A Lagrange Multiplier-based Regularization Algorithm for Image Super-resolution

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Single-image Super-resolution (SR) Problem

- To restore a high-resolution (HR) image from a single low resolution (LR) input image
- The image super-resolution is an ill-posed inverse problem
- Y = HX

  X = HR image
  Y = LR image
  H = down-sampling matrix

Regularization Model

$$\min(X)\ \|Y - HX\|^2 + \lambda\|CX\|^2$$

$\lambda=$ Regularization parameter
C = High Pass Filtering Matrix

The Lagrange multiplier-based model

The constrained optimization model can be described as
min(X) $\|Y - HX\|^2$
$s.t. \|CX\|^2 = \delta$

δ = Constrained Parameter
By means of Lagrange multiplier method
X* = argmin(X) $\|Y - HX\|^2 + \mu\|CX\|^2 - \mu\delta$

The Lagrange multiplier-based model

$$\mathrm{L}(\mathrm{X},\mu) = \|Y - HX\|^2 + \mu\|CX\|^2 - \mu\delta$$

$$\nabla L(x, \mu) = 0$$

$$\nabla\|Y - HX\|^2 + \mu\nabla\|CX\|^2 = 0$$
$$\text{s.t.}\|CX\|^2 = \delta$$

The two-phase iterative method

Problem

How to determine the value of μ according to the value of δ needs to be solved.

The first phase:

Step 1

Step 1.1: If $\|CX\|^2 - \delta = 0$, then set indicator(k) $=$ 0

Step 1.2: If $\|CX\|^2 - \delta < 0$, then set indicator(k) $=$ -1, $\alpha = 0.1$ and go to step 2

Step 1.3: If $\|CX\|^2 - \delta > 0$, then set indicator(k) $= +1$, $\alpha = 10$ and go to step 2

Step 2 $=$ Set $\mu = \alpha\mu$ compute X by the conjugate gradient method based on the value of μ

Step 3 $=$ Repeat the step 1 and 2

Step 4 $=$ Calculate the product of indicator functions from step 3 step 1

The two-phase iterative method

The Second Phase:

Step 1: Define the variable ϵ, μ_{+-} and $\text{X}_{+-} andthensetthevalue \epsilon$
Step 2: Compare the values of μ_- - μ_+ and ϵ
Step 2.1: If μ_- - $\mu_+ > \epsilon$ set $\mu_{+-} = 0.5\mu_+ + 0.5\mu_-$ and compute X_{+-} by the conjugate gradient method based on the value of μ_{+-}
Step 2.2: If μ_- - $\mu_+ < \epsilon$ set $\mu_{+-} = 0.5\mu_+ + 0.5\mu_-$ and go to step 4
Step 3.1: Compare the values of $\|CX\|^2$ and δ
Step 3.1: If $\|CX\|^2 - \delta = 0$, then set $\mu_{+-} = \mu$ and go to step 4
Step 3.2: If $\|CX\|^2 - \delta < 0$, then set $\mu_- = \mu_{+-}$ and go to step 2
Step 3.3: If $\|CX\|^2 - \delta > 0$, then set $\mu_+ = \mu_{+-}$ and go to step 2
Step 4: return μ

Conclusion

- This algorithm can be applied to LR image so that it can be identified by recognition equipment
- We know LR image(Y), DownSampling Filter (H) and constraint on squared norm of HR image
- The unknowns are μ and X
- We can find μ from two-phase iterative method
- By using lagrange multiplier method we can find X