Optimization - EE5327

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Sensor Selection via Convex Optimization

Abstract—We consider the problem of choosing a set of 'k' sensor measurements, from a set of 'm' possible or potential sensor measurements, that minimizes the error in estimating some parameters. Solving this problem by evaluating the performance for each of ${}^{m}C_{k}$ the possible choices of sensor measurements is not practical unless m and k are small.
In this paper, we describe a heuristic, based on convex optimization, for approximately solving this problem. Our heuristic gives a subset selection as well as a bound on the best performance that can be achieved by any selection of 'k' sensor measurements.

Prior and Related Work:

The sensor selection problem arises in various applications, including robotics, sensor placement for structures , target tracking, chemical plant control, and wireless networks etc.

Introduction

We study the problem of selecting 'k' sensors, from among potential sensors. Each sensor gives a linear function of a parameter vector 'x' , plus an additive noise; we assume these measurement noises are independent identically distributed zero-mean Gaussian random variables. The sensor selection, i.e., the choice of the subset of 'k' sensors to use, affects the estimation error covariance matrix. Our goal is to choose the sensor selection to minimize the determinant of the estimation error covariance matrix, which is equivalent to minimizing the volume of the associated confidence ellipsoid. One simple method for solving the sensor selection problem is to evaluate the performance for all ${}^{m}C_{k}$ choices for the sensor selection, but evidently this is not practical unless or is very small. For example, with 100 potential sensors, from which we are to choose , there are on the order of possible choices, so direct enumeration is clearly not possible.

In this paper we describe a new method for approximately solving the sensor selection problem. Our method is based on convex optimization, and is therefore tractable,ble, with computational complexity growing m^3.

Sensor selection : Parameter Estimation

Suppose we are to estimate a vector x$\in R^n$ from linear measurements, corrupted by additive noise,

$$y_i = a_i^T x + v_i, i = 1, ..., m \qquad (1)$$

where x$\in R^n$ is a vector of parameters to estimate, and v_i are IID Random variables.

We assume a_i's span R^n. The maximum-likelihood estimate of x is

$$\hat{x} = \left(\sum_{i=1}^{m} a_i a_i^T \right)^{-1} \sum_{i=1}^{m} y_i a_i. \qquad (2)$$

Parameter Estimation

The estimation error "x - $\hat{x}$" has zero mean and covariance

$$\Sigma = \sigma^2 \Big(\sum_{i=1}^{m} a_i a_i^T \Big)^{-1}.$$

The η -confidence ellipsoid for error , which is the minimum volume ellipsoid that error with probability η , is given by

$$\varepsilon_\alpha = \{ z | z^T \Sigma^{-1} z \leq \alpha \} \qquad (3)$$

Parameter Estimation

A scalar measure of the quality of estimation is the volume of the η-confidence ellipsoid

$$vol(\varepsilon_\alpha) = \frac{(\alpha\pi)^{n/2}}{\Gamma(\frac{n}{2+1})} det\Sigma^{1/2} \qquad (4)$$

where is Γ the Gamma function. Another scalar measure of uncertainty, that has the same units as the entries in the parameter x , is the mean radius, defined as the geometric mean of the lengths of the semi-axes of the η-confidence ellipsoid

$$\rho(\varepsilon_\alpha) = \sqrt{\alpha}(det\Sigma)^{1/2n} \qquad (5)$$

Parameter Estimation

We will be interested in volume ratios, so it is convenient to work with the log of the volume

$$\log vol(\varepsilon_\alpha) = \beta - (\frac{1}{2}) \log det(\sum_{i=1}^{m} a_i a_i^T) \qquad (6)$$

where β is a constant that depends only on σ , n and η. The log volume of the confidence ellipsoid, given in (6), gives a quantitative measure of how informative the collection of 'm' measurements is.

