Technical Report

A survey of multi-agent formation control: Position-, displacement-, and distance-based approaches

Number: GIST DCASL TR 2012-02

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December 17, 2012

A survey of multi-agent formation control: Position-, displacement-, and distance-based approaches

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Abstract—**This report presents a brief survey of formation control of multi-agent systems. Focusing on types of actively controlled variables to achieve prescribed desired formations, we categorize the existing results on formation control into position-, displacement-, and distance-based approaches. We summarize typical problem formulations in the approaches and review the existing results. Then, we provide some comments on these approaches as well as possible future research directions. Some other important recent results are also briefly reviewed.**

I. INTRODUCTION

Recently, a significant amount of research interest has been focused on multi-agent systems due to both their practical potential in various applications and theoretical challenges arising in coordination and control of such systems. Applications of multi-agent systems include, for instance, mobile robots, unmanned aerial vehicles, satellites, sensor networks, power grids, etc. Theoretical challenges mainly arise from that agents need to be coordinated and controlled to achieve their objective cooperatively without a centralized coordinator while they have limited sensing and computational capabilities.

Among various research topics within the realm of multi-agent systems, formation control is the one of the most dominantly studied topics. In formation control problems, agents aim to control their states so that a prescribed geometric shape of the states is achieved. Excellent reviews on formation control problems are found in Chen and Wang (2005); Mesbahi and Egerstedt (2010); Olfati-Saber *et al.* (2007); Ren and Cao (2010); Ren *et al.* (2005, 2007*b*). However, Chen and Wang (2005); Mesbahi and Egerstedt (2010); Olfati-Saber *et al.* (2007); Ren *et al.* (2005, 2007*b*) have mainly focused on formation control problems related with consensus, and thus some important results, particularly on inter-agent distance-based formation control, have not been extensively reviewed in such works. Scharf *et al.* (2004) have presented a survey of formation control, but their work has focused on formation flying of spacecrafts. An excellent introduction of inter-agent distance-based formation control is found in Anderson *et al.* (2008), but many results have been published thereafter.

The aim of this survey is to briefly review the existing results on formation control. From our perspective, research interest on formation control has migrated from centralized to decentralized approaches. Here, the centralized approach means that a desired formation in a multi-agent system is achieved by using global sensing information, while in decentralized approach, only local sensing information is utilized to achieve a desired formation. It is supposed that the desired formation in both centralized and decentralized approaches is given initially or may be assigned during the control task through a communication network. We believe it is timely to overview such research tendency while enumerating recent issues. Moreover, many formation control frameworks have been proposed to attack various problems formulated in decentralized approaches. We expect that it is beneficial at this point to review such frameworks for further research. Finally, measurement and control structures of agents (i.e., what variables are actively measured and controlled to achieve prescribed desired formations) play a crucial role, particularly, in decentralized formation control approaches. It is of interest to see how such structures affect problem formulations and control strategies in decentralized formation control.

Due to the vast amount of literature on formation control, it is a huge and challenging task to exhaustively review the existing results. In this survey, rather than an exhaustive review, we focus on measurement and control structures of agents. Depending on the types of actively controlled variables, we categorize the existing results into the following three approaches:

- Position-based approach: Agents measure their own positions with respect to a global coordinate system. The desire formation is prescribed by the desired positions for the agents. Then the desired formation is achieved by actively controlling the positions of the individual agents.
- Displacement-based approach: Each agent measures the relative positions (displacements) of its neighboring agents with respect to a global coordinate system. The desired formation is specified by the desired displacements between pair of the agents. Then the agents, without any knowledge of their positions, achieve the desired formation by actively controlling the displacements of their neighboring agents.
- Distance-based approach: Each agent measures the relative positions of its neighboring agents with respect to its own local coordinate system without any knowledge on a global coordinate system. The desired formation is then specified by the desired distances between pair of the agents. The agents achieve the desired formation by actively controlling the distances of their neighboring agents.

By categorizing the existing results into the three approaches, we can clearly see how the measurement and control structures of agents affect problem formulations, objectives of formation control, and control strategies. However, since there have been a variety of formation problem formulations, it is

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not easy to cover all the existing results by the categorization based on a criterion. Thus, some other categorization criteria are additionally discussed and some important results that are not categorized by the criterion in this report are also reviewed. Further, we provide an extensive summary of the recent results in distance-based approach. Distance-based approach is of interest because it does not require agents to share any global information such as a common sense of orientation.

The remainder of this report is organized as follows. In Section II, we provide a brief summary of graph theory required for further discussions in this report. In Section III, we formulate a general formation control problem and discuss various classifications of the existing approaches to formation control. In Sections IV, V, and VI, we summarize the formation problem formulations in position-, displacement-, and distance-based approaches and review some recent significant results in each approach, respectively. Discussions, summary, and further issues are provided in Section VII. In Section VIII, an overview on network localization problems under formation control problem setups is provided considering the close relationship between network localization and formation control. Some other important results that do not belong to one of the three approaches are reviewed in Section IX. Finally, concluding remarks and future works are discussed in Section X.

II. Preliminaries

A. Notations

The set of non-negative (respectively, positive) real numbers is denoted by $\bar{\mathbb{R}}_+$ (respectively, $\mathbb{R}_+$). Given a set S, $|S|$ denotes the cardinality of S. Given a vector x, $\|x\|$ denotes the Euclidean norm of x. Given a matrix A, $\mathrm{Im}(A)$ denotes the image of A. The rank of A is denoted by $\mathrm{Rank}(A)$. The matrix I_n denotes the n-dimensional identity matrix. Given two matrices A and B, $A \otimes B$ denotes the Kronecker product of the matrices.

B. Graph theory

Sensing, communication, and other interaction topologies in multi-agent systems are naturally modeled by directed or undirected graphs. In this subsection, we briefly summarize basic notions in graph theory. Details are found in Godsil and Royle (2001).

A directed graph is defined as a pair $\mathcal{G} := (\mathcal{V}, \mathcal{E})$, where $\mathcal{V}$ is the set of nodes and $\mathcal{E} \subset \mathcal{V} \times \mathcal{V}$ is the set of ordered pairs of the nodes, called edges. We assume that there is no self-edge, i.e., $(i,i) \notin \mathcal{E}$ for $i \in \mathcal{V}$. The set of neighbors of $i \in \mathcal{V}$ is defined as $\mathcal{N}_i := \{j \in \mathcal{V} : (i,j) \in \mathcal{E}\}$. A directed path of $\mathcal{G}$ is a sequence of edges of the form $(v_{i_1}, v_{i_2}), (v_{i_2}, v_{i_3}), \ldots, (v_{i_{n-1}}, v_{i_n})$, where $v_{i_1}, \ldots, v_{i_n} \in \mathcal{V}$. A directed graph is said to be strongly connected if there is a path from any node to any other nodes. For $(i,j) \in \mathcal{E}$, i is the head node and j is the tail node of $(i,j) \in \mathcal{E}$. If $(i,j) \in \mathcal{E}$, j is called a parent of i and i is called a child of j. A tree is a directed graph where a node, called the root, has no parent and the other nodes have only one parent. A spanning tree of $\mathcal{G}$ is a directed tree containing every node of $\mathcal{G}$.

We assume that the edges of $\mathcal{G}$ are weighted by positive real numbers, i.e., a positive real number is associated with each edge. Let a_{ij} be the weight associated with $(i,j) \in \mathcal{E}$. Then the Laplacian matrix $L = [l_{ij}] \in \mathbb{R}^{|\mathcal{V}| \times |\mathcal{V}|}$ of $\mathcal{G}$ is defined as

$$l_{ij} = \begin{cases} -\sum_{k \in \mathcal{N}_i} a_{ik}, & \text{if } i = j; \\ a_{ij}, & \text{if } (i,j) \in \mathcal{E}; \\ 0, & \text{otherwise.} \end{cases}$$

The following properties are well known for any Laplacian matrix $L \in \mathbb{R}^{N \times N}$ (Ren *et al.*, 2004):

- All eigenvalues of L are in the closed right half complex plane;
- There exists a zero eigenvalue of L with its corresponding eigenvector 1_N.

If the graph associated with L has a spanning tree, the following is true (Ren *et al.*, 2004):

- There exists a distinct zero eigenvalue of L with its corresponding eigenvector 1_N;
- All nonzero eigenvalues of L are in the open right half complex plane;
- There exists a right eigenvector $v \in \mathbb{R}^N$ of L such that $\lim_{t \to \infty} e^{-Lt} = 1_N v^T$;

When edges of a graph are time-varying, the graph is denoted by $\mathcal{G}(t) = (\mathcal{V}, \mathcal{E}(t))$. Following the notion in Scardovi and Sepulchre (2009), we assume that $a_{ij}(t) \in [a_{min}, a_{max}]$ for all $(i,j) \in \mathcal{E}(t)$, where $0 < a_{min} < a_{max}$ and a_{max} are finite, for any $t \geq t_0$. The graph $\mathcal{G}(t) = (\mathcal{V}, \mathcal{E}(t))$ is said to be uniformly connected if, for any $t \geq t_0$, there exists a finite time T and a node $i \in \mathcal{V}$ such that i is the root of a spanning tree of the graph $(\mathcal{V}, \cup_{\tau \in [t,T]} \mathcal{E}(\tau), \int_t^T A(\tau) d\tau)$.

A undirected graph is defined as a pair $\mathcal{G} := (\mathcal{V}, \mathcal{E})$, where $\mathcal{V}$ is the set of nodes and $\mathcal{E}$ is the set of unordered pairs of the nodes, called edges. For our purpose, it is convenient to view a undirected graph as a directed graph with a special structure. That is, $\mathcal{G} = (\mathcal{V}, \mathcal{E})$ is said to be undirected if $(i,j) \in \mathcal{E}$ implies $(j,i) \in \mathcal{E}$ and $a_{ij} = a_{ji}$ for all $(i,j) \in \mathcal{E}$. If there is a path from any node to any other nodes, $\mathcal{G}$ is said to be connected. The Laplacian matrix L of $\mathcal{G}$ is symmetric and positive-semidefinite. When $\mathcal{G}$ is connected, all nonzero eigenvalues of L are positive.

III. Formation control problems

A. A general formation control problem

In this subsection, we formulate a general formation control problem and then describe position-, displacement-, and distance-based approaches based on the problem. Though formation control involves primarily mobile agents, it can be generalized to control and coordination of general multi-agents. In a general formulation, a formation control problem can be described as follows. Consider the following N-agents:

$$\begin{cases} \dot{x}_i = & f_i(x_i, u_i), \\ \dot{y}_i = & g_i(x_1, \ldots, x_N), \quad i = 1, \ldots, N, \\ z_i = & h_i(x_i, u_i), \end{cases} \tag{1}$$

where $x \in \mathbb{R}^{n_i}$, $u_i \in \mathbb{R}^{p_i}$, $y_i \in \mathbb{R}^q$, $z_i \in \mathbb{R}^r$, $f_i : \mathbb{R}^{n_i} \times \mathbb{R}^{p_i} \to \mathbb{R}^{n_i}$, $g_i : \mathbb{R}^{n_1} \times \mathbb{R}^{n_2} \times \cdots \times \mathbb{R}^{n_N} \to \mathbb{R}^q$, and $h_i : \mathbb{R}^{n_i} \times \mathbb{R}^{p_i} \to \mathbb{R}^r$. Here, x_i, y_i, and z_i are the state, the measurement, and the output of agent $i \in \{1, \ldots, N\}$. Defining $z = [z_1^T \cdots z_N^T]^T \in \mathbb{R}^{rN}$, the desired formation for the agents can be prescribed by M-constraints as:

$$F(z) = F(z^*), \tag{2}$$

where $F : \mathbb{R}^{rN} \to \mathbb{R}^M$, for some given $z^* \in \mathbb{R}^{rN}$. Then, a formation control problem is stated as follows:

Problem 3.1 (A general formation control problem): For the agents (1), design a control law $u_i, i \in \{1, \ldots, N\}$ such that the set

$$E_{z^*} = \{[x_1^T \cdots x_N^T]^T : F(z) = F(z^*)\}$$

becomes asymptotically stable under the control law.

We emphasize that the objective of *Problem 3.1* is not to achieve $z_i \to z_i^*$ but to achieve $F(z) \to F(z^*)$ in general. Further, the constraint (2) might be different dependent on problem setups.

