# EE5327 : Optimization

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# Question

Plot the circles

$$f(\mathbf{x}) = (x_1 - 8)^2 + (x_2 - 6)^2 = r^2 \tag{1}$$

$\mathbf{x} = (x_1, x_2)^T$, for different values of $r$ along with the line

$$g(\mathbf{x}) = x_1 + x_2 - 9 = 0 \tag{2}$$

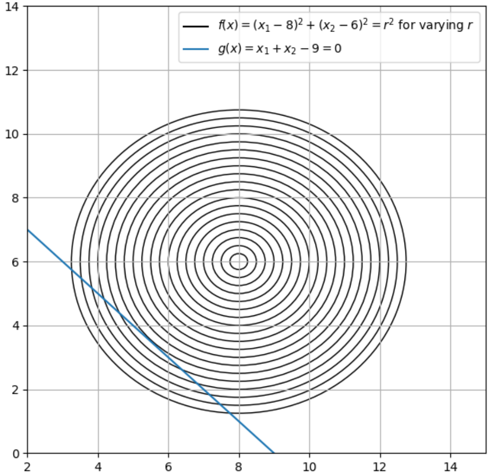
From the graph, find

$$\min_{\mathbf{x}} f(\mathbf{x}) \quad \text{s.t.}$$
$$g(\mathbf{x}) = x_1 + x_2 - 9 = 0$$

Also obtain a theoretical solution for the problem above using coordinate geometry.

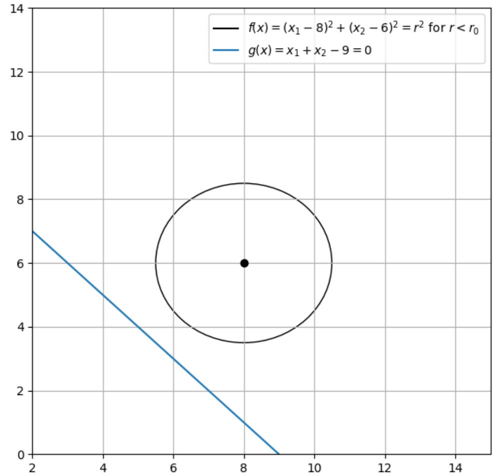
# Graphical Approach

Plotting for different values of r



# GRAPHICAL APPROACH

From the graph, find minimum value of $f(\mathrm{x})$ satisfying $g(\mathrm{x}) = 0$



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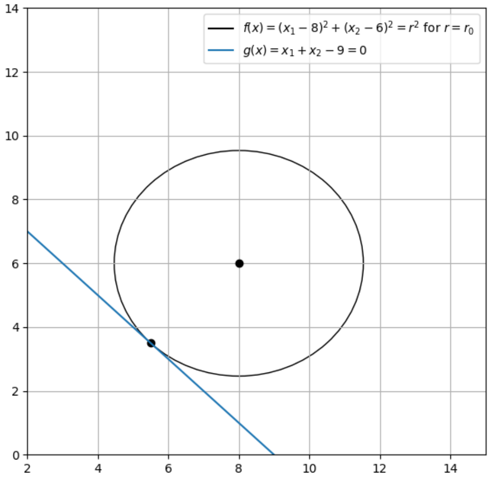
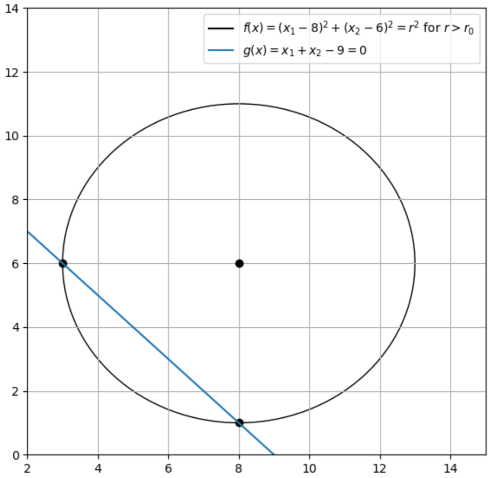


From the graph, find minimum value of $f(\mathrm{x})$ satisfying $g(\mathrm{x}) = 0$



From graph,
Since the optimal point lies on the line $g(\mathbf{x}) = 0$, it's coordinates satisfy the relation:

$$x_1 + x_2 = 9 \tag{3}$$

Using this relation in $f(\mathbf{x})$, we get

$$r^2 = (x_1 - 8)^2 + (3 - x_1)^2 \tag{4}$$

$$= 2x_1^2 - 22x_1 + 73 \tag{5}$$

$$\Rightarrow r^2 = \frac{(2x_1 - 11)^2 + 5^2}{2} \tag{6}$$

This is minimum at $x_1 = \frac{11}{2}$.
To find $x_2$, we use equation (3) $\Rightarrow x_2 = 9 - x_1 = \frac{7}{2}$.

