Optimization

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Simplex Method

Problem 3

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Problem

Solve the linear programming problem using the Simplex Method.

$$\text{Minimize } f = -40x_1 - 100x_2$$

Subject to:

$$10x_1 + 5x_2 \leq 2500, \qquad (1)$$

$$4x_1 + 10x_2 \leq 2000, \qquad (2)$$

$$2x_1 + 3x_2 \leq 900, \qquad (3)$$

$$x_1 \geq 0, x_2 \geq 0$$

Solution

The Problem is converted to canonical form by adding slack,surplus and artificial variables as appropriate.

$\Rightarrow$ As the constraint - 1 is of type '$\leq$' we should add slack variable S_1

$\Rightarrow$ As the constraint - 2 is of type '$\leq$' we should add slack variable S_2

$\Rightarrow$ As the constraint - 3 is of type '$\leq$' we should add slack variable S_3

After introducing slack variables, we have

f $= 40x_1 + 100x_2 + 0S_1 + 0S_2 + 0S_3$

subject to: $10x_1 + 5x_2 + S_1 = 2500$

$4x_1 + 10x_2 + S_2 = 2000$

$2x_1 + 3x_2 + S_3 = 900$

and $x1, x2, S1, S_2, S_3 \geq 0$

Table

Iteration-1		C_j **40**	**100**	**0**	**0**	**0**	
B	X_B	x_1	x_2	S_1	S_2	S_3	Min Ratio$=\frac{X_B}{x_2}$
S_1	2500	10	5	1	0	0	$\frac{2500}{5}$=500
S_2	2000	4	(10)	0	1	0	$\frac{2000}{10}$=200
S_3	900	2	3	0	0	1	$\frac{900}{3}$=300
Z = 0	Z_j	0	0	0	0	0	
	$Z_j - C_j$	-40	-100	0	0	0	

The minimum $Z_j - C_j$ is -100 and its column index is 2.So, the entering variable is x_2.
The minimum ratio is 200 and its row index is 2. So, the leaving basis variable is s_2.
$\therefore$ The pivot element is 10.

Entering = x_2, Departing = S_2 , Key Element = 10.

$R_2(new) = R_2(old)/10$

$R_1(new) = R_1(old)$ - $5R_1(new)$

$R_3(new) = R_3(old)$ - $3R_2(new)$

Table

Iteration-2		C_j **40**	**100**	**0**	**0**	**0**	
B	X_B	x_1	x_2	S_1	S_2	S_3	Min Ratio$=\frac{X_B}{x_2}$
S_1	1500	8	0	1	$-\frac{1}{2}$	0	
x_2	200	$\frac{2}{5}$	1	0	$\frac{1}{10}$	0	
S_3	300	$\frac{4}{5}$	0	0	$-\frac{3}{10}$	1	
$Z = 20000$	Z_j	40	100	0	10	0	
	$Z_j - C_j$	0	0	0	10	0	

Since all $Z_j - C_j \geq 0$

Hence optimal solution arrived with value of variables as:

$x_1 = 0, x_2 = 200$

Max Z $=$ 20000

Min -Z $=$ -20000

Hence, the minimum of $f = -40x_1 - 100x_2$ subject to the given conditions is -20000

Python Code

Example

```
   from scipy.optimize import linprog

C = [-40,-100]      #cost function
A = [[10,5],[4,10],[2,3]]   #Constraint matrix
B = [2500,2000,900]     #RHS of constraints
x0_bounds = (0,None)    #making sure x1 and x2 are >=0
x1_bounds = (0,None)
#call the lin prog function from the library we imported
res = linprog(C,A_ub=A,b_ub=B,bounds=(x0_bounds,x1_bounds)
,options={"disp":True})
```

The End

Thank You!