Problems 3.17

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February 21, 2019

Question 3.17

$$\min_{x} \mathbf{x1} + \mathbf{x2}$$

With constraints

$$x_1^2 - x_1 + x_2^2 \leq 0$$

$$\text{where } x = \begin{pmatrix} x_1 \\ x_2 \end{pmatrix}$$

Solution

Using the method of Lagrangian multipliers,

$$\nabla f(x) + \mu g(x) = 0, \mu \geq 0$$

resulting in the equations

$$2x_1\mu - \mu + 1 = 0$$
$$2x_2\mu + 1 = 0$$
$$x_1^2 - x_1 + x_2^2 = 0$$

which can be simplified to obtain

$$\frac{1-\mu}{2\mu}^2 + \frac{1}{2\mu}^2 + \frac{1-\mu}{2\mu} = 0 \tag{1}$$

$$\Rightarrow 1 + \mu^2 - 2\mu + 1 + 2\mu 1 - \mu = 0 \tag{2}$$

$$\Rightarrow \mu^2 = 2, or \mu = \pm\sqrt{2} \tag{3}$$

$$\mu \geq 0 \Rightarrow \mu = \sqrt{2}.$$

Solution

The desired solution is

$$x = \begin{pmatrix} \frac{\sqrt{2}-1}{2\sqrt{2}} \\ -\frac{1}{2\sqrt{2}} \end{pmatrix} \tag{4}$$

Graphical solution: The constraint can be expressed as

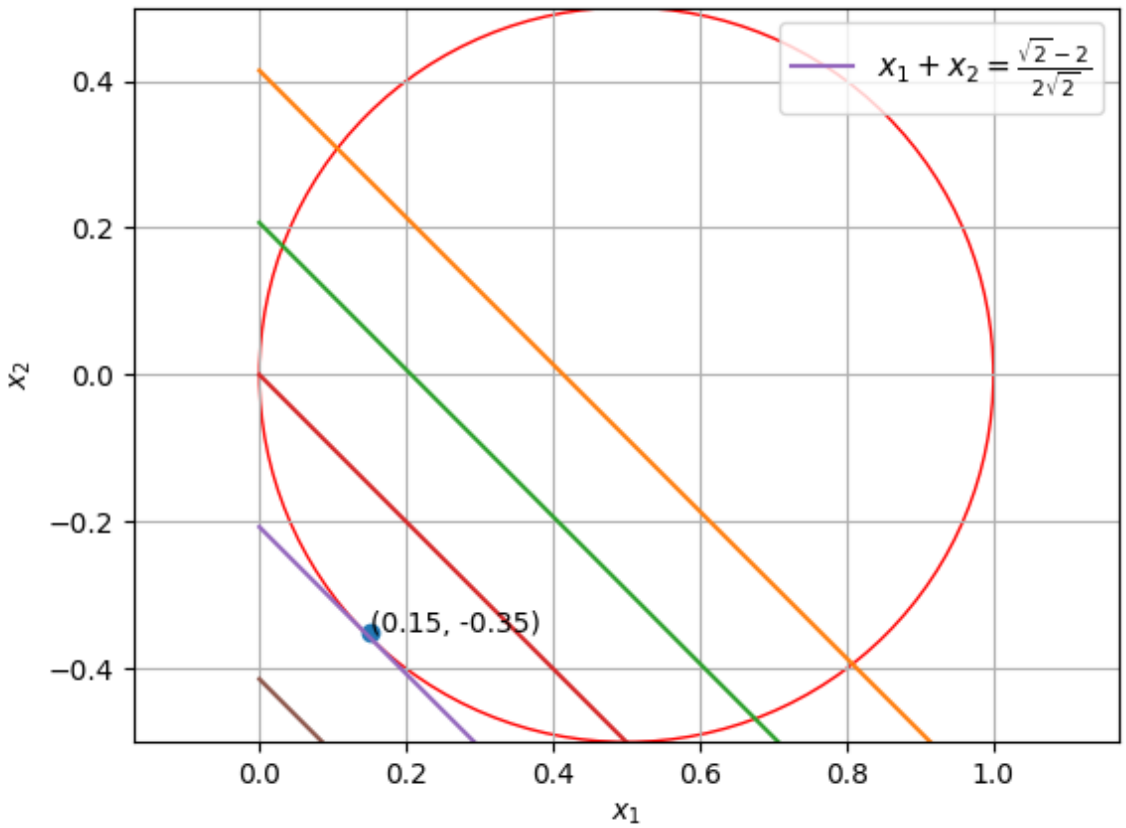
$$x_1^2 - x_1 + x_2^2 \leq 0 \tag{5}$$

$$\Rightarrow x_1 - \frac{1}{2}^2 + x_2^2 \leq \frac{1}{2}^2 \tag{6}$$

Solution

Figure: Optimal solution is the lower tangent to the circle

Solution


$x_1 + x_2 = \frac{\sqrt{2}-2}{2\sqrt{2}}$
(0.15, -0.35)
0.4
0.2
0.0
−0.2
−0.4
x_2
0.0
0.2
0.4
0.6
0.8
1.0
x_1

And last

Thank You