

# Slopes of $U$ -operators

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This document contains further results of the computations explained in [BGK25]. Throughout, bold exponents denote multiplicities of the slopes. If for  $\Gamma \in \{\mathrm{GL}_3(A), \Gamma_0^P, \Gamma_2^P\}$  the space of  $\Gamma$ -invariant harmonic cocycles is zero, the corresponding field in the table is empty. If for some  $k$  the space of  $\Gamma_0(t)$ -invariant harmonic cocycles is zero, the smaller spaces are zero as well and the entire row is omitted.

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Table 1: Slopes for  $q = 2$ ,  $i = 1$ . Slopes of the form  $\frac{2k}{3}$  are marked in blue.

$k$	$T_1$ -Slopes	$U_1^{\Gamma_0^P}$ -Slopes	$U_1^{\Gamma_2^P}$ -Slopes	$U_1^{\Gamma_0(t)}$ -Slopes
0				$0^1$
1		$\infty^1$	$0^1$	$0^1, \infty^2$
2		$1^1, \infty^1$	$0^1, 1^1$	$0^1, 1^2, \infty^3$
3		$\frac{3}{2}^2, \infty^1$	$0^1, \frac{3}{2}^2$	$0^1, \frac{3}{2}^4, 2^1, \infty^4$
4	$1^1$	$1^1, 2^1, \infty^3$	$0^1, 1^2, 2^1, \infty^1$	$0^1, 1^2, 2^2, \frac{8}{3}^3, \infty^7$
5	$2^1$	$2^1, \frac{5}{2}^2, \infty^4$	$0^1, 2^3, \frac{5}{2}^2, \infty^1$	$0^1, 2^3, \frac{5}{2}^4, \frac{10}{3}^3, \infty^{10}$
6	$1^1$	$1^1, 3^3, \infty^5$	$0^1, 1^2, 3^5, \infty^1$	$0^1, 1^2, 3^8, 4^4, \infty^{13}$
7	$\frac{3}{2}^2$	$\frac{3}{2}^2, \frac{7}{2}^2, 4^1, \infty^7$	$0^1, \frac{3}{2}^4, 2^1, \frac{7}{2}^2, 4^2, \infty^2$	$0^1, \frac{3}{2}^4, 2^1, \frac{7}{2}^4, 4^3, \frac{14}{3}^6, \infty^{17}$
8	$1^1, 2^1$	$1^1, 2^1, 4^3, 5^1, \infty^9$	$0^1, 1^2, 2^2, \frac{8}{3}^3, 4^3, 5^1, 8^1, \infty^2$	$0^1, 1^2, 2^2, \frac{8}{3}^3, 4^6, 5^2, \frac{16}{3}^6, 8^1, \infty^{22}$
9	$2^1, 4^1$	$2^1, 4^1, \frac{9}{2}^4, 5^2, \infty^{10}$	$0^1, 2^3, \frac{10}{3}^3, 4^2, \frac{9}{2}^4, 5^2, 8^1, \infty^2$	$0^1, 2^3, \frac{10}{3}^3, 4^2, \frac{9}{2}^8, 5^4, 6^7, 8^1, \infty^{26}$
10	$1^1, 3^1, 4^1$	$1^1, 3^1, 4^1, 5^3, \frac{23}{4}^4, \infty^{12}$	$0^1, 1^2, 3^4, 4^3, 5^5, \frac{23}{4}^4, \infty^3$	$0^1, 1^2, 3^4, 4^3, 5^8, \frac{23}{4}^8, \frac{20}{3}^9, \infty^{31}$
11	$\frac{3}{2}^2, 4^2$	$\frac{3}{2}^2, 4^2, \frac{11}{2}^4, 6^1, \frac{13}{2}^2, \infty^{15}$	$0^1, \frac{3}{2}^4, 2^1, 4^5, \frac{11}{2}^8, 6^1, \frac{13}{2}^2, \infty^4$	$0^1, \frac{3}{2}^4, 2^1, 4^5, \frac{11}{2}^{12}, 6^2, \frac{13}{2}^4, \frac{22}{3}^{12}, \infty^{37}$
12	$1^1, 2^1, 5^2$	$1^1, 2^1, 5^2, 6^5, \frac{20}{3}^3, 9^1, \infty^{17}$	$0^1, 1^2, 2^2, \frac{8}{3}^3, 5^6, 6^7, \frac{20}{3}^3, 9^1, 16^1, \infty^4$	$0^1, 1^2, 2^2, \frac{8}{3}^3, 5^6, 6^{12}, \frac{20}{3}^6, 8^{13}, 9^2, 16^1, \infty^{43}$
13	$2^1, \frac{5}{2}^2, 6^2$	$2^1, \frac{5}{2}^2, 6^2, \frac{13}{2}^4, 7^2, \frac{22}{3}^3, 10^1, \infty^{20}$	$0^1, 2^3, \frac{5}{2}^4, \frac{10}{3}^3, 6^7, \frac{13}{2}^4, 7^2, \frac{22}{3}^3, 8^1, 10^1, 16^1, \infty^5$	$0^1, 2^3, \frac{5}{2}^4, \frac{10}{3}^3, 6^7, \frac{13}{2}^8, 7^4, \frac{22}{3}^6, 8^1, \frac{26}{3}^{15}, 10^2, 16^1, \infty^{50}$
14	$1^1, 3^3, 7^2$	$1^1, 3^3, 7^7, \frac{31}{4}^4, 8^1, 9^1, \infty^{23}$	$0^1, 1^2, 3^8, 4^4, 7^{11}, \frac{31}{4}^4, 8^2, 9^1, 16^1, \infty^6$	$0^1, 1^2, 3^8, 4^4, 7^{16}, \frac{31}{4}^8, 8^3, 9^2, \frac{28}{3}^{18}, 16^1, \infty^{57}$
15	$\frac{3}{2}^2, \frac{7}{2}^2, 4^1, 8^1$	$\frac{3}{2}^2, \frac{7}{2}^2, 4^1, \frac{15}{2}^6, 8^4, \frac{17}{2}^2, \frac{19}{2}^2, \infty^{26}$	$0^1, \frac{3}{2}^4, 2^1, \frac{7}{2}^4, 4^3, \frac{14}{3}^6, \frac{15}{2}^6, 8^8, \frac{17}{2}^2, \frac{19}{2}^2, 16^2, \infty^6$	$0^1, \frac{3}{2}^4, 2^1, \frac{7}{2}^4, 4^3, \frac{14}{3}^6, \frac{15}{2}^{12}, 8^{11}, \frac{17}{2}^4, \frac{19}{2}^4, 10^{19}, 16^2, \infty^{65}$
16	$1^1, 2^1, 4^3, 5^1, 9^1$	$1^1, 2^1, 4^3, 5^1, 8^5, \frac{26}{3}^6, 9^2, 10^3, \infty^{29}$	$0^1, 1^2, 2^2, \frac{8}{3}^3, 4^6, 5^2, \frac{16}{3}^6, 8^6, \frac{26}{3}^6, 9^5, 10^3, 16^2, \infty^7$	$0^1, 1^2, 2^2, \frac{8}{3}^3, 4^6, 5^2, \frac{16}{3}^6, 8^{11}, \frac{26}{3}^{12}, 9^6, 10^6, \frac{32}{3}^{21}, 16^2, \infty^{73}$