For *Problem 3.1*, position-, displacement-, and distance-based approaches can be described as follows:

- In position-based approach, the measurement y_i is assumed to be the state (or partial state) of agent $i \in \{1, \ldots, N\}$. The constraint (2) specifies the desired values for the states measured by the agents.
- In displacement-based approach, it is assumed that each agent has some neighboring agents and it measures the relative states (or partial relative states) of the neighboring agents. The constraint (2) specifies the desired values for the measured relative states.
- In distance-based approach, it is assumed that each agent maintains its own local coordinate system, of which orientation is not aligned to those of the other agents. Each agent measures the relative states (or partial relative states) of its neighboring agents. The constraint (2) specifies the desired magnitude of the relative states.

Consensus can be viewed as a special case of formation control. Let $z^* = 0_{rN}$ and $F(z) = [\cdots z_i^T - z_j^T \cdots]^T$ for all $i, j \in \{1, \ldots, N\}$, where 0_{rN} is the length rN vector with zero elements. Then it is obvious that *Problem 3.1* becomes a general consensus problem, which is called rendezvous in formation control.

B. Classification of formation problems

Depending on the problem setups, one can formulate a variety of formation control problems. Thus, though we focus on the classification based on the types of actively controlled variables in this survey, formation control problems can be classified based on various criteria. In this subsection, we review several important classifications found in the literature.

Depending on whether or not desired formation shapes are time-varying, Ren and Cao (2010) have classified the formation control problems as follows:

- Formation producing problem: In this problem, the objective of the agents is to achieve a prescribed geometric formation shape by controlling their positions. Approaches to this problem include matrix theory based approach, Lyapunov based approach, graph rigidity approach, and receding horizon approach (Ren and Cao, 2010).
- Formation tracking problem: In this problem, reference trajectories are prescribed and the agents are controlled to track the trajectory while maintaining their formation shape. Approaches to this problem include matrix theory based approach, potential function based approach, Lyapunov based approach, and some other approaches (Ren and Cao, 2010).

Beard *et al.* (2001); Scharf *et al.* (2004) have classified approaches to formation control as follows:

- Leader-follower approach: In this approach, at least one agent plays a role as a leader and the rest of the agents are designated as followers. The followers track the position of the leader with some prescribed offset while the leader tracks its desired trajectory.
- Behavioral approach: In this approach, several desired behaviors are prescribed for the agents. Such desired behaviors might include collision avoidance, obstacle avoidance, and formation keeping.
- Virtual structure approach: In virtual structure approach, the desired dynamics of the virtual structure is defined by treating the formation as a single structure, the motion of the virtual structure is translated into the desired motion for each agent appropriately, and then control laws for the agents are designed.

In Section I, centralized and decentralized approaches are classified based on whether or not global information is utilized. In a stricter sense, one may want to classify the existing results based on whether a centralized coordinator exists or not as follows:

- Centralized approach: At least one centralized coordinator gathers state information from agents and/or generates control inputs for the agents.
- Decentralized approach: There is no centralized coordinator in this approach. Agents behave based on their local information to achieve their prescribed objective.

Depending on whether or not desired formation shapes are explicitly prescribed, one may classify formation control problems as follows:

- Morphous formation control: Desired formation shapes are explicitly specified by desired positions of agents, desired inter-agent displacements, desired inter-agent distances, etc.
- Amorphous formation control: Without explicitly specified desired formation shapes, desired behavior patterns such as velocity alignment, collision avoidance, etc., are prescribed for agents.

In this survey, we mainly focus on decentralized approaches for morphous formation control problems.

IV. Position-based formation control

In position-based approach, it is assumed that agents measure their positions with respect to a global coordinate system and the desired formation for the agents are specified by the prescribed desired destinations of the agents. The primary objective of formation control in this approach is to drive the agents to their destinations. Thus the objective can be achieved by actively controlling the position of each individual agent in this approach.

Though the desired formation can be achieved by controlling positions of individual agents in this approach, interconnections among the agents are beneficial when there is an additional objective to maintain a prescribed formation shape during the transition from initial locations to the destinations. To see this, consider N single-integrator modeled agents in n-dimensional space, i.e., $\dot{p}_i = u_i$, where $p_i \in \mathbb{R}^n$ and $u_i \in \mathbb{R}^n$ for $i \in \{1, \ldots, N\}$. Suppose that the objective of the agents is to move from their initial positions to the prescribed destinations while keeping their formation shape. The destinations and the desired formation shape are specified by $p^* = [p_1^{*T} \cdots p_N^{*T}]^T \in \mathbb{R}^{nN}$. That is, the objective of the agents is to achieve $p \to p^*$ while satisfying $p_i - p_j = p_i^* - p_j^*$ for all $i, j \in \{1, \ldots, N\}$ during their transition. Assuming that each agent measures its actual position, we can consider the following control law:

$$u_i = k_p(p_i^* - p_i),$$

where $k_p > 0$. Defining $e_p := [p_1^T \cdots p_N^T]^T - p^*$, we have the overall error dynamics, $\dot{e}_p = -k_p e_p$. While it is obvious that p exponentially converges to p^*, the desired formation shape is not preserved during transition unless $p_i - p_j = p_i^* - p_j^*$ for all $i, j \in \{1, \ldots, N\}$ at initial instant. To add some control effort to preserve the desired formation shape during the transition, assume that each agent has some neighboring agent and measures the relative positions of the agents. The sensing topology is given by a directed graph $\mathcal{G}$ having a spanning tree. We then introduce additional control input based on the relative position errors to the original control input to obtain

$$\dot{e}_p = -k_p e_p - (L \otimes I_n) e_p,$$

where L is the Laplacian matrix of $\mathcal{G}$. Based on the Geršgorin's disc theorem (Horn and Johnson, 1990), it can be easily shown that the eigenvalues of $k_p I_n + L$ is greater than or equal to k_p. This shows that the agents reach their destinations. Further, it is expected that the additional control input $-(L \otimes I_n)e_p$ is beneficial for achieving the desired formation shape during the transition.

A similar formation control problem for double-integrator modeled agents has been studied in Ren and Atkins (2007). The agents are modeled by $\ddot{p}_i = u_i$, where $p_i \in \mathbb{R}^2$ and $u_i \in \mathbb{R}^2$ for $i \in \{1, 2, 3, 4\}$. The sensing topology for the agents is given by a directed graph $\mathcal{G}$ having a spanning tree. The objective of the agents is to move from their initial locations to the prescribed destinations while keeping their formation shape during the movements. The destinations and the desired formation for the agents are specified by $p^* = [p_1^{*T} \cdots p_4^{*T}]^T$. Assuming that each agent measures its position, velocity, and the relative positions of its neighbors, Ren and Atkins (2007) have proposed the following control law for the agents:

$$u_i = u_i^a + u_i^r$$

with

$$\begin{aligned} u_i^a =& - k_a k_p (p_i - p_i^*) - k_a k_v \dot{p}_i, \\ u_i^r =& - k_p \sum_{j \in \mathcal{N}_i} w_{ij} (p_i - p_j - p_i^* + p_j^*) \\ & - k_v \sum_{j \in \mathcal{N}_i} w_{ij} (\dot{p}_i - \dot{p}_j), \end{aligned}$$

where $k_a > 0$, $k_p > 0$, and $k_v > 0$. Defining $e_p := p - p^*$, the overall dynamics for the agents can be written as

$$\ddot{e}_p = -k_v (L \otimes I_2 + k_a I_8) \dot{e}_p - k_p (L \otimes I_2 + k_a I_8) e_p,$$

where L is the Laplacian matrix of $\mathcal{G}$. Then, based on simulation, Ren and Atkins (2007) have showed that the agents move to their destinations while preserving their desired formation by suitably taking k_a, k_p, and k_v. Further, they have mentioned that u_i^a ensures the agents to reach their destinations while u_i^r ensures the achievement of the desired formation shape.

Based on a similar idea, Dong and Farrell (2008*a*,*b*) have studied formation control of nonholonomic mobile agents described by a canonical chained form. Under the assumption that each agent knows its own state and desired trajectory and measures relative states of its neighbors, they have proposed a control law to drive the agents to track the desired trajectories. They have applied the proposed control law to unicycles to show that the desired formation is satisfactorily achieved. Further, Dong and Farrell (2008*b*) have addressed the time-varying sensing graph case.

A similar result for general linear agents is found in Fax and Murray (2002, 2004). They have considered the following identical, linear time-invariant systems over a directed graph $\mathcal{G}$:

$$\left\{ \begin{array}{rl} \dot{x}_i = & A_P x_i + B_P u_i, \\ y_i = & C_{P_a} x_i, \\ z_i = & \sum_{j \in \mathcal{N}_i} C_{P_r} (x_i - x_j), \end{array} \right. \quad i = 1, \ldots, N, \qquad (3)$$

where $x_i \in \mathbb{R}^n$, $u_i \in \mathbb{R}^p$, $y_i \in \mathbb{R}^{q_a}$, and $z_i \in \mathbb{R}^{q_r}$ are the state, the control input, the absolute measurement, and relative measurement of agent i, respectively. The matrices A_P, B_P, C_{P_a}, and C_{P_r} are constant matrices with appropriate dimensions. Then, for the agents (3), Fax and Murray (2002, 2004) have proposed the following dynamic control law to stabilize the states:

$$\left\{ \begin{array}{rl} \dot{\xi}_i = & A_K \xi_i + B_{K_a} y_i + B_{K_r} z_i, \\ u_i = & C_K \xi_i + D_{K_a} y_i + D_{K_r} z_i, \end{array} \right. \quad i = 1, \ldots, N, \qquad (4)$$

where $\xi_i \in \mathbb{R}^r$ and the matrices A_K, B_{K_a}, B_{K_r}, C_K, D_{K_a}, and D_{K_r} are constant matrices with appropriate dimensions. When the controller (4) are based on the absolute measurements y_i, this problem can be considered as a position-based problem. In the case that (4) does not depend on the absolute measurements, this problem becomes a displacement-based problem.

It might be a huge task to check the stability of a given (3) and (4) when N is large so that the dimension of the overall closed loop system is huge. Based on a Jordan decomposition of the Laplacian matrix L of $\mathcal{G}$, Fax and Murray (2002, 2004) have shown that the overall closed loop system is asymptotically stable if and only if the following N-systems are asymptotically stable:

$$\begin{cases} \dot{x}_i = & A_P x_i + B_P u_i, \\ y_i = & C_{P,a} x_i, \qquad\qquad i = 1, \ldots, N, \\ z_i = & \lambda_i C_{P_r} x_i, \end{cases}$$

where λ_i are the eigenvalues of L. Further, they have provided a Nyquist criterion that uses the eigenvalues of L to check the stability.

Different from the above cases, it is possible to drive agents to their destinations while maintaining their formation shape by assigning desired trajectories to the agents as in Lewis and Tan (1997); Tan and Lewis (1996). By treating the desired formation as a single rigid body called a virtual structure, Lewis and Tan (1997); Tan and Lewis (1996) have generated desired trajectories from the dynamics of the virtual structure. Then, based on the local controller, each agent follows its desired trajectory generated by the virtual structure. This approach can be considered position-based approach in the sense that the positions of the agents are actively controlled by the local controllers to achieve the desired formation.

In the presence of disturbances to agents and/or capability limitation of agent actuators, agents might not be able to follow their prescribed desired trajectories. To consider such situations, Beard *et al.* (2001) have proposed an architecture for formation flying of spacecrafts as depicted in Figure 1. In the architecture, the formation coordinator denoted by F gathers the performance data from the spacecrafts and generates coordinate variables for the spacecrafts to broadcast them. Then, based on the broadcast coordinate variables, each spacecraft maneuvers by using its own local controller. Since the spacecrafts measure their outputs, this architecture suggests a position-based formation control scheme. Though the formation coordinator is implemented at a centralized location in Figure 1, as mentioned in Beard *et al.* (2001), it can be implemented in a decentralized way by allowing each spacecraft to instantiate a local copy. Young *et al.* (2001) have utilized the architecture proposed in Beard *et al.* (2001) for formation tracking of unicycle type mobile agents. A similar architecture has been proposed in Ren *et al.* (2007*b*).