Hence, $f(\mathbf{x})_{min} = \frac{25}{2}$ at $(\frac{11}{2}, \frac{7}{2})$ and $r_{min} = \frac{5}{\sqrt{2}}$.

# Another Graphical Approach

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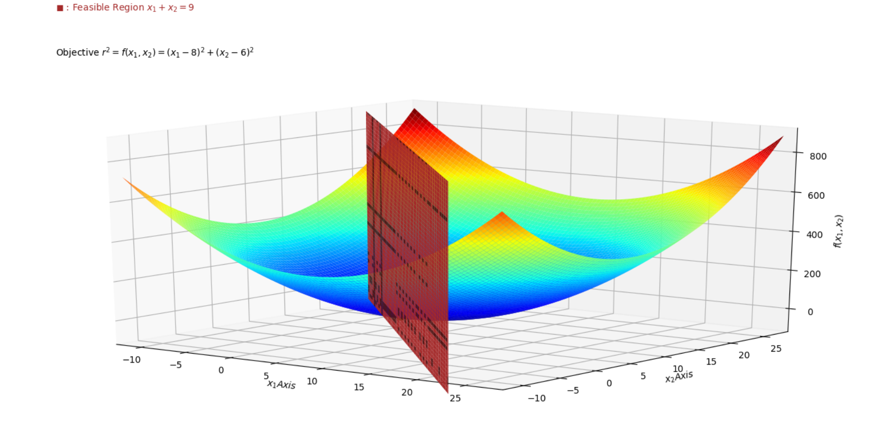
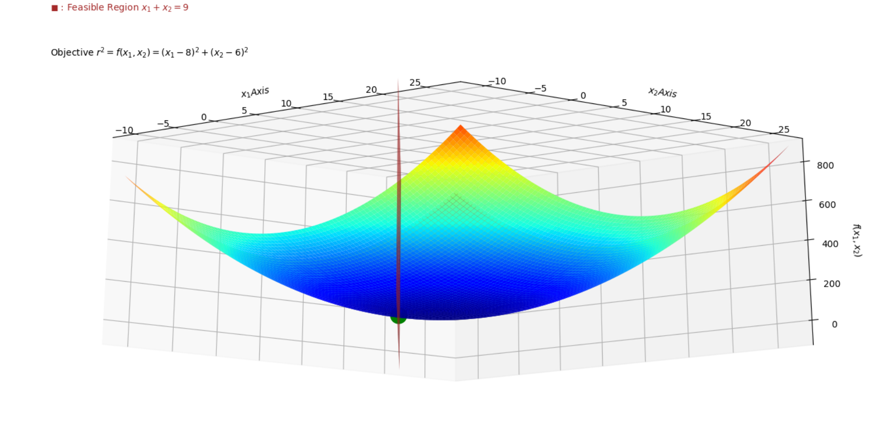
Plot Objective z=f($x$) and Feasible Region g($x$)=0.



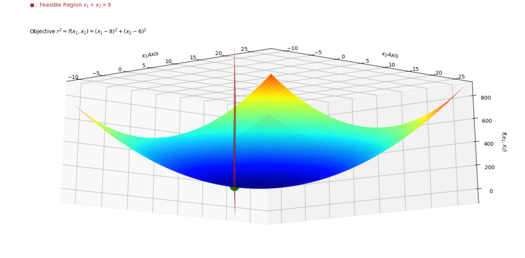
# Another Graphical Approach

From graph, find minimum z=f($\boldsymbol{x}$) such that point ($\boldsymbol{x}$,z) lies on feasible region g($\boldsymbol{x}$)=0.



# Another Graphical Approach

Finding Constraints for Solving :




As $\boldsymbol{x}$ lies on $g(\boldsymbol{x}) = 0$, $\qquad x_1 + x_2 = 9$ -(i)
As f($\boldsymbol{x}$) is decreasing along $-\nabla$ f($\boldsymbol{x}$), and f($\boldsymbol{x}$) cannot decrease in any neighbourhood of ($\boldsymbol{x}$,z) in plane, $-\nabla$ f($\boldsymbol{x}$) should be perpendicular to plane g($\boldsymbol{x}$)=0. $\qquad -\nabla f(\boldsymbol{x})^T . \begin{bmatrix} 9 \\ -9 \end{bmatrix}$=0-(ii)

Solve (i) and (ii) to get $x_1 = \frac{11}{2}, x_2 = \frac{7}{2}, f(\boldsymbol{x})_{min} = \frac{25}{2}$, $r_{min} = \frac{5}{\sqrt{2}}$