Consulting Project

# Compound events in Bavaria: Multivariate analysis of climatological and hydrological drivers of low-flow events


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## Abstract

As the world faces the dire reality of climate change, hydrological droughts have become a major concern, with devastating consequences for nature and humans. In Bavarian rivers, low-flow events, i.e. days where drainage falls below the NM7Q on three consecutive days, have become more frequent in recent years. Therefore, this research project aims to quantify the primary drivers of these events in terms of hydrological and meteorological variables.

In climatology, large ensemble climate projections of meteorological and hydrological variables are commonly used to understand the effects of climate change and to predict possible climate developments and their consequences. In this project, ten different realisations of climate simulations of the water balance simula- tion model (WaSiM) form the basis for the statistical analysis. In order to explain the occurrence of low-flows for the simulated data at hand, logistic regression is applied. The resulting models are the staring point to evaluate the effect sizes together with their significances. In a next step, the fitted models are used to predict low-flow scenarios under more extreme climate conditions. Subsequently, a K-means algorithm clusters the estimated coefficients to find regional patterns in combinations of drivers.

The analysis reveals large regional differences between the effects and significance of drivers such as precipitation, soil water, snow storage and temperature on the occurrence of low-flow in "hydrological Bavaria". In terms of significance, soil water and precipitation in particular provide striking results, while influencing factors such as temperature and snow storage have no significant effect in most regions. More extreme climate conditions lead to a partially severe increase of low-flow events in summer and a decrease in winter. The clustering of the estimated coefficients divides the area into four subregions with similar effects of drivers and their combinations leading to low-flow.

This project lays the foundation for the investigation of the effects of climate change on extreme events and implications for water management in Bavaria.

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# 1 Introduction

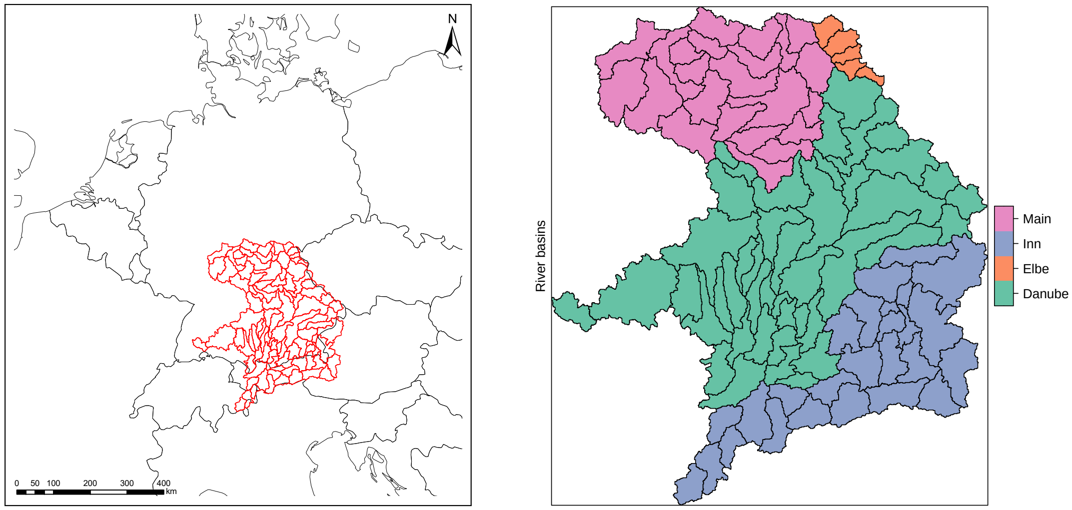
Drying rivers and hydrological droughts in Europe - in recent years these impacts of climate change have become more frequent and their consequences for people and nature are becoming increasingly devastating (Henley (2022); Horton (2022); Guardian (2023); BR24 (2022)). Water is becoming scarce, shipping is hampered and agriculture is unable to secure food supplies. Also in Bavaria, there is a worrying lack of water in rivers and other water bodies (Sebald (2023)). As a result, low-flow events, where drainage falls below a certain threshold for several consecutive days, are occurring with greater frequency and intensity.

Hydrological droughts leading to low-flow events are a type of drought that is characterized by a prolonged and unusual reduction in the availability of water in for instance rivers, lakes, and groundwater. But how can the occurrence of low-flow in Bavarian rivers be explained and which hydrological and meteorological drivers such as precipitation and temperature are causing low-flow? One possible explanatory approach could be that the emergence results from compound events, i.e. a combination of moderately pronounced drivers that amplify the hazard for an extreme event (Zscheischler et al. (2020)). This possibility is considered and analyzed in this research project which is embedded in the Climate Change and Hydrological Extremes (ClimEx) Project, established in 2015. The ClimEx scientists study the role of climate change in hydrological droughts and other extreme events in Bavaria and Quebec (Willkofer et al. (2020)). The project aims to provide recommendations for effective water management practices, taking into account the potential impacts of climate change on the region's water resources. For this purpose, ensembles of climatological and hydrological simulation models such as CanESM2, CRCM5 or WaSim are generated (*The ClimEx Project* (2015)).

The following analysis works with a part of the WaSim data set and aims to answer the following research questions:

1. How can the occurrence of low-flow events be explained?
2. Are the drivers of an extreme event themselves extreme? Or is it a compound event that leads to extreme low-flows?
3. Which drivers are relevant? Does their significance differ depending on the catchment?
4. What happens for more extreme climate conditions?
5. Is it possible to group catchments according to the driver's effects?

To achieve this objective, the first step involves establishing the data structure, followed by a descriptive analysis of the data. Since logistic regression is used to answer the main research questions, the theoretical concepts are presented before the results of the analysis. In the next step, a threshold analysis is carried out to find an appropriate threshold for the classification of low-flow. A scenario analysis gives insights into possible future outcomes by assessing the number of low-flow events for more extreme climate conditions. For the final research question, K-means clustering is employed to identify regional clusters of drivers. This paper concludes with a summary and discussion of possible analyses in the future.




(a) (b)

Figure 1: Map of hydrological Bavaria (a) and the corresponding river basins (b)

# 2 Data

The data set is composed of 10 hydrological simulations of the water balance simulation model (WaSiM), each of which is driven by a member of the single model initial condition large ensemble CRCM5-LE (Leduc et al. (2019)). The members are defined by their unique starting condition induced by perturbations that account for natural variability. The geographical area under consideration, hydrological Bavaria (see Figure 1a), comprises the Bavarian river basins of the upper Danube, the Inn, the Main and a few parts of the Elbe as visualized in Figure 1b. The area is divided into 98 catchments with virtual gauges. The information that is gathered in the gauges is then further averaged over the catchment for each variable.

The WaSiM data set consists of a time series of three-hourly data from 1990 to 2020 and provides information regarding several hydrological and meteorological drivers such as precipitation, temperature and snow storage (see Table 1). For this analysis, the data is aggregated into daily averages, except for precipitation which is transformed to daily sums. In total, the data set comprises of 11,088,700 data points, which makes 1,108,870 data points per member and 11,315 per catchment. Each data point pertains to a particular time, catchment and member.

The variable of interest low-flow is classified as one when drainage falls below a season-, catchment- and member-specific threshold of NM7Q for at least 3 days in a row (see Figure 2). The NM7Q is defined as the lowest 7-day mean of drainage averaged over 31 years and all members and acts as an indicator for dryness. The seasons refer to the hydrological half-year, with summer covering the months from May to October and winter the months from November to March.

| **Variable** | **Explanation** | **Unit** |
|---|---|---|
| Drainage | Daily mean of drainage | $m^3/s$ |
| Low-flow | Dummy variable of low-flow event | 0 = no event or 1 = event |
| Precipitation | Daily sum of precipitation | mm |
| Temperature | Daily mean of Temperature | °C |
| Soil water | Daily mean of Soil water | % |
| Snow storage | Daily mean of Snow storage | mm |
| Relative humidity | Daily mean of Humidity | % |
| Radiation | Daily mean of Radiation | $Wh/m^2$ |
| Latitude | Geographical latitude | decimal degree |
| Longitude | Geographical longitude | decimal degree |
| DGM | Ground elevation | m above sea level |
| Land use | Largest land use class in catchment area | wetlands, water bodies, grassland, forest, semi-natural areas, artificial surfaces, agricultural areas |
| Slope | Terrain slope | degree |

Table 1: Explanation of variables in the data set with corresponding unit.

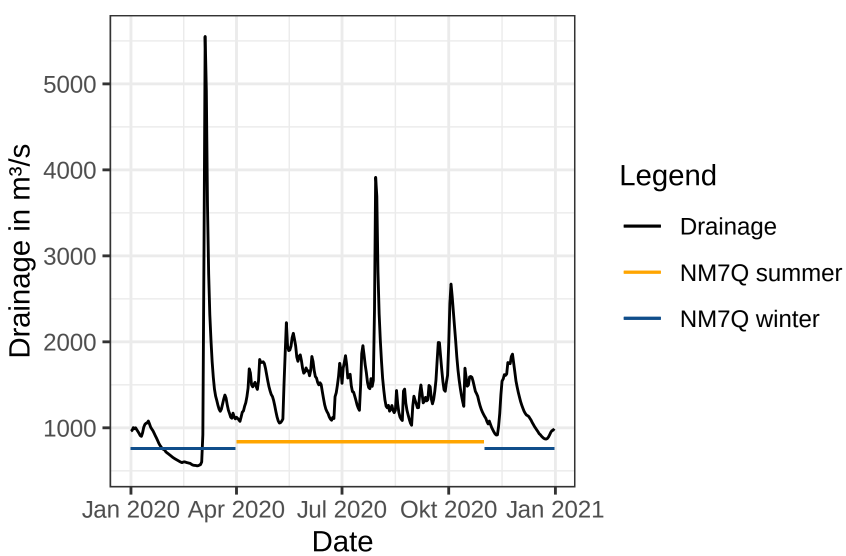



Figure 2: Time series plot of drainage in 2020 plotted against threshold of NM7Q in summer (yellow line) and winter (blue line) for member kbt and catchment "Donau-Achleiten". Drainage that falls below a line is classified as low-flow. In this example, only a few days in the beginning of the year are classified as low-flow events.

| Variable | N | Mean | SD | Min | Q1 | Median (Q2) | Q3 | Max |
|---|---|---|---|---|---|---|---|---|
| Hydrological half-year: summer | | | | | | | | |
| Drainage | 558992 | 105.92 | 236.316 | 0.211 | 7.885 | 27.321 | 93.229 | 6458.737 |
| Nr. low-flow days | 558992 | 3.23 | 13.567 | 0 | 0 | 0 | 0 | 306 |
| Low-flow intensity | 558992 | 54.545 | 148.627 | -248.973 | 1.401 | 9.409 | 42.224 | 5621.135 |
| Precipitation | 558992 | 3.456 | 5.955 | 0 | 0.002 | 0.813 | 4.47 | 146.299 |
| Temperature | 558992 | 12.723 | 5.306 | -10.152 | 9.159 | 12.99 | 16.509 | 29.634 |
| Radiation | 558992 | 528.739 | 244.148 | 31.637 | 331.063 | 515.648 | 711.033 | 1653.193 |
| Humidity | 558992 | 0.777 | 0.113 | 0.234 | 0.707 | 0.795 | 0.861 | 1 |
| Snow storage | 558992 | 8.037 | 33.864 | 0 | 0 | 0 | 0.391 | 677.832 |
| Soil water | 558992 | 0.337 | 0.059 | 0.07 | 0.305 | 0.346 | 0.38 | 0.477 |
| Hydrological half-year: winter | | | | | | | | |
| Drainage | 549878 | 88.882 | 198.03 | 0.221 | 9.776 | 23.803 | 73.93 | 5551.362 |
| Nr. low-flow days | 549878 | 2.606 | 15.485 | 0 | 0 | 0 | 0 | 291 |
| Low-flow intensity | 549878 | 42.819 | 119 | -223.542 | 2.302 | 8.822 | 31.144 | 4790.992 |
| Precipitation | 549878 | 2.421 | 4.67 | 0 | 0 | 0.42 | 2.882 | 120.33 |
| Temperature | 549878 | 1.021 | 5.579 | -25.801 | -2.575 | 1.109 | 4.789 | 19.874 |
| Radiation | 549878 | 245.486 | 197.537 | 11.995 | 97.203 | 178.034 | 335.13 | 1395.675 |
| Humidity | 549878 | 0.814 | 0.11 | 0.133 | 0.755 | 0.832 | 0.895 | 1 |
| Snow storage | 549878 | 44.278 | 78.309 | 0 | 0.001 | 4.246 | 52.953 | 643.303 |
| Soil water | 549878 | 0.36 | 0.044 | 0.124 | 0.334 | 0.365 | 0.391 | 0.507 |

Table 2: Descriptive statistics for each variable and each hydrological half-year comprising of the mean, standard deviation, minimum, first quartile (Q1), median, third quartile (Q3) and the maximum.

# 3 Descriptive Analysis

Having established the structure of the data in the previous chapter, the aim now is to give an intuition of the data at hand through a descriptive analysis. In order to illustrate the trends and patterns of the variables, the analysis will focus on the randomly selected member "kbt" and in some instances on the catchment Donau-Achleiten (see Figure 3a).

Table 2 summarizes all descriptive statistics such as the mean and quartiles for each hydrological half-year in one table. This illustrates how the variables vary depending on the season. In the following, these statistics are evaluated in detail for each variable, starting with low-flow.

## 3.1 Time-Varying Variables

The data set consists of several time-varying variables, including drainage and thus the binary indicator low-flow yes/no as well as drivers, all of which are examined in detail in this section.

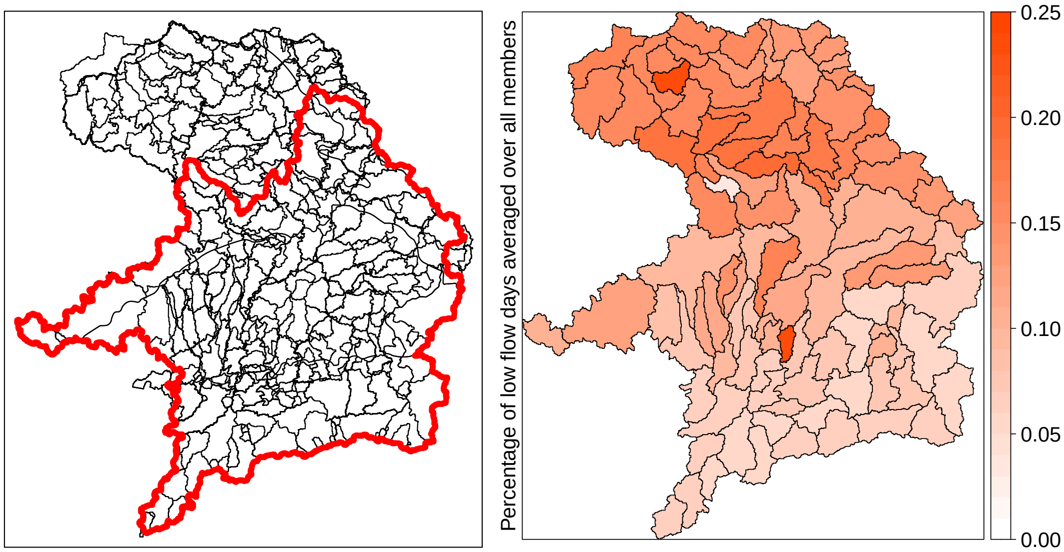



Figure 3: Map of catchment "Donau Achleiten" (a) and percentage of days of low-flow for each catchment averaged over all members (b).

### 3.1.1 Low-Flow Events

Looking at hydrological Bavaria as a whole, a day is classified as low-flow event on about 11 % of all days. This result is similar for each member and thus for the entire data set. When considering the percentage of days of low-flow averaged over members for each catchment, there are noticeable regional differences. Overall, the percentage ranges from close to 0 % to 25 %, with the proportion being higher in the north than in the south of hydrological Bavaria (see Figure 3b). In addition to these spatial differences, also a trend over time and a seasonal trend can be observed. Figure 4 shows that over the years, the annual number of low-flow events increases remarkably. The same applies to the number of catchments in which these events occur. Taking a closer look at 2020 reveals seasonal differences for the number of low-flow events as depicted in Figure 5. In the summer months there are fewer catchments where low-flow events occur, but the catchments affected by low-flow have a higher number of low-flow days than in the winter months.

These patterns are also observable in Table 2. When looking at the summer and winter drainage over whole hydrological Bavaria, the differences in mean and median are small, but nevertheless less drainage is detectable in winter than in summer. This is reflected in the number of low-flow events: in winter there is a weak tendency towards more low-flow than in summer. However, looking at the intensity of the low-flow, i.e. the difference between drainage and NM7Q, the comparison of the minimum value shows that the events tend to be stronger in summer than in winter. That is, the drainage is more distant from the threshold than in winter.

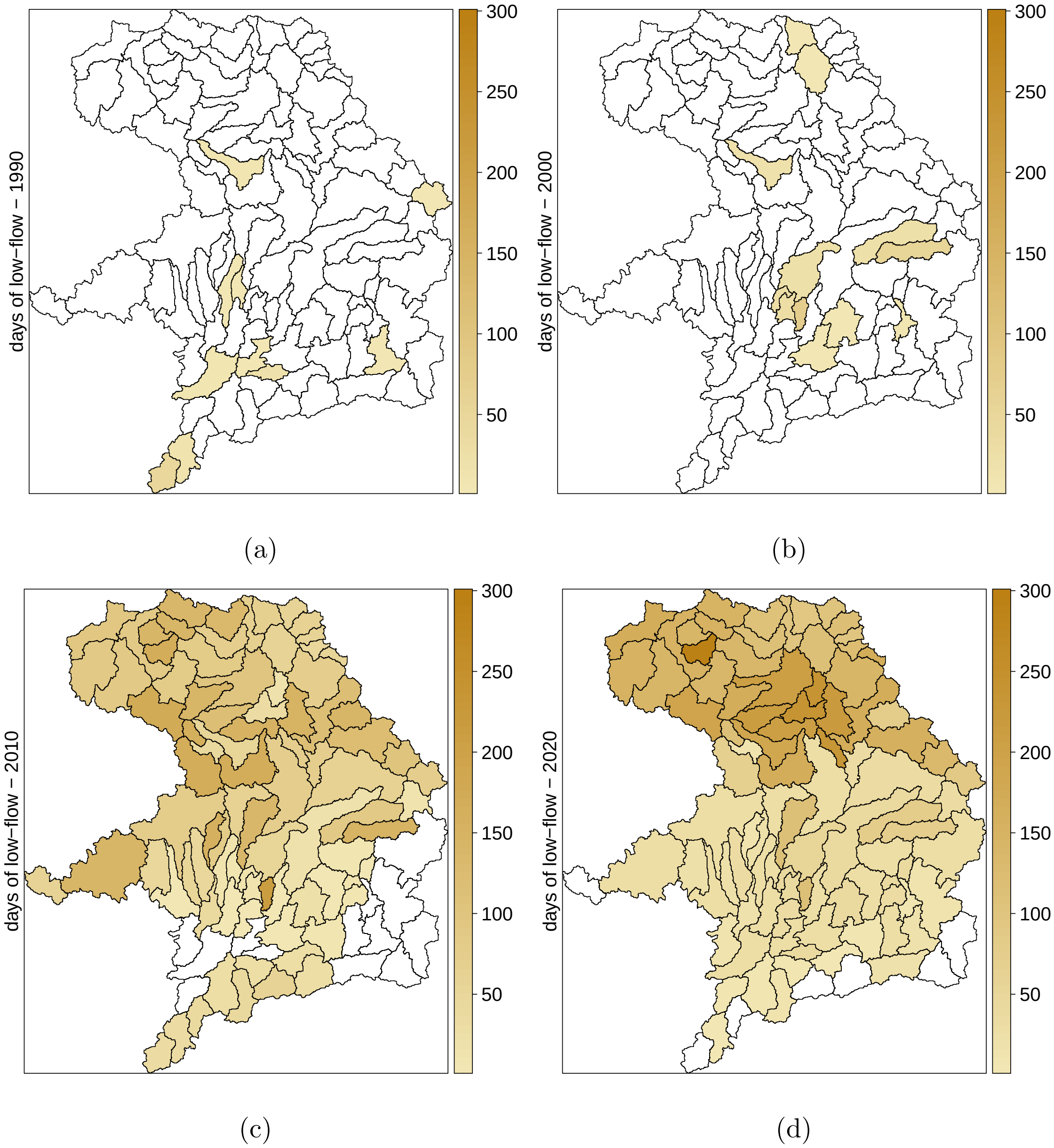
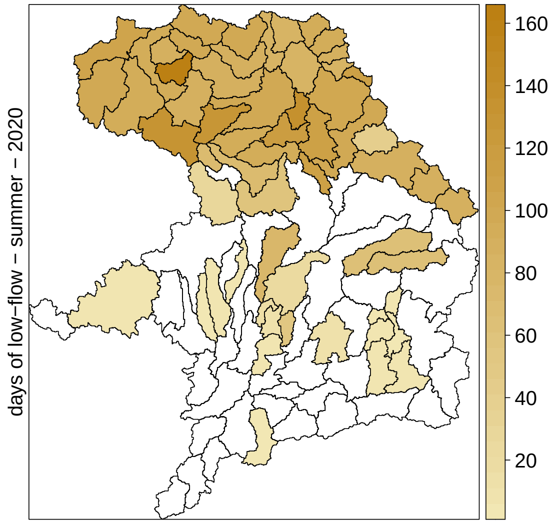
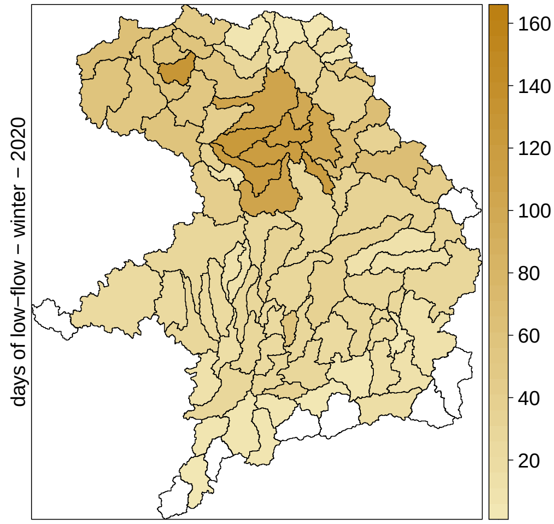



Figure 4: Exemplary number of low-flow events in 1990, 2000, 2010 and 2020 for member "kbt" for each catchment. Catchments visualized in white are not subject to any low-flow event.




(a)




(b)

Figure 5: Exemplary number of low-flow events in summer (a) and winter (b) in 2020 for member "kbt" for each catchment. Catchments visualized in white are not subject to any low-flow event.

### 3.1.2 Drivers

The aim of this project is to explain the occurrence of low-flows in terms of climatological and hydrological variables, so called drivers. To get an intuition of how these variables are distributed across the different catchments, each driver is analysed for a randomly chosen year, 2020.

According to National Geographic Education (2021), **precipitation** means any liquid or frozen water formed in the atmosphere which returns to the earth. It comes in a variety of forms, such as rain, sleet and snow. Precipitation is one of the three main parts of the global water cycle, along with evaporation and condensation. Figure 6a shows the average precipitation in millimetres for each catchment in 2020, with more precipitation in the southern catchments than in northern ones. Table 2 also reveals some seasonal differences. In terms of mean, median and maximum values, there tends to be more precipitation in the summer months than in the winter months. It should be noted that precipitation, especially in winter, may fall as snow and is therefore stored.

Figure 6b shows the average **temperature** measured in °C in the year 2020. Alpine regions tend to be colder than the rest of hydrological Bavaria. Table 2 reveals a clear seasonal pattern with increased temperatures in summer.

According to Western, Grayson, and Bloschl (1999), near-surface relative **soil water** is a measure of the moisture content of the soil in the upper layers. It usually refers to the percentage of the volume of soil that is filled with water relative to the total volume of soil

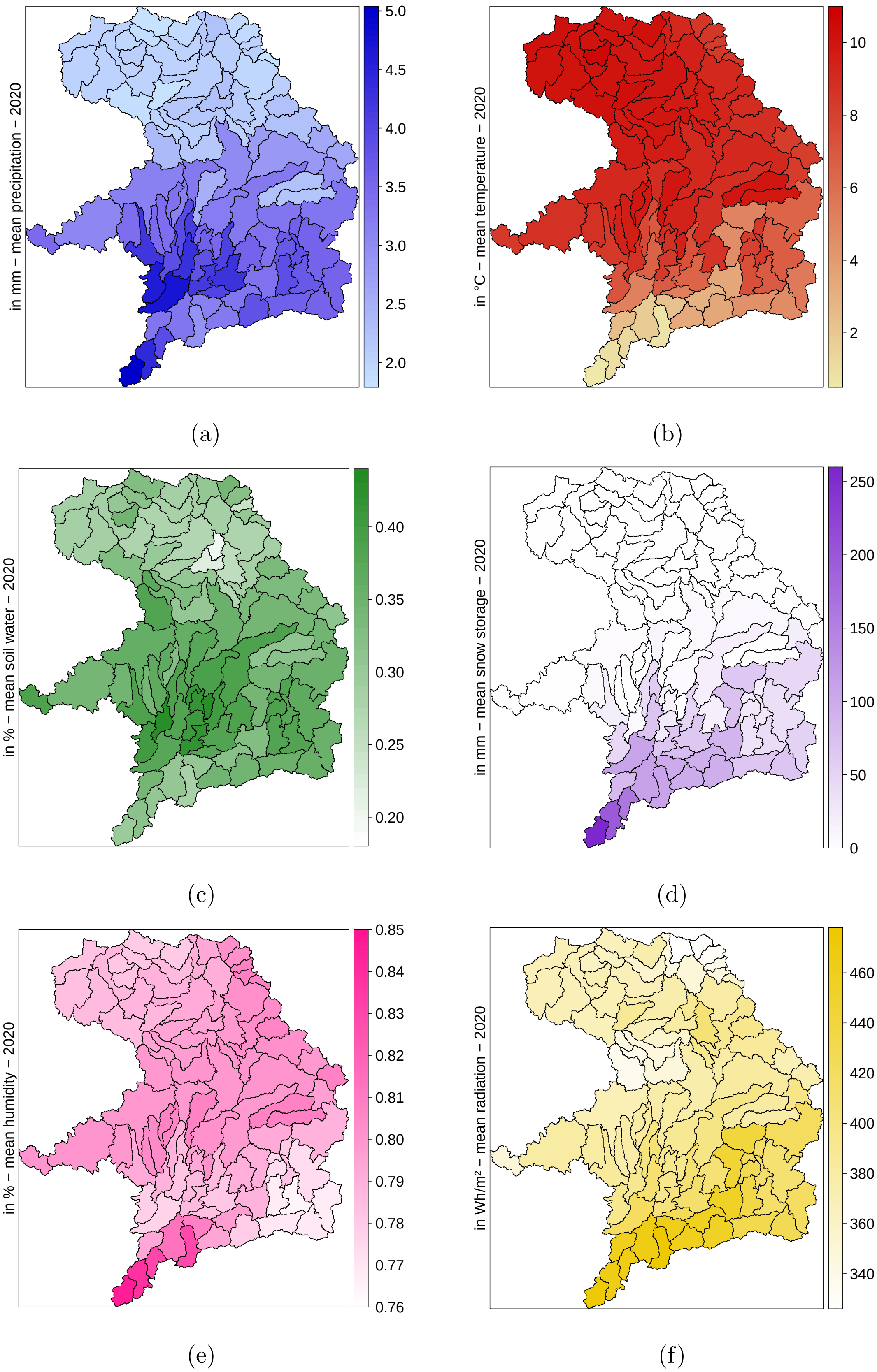



Figure 6: Spatial distribution of each driver in 2020. Figure (a) shows average precipitation, (b) temperature, (c) depicts soil water, (d) snow storage, (e) humidity and (f) radiation.

in a given area. It is important for monitoring soil dryness and drought conditions that can affect agriculture and ecosystems. In hydrological Bavaria there are moderately pronounced regional differences for soil water, as shown in Figure 6c. Here, the average percentage of soil water in 2020 is displayed, with a higher percentage in the south. Table 2 suggests only a small seasonal variation in terms of mean and median, and therefore soil water is considered to be a storage variable indicating dryness.

