mathbf{4}}$, $2^{\mathbf{4}}$, $\frac{7}{2}^{\mathbf{2}}$, $4^{\mathbf{1}}$, $8^{\mathbf{1}}$, $\infty^{\mathbf{23}}$	$0^{\mathbf{15}}$, $\frac{3}{2}^{\mathbf{12}}$, $2^{\mathbf{9}}$, $\frac{7}{2}^{\mathbf{4}}$, $4^{\mathbf{3}}$, $\frac{14}{3}^{\mathbf{18}}$, $8^{\mathbf{2}}$, $\infty^{\mathbf{57}}$
15	$0^{\mathbf{4}}$, $1^{\mathbf{1}}$, $2^{\mathbf{1}}$	$0^{\mathbf{15}}$, $1^{\mathbf{6}}$, $2^{\mathbf{5}}$, $\frac{7}{3}^{\mathbf{6}}$, $4^{\mathbf{5}}$, $8^{\mathbf{2}}$, $\infty^{\mathbf{6}}$	$0^{\mathbf{5}}$, $1^{\mathbf{2}}$, $2^{\mathbf{2}}$, $\frac{7}{3}^{\mathbf{6}}$, $4^{\mathbf{3}}$, $8^{\mathbf{2}}$, $\infty^{\mathbf{25}}$	$0^{\mathbf{16}}$, $1^{\mathbf{7}}$, $2^{\mathbf{6}}$, $\frac{7}{3}^{\mathbf{12}}$, $4^{\mathbf{8}}$, $5^{\mathbf{19}}$, $8^{\mathbf{4}}$, $\infty^{\mathbf{64}}$
16	$0^{\mathbf{5}}$, $2^{\mathbf{1}}$, $4^{\mathbf{1}}$	$0^{\mathbf{16}}$, $\frac{4}{3}^{\mathbf{9}}$, $2^{\mathbf{3}}$, $\frac{8}{3}^{\mathbf{6}}$, $4^{\mathbf{6}}$, $\frac{9}{2}^{\mathbf{2}}$, $8^{\mathbf{2}}$, $\infty^{\mathbf{7}}$	$0^{\mathbf{6}}$, $\frac{4}{3}^{\mathbf{3}}$, $2^{\mathbf{1}}$, $\frac{8}{3}^{\mathbf{6}}$, $4^{\mathbf{2}}$, $\frac{9}{2}^{\mathbf{2}}$, $8^{\mathbf{2}}$, $\infty^{\mathbf{29}}$	$0^{\mathbf{17}}$, $\frac{4}{3}^{\mathbf{12}}$, $2^{\mathbf{3}}$, $\frac{8}{3}^{\mathbf{12}}$, $4^{\mathbf{7}}$, $\frac{9}{2}^{\mathbf{4}}$, $\frac{16}{3}^{\mathbf{21}}$, $8^{\mathbf{4}}$, $\infty^{\mathbf{73}}$

Table 2: Slopes for $q = 2$, $i = 2$. Slopes of the form $\frac{k}{3}$ are marked in blue.

k	T_1-Slopes	$U_1^{\Gamma_0^P}$-Slopes	$U_1^{\Gamma_2^P}$-Slopes	$U_1^{\Gamma_0(t)}$-Slopes
3				$2^{\mathbf{1}}$
5				$\frac{10}{3}^{\mathbf{3}}$
7		$\infty^{\mathbf{1}}$	$2^{\mathbf{1}}$	$2^{\mathbf{1}}$, $\frac{14}{3}^{\mathbf{3}}$, $\infty^{\mathbf{2}}$
9		$\infty^{\mathbf{2}}$	$2^{\mathbf{1}}$, $6^{\mathbf{1}}$	$2^{\mathbf{1}}$, $6^{\mathbf{5}}$, $\infty^{\mathbf{4}}$
11		$6^{\mathbf{1}}$, $\infty^{\mathbf{2}}$	$4^{\mathbf{2}}$, $6^{\mathbf{1}}$	$4^{\mathbf{2}}$, $6^{\mathbf{2}}$, $\frac{22}{3}^{\mathbf{6}}$, $\infty^{\mathbf{5}}$
13		$7^{\mathbf{2}}$, $\infty^{\mathbf{2}}$	$2^{\mathbf{1}}$, $6^{\mathbf{1}}$, $7^{\mathbf{2}}$	$2^{\mathbf{1}}$, $6^{\mathbf{1}}$, $7^{\mathbf{4}}$, $\frac{26}{3}^{\mathbf{9}}$, $\infty^{\mathbf{6}}$
15		$8^{\mathbf{1}}$, $10^{\mathbf{1}}$, $\infty^{\mathbf{4}}$	$2^{\mathbf{1}}$, $4^{\mathbf{2}}$, $8^{\mathbf{1}}$, $10^{\mathbf{1}}$, $18^{\mathbf{1}}$	$2^{\mathbf{1}}$, $4^{\mathbf{2}}$, $8^{\mathbf{2}}$, $10^{\mathbf{12}}$, $18^{\mathbf{1}}$, $\infty^{\mathbf{10}}$

Table 3: Slopes for $q = 3$, $i = 1$. Slopes of the form $\frac{2k}{3}$ are marked in blue.

k	T_2-Slopes	$U_2^{\Gamma_0^P}$-Slopes	$U_2^{\Gamma_2^P}$-Slopes	$U_2^{\Gamma_0(t)}$-Slopes
3				$1^{\mathbf{1}}$
5				$\frac{5}{3}^{\mathbf{3}}$
7		$1^{\mathbf{1}}$	$1^{\mathbf{1}}$	$1^{\mathbf{2}}$, $\frac{7}{3}^{\mathbf{3}}$, $\infty^{\mathbf{1}}$
9		$1^{\mathbf{1}}$, $3^{\mathbf{1}}$	$1^{\mathbf{1}}$, $3^{\mathbf{1}}$	$1^{\mathbf{2}}$, $3^{\mathbf{6}}$, $\infty^{\mathbf{2}}$
11		$1^{\mathbf{1}}$, $2^{\mathbf{2}}$	$2^{\mathbf{2}}$, $\infty^{\mathbf{1}}$	$1^{\mathbf{1}}$, $2^{\mathbf{4}}$, $\frac{11}{3}^{\mathbf{6}}$, $\infty^{\mathbf{4}}$
13		$1^{\mathbf{3}}$, $3^{\mathbf{1}}$	$1^{\mathbf{1}}$, $3^{\mathbf{1}}$, $\infty^{\mathbf{2}}$	$1^{\mathbf{4}}$, $3^{\mathbf{2}}$, $\frac{13}{3}^{\mathbf{9}}$, $\infty^{\mathbf{6}}$
15		$1^{\mathbf{2}}$, $2^{\mathbf{2}}$, $5^{\mathbf{1}}$, $9^{\mathbf{1}}$	$1^{\mathbf{1}}$, $2^{\mathbf{2}}$, $9^{\mathbf{1}}$, $\infty^{\mathbf{2}}$	$1^{\mathbf{3}}$, $2^{\mathbf{4}}$, $5^{\mathbf{11}}$, $9^{\mathbf{2}}$, $\infty^{\mathbf{8}}$

Table 4: Slopes for $q = 3$, $i = 2$. Slopes of the form $\frac{k}{3}$ are marked in blue.

References

[BGK25] Gebhard Boeckle, Peter Mathias Graef, and Theresa Kaiser. *U-Operators Acting on Harmonic Cocycles for* GL_3 *and Their Slopes*. 2025. arXiv: 2503.00141 [math.NT]. URL: https://arxiv.org/abs/2503.00141.