17	$2^1, 4^1, \frac{9^2}{2}, 5^2, 8^1, 10^1$	$2^1, 4^1, \frac{9^2}{2}, 5^2, 8^1, \frac{17^6}{2}, 9^2, \frac{28^6}{3}, 10^2, \frac{21^2}{2}, \infty^{32}$	$0^1, 2^3, \frac{10^3}{3}, 4^2, \frac{9^4}{2}, 5^4, 6^7, 8^3, \frac{17^6}{2}, 9^2, \frac{28^6}{3}, 10^4, \frac{21^2}{2}, 16^2, \infty^8$	$0^1, 2^3, \frac{10^3}{3}, 4^2, \frac{9^4}{2}, 5^4, 6^7, 8^3, \frac{17^{12}}{2}, 9^4, \frac{28^{12}}{3}, 10^5, \frac{21^4}{2}, \frac{34^{24}}{3}, 16^2, \infty^{81}$
18	$1^1, 3^1, 4^1, 5^1, \frac{23^4}{4}, 8^1$	$1^1, 3^1, 4^1, 5^1, \frac{23^4}{4}, 8^1, 9^7, \frac{39^4}{4}, 10^2, \frac{41^4}{4}, 11^1, \infty^{36}$	$0^1, 1^2, 3^4, 4^3, 5^4, \frac{23^8}{4}, \frac{20^6}{3}, 8^2, 9^9, \frac{39^4}{4}, 10^2, \frac{41^4}{4}, 11^3, 16^2, \infty^9$	$0^1, 1^2, 3^4, 4^3, 5^4, \frac{23^8}{4}, \frac{20^6}{3}, 8^2, 9^{16}, \frac{39^8}{4}, 10^4, \frac{41^8}{4}, 11^4, \frac{12^{28}}{3}, 16^2, \infty^{90}$
19	$\frac{3^2}{2}, 4^2, \frac{11^2}{2}, 6^1, \frac{13^2}{2}, 8^1$	$\frac{3^2}{2}, 4^2, \frac{11^2}{2}, 6^1, \frac{13^2}{2}, 8^1, \frac{19^6}{2}, 10^3, \frac{21^2}{2}, \frac{87^8}{8}, 12^1, \infty^{40}$	$0^1, \frac{3^4}{2}, 2^1, 4^5, \frac{11^8}{2}, 6^2, \frac{13^4}{2}, \frac{22^6}{3}, 8^2, \frac{19^{10}}{2}, 10^3, \frac{21^2}{2}, \frac{87^8}{8}, 12^2, 16^1, 18^1, \infty^{10}$	$0^1, \frac{3^4}{2}, 2^1, 4^5, \frac{11^8}{2}, 6^2, \frac{13^4}{2}, \frac{22^6}{3}, 8^2, \frac{19^{16}}{2}, 10^6, \frac{21^4}{2}, \frac{87^{16}}{8}, 12^3, \frac{38^{30}}{3}, 16^1, 18^1, \infty^{100}$
20	$1^1, 2^1, 5^2, 6^1, \frac{20^3}{3}, 8^1, 9^2$	$1^1, 2^1, 5^2, 6^1, \frac{20^3}{3}, 8^1, 9^2, 10^7, \frac{32^3}{3}, 11^2, \frac{45^4}{4}, \frac{23^4}{2}, 13^1, 17^1, \infty^{44}$	$0^1, 1^2, 2^2, \frac{8^3}{3}, 5^6, 6^4, \frac{20^6}{3}, 8^9, 9^6, 10^9, \frac{32^3}{3}, 11^2, \frac{45^4}{4}, \frac{23^4}{2}, 13^1, 16^1, 17^1, 20^1, 32^1, \infty^{11}$	$0^1, 1^2, 2^2, \frac{8^3}{3}, 5^6, 6^4, \frac{20^6}{3}, 8^9, 9^6, 10^{16}, \frac{32^6}{3}, 11^4, \frac{45^8}{4}, \frac{23^8}{2}, 13^2, \frac{40^{33}}{3}, 16^1, 17^2, 20^1, 32^1, \infty^{110}$
