Optimization - EE5327

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Question 3.3

- Define L $(\boldsymbol{x}, \lambda) = \mathrm{f}(\boldsymbol{x}) - \lambda \mathrm{g}(\boldsymbol{x})$ and $\nabla = \begin{bmatrix} \frac{\partial}{\partial x_1} \\ \frac{\partial}{\partial x_2} \\ \frac{\partial}{\partial \lambda} \end{bmatrix}$.

  Solve the equations $\nabla \mathrm{L}(\boldsymbol{x}, \lambda) = 0$.

  where $f(\boldsymbol{x}) = (x_1 - 8)^2 + (x_2 - 6)^2$ and
  $g(\boldsymbol{x}) = x_1 + x_2 - 9 = 0$, $\boldsymbol{x} = (x_1\ x_2)^T$.

  How is this related to problem 3.1? What is the sign of λ?

  L is known as the Lagrangian and the above technique is known as the Method of Lagrange Multipliers,

Solution

$$L\ (\boldsymbol{x},\ \lambda) = (x_1 - 8)^2 + (x_2 - 6)^2 - \lambda(x_1 + x_2 - 9)$$

$$\nabla L(\boldsymbol{x},\ \lambda) = \begin{bmatrix} 2x_1 - 16 - \lambda \\ 2x_2 - 12 - \lambda \\ x_1 + x_2 - 9 \end{bmatrix}$$

Equating $\nabla L\ (\boldsymbol{x},\ \lambda)$ *to* 0

Solution

$$\begin{bmatrix} 2 & 0 & -1 \\ 0 & 2 & -1 \\ 1 & 1 & 0 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ \lambda \end{bmatrix} = \begin{bmatrix} 16 \\ 12 \\ 9 \end{bmatrix}$$

This is of the form $\mathrm{A}\boldsymbol{x} = \mathrm{b}$

Solving the above equations, we get $\boldsymbol{x} = \begin{bmatrix} x_1 \\ x_2 \\ \lambda \end{bmatrix} = \begin{bmatrix} \frac{11}{2} \\ \frac{7}{2} \\ -5 \end{bmatrix}$

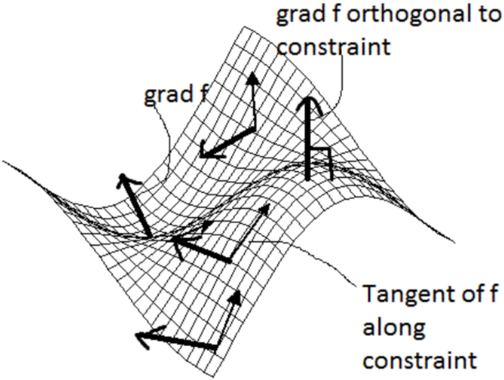
$\lambda = -5 i.e.\ it\ is\ negative$

Solution

- Let, maximize or minimize a function f(x,y) under a constraint path g(x,y)=c.
- Now,
  Gradient is the multidimensional rate of change of given function.

Figure

(27).png



Solution

- So, ∇f(x,y) is a vector that indicates direction to maximize f(x,y) . But due to constraint g(x,y)=c allowed direction is to move along the tangents to constraint g(x,y)=c which is orthogonal to ∇g(x,y) .
- Therefore, to maximize f(x,y) , directional derivative along the path df must be zero, which means ∇f(x,y,z) must be orthogonal to g(x,y,z)=c i.e. parallel to ∇g(x,y,z).

Solution

- Now, if two vectors are parallel they can be defined linearly(scalar multiply). Thus, $\nabla f = \lambda \nabla g$ where λ is the Lagrange multiplier Lagrange function is defined by L(x,y,) = f(x,y) - λg(x,y)
- So, $\nabla L = 0$