Extending the results in Beard *et al.* (2001); Lewis and Tan (1997); Tan and Lewis (1996), Ren and Beard (2004) have considered formation feedback from agents to their virtual structure. In the absence of feedback from agents to the virtual structure, when the virtual structure evolves too fast, some agents might not be able to track their desired trajectories accurately because of their limited actuation capability. Based on this observation, Ren and Beard (2004) have designed the dynamics of the virtual structure considering the state errors of individual agents and demonstrated the advantages of their approach by simulation.

Do and Pan (2007) have addressed a formation tracking problem for two-wheel driven mobile agents based on the architecture proposed in Beard *et al.* (2001). They have defined the reference trajectory parameterized by a path parameter and generated the desired trajectories for individual agents by adding some offset to the reference trajectory. The reference

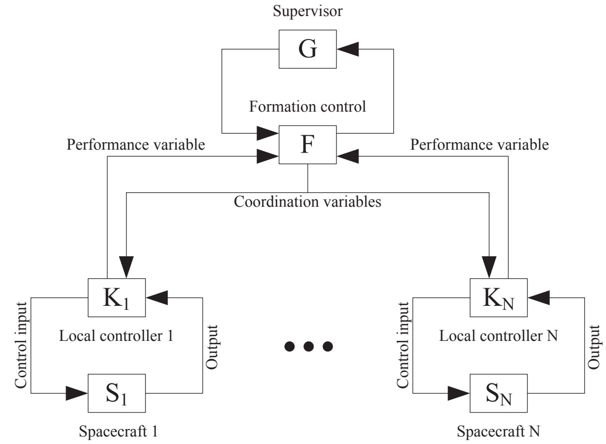


Fig. 1: Architecture for formation flying taken from Beard *et al.* (2001).

trajectory and the offsets defines a virtual structure (Lewis and Tan, 1997; Tan and Lewis, 1996) . Assuming that each agent measures its own position and heading angle with respect to a global coordinate system, they have designed an output feedback control law for the trajectory tracking and proved that the trajectory tracking errors asymptotically converge. Further, they have claimed that the mobility of each agent can be taken into account by appropriately designing the path parameter for the reference trajectory as a function of the tracking errors. That is, the path parameter can play a role as a formation coordinator. The approach in Do and Pan (2007) can be considered centralized in the sense that a centralized coordinator needs to gather all the error values from the agents to generate the path parameter.

van den Broek *et al.* (2009*a*,*b*) have studied a trajectory tacking problem for the agents modeled by unicycle-type kinematic model in the plane under the assumption that each agent measures its own position and heading angle with respect to a global Cartesian coordinate system. While the desired formation of the agents is maintained provided that each agent tracks its own desired trajectory under their problem setup, van den Broek *et al.* (2009*a*,*b*) have introduced additional, undirected coupling inputs based on the relative tracking errors after designing the trajectory tracking control law. For two agent case, they have showed the local asymptotic stability of the origin of the overall tracking error dynamics under the proposed control law. Then they have claimed that the additional coupling inputs enhance the robustness of the formation with respect to disturbances based on an experimental result.

V. Displacement-based formation control

In displacement-based approach, rather than the absolute positions of agents, the displacements (relative positions) of neighboring agents are actively controlled to achieve the prescribed desired formation, which is specified by constraints on the relative positions. This control strategy can be justified by the usual assumption in displacement-based approach that the agents do not measure their positions with respect to a global coordinate system.

In displacement-based approach, it is assumed that agents have their own local coordinate systems whose orientations are aligned to a global coordinate system and they measure the relative positions of their neighbors with respect to the local coordinate systems. Under these assumptions, the absolute positions of the agents cannot be actively controlled in general.

In stability analysis in displacement-based approach, properties of graph Laplacian matrix play a crucial role. In many cases, formation control problems in this approach can be reduced into consensus problems.

A. Single-integrator modeled agent case

Consider the following single-integrator modeled agents in n-dimensional space over a graph $\mathcal{G}$:

$$\dot{p}_i = u_i,\ i = 1, \ldots, N, \tag{5}$$

where $p_i \in \mathbb{R}^n$ and $u_i \in \mathbb{R}^n$ are the position and the control input of agent i. Assume that each agent measures the relative positions of some other agents with respect to a global reference frame and the sensing topology is modeled by $\mathcal{G} = (\mathcal{V}, \mathcal{E})$. Then agent $i \in \mathcal{V}$ has the following relative position measurements:

$$p_{ji} := p_j - p_i,\ j \in \mathcal{N}_i. \tag{6}$$

In this problem setup, each agent can be assumed to have its own local coordinate system whose orientation is aligned to that of the global coordinate system. Figure 2a illustrates an example of this problem setup in the plane.

For given $p^* = [p_1^{*T} \cdots p_N^{*T}]^T \in \mathbb{R}^{nN}$, the objective of the agents is to satisfy the following constraints for inter-agent displacements:

$$p_i - p_j = p_i^* - p_j^*,\ i, j \in \mathcal{V}.$$

In general, p_i^* are not the destinations of the agents. It specifies only the desired relative displacements. The desired formation for the agents is defined as a set

$$E_{p^*} := \{p \in \mathbb{R}^{nN} : p_j - p_i = p_j^* - p_i^*,\ i, j \in \mathcal{V}\}. \tag{7}$$

That is, the objective of the formation control is to drive p to p^* up to translation.

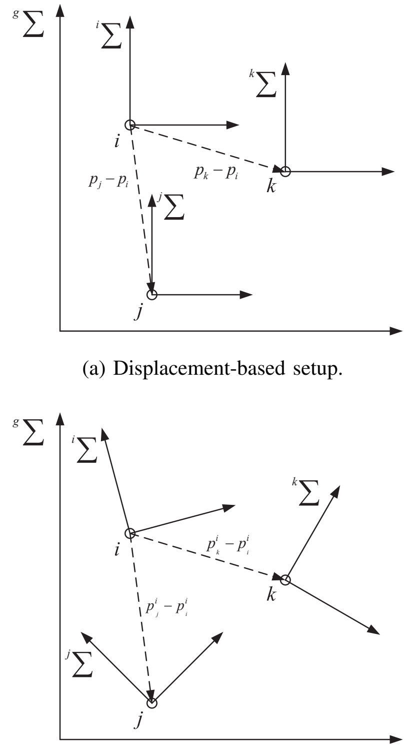
It has been known that this formation control problem can be solved based on consensus. Consider the following control law for agent $i \in \mathcal{V}$:

$$u_i = k_p \sum_{j \in \mathcal{N}_i} a_{ij}(p_j - p_i - p_j^* + p_i^*), \tag{8}$$

where $k_p > 0$. Defining $e_p := p - p^*$, we have the following first-order consensus dynamics:

$$\dot{e}_p = -k_p(L \otimes I_n)e_p, \tag{9}$$

which has been studied in Olfati-Saber and Murray (2004) for undirected graph case, Jadbabaie *et al.* (2003) for discrete-time agent case, and Ren *et al.* (2004) for directed graph case. According to Ren *et al.* (2004), E_{p^*} is exponentially stable if and only if $\mathcal{G}$ has a spanning tree. Further, p exponentially converges to a finite point in E_{p^*}. In the case that edges of $\mathcal{G}$ are time-varying, E_{p^*} is uniformly exponentially stable and p asymptotically converges to a finite point in E_{p^*} if $\mathcal{G}$ is uniformly connected (Moreau, 2004, 2005).



(a) Displacement-based setup.

(b) Distance-based setup.

Fig. 2: Formation control problem setups.

In the above problem formulation, p^* is assumed to be given to specify the desired formation. Rather than p^*, one can specify the desired formation by prescribing the desired displacements δ_{ji}^* for all $(i, j) \in \mathcal{E}$. By ordering δ_{ji}^* in a way, let $\delta^* := [\cdots \delta_{ji}^{*T} \cdots]^T \in \mathbb{R}^{n|\mathcal{E}|}$ for all $(i, j) \in \mathcal{E}$. We define the realizability of the pair $(\mathcal{G}, \delta^*)$ as follows. The pair $(\mathcal{G}, \delta^*)$ is said to be realizable in $\mathbb{R}^n$ if there exists $p^* \in \mathbb{R}^{nN}$ such that $p_j^* - p_i^* = \delta_{ji}^*$ for all $(i, j) \in \mathcal{E}$ (Ji and Egerstedt, 2007). In the case that $(\mathcal{G}, \delta^*)$ is realizable in $\mathbb{R}^n$, the agents achieve the desired formation under the following control law

$$u_i = k_p \sum_{j \in \mathcal{N}_i} a_{ij}(p_j - p_i - \delta_{ji}^*). \tag{10}$$

if and only if $\mathcal{G}$ has a spanning tree.

In majority of the existing results in displacement-based approach, it has been assumed that $(\mathcal{G}, \delta^*)$ is realizable. What happens if it is not realizable? According to Dimarogonas and Kyriakopoulos (2008), the velocities of the agents converge to $(1/N) \sum_{i=1}^{N} \sum_{j \in \mathcal{N}_i} \delta_{ji}^*$ under the control law (10) when $(\mathcal{G}, \delta^*)$ is not realizable. This shows that the agents behave as a flock when $(\mathcal{G}, \delta^*)$ is not realizable.

It is assumed in (8) that edges of the sensing graph are static. However, in practice, the agents have limited sensing ranges and thus edges of $\mathcal{G}$ are given by functions of the agent positions. Then the connectedness of $\mathcal{G}$ needs to be preserved by an appropriate control law rather than assumed. Ji and Egerstedt (2007) have studied connectedness preservation in formation control of single-integrator modeled agents. Assuming that the agents have a limited sensing range, they have defined a dynamic sensing graph that depends on time and the agent positions. Then they have proposed a control law of the form

$$u_i = -\sum_{(i,j)\in\mathcal{E}(t,p)} a(p_j - p_i, p_j^* - p_i^*)(p_j - p_i - p_j^* + p_i^*),$$

where a is a nonlinear function of $p_j - p_i$ and $p_j^* - p_i^*$, to show the asymptotic convergence of p to p^* up to translation under a certain condition.

B. Double-integrator modeled agent case

Consider the following N double-integrator modeled agents in n-dimensional space over a graph $\mathcal{G}$:

$$\begin{cases} \dot{p}_i = v_i, \\ \dot{v}_i = u_i, \end{cases} \quad i = 1, \ldots, N, \tag{11}$$

where $p_i \in \mathbb{R}^n$, $v_i \in \mathbb{R}^n$, and $u_i \in \mathbb{R}^n$ denote the position, the velocity, and the control input, respectively, of agent i. Suppose that $p^* \in \mathbb{R}^{nN}$ and $v^* \in \mathbb{R}^{nN}$ specify the desired formation. Here it is assumed that $\dot{p}^* = v^*$ and $\dot{v}^* = 0$. Then a formation control law can be designed as follows:

$$\begin{aligned} u_i = &- k_p \sum_{j\in\mathcal{N}_i} (p_i - p_j - p_i^* + p_j^*) \\ &- k_v \sum_{j\in\mathcal{N}_i} (v_i - v_j + v_i^* - v_j^*), \end{aligned} \tag{12}$$

where $k_p > 0$ and $k_v > 0$. Defining $e_p := p - p^*$ and $e_v := v - v^*$, the overall dynamics for the agents can be written as

$$\begin{bmatrix} \dot{e}_p \\ \dot{e}_v \end{bmatrix} = \underbrace{\begin{bmatrix} 0 & I_{nN} \\ -k_p(L \otimes I_n) & -k_v(L\otimes I_n) \end{bmatrix}}_{=:\Gamma} \begin{bmatrix} e_p \\ e_v \end{bmatrix}.$$

Let $\lambda_1, \ldots, \lambda_N$ be the eigenvalues of L. Then, due to properties of Kronecker product (Laub, 2004), the eigenvalues of Γ are given by

$$\mu_{i\pm} = \frac{k_v \lambda_i \pm \sqrt{k_v^2 \mu_i^2 + 4k_p\lambda_i}}{2}$$

with multiplicity n. According to Ren and Atkins (2007), $\|p_i - p_j\| \to \|p_i^* - p_j^*\|$ and $\|v_i - v_j\| \to \|v_i^* - v_j^*\|$ asymptotically if and only if Γ has exactly n zero eigenvalues and all the other eigenvalues have negative real parts.