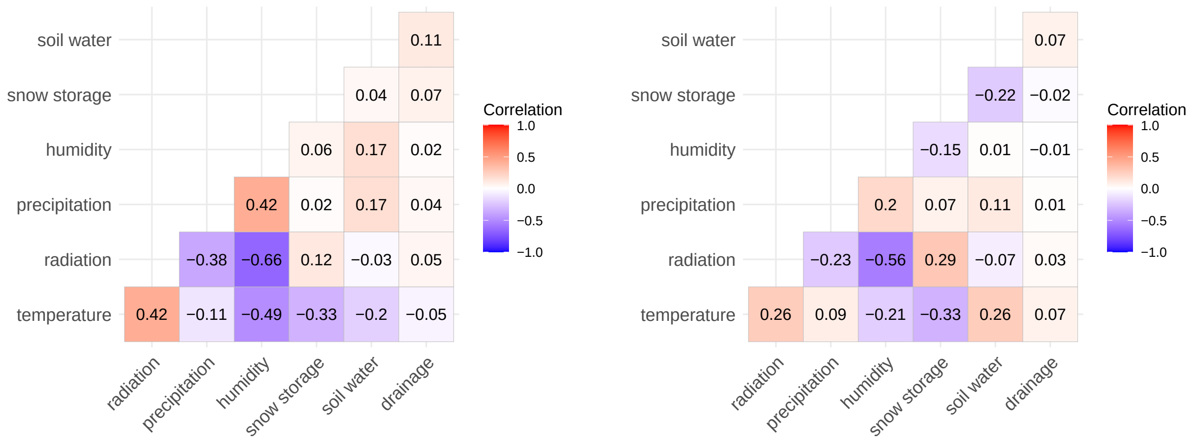
**Snow storage** refers to the seasonal accumulation and storage of snow and is an important component of the hydrological cycle, where it acts as a natural water reservoir. The snow mass can store large quantities of water, which is slowly released as the snow melts (see Marty et al. (2017)). Due to regional conditions such as ground elevation and mountainousness, snow storage is highly region-dependent in hydrological Bavaria as shown in Figure 6d. This figure shows the average snow storage in 2020 in millimetres, with snow storage almost only being located in the Alps and Pre-Alps. Looking at Table 2, there are clear seasonal differences in terms of mean, median and maximum, which can be explained by northern catchments with negligible snow storage during the summer months.

**Relative humidity** is a measure of the amount of moisture present in the air relative to the maximum amount of moisture the air can hold at a specific temperature, as explained by the American Meteorological Society (2021). Figure 6e depicts the average humidity in 2020. There is no clear spatial pattern, but the south tends to be slightly more humid than the north. The evaluation of the differences between summer and winter in Table 2 shows only minor differences in terms of mean and median, indicating that the relative humidity is slightly higher in winter than in summer.

**Solar radiation** is the radiation emitted by the sun due to various physical effects. Depending on the wavelength, radiation is absorbed by the atmosphere to a greater or lesser extent. The intensity reaching the earth's surface also depends strongly on the weather and the position of the sun (see Iberdrola (2022)). In hydrological Bavaria, Figure 6f reveals regional differences in average radiation in 2020, since it increases from north to south. In addition, the intensity of radiation varies strongly between seasons, as shown in Table 2. In summer, the mean radiation is more than twice as high as in winter, and the median is almost three times as high.

### 3.1.3 Relationship Between Variables

In addition to the descriptive analysis of each covariate, examining the relationship between drainage and the drivers is also of interest. Correlation matrices in Figure 7 reveal that the correlations between drainage and the drivers are close to 0 in both summer and winter, implying no linear relationship. However, the correlations between humidity and radiation and between humidity and temperature are quite negative, while there is a strong correlation between temperature and radiation. It is important to note that these matrices only show the relationships between variables on the same day. In the case of drainage, the influence of the covariate may be time-lagged, i.e. the values may be important some days before the day under consideration. When comparing the time series plot of drainage alongside pre-




(a) (b)

Figure 7: Correlation matrices for summer (a) and winter (b). The right column indicates the correlation between drainage and the drivers.

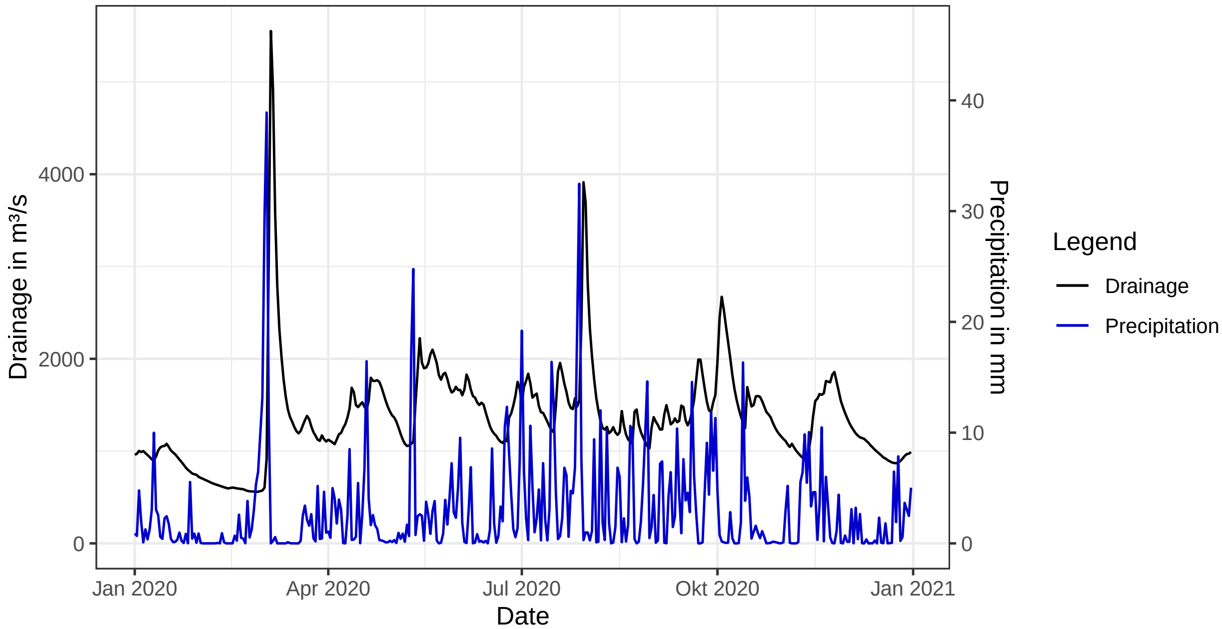
cipitation in Figure 8a in catchment "Donau-Achleiten" in 2020, a time-shifted relationship can be noticed. Shortly after an increase in precipitation, a corresponding surge in drainage is observed. However, no such direct relationship can be established for temperature, as shown in Figure 8b. In Appendix A, Figure 35 and Figure 36, drainage is plotted against all remaining drivers.

Overall, this time lag structure should be taken into account in the variable specification when it comes to modelling in section 5.

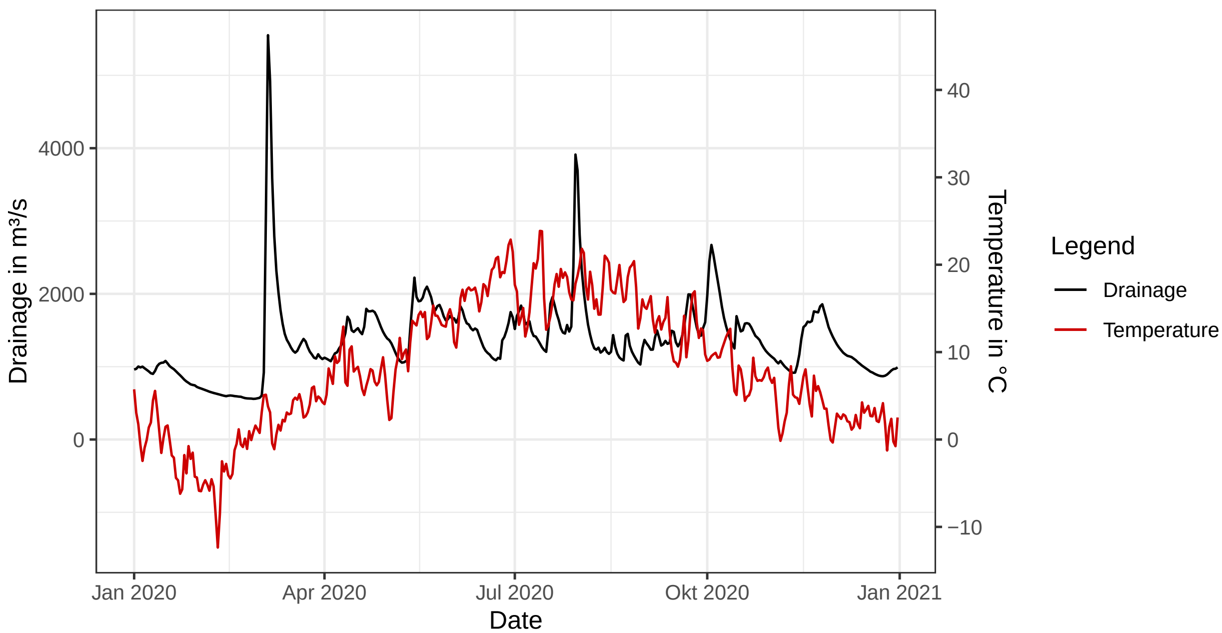
#### 3.1.4 Differences Between Members

Natural variability is accounted for by perturbations of the initial conditions in the WaSim data set of the 10 randomly selected members, as elaborated in section 2. This yields different realizations of the time-varying variables between the members. Figure 9 shows exemplary box plots for the total annual sum of days of low-flow (a), average precipitation (b) and average temperature (c) for 1990, 2000, 2010 and 2020 in catchment "Donau-Achleiten" (see Figure 3a) over all 10 members. It appears that the annual number of low-flow events varies between members, with some notable outliers in 1990, 2000 and 2020. This also applies to precipitation, where the range of values varies widely over the years. These differences are less pronounced for temperature, as no outliers can be detected. Overall, these box plots indicate partly striking differences in the time-varying variables between the members. This result is also valid for the other catchment areas.

Note that in addition to the member-specific differences, the box plots also give an intuition of how the respective variables develop over time. While there is no clear pattern for the sum of low-flow days and precipitation, Figure 9c shows an increasing trend in average temperature over the years.

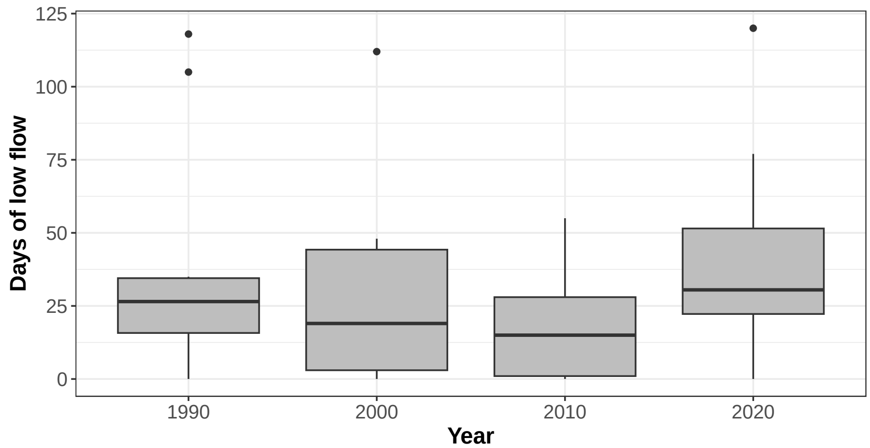



(a)

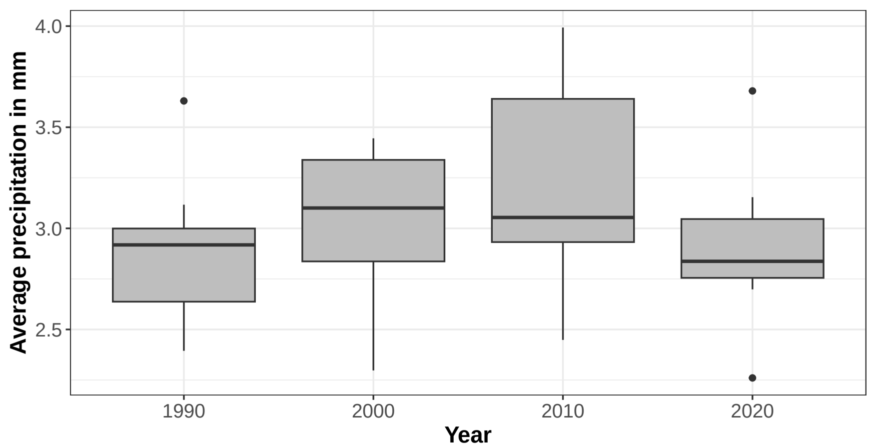



(b)

Figure 8: Time series of drainage and precipitation (a) and drainage and temperature (b) in 2020 for catchment "Donau-Achleiten" (see Figure 3a).




(a)

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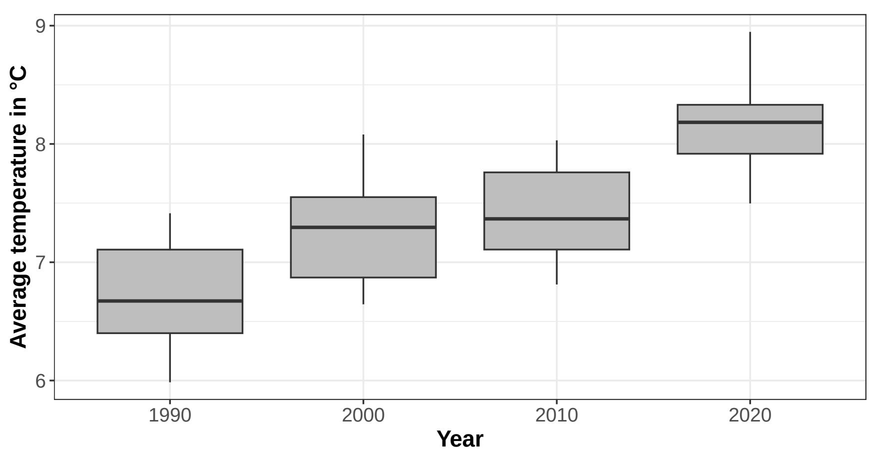
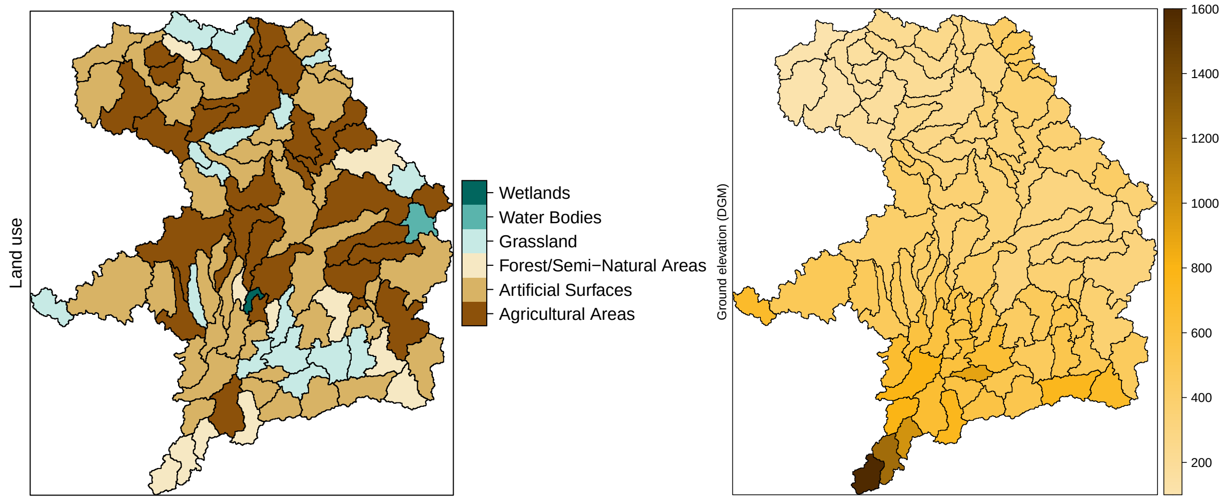



(c)

Figure 9: Box plots of the total sum of annual low-flow days (a), average precipitation (b) and average temperature (c) for 1990, 2000, 2010 and 2020 in catchment "Donau-Achleiten" (see Figure 3a) of all 10 members.




(a) (b)

Figure 10: Land use (a) and ground elevation (b) of each catchment in hydrological Bavaria

## 3.2 Time-Constant Variables

Along with the target and driver variables, the data set provides information about geographic properties of the catchments. Figure 10a shows the predominant land use in each catchment in hydrological Bavaria. It is determined by the Copernicus Land Cover (CLC), a European program that provides detailed information on the land use across the European Union. The CLC uses satellite imagery and ground surveys, in this project's case of the year 2006, to produce land use maps at a resolution of 100 metres. The maps are based on a classification system called Coordination of Information on the Environment (CORINE) land cover classification system (*Copernicus Land Monitoring Service* (2014)). Almost 40 % of Bavaria's hydrological area consists of artificial surfaces, 30 % is used for agriculture, especially in the north and centre. In addition to 16 % of grasslands, forest and semi-natural areas predominate in the south and in some other isolated catchments (11 %). Only two catchments are used as water bodies or wetlands.

The regions in hydrological Bavaria differ widely in terms of topology. This becomes apparent looking at the ground elevation above sea level in Figure 10b. From north to south, the altitude increases and reaches a peak in the Alps.

A brief look ahead at section 5: although the analysis here shows the heterogeneity of geographical characteristics of the catchments, the chosen modelling approach does not allow for time-constant variables.

# 4 Methods

Recall that the aim of this research project is to explain the occurrence of low-flow events by hydrological and meteorological drivers and their interactions. Given that the variable of interest - whether or not a low-flow event occurs - is binary, logistic regression is appropriate for the data at hand.

In this section, the necessary theory for the methods used in the analysis of low-flow events is presented. This includes some background information on logistic regression and its evaluation using goodness of fit measures, as well as an overview of $K$-means clustering.

All the following theory is based on Fahrmeir et al. (2013), Held and Bové (2013) and Wu (2012).

## 4.1 Logistic Regression

In general, regression aims to model the expected value of the response variable in the presence of covariates. For continuous outcomes *linear regression models* as

$$Y_i = \beta_0 + \beta_1 x_{i1} + ... + \beta_k x_{ik} + \epsilon_i, \quad i = 1, ..., n, \tag{1}$$

for $n$ response variables, $k$ covariates and independently and identically distributed (i.i.d.) errors $\epsilon_i \sim \mathcal{N}(0, \sigma^2)$ represent a good starting point for analyses. However, for binary response variables, this model is no longer appropriate since the expected value of a binary variable $Y_i$ is given by

$$\mathbb{E}(Y_i) = \mathbb{P}(Y_i = 0) \cdot 0 + \mathbb{P}(Y_i = 1) \cdot 1 = \mathbb{P}(Y_i = 1) = \mathbb{P}(Y_i = 1 | x_{i1}, ..., x_{ik}) = \pi_i. \tag{2}$$

The classical linear regression model in Equation 1 faces several problems since in contrary to $Y_i$, the right-hand side is not binary and the error variance can no longer be homoscedastic. The latter follows from the distributional assumption: The response variables $Y_i$ are now independently binomial distributed with success probability $\pi_i$ and therefore the variance is $\text{Var}(Y_i) = \pi_i(1 - \pi_i)$. Moreover, $\pi_i$ in a linear model could take on values less than 0 or greater than 1, which would contravene Kolmogorov's axioms of probability theory.

To circumvent these issues and to get the output of the model in the desired range between 0 and 1, the *logit model* uses the logistic distribution function as *response function*:

$$E(Y_i | x_{i1}, ..., x_{ik}) = \pi_i = \frac{exp(\eta_i)}{1 + exp(\eta_i)} \tag{3}$$

with *linear predictor*

$$\eta_i = \beta_{i0} + x_{i1}\beta_{i1} + ... + x_{ik}\beta_{ik}. \tag{4}$$

This means in particular that the linear predictor is estimated as a linear combination of effects and covariates, and is then mapped to a probability between 0 and 1 by the logistic distribution function shown in Figure 11. Note that the output is not a binary expected

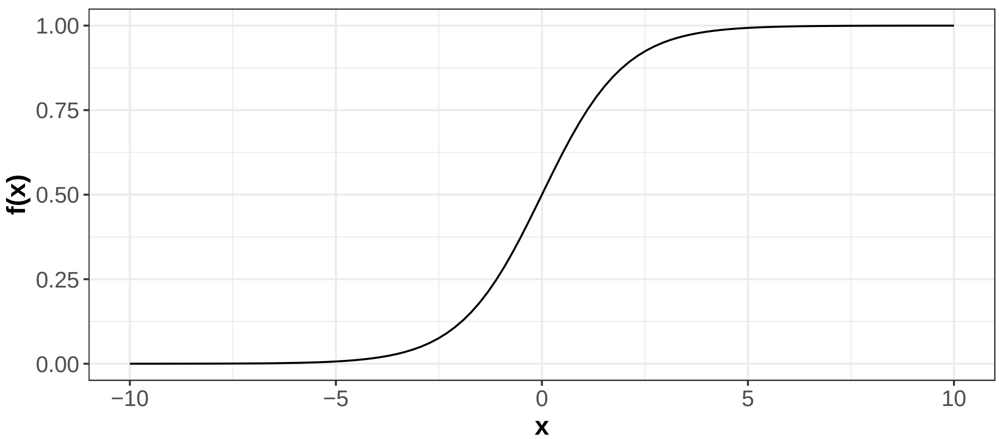



Figure 11: Plot of the logistic distribution function $f(x) = \frac{exp(x)}{1+exp(x)}$. Every $x \in \mathbb{R}$ is mapped to $f(x) \in (0, 1)$ that can be interpreted as probabilites.

value but the estimated probability that an event occurs for the specific sample.

Logit models are interpreted in terms of the *odds*, i.e. the proportion of an event's chance of happening to its chance of not happening:

$$\frac{P(Y_i = 1)}{1 - P(Y_i = 1)} = exp(\beta_{i0}) \cdot exp(x_{i1}\beta_{i1}) \cdot ... \cdot exp(x_{ik}\beta_{ik}) \tag{5}$$

Hence, this interpretation is based on a multiplicative model for the odds. For an interpretation as a linear model, the *log odds* represent a logarithmic transformation of the odds:

$$\log\left(\frac{P(Y_i = 1)}{1 - P(Y_i = 1)}\right) = \beta_{i0} + x_{i1}\beta_{i1} + ... + x_{ik}\beta_{ik} \tag{6}$$

The interpretation of the estimated coefficients is now straightforward:

- $\beta_i > 0 \Rightarrow exp(\beta_i) > 1$: the odds of observing the event increase with increasing covariate $x_i$
- $\beta_i < 0 \Rightarrow exp(\beta_i) < 1$: the odds of observing the event decrease with increasing covariate $x_i$
- $\beta_i = 0 \Rightarrow exp(\beta_i) = 1$: the odds of observing the event don't change with increasing covariate $x_i$.

Estimated coefficients are often evaluated by their *significance*. A significant effect in logistic regressions means that there is evidence to assume that a covariate has a meaningful impact on the likelihood of the response variable. The significance of an effect is typically determined using a hypothesis test, such as a Wald test, a likelihood ratio test, or by calculating the z-value, i.e. the estimated coefficient divided by its estimated standard error. A p-value less than the chosen significance level $\alpha$ (usually $\alpha = 0.05$) indicates that the effect of the covariate is statistically significant, meaning that the probability of observing the estimated effect size due to chance alone is very low. On the contrary, a p-value larger than $\alpha$ suggests that the effect is not statistically significant and may be caused by random variation in the data.

For more details to logistic regression and its interpretation see Fahrmeir et al. (2013).

## 4.2 Goodness of Fit Criteria

For model choice and evaluation of the goodness of fit of logistic regression models, several criteria exist. In the following, the three main measures used in this project are presented.

### 4.2.1 Akaike Information Criterion

For model selection in the context of regression analysis, a popular evaluation technique is the *Akaike information criterion (AIC)*. It measures the relative quality of a statistical model in terms of goodness of fit, reflected as log-likelihood function $l$, and the complexity of the model, i.e. the number of parameters $k$ to be estimated:

$$\begin{aligned} AIC &= -2l + 2k \\ &= -2\sum_{i=1}^{n}(y_i \ln(\pi_i) + (1-y_i)\ln(1-\pi_i)) + 2k, \end{aligned} \tag{7}$$

where in the second step the log-likelihood function of the logistic regression is plugged in. The goal is to select the model that provides the best balance between complexity and goodness of fit. Models with lower AIC are generally preferred over models with higher AIC, as they are considered to be more sparse while still fitting the data well. Sparsity ensures that the model is less likely to overfit the data and is easier to interpret. Note that the AIC criterion is only able to compare different models, as it does not allow any conclusions regarding the goodness of fit of a single model.

### 4.2.2 Deviance Explained

The *deviance* is another measurement to compare the observed data and the values predicted by the model. It is calculated as the difference between the log-likelihood of the fitted model and the log-likelihood of a reference model, typically a null model. The null model has no predictors and represents the overall distribution of the response variable.