21	$2^1, \frac{5^2}{2}, 6^2, \frac{22^3}{3}, 8^1, 10^3$	$2^1, \frac{5^2}{2}, 6^2, \frac{22^3}{3}, 8^1, 10^3, \frac{21^8}{2}, 11^4, \frac{34^3}{3}, 12^6, \frac{25^2}{2}, 18^1, \infty^{48}$	$0^1, 2^3, \frac{5^4}{2}, \frac{10^3}{3}, 6^7, \frac{22^6}{3}, 8^3, \frac{26^6}{3}, 10^8, \frac{21^{12}}{2}, 11^4, \frac{34^3}{3}, 12^6, \frac{25^2}{2}, 16^1, 18^2, 32^1, \infty^{12}$	$0^1, 2^3, \frac{5^4}{2}, \frac{10^3}{3}, 6^7, \frac{22^6}{3}, 8^3, \frac{26^6}{3}, 10^8, \frac{21^{20}}{2}, 11^8, \frac{34^6}{3}, 12^{12}, \frac{25^4}{2}, \frac{14^{37}}{3}, 16^1, 18^3, 32^1, \infty^{120}$
22	$1^1, 3^3, 7^3, 8^2, 9^2, 11^2$	$1^1, 3^3, 7^3, 8^2, 9^2, 11^9, \frac{47^8}{4}, 12^4, \frac{51^4}{4}, 13^2, 17^1, \infty^{53}$	$0^1, 1^2, 3^8, 4^4, 7^8, 8^5, 9^6, \frac{28^6}{3}, 11^{17}, \frac{47^8}{4}, 12^4, \frac{51^4}{4}, 13^2, 17^1, 19^2, 32^1, \infty^{13}$	$0^1, 1^2, 3^8, 4^4, 7^8, 8^5, 9^6, \frac{28^6}{3}, 11^{24}, \frac{47^{16}}{4}, 12^8, \frac{51^8}{4}, 13^4, \frac{44^{39}}{3}, 17^2, 19^2, 32^1, \infty^{132}$
23	$\frac{3^2}{2}, \frac{7^2}{2}, 4^1, 8^3, \frac{19^4}{2}, 10^1, 12^1$	$\frac{3^2}{2}, \frac{7^2}{2}, 4^1, 8^3, \frac{19^4}{2}, 10^1, \frac{23^8}{2}, 12^4, \frac{25^4}{2}, \frac{38^6}{3}, \frac{27^6}{2}, \frac{35^2}{2}, \infty^{57}$	$0^1, \frac{3^4}{2}, 2^1, \frac{7^4}{2}, 4^3, \frac{14^6}{3}, 8^9, \frac{19^{12}}{2}, 10^6, \frac{23^{12}}{2}, 12^7, \frac{25^4}{2}, \frac{38^6}{3}, \frac{27^6}{2}, \frac{35^2}{2}, 20^1, 32^2, \infty^{14}$	$0^1, \frac{3^4}{2}, 2^1, \frac{7^4}{2}, 4^3, \frac{14^6}{3}, 8^9, \frac{19^{12}}{2}, 10^6, \frac{23^{20}}{2}, 12^{10}, \frac{25^8}{2}, \frac{38^{12}}{3}, \frac{27^{12}}{2}, \frac{46^{42}}{3}, \frac{35^4}{2}, 20^1, 32^2, \infty^{143}$
24	$1^1, 2^1, 4^3, 5^1, 9^3, 10^2, \frac{32^3}{3}, 16^1$	$1^1, 2^1, 4^3, 5^1, 9^3, 10^2, \frac{32^3}{3}, 12^9, \frac{38^6}{3}, 13^1, \frac{40^6}{3}, 14^4, \frac{57^4}{4}, 16^1, 17^1, 18^1, \infty^{61}$	$0^1, 1^2, 2^2, \frac{8^3}{3}, 4^6, 5^2, \frac{16^6}{3}, 8^1, 9^{10}, 10^6, \frac{32^9}{3}, 12^{15}, \frac{38^6}{3}, 13^3, \frac{40^6}{3}, 14^4, \frac{57^4}{4}, 16^2, 17^1, 18^1, 24^1, 32^2, \infty^{15}$	$0^1, 1^2, 2^2, \frac{8^3}{3}, 4^6, 5^2, \frac{16^6}{3}, 8^1, 9^{10}, 10^6, \frac{32^9}{3}, 12^{24}, \frac{38^{12}}{3}, 13^4, \frac{40^{12}}{3}, 14^8, \frac{57^8}{4}, \frac{16^{48}}{3}, 17^2, 18^2, 24^1, 32^2, \infty^{154}$