C. General linear agent case

Consider the following N-agents modeled by identical linear time-invariant systems over a graph $\mathcal{G}$:

$$\dot{x}_i = Ax_i + Bu_i,\ i = 1, \ldots, N, \tag{13}$$

where $x_i \in \mathbb{R}^n$ is the state and $u_i \in \mathbb{R}^m$ is the control input of agent i. Constant matrices A and B are of proper dimensions. Assume that agent i measures the following relative partial states of its neighbors:

$$y_{ji} = C(x_j - x_i),\ j \in \mathcal{N}_i, \tag{14}$$

where $C \in \mathbb{R}^{p\times n}$, and $x^* \in \mathbb{R}^{nN}$ is given such that

$$\dot{x}^* = (I_N \otimes A)x^*. \tag{15}$$

The objective of the agents is to achieve $x_j - x_i \to x_j^* - x_i^*$ for all $i, j \in \mathcal{V}$. An immediate generalization of (8) and (12) for (13) is

$$u_i = KC \sum_{j\in\mathcal{N}_i} a_{ij}(x_j - x_i), \tag{16}$$

where $K \in \mathbb{R}^{m\times p}$.

Defining $e_x := x - x^*$, the overall error dynamics is arranged as

$$\dot{e}_x = (I_N \otimes A)e_x - (L \otimes BKC)e_x. \tag{17}$$

By means of a coordinate transformation, the error dynamics (17) can be decomposed based on Jordan blocks of L. Then $x_i - x_j$ exponentially converge to $x_j^* - x_i^*$ provided that $\mathcal{G}$ has a spanning tree and $A - \lambda_i BKC$ are Hurwitz, where λ_i are nonzero eigenvalues of L (Fax and Murray, 2002, 2004). Under the assumption that relative states are measured, i.e., $C = I_n$ in (14), Tuna (2008) has proposed a linear matrix inequality condition to design K. The dynamic consensus law proposed in Li *et al.* (2010) can be applied to formation control of agents (13).

D. Nonholonomic constrained agent model case

Consider the following unicycles in the plane over $\mathcal{G}$:

$$\begin{cases} \dot{x}_i &= v_i \cos\theta_i, \\ \dot{y}_i &= v_i \sin\theta_i, \quad i = 1, \ldots, N, \\ \dot{\theta}_i &= \omega_i, \end{cases} \tag{18}$$

where $p_i = [x_i\ y_i]^T \in \mathbb{R}^2$ and $\theta_i \in (-\pi, \pi]$ are the coordinate for the position and the heading angle of agent i with respect to a global Cartesian coordinate system and $v_i \in \mathbb{R}$ and $\omega \in \mathbb{R}$ are the control inputs of agent i. Each agent measures the relative positions of its neighbors with respect to the global coordinate system.

Displacement-based formation control of unicycles has been studied in Dimarogonas and Kyriakopoulos (2008); Lin *et al.* (2005). For given $p^* \in \mathbb{R}^{2N}$, the control law proposed by Lin *et al.* (2005) can be written as follows[1]:

$$\begin{aligned} v_i &= k[\cos\theta_i\ \sin\theta_i] \textstyle\sum_{j\in\mathcal{N}_i} (p_j - p_i - p_j^* + p_i^*), \\ \omega_i &= \cos t, \end{aligned}$$

which shows that the displacements are actively controlled. Then they have showed that p globally exponentially converge to p^* up to translation by suitably taking the value of k if $\mathcal{G}$ has a spanning tree.

[1]The control law proposed by Lin *et al.* (2005) is more complicated because they have assumed that the orientations of the local coordinate systems of the agents are not aligned with that of the global coordinate system. For simplicity, we assume that they are aligned here.

Dimarogonas and Kyriakopoulos (2008) have also studied formation control for unicycles (18) over a undirected graph $\mathcal{G}$. The desired formation is specified by desired inter-agent displacements δ^*. Under the proposed non-smooth control law

$$\begin{aligned} v_i = & \ \mathrm{sgn}\left([\cos\theta_i \ \sin\theta_i]\sum_{j\in\mathcal{N}_i}(p_j - p_i - \delta^*_{ji})\right) \\ & \times\left\|\sum_{j\in\mathcal{N}_i}(p_j - p_i - \delta^*_{ji})\right\|, \\ \omega_i = & \ -\left(\theta_i - \arctan(\sum_{j\in\mathcal{N}_i}(p_j - p_i - \delta^*_{ji})\right), \end{aligned}$$

which shows that the inter-agent displacements are actively controlled, they have proved that inter-agent displacements asymptotically converge to the desired values and the heading angles of the agents converge to zero if $\mathcal{G}$ is connected and $(\mathcal{G}, \delta^*)$ is realizable. Further, they have considered the the case that $(\mathcal{G}, \delta^*)$ is not realizable.

E. Displacement-based formation transition

It is often the case that the objective of agents is to move to prescribed destinations. Such an objective cannot be achieved when no agent measures its current position. Thus assume that at least one agent knows its current position with respect to the global coordinate system. Different from position-based approach, we assume that the number of such agents is far less than the total number of agents. Then majority of agents are under a displacement-based formation control law though some agents actively control their absolute positions.

For single-integrator modeled agents (20), we assume that at least one agent measures its current position and p^* specifies the destinations of the agents. Then, (8) can be modified as

$$u_i = k_p \sum_{j\in\mathcal{N}_i} a_{ij}(p_j - p_i - p^*_j + p^*_i) + g_{ii}k_p(p^*_i - p_i),$$

where $g_{ii} > 0$ if agent i measures p_i and $g_{ii} = 0$ otherwise, to obtain the following error dynamics:

$$\dot{e}_p = -k_p[(L + G) \otimes I_n]e_p,$$

where $G = \mathrm{diag}(g_{11}, \ldots, g_{NN})$. When $\mathcal{G}$ is directed, $-(L + G)$ is Hurwitz, which means that $e_p \to 0$ exponentially, if $\mathcal{G}$ has a spanning tree and the agent located at the root of such a tree measures its current position (Li *et al.*, 2010; Ren, 2007). If $\mathcal{G}$ is undirected and connected, $-(L+G)$ is Hurwitz (Hong *et al.*, 2006). Though at least one agent is required to know its current position, majority of the agents are able to move to their desired positions without position measurements different from the assumption in Fax and Murray (2002, 2004).

For linear agents (13), assume that the objective of the agents is to achieve $x \to x^*$ and at least one agent measures its state. Modify the control law (16) as

$$u_i = KC \sum_{j\in\mathcal{N}_i} a_{ij}(x_j - x_i) + g_{ii}KC(x^*_i - x_i),$$

where $g_{ii} > 0$ if agent i measures x_i and $g_{ii} = 0$ otherwise, to obtain the following error dynamics:

$$\dot{e}_x = (I_N \otimes A)e_x - [(L + G) \otimes BKC]e_x,$$

where $G = \mathrm{diag}(g_{11}, \ldots, g_{NN})$. Then $x \to x^*$ asymptotically if $A-\lambda_i BKC$ are Hurwitz, where λ_i are eigenvalues of $L+G$ (Zhang *et al.*, 2011). The dynamic consensus law proposed in Li *et al.* (2010) can be applied to this formation tracking. Ren *et al.* (2007*a*) have addressed a similar problem for high-order integrators.

Tanner *et al.* (2002, 2004) have studied input to state stability of displacement-based formation control systems. Consider a team of agents whose sensing graph is directed and acyclic. Any agents having no neighbor are leaders of the team and they know their desired states. All the other agents are followers and they know the desired relative states of their neighbors. Let $x_i \in \mathbb{R}^n$ be the state of agent i. If agent i is a leader, e_i is defined as $e_i := x^*_i - x_i$, where x^*_i is the desired state of agent i. Otherwise $e_i := \sum_{j\in\mathcal{N}_i} S_{ji}(x_j - \delta^*_{ji}) - x_i$, where δ^*_{ji} is the desired displacement of agent $j \in \mathcal{N}_i$ and S_{ji} are projection matrices with $\sum_{j\in\mathcal{N}_i} \mathrm{Rank}(S_{ij}) = n$. Denoting the set of the indices of the leaders by $\mathcal{L}$, they then have presented conditions for the following input to state stability:

$$\|e(t)\| \le \beta(\|e(0)\|, t) + \sum_{i\in\mathcal{L}} \gamma(\sup_{0\le\tau\le t} \|e_i(\tau)\|),$$

where $e := [\cdots e_i^T \cdots]^T$, β is a class $\mathcal{KL}$ function, and γ is a class $\mathcal{K}$ function. Functions β and γ can be understood as gain estimates quantifying the effect of initial formation errors and the leader formation errors, respectively.

VI. Distance-based formation control

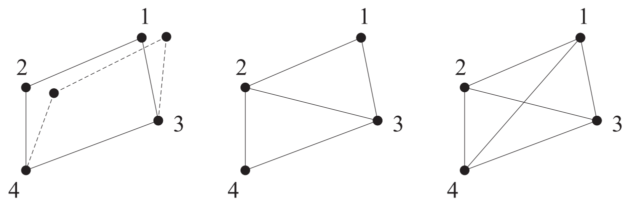
In displacement-based approach, agents are able to measure the relative positions of their neighboring agents with respect to a global coordinate system. This implies that the agents share a common sense of orientation.

What if agents do not share a common sense of orientation? More precisely, assume that the agents maintain their own local coordinate systems, whose orientations are not aligned with each other, and they do not know the orientations of their local coordinate systems with respect to a global coordinate system. In this case, even when the agents measure the relative positions of their neighbors with respect to their own local coordinate systems, the displacements cannot be directly controlled due to the lack of a common sense of orientation.

Subsequently, the Euclidean norms of the relative position measurements are controlled in the literature under the assumption that the agents do not share a common sense of orientation. Thus the directional information contained in relative positions are not actively exploited in distance-based approach. Further, different from position- and displacement-based approaches, control laws in distance-based approach are usually nonlinear. Due to such reasons, formation control problems in distance-based approach are complicated and thus simple agent models such as single-integrators have been mainly studied in the literature.

A. Distance-based undirected formation

A natural question for distance-based approach is what is the best formation p that can be achieved by actively controlling inter-agent distances for given p^*. In the literature, agents are usually aimed to achieve $p \to p^*$ up to congruence, i.e.,



(a) Not rigid. (b) Rigid. (c) Globally rigid.

Fig. 3: Examples of undirected frameworks taken from Hendrickx *et al.* (2007).

$\|p_i - p_j\| \to \|p_i^* - p_j^*\|$ for all $i, j \in \{1, \dots, N\}$. Since only inter-agent distances (not displacements) are actively controlled in distance-based approach, connectedness of a sensing graph does not ensure such congruence and thus it is expected that heavier connectivity condition is required. It has been known that rigidity or persistence involves with such a condition.

B. Graph rigidity

If $\mathcal{G} = (\mathcal{V}, \mathcal{E})$ is undirected, the edge set $\mathcal{E}$ can be partitioned as $\mathcal{E} = \mathcal{E}_+ \cup \mathcal{E}_-$ such that $\mathcal{E}_+$ and $\mathcal{E}_-$ are disjoint and $(i, j) \in \mathcal{E}_+$ if and only if $(j, i) \in \mathcal{E}_-$. Let $\mathcal{V} = \{1, \dots, N\}$ and $\mathcal{E}_+ = \{\epsilon_{+,1}, \dots, \epsilon_{+,M}\}$. Matrix $H_+ = [h_{+,ij}] \in \mathbb{R}^{M \times N}$ is defined as

$$h_{+,ij} := \begin{cases} 1, & \text{if } j \text{ is the tail node of } \epsilon_{+,i}, \\ -1, & \text{if } j \text{ is the head node of } \epsilon_{+,i}, \\ 0, & \text{otherwise.} \end{cases}$$

Let $p_i \in \mathbb{R}^n$ be a point that is assigned to $i \in \mathcal{V}$. Then $p = [p_1^T \cdots p_N^T]^T \in \mathbb{R}^{nN}$ is said to be a realization of $\mathcal{G}$ in $\mathbb{R}^n$. The pair $(\mathcal{G}, p)$ is said to be a framework of $\mathcal{G}$ in $\mathbb{R}^n$. By ordering edges in $\mathcal{E}_+$, an edge function $g_\mathcal{G} : \mathbb{R}^{nN} \to \mathbb{R}^M$ associated with $(\mathcal{G}, p)$ is defined as

$$g_\mathcal{G}(p) := \frac{1}{2}[\cdots \|p_i - p_j\|^2 \cdots]^T, \ \forall (i, j) \in \mathcal{E}_+. \tag{19}$$

The rigidity of frameworks is then defined as follows:

Definition 6.1: Asimow and Roth (1979) A framework $(\mathcal{G}, p)$ is rigid in $\mathbb{R}^n$ if there exists a neighborhood U_p of $p \in \mathbb{R}^{nN}$ such that $g_\mathcal{G}^{-1}(g_\mathcal{G}(p)) \cap U_p = g_\mathcal{K}^{-1}(g_\mathcal{K}(p)) \cap U_p$, where $\mathcal{K}$ is the complete graph on N-nodes. Further, the framework $(\mathcal{G}, p)$ is globally rigid in $\mathbb{R}^n$ if $g_\mathcal{G}^{-1}(g_\mathcal{G}(p)) = g_\mathcal{K}^{-1}(g_\mathcal{K}(p))$.