The *deviance explained* is calculated as the difference between the deviance of the null model and the deviance of the fitted model, expressed as the proportion of the deviance of the null model between 0 and 1. It represents an indicator of the percentage of the total variation in the data that is accounted for by the covariates. A higher deviance explained indicates a better fit of the model to the data, while a lower deviance explained suggests that the model is not capturing all the important features of the data.

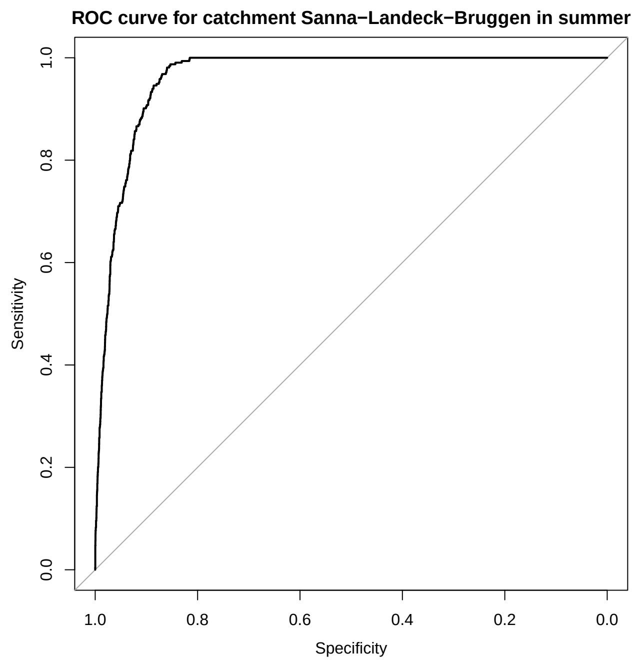



Figure 12: Example of a ROC curve with an AUC of 0.97 for a randomly chosen model.

Note that in order to assess whether the value of the deviance explained is at an acceptable level, expert knowledge of the subject area concerned is necessary.

### 4.2.3 Area Under the Curve

The *area under the curve (AUC)* is a commonly used evaluation measure for assessing the performance of binary classification models such as logistic regressions. It represents to which extent a model is able to distinguish between positive and negative outcomes.

In binary classification, the *true positive rate (TPR)*, the *false positive rate (FPR)* and the *true discovery rate (TDR)* are important evaluation measures:

- **TPR:** Proportion of correctly predicted outcomes as positive when the actual outcome is positive.
- **FPR:** Proportion of correctly predicted outcomes as negative when the actual outcome is negative.
- **TDR:** Proportion of correctly predicted outcomes as positive out of all predicted outcomes as positive.

TPR is also referred to as *sensitivity* and 1-FPR is called *specificity*. Both can be plotted against each other by varying the classification threshold, i.e. the threshold above which an event is classified as such, resulting in a curve known as the *receiver operating characteristic (ROC) curve*. One example of a ROC curve is portrayed in Figure 12.

The AUC is the area under this ROC curve, ranging from 0 to 1, with a higher value indicating a better discriminatory ability of the model. A model that randomly classifies events has an AUC of 0.5, while a perfect classification model has an AUC of 1.0. In the example model in Figure 12, a AUC of 0.97 indicates a very good detection of (low-flow) events.

The AUC is a useful metric for the comparison of performance differences between logit models or selecting the best threshold for a given model (see subsection 6.2).

## 4.3 K-Means Clustering

*K-Means clustering* is an unsupervised machine learning technique that is employed for detecting patterns within a data set. The algorithm aims to find homogeneous subgroups such that points are assigned to a cluster if they have similar features. Similarity is assessed based on distance measures or correlation based measures between the data points and a centroid, i.e. the cluster mean of data points within a cluster. This analysis uses the euclidean distance as the similarity measure, which is the squared distance between a data point and the clusteroid. The number of clusters $K$ is defined prior to the algorithm by use of the *elbow method*, where the within-cluster sum of squares is plotted against a row of number of clusters $K = (1, ..., 10)$. The presence of a bend in the plot indicates the optimal number of clusters, suggesting a notable drop in the rate of change of the within-cluster sum of squares beyond that point. In Figure 13 an example of an elbow plot is portrayed. Since a bend is observable between $K = 3$ and $K = 4$, the optimal number of clusters would be 4.

Algorithm 1 shows the pseudo code of the clustering. The procedure consists of two steps: the assignment to clusters and an updating step. First, the algorithm picks $K$ centroids randomly. The distances between the data points and these cluster centroids are calculated to assign the data points to the closest centroid. Second, the mean of the data points belonging to each cluster is calculated which serves as updated centroid. This process is repeated until convergence, i.e. the assignment does not change after an iteration. By then, the data points are assigned to the clusters with highest similarity (Wu (2012)).

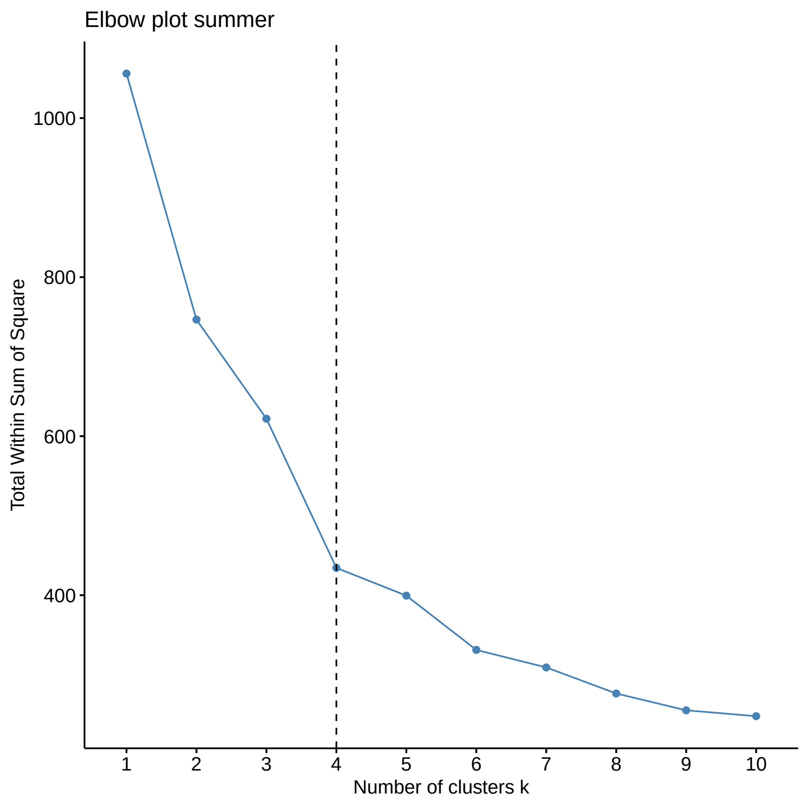



Figure 13: Example of an elbow plot.

**Algorithm 1:** K-Means Clustering

**Data:** Data set $x_1, x_2, ..., x_n$, Number of clusters $K$
**Result:** Cluster labels for each data point
Initialize centroids $c_1, c_2, ..., c_K$ randomly;
**while** *not converged* **do**
  **for** $i = 1$ *to* $N$ **do**
    Find the nearest centroid $c_j$ to $x_i$:

$$j = \arg\min_k ||x_i - c_k||_2^2$$

    Assign $x_i$ to cluster $j$;
  **end**
  **for** $j = 1$ *to* $K$ **do**
    Update clusteroids:

$$c_j = \frac{1}{n_j} \sum_{i=1}^{n_j} x_i$$

  **end**
**end**

# 5 Modelling Approach

Now that an initial intention for the data has been given in section 2 and section 3, and the required theory has been explained in section 4, this part captures the modelling approach in detail. First, some challenges associated with the data set and their elaborated solutions are presented, before the final model is specified in a step-by-step derivation in the second part.

## 5.1 Challenges

In order to explain the occurrence of low-flow events, logistic regression is a suitable modelling approach. However, looking at the data at hand, several challenges arise that must be taken into account when specifying the model:

**Time lag:** As displayed exemplary in Figure 8a in the descriptive analysis, driver variables influence drainage with a time delay of several days or months. To account for this lag structure, each variable is included as a rolling mean, i.e. an unweighted mean of previous time points. Based on statistical analyses and expert knowledge, the following time periods are chosen:

- 7-day rolling averages for temperature, precipitation, radiation and relative humidity
- 30-day rolling average for snow storage
- 60-day rolling average for soil water

The idea behind this is to classify snow storage and soil water as storage variables that reflect long-term trends, while the remaining drivers capture short-term variations in the weather.

**Seasonality:** The descriptive analyses lead to the conclusion that the effects of some drivers may differ between the hydrological half-years (see Table 2). Therefore, separate models are fitted for summer and winter. In addition, Figure 4 shows a trend over time that is accounted for by including year as a covariate in each model.

**Catchment-specific effects:** One of the main research objectives is to investigate differences between catchments in terms of effect sizes. Therefore, a logit model is fitted to each catchment separately. This allows the estimated effect of each driver to be compared between catchments.

**Compound events:** In order to adequately model compound events, i.e. the combination of moderately pronounced drivers resulting in extreme impacts, interactions are included in each model. The intention behind this is based on the convenient statistical interpretation of interactions: For instance, if the covariates temperature and precipitation are included as drivers, then the interaction between the two represents the change in the effect of precipitation as temperature varies. Based on expert knowledge interactions between temperature and precipitation, temperature and soil water as well as temperature and snow storage are included in the model.

**Snow storage in summer:** In the summer half-year, only a negligible amount of snow storage is observed in some of the catchments. Consequently, including it as a covariate in the model is only reasonable if snow storage is greater than a certain threshold. In the following, snow storage is only considered if the maximum snow storage in the entire time series is at least 1 cm for each member.

**10 different members:** The data consists of 10 different simulated data sets, i.e. members, which consequently leads to fitting a separate logit model for each member. This results in 10 different estimated coefficients for the same catchment in summer and winter. To make the effects comparable, i.e. member-independent, and to counteract outliers from the estimated effects, the coefficients are averaged over the 10 members.

In conclusion, a logistic model with drivers transformed to rolling averages is fitted to each catchment and to each member for summer and winter separately. In addition, year and several interactions are included as covariates. Note that one single catchment, "Altmühl-Aha", is not subject to any low-flow events in 9 out of 10 members in summer and consequently, no summer model is estimated. This is due to the definition of low-flow: although in 9 out of 10 members the drainage falls below NM7Q on individual days, these undershoots do not last for at least three consecutive days. Overall, this results in $2 \cdot 98 \cdot 10 - 1 = 1959$ fitted models in total. In the second step, the 10 member-specific estimated coefficients are averaged to finally obtain one estimated effect for each catchment in summer and winter.

## 5.2 Model Specification

In this section, the actual model specification is explained and evaluated based on one example model.

A general task in modelling is to assess whether a covariate is included as a linear or non-linear effect, i.e. if the effect of the considered covariate is assumed to be constant across all values of the covariate, or if it changes depending on the covariate's value. Regarding the drivers that can be possibly included in the logit models for predicting low-flow events, one can observe that only the effect of temperature is depending on its values (i.e. the effects differ between heat and cold). Since seasonality and therefore different levels of the temperature are already accounted for by the winter/summer split, the effect for temperature is once again close to linear. Consequently, no non-linear trends are included in the models. Furthermore, to ensure interpretability of the interactions, each driver is centred before inclusion in the models (i.e. subtraction of the mean), and the total number of interactions is restricted to three.

The combination of statistical considerations and expert domain knowledge lead to the fol-

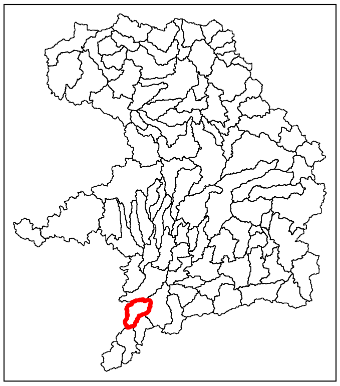


Figure 14: Catchment "Sanna-Landeck-Bruggen"

| **Model** | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 |
|---|---|---|---|---|---|---|---|---|---|---|
| Year | x | x | x | x | x | x | x | x | x | x |
| Precipitation | | x | x | x | x | x | x | x | x | x |
| Temperature | | | x | x | x | x | x | x | x | x |
| Soil water | | | | x | x | x | x | x | x | x |
| Snow storage | | | | | x | x | x | x | x | x |
| Humidity | | | | | | x | x | x | x | x |
| Radiation | | | | | | | x | x | x | x |
| Temperature:Precipitation | | | | | | | | x | x | x |
| Temperature:Snow storage | | | | | | | | | x | x |
| Temperature:Soil water | | | | | | | | | | x |

Table 3: Covariate configuration for a step-by-step derivation of the final model (model 10). Starting point is model 1 with covariate year only.

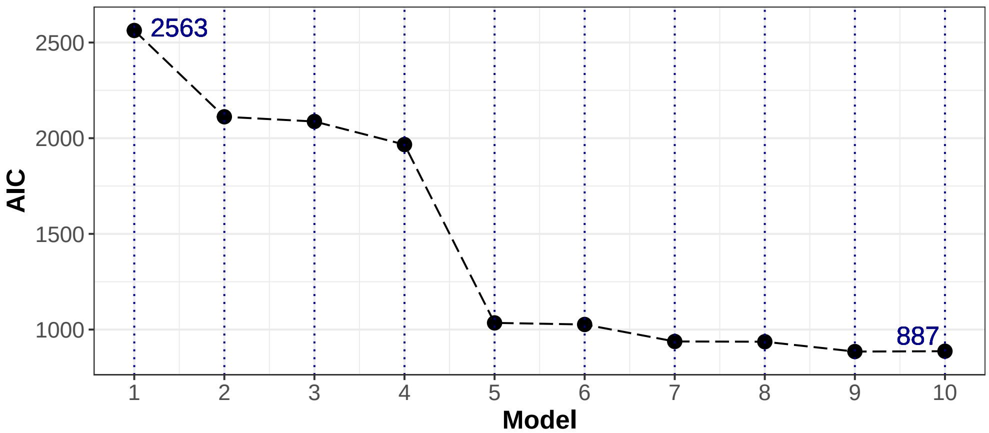
lowing final model equation:

$$\begin{aligned}\log\left(\frac{P(Y=1)}{1-P(Y=1)}\right) = \beta_0 &+ \beta_1\texttt{year} + \beta_2\texttt{precipitation} + \beta_3\texttt{temperature}\\ &+ \beta_4\texttt{soil water} + \beta_5\texttt{snow storage} + \beta_6\texttt{humidity} + \beta_7\texttt{radiation}\\ &+ \beta_8\texttt{temperature:precipitation}\\ &+ \beta_9\texttt{temperature:soil water}\\ &+ \beta_{10}\texttt{temperature:snow storage}\end{aligned} \tag{8}$$

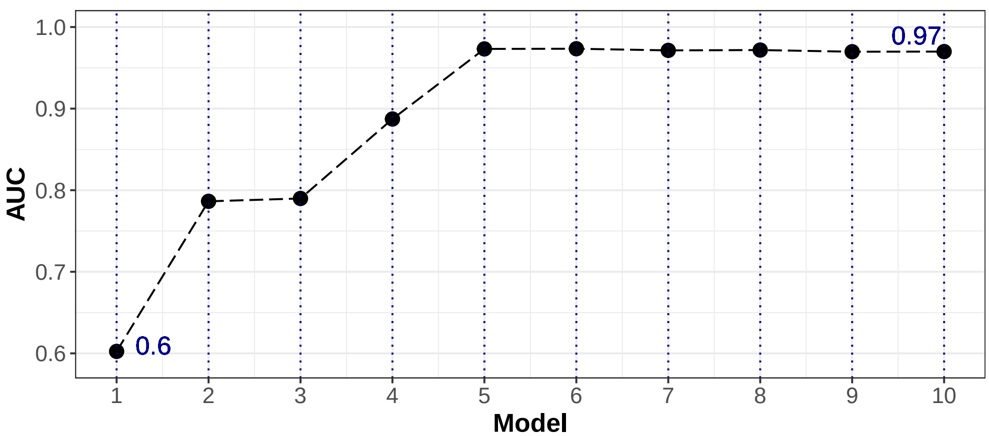
where $Y$ is representing the response variable of the logistic regression, i.e. a binary indicator of a low-flow event, and a colon between covariates stands for an interaction term. Note that the covariate snow storage and the corresponding interaction with temperature is only part of the equation if the inclusion criterion presented in subsection 5.1 is met.

To get an impression of the goodness of fit, the summer model for catchment "Sanna-Landeck-Bruggen" (see Figure 14) is built exemplary step by step. This means that a simple model with only one covariate (year) serves as a starting point (model 1) and the number of covariates is gradually increased by one at a time. Since the final model consists of 10 covariates in total, 10 models are iteratively fitted and evaluated by the goodness of fit criteria presented in subsection 4.2. The procedure is portrayed in Table 3. Model 1 starts with covariate year only, model 2 includes covariates year and precipitation etc. until the final model 10 is reached.

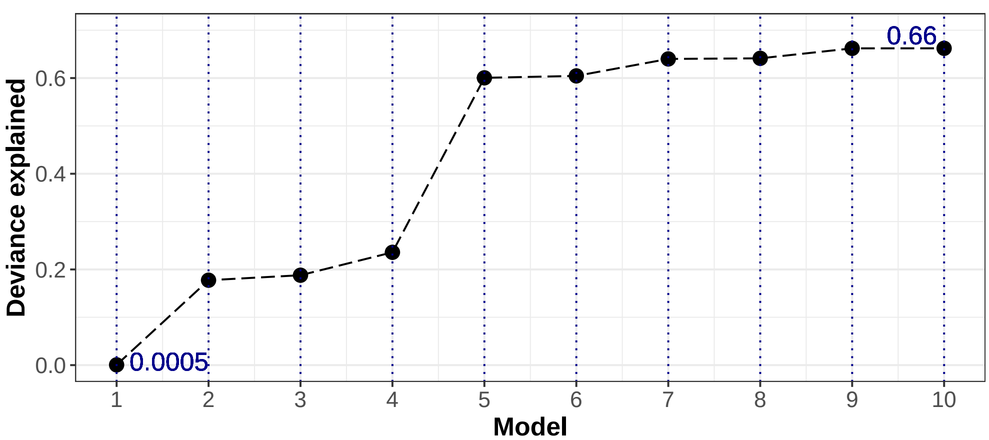
Figure 15 shows goodness of fit criteria for models 1 to 10. As portrayed in Figure 15a, the AIC of the model steadily decreases indicating that the model fits better with a higher number of covariates. It is interesting to note that the greatest decrease is between model 4 and model 5, which represents the inclusion of snow storage in the model. After that, considering humidity, radiation and interactions only slightly improves the fit. The same pattern can be seen in Figure 15b. The largest differences in AUC are caused by the inclusion of




(a)




(b)




(c)

Figure 15: Goodness of fit criteria for different models (according to Table 3) for catchment "Sanna-Landeck-Bruggen" in summer. Figure (a) shows the AIC for the step-by-step derivation of final model 10, figure (b) and (c) show the AUC and deviance explained, respectively.

precipitation (model 2) as well as soil water and snow storage (model 3 and 4), but the AUC remains nearly constant afterwards. Note that an AUC of 0.97 for the final model is an indicator of a very good detection of low-flow events. Recall that the corresponding ROC curve in Figure 12 has already suggested that result. Regarding the deviance explained in Figure 15c, the starting model 1 indicates a very poor fit. Similarly to before, the inclusion of snow storage in the model drastically improves the explained deviation, after which the fit increases steadily but slowly to an acceptable value of 0.68.

All three criteria show that the chosen interactions improve the fit only very slightly. The selection of these three terms is based on the one hand, on expert knowledge, and on the other hand on the fact that taking into account other interactions, e.g. precipitation and soil water or `snow storage`, does not lead to a better model performance as further analyses reveal.

Although this is one model out of 1959, the performance is similar across all fitted models. In conclusion, the logit models presented in Equation 8 fit the data very well.

## 5.3 Analysis of Significance

Since the model procedure presented in the previous sections includes averaging over 10 coefficients to get a final estimate for each catchment and hydrological half-year, one has to be careful when it comes to assessing the significance of estimated coefficients. When multiple hypothesis tests are performed for the same catchment, the probability of at least one significant result due to chance alone increases with the number of tests. This phenomenon is known as *multiple testing problem*. To circumvent this issue, several techniques exist. The method used in this project is called *Bonferroni correction*. It is based on the idea that the significance level decreases with the number of hypothesis tests performed to reduce the likelihood of erroneously obtaining a significant result. With 10 tests as in this project's models, the significance level drops to $\alpha_B = \frac{\alpha}{10}$. Note however, that Bonferroni is a rather conservative method and tends to reject less hypotheses as other correction techniques.

In the following, a member-averaged effect is called significant if each of the 10 p-values associated with the 10 estimated effects is smaller than $\frac{0.05}{10} = 0.005$.

# 6 Results

In this chapter the results of the previously introduced models are presented. This includes the estimated effect sizes of the drivers and a ROC analysis to determine an appropriate threshold for predictions as introduced in subsection 4.2 to use the fitted models for predicting climate scenarios. In addition, the estimated coefficients get clustered to detect regional patterns.

## 6.1 Effects

In the following sections, for each driver and hydrological half-year, the member-averaged effects are visualized as log-odds by means of maps of hydrological Bavaria, in order to assess where the respective driver contributes to the occurrence of low-flows. Due to the lack of a model for summer, the catchment "Altmühl-Aha" is shown in white in the corresponding figures.

### 6.1.1 Precipitation

Before diving deeper into regional patterns, consider once more catchment "Sanna-Landeck-Bruggen" (see Figure 14) to get a better understanding on how log-odds are interpreted. Here, the estimated effect is -0.75 in summer for a 1 mm increase in precipitation, and a possible interpretation could be:

> Keeping all other variables constant, for average temperature, increasing the 7-day mean of precipitation by 1 mm, the log-odds of occurring low-flow decrease by 0.75 additively on average in summer.

This in turn means a decrease of the odds by $\exp(-0.75) = 0.47$ multiplicatively. Loosely speaking, the more precipitation falls, the less days of low-flow are expected on average for keeping everything else constant and average temperature. The restriction to average temperature is due to the interaction between precipitation and temperature and the aforementioned centring of the covariates.

Looking at the entire effect map for summer in Figure 16a, a negative impact of precipitation on the log-odds can be seen in almost all catchments. Interestingly, the effects are stronger in absolute terms in the southern and eastern regions for a 1 mm precipitation increase. Figure 16b shows the significance of the effect where catchments with a non-significant effect are displayed in white. In large parts of hydrological Bavaria, precipitation has a significant effect and only a few catchments in the north-east, centre and south fall out. Note that the only counter-intuitive positive effect is not significant.

In order to assess whether precipitation and temperature together result in a compound event, Figure 16c and Figure 16d show the effect of precipitation for 5 °C colder and warmer than on average, respectively. If they depicted a compound event, the effect would change when varying the temperature. Comparing both figures with Figure 16a, one can observe

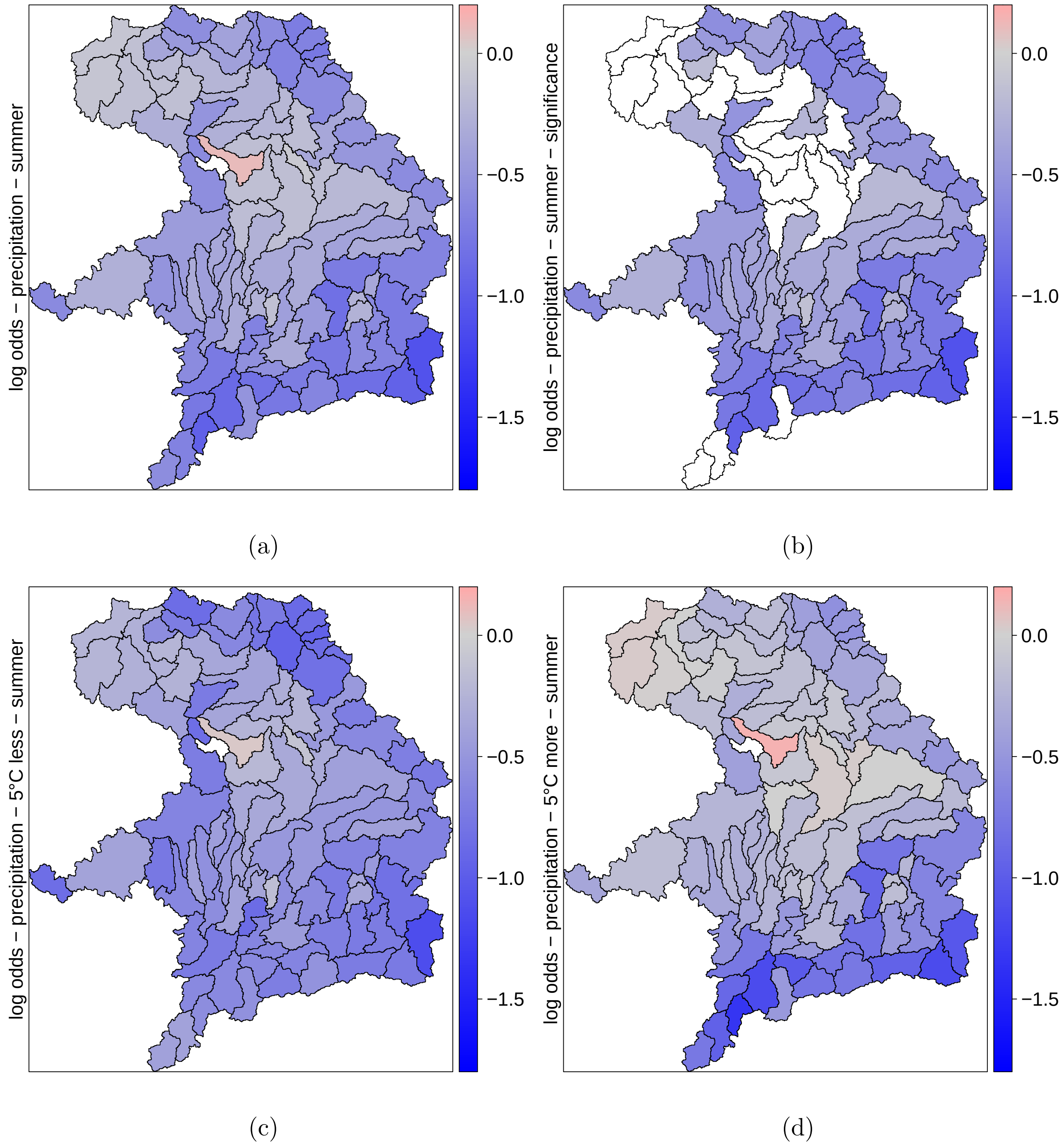



Figure 16: Effects on the log-odds of precipitation in summer (a) for a 1 mm increase and the corresponding significance (b). Non-significant effects are displayed in white. Figure (c) and (d) show the effect for varying temperature, namely a 5 °C decrease and a 5 °C increase, respectively.

some regional differences, but the effects stay mainly similar. In Appendix A, Figure 37b, the significance of this interaction term is portrayed and besides three catchments, no effect is significant. As a consequence, one has to be careful when it comes to the interpretation. To conclude, for summer, no compound event of precipitation and temperature can be identified.