25	$2^1, 4^1, \frac{9^4}{2}, 5^2, 10^4, \frac{34^3}{3}, 12^1, 16^1$	$2^1, 4^1, \frac{9^4}{2}, 5^2, 10^4, \frac{34^3}{3}, 12^1, \frac{25^8}{2}, 13^4, \frac{40^6}{3}, 14^7, \frac{29^4}{2}, 15^2, 16^1, 18^1, 20^1, \infty^{67}$	$0^1, 2^3, \frac{10^3}{3}, 4^2, \frac{9^8}{2}, 5^4, 6^7, 8^1, 10^{11}, \frac{34^9}{3}, 12^4, \frac{25^{12}}{2}, 13^8, \frac{40^6}{3}, 14^7, \frac{29^4}{2}, 15^2, 16^3, 18^1, 20^1, 24^1, 32^2, \infty^{17}$	$0^1, 2^3, \frac{10^3}{3}, 4^2, \frac{9^8}{2}, 5^4, 6^7, 8^1, 10^{11}, \frac{34^9}{3}, 12^4, \frac{25^{20}}{2}, 13^{12}, \frac{40^{12}}{3}, 14^{14}, \frac{29^8}{2}, 15^4, 16^3, \frac{50^{51}}{3}, 18^2, 20^2, 24^1, 32^2, \infty^{167}$
26	$1^1, 3^1, 4^1, 5^3, \frac{23^4}{4}, 11^4, 12^2, 13^2$	$1^1, 3^1, 4^1, 5^3, \frac{23^4}{4}, 11^4, 12^2, 13^{11}, \frac{55^8}{4}, 14^2, \frac{57^4}{4}, \frac{44^6}{3}, \frac{63^4}{4}, 17^1, 19^1, 20^1, \infty^{72}$	$0^1, 1^2, 3^4, 4^3, 5^8, \frac{23^8}{4}, \frac{20^9}{3}, 11^{12}, 12^7, 13^{15}, \frac{55^{16}}{4}, 14^2, \frac{57^4}{4}, \frac{44^6}{3}, \frac{63^4}{4}, 16^1, 17^1, 19^1, 20^1, 32^3, \infty^{18}$	$0^1, 1^2, 3^4, 4^3, 5^8, \frac{23^8}{4}, \frac{20^9}{3}, 11^{12}, 12^7, 13^{24}, \frac{55^{24}}{4}, 14^4, \frac{57^8}{4}, \frac{44^{12}}{3}, \frac{63^8}{4}, 16^1, 17^2, \frac{52^{54}}{3}, 19^2, 20^2, 32^3, \infty^{180}$
27	$\frac{3^2}{2}, 4^2, \frac{11^4}{2}, 6^1, \frac{13^2}{2}, 12^4, \frac{27^4}{2}$	$\frac{3^2}{2}, 4^2, \frac{11^4}{2}, 6^1, \frac{13^2}{2}, 12^4, \frac{27^{14}}{2}, 14^5, \frac{29^4}{2}, \frac{119^8}{8}, \frac{46^6}{3}, \frac{33^2}{2}, \frac{35^2}{2}, 20^2, \infty^{77}$	$0^1, \frac{3^4}{2}, 2^1, 4^5, \frac{11^{12}}{2}, 6^2, \frac{13^4}{2}, \frac{22^{12}}{3}, 12^{13}, \frac{27^{22}}{2}, 14^7, \frac{29^8}{2}, \frac{119^8}{8}, \frac{46^6}{3}, 16^1, \frac{33^2}{2}, \frac{35^2}{2}, 20^2, 32^3, 34^1, \infty^{19}$	$0^1, \frac{3^4}{2}, 2^1, 4^5, \frac{11^{12}}{2}, 6^2, \frac{13^4}{2}, \frac{22^{12}}{3}, 12^{13}, \frac{27^{32}}{2}, 14^{12}, \frac{29^{12}}{2}, \frac{119^{16}}{8}, \frac{46^{12}}{3}, 16^1, \frac{33^4}{2}, \frac{35^4}{2}, \frac{18^{58}}{3}, 20^4, 32^3, 34^1, \infty^{193}$