Sensor Selection Problem

Now we can describe the sensor selection problem. We consider a set of m potential measurements, characterized by $a_i \in R^n$; we are to choose a subset of k ($\geq$ n) of them that minimizes the log volume (or mean radius) of the resulting confidence ellipsoid. This can be expressed as the optimization problem

$$\begin{aligned} &maximize \quad && \log det\left(\sum_{i \in S} a_i a_i^T\right) \\ &subject\ \ to \quad && |S| = k \end{aligned} \qquad (7)$$

Sensor Selection Problem

where S is optimization variable. This can be rewritten as

$$
\begin{aligned}
&\textit{maximize} \qquad && \log det\left(\sum_{i=1}^{m} z_i a_i a_i^T\right) \\
&\textit{subject to} \qquad && 1^T z = k \\
& && z_i \in \{0, 1\}, i = 1, ..., m \qquad (8)
\end{aligned}
$$

with variable $z \in R^m$

Convex Relaxation : The Relaxed Sensor Selection Problem

Now in above optimization by replacing the non-convex constraints relax the domain from z to 0 or 1 to z $\in$ [0,1]

$$\begin{aligned} & \textit{maximize} && \log \det\Big(\sum_{i=1}^{m} z_i a_i a_i^T\Big) \\ & \textit{subject to} && 1^T z = k \\ & && 0 \leq z_i \leq 1, i = 1, ..., m \qquad (9) \end{aligned}$$

This problem, unlike the original sensor selection problem (7), is convex, since the objective (to be maximized) is concave, and the equality and inequality constraints on z are linear. It can be solved efficiently, for example, using interior-point methods. These methods typically require a few tens of iterations; each iteration can be carried out (as we will see below) with a complexity O(m^3) of operations, so the overall complexity is O(m^3). The relaxed sensor selection problem (9) is not equivalent to the original sensor selection problem (7); in particular,z_i^* can be fractional.

Convex Relaxation : The Relaxed Sensor Selection Problem

We can say, however, that the optimal objective value of the relaxed sensor selection problem (9), which we denote 'U', is an upper bound on p^*, the optimal objective value of the sensor selection problem (8).

The Relaxed Sensor selection Problem

We can also use the solution z^* of the relaxed problem (9) to generate a suboptimal subset selection $\hat{S}$. There are many ways to do this; but we describe here the simplest possible method. Let $z^*_{i1}, ...z^*_{im}$ denote the elements of z^* rearranged in descending order. (Ties can be broken arbitrarily.) Our selection is then $\hat{S} = i_1, ..., i_k$.
i.e., the indexes corresponding to the k largest elements of z^*.
We let be the corresponding 0–1 vector. The point $\hat{z}$ is feasible for the sensor selection problem (8); the associated objective value

$$L = \log det\left(\sum_{i=1}^{m} \hat{z}_i a_i a_i^T\right)$$

is then a lower bound on p^* , the optimal value of the sensor selection problem (8).

Sensor Selection Problem

The difference between the upper and lower bounds on p^*,

$$\delta = U - L$$

$$= \log det\left(\sum_{i=1}^{m} z_i^* a_i a_i^T\right) - \log det\left(\sum_{i=1}^{m} \hat{z}_i a_i a_i^T\right)$$

is called the gap. The gap is always non-negative; if it is zero, then $\hat{z}$ is actually optimal for the sensor selection problem (8); more generally, we can say that the subset selection $\hat{z}$ is no more than δ-suboptimal. . The gap is, however, very useful when evaluated for a given problem instance.

Approximate Relaxed Sensor Selection

It is not necessary to solve the relaxed sensor selection problem (9) to high accuracy, since we use it only to get the upper bound U, and to find the indexes associated with the largest k values of its solution. In this section we describe a simple method for solving it approximately but very efficiently, while retaining a provable upper bound on $\hat{p}$. This can be done by solving a smooth convex problem, which is closely related to the subproblems solved in an interior-point method for solving the relaxed problem.

$$\begin{array}{ll} maximize & \psi(z) = \log det\left(\sum_{i=1}^{m} z_i a_i a_i^T\right) + \kappa \sum_{i=1}^{m} (\log z_i + \log(1 - z_i)) \\ subject\ \ to & 1^T z = k \end{array}$$

Example

In this section, we illustrate the sensor selection method with a numerical example. We consider an example instance with m = 100 potential sensors and n = 20 parameters to estimate. The measurement vectors $a_1, .., a_m$ are chosen randomly, and independently, from an $N(0, I/\sqrt{n})$ distribution. We solve the relaxed problem (11), with k = 10^{-3} , and find suboptimal subset selections, with and without local optimization, for k = 21,...,40.