The effects in winter, as shown in Figure 17a, are all negative and particularly pronounced in the northern catchments in contrast to the effects in summer. Instead, the effects in the Alpine regions in the south are very small and looking at Figure 17b, they are not significant in comparison to the rest of hydrological Bavaria. This may be due to precipitation in the form of snow in the Alps in winter, which remains as snow storage and does not have an immediate effect on drainage and therefore low-flows. In total, this clearly shows that the effects of precipitation differ between summer and winter.

Also in winter, the interaction term is of negligible size for varying temperature by 5 °C since Figure 17c and Figure 17d do not differ remarkably from the original effects in Figure 17a. Furthermore, it is shown in Appendix A, Figure 37d, that there is not a single significant interaction term. So, just as in summer, a compound event of precipitation and temperature is not discernible.

### 6.1.2 Temperature

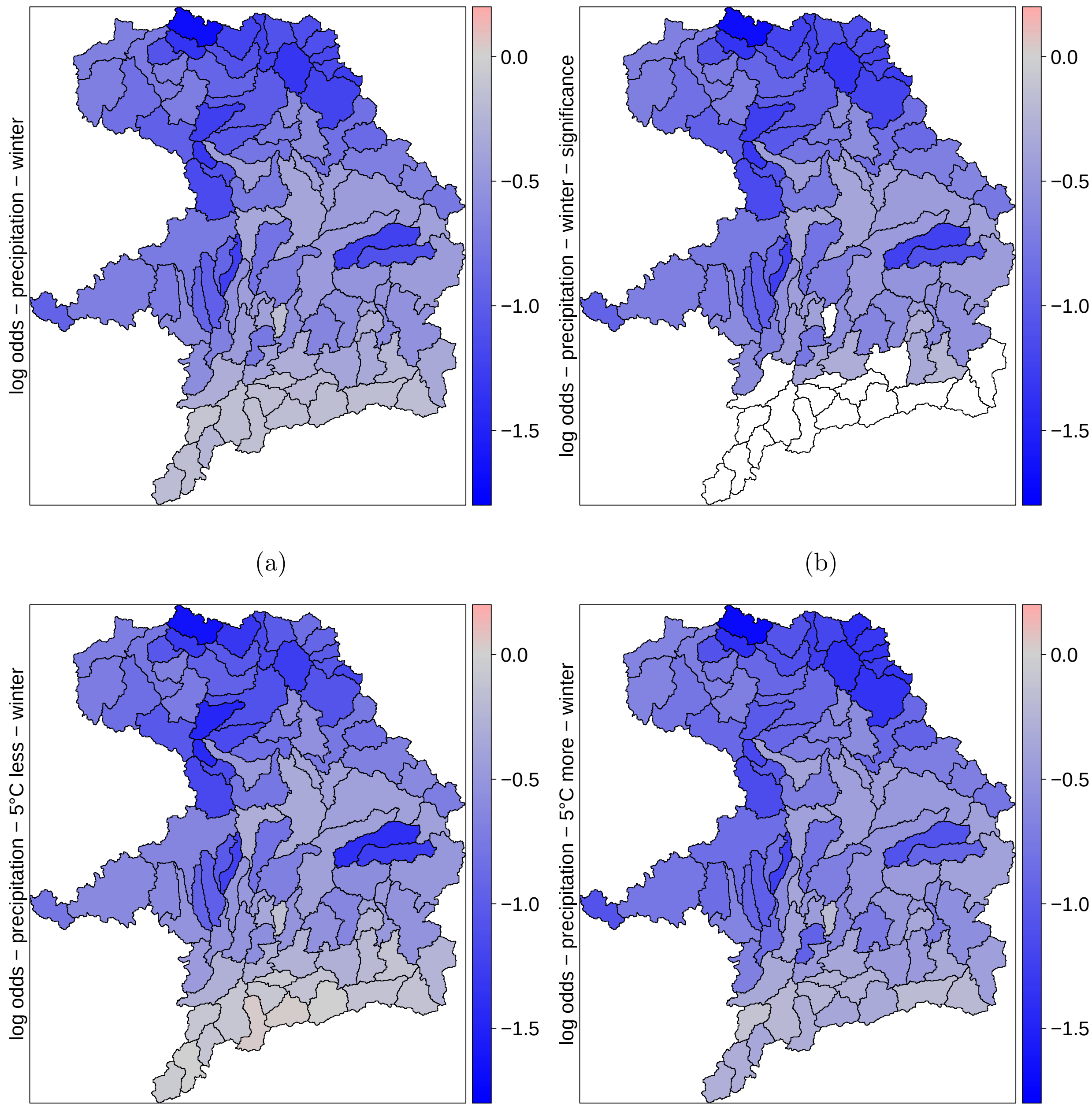
As opposed to the effects just shown, there are both negative and positive effects for temperature depending on the region in summer, as visualized in Figure 18a. Again, before analyzing the effects in detail, an example interpretation for catchment "Sanna-Landeck-Bruggen" is given as:

> Keeping all other variables constant, for average precipitation, snow storage and soil water, increasing the 7-day mean of temperature by 1 °C, the log-odds of occurring low-flow decrease by 1.67 additively on average in summer.

This means that the odds of occurring low-flow decrease by exp(-1.67) = 0.19 multiplicatively, i.e. one would expect a lower chance of low-flow with increasing temperature. Note that this interpretation is for average precipitation, soil water and snow storage because of the included interactions.

Looking at hydrological Bavaria as a whole, clear regional differences become apparent. In large parts of the north only negligible effects are estimated. In some catchments in the centre, the effect of temperature seems to be strongly positive, while in the Alpine regions the effect on the occurrence of low-flow is once again negative. However, the strong positive effects should not be overrated since Figure 18b suggests that none of the positive effects are significant. To be precise, temperature only has a significant effect on a few catchments in the south. This leads to the conjecture that temperature is not a central driver explaining the occurrence of low-flow events in summer.

In contrast to positive and negative effects in summer, almost all effects of temperature




(a) (b)

(c) (d)

Figure 17: Effects on the log-odds of precipitation in winter (a) for a 1 mm increase and the corresponding significance (b). Non-significant effects are displayed in white. Figure (c) and (d) show the effect for varying temperature, namely a 5 °C decrease and a 5 °C increase, respectively.

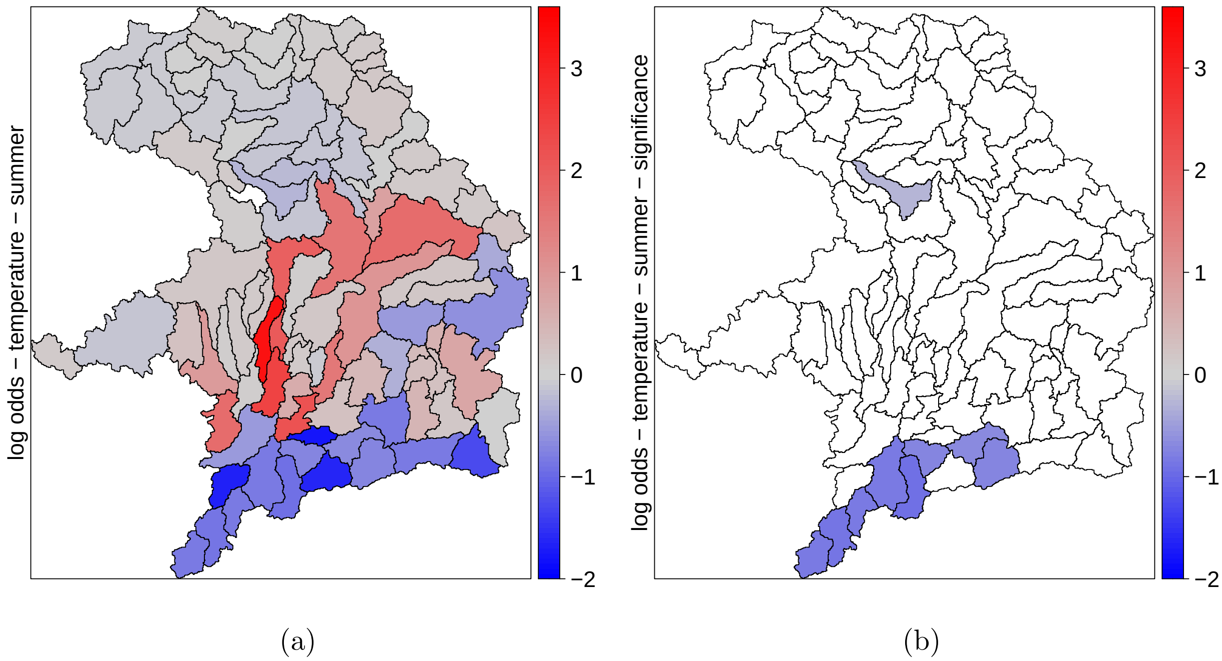


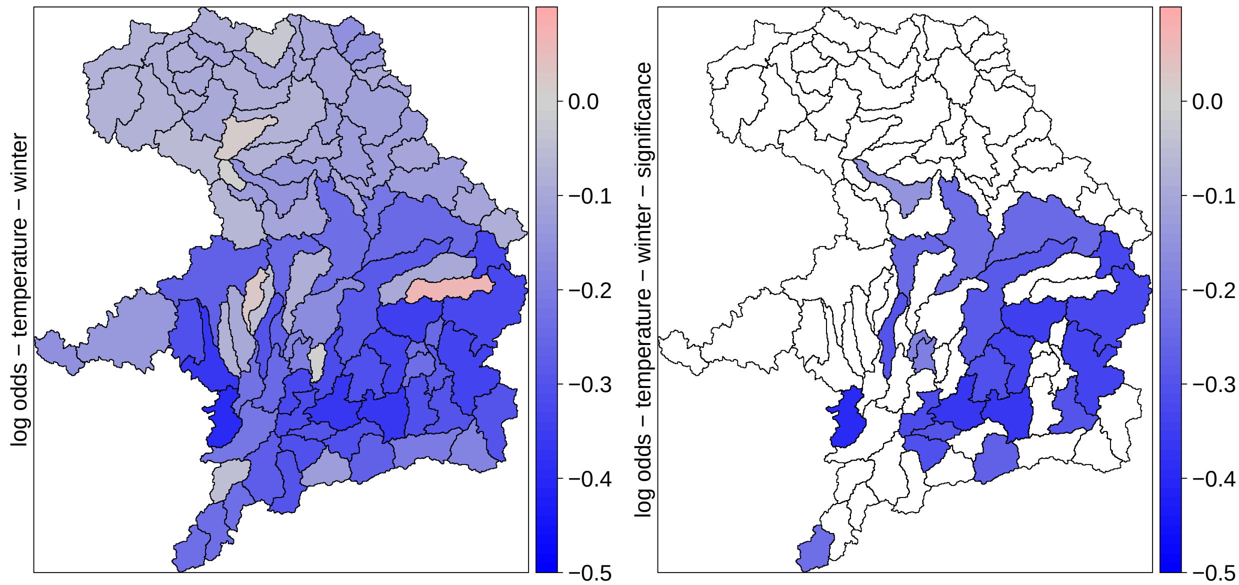

Figure 18: Effects on the log-odds of temperature in summer (a) for a 1 °C increase and the corresponding significance of the effects (b). Non-significant effects are displayed in white.

are negative in winter as depicted in Figure 19a. Especially in the centre and south of hydrological Bavaria, negative effects of temperature on the occurence of low-flow events can be observed. Note however, given the scale, the effects are less pronounced than in summer. That means in particular that a 1 °C change in temperature in winter has less of an effect than a 1 °C change in summer. When looking at the significance in Figure 19b, more pronounced effects in the centre appear to be significant while in the north and west temperature has no significant effect. Although in winter there are more catchments with a significant effect of temperature than in summer, the total number is still small and therefore, temperature may not be a an important driver for low-flow events overall.

### 6.1.3 Snow Storage

Recall that in summer, due to lack of snow storage, not every catchment-specific model specification includes the covariate snow storage. Overall, only 47 out of 98 catchments meet the inclusion criterion introduced in subsection 5.1, especially those in the southern half of hydrological Bavaria. The catchments without estimated effect for snow storage are visualized in white in the following figures. As with the previous drivers, an exemplary interpretation for catchment "Sanna-Landeck-Bruggen" is presented:

> Keeping all other variables constant, for average temperature, increasing the 30-day mean of snow storage by 1 mm, the log-odds of occurring low-flow that day decrease by 0.47 additively on average in summer.




(a) (b)

Figure 19: Effects on the log-odds of temperature in winter (a) for a 1 °C increase and the corresponding significance of the effects (b). Non-significant effects are displayed in white.

In particular, the odds for a low-flow event decrease by $\exp(-0.47) = 0.63$ multiplicatively, i.e. one would expect less low-flow for increasing snow storage.

Considering the effect of snow storage in entire hydrological Bavaria displayed in Figure 20a, it is important to note that the catchments in the centre are strongly negatively affected, while the effects on the catchments in the Alpine regions are very low for a 1 mm increase in snow storage. This is due to the distribution of snow storage: in the Alps, one can observe more snow fall and therefore more snow storage than in the centre. As a result, a change in snow storage of 1 mm in the Alps does not have the same effect on the occurrence of low-flow as in regions with a lower snow storage from the very start. To take a deeper look at how the log odds change for a 10 cm increase instead of a 1 mm increase in the Alpine regions in summer, Figure 21a is considered. Still, the effects on most of the Alpine catchments are slightly negative with some catchment with stronger negative effects.

Coming back to effects displayed in Figure 20a and taking significance into account, Figure 20b shows that especially the effects of snow storage on Alpine catchments are rarely significant in summer. In Figure 20c and Figure 20d the effect of snow storage for varying temperature is visualized, namely 5 °C less and more on average, respectively. At lower temperatures, the effects are amplified in the centre, while at higher temperatures the effects are weakened, but for the rest of the southern half they remain virtually the same. In Appendix A, Figure 38b, it is shown that not a single estimated interaction term is significant in summer, so again, no compound effect can be detected.

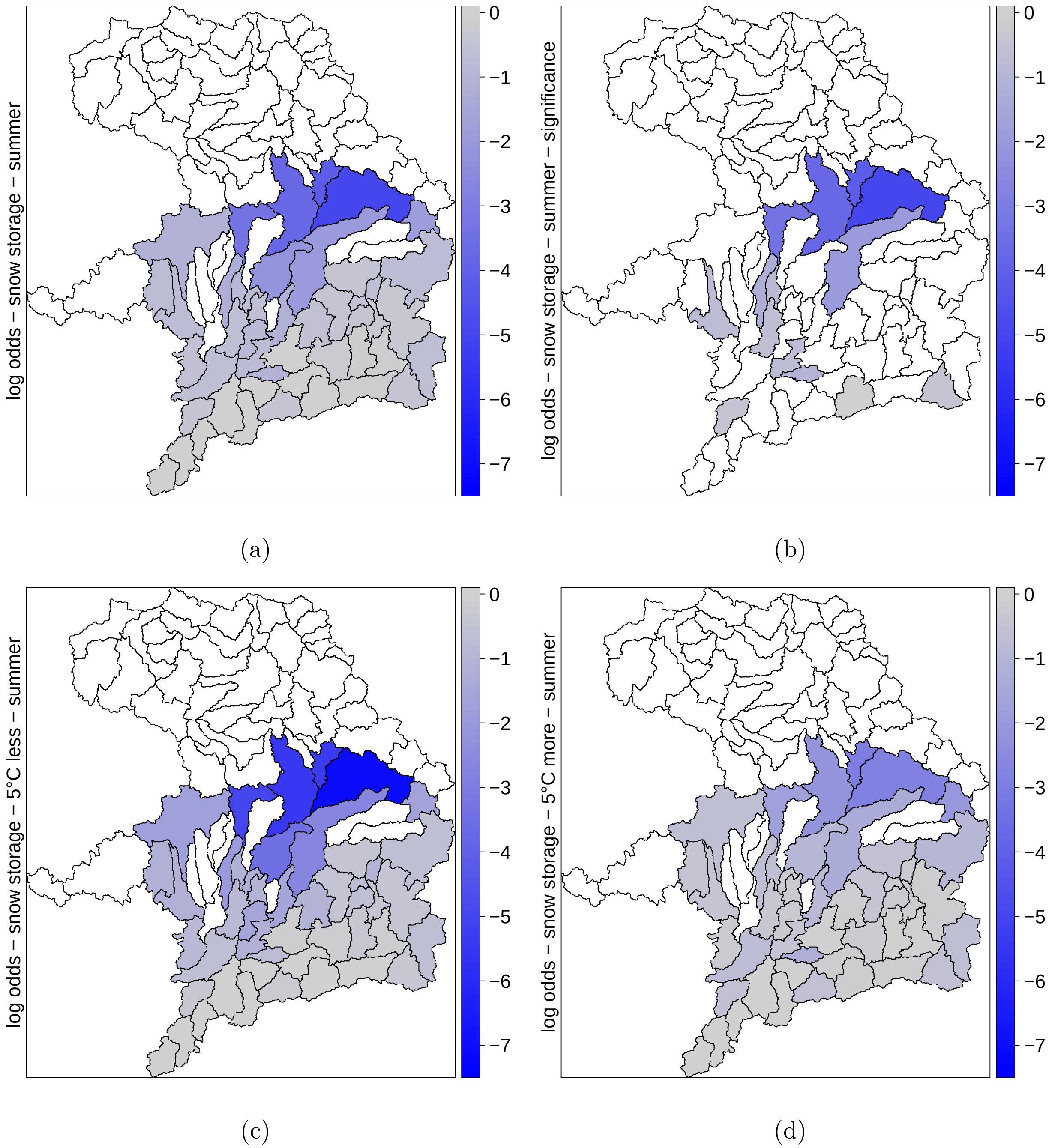
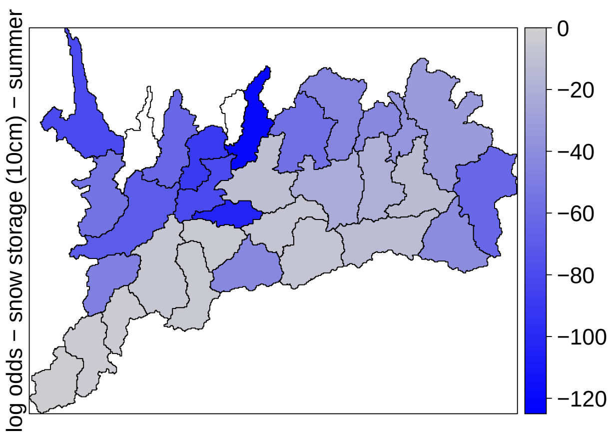
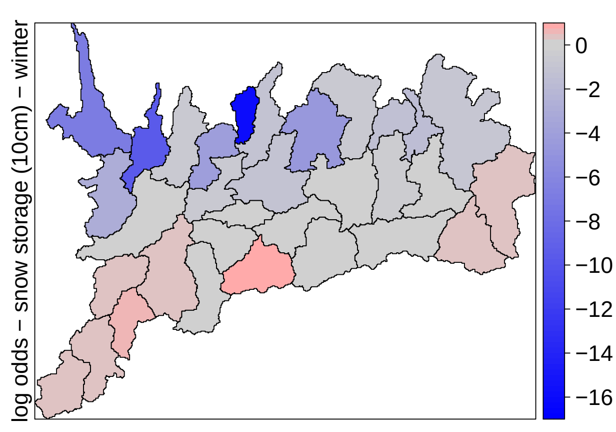



Figure 20: Effects on the log-odds of snow storage in summer (a) for a 1 mm increase and the corresponding significance (b). Non-estimated and non-significant effects are displayed in white. Figure (c) and (d) show the effect for varying temperature, namely a 5 °C decrease and a 5 °C increase, respectively.




(a) (b)

Figure 21: Effects of snow storage in summer (a) and winter (b) for a 10 cm increase on the log-odds.

On the contrary to summer, in winter, for every catchment an effect of snow storage is estimated. Figure 22a shows the estimated effect sizes. Interestingly, besides two catchments, only catchments in the north-western part of hydrological Bavaria are strongly negative influenced by snow storage while in the southern regions the effect is negligible. Once again, if one considers a 10 cm increase in snow storage rather than a 1 mm increase in the Alps as displayed in Figure 21b, the effects are still very small but are slightly positive for some catchments. This effect perhaps stems from the fact that in winter the snow reservoirs in higher regions do not melt and therefore less water reaches the rivers. However, caution is needed in the interpretation, as Figure 22b points out that only a few estimated member-averaged coefficients are significant at all.

In order to assess whether snow storage and temperature together build a compound event, the effect of snow storage for varying temperature is displayed in Figure 22c for 5 °C colder and in Figure 22d for 5 °C warmer than on average. The effects in the north-western part are slightly weakened (colder) and magnified (warmer), but overall the effects do not change remarkably and since not a single estimated member-averaged interaction term is significant as portrayed in Appendix A, Figure 38d, also in winter no compound event is identifiable.

To conclude, taking effect sizes and significance into account, for both summer and winter, snow storage is an important driver for only a few catchments.

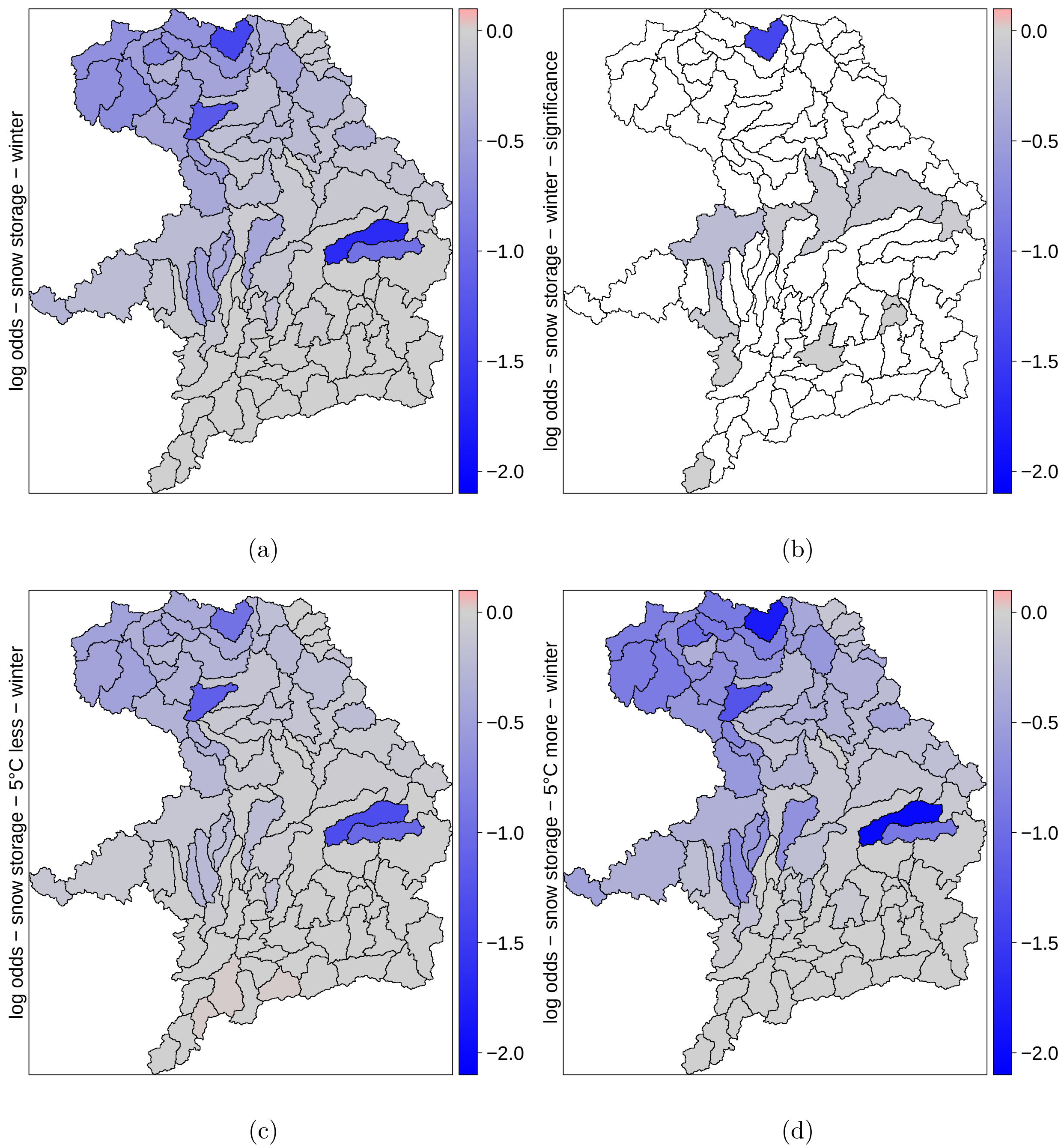



Figure 22: Effects on the log-odds of snow storage in winter (a) for a 1 mm increase and the corresponding significance (b). Non-estimated and non-significant effects are displayed in white. Figure (c) and (d) show the effect for varying temperature, namely a 5 °C decrease and a 5 °C increase, respectively.

### 6.1.4 Soil Water

Next, the effect of soil water on the occurrence of low-flow is evaluated. Starting with the effects in the summer months, Figure 23a shows the estimated member-averaged effects. Clearly, regional patterns are recognisable since the strongest negative effects are primarily in the southern Pre-Alps. Note that every single effect is negative. One of the catchments with a rather strong negative effect of soil water is "Sanna-Landeck-Bruggen". A possible interpretation is given as:

> Keeping all other variables constant, for average temperature, increasing the 60-day mean of soil water by 1 %, the log-odds of occurring low-flow that day decrease by 1.72 additively on average in summer.

In turn, the odds for the occurrence of a low-flow event decrease by 0.18 multiplicatively.

Soil water has a significant influence on the log-odds in all but five catchments in summer, as shown in Figure 23b, which is a remarkable result. Inversely, looking at the interaction term, only three estimated effects appear to be significant (see Appendix A, Figure 39b), and in addition, the effect for varying temperature, Figure 23c (5 °C less) and Figure 23d (5 °C more) show only small changes of the main effect. From this it can be concluded that no compound event of soil water and temperature can be determined from the data.