Table 2: Slopes for  $q = 2, i = 2$ . Slopes of the form  $\frac{k}{3}$  are marked in blue.

$k$	$T_2$ -Slopes	$U_2^{\Gamma_0^P}$ -Slopes	$U_2^{\Gamma_2^P}$ -Slopes	$U_2^{\Gamma_0(t)}$ -Slopes
0				$0^1$
1		$0^1$	$0^1$	$0^2, \infty^1$
2		$0^2$	$0^1, \infty^1$	$0^3, \infty^3$
3		$0^3$	$0^1, \infty^2$	$0^4, 1^1, \infty^5$
4	$0^1$	$0^4, \infty^1$	$0^2, \infty^3$	$0^5, \frac{4}{3}^3, \infty^7$
5	$0^1$	$0^5, 1^1, \infty^1$	$0^2, 1^1, \infty^4$	$0^6, 1^2, \frac{5}{3}^3, \infty^{10}$
6	$0^1$	$0^6, \frac{3}{2}^2, \infty^1$	$0^2, \frac{3}{2}^2, \infty^5$	$0^7, \frac{3}{2}^4, 2^4, \infty^{13}$
7	$0^2$	$0^7, 1^2, 2^1, \infty^2$	$0^3, 1^1, 2^1, \infty^7$	$0^8, 1^3, 2^2, \frac{7}{3}^6, \infty^{17}$
8	$0^2$	$0^8, \frac{4}{3}^3, 2^1, 4^1, \infty^2$	$0^3, \frac{4}{3}^3, 4^1, \infty^8$	$0^9, \frac{4}{3}^6, 2^1, \frac{8}{3}^6, 4^2, \infty^{21}$
9	$0^2$	$0^9, 1^3, \frac{5}{3}^3, 4^1, \infty^2$	$0^3, 1^1, \frac{5}{3}^3, 4^1, \infty^{10}$	$0^{10}, 1^4, \frac{5}{3}^6, 3^7, 4^2, \infty^{26}$
10	$0^3$	$0^{10}, \frac{3}{2}^6, 2^1, \frac{5}{2}^2, \infty^3$	$0^4, \frac{3}{2}^2, 2^1, \frac{5}{2}^2, \infty^{13}$	$0^{11}, \frac{3}{2}^8, 2^2, \frac{5}{2}^4, \frac{10}{3}^9, \infty^{32}$
11	$0^3, 1^1$	$0^{11}, 1^4, 2^3, \frac{11}{4}^4, \infty^4$	$0^4, 1^2, 2^1, \frac{11}{4}^4, \infty^{15}$	$0^{12}, 1^5, 2^4, \frac{11}{4}^8, \frac{11}{3}^{12}, \infty^{37}$
12	$0^3, 2^1$	$0^{12}, \frac{4}{3}^6, 2^2, \frac{5}{2}^2, 3^2, 6^1, 8^1, \infty^4$	$0^4, \frac{4}{3}^3, 2^1, \frac{5}{2}^2, 3^2, 8^1, \infty^{17}$	$0^{13}, \frac{4}{3}^9, 2^2, \frac{5}{2}^4, 3^4, 4^{13}, 6^1, 8^2, \infty^{43}$
13	$0^4, 1^1$	$0^{13}, 1^5, \frac{5}{3}^6, 3^3, 4^1, 7^1, 8^1, \infty^5$	$0^5, 1^2, \frac{5}{3}^3, 3^3, 4^1, 8^1, \infty^{20}$	$0^{14}, 1^6, \frac{5}{3}^9, 3^6, 4^2, \frac{13}{3}^{15}, 7^1, 8^2, \infty^{50}$
14	$0^4, \frac{3}{2}^2$	$0^{14}, \frac{3}{2}^{10}, 2^5, \frac{7}{2}^2, 4^2, 8^1, \infty^6$	$0^5, \frac{3}{2}^4, 2^4, \frac{7}{2}^2, 4^1, 8^1, \infty^{23}$	$0^{15}, \frac{3}{2}^{12}, 2^9, \frac{7}{2}^4, 4^3, \frac{14}{3}^{18}, 8^2, \infty^{57}$
15	$0^4, 1^1, 2^1$	$0^{15}, 1^6, 2^5, \frac{7}{3}^6, 4^5, 8^2, \infty^6$	$0^5, 1^2, 2^2, \frac{7}{3}^6, 4^3, 8^2, \infty^{25}$	$0^{16}, 1^7, 2^6, \frac{7}{3}^{12}, 4^8, 5^{19}, 8^4, \infty^{64}$
16	$0^5, 2^1, 4^1$	$0^{16}, \frac{4}{3}^9, 2^3, \frac{8}{3}^6, 4^6, \frac{9}{2}^2, 8^2, \infty^7$	$0^6, \frac{4}{3}^3, 2^1, \frac{8}{3}^6, 4^2, \frac{9}{2}^2, 8^2, \infty^{29}$	$0^{17}, \frac{4}{3}^{12}, 2^3, \frac{8}{3}^{12}, 4^7, \frac{9}{2}^4, \frac{16}{3}^{21}, 8^4, \infty^{73}$
17	$0^5, 1^2, 4^1$	$0^{17}, 1^7, \frac{5}{3}^9, 3^8, 4^5, 5^1, 8^2, \infty^8$	$0^6, 1^3, \frac{5}{3}^3, 3^7, 4^2, 5^1, 8^2, \infty^{33}$	$0^{18}, 1^8, \frac{5}{3}^{12}, 3^{15}, 4^6, 5^2, \frac{17}{3}^{24}, 8^4, \infty^{82}$
18	$0^5, \frac{3}{2}^4$	$0^{18}, \frac{3}{2}^{14}, 2^3, \frac{5}{2}^6, \frac{10}{3}^6, 4^1, \frac{9}{2}^2, \frac{11}{2}^2, 8^2, \infty^9$	$0^6, \frac{3}{2}^6, 2^1, \frac{5}{2}^2, \frac{10}{3}^6, \frac{9}{2}^2, \frac{11}{2}^2, 8^2, \infty^{36}$	$0^{19}, \frac{3}{2}^{16}, 2^4, \frac{5}{2}^8, \frac{10}{3}^{12}, 4^1, \frac{9}{2}^4, \frac{11}{2}^4, 6^{28}, 8^4, \infty^{90}$
19	$0^6, 1^2, 2^2$	$0^{19}, 1^8, 2^7, \frac{11}{4}^{12}, \frac{11}{3}^6, \frac{19}{4}^4, 5^1, 6^1, 8^1, 9^1, \infty^{10}$	$0^7, 1^3, 2^3, \frac{11}{4}^4, \frac{11}{3}^6, \frac{19}{4}^4, 6^1, 8^1, 9^1, \infty^{40}$	$0^{20}, 1^9, 2^8, \frac{11}{4}^{16}, \frac{11}{3}^{12}, \frac{19}{4}^8, 5^1, 6^2, \frac{19}{3}^{30}, 8^2, 9^2, \infty^{100}$