Two frameworks $(\mathcal{G}, p)$ and $(\mathcal{G}, q)$ are said to be equivalent if $g_\mathcal{G}(p) = g_\mathcal{G}(q)$, i.e., $\|p_i - p_j\| = \|q_i - q_j\|$ for all $(i, j) \in \mathcal{E}_+$. Further, they are said to be congruent if $\|p_i - p_j\| = \|q_i - q_j\|$ for all $i, j \in \mathcal{V}$. Thus the framework $(\mathcal{G}, p)$ is rigid if there exists a neighborhood U_p of $p \in \mathbb{R}^{nN}$ such that, for any $q \in U_p$, if $(\mathcal{G}, p)$ and $(\mathcal{G}, q)$ are equivalent, then they are congruent.

Let m be the dimension of convex hull of $\{p_1, \dots, p_N\}$. The framework $(\mathcal{G}, p)$ is then said to be infinitesimally rigid in $\mathbb{R}^n$ if $\mathrm{Rank}(\partial g_\mathcal{G}(p)/\partial p) = nN - (m+1)(2n-m)/2$.

As an example, consider four agents moving in the plane. As depicted in Figure 3, each agent has some edges and it tries to control the edge lengths. In the case of Figure 3a, the formation $p = [p_1^T \cdots p_4^T]^T$ is not unique up to congruence even when all the edge lengths are fixed. As showed by dotted line in Figure 3a, p can be deformed in this case. By adding an edge between agents 2 and 3, p becomes locally unique up to congruence when all the edge lengths are fixed as depicted in Figure 3b. By adding one more edge between agents 1 and 4, p becomes globally unique up to congruence.

1) Single-integrator case: Consider the following N single-integrator modeled agents in n-dimensional space.

$$\dot{p}_i = u_i, \ i = 1, \dots, N, \tag{20}$$

where $p_i \in \mathbb{R}^n$ and $u_i \in \mathbb{R}^n$ denote the coordinates of the position and the control input, respectively, of agent i with respect to the global Cartesian coordinate system ${}^g\sum$. Assume that the agents do not share a common sense of orientation. Due to the absence of a common sense of orientation, each agent i maintains its own local Cartesian coordinate system, whose origin is located at p_i and orientation is not aligned with ${}^g\sum$ as depicted in Figure 2b. Further, the orientations of the local coordinate systems are not aligned with each other. We denote the local coordinate system of agent i by ${}^i\sum$. By adopting a notation in which superscripts are used to denote coordinate systems, the position dynamics of the agents can be written as

$$\dot{p}_i^i = u_i^i, \ i = 1, \dots, N, \tag{21}$$

where $p_i^i \in \mathbb{R}^n$ and $u_i^i \in \mathbb{R}^n$ are the coordinates of the position and the control input, respectively, of agent i with respect to ${}^i\sum$.

Agent i has some neighboring agents and it measures the relative positions of the neighbors with respect to ${}^i\sum$. The sensing topology among the agents is modeled by a undirected graph $\mathcal{G} = (\mathcal{V}, \mathcal{E})$. Thus the following measurements are available to agent $i \in \mathcal{V}$,

$$p_{ji}^i := p_j^i - p_i^i \equiv p_j^i, \ \forall j \in \mathcal{N}_i, \tag{22}$$

where p_j^i is the coordinate of the position of agent j with respect to ${}^i\sum$.

Given $p^* = [p_1^{*T} \cdots p_N^{*T}]^T \in \mathbb{R}^{nN}$, the desired formation for the agents is defined as the set of realizations that are congruent to p^*:

$$E_{p^*} := \{p \in \mathbb{R}^{nN} : \|p_j - p_i\| = \|p_j^* - p_i^*\|, \ \forall i, j \in \mathcal{V}\}. \tag{23}$$

In the literature, gradient-based control laws have been dominantly proposed to achieve the desired formation. For agent $i \in \mathcal{V}$, define a local potential $\phi_i : \mathbb{R}^{n(|\mathcal{N}_i|+1) \to \bar{\mathbb{R}}_+}$ as

$$\phi_i(p_i^i, \dots, p_j^i, \dots) := \frac{k_p}{2} \sum_{j \in \mathcal{N}_i} \gamma_{ij} \left(\|p_j^i - p_i^i\| \right), \tag{24}$$

where $k_p > 0$ and $\gamma_{ij} : \mathbb{R} \to \bar{\mathbb{R}}_+$ is differentiable. A control input for agent $i \in \mathcal{V}$ can be designed as

$$\begin{aligned} u_i^i &= -\nabla_{p_i^i} \phi_i(p_i^i, \dots, p_j^i, \dots) \\ &= k_p \sum_{j \in \mathcal{N}_i} \frac{\partial \gamma_{ij}(\|p_j^i - p_i^i\|)}{\partial \|p_j^i - p_i^i\|} \frac{p_j^i - p_i^i}{\|p_j^i - p_i^i\|}, \end{aligned} \tag{25}$$

Control law (25) can be implemented in the local coordinate systems of the agents by using only the measurements (22), which is obvious because γ_{ij} is the function of $\|p_j^i - p_i^i\|$.

Though control law (25) is implemented in ${}^i\sum$ in practice, it is convenient to describe the agents with respect to ${}^g\sum$ for stability analysis. This can be done based on a suitable coordinate transformation to obtain

$$u_i = -\nabla_{p_i}\phi_i(p_i, \ldots, p_j, \ldots). \tag{26}$$

Under gradient control law (26), the agents can be described as a gradient system. To see this, define a global potential function $\phi : \mathbb{R}^{nN} \to \bar{\mathbb{R}}_+$ as

$$\phi(p) := \sum_{(i,j)\in\mathcal{E}_+} \gamma_{ij}(\|p_j^i - p_i^i\|). \tag{27}$$

From the fact that

$$\nabla_{p_i}\phi_i(p_i, \ldots, p_j, \ldots) = \nabla_{p_i}\phi(p),$$

we obtain

$$\dot{p} = u = -\nabla\phi(p). \tag{28}$$

Obviously, it is required that the set of the critical points of $\phi(p)$ include E_{p^*}.

Krick *et al.* (2008, 2009) have extensively studied distance-based formation control of the agents by adopting the following γ_{ij}:

$$\gamma_{ij}(\|p_j - p_i\|) := k_p\left(\|p_j - p_i\|^2 - \|p_j^* - p_i^*\|^2\right)^2, \tag{29}$$

where $k_p > 0$. Under the assumption that $(\mathcal{G}, p^*)$ is infinitesimally rigid in $\mathbb{R}^2$, they have showed E_{p^*} is three dimensional manifold. To avoid the complicatedness arising from the non-compactness of E_{p^*} in stability analysis, they have separated the centroid dynamics of the agents, which is stationary under the gradient law, from the rest of the dynamics to obtain the reduced state equation. Then, by exploiting the compactness of the equilibrium set characterized by the reduced state, they have applied the center manifold theory with respect to finite numbers of equilibrium points in E_{p^*} to show the local asymptotic stability of E_{p^*}.

By means of a Lyapunov stability analysis, Dörfler and Francis (2009) have provided a stability result under the gradient control law adopted in Krick *et al.* (2008, 2009). To avoid the complexity arising from the non-compactness of E_{p^*}, they have described the system (28) by $e = [e_1^T \cdots e_N^T]^T := (H_+^T \otimes I_2)p \in \mathbb{R}^{2M}$. The desired formation parameterized by e is compact. Taking the Lyapunov function as $V(e) := \sum_{i=1}^M \left(\|e_i\|^2 - \|e_i^*\|^2\right)$, where $[e_1^{*T} \cdots e_N^{*T}]^T := (H_+^T \otimes I_2)p^*$, they have showed that the time-derivative of V is negative definite in some neighborhood of the compact equilibrium set if $(\mathcal{G}, p^*)$ is minimally, infinitesimally rigid in $\mathbb{R}^2$. Further, they have showed that the gradient control law is the optimal control law with respect to a cost functional. Extending the result in Dörfler and Francis (2009) to n-dimension, Oh and Ahn (2012*a*) have showed that the infinitesimal rigidity of $(\mathcal{G}, p^*)$ is not crucial for the local asymptotic stability of E_{p^*}. They have showed that if $(\mathcal{G}, p^*)$ is rigid in $\mathbb{R}^n$, E_{p^*} is locally asymptotically stable with respect to (28) by exploiting a property of gradient systems found in Lojasiewicz (1970).

The following potential function has been proposed in Dimarogonas and Johansson (2008, 2010):

$$\gamma_{ij}(\|p_j - p_i\|) := k_p\frac{\left(\|p_j - p_i\|^2 - \|p_j^* - p_i^*\|^2\right)^2}{\|p_j - p_i\|^2}, \tag{30}$$

where $k_p > 0$. Since the value of function (30) approaches infinity as $p_i - p_j \to 0$ for any $(i,j) \in \mathcal{E}_+$, the gradient control law based on (30) ensures collision avoidance between neighboring agents (Dimarogonas and Johansson, 2008). Dimarogonas and Johansson (2008, 2010) have showed that the equilibrium set,

$$E'_{p^*} := \{p \in \mathbb{R}^{nN} : \|p_j - p_i\| = \|p_j^* - p_i^*\|,\ \forall (i,j) \in \mathcal{E}_+\},$$

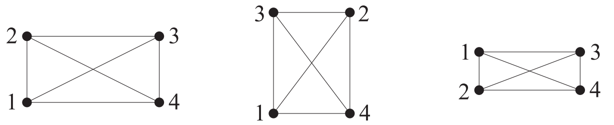
is locally asymptotically stable and p always converges to E'_{p^*} asymptotically whenever any agents are not collocated initially if and only if $\mathcal{G}$ is a tree. In the case that $\mathcal{G}$ has a cycle, the global stability property is not valid any more as studied in Dimarogonas and Johansson (2009, 2010). While $\mathcal{G}$ has been assumed to be a tree for the investigation of the global stability property of E'_{p^*} in Dimarogonas and Johansson (2008, 2010), it can be showed that if $(\mathcal{G}, p^*)$ is rigid, E_{p^*} is locally asymptotically stable with respect to (28) under the gradient control law based on (30) (Oh and Ahn, 2012*a*).

Oh and Ahn (2011*b*,*c*,*e*) have proposed a control law, which is not described as the gradient of a potential function. Attempting to control the squared inter-agent distances in a prescribed manner, they have designed a control law and showed the local asymptotic stability of E_{p^*} under the control law. Interestingly, the control law is related with that in Dörfler and Francis (2009); Krick *et al.* (2008, 2009) by multiplication of a positive definite matrix though it has been derived from different idea.

Technically, E_{p^*} is not globally asymptotically stable with respect to (28) under gradient control laws based on functions γ_{ij} defined in (29) or (30). This is obvious due to the existence of trivial undesired equilibrium points of (28). For instance, if all the agents are located at a common point, then the control inputs are zero or not defined. Further, whenever all the agents are located in a common straight line, they cannot escape from the line. Thus if the dimension of the affine hull of $p_1, \ldots, p_N$ is n, E_{p^*} is not globally asymptotically stable.