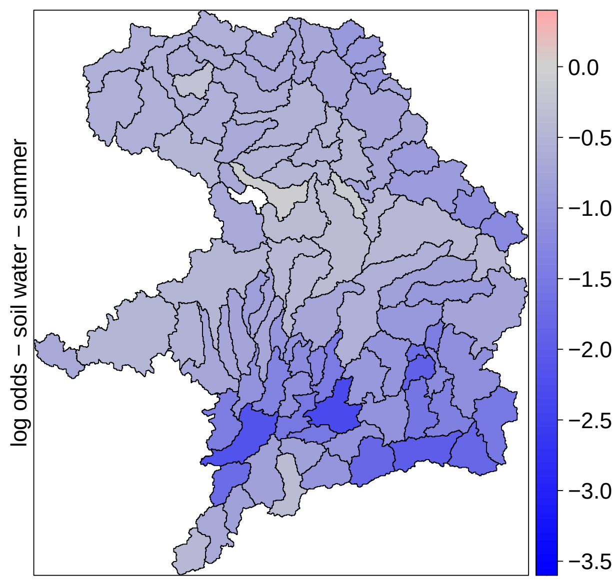
Compared to the effects estimated in summer, the effects in winter hardly change as visualized in Figure 24a. Again, all effects are negative and most pronounced in the Pre-Alps region. Most noteworthy is the significance of the estimated coefficients: every single coefficient is significant on a significance level of 0.005. In contrast, none of the estimated interaction terms for the compound event of temperature and soil water is significant (see Appendix A, Figure 39d). Nevertheless, consider the effect sizes of the interaction terms. A reduction or increase in the average temperature by 5 °C, as shown in Figure 24c and Figure 24d, respectively, leads to a slightly stronger effect at colder temperatures and a slightly weaker effect at warmer temperatures.

In summary, soil water as driver contributes well to explaining low-flow events in both summer and winter, but a compound event with temperature is not detectable.

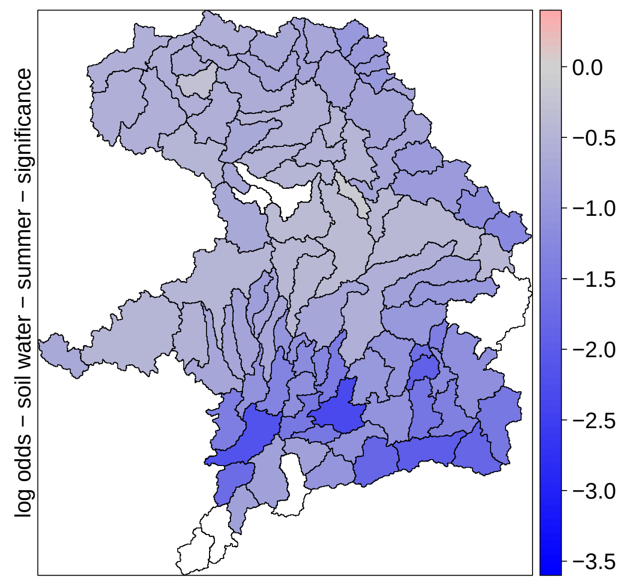
### 6.1.5 Humidity

This part takes a deeper look into how relative humidity affects low-flow events. In Figure 25a the effects on the log-odds for an increase of 10 % in hydrological Bavaria are displayed. Note that this scaling is due to a weak influence for a 1 % increase on the development of low-flow. Every coefficient is negative, so loosely speaking, a 10 % increase in relative humidity leads to a smaller chance of low-flow. A precise interpretation for catchment "Sanna-Landeck-Bruggen" could be as this:

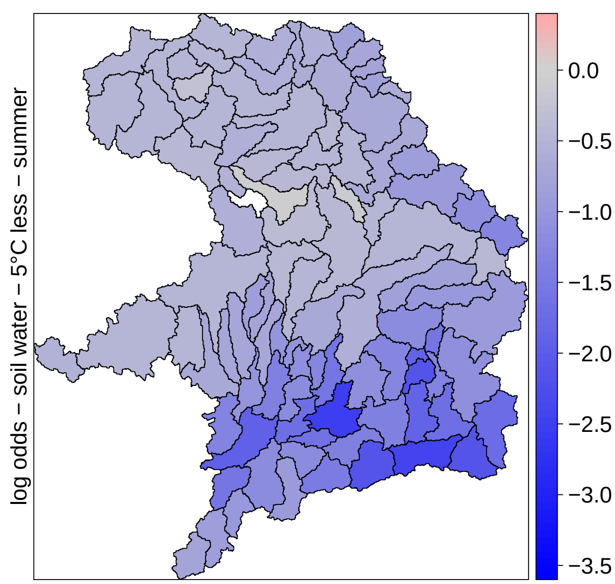
> Keeping all other variables constant, increasing the 7-day mean of relative



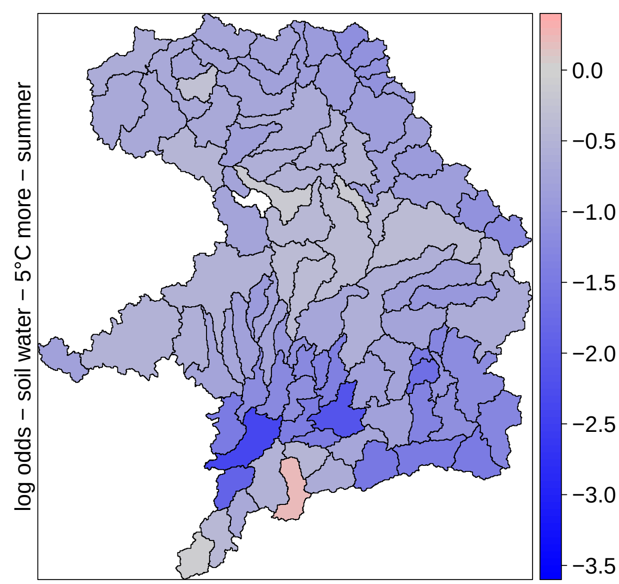

(a)




(b)




(c)




(d)

Figure 23: Effects on the log-odds of soil water in summer (a) for a 1 % increase and the corresponding significance (b). Non-significant effects are displayed in white. Figure (c) and (d) show the effect for varying temperature, namely a 5 °C decrease and a 5 °C increase, respectively.

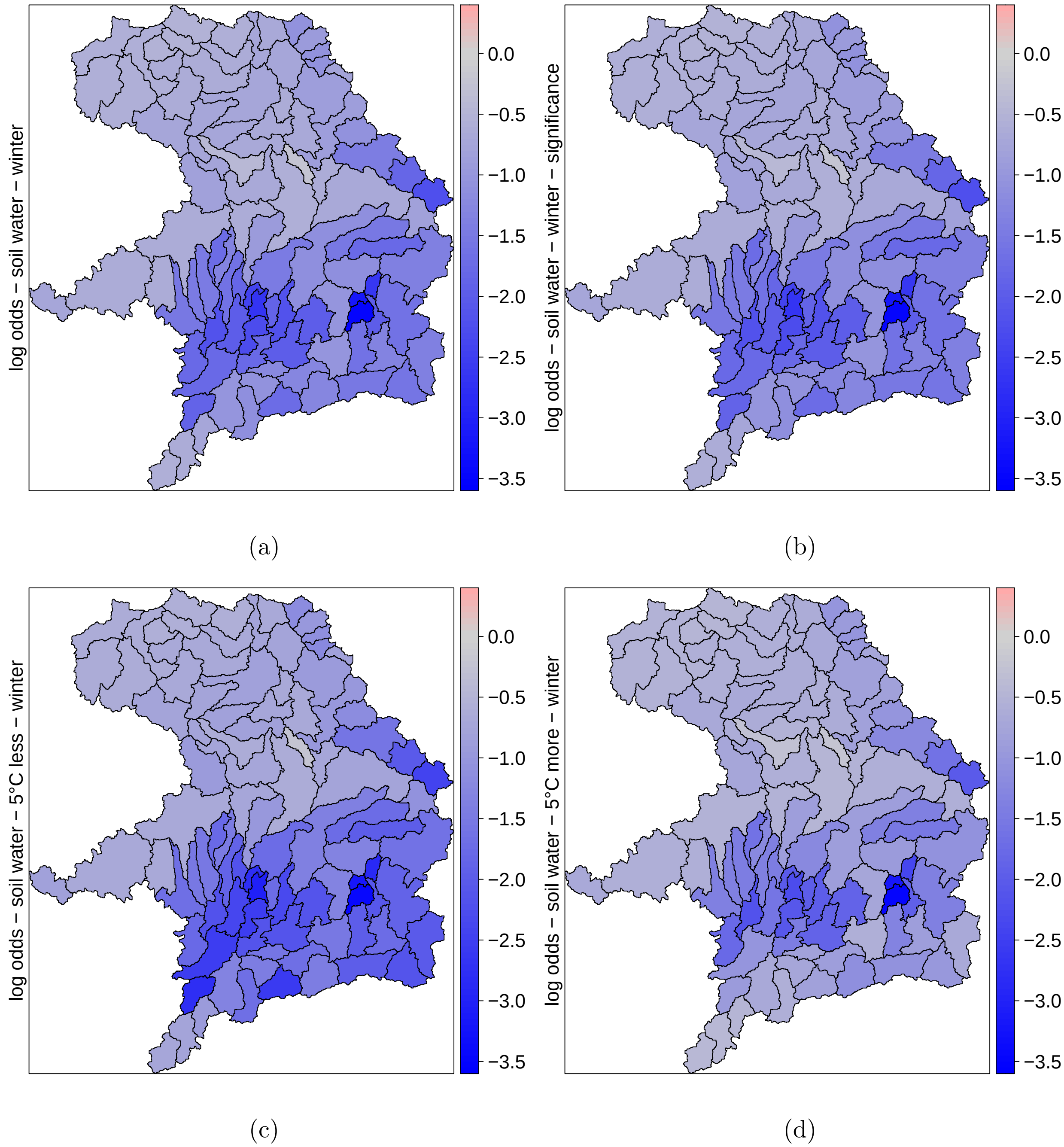
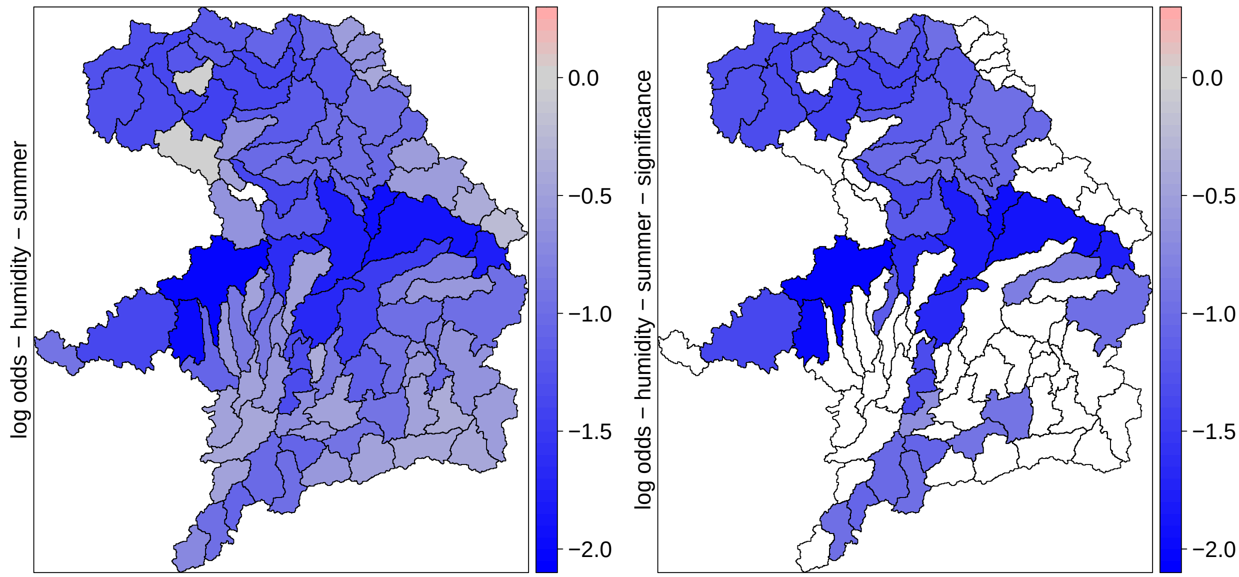



Figure 24: Effects on the log-odds of soil water in winter (a) for a 1 % increase and the corresponding significance (b). Non-significant effects are displayed in white. Figure (c) and (d) show the effect for varying temperature, namely a 5 °C decrease and a 5 °C increase, respectively.




(a)

(b)

Figure 25: Effects on the log-odds of relative humidity in summer (a) for a 10 % increase and the corresponding significance of the effects (b). Non-significant effects are displayed in white.

> humidity by 10 %, the log-odds of occurring low-flow decrease by 0.46 additively on average in summer.

In particular, this means that the odds decrease multiplicatively by 0.56.

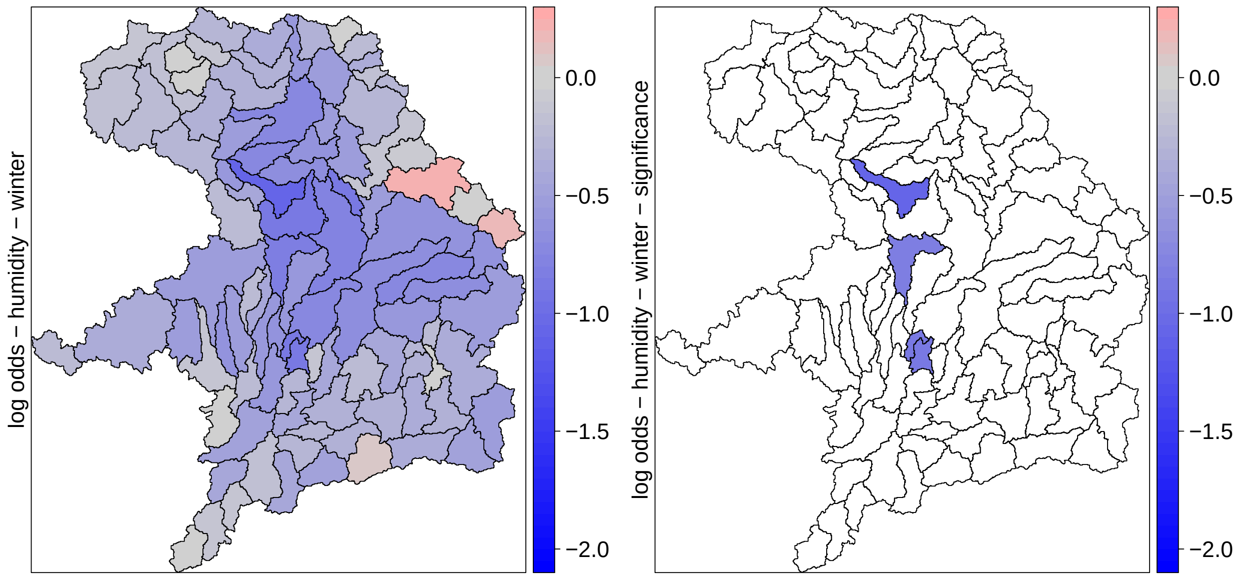
In Figure 25b the significance of the estimated averaged coefficients is shown. It clearly follows some regional pattern, since in many parts of hydrological Bavaria humidity has a significant effect on low-flow events, especially in the north and centre.

In comparison to the effect sizes in summer, the effects in winter are by far less pronounced in absolute terms as visualized in Figure 26a. In some eastern and southern catchments the effect is even positive, but looking at the significance in Figure 26b, all this positive effects are non-significant. In fact, except for four catchments, no effect is significant in winter.

Consequently, relative humidity seems to negatively influence the occurrence of low-flow in large parts of hydrological Bavaria in summer, but this driver is of less importance in winter.

### 6.1.6 Radiation

Lastly, the impact of radiation on low-flow events is assessed. In Figure 27a the estimated member-averaged coefficients are shown. In the northern and western catchments, the effects appear to be positive, i.e. an increase in radiation leads to more low-flow on average,




(a) (b)

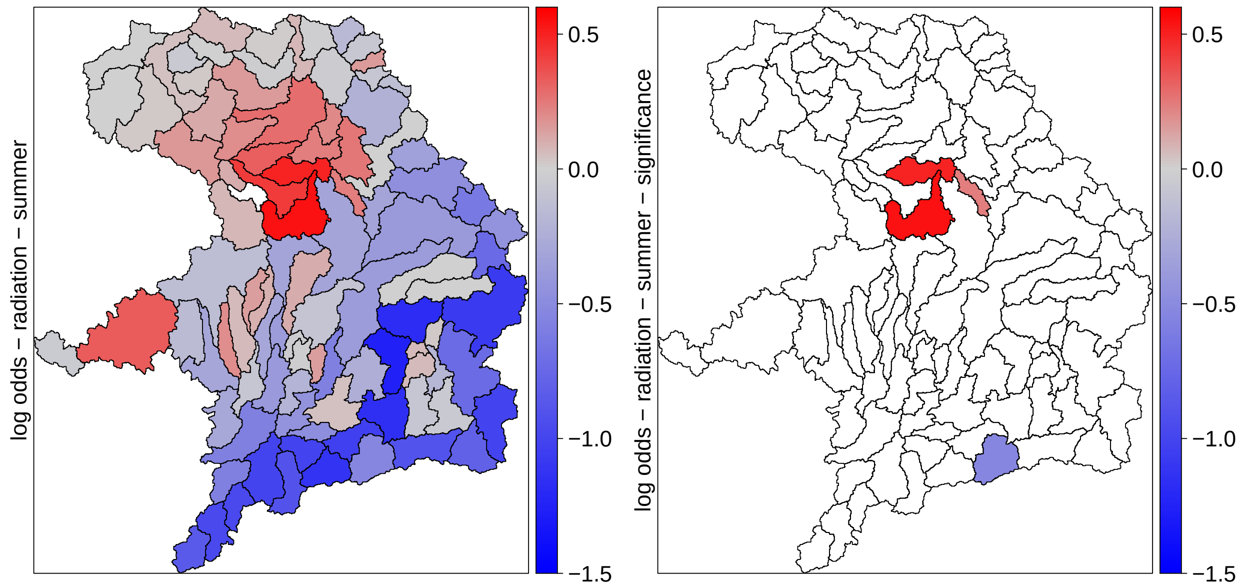
Figure 26: Effects on the log-odds of relative humidity in winter (a) for a 10 % increase and the corresponding significance of the effects (b). Non-significant effects are displayed in white.

while in the south and west of hydrological Bavaria the estimated effects are negative. For catchment "Sanna-Landeck-Bruggen", an example interpretation could be:

> Keeping all other variables constant, increasing the 7-day mean of radiation by 100 Wh/m$^2$, the log-odds of occurring low-flow decrease by 0.58 additively on average in summer.

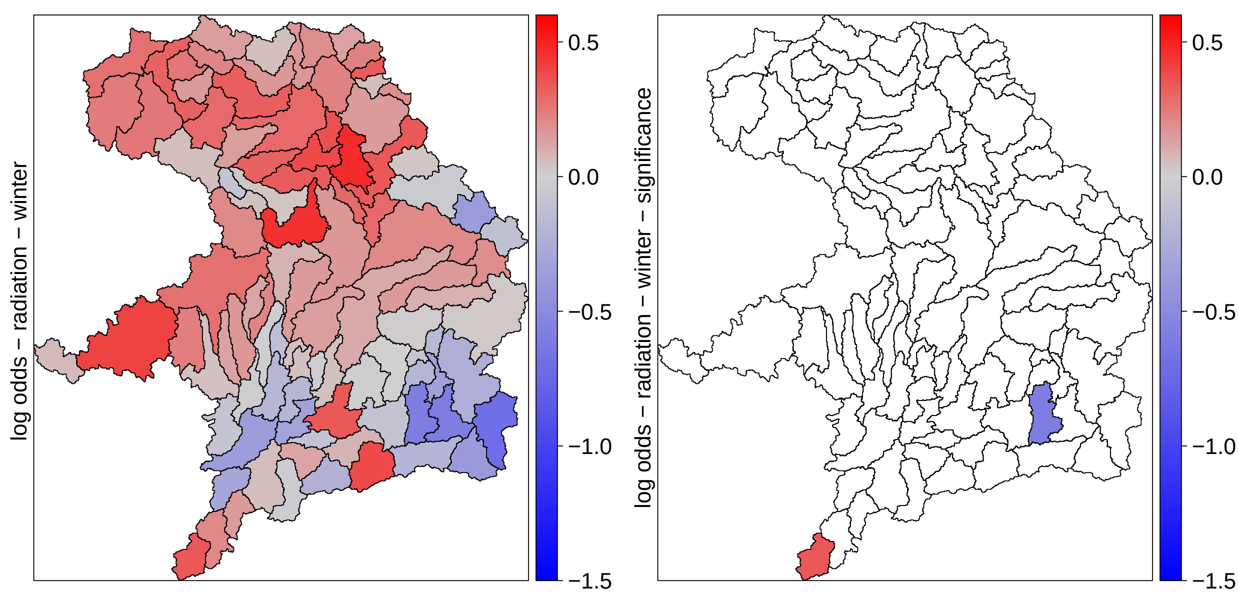
As previously seen for humidity, an interpretation in terms of 100 units, i.e. here 100 Wh/m$^2$, is preferred over an interpretation for an increase by 1 Wh/m$^2$, as one unit does not considerably influence low-flow events.

According to Figure 27b however, only in four catchments the effect of radiation is significant. The number of catchments with a significant effect of radiation is even lower in winter: in Figure 28a the effects in winter are displayed and the corresponding significance is shown in Figure 28b. In comparison to the effects in summer, in large parts of hydrological Bavaria positive effects are estimated. Only in a few catchments in the south and west, some moderately pronounced negative effects exist. But, again, caution is important when it comes to the interpretation, as only two effects are in fact significant. To conclude, regarding the significance, radiation is less crucial for the explanation of low-flow events.




(a) (b)

Figure 27: Effects on the log-odds of radiation in summer (a) for a 100 Wh/m² increase and the corresponding significance of the effects (b). Non-significant effects are displayed in white.




(a) (b)

Figure 28: Effects on the log-odds of radiation in winter (a) for a 100 Wh/m² increase and the corresponding significance of the effects (b). Non-significant effects are displayed in white.

In summary, the following coefficients and significances are established exemplary for catchment "Sanna-Landeck-Bruggen":

| Covariate | Coefficient | Significance | Unit |
|---|---|---|---|
| Precipitation | -0.75 | yes | 1 mm |
| Temperature | -1.67 | no | 1 °C |
| Snow Storage | -0.47 | no | 1 mm |
| Soil Water | -1.72 | yes | 1 % |
| Humidity | -0.45 | no | 10 % |
| Radiation | -0.58 | no | 100 Wh/m² |

Table 4: Member-averaged driver effects for catchment "Sanna-Landeck-Bruggen".

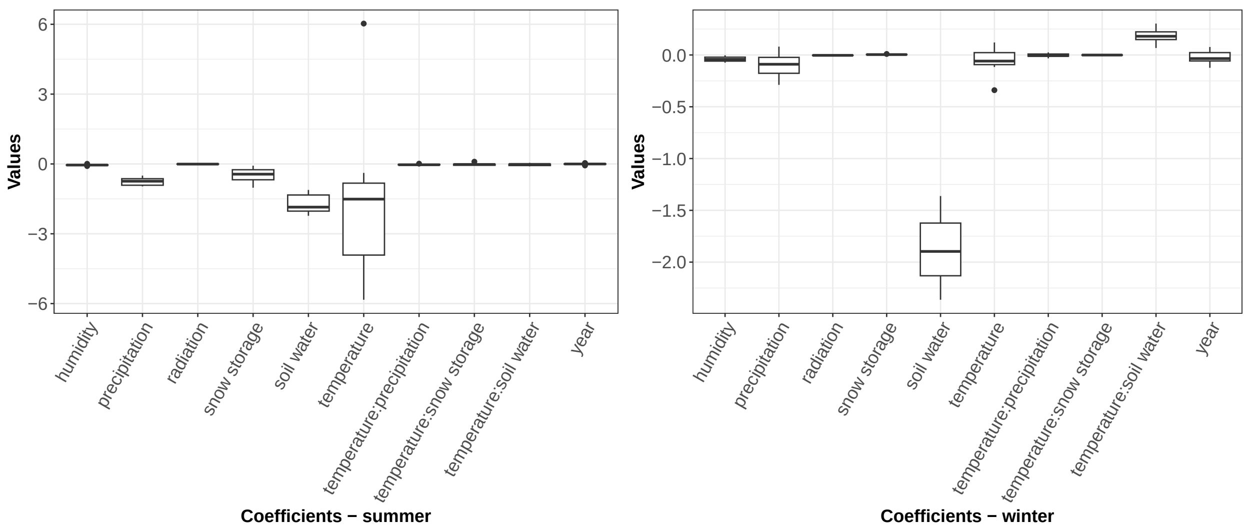
In order to assess which driver has the strongest influence on the occurrence of low-flow, the question arises as to how the estimated coefficients can be compared. With regard to the present coefficients, one could argue that a 1 % increase in soil water has a stronger influence on low-flow than a 1 mm increase in precipitation. But is a 1 % increase in soil water equivalent to a 1 mm increase in precipitation, i.e. are these increases comparable? This is a question where statistics reach their limits and expertise from the field is needed.

### 6.1.7 Differences Between Members

Recall that 10 different coefficients are initially estimated, i.e. for each member separately, before being averaged to obtain a final estimated effect of the respective driver in the respective catchment for summer and winter. Even if the data of the different members differ remarkably, as shown in Figure 9 in the descriptive analysis, the hydrological patterns leading to low-flow events should be similar and therefore similar effects on the same catchment should be estimated for each member.

To give an intuition, that this is indeed the case for most of the estimated coefficients, Figure 29 shows box plots over the 10 estimated coefficients for summer (a) and winter (b) separately for the example catchment "Sanna-Landeck-Bruggen". In summer, all covariates except one are very similar between the different members. However, looking at the 10 estimated effects for temperature, all but one estimated coefficient are negative. Interestingly, this one positive effect is strongly pronounced. But since every other coefficient for temperature is negative, the method of averaging over members makes sure to reduce the influence of outliers. In winter (Figure 29b) the 10 estimated effects are very similar and only for covariate soil water, some higher variability is recognisable. Since the coefficients here all have the same sign and lie in a similar range, averaging over the members does not lead to a dilution of the effects.

In total, the estimated coefficients are quite similar as expected for most of the drivers. Unlikely coefficients are reduced and therefore, the final results are more reliable and member-independent.




(a) (b)

Figure 29: Differences in estimated effects between the 10 members for summer (a) and winter (b) for the example catchment "Sanna-Landeck-Bruggen" visualized as box plots.