20	$0^6, \frac{4}{3}^3, 2^1, 6^1$	$0^{20}, \frac{4}{3}^{12}, 2^4, \frac{5}{2}^6, 3^6, 4^7, \frac{9}{2}^2, 5^2, 6^3, 8^1, 10^1, 14^1, 16^1, \infty^{11}$	$0^7, \frac{4}{3}^6, 2^1, \frac{5}{2}^2, 3^2, 4^7, \frac{9}{2}^2, 5^2, 6^1, 8^1, 10^1, 16^1, \infty^{44}$	$0^{21}, \frac{4}{3}^{15}, 2^4, \frac{5}{2}^8, 3^8, 4^{14}, \frac{9}{2}^4, 5^4, 6^3, \frac{20}{3}^{33}, 8^2, 10^2, 14^1, 16^2, \infty^{110}$
21	$0^6, 1^2, \frac{5}{3}^3, 7^1$	$0^{21}, 1^9, \frac{5}{3}^{12}, 3^9, 4^3, \frac{13}{3}^6, 5^2, \frac{21}{4}^4, 7^2, 8^1, 9^1, 15^1, 16^1, \infty^{12}$	$0^7, 1^3, \frac{5}{3}^6, 3^3, 4^1, \frac{13}{3}^6, 5^2, \frac{21}{4}^4, 7^1, 8^1, 9^1, 16^1, \infty^{48}$	$0^{22}, 1^{10}, \frac{5}{3}^{15}, 3^{12}, 4^4, \frac{13}{3}^{12}, 5^4, \frac{21}{4}^8, \frac{7}{3}^{39}, 8^2, 9^2, 15^1, 16^2, \infty^{120}$
22	$0^7, \frac{3}{2}^4, 2^1, 4^1$	$0^{22}, \frac{3}{2}^{18}, 2^{10}, \frac{7}{2}^6, 4^5, \frac{9}{2}^2, \frac{14}{3}^6, \frac{11}{2}^6, \frac{19}{2}^2, 12^1, 16^1, \infty^{13}$	$0^8, \frac{3}{2}^6, 2^5, \frac{7}{2}^2, 4^2, \frac{9}{2}^2, \frac{14}{3}^6, \frac{11}{2}^6, \frac{19}{2}^2, 16^1, \infty^{52}$	$0^{23}, \frac{3}{2}^{20}, 2^{14}, \frac{7}{2}^8, 4^6, \frac{9}{2}^4, \frac{14}{3}^{12}, \frac{11}{2}^{12}, \frac{22}{3}^{39}, \frac{19}{2}^4, 12^1, 16^2, \infty^{131}$
23	$0^7, 1^3, 2^2, 4^2$	$0^{23}, 1^{10}, 2^9, \frac{7}{3}^{12}, 4^{13}, \frac{19}{4}^4, 5^4, \frac{23}{4}^4, 6^2, 10^1, 12^2, 16^2, \infty^{14}$	$0^8, 1^4, 2^3, \frac{7}{3}^6, 4^5, \frac{19}{4}^4, 5^4, \frac{23}{4}^4, 6^2, 10^1, 16^2, \infty^{57}$	$0^{24}, 1^{11}, 2^{10}, \frac{7}{3}^{18}, 4^{16}, \frac{19}{4}^8, 5^8, \frac{23}{4}^8, 6^4, \frac{23}{3}^{42}, 10^2, 12^2, 16^4, \infty^{143}$
