Optimization EE5327

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Question 5.5

Maximize 5x1 + 3x2 w.r.t the constraints
using Lagrangian multiplier
$x1 + x2 \leq 2$
$5x1 + 2x2 \leq 10$
$3x1 + 8x2 \leq 12$
where $x1, x2 \geq 0$

Solution

$$f(x) = 5x1 + 3x2$$
$$g_1(x) = x1 + x2 - 2 \leq 0$$
$$g_2(x) = 5x1 + 2x2 - 10 \leq 0$$
$$g_3(x) = 3x1 + 8x2 - 12 \leq 0$$

Solution

By Lagrange Multiplier Method
Since all constraints g(x) $\leq$ 0 So λ *is positive*

$$L(x, \lambda) = f(x) + \lambda_1 g_1(x) + \lambda_2 g_2(x) + \lambda_3 g_3(x)$$

$$L(x, \lambda) =$$
$$(5x1 + 3x2) + \lambda_1(x1 + x2 - 2) + \lambda_2(5x1 + 2x2 - 10) + \lambda 3(3x1 + 8x2 - 12)$$

$$\nabla L(x, \lambda) = 0$$

Solution

$$\nabla L(x,\lambda) = \begin{bmatrix} 5+\lambda_1+5\lambda 2+3\lambda_3 \\ 3+\lambda_1+2\lambda 2+8\lambda_3 \\ x1+x2+-2 \\ 5x1+2x2-10 \\ 3x1+8x2-12 \end{bmatrix}$$

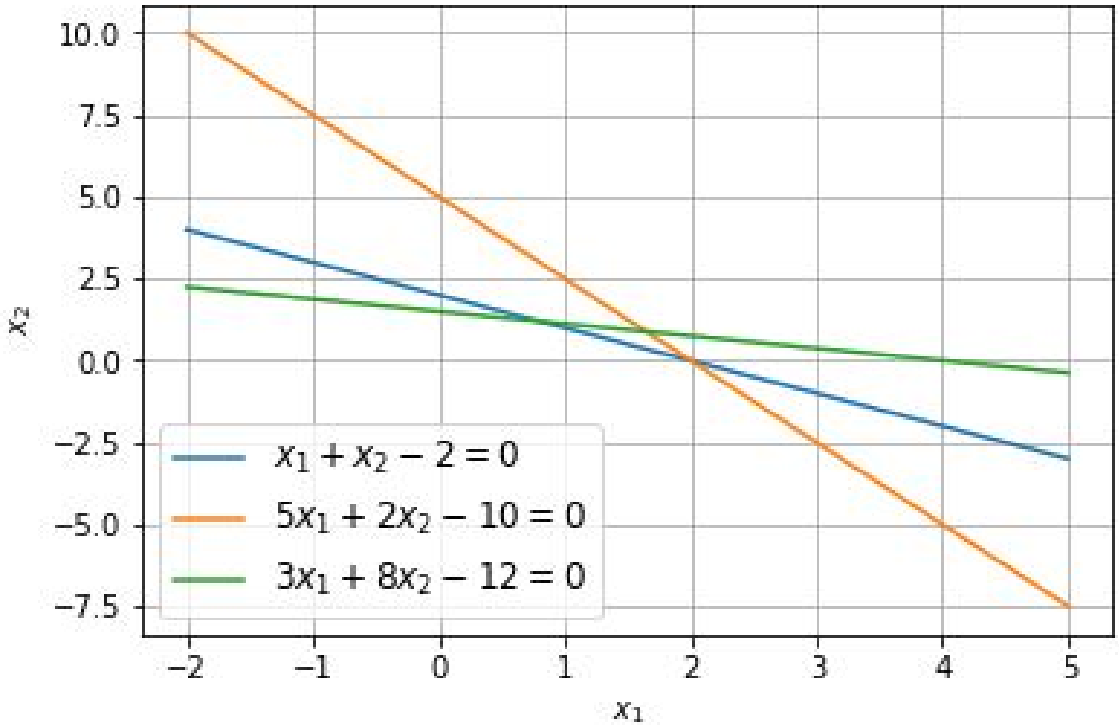
Solution

$$\begin{bmatrix} 0 & 0 & 1 & 5 & 3 \\ 0 & 0 & 1 & 2 & 8 \\ 1 & 1 & 0 & 0 & 0 \\ 5 & 2 & 0 & 0 & 0 \\ 3 & 8 & 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ \lambda_1 \\ \lambda_2 \\ \lambda_3 \end{bmatrix} = \begin{bmatrix} -5 \\ -3 \\ 2 \\ 10 \\ 12 \end{bmatrix}$$

- By observation, x1 x2 are independent of λ

Solution

- For Finding values of x1 x2, we have 3 equations and 2 unknowns
- therefore It has no common solution.
- By solving, we get 3 points.
  P1 = (0.8, 1.2), P2 = (2, 0), P3 = (1.647, 0.882)



Solution

- Substituting P1, P2, P3 in f(x) we get maximum at P2 =(2,0)
  $f(x)_{x=P2} = 10$

Solution

```
1
2 import cvxpy as cp
3 import numpy as np
4
5 x1 = cp.Variable()
6 x2 = cp.Variable()
7 constraints = [x1+x2<=2,
8                 5*x1+2*x2<=10,
9                 3*x1+8*x2<=12
10                ]
11 obj = cp.Maximize(5*x1+3*x2)
12
13 prob = cp.Problem(obj, constraints)
14 prob.solve()  # Returns the optimal value.
15 print("optimal value", prob.value,"found at",x1.value,"and",np.round(x2.value
16
17 #print("optimal var", x.value, y.value)
```

Solution

```
In [5]: runfile('/Users/ruchiakole/Downloads/Optimization/5.Scvx.py', wdir='/Users/
ruchiakole/Downloads/Optimization')
optimal value 10.000000000000002 found at 2.0 and 0.0

In [6]:
```

The End