## 6.2 Threshold Analysis

Recall the idea of the AUC and the corresponding ROC curve is to evaluate the fit, but also to determine the threshold for the classification of events, as presented in subsubsection 4.2.3. The fitted models introduced in section 5 can be used to predict daily probabilities for low-flow events given new data. In general, the closer this predicted probability is to 1, the more likely a day of low-flow occurs. The task of a threshold analysis using ROC curves is to determine a suitable threshold value between 0 and 1 above which a low-flow event is classified as such.

Applied to the project at hand, that means in particular that the models are trained on a specific member and then evaluated on the remaining 9 members. Therefore, TPR, FPR and TDR are calculated for different thresholds $c \in \{0.1, 0.2, 0.3, 0.4, 0.5, 0.6, 0.7, 0.8, 0.9\}$ for all 9 test sets, and the resulting rates are then averaged over the 9 values. In Figure 30, the rates are plotted for four randomly chosen catchments in summer. Clearly, when looking at the specificity visualized in red, all four models seem to correctly identify days of no low-flow for nearly every threshold $c$. Especially above $c = 0.4$, the specificity is higher than 0.8 in all four catchments, indicating a good discrimination. The TPR (black curve) decreases almost linearly with increasing threshold. This is due to the fact that for a low threshold, nearly every day is classified as low-flow event and among them, there are a plethora of false positives, as the increase of 1-FPR with increasing threshold indicates. Considering TDR in blue, the less days are classified as low-flow events, the proportion of correctly predicted days of low-flow out of all predicted ones increases with increasing threshold.

To finally pick a threshold, note that there is no clear statistical "right choice". It always depends on the context of the domain and on which evaluation measure is of great impor-

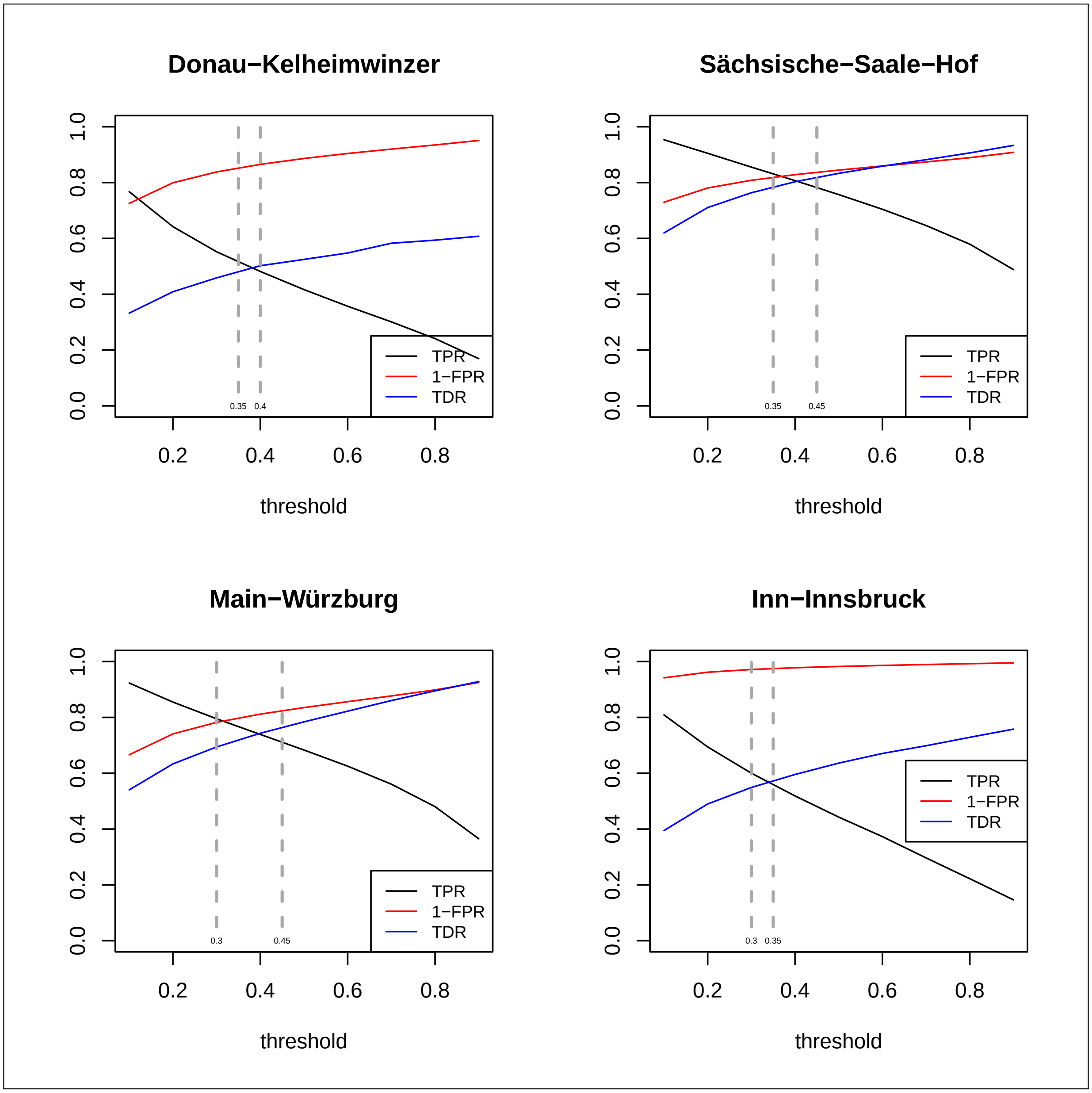



Figure 30: Threshold analysis including TPR, FPR and TDR for four different catchments in summer. The range of the intersection points of TPR and TDR are marked as grey dotted lines.

tance. For this project, the intersection points of TPR and TDR seem to be an appropriate threshold, as the proportion of true positives and true discoveries outweighs the fact that not too many false positives are detected. According to the figures, the threshold could be chosen between 0.35 and 0.45. As a result, for all future analyses, the threshold is set to $c = 0.4$.

In order to give an intuition, that a cutoff of $c = 0.4$ is appropriate, Figure 31 is considered. Here, for summer 2020, the fitted models for one random member (member "kbt") were used to predict the probability of low-flow for all remaining members. The resulting 9 probabilities are averaged and based on this value the number of low-flow days in Mai to October 2020 is calculated for different thresholds $c$. In Figure 31a, the true number of days of low-flows in the data is portrayed. Clearly, the north of hydrological Bavaria was more subject to low-flow events than the centre and south. Figure 31b shows the predicted number of low-flow events for a threshold of $c = 0.3$. In the southern and central regions, clearly too many days of low-flow are predicted. For a threshold of $c = 0.4$ in Figure 31c, the amount of falsely detected low-flow events decreases. But looking at $c = 0.5$ in Figure 31d, the number of predicted low-flows is too low in some catchments.

So far, only models for summer are taken into account. Since for winter, the results are similar, the same threshold is used in the following. See Appendix A , subsection A.3, for a brief analysis of the winter threshold.

## 6.3 Climate Scenarios

Now that an appropriate threshold is found, the fitted models can be utilized for prediction on new data. So far, only data from members not taken for training were used for testing. This includes rather similar basic conditions in terms of climate. But what happens, if the climate changes? The goal of this section is to compare model predictions for different climate scenarios. In detail, the following data adjustments are explored:

- Increase in temperature by 3 °C
- Reduction of precipitation in summer by 50 %
- Increase in precipitation in winter by 50 %
- No occurrence of snow storage

The selection of +3 °C roughly reflects the projected warming by mid-century over Bavaria (assuming climate change scenario RCP8.5). For precipitation, it is an extreme increase or decrease: -50 % in relation to the reference would mean a severe drought, for instance.

The scenarios presented in this section build on each other. Intermediate scenarios can be found in Appendix A, subsection A.4. To account for all members, predictions for every model with adjusted test data from all members separately not used for training are calculated for summer and winter of 2010. Note that the year 2010 is chosen as it appears to be an average year in terms of the number of low-flow events. According to the previous section,

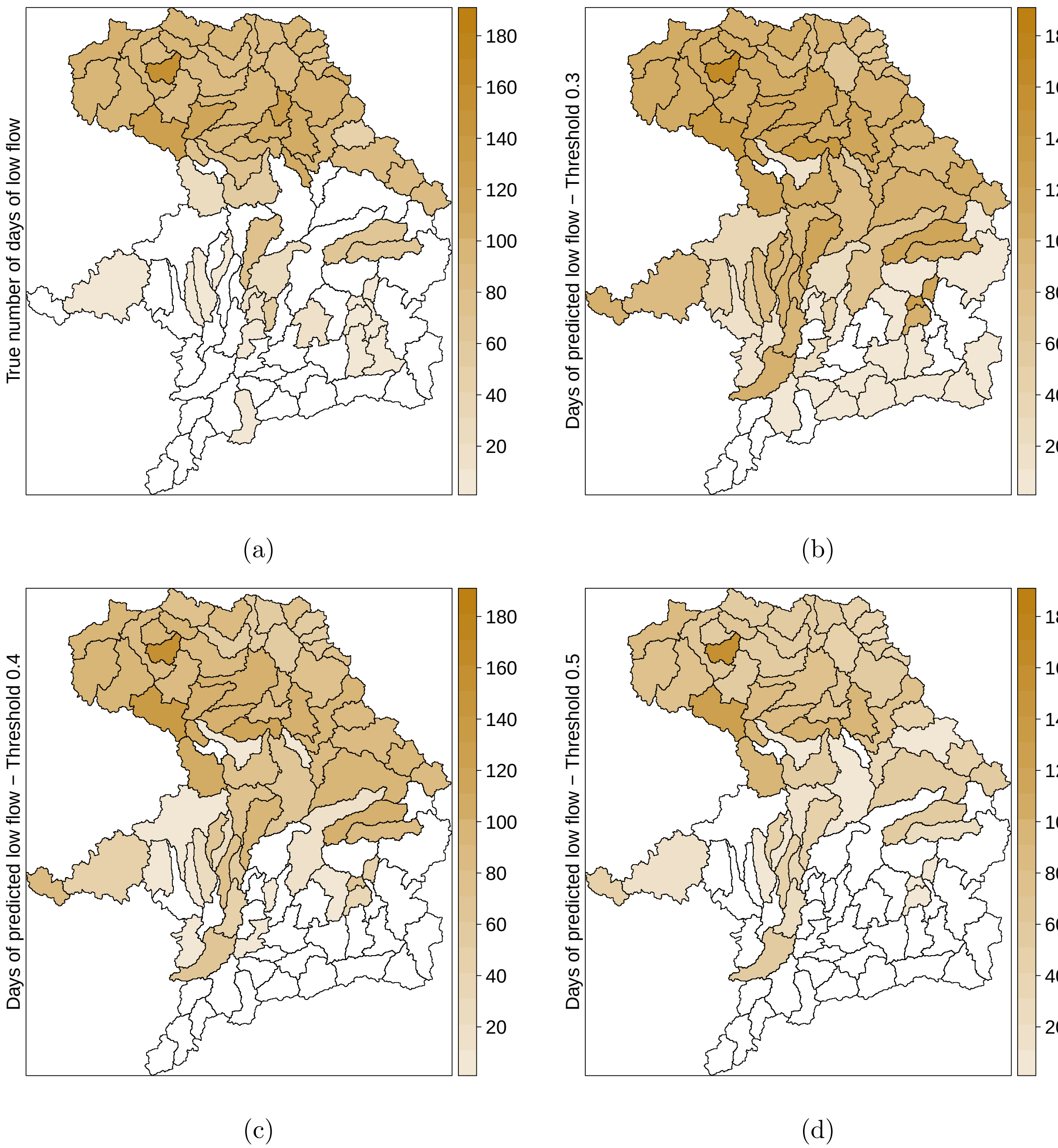
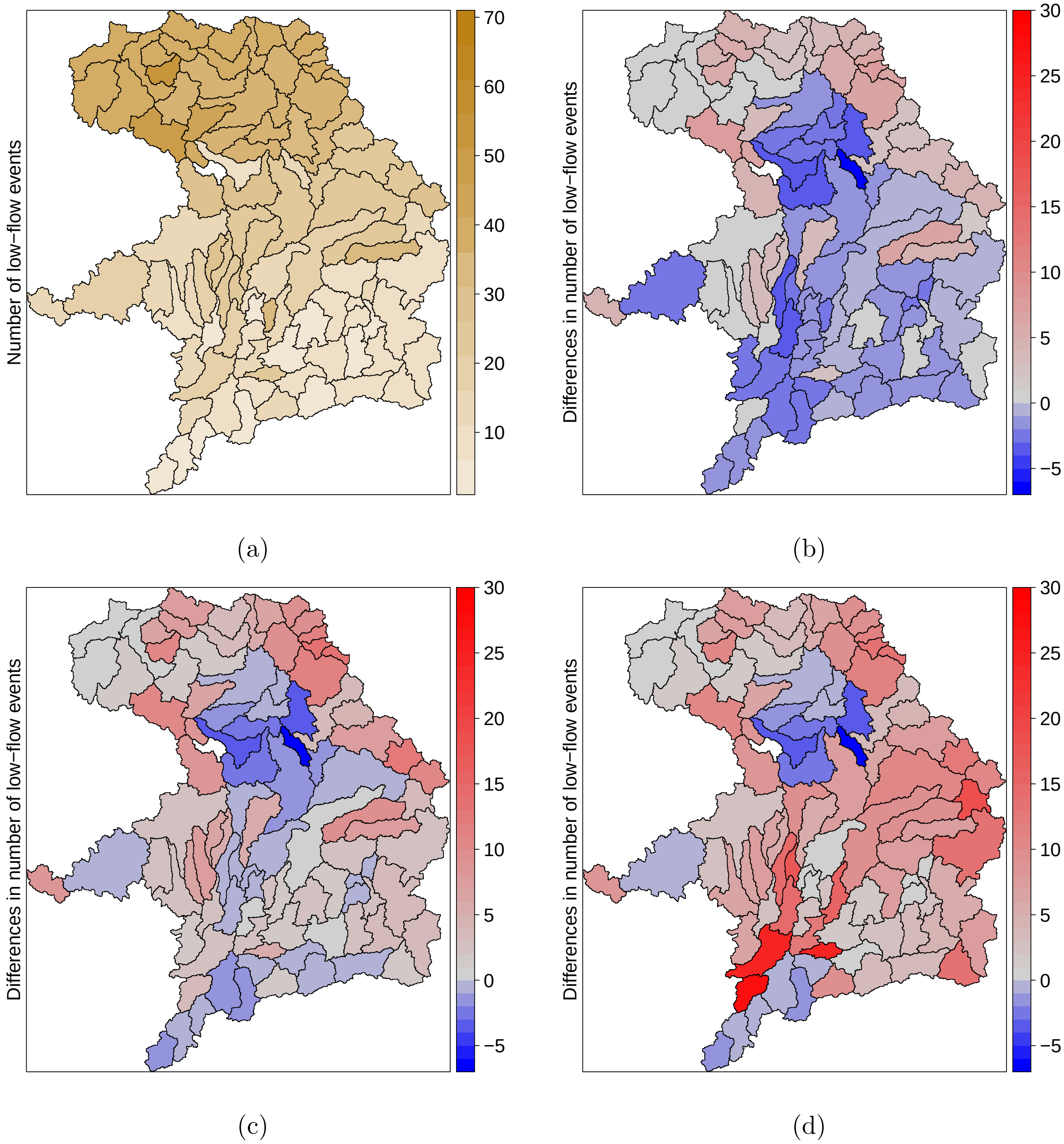



Figure 31: Predicted days of low-flow in summer 2020 for different thresholds. Figure (a) shows the true number of low-flow events for member "kbt". Figure (b), (c) and (d) depict the number of predicted days of low-flow for a threshold of 0.3, 0.4 and 0.5, respectively.

the threshold for the classification of low-flow events is set to $c = 0.4$. In the last step the resulting counts of low-flow days are averaged over all 9 members for each catchment.

In Figure 32 the results for summer are displayed. In Figure 32a, the number of predicted days of low-flow for unmodified data is shown. This serves as a baseline for comparison with more extreme climate scenarios. As in previous analyses, it is mainly the north that is affected by low-flow events, while low-flow only occurs on a few days in the south. The other three figures show the differences in days for three mentioned scenarios, where the colour blue indicates less days than in the baseline, and the colour red more days of predicted low-flow. A 3 °C increase in temperature (see Figure 32b) leads to slightly more days with low-flow in the north and north-east of hydrological Bavaria, while a few days less are observed in the rest. In Figure 32c, in addition to the temperature, precipitation is reduced by 50 %. The overall number of low-flow events increases slightly in many catchments. Changing only precipitation (see Appendix A, Figure 42c) leads to more or the same number of low-flow events in each catchment. If in addition to the modification of temperature and precipitation no snow storage occurs, Figure 32d reveals that in most regions, more low-flow events are expected which exceptions in the north and some isolated catchments in the Alps. In a few catchments, up to 27 more days are observed than in the original forecasts, which represents a drastic increase in low-flow. Overall, these hypothetical climate changes result in additional 519 low-flow days across all catchments in the summer half-year. This is due to a strong effect of the non-occurrence of snow storage (see Appendix A, Figure 42d) on several southern catchments.

In winter, in contrast, the effects of changes in temperature, precipitation and snow storage differ noticeably from the effects in summer. Figure 33a shows once again the baseline number of predicted low-flow events in 2010. As compared to summer, there are less events and the differences between north and south are less pronounced. As visualized in Figure 33b, a 3 °C increase in temperature leads to less low-flows in almost all catchments, with the effects being strongest in southern regions. This impact gets amplified by an increase of precipitation by 50 % as displayed in Figure 33c. In Appendix A, Figure 43c, the impact of the increase in precipitation only can be found. If additionally the snow storage is set to zero, as in summer, the total number of days of low-flow decreases in the Alpine regions (Figure 33d) in comparison to the previous scenario. However, the number of low-flow events in the rest of hydrological Bavaria increase. This counteracts the effect of more precipitation in the previous scenario, some isolated catchments experience even more days of low-flow as in the baseline. Overall, this final climate scenario leads to 519 fewer days with low-flow compared to the predicted events with unchanged data. The intermediate result for modifying snow storage only can be found in the Appendix A, Figure 43d.




(a) (b) (c) (d)

Figure 32: Number of low-flow events in summer of 2010 for unmodified data (a). Figure (b) to (d) show the differences in number of days for three different scenarios. The colour red indicates more, the colour blue less days of low-flow, respectively. In Figure (b), a 3 °C increase in temperature is assessed, while in (c) in addition, precipitation is reduced by 50 %. Figure (d) depicts the difference to the final scenario, where additionally no snow storage occurs.

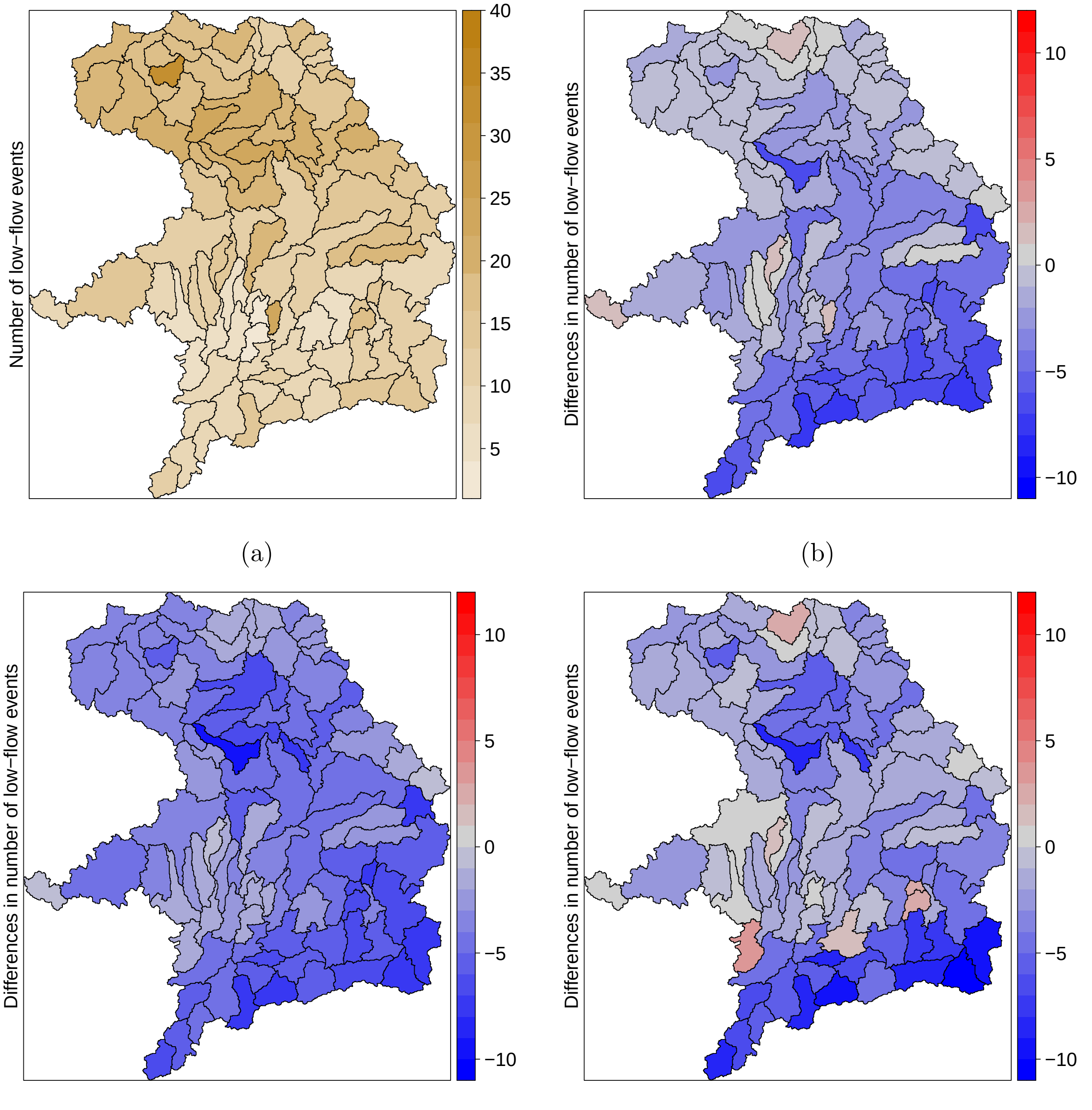



Figure 33: Number of low-flow events in winter of 2010 for unmodified data (a). Figure (b) to (d) show the differences in number of days for three different scenarios. The colour red indicates more, the colour blue less days of low-flow, respectively. In Figure (b), a 3 °C increase in temperature is assessed, while in (c) in addition, precipitation is reduced by 50 %. Figure (d) depicts the difference to the final scenario, where additionally no snow storage occurs.

## 6.4 Clustering

For the last research question, which asks to group the catchments according to the drivers, a K-means algorithm on the averaged coefficients is applied. The optimal number of clusters is obtained by the visual analysis of an elbow plot where based on the summer coefficients K is set to 4 (Figure 13). For the benefit of consistency and comparability, the same number of clusters is used for the winter coefficients, even though the elbow plot lacks a discernible bend (see Appendix A, Figure 44). This results in regional patterns that summarize the effects of the drivers on the occurrence of low-flow events. The clusteroids in Table 5 and Table 6 show the mean effect size for each coefficient within a cluster for the respective season.

Unusually for K-means clustering, the cluster sizes for the summer coefficients vary considerably, ranging from 6 to 43 catchments within a cluster. According to Figure 34a the catchments belonging to cluster 1 (orange) and 4 (purple) are somewhat scattered, while the other two clusters (blue and green) are regionally contiguous. The largest cluster is located in the north of hydrological Bavaria (cluster 1). Moreover, the cluster is defined by its low effects of temperature and by its lack of snow storage, consisting of catchments that do not meet the inclusion criterion for snow storage in summer. Conversely, the smallest cluster, which is located in the centre (cluster 3), has the highest mean effect size of snow storage as well as temperature. The Alpine region summarized by cluster 2 has the highest mean effect size for precipitation and the second highest effect size in soil water. Both of these effects can also be seen when analyzing the results. It is the only cluster where the average temperature coefficient is negative, i.e. as the in temperature increases a decreased odds for low-flow can be expected which might be due to the colder temperatures in this region. The same is true for the interaction between temperature and precipitation as well as snow storage. Cluster 4 is the second largest cluster with 31 catchments and can be found scattered across the centre of hydrological Bavaria. It has the highest mean effect size for soil water.