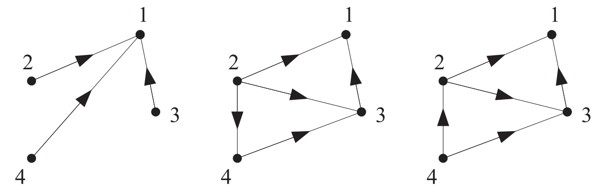
For three-agent case, global stability properties have been revealed. Consider three agents moving in the plane and assume that $\mathcal{G}$ is a undirected complete graph and p_1^*, p_2^*, p_3^* are not collinear. Under the gradient law based on (29), E_{p^*} is locally asymptotically stable. Notice that the undesired equilibrium points in this case correspond to the critical points of $\phi(p)$ involving the collinearity of the three points. It can be showed that if p_1^*, p_2^*, p_3^* are not collinear initially, then p does not approach the undesired equilibrium points based on the analysis found in Cao *et al.* (2008*b*).

According to Krick *et al.* (2009), there exist non-trivial undesired equilibrium points for a four agent formation with a undirected complete graph under the gradient control law



(a) Desired. (b) Undesired. (c) Undesired.

Fig. 4: Desired and undesired rectangular equilibrium formations taken from Summers *et al.* (2009).



(a) Not rigid. (b) Not persistent. (c) Persistent.

Fig. 5: Examples of directed frameworks taken from Hendrickx *et al.* (2007).

based on (29). Suppose that the desired formation for the agents is given by the rectangular formation shape depicted in Figure 4a with $\|p_2^* - p_1^*\| = a$ and $\|p_4^* - p_1^*\| = b$, where $a > 0$ and $b > 0$. By simulation, Krick *et al.* (2009) have showed that p can converge to the rectangular formation shape depicted in Figure 4b with $\|p_3 - p_1\| = \sqrt{a^2 + b^2/3}$ and $\|p_4 - p_1\| = \sqrt{b^2/3}$.

Motivated by the example in Krick *et al.* (2009), global stability properties of a undirected four-agent formation have been investigated in Anderson *et al.* (2010); Dasgupta *et al.* (2011); Summers *et al.* (2009). Summers *et al.* (2009) have showed that there exists another rectangular formation shape involving undesired equilibria (Figure 4c) and the equilibrium points associated with the two undesired rectangular formations are saddle and therefore unstable. Anderson *et al.* (2010) have showed that a rectangular desired formation has two different associated undesired rectangular equilibria and they are necessarily saddle points. Dasgupta *et al.* (2011) have proved that every undesired equilibrium formation is unstable if the desired formation is given by a rectangle. Though global stability properties have been revealed for rectangular formations, as remarked in Anderson *et al.* (2010); Dasgupta *et al.* (2011), it is a still challenging open problem to show whether there exists a undesired, attractive equilibrium for a general quadrilateral formation.

In general, the global stability properties of distance-based formations remain open. The major difficulty arises from the fact that the set of critical points of $\phi(p)$ are not analytically found.

2) Double-integrator case: Distance-based control of double-integrator modeled agents has been studied in Oh and Ahn (2012*a*); Olfati-Saber and Murray (2002). Consider the following N double-integrator modeled agents in n-dimensional space:

$$\begin{cases} \dot{p}_i = v_i, \\ \dot{v}_i = u_i, \end{cases} \quad i = 1, \dots, N, \tag{31}$$

where $p_i \in \mathbb{R}^n$, $v_i \in \mathbb{R}^n$, and $u_i \in \mathbb{R}^n$ denote the position, the velocity, and the control input, respectively, of agent i with respect to ${}^g\sum$. Assume that each agent measures its own velocity and the relative positions of its neighbors with respect to its own local coordinate system. Further, given a realization $p^* = [p_1^{*T} \cdots p_N^{*T}]^T \in \mathbb{R}^{nN}$, we define the desired formation E_{p^*,v^*} of the agents as

$$\begin{aligned} E_{p^*,v^*} := \{[p^T\ v^T]^T \in \mathbb{R}^{2nN} : \|p_j - p_i\| = \|p_j^* - p_i^*\|, \\ v = 0,\ \forall i, j \in \mathcal{V}\}. \end{aligned}$$

For agents (31) in the plane, Olfati-Saber and Murray (2002) have proposed the following control law:

$$u = -\nabla_p \phi(p) - D(p, v), \tag{32}$$

where ϕ is defined in (27) based on (29) and $D(p, v)$ satisfies the following properties: $\langle D(p, v), v\rangle > 0$ for all $v \neq 0$ and $D(p, 0) = 0$. Then they have showed the local asymptotic stability of E_{p^*,v^*} with respect to (31) under the proposed control law.

Oh and Ahn (2012*a*) have also studied distance-based formation control of agents (31). Defining

$$\psi(p, v) := \frac{1}{2}\sum_{i\in\mathcal{V}} \|v_i\|^2 + \sum_{i=1}^{M} \gamma_{ij}(\|p_j - p_i\|^2),$$

where γ_{ij} is defined in (29), they have proposed a gradient based control law to obtain the following overall dynamics, which is a dissipative Hamiltonian system:

$$\dot{p} = \nabla_v \psi, \tag{33a}$$

$$\dot{v} = -k_v \nabla_v \psi - \nabla_p \psi, \tag{33b}$$

where $k_p > 0$ and $k_v > 0$. Based on the results in Dorfler and Bullo (2011), which have revealed the topological equivalence of (33) to the following system,

$$\begin{aligned} \dot{p} &= -\nabla_p \psi, \\ \dot{v} &= -k_v \nabla_v \psi, \end{aligned}$$

they have showed the local asymptotic stability of E_{p^*,v^*} with respect to (31) under the proposed control law.

C. Distance-based directed formation

We next review distance-based directed formation control problems. The problem setup is basically the same as the undirected case except that the graph $\mathcal{G}$ representing the sensing topology among agents is directed.

Though the rigidity of the underlying undirected graph of $\mathcal{G}$ is a necessary condition for the stability of E_p^* defined in (23), there is subtlety in directed formation case. To see this, consider the sensing graphs for four agents showed in Figure 5. Suppose that the agents are in the plane. First, the underlying undirected graph of Figure 5a is not rigid. Thus E_{p^*} cannot be stable in this case. Second, though the underlying undirected graph of Figure 5b is rigid, agent 2 has too much responsibility. Notice that agents 1, 2, and 3 do not care the lengths of the edges $(1, 2)$, $(3, 2)$, and $(4, 2)$, and thus agent 2 is responsible for controlling the lengths of the three edges. Since agent 2 is moving in the plane, it cannot control the three edge lengths independently. This shows that the graph rigidity of the underlying undirected graph is not sufficient for the stability of E_p^*. Third, the responsibility for controlling edge lengths is well distributed in the case of Figure 5c. That is, every agent can control its outgoing edge lengths to the appropriately assigned desired values, provided that all the other edges have the desired lengths.

As depicted in Figure 5, it is required to appropriately assign the responsibility for controlling edge lengths. This condition has been observed in Baillieul and Suri (2003) and then has been extensively studied in Hendrickx *et al.* (2007); Yu *et al.* (2007).

1) Graph persistence: Let $\mathcal{G} = (\mathcal{V}, \mathcal{E})$ be a directed graph. Suppose that, for a pair $(\mathcal{G}, p)$, desired squared distances d_{ij}^* for all $(i, j) \in \mathcal{E}$ are given. Then the edge $(i, j) \in \mathcal{E}$ is active if $\|p_i - p_j\|^2 = d_{ij}^*$. Given $p \in \mathbb{R}^{nN}$, the position $p_i \in \mathbb{R}^n$ is fitting for the desired squared distances if there is no $p_i' \in \mathbb{R}^n$ such that the following strict inclusion holds:

$$\{j \in \mathcal{N}_i : \|p_i - p_j\|^2 = d_{ij}^*\} \subset \{j \in \mathcal{N}_i : \|p_i' - p_j\|^2 = d_{ij}^*\}.$$

This means that p_i is one of the best positions to maximize the number of active outgoing edges of node i when the positions of the other nodes remain unchanged. The framework $(\mathcal{G}, p)$ is fitting for the desired squared distances if all the vertices of $\mathcal{G}$ are at fitting positions for the desired squared distances. Then the persistence of frameworks is defined as follows (Hendrickx *et al.*, 2007):

Definition 6.2: (Hendrickx *et al.*, 2007) Let $\mathcal{G} = (\mathcal{V}, \mathcal{E})$ be a directed graph. A framework $(\mathcal{G}, p)$ is persistent if there exists $\epsilon > 0$ such that every realization q fitting for the distance set induced by p and satisfying $d(p, q) < \epsilon$, where $d(p, q) = \max_{i \in \mathcal{V}} \|p_i - q_i\|$, is congruent to p.
That is, if $(\mathcal{G}, p)$ is persistent, there exists a neighborhood of p such that every realization q fitting to p is congruent to p in the neighborhood. Analogous to minimal rigidity, the notion of minimal persistence is defined as follows (Hendrickx *et al.*, 2007):

Definition 6.3: (Hendrickx *et al.*, 2007) Let $\mathcal{G} = (\mathcal{V}, \mathcal{E})$ be a directed graph. A framework $(\mathcal{G}, p)$ is minimally persistent if it is persistent and no edge can be removed without losing persistence.

2) Distance-based control of persistent formations: The existing results on distance-based directed formation control have focused on single-integrator modeled agents in the plane under persistence assumption. An early work on distance-based directed formation control is found in Baillieul and Suri (2003). For a single-integrator modeled agents in the plane, they have proposed a control law u_i as

$$u_i = k_p \sum_{j \in \mathcal{N}_i} (\|p_j - p_i\| - \|p_j^* - p_i^*\|)(p_j - p_i), \tag{34}$$

and they have raised the possibility of instability of directed formations having cyclic sensing topology at least in the presence of measurement noises and/or biases based on a cyclic triangular formation example, which motivated research interest on the control of directed formations having cyclic sensing topology. Further, Baillieul and Suri (2003) have provided three conditions under which a directed formation is stably rigid. They call a formation stably rigid under a control law if for any sufficiently small perturbation in the relative positions of the agents, the control law steers them asymptotically back into the prescribed formation. Eren *et al.* (2005) have claimed that one of the three conditions provided in Baillieul and Suri (2003) is redundant and showed that a directed formation is stably rigid under a control law similar to (34) if the following conditions hold: (i) the underlying undirected graph of a directed formation is infinitesimally, minimally rigid; (ii) the directed formation graph is acyclic.

Distance-based control of acyclic persistent formations have been studied in Krick *et al.* (2009); Oh and Ahn (2011*a*). Any acyclic persistent formations, which are the simplest type of persistent formations, can be constructed by Henneberg vertex addition sequence (Tay and Whiteley, 1985). Under control law (26) with γ_{ij} in (29), Krick *et al.* (2009) have applied the center manifold theory to show the local asymptotic stability of an acyclic, minimally persistent formation. Oh and Ahn (2011*a*) have showed the local convergence of a acyclic persistent formation by means of input-to-state stability properties (Khalil, 1996).

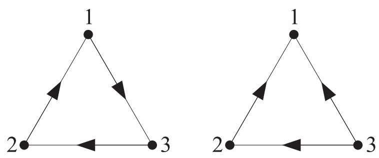
Minimally persistent formation control has been studied in Summers *et al.* (2011); Yu *et al.* (2009). A directed framework $(\mathcal{G}, p)$ is minimally persistent in $\mathbb{R}^2$ if and only if it is persistent and $|\mathcal{E}| = 2|\mathcal{V}| - 3$ (Hendrickx *et al.*, 2007). From this property, any minimally persistent formation belongs to one of the following types (Summers *et al.*, 2011):

- Leader-first-follower (LFF): One leader agent has no out-going edge. One first follower has one out-going edge to the leader agent. The other agents have two out-going edges;
- Leader-remote-follower (LRF): One leader agent has no out-going edges. One remote follower has one out-going edge to an agent other than the leader agent. The other agents have two out-going edges;
- Co-leader: Each of three agents known as co-leaders has one out-going edge. The other agents have two out-going edges.

Yu *et al.* (2009) have proposed the following control for N single-integrator modeled agents in the plane to achieve an LFF type desired formation:

$$u_i = K_i(\bar{p}_i - p_i), \tag{35}$$

where $K_i \in \mathbb{R}^{2\times 2}$ is a gain matrix and $\bar{p}_i$ is the closest position for agent $i \in \{1, \ldots, N\}$ such that $\|p_j - \bar{p}_i\| = \|p_j^* - p_i^*\|$ for all $j \in \mathcal{N}_i$, where $p^* \in \mathbb{R}^{2N}$ is given for specifying the



(a) Co-leader type. (b) LFF type.

