Optimization

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Problem

Four jobs I,II,III,IV are assigned to four workers A,B,C and D. The time taken by different workers (in hours) in completing different jobs is given below:

		I	II	III	IV
	A	5	3	2	5
Workers	B	7	9	2	3
	C	4	2	3	2
	D	5	7	7	5

Find the minimum time taken by the workers to complete the job

Conversion to LPP

	I	II	III	IV	
A	X_{11}	X_{12}	X_{13}	X_{14}	1
B	X_{21}	X_{22}	X_{23}	X_{24}	1
C	X_{31}	X_{32}	X_{33}	X_{34}	1
D	X_{41}	X_{42}	X_{43}	X_{44}	1
	1	1	1	1	

The variables either take the value 0 or 1.

The sum of each element row-wise is 1.

$$\sum_{j=1}^{4} X_{ij} = 1$$

The sum of each element column-wise is 1.

$$\sum_{i=1}^{4} X_{ij} = 1$$

The constraints are

$$X_{11} + X_{12} + X_{13} + X_{14} = 1$$

$$X_{21} + X_{22} + X_{23} + X_{24} = 1$$

$$X_{31} + X_{32} + X_{33} + X_{34} = 1$$

$$X_{41} + X_{42} + X_{43} + X_{44} = 1$$

$$X_{11} + X_{21} + X_{31} + X_{41} = 1$$

$$X_{12} + X_{22} + X_{32} + X_{42} = 1$$

$$X_{13} + X_{23} + X_{33} + X_{43} = 1$$

$$X_{14} + X_{24} + X_{34} + X_{44} = 1$$

$$X_{ij} \geq 0 \ \ \forall i, j \in \{1, 2, 3, 4\}$$

Code using cvxpy

```
import cvxpy as cp
import numpy as np

x = cp.Variable((4,4),nonneg=True)

constraints = [x[0,0]+x[0,1]+x[0,2]+x[0,3]==1,
               x[1,0]+x[1,1]+x[1,2]+x[1,3]==1,
               x[2,0]+x[2,1]+x[2,2]+x[2,3]==1,
               x[3,0]+x[3,1]+x[3,2]+x[3,3]==1,
               x[0,0]+x[1,0]+x[2,0]+x[3,0]==1,
               x[0,1]+x[1,1]+x[2,1]+x[3,1]==1,
               x[0,2]+x[1,2]+x[2,2]+x[3,2]==1,
               x[0,3]+x[1,3]+x[2,3]+x[3,3]==1]
```

```
obj = cp.Minimize(
    5*x[0,0]+3*x[0,1]+2*x[0,2]+5*x[0,3]+
    7*x[1,0]+9*x[1,1]+2*x[1,2]+3*x[1,3]+
    4*x[2,0]+2*x[2,1]+3*x[2,2]+2*x[2,3]+
    5*x[3,0]+7*x[3,1]+7*x[3,2]+5*x[3,3])

prob = cp.Problem(obj,constraints)
prob.solve()

print np.round(prob.value)

for i in x:
    for j in i:
        print np.round(j.value),
    print
```

Result

The minimum number of hours required is **12 hours**.

$$\mathbf{X}^* = \begin{bmatrix} 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 1 & 0 & 0 \\ 1 & 0 & 0 & 0 \end{bmatrix}$$

A does task III
B does task IV
C does task II
D does task I