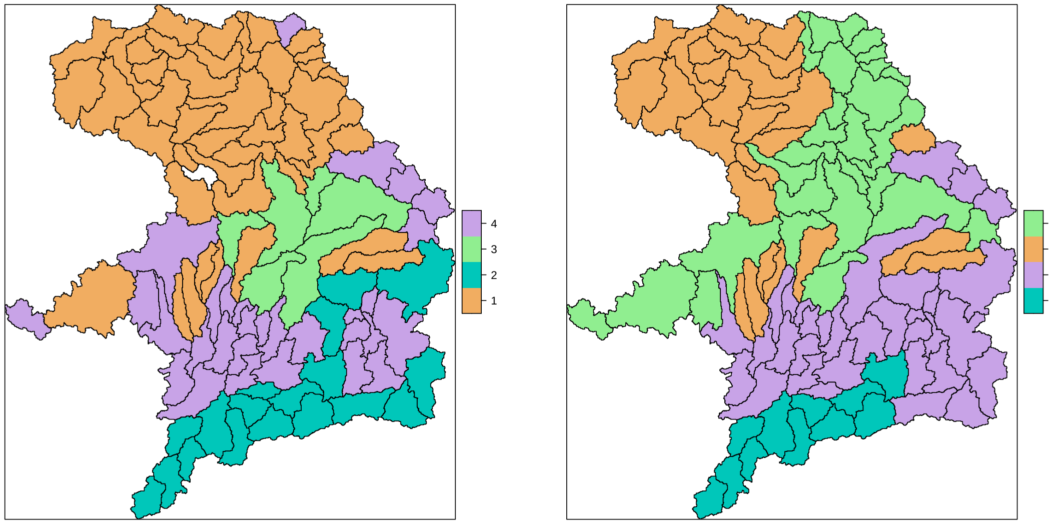
For the winter coefficients the cluster sizes do not vary as much with a range of 11 to 31 catchments within each cluster. Once again only the catchments in purple and orange belonging to cluster 2 and 4 are not regionally contiguous (Figure 34b). The cluster with the strikingly fewest catchments is the one in the Alpine region (cluster 1). Here the snow storage effects are noticeably low, which may be due to the scaling as previously explained. This is also transferable to the interaction term between snow storage and temperature, which is roughly equivalent to a null effect. Compared to the summer coefficients, the ranking of the mean effect size of precipitation is reversed from the highest to lowest mean. For the northern clusters (cluster 3) the mean effect size of precipitation and of snow storage is the highest and of temperature the lowest among the clusters. The mean soil water coefficients in absolute terms are quite high in all clusters, with cluster 2 comprising the catchments with the highest soil water coefficients. This cluster is located in the Pre-Alps and is further characterized by comprising the highest mean effect size for temperature. In contrast to that, the lowest temperature effect can be found in the north in cluster 3. This cluster has the lowest mean effect for precipitation and the highest for snow storage as could also be noticed in the section introducing the coefficients.

| Cluster | | 1 | | 2 | | 3 | | 4 |
|---|---|---|---|---|---|---|---|---|
| Variable | N | Mean | N | Mean | N | Mean | N | Mean |
| Intercept | 43 | -27.575 | 18 | 37.641 | 6 | 2.331 | 30 | 20.733 |
| Precipitation | 43 | -0.356 | 18 | -0.759 | 6 | -0.22 | 30 | -0.474 |
| Temperature | 43 | 0.019 | 18 | -0.878 | 6 | 1.209 | 30 | 0.627 |
| Radiation | 43 | 0.001 | 18 | -0.009 | 6 | -0.003 | 30 | -0.002 |
| Humidity | 43 | -0.094 | 18 | -0.077 | 6 | -0.172 | 30 | -0.083 |
| Snow storage | 43 | 0 | 18 | -0.296 | 6 | -3.402 | 30 | -0.535 |
| Soil water | 43 | -0.63 | 18 | -1.125 | 6 | -0.492 | 30 | -1.204 |
| Temperature:Precipitation | 43 | 0.029 | 18 | -0.022 | 6 | 0.035 | 30 | 0.025 |
| Temperature:Snow storage | 43 | 0 | 18 | -0.013 | 6 | 0.282 | 30 | 0.044 |
| Temperature:Soil water | 43 | -0.014 | 18 | 0.053 | 6 | 0.002 | 30 | 0.008 |

Table 5: Mean effect sizes per cluster and covariate for the averaged coefficients in summer.

| Cluster | | 1 | | 2 | | 3 | | 4 |
|---|---|---|---|---|---|---|---|---|
| Variable | N | Mean | N | Mean | N | Mean | N | Mean |
| Intercept | 11 | 69.826 | 33 | -7.05 | 27 | 0.475 | 27 | 25.332 |
| Precipitation | 11 | -0.183 | 33 | -0.459 | 27 | -1.018 | 27 | -0.799 |
| Temperature | 11 | -0.242 | 33 | -0.258 | 27 | -0.066 | 27 | -0.158 |
| Radiation | 11 | 0.001 | 33 | -0.001 | 27 | 0.002 | 27 | 0.002 |
| Humidity | 11 | -0.023 | 33 | -0.034 | 27 | -0.036 | 27 | -0.052 |
| Snow storage | 11 | 0.003 | 33 | -0.032 | 27 | -0.594 | 27 | -0.171 |
| Soil water | 11 | -1.109 | 33 | -1.859 | 27 | -0.868 | 27 | -0.746 |
| Temperature:Precipitation | 11 | -0.023 | 33 | -0.01 | 27 | 0.009 | 27 | -0.013 |
| Temperature:Snow storage | 11 | -0.001 | 33 | -0.003 | 27 | -0.035 | 27 | -0.017 |
| Temperature:Soil water | 11 | 0.084 | 33 | 0.052 | 27 | 0.014 | 27 | 0.017 |

Table 6: Mean effect sizes per cluster and covariate for the averaged coefficients in winter.




(a) (b)

Figure 34: Map of clusters for averaged coefficients in summer (a) and winter (b). The figures illustrates the regional and seasonal patterns of the drivers.

In conlusion, the cluster analysis confirms the regionality of the effects that emerge in the analysis of the coefficients. Also, the marginal effects of radiation, humidity and the interaction terms are reiterated by their clusteroids.

# 7 Discussion

In this project, the occurrence of low-flow events in hydrological Bavaria is explained by a logistic model for each catchment in each member and each season with a linear trend of year, precipitation, temperature, soil water, snow storage, humidity and radiation as well as interaction terms between temperature and precipitation, snow storage and soil water, respectively. Through this process, regional and seasonal patterns for both the effects and their significance are identified. The covariate soil water shows the most amount of significant effects while the least amount is with the interactions. Based on these findings, it can be concluded that the effects of the interaction terms are not of major importance in modelling low-flow events. Hence, hydrological droughts are rather a result of extreme drivers than of compound events. Additionally, a climate scenario analysis reveals that the scenario without snow storage, increased temperature and 50% less or more precipitation for summer and winter respectively shows a regionally dependent, partly drastic increase in low-flows in summer and decrease in winter. This regionality is further analysed in a K-means clustering, which groups the catchments according to the impact of the drivers. The algorithm identifies four clusters for each season and provides information on similar catchments in terms of driver effects.

The evaluation of goodness-of-fit criteria shows that the models fit the data well. Nevertheless, the approach presented is subject to some limitations. Each driver is included as a linear trend that sufficiently captures its impact. However, adding non-linear trends to the models could possibly improve the fit even further. A disadvantage of this extension is the difficulty of including interaction terms, and averaging the effects across members is not straightforward. Moreover, the structure of the time lag is rather simple and might lack some flexibility. Due to the small-scale modelling approach, catchment-specific time-constant variables such as land use can not be taken into account, although they might be influencing factors in explaining low-flow events. As for estimation, the paucity of number of low-flow events in some catchments is at times challenging. Particularly in the south, only a small percentage of days are classified as events, making it more difficult to fit an appropriate model. Nevertheless, the ROC analysis shows a sufficient level of accuracy. Caution is also needed in assessing the realism of the climate scenarios, as the results should not be overestimated. For instance, an increase in temperature would have many knock-on effects on other drivers such as relative humidity and radiation. However, this is not something that the model presented here is able to do on the basis of the data at hand. With regard to clustering, the elbow plot for the winter coefficients gives no indication of an optimal number of clusters. Thus, $K$ is set based on practical reasons rather than statistical considerations. This allows a direct comparison between the regional groupings of the averaged coefficients and their seasonal developments.

One possible future modelling approach to circumvent this rigid lag structure is a distributed lag model (Aßenmacher et al. (2019)). These provide a flexible determination of a time lag between each covariate and the response variable. Another modelling approach that could make estimation more accurate is a generalized linear mixed model where the dependency structure between the catchments and the members could be included via random intercepts.

However, in order to take the regional differences of drivers into account random slopes for the drivers are needed, which is computationally difficult.

To conclude, this project is able to develop modelling approaches for the explanation of hydrological droughts. Further research could entail a comparison between simulated and real data.

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# A Appendix

Here is a collection of additional material consisting of drainage-driver relation plots, effect plots for interaction terms, ROC analysis for the winter models, intermediate results of the scenarios and an elbow plot for the clustering of winter coefficients.

## A.1 Descriptive Analysis: Drainage-Driver Relationships

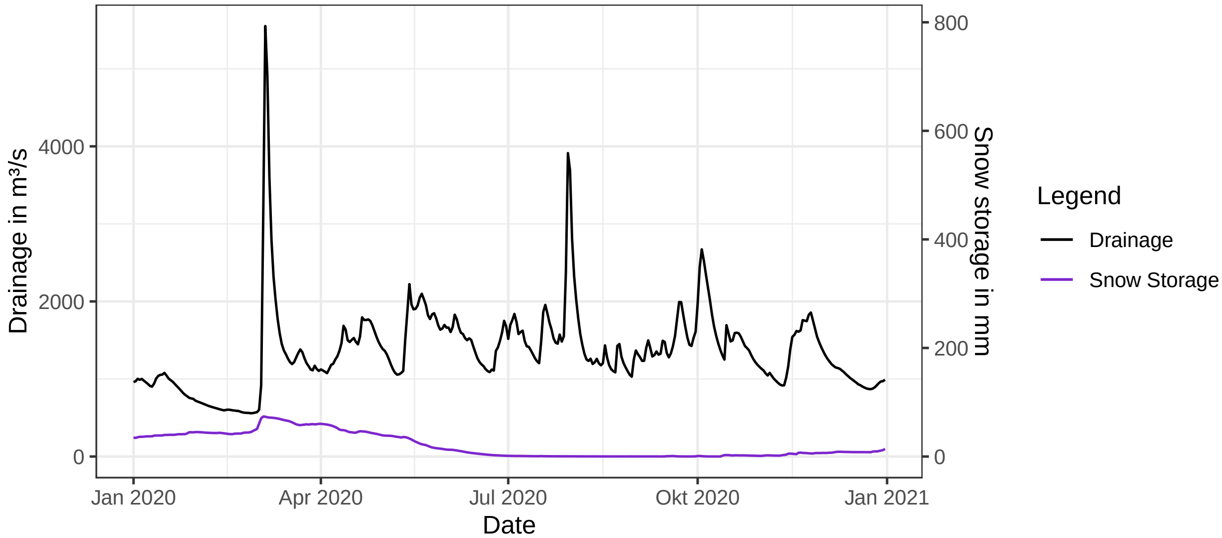
In subsubsection 3.1.3, the relationship between drainage and two drivers, precipitation and temperature, is established. Below, the relationship between drainage and the remaining drivers in the data set is examined in more detail.

The following figures still refer to the catchment area "Donau-Achleiten" (see Figure 3a) in 2020 for member "kbt". Figure 35a shows drainage in black and snow storage in purple. Obviously, snow storage is greatest in winter and spring, indicating a strong seasonality of occurrence. At a certain point in March, both drainage and snow storage seem to peak. As this peak also occurs in precipitation (see Figure 8a), it is likely to be due to precipitation in the form of snowfall, which then remains as snow storage. Otherwise there is no clear direct relation.

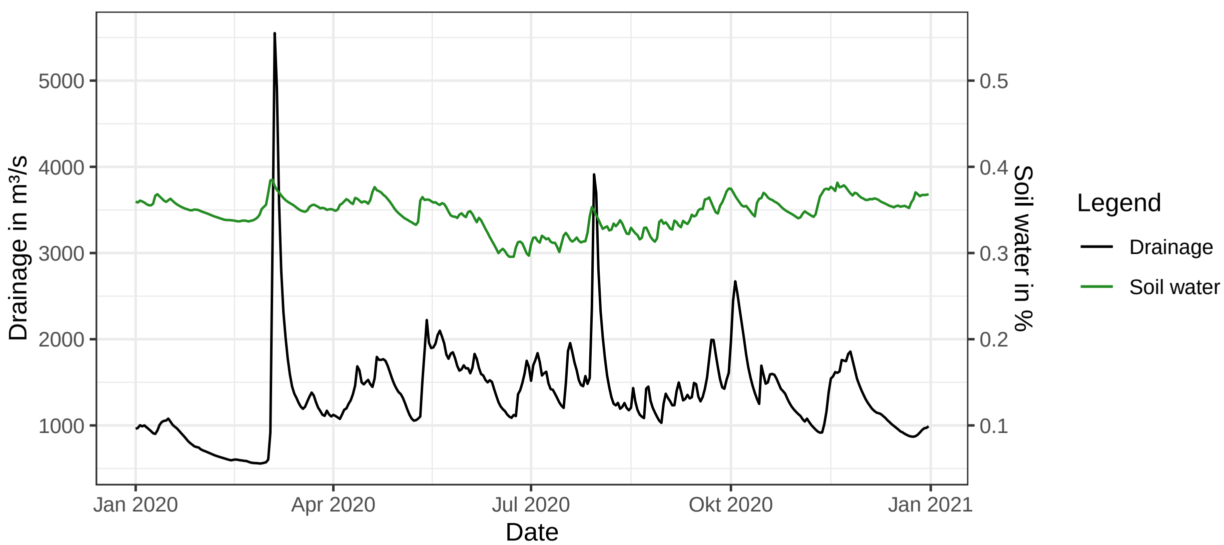
In Figure 35b drainage is displayed in black while the green curve depicts soil water. Almost every high and low point of the drainage is captured by soil water with a delay of a few days, indicating a direct time-lagged relationship just as established for precipitation.

Looking at relative humidity and drainage in Figure 36a reveals no visible direct relationship. Nevertheless, the figure provides information about the relative humidity over the course of the year. As can also be seen from Table 2, humidity tends to be higher in winter than in summer.

The same applies to radiation in Figure 36b. Although no direct relationship can be inferred from the graph, the radiation follows a bell shape over the course of the year, with its peak in the middle of the year.

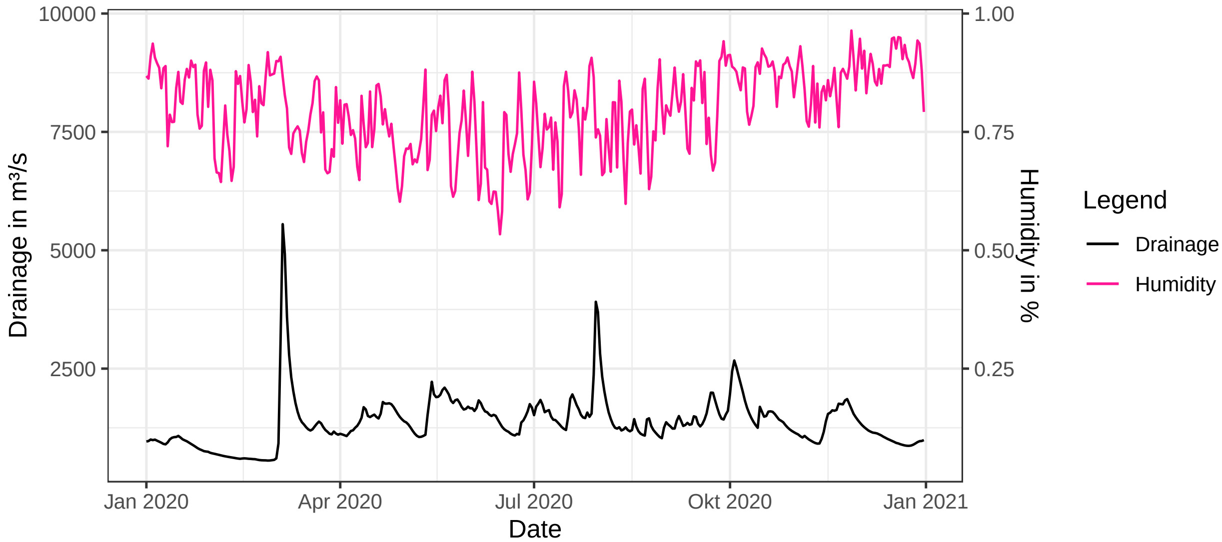



(a)

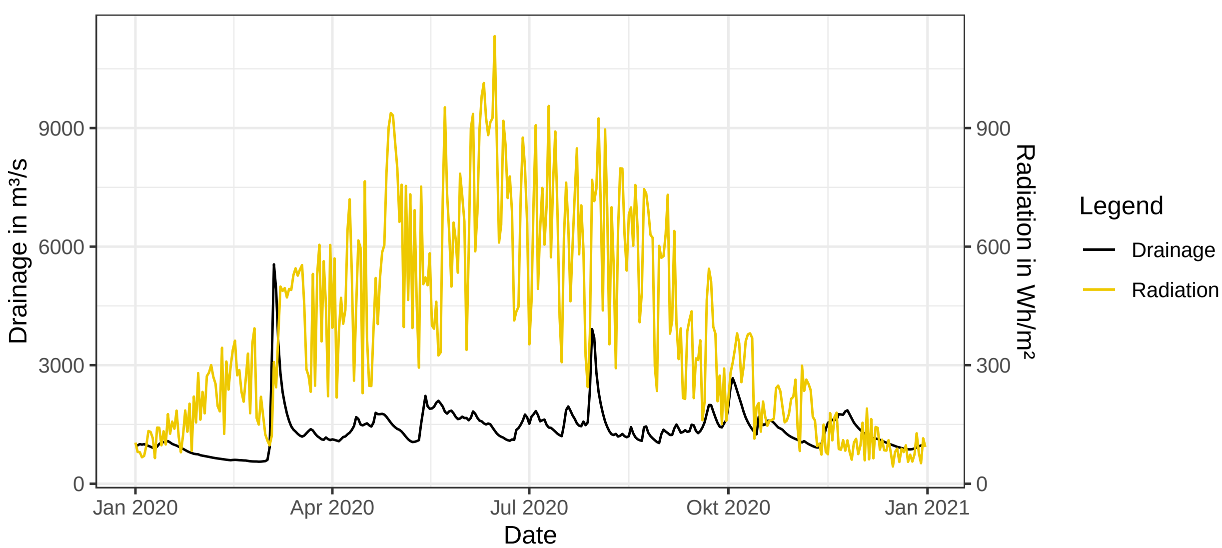



(b)

Figure 35: Time series of drainage and snow storage (a) and drainage and soil water (b) in 2020 for catchment "Donau-Achleiten" (see Figure 3a).




(a)




(b)

Figure 36: Time series of drainage and relative humidity (a) and drainage and radiation (b) in 2020 for catchment "Donau-Achleiten" (see Figure 3a).

## A.2 Effects: Interaction Terms

The models introduced in section 5 include three interaction terms: temperature with precipitation, snow storage and soil water, respectively. So far, only the changing main effect has been considered in detail. The following Figure 37, Figure 38 and Figure 39 show the pure interaction effects and their significance.

As with the main effects, there are clear seasonal and regional differences. Caution is warranted in two respects: first, the estimated effects are very small and thus have little influence on the prediction of low-flow; and second, significant effects are only detectable in very isolated cases, if at all. The conclusion can therefore be that there is no reason for the assumption that low-flow events are compound events.

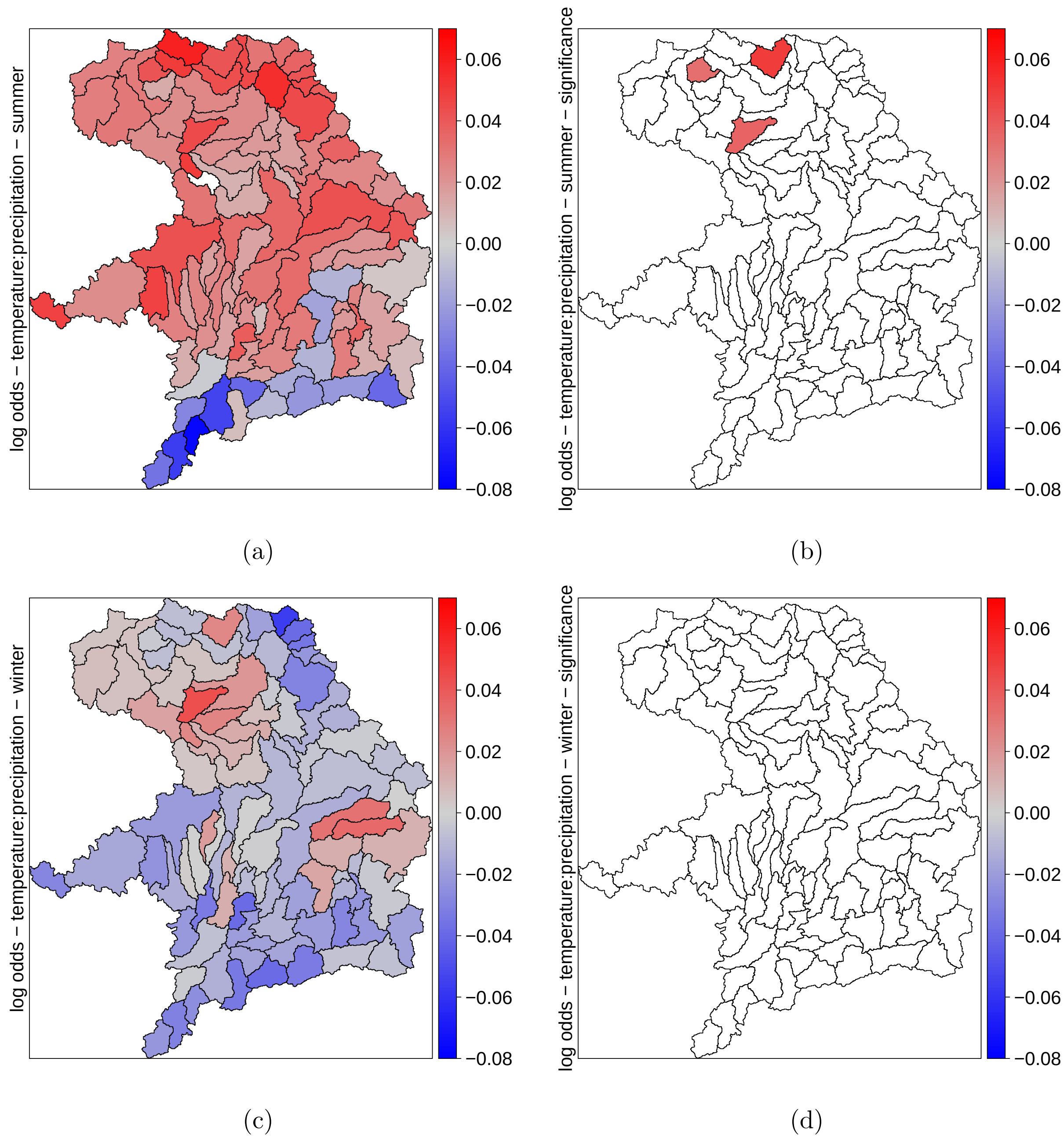



Figure 37: Effects of the interaction of temperature and precipitation in summer (a) and winter (c) and the corresponding significance (b) and (d), respectively. Non-significant effects are displayed in white.

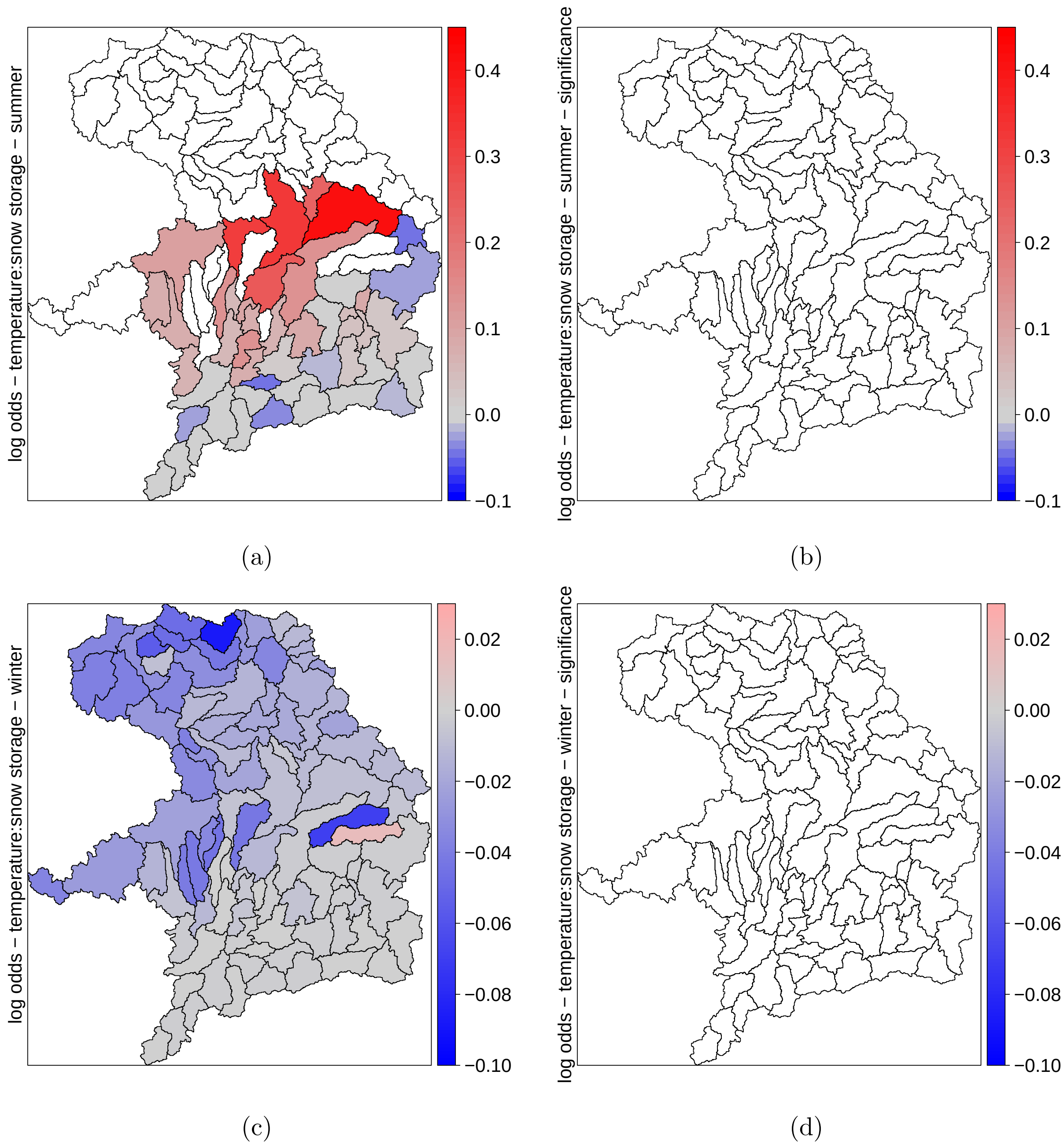



Figure 38: Effects of the interaction of temperature and snow storage in summer (a) and winter (c) and the corresponding significance (b) and (d), respectively. Non-significant effects are displayed in white.