Fig. 6: Minimally persistent triangular formations.

desired formation. When the perturbation of p from the desired shape is sufficiently small, $\bar{p}$ is well-defined and unique (Yu *et al.*, 2009). By choosing a global coordinate system so that the x-axis coincides with the line joining the leader and the first-follower, Yu *et al.* (2009) have obtained a hyperbolic linearized system and they further showed that the system matrix of the linearized system becomes Hurwitz by suitably designing K_i, which is possible for LFF type formations.

Under control law (35), Summers *et al.* (2011) have studied local stability of LRF and co-leader type formations of single-integrator modeled agents in the plane. After proving the desired formation of the agents is indeed a center manifold, they have used the result to show that LRF and co-leader type, directed formations become locally asymptotically stable by suitably taking K_i.

Minimally persistent triangular formations depicted in Figure 6 have been studied in Anderson *et al.* (2007); Cao *et al.* (2007, 2011*a*, 2008*a*,*b*); Dörfler and Francis (2010); Smith *et al.* (2006). In such works, it is assumed that p_1^*, p_2^*, p_3^* are not collinear. Then it has been showed that p exponentially converges to E_{p^*} provided that p_1, p_2, p_3 are initially not collinear.

Anderson *et al.* (2007) have proposed the following control law for co-leader type triangular formations in the plane:

$$u_i = (\|p_j - p_i\| - \|p_j^* - p_i^*\|)\frac{(p_j - p_i)}{\|p_j - p_i\|},\ j \in \mathcal{N}_i.$$

Then they have showed that the inter-agent distances exponentially converge to the desired values unless p_1, p_2, p_3 are collinear initially. Cao *et al.* (2007, 2011*a*) have also studied the local asymptotic stability of co-leader type triangular formations. They have proved that p asymptotically converges to a finite realization in E_{p^*} under control law (26) with γ_{ij} in (29), provided that p is not collinear initially, by showing that all the edge lengths exponentially converges to the desired values. Further, they have showed that all the trajectories of p starting from the outside of the collinear realization set do not approach the undesired equilibrium set involving the collinearity. The same stability property has been obtained in Cao *et al.* (2008*a*) for LFF type triangular formations based on a similar stability analysis. Extending the result in Cao *et al.* (2007), Cao *et al.* (2011*a*, 2008*b*) have showed that the stability properties of co-leader type triangular formations are also obtained under a generalized control law. Dörfler and Francis (2010) have showed that all invariant sets other than the desired formation is unstable for a co-leader type triangular formation by means of a differential geometric analysis. Further, they have remarked that stability properties of a triangular formation is not dependent upon whether the sensing graph is undirected or directed. Park *et al.* (2012) have proposed a control law for a LFF type triangular formation to allow the agents to escape from collinear positions.

VII. Discussions and further issues

A. Position-based approach

In position-based approach, agents measure their positions with respect to a global coordinate system and they actively control their positions to achieve their desired formation, which is given by the desired destinations for them. Thus, based on the concept in Section I, since it relies upon global sensing information, the position-based formation control can be considered as a centralized approach. The desired formation can be achieved without any interactions among the agents under ideal conditions. Some interactions among the agents have been introduced in the literature to consider practical issues such as disturbances, actuator saturation, etc. Different from displacement- and distance-based approaches, realistic agent models have been studied in position-based approach. This approach might not be cost effective because agents need to carry positioning sensors such as GPS receivers to measure their absolute positions. However, this approach might provide effective solutions to practical formation control applications.

B. Displacement-based approach

In displacement-based approach, agents measure the relative positions of their neighboring agents with respect to a global coordinate system and they actively control the measured displacements to achieve their desired formation, which is characterized by the desired values for the displacements. It is assumed that majority of the agents do not measure their absolute positions with respect to the global coordinate system. Thus, since it uses global and local sensing information for formation control, the displacement-based approach is a kind of combination of centralized and decentralized approaches. As seen in Section V, many formation control problems in this approach can be formulated as consensus problems. That is why stability analyses in this approach are closely related with properties of graph Laplacians.

Some future research directions in this approach can be summarized as follows. First, heterogeneous agents need to be studied. Majority of the existing results in this approach have been focused on formation control of identical agents. In applications, heterogeneous agents might consist of a team due to various reasons. Second, connectivity preservation and collision avoidance issues are important in applications. Further, it is desirable to consider both issues simultaneously to ensure satisfactory performance.

C. Distance-based approach

In distance-based approach, agents measure the relative positions of their neighboring agents with respect to their local coordinate systems under the assumption that the agents do not share a common sense of orientation and they actively control the lengths of the relative positions to achieve their desired formation, which is characterized by the desired values for the lengths. Since only the inter-agent distances are actively controlled in this approach, the sensing graph for the agents is required to be rigid or persistent. A main advantage of this approach is that agents need not share any global information such as knowledge on a global coordinate system. Thus, it is a decentralized approach based on the concept in Section I. A disadvantage of this approach is that global stability properties have yet to be investigated for general cases.

Future research directions in this approach can be summarized as follows. First, stability properties of general rigid or persistent formations need to be investigated. Though global stability properties of triangular formations have been satisfactorily investigated, those of general rigid or persistent formations have yet to be investigated. Further, even local stability properties of general persistent formations have not been fully revealed. Second, it is desirable to consider more practical agent model. Majority of the existing results in this approach have been focused on single-integrator modeled agents in the plane. Though such simple agents are beneficial for investigating the fundamental properties, reasonably complicated agent models need to be studied to enhance the practicalty of this approach. Finally, some practical issues such as connectivity preservation and collision avoidance should be considered.

VIII. Network localization

While agents are assumed to know their own absolute positions in position-based approach, such position information is not available in displacement- and distance-based approaches in general. Though absolute positions are not crucial for formation shape control, it might be desirable to allow the agents to know their positions at least with respect to a common coordinate system, depending on the mission of the agents. For instance, suppose that the agents are committed to monitor some physical characteristics over a certain area. In this case, the agents are essentially intended to provide spatial characteristics of the area, which requires location stamps for gathered data (Krishnamachari, 2005).

A variety of approaches have been proposed for network localization. Several excellent surveys on general localization problems are found, for instance, in Mao and Fidan (2009); Mao *et al.* (2007); Patwari *et al.* (2005). While agents are assumed to have quite limited sensing capabilities in common localization problems setup, we focus on localization algorithm in displacement- and distance-based formation control problem setups in this section.

A. Displacement-based localization

Consider single-integrator modeled agents (20). Under the assumption that the agents measure the relative-positions of their neighbors with respect to a global coordinate system and any neighboring agents are able to communicate to exchange their local information, a relative position position estimation law can be designed as follows (Oh and Ahn, 2011*d*, 2012*b*):

$$\dot{\hat{p}}_i = u_i + k_o \sum_{j\in\mathcal{N}_i} a_{ij}\left[(\hat{p}_j - \hat{p}_i) - (p_j - p_i)\right],$$

where $k_o > 0$ and $\hat{p}_i$ is the estimated position of agent $i \in \mathcal{V}$ with respect to a global coordinate system. Defining $\tilde{p}_i = p_i - \hat{p}_i$, the overall estimation error dynamics can be arranged as

$$\dot{\tilde{p}} = -k_o(L \otimes I_n)\tilde{p}, \tag{36}$$

where L is the Laplacian matrix of $\mathcal{G}$. Then $\hat{p}$ asymptotically converges to p up to translation if $\mathcal{G}$ is uniformly connected. Based on the estimated positions, Oh and Ahn (2011*d*, 2012*b*) have proposed a formation control law. Further, they have applied this position estimation approach to unicycles.

It is well known that there is duality between controllability and observability of a linear time-invariant system (Chen, 1998). The similarity between (9) and (36) shows such duality between formation control and localization, which has been investigated in Tuna (2008); Zhang *et al.* (2011). For agents (13) over $\mathcal{G}$, consider the following localization law:

$$\dot{\hat{x}}_i = A\hat{x}_i + Bu_i - F\sum_{j\in\mathcal{N}_i} a_{ij}(\hat{y}_i - \hat{y}_j - y_i + y_j),$$

where $\hat{x}_i$ is the estimated state of agent $i \in \mathcal{V}$. Define $\tilde{x} := x - \hat{x}$ to obtain

$$\dot{\tilde{x}} = (I_N \otimes A - L \otimes FC)\tilde{x}. \tag{37}$$

Let $\mathcal{G}'$ be the reversed graph $\mathcal{G}$ and consider the following agents over a graph $\mathcal{G}'$:

$$\dot{x}_i = A^T x_i + u_i,\ i = 1, \ldots, N.$$

Let $u_i = -F^T B^T \sum_{j\in\mathcal{N}'_i} a'_{ij}(x_i - x_j)$ to obtain

$$\dot{e}_x = (I_N \otimes A^T - L^T \otimes FB^T)e_x. \tag{38}$$

In view of (37) and (38), localization is achieved for (A, C) by F over $\mathcal{G}$ if and only if the desired formation is achieved for (A^T, B^T) by F^T over $\mathcal{G}'$ (Zhang *et al.*, 2011). This shows duality between formation control and localization in displacement-based approach. Tuna (2008) has also pointed out such duality.

B. Distance-based localization

Distance-based localization problems have been usually formulated as optimization problems in the literature, e.g., Alfakih *et al.* (1999); Biswas and Ye (2004); Ding *et al.* (2010); Shang *et al.* (2003); So and Ye (2007). In such works, agents are assumed to measure only inter-agent distances. Thus the graph representing the sensing and communication topology among the agents needs to be globally rigid as remarked in Aspnes *et al.* (2006). Most of the localization algorithms proposed in the works need to be implemented in a central data center. When the sensing topology of agents has a special structure, the agents can be easily localized by using a tri-lateration based algorithm, which can be implemented in a distributed way Fang *et al.* (2009).

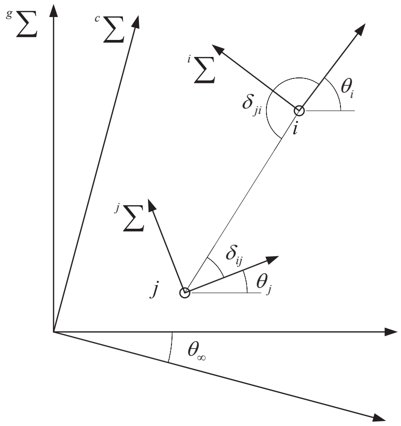


Fig. 7: Measurement of relative orientation angle.

As reviewed in Section VI, in distance-based approach, it is usually assumed that agents measure the relative positions of their neighbors with respect to their own local coordinate systems. Then, under the assumption that neighboring agents are able to exchange their estimated positions with each other, we can consider the following localization law for N stationary agents over a undirected graph $\mathcal{G} = (\mathcal{V}, \mathcal{E})$:

$$\dot{\hat{p}}_i = k_o \sum_{j \in N_i} (\|\hat{p}_j - \hat{p}_i\|^2 - \|p_j - p_i\|^2)(\hat{p}_j - \hat{p}_i), \qquad (39)$$

where $k_o > 0$, for $i \in \{1, \dots, N\}$. Interestingly, (39) has the same form as the formation dynamics in Krick *et al.* (2008, 2009). This similarity also shows that there is duality between localization and formation control. Based on the results from Krick *et al.* (2008, 2009), it can be concluded that $[\hat{p}_1^T \cdots \hat{p}_N^T]$ locally asymptotically converges to $[p_1^T \cdots p_N^T]$ up to congruence if $(\mathcal{G}, p)$ is rigid.

Another approach is found in Oh and Ahn (2012*c*). For stationary agents in the plane over a directed graph, they have proposed a localization algorithm based on orientation estimation. Let θ_i be the orientation angle of the local coordinate system of agent i as depicted in Figure 7. Then agent i calculates the relative orientation angles $\theta_j - \theta_i$ for $j \in \mathcal{N}_i$ by using its measurements and communicating with its neighbors. By using the relative orientation angles, the agents estimate their orientation angles up to translation. Then the positions can be estimated by a displacement-based localization algorithm proposed in Oh and Ahn (2011*d*, 2012*b*). Thus, the distance-based localization proposed in Oh and Ahn (2012*c*) is a decentralized scheme since it does not use any global sensing information.

