Solutions to Problems 3.8-3.11

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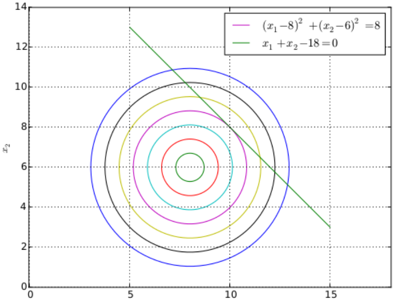
Question 3.8

Find graphically,

$$\min_{x} f(\mathbf{x}) = (x_1 - 8)^2 + (x_2 - 6)^2$$

Subject to the constraint $g(\mathbf{x}) = x_1 + x_2 - 18 = 0$
Solution:



Question 3.9

Solve Using Lagrangian,

$$\min_{x} f(\mathbf{x}) = (x_1 - 8)^2 + (x_2 - 6)^2$$

Subject to the constraint $g(\mathbf{x}) = x_1 + x_2 - 18 = 0$
Solution:

$$L(\mathbf{x}, \lambda) = (x_1 - 8)^2 + (x_2 - 6)^2 + \lambda(x_1 + x_2 - 18)$$

By Stationarity,

$$\nabla_x L(\mathbf{x}^{opt}, \lambda) = 0$$

This gives the 2 equations $2x_1 - 16 - \lambda = 0$,$2x_2 - 12 - \lambda = 0$.

Question 3.9

2 equations $2x_1 - 16 - \lambda = 0$, $2x_2 - 12 - \lambda = 0$.
From primal feasibility, we get the third equation: $x_1 + x_2 - 18 = 0$.

$$\begin{bmatrix} 2 & 0 & -1 \\ 0 & 2 & -1 \\ 1 & 1 & 0 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ \lambda \end{bmatrix} = \begin{bmatrix} 16 \\ 12 \\ 18 \end{bmatrix}$$

Solving, we get:

$$\begin{bmatrix} x_1 \\ x_2 \\ \lambda \end{bmatrix} = \begin{bmatrix} 10 \\ 8 \\ 4 \end{bmatrix}$$

Note That Dual Feasibility and Complimentary Slackness are trivially true as there are no inequality constraints.

Question 3.10

Solve the same minimization problem as in 3.9, subject to the constraint
$g(\mathbf{x}) = x_1 + x_2 - 18 \geq 0$
Solution:

$$L(\mathbf{x}, \mu) = (x_1 - 8)^2 + (x_2 - 6)^2 + \mu(x_1 + x_2 - 18)$$

By Dual Feasibility,$\mu \geq 0$.
By complimentary slackness: $\mu(x_1 + x_2 - 18) = 0$
By Stationarity,

$$\nabla_x L(\mathbf{x}^{opt}, \mu) = 0$$

This gives the 2 equations $2x_1 - 16 - \mu = 0, 2x_2 - 12 - \mu = 0$.

Question 3.10

2 equations $2x_1 - 16 - \mu = 0$, $2x_2 - 12 - \mu = 0$.
From primal feasibility, we get the third equation: $x_1 + x_2 - 18 = 0$.

$$\begin{bmatrix} 2 & 0 & -1 \\ 0 & 2 & -1 \\ 1 & 1 & 0 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ \mu \end{bmatrix} = \begin{bmatrix} 16 \\ 12 \\ 18 \end{bmatrix}$$

Solving, we get:

$$\begin{bmatrix} x_1 \\ x_2 \\ \mu \end{bmatrix} = \begin{bmatrix} 10 \\ 8 \\ 4 \end{bmatrix}$$

Note That Dual Feasibility is satisfied as $\mu \geq 0$
Note That Complimentary slackness is also satisfied as

$$\mu(x_1 + x_2 - 18) = 4(10 + 8 - 18) = 0$$

Question 3.11

Generalize Lagrangian Multiplier Method for

$$\min_x f(\mathbf{x})$$

Subject to the constraint $g(\mathbf{x}) \geq 0$
Solution:

$$L(\mathbf{x}, \lambda) = f(\mathbf{x}) - \mu g(\mathbf{x})$$

$$\nabla_x f(\mathbf{x}^{opt}), \mu) = \mu \nabla_x g(\mathbf{x}^{opt})$$

$$\mu \geq 0, \mu g(\mathbf{x}^{opt}) = 0$$