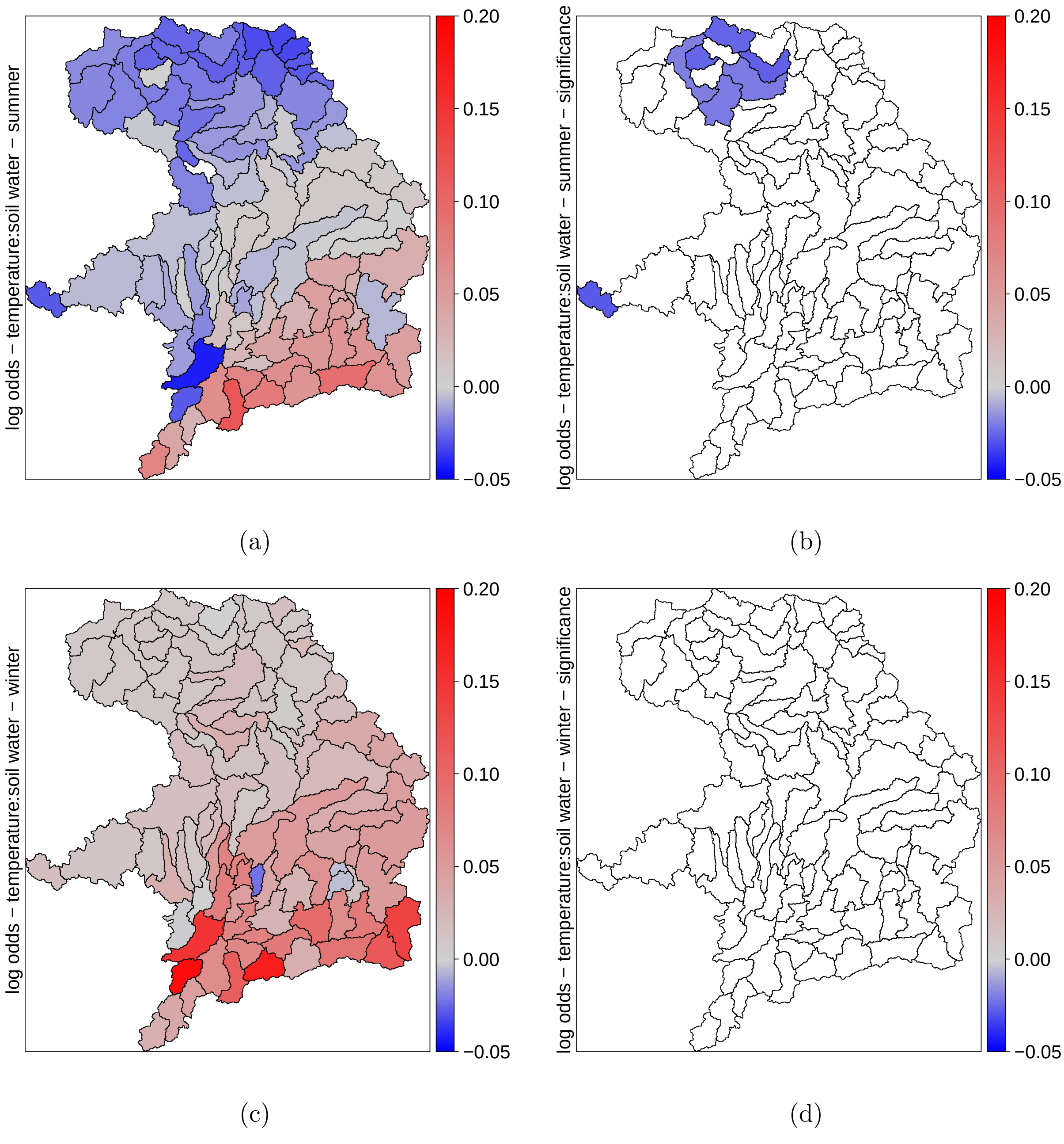



Figure 39: Effects of the interaction of temperature and soil water in summer (a) and winter (c) and the corresponding significance (b) and (d), respectively. Non-significant effects are displayed in white.

## A.3 Threshold Analysis for Winter Models

In the threshold analysis carried out in subsection 6.2, only the summer models were taken into account. Now the winter models and their corresponding threshold for the prediction are considered in more detail.

Analogous to summer, Figure 40 shows the sensitivity and specificity together with the TDR for four example catchments. Looking at the intersection points of TPR and TDR, they are all between 0.25 and 0.4 for these examples. Hence, the threshold tends to be slightly lower than for the summer models. For consistency, the threshold is nevertheless set to $c = 0.4$ for each model. Note that the FPR is very low for every possible threshold $c$, as it is in summer, indicating a good determination of the days without low-flow.

Comparing the number of true low-flow days in winter 2020 for member "kbt" in Figure 41 on the left with the number of predicted low-flow days for a threshold of 0.4 on the right, we see that although some catchments with low-flow events remain undetected, the most important catchments with low-flow are identified in the north of hydrological Bavaria.

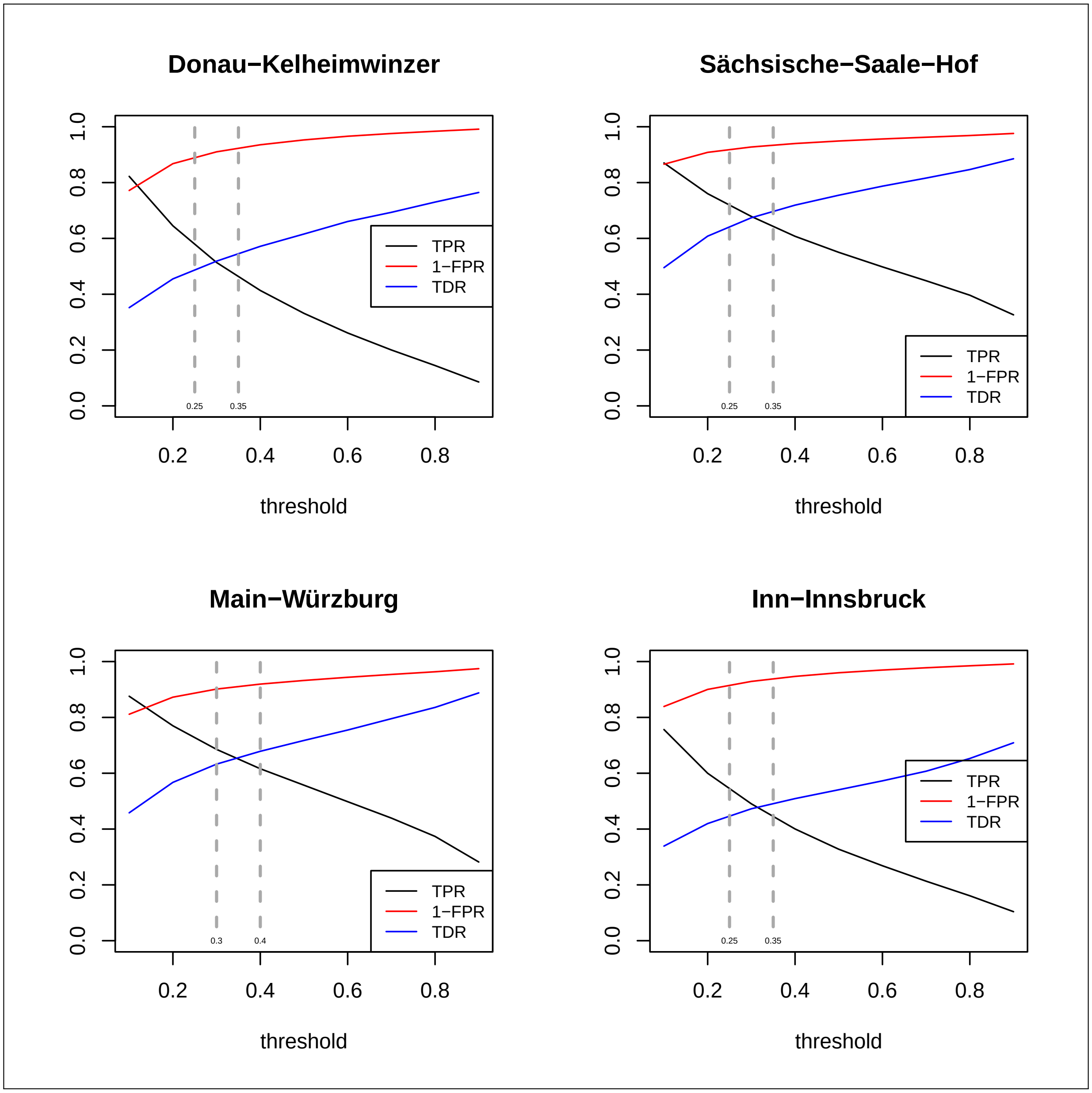
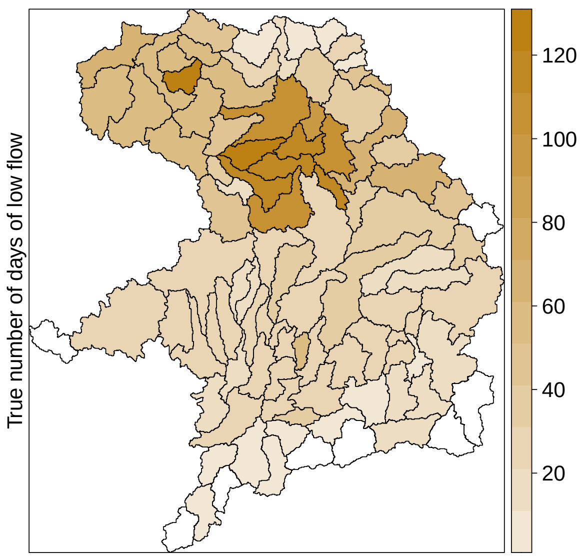
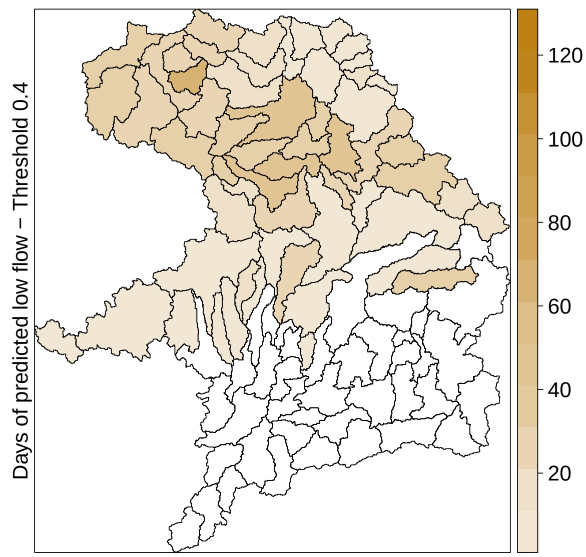



Figure 40: Threshold analysis including TPR (black), FPR (red) and TDR (blue) for four different catchments in winter. The range of the intersection points of TPR and TDR are marked as grey dotted lines.




(a)




(b)

Figure 41: Figure (a) shows the true number of low-flow events for member "kbt" in winter 2020, while Figure (b) depicts the number of predicted days of low-flow for a threshold of 0.4.

## A.4 Scenario Analysis: Intermediate Results

In subsection 6.3, the analysis of the climate scenarios consists of three steps: first, the temperature is increased by 3°C, second, precipitation is additionally reduced or increased by 50%, and third, the scenario is extended to no snow storage. In order to better disentangle which driver contributes to which difference in low-flow days, the intermediate results of the change in individual drivers are presented below. This means in particular the number of more or fewer days for the change in temperature, precipitation and snow storage separately.

Figure 42a shows the number of low-flow events predicted for the unmodified data set, while Figure 42b shows the differences in the number of days for a 3°C temperature increase. See subsection 6.3 for a detailed interpretation. In Figure 42c the difference in the number of low-flow days for a 50% decrease in precipitation is visualized. As expected, the number of events increases by a few days or remains constant. It is notable that there is no clear regionality. The effects in this scenario are only moderately pronounced, but considering the change in snow storage in Figure 42d, there is a drastic increase in low-flow days in summer in some catchments in the southern half of hydrological Bavaria. The partly strong increase in the number of events in Figure 32d is therefore mainly caused by precipitation and snow storage.

A look at the climate scenarios in winter shows a different picture. The number of predicted low-flow events for unchanged data in Figure 43a shows only a weak regional trend towards the north. A 3 °C increase in temperature leads to fewer days with low-flow, especially in the Alpine regions as visualized in Figure 43b. This effect is reversed by a 50 % increase in precipitation (Figure 43c). The number of events decreases especially in the north of hydrological Bavaria, while the south is less affected and there are hardly any differences. Finally, looking at the absence of snow storage in Figure 43d, a clear regional pattern becomes apparent. While the number of low-flow events decreases in the Alpine regions, the same number or more days with low-flow are observed in the other areas. For the winter, it is not possible to identify a clear driver for the combined effects in Figure 33d.

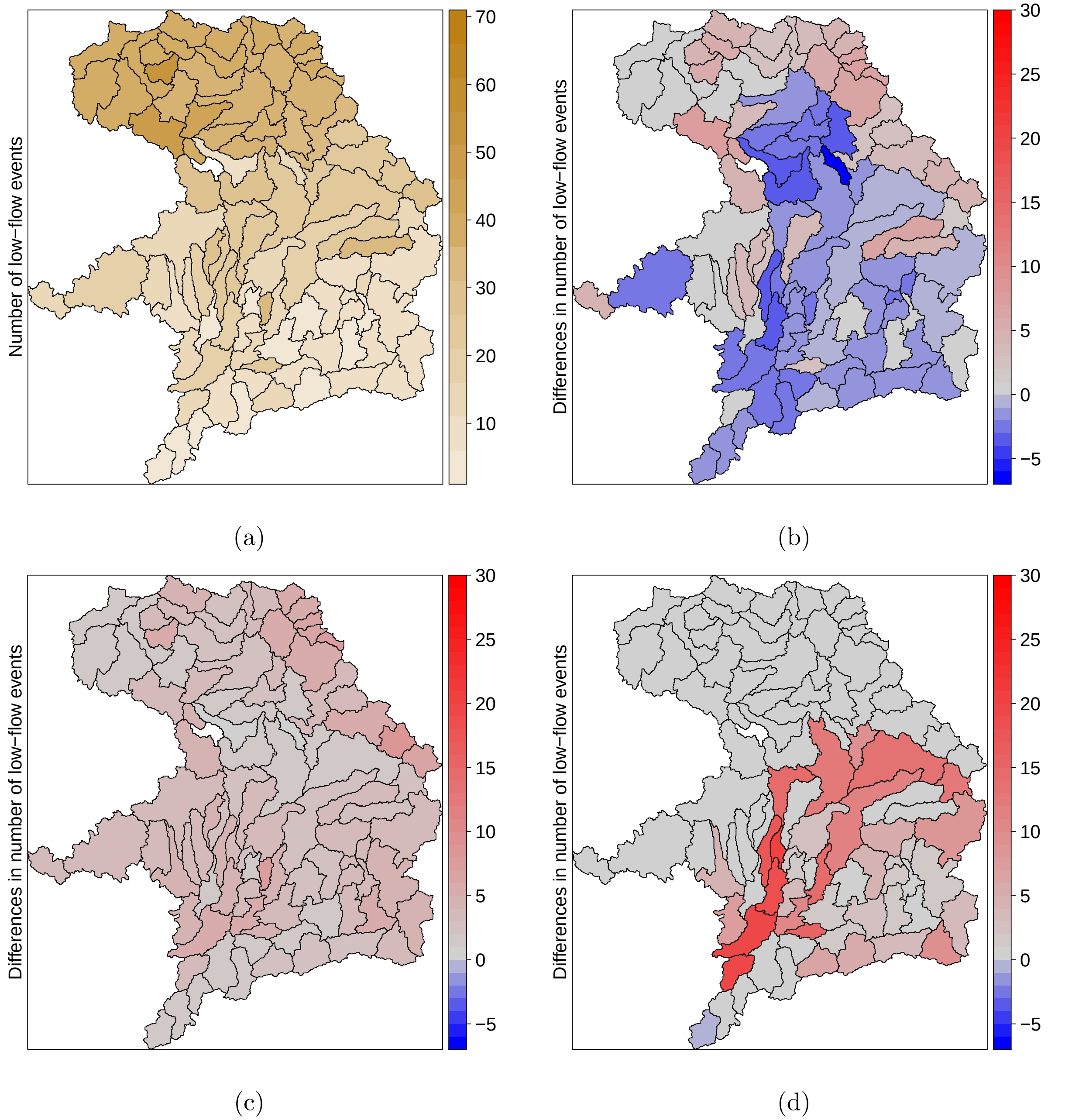



Figure 42: Number of low-flow events in summer of 2010 for unmodified data (a). Figure (b) to (d) show the differences in number of days for three different scenarios. The colour red indicates more, the colour blue less days of low-flow, respectively. In Figure (b), a 3 °C increase in temperature is assessed, while in (c) precipitation is reduced by 50 %. Figure (d) depicts the difference to the scenario, where no snow storage occurs.

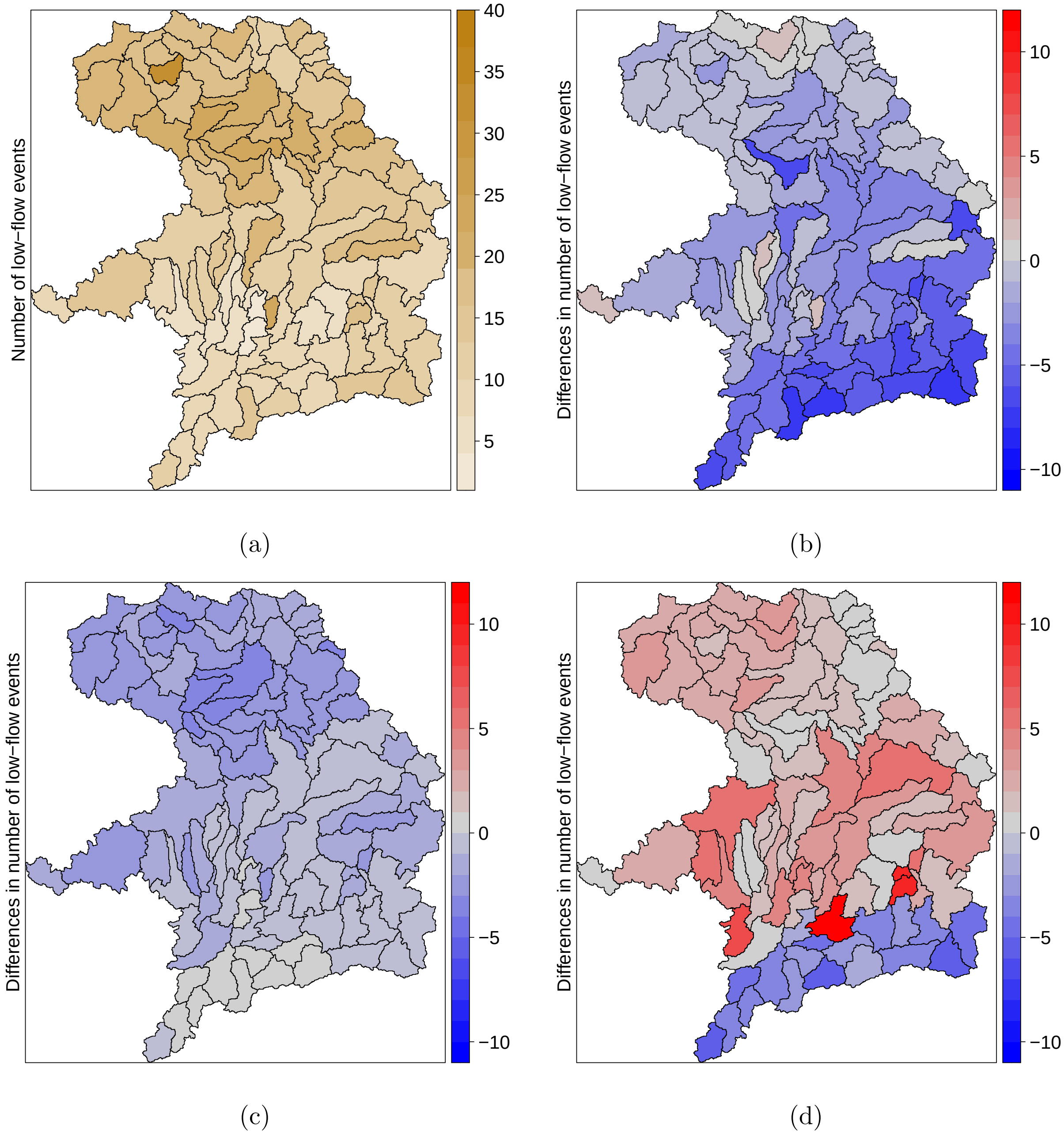



Figure 43: Number of low-flow events in winter of 2010 for unmodified data (a). Figure (b) to (d) show the differences in number of days for three different scenarios. The colour red indicates more, the colour blue less days of low-flow, respectively. In Figure (b), a 3 °C increase in temperature is assessed, while in (c) precipitation is reduced by 50 %. Figure (d) depicts the difference to the scenario, where no snow storage occurs.

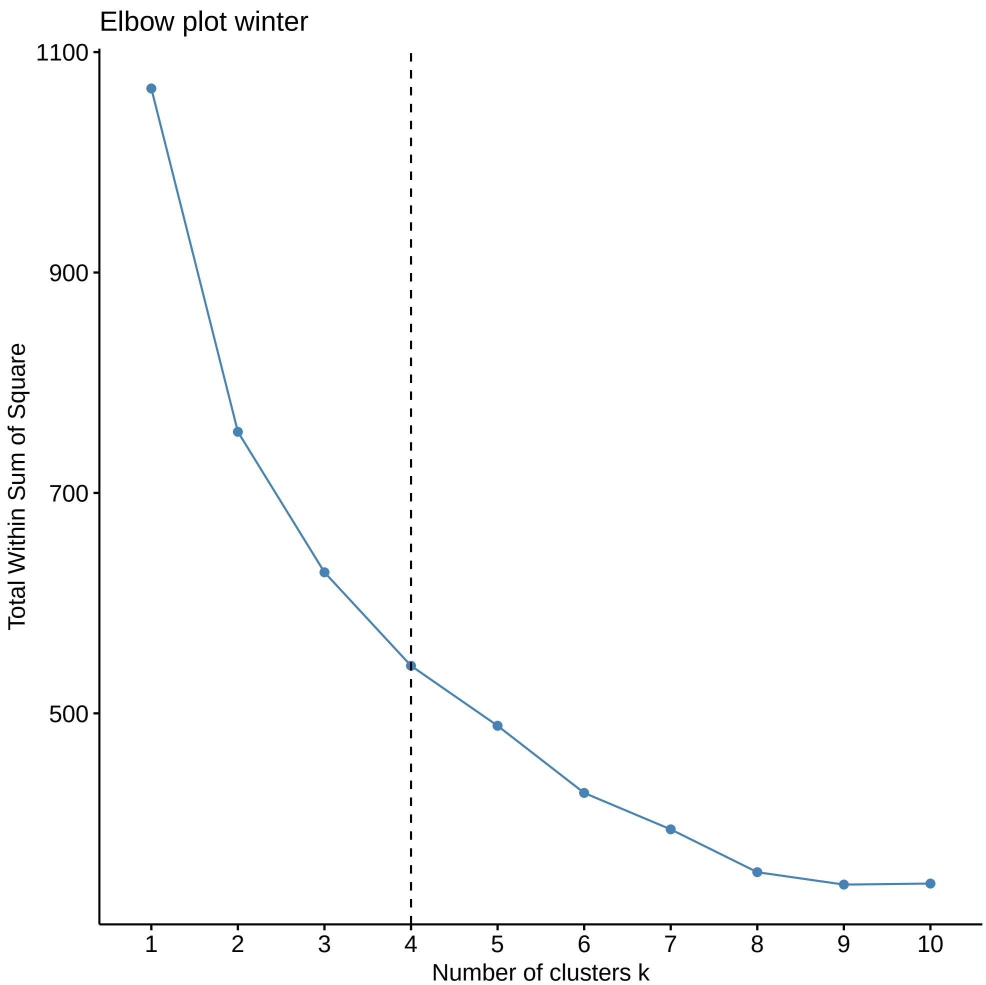



Figure 44: Elbow plot for member-averaged coefficients in winter.

## A.5 Clustering: Elbow Plot for Winter

The elbow method is used to determine the optimal number of cluster $K$ in the clustering procedure. Figure 44 shows no indication for $K$. Hence, $K$ is set to 4 just as for summer.

# B Electronic Appendix

All codes, figures, documents and tables can be found on GitHub[1]. While the **Plots** folder contains all the figures used in this report and the oral presentation as pdf files, including plots of the descriptive analysis, effects, ROC analysis, scenarios and clustering, the **Documents** folder consists of this report and the oral presentation slides. The folder **Models** stores all the 1959 models as RDS files.

In the **Codes** folder is a collection of all R files used for data pre-processing, modelling, visualisation and further analyses such as clustering, etc. In order to reproduce this study, the steps below are necessary:

**01_Data_Preparation:** In this folder two files are stored, one consisting of functions (`01_Data_preparation_functions.R`) that are called in the main file (`02_Data_generation.R`). The principal outcomes of the codes are the following:

- Transform data to long format
- Merge time-varying and time-constant variables to get a full data set
- Aggregate data to daily means and sums
- Calculate low-flow variable
- Create summer and winter data sets with rolling averages of driver variables

To fully reproduce the code, the path has to be adjusted and the raw data needs to be provided.

**02_Descriptive_Analysis:** The folder contains files to generate the plots in section 3, namely `Descriptive_Analysis.R` for the analysis of member "kbt" and `Comparison_Member.R` for the comparison of the members. In order to reproduce the code, the complete data set generated in the previous step is necessary as well as the shape file of hydrological Bavaria.

**03_Modelling:** In the folder **Models** the code for the summer and winter models (`Logit_summer_models.R` and `Logit_winter_models.R`) is stored next to the step-by-step derivation (see subsection 5.2, `Logit_step_by_step.R`) and the analysis of significance (`Significance_Effects.R`). To fully reproduce the code, the paths need to be adapted and the generated data set needs to be available. The resulting models are all contained in the **Models** folder. The effect plots and box plots shown in section 6 are all obtained by running files `Visualization_effects.R` and `Boxplots_members_effects.R` in folder **Visualization**. In order to rerun this code, the tables stored in folder **tables** are required.

**04_ROC_Analysis:** The ROC Analysis is divided into two parts: first, `ROC_analysis.R` is the code that is used for the initial analysis to determine an appropriate threshold and its

[1] https://github.com/TheresaMeier/Consulting_Hydrology

visualization is done in `ROC_analysis_visualization.R` (see e.g. Figure 30). The example predictions in Figure 31 are based on analyses in `ROC_analysis_events.R` and its visualization `ROC_analysis_events_visualization.R`. For the reproduction of this code, the summer and winter data set is needed together with evaluation tables stored in folder **Tables**.

**05_Clustering:** This folder contains the file for the cluster analysis, namely `Clustering.R`. To reproduce the code the averaged coefficients for summer and winter from the folder **Tables**, the map for visualisation and the data set is needed. Also, the paths should be adjusted.

**06_Scenarios:** The analysis of all scenarios is contained in file `Scenario_analysis.R` and its visualization in `Scenario_visualization.R`. To fully reproduce the code, the data sets and the models are required and the paths have to be adjusted. The tables for the visualization are stored in **Tables**.

# Declaration of Authorship

We hereby declare that the report submitted is our own unaided work. All direct or indirect sources used are acknowledged as references. We are aware that the report in digital form can be examined for the use of unauthorized aid and in order to determine whether the report as a whole or parts incorporated in it may be deemed as plagiarism. For the comparison of our work with existing sources we agree that it shall be entered in a database where it shall also remain after examination, to enable comparison with future reports submitted. Further rights of reproduction and usage, however, are not granted here. This paper was not previously presented to another examination board and has not been published.

Munich, April 3rd, 2023

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