24	$0^7, \frac{4}{3}^3, 2^2, 4^3$	$0^{24}, \frac{4}{3}^{15}, 2^5, \frac{8}{3}^{12}, 4^{11}, \frac{9}{2}^8, 5^2, \frac{16}{3}^3, 6^6, \frac{13}{2}^2, 10^1, 12^2, 16^2, \infty^{15}$	$0^8, \frac{4}{3}^6, 2^2, \frac{8}{3}^6, 4^4, \frac{9}{2}^4, 5^2, \frac{16}{3}^3, 6^6, \frac{13}{2}^2, 12^1, 16^2, \infty^{62}$	$0^{25}, \frac{4}{3}^{18}, 2^5, \frac{8}{3}^{18}, 4^{12}, \frac{9}{2}^{12}, 5^4, \frac{16}{3}^6, 6^{12}, \frac{13}{2}^4, \frac{8}{46}, 10^1, 12^3, 16^4, \infty^{155}$
25	$0^8, 1^3, \frac{5}{3}^3, 3^1, 4^2$	$0^{25}, 1^{11}, \frac{5}{3}^{15}, 3^{16}, 4^9, 5^5, \frac{17}{3}^3, 6^2, \frac{25}{4}^4, \frac{13}{2}^4, 8^1, 11^1, 12^1, 15^1, 16^2, \infty^{17}$	$0^9, 1^4, \frac{5}{3}^6, 3^8, 4^3, 5^3, \frac{17}{3}^3, 6^2, \frac{25}{4}^4, \frac{13}{2}^4, 8^1, 12^1, 16^2, \infty^{67}$	$0^{26}, 1^{12}, \frac{5}{3}^{18}, 3^{23}, 4^{10}, 5^8, \frac{17}{3}^6, 6^4, \frac{25}{4}^8, \frac{13}{2}^8, 8^2, \frac{25}{3}^{51}, 11^1, 12^2, 15^1, 16^4, \infty^{167}$
26	$0^8, \frac{3}{2}^6, 2^1, \frac{5}{2}^2, 4^1$	$0^{26}, \frac{3}{2}^{22}, 2^5, \frac{5}{2}^{10}, \frac{10}{3}^{15}, 4^2, \frac{11}{2}^8, 6^3, \frac{13}{2}^2, \frac{55}{8}^8, 8^2, 12^1, 14^1, 16^3, \infty^{18}$	$0^9, \frac{3}{2}^8, 2^2, \frac{5}{2}^4, \frac{10}{3}^9, 4^1, \frac{11}{2}^4, 6^3, \frac{13}{2}^2, \frac{55}{8}^8, 8^1, 16^3, \infty^{72}$	$0^{27}, \frac{3}{2}^{24}, 2^6, \frac{5}{2}^{12}, \frac{10}{3}^{24}, 4^2, \frac{11}{2}^{12}, 6^6, \frac{13}{2}^4, \frac{55}{8}^{16}, 8^3, \frac{26}{3}^{54}, 12^1, 14^1, 16^6, \infty^{180}$
27	$0^8, 1^3, 2^3, \frac{11}{4}^4, 5^1$	$0^{27}, 1^{12}, 2^{11}, \frac{11}{4}^{20}, \frac{11}{3}^{18}, 5^2, 6^7, \frac{27}{4}^4, 7^2, \frac{29}{4}^4, 8^3, 13^2, 16^3, 17^1, \infty^{19}$	$0^9, 1^4, 2^4, \frac{11}{4}^8, \frac{11}{3}^{12}, 5^1, 6^5, \frac{27}{4}^4, 7^2, \frac{29}{4}^4, 8^1, 16^3, 17^1, \infty^{77}$	$0^{28}, 1^{13}, 2^{12}, \frac{11}{4}^{24}, \frac{11}{3}^{30}, 5^2, 6^{12}, \frac{27}{4}^8, 7^4, \frac{29}{4}^8, 8^4, \frac{9}{58}, 13^2, 16^6, 17^2, \infty^{193}$