IX. Other approaches

Though we focus on position-, displacement-, and distance-based approaches in this survey, some important results do not belong to such approaches. In this section, we briefly review some important results that do not belong to the three approaches.

A. Morphous formation control

1) Formation control based the controllability of consensus networks: Ji *et al.* (2006); Olfati-Saber and Shamma (2005); Rahmani and Mesbahi (2006); Rahmani *et al.* (2009); Tanner (2004) have investigated the controllability of consensus networks, which can be exploited for formation control. Tanner (2004) have considered a single-integrator consensus network with a leader agent. The state of the leader agent can be viewed as the control input to the states of the remaining agents. Then they have provided a necessary and sufficient condition for the controllability based on properties of network topology of the agents. According to the analysis in Tanner (2004), an increased connectivity might show an adverse effect on the controllability of the network. More general results are found in Rahmani *et al.* (2009). Generalizing the results in Tanner (2004), Rahmani and Mesbahi (2006); Rahmani *et al.* (2009) have presented a necessary and sufficient condition for the controllability in the presence of multiples of leader agents and showed some further properties of controlled consensus networks based on the symmetry structure of the network. Ji *et al.* (2006) have proposed a centralized optimal algorithm for a controlled consensus networks from a quasi-static equilibrium point to another. More detailed review on these results is found in Mesbahi and Egerstedt (2010).

2) Angle-based approach: Basiri *et al.* (2010); Bishop (2011*b*) have studied a formation control based on bearing measurements for three agents in the plane. The desired formation for the agents is specified by $\alpha^* = [\alpha_1^* \ \alpha_2^* \ \alpha_3^*]^T$, where α_i^* is the desired angle subtended at agent $i \in \{1, 2, 3\}$ by the other two agents. Notice that only the formation shape, not scale, is specified by α^*. Assuming that agent $i \in \{1, 2, 3\}$ measures the subtended angle α_i, they have proposed the control law as

$$\dot{p}_i = (\alpha_i^* - \alpha_i)[\cos\beta_i \ \sin\beta_i]^T, \qquad (40)$$

where β_i is the angle of the line that passes through p_i bisecting the subtended angle α_i with respect to the x-axis of the global coordinate system.Then they have proved that α^* is globally asymptotically stable with respect to (40) by showing that α^* is the only equilibrium and there is no periodic orbit. Based on the control law (40), Bishop and Basiri (2010) have extended the result of Basiri *et al.* (2010); Bishop (2011*b*) to the three agents on a sphere. Bishop (2011*a*) has extended the result to the four agents whose sensing topology is a undirected cycle to show that the desired formation is globally asymptotically stable based on a Lyapunov stability analysis.

Generalizing the previous results for three and four agent cases, Bishop *et al.* (2011) have proposed a gradient control law based on inter-agent bearing measurements under the assumption that agents share a common direction. Defining $\alpha := [\alpha_1 \cdots \alpha_N]^T$ by ordering the subtended angles in some way, they have proposed a control law as

$$u = -(\nabla(\alpha - \alpha^*))^T(\alpha - \alpha^*), \qquad (41)$$

where α^* is given. Then they have showed the local asymptotic stability of the desired formation under certain conditions.

When the desired formation shape is specified by subtended angles, its scale is not uniquely determined. To remove this scaling ambiguity, Bishop *et al.* (2012) have proposed a formation control strategy based on a mix of angle and distance measurements to three agents in the plane. Assuming that agent 1 measures the relative position of agent 2 while agents 2 and 3 measure their subtended angles, the desired formation is specified by the prescribed distance between agents 1 and 2 and subtended angles α_2^* and α_3^*. Designing a control law for agent 1 to control the distance of agent 2 while adopting (40) as the control law for agents 2 and 3, they have presented a condition for the local convergence of the triangular formation to the desired one.

3) Formation control based on only distance measurements: In distance-based formation control problems reviewed in section VI, agents are required to measure the relative positions of their neighbors though desired formations are achieved by controlling the distances of the neighbors. Motivated by this observation, Cao *et al.* (2011*b*) have proposed a formation shape control strategy based only on distance measurements. Though the distances among the neighboring agents are actively controlled in this work, the problem setup is different from the typical one discussed in VI.

In Cao *et al.* (2011*b*), the agents are suitably partitioned into at most four subgroups, and each agent i belonging to a subgroup localizes the relative positions of its neighboring agents with respect to its own local coordinate system and moves to reduce the value of its potential function $\phi_i(p_i, \ldots, p_j, \ldots) = \sum_{j\in\mathcal{N}_i}\left(\left(\|p_j - p_i\|^2 - \|p_j^* - p_i^*\|^2\right)^2\right.$ while all the agents belonging to the other subsets remain stationary. Then Cao *et al.* (2011*b*) have showed that all inter-agent distances converge to the desired values by repeating the procedure cyclically with respect to the subgroups, under the assumption that $(\mathcal{G}, p^*)$ is minimally, infinitesimally rigid. Further, Cao *et al.* (2011*b*) have proposed a strategy for determining the velocity of a leader agent moving at a constant velocity based only on distance measurements.

B. Amorphous formation control

In amorphous formation control problems, the desired formation of agents is not explicitly specified. One of typical amorphous formation control is flocking. In a flocking problem, some behavioral rules are assigned to agents to show some collective behavior. Another amorphous formation control is containment control. In a containment control problem, follower agents are driven by multiple leader agents to be in the convex hull spanned by the leaders. An extensive review on containment control is found in Ren and Cao (2010). Thus we focus on flocking in this subsection.

For decades, scientists have revealed that collective behaviors discovered in various fields are indeed based on relatively simple interactions among individuals without any intervention of a central coordinator (Strogatz, 2003). Inspired by such collective behaviors, Reynolds (1987) has proposed an agent model based on the following three basic rules, which are known as Reynolds rules:

- Cohesion: stay close to nearby neighbors;
- Separation: avoid collisions with nearby neighbors;
- Alignment: match velocity with nearby neighbors,

Based on these simple rules, the agent model shows an extremely realistic motion, which might be hard to be created otherwise.

Many control laws have been proposed to achieve collective behaviors in the literature. Majority of them can be viewed as implementations of Reynolds rules. As reviewed below, cohesion/separation rules have been usually implemented by means of of artificial potential functions of inter-agent distances. Alignment rule has been implemented by means of velocity consensus of agents. Further, virtual leaders have been introduced to allow agents to move to their prescribed destinations in some works.

Leonard and Fiorelli (2001) have proposed a framework for achieving collective motions of double-integrator modeled agents by means of artificial potential functions and virtual leaders. Assuming that the agents know the positions of virtual leaders and measure the relative positions of other agents locating within some range, they have attempted to encode the behavioral rules suggested by Okubo (1986), which are similar to Reynolds rules. In their framework, artificial potential functions define interaction forces between neighboring agents to maintain inter-agent distances properly. Virtual leaders provide moving reference points that influence the agents by means of additional potential functions. Their proposed control law consists of interaction forces associated with neighboring agents and virtual leaders and some dissipative force. The dissipative force is responsible for matching velocities of the agents. Leonard and Fiorelli (2001) have illustrated various collective motions can be achieved in their framework. Based on the kinetic and the artificial potential energies of the agents, a Lyapunov function can be constructed for the stability analysis of of such motions.

Tanner *et al.* (2003*a*,*b*, 2007) have studied collective motions of double-integrator modeled agents in the plane based on Reynolds rules. They have assumed that the relative position and velocity sensing topologies might be nonidentical. Assuming that the topologies are characterized by undirected graphs $\mathcal{G}^p$ and $\mathcal{G}^v$, respectively, under their proposed control law, the agents are described as

$$\ddot{p}_i = -\sum_{j\in\mathcal{N}^v}\nabla_{p_i}\phi(\|p_j - p_i\|) - \sum_{j\in\mathcal{N}_i^p}(\dot{p}_i - \dot{p}_j),$$

where $\phi : \mathbb{R}_{\geq 0} \to \mathbb{R}_{\geq 0}$ is a differentiable function such that $\phi(x) \to \infty$ as $x \to 0$, $\phi(x) = 0$ if and only if $x = d^* > 0$, and $d\phi(x)/dx = 0$ for all $x > r^* > 0$. Here d^* is the desired inter-agent distance and r^* is the range for relative position sensing. The gradient-based term is used for separation and cohesion and the velocity consensus term is for alignment. Based on a non-smooth Lyapunov stability analysis, Tanner *et al.* (2007) have showed that the velocities are asymptotically aligned, collisions between neighboring agents are avoided, and the formation approaches an extremum of the potential function $\sum_{(i,j)\in\mathcal{E}^p}\phi(\|p_j - p_i\|)$ if $\mathcal{G}^p$ and $\mathcal{G}^v$ are arbitrarily switching but always connected by means of a non-smooth Lyapunov analysis. Fixed topology case has been addressed in Tanner *et al.* (2003*a*).

Based on Reynolds rules, Olfati-Saber (2006); Olfati-Saber and Murray (2003) have addressed collective motions of double-integrator modeled agents in n-dimensional space under the assumption that the sensing topology of the agents are dependent on relative positions. For sensing range $r^* > 0$, the topology is modeled by a undirected graph $\mathcal{G}(p) = (\mathcal{V}, \mathcal{E}(p), \mathcal{W})$, where

$$\mathcal{E}(p) = \{(i,j) \in \mathcal{V} \times \mathcal{V} : \|p_i - p_j\| < r^*, i \neq j\}.$$

Then, under the proposed control law, the agents can be described as

$$\ddot{p}_i = -\sum_{j\in\mathcal{N}(p)} \nabla_{p_i}\phi(\|p_j - p_i\|) - \sum_{j\in\mathcal{N}_i(p)} a_{ij}(p)(\dot{p}_i - \dot{p}_j),$$

where $\phi : \mathbb{R}_{\geq 0} \to \mathbb{R}_{\geq 0}$ is a appropriately defined smooth potential function with a global minimum at $x = d^*$ for some $d^* > 0$, which is the desired inter-agent distance. Then Olfati-Saber (2006) has analyzed stability of the overall dynamics based on LaSalle's invariance principle to show that inter-agent distances asymptotically converge to zero, the velocities of the agents converge to a common vector, and no inter-agent collisions occur. Further, the following additional control input term has been considered to provide the agents a common reference point:

$$u_i^r = -c_1(p_i - p_r) - c_2(\dot{p}_i - \dot{p}_r),$$

where $c_1 > 0$, $c_2 > 0$, and $p_r \in \mathbb{R}^n$ denotes the positions of a common reference point. While the connectivity of $\mathcal{G}(p)$ is not ensured in general in the absence of u_i^r, Olfati-Saber (2006) has conjectured that u_i^r ensures the connectivity and inter-agent distances converge to some values close to d^*. Moreover, Olfati-Saber (2006); Olfati-Saber and Murray (2003) have proposed an obstacle avoidance scheme in which the agents generate repulsive force against obstacles.

Gazi and Passino (2003) have studied cohesive behavior of the following N-agents in n-dimensional space:

$$\dot{p}_i = \sum_{j=1}^{N} \left(a - b\exp\left(-\frac{\|p_j - p_i\|^2}{c}\right)\right)(p_j - p_i),$$

where a, b, and c are positive constants and $b > a$. Intuitively, any two agents i and j are attractive to each other if $\|p_j - p_i\| > \sqrt{c\ln(b/a)}$ and repulsive otherwise in their model. Based on a Lyapunov stability analysis, they have showed that the agents globally asymptotically approach to finite points in a hyperball with $(b/a)\sqrt{c/2}\exp(-1/2)$ as its radius.

X. Conclusion

We presented a brief survey of formation control in this report. Categorizing the existing results into position-, displacement-, and distance-based approaches, we provided fundamental problem setups in the approaches and reviewed some important results. Further some other existing results related with multi-agent formation control were also reviewed. From the literature review, it is shown that the research trend has been changed from centralized (i.e., position-based approaches) to decentralized approaches (i.e., distance-based approaches). It is remarkable that this survey is far from an exhaustive literature review; and thus, many important results might be missed in this report. Nevertheless we believe that this survey provides a helpful overview of formation control.

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