Solution.

To solve each approximate relaxed problem requires 11 Newton steps, which would take a few milliseconds in a C implementation, run on a typical 2-GHz personal computer. For each problem instance, the (basic) local search checks 4000–12 000 sensor swaps, and around 3–20 swaps are taken before a 2-opt solution is found. We also the run the restricted version on the local search, which only considers sensors with z_i^* value in the interval [0.1,0.9] . This local search produces an equally good final sensor selection, while checking a factor 10–15 times fewer swaps than the basic method. (In any case, the basic local search only takes milliseconds to complete, on a typical personal computer, for a problem instance of this size.)

Results

The Quality of chosen selected sensor subsets are :

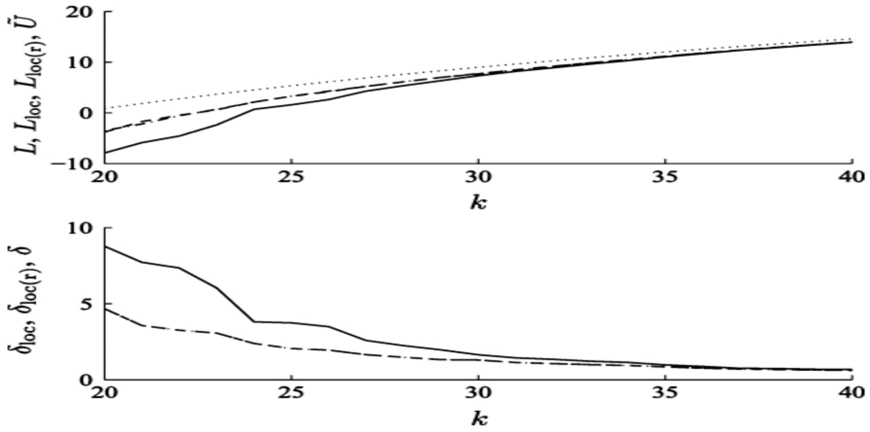

Fig. 1. Top: Upper bound $\tilde{U}$ (top curve); lower bounds L_{loc} and $L_{loc(r)}$ (middle curves); lower bound L (bottom curve). Bottom: Gap δ (top curve); δ_{loc} and $\delta_{loc(r)}$ (bottom curves).

Results

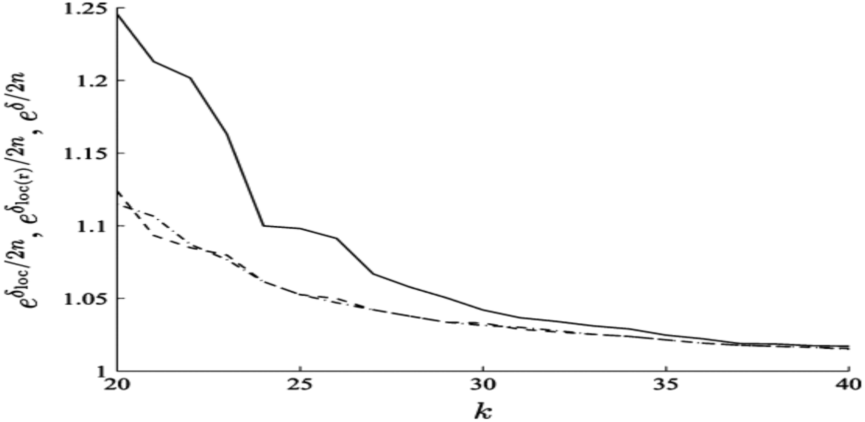

Fig. 2. Gaps expressed as ratios of mean radii: $\exp(\delta/2n)$ (top curve); $\exp(\delta_{\text{loc}}/2n)$ and $\exp(\delta_{\text{loc(r)}}/2n)$ (bottom curves).