Table 3: Slopes for  $q = 3$ ,  $i = 1$ . Slopes of the form  $\frac{2k}{3}$  are marked in blue.

$k$	$T_1$ -Slopes	$U_1^{\Gamma_0^P}$ -Slopes	$U_1^{\Gamma_2^P}$ -Slopes	$U_1^{\Gamma_0(t)}$ -Slopes
3				$2^1$
5				$\frac{10^3}{3}$
7		$\infty^1$	$2^1$	$2^1, \frac{14^3}{3}, \infty^2$
9		$\infty^2$	$2^1, 6^1$	$2^1, 6^5, \infty^4$
11		$6^1, \infty^2$	$4^2, 6^1$	$4^2, 6^2, \frac{22^6}{3}, \infty^5$
13		$7^2, \infty^2$	$2^1, 6^1, 7^2$	$2^1, 6^1, 7^4, \frac{26^9}{3}, \infty^6$
15		$8^1, 10^1, \infty^4$	$2^1, 4^2, 8^1, 10^1, 18^1$	$2^1, 4^2, 8^2, 10^{12}, 18^1, \infty^{10}$
17		$9^2, 10^1, \infty^5$	$\frac{10^3}{3}, 6^1, 9^2, 10^1, 18^1$	$\frac{10^3}{3}, 6^1, 9^4, 10^2, \frac{34^{12}}{3}, 18^1, \infty^{13}$
19		$10^4, 14^1, \infty^5$	$2^1, \frac{14^3}{3}, 10^4, 14^1, 18^1$	$2^1, \frac{14^3}{3}, 10^8, \frac{38^{15}}{3}, 14^2, 18^1, \infty^{15}$
21		$11^2, 12^2, \frac{38^3}{3}, \infty^5$	$2^1, 4^2, 6^1, 11^2, 12^2, \frac{38^3}{3}, 18^1$	$2^1, 4^2, 6^1, 11^4, 12^4, \frac{38^6}{3}, 14^{19}, 18^1, \infty^{17}$
23	$6^1$	$6^1, 12^1, \frac{25^4}{2}, 18^1, \infty^8$	$\frac{10^3}{3}, 6^3, 12^1, \frac{25^4}{2}, 18^3, \infty^1$	$\frac{10^3}{3}, 6^3, 12^2, \frac{25^8}{2}, \frac{46^{24}}{3}, 18^4, \infty^{22}$
25	$8^1$	$8^1, 13^4, 14^3, \infty^{10}$	$2^1, \frac{14^3}{3}, 8^4, 13^4, 14^3, 18^2, \infty^1$	$2^1, \frac{14^3}{3}, 8^4, 13^8, 14^6, \frac{50^{27}}{3}, 18^2, \infty^{27}$
27	$10^1$	$10^1, 14^3, \frac{29^4}{2}, 15^2, \infty^{11}$	$2^1, 6^5, 10^3, 14^3, \frac{29^4}{2}, 15^2, 18^2, \infty^1$	$2^1, 6^5, 10^3, 14^6, \frac{29^8}{2}, 15^4, 18^{33}, \infty^{31}$

Table 4: Slopes for  $q = 3, i = 2$ . Slopes of the form  $\frac{k}{3}$  are marked in blue.

$k$	$T_2$ -Slopes	$U_2^{\Gamma_0^P}$ -Slopes	$U_2^{\Gamma_2^P}$ -Slopes	$U_2^{\Gamma_0(t)}$ -Slopes
3				$1^1$
5				$\frac{5}{3}^3$
7		$1^1$	$1^1$	$1^2, \frac{7}{3}^3, \infty^1$
9		$1^1, 3^1$	$1^1, 3^1$	$1^2, 3^6, \infty^2$
11		$1^1, 2^2$	$2^2, \infty^1$	$1^1, 2^4, \frac{11}{3}^6, \infty^4$
13		$1^3, 3^1$	$1^1, 3^1, \infty^2$	$1^4, 3^2, \frac{13}{3}^9, \infty^6$
15		$1^2, 2^2, 5^1, 9^1$	$1^1, 2^2, 9^1, \infty^2$	$1^3, 2^4, 5^{11}, 9^2, \infty^8$
17		$1^2, \frac{5}{3}^3, 3^2, 9^1$	$\frac{5}{3}^3, 3^1, 9^1, \infty^3$	$1^2, \frac{5}{3}^6, 3^3, \frac{17}{3}^{12}, 9^2, \infty^{11}$
19		$1^5, \frac{7}{3}^3, 5^1, 7^1$	$1^1, \frac{7}{3}^3, 9^1, \infty^5$	$1^6, \frac{7}{3}^6, \frac{19}{3}^{15}, 9^3, \infty^{15}$
21		$1^3, 2^2, 3^5, 5^2$	$1^1, 2^2, 3^1, 9^1, \infty^7$	$1^4, 2^4, 3^4, \frac{13}{3}^3, 7^{19}, 9^2, \infty^{19}$
23	$1^1$	$1^3, \frac{5}{3}^3, 2^4, 3^1, 5^2, 9^2$	$1^1, \frac{5}{3}^3, 3^1, 9^2, \infty^8$	$1^3, \frac{5}{3}^6, 2^4, 3^2, \frac{23}{3}^{24}, 9^4, 13^1, \infty^{22}$
25	$1^1$	$1^7, \frac{7}{3}^3, 3^3, 4^2, 7^1, 8^2$	$1^2, \frac{7}{3}^3, 4^2, 9^2, \infty^9$	$1^8, \frac{7}{3}^6, 3^3, 4^4, \frac{25}{3}^{27}, 9^4, \infty^{26}$
27	$3^1$	$1^4, 2^4, 3^7, 4^4, 5^1, 7^1$	$1^1, 3^6, 5^1, 9^2, \infty^{11}$	$1^5, 2^4, 3^{14}, 5^2, 9^{35}, \infty^{31}$

## References

- [BGK25] Gebhard Boeckle, Peter Mathias Graef, and Theresa Kaiser. *U-Operators Acting on Harmonic Cocycles for  $GL_3$  and Their Slopes*. 2025. arXiv: 2503.00141 [math.NT]. URL: <https://arxiv.org/abs/2503.00141>.