Optimization

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Problem

Problem Statement

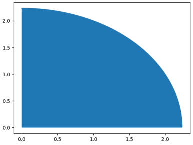
Maximize

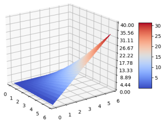
$$f(\mathbf{X}) = \sqrt{x_1 * x_2}$$

subject to the constraints

$$x_1^2 + x_2^2 \leq 5$$

$$x_1 \geq 0, x_2 \geq 0$$





Lagrange Multiplier

Consider the problem optimization problem

$$\min_{x} f(\mathbf{x}), g(\mathbf{x}) \leq 0$$

The Lagrangian is

$$\mathcal{L}(\mathbf{x}, \lambda) = f(\mathbf{x}) + \lambda g(\mathbf{x})$$

We find $\nabla \mathcal{L}(\mathbf{x}, \lambda)$ and set it to 0.
If λ obtained after solving is positive, the solution is correct.
Else set $\lambda = 0$

Solution

Considering the Lagrangian

$$\mathcal{L}(\mathbf{X}, \lambda) = -\sqrt{x_1 x_2} + \lambda(x_1^2 + x_2^2 - 5)$$

$$\nabla\mathcal{L}(\mathbf{X}, \lambda) = \begin{bmatrix} \dfrac{-\sqrt{x_2}}{2\sqrt{x_1}} + 2\lambda x_1 \\ \\ \dfrac{-\sqrt{x_1}}{2\sqrt{x_2}} + 2\lambda x_2 \\ \\ x_1^2 + x_2^2 - 5 \end{bmatrix}$$

To find optimal values of x_1 and x_2 we set

$$\nabla\mathcal{L}(\mathbf{X}, \lambda) = 0$$

$$\frac{\sqrt{x_2}}{2\sqrt{x_1}} = 2\lambda x_1 \tag{1}$$

$$\implies \lambda = \frac{\sqrt{x_2}}{4x_1^{\frac{3}{2}}}$$

$$\frac{-\sqrt{x_1}}{2\sqrt{x_2}} + 2\lambda x_2 \tag{2}$$

$$\implies \lambda = \frac{\sqrt{x_1}}{4x_2^{\frac{3}{2}}}$$

$$\frac{\sqrt{x_2}}{4x_1^{\frac{3}{2}}} = \frac{\sqrt{x_1}}{4x_2^{\frac{3}{2}}}$$

$$x_1^2 = x_2^2 \implies x_1 = x_2$$

Substituting $x_1 = x_2$ in the equation

$$x_1^2 + x_2^2 = 5 \tag{3}$$

We get

$$2x_1^2 = 5$$

$$\implies x_1 = x_2 = \sqrt{\frac{5}{2}}$$

Maximum value of f($\mathbf{X}$) is

$$f_{max}(\mathbf{X}) = \sqrt{\sqrt{\frac{5}{2}} * \sqrt{\frac{5}{2}}} = \sqrt{\frac{5}{2}} = 1.5811$$

Maximum value is 1.5811

Solution using CVXPY

```
import cvxpy as cp

x = cp.Variable(2)

constraints=[cp.norm(x,2)<=5**0.5,x>=0]
obj = cp.Maximize(cp.geo_mean(x))

prob = cp.Problem(obj, constraints)
prob.solve()
print prob.value
print x.value
```

Maximum value = 1.581138

$x_1 = x_2 = 1.581138$