

# Implementation of Low Density Parity Check (LDPC) codes

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## CONTENTS

<b>1</b>	<b>Introduction</b>	<b>1</b>
<b>2</b>	<b>Encoding</b>	<b>1</b>
<b>3</b>	<b>Decoding</b>	<b>2</b>
3.1	Useful Calculations for proceeding LDPC Decoding . .	2
3.2	Message Passing Algorithm using min-sum Approximation	2
<b>4</b>	<b>Results</b>	<b>3</b>
	<b>References</b>	<b>3</b>
	<i>Abstract</i> —A brief description about the design and implementation of LDPC codes using DVB-S2 [1] standard.	

## 1. INTRODUCTION

Let the Channel model be,

$$Y_k = X_k + V_k, \quad k = 0, \dots, 6 \quad (1.1)$$

where  $X_k$  is the transmitted symbol in the  $k$ th time slot using the BPSK modulation and  $V_k(m) \sim \mathcal{N}(0, \sigma^2)$ .

## 2. ENCODING

LDPC codes are popular linear block codes with closest shannon limit channel capacity [2]. As an example Lets take (7,4) Hamming parity check matrix.

$$H = \begin{bmatrix} 1 & 1 & 1 & 0 & 1 & 0 & 0 \\ 1 & 0 & 1 & 1 & 0 & 1 & 0 \\ 1 & 1 & 0 & 1 & 0 & 0 & 1 \end{bmatrix} \quad (2.1)$$

Above  $H$  matrix having the parameters, information bits  $k = 4$  i.e  $m = m_0, \dots, m_3$ , parity bits are  $m = n - k = 3$  i.e  $p = [p_0, p_1, p_2]$  and code word length  $n = 7$  i.e  $c = [m \ p]$ . Encoding can be carried

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variable nodes                      check nodes

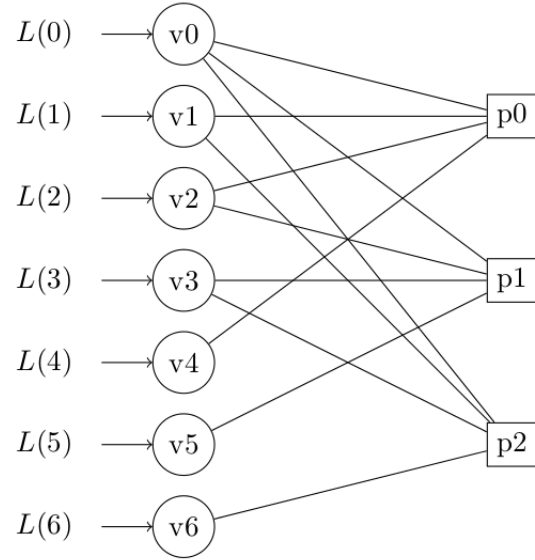


Fig. 1: Tanner Graph Representation for (7,4) Hamming parity check matrix

out by using

$$H \times c^T = 0 \quad (2.2)$$

$$\begin{bmatrix} 1 & 1 & 1 & 0 & 1 & 0 & 0 \\ 1 & 0 & 1 & 1 & 0 & 1 & 0 \\ 1 & 1 & 0 & 1 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} m_0 \\ m_1 \\ m_2 \\ m_3 \\ p_0 \\ p_1 \\ p_2 \end{bmatrix} = 0 \quad (2.3)$$

solving we get

$$p_0 = m_0 \oplus m_1 \oplus m_2 \quad (2.4)$$

$$p_1 = m_0 \oplus m_2 \oplus m_3 \quad (2.5)$$

$$p_2 = m_0 \oplus m_1 \oplus m_3 \quad (2.6)$$

This is called Systematic Encoding.i.e Encoder will

ensures information bits followed by parity bits.

### 3. DECODING

#### A. Useful Calculations for proceeding LDPC Decoding

- 1) Calculation of Input Channel Log Likelihood Ratio LLR

$$L(x_j) = \log \left( \frac{Pr(x_j = 1|y)}{Pr(x_j = -1|y)} \right) \quad X = 1 - 2c \quad (3.1)$$

$$= \log \left( \frac{f(y|x_j = 1)Pr(x_j = 1)}{f(y|x_j = -1)Pr(x_j = -1)} \right) \quad (3.2)$$

$$= \log \left( \frac{\frac{1}{\sqrt{2\pi\sigma^2}} e^{-\frac{(y_j-1)^2}{2\sigma^2}}}{\frac{1}{\sqrt{2\pi\sigma^2}} e^{-\frac{(y_j+1)^2}{2\sigma^2}}} \right) \quad (3.3)$$

$$= \log \left( e^{\frac{2y_j}{\sigma^2}} \right) \quad (3.4)$$

$$L(x_j) = \frac{2y_j}{\sigma^2} \quad (3.5)$$

- 2) Check Node Operation :

Lets assume that we have initilized all LLR values to variable nodes and we sent to check nodes.

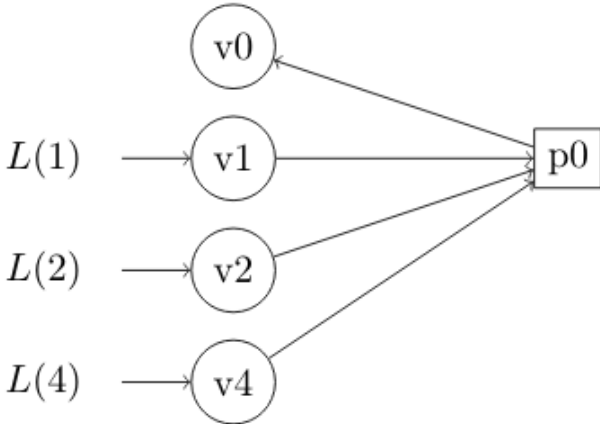


Fig. 2: Check node operation

- 3) Variable Node Operation :

Lets take the first variable node  $v_0$  which takes input LLR, and check to variable message node messages  $Q_{vc}$  . i.e  $Q_{vc}(1,0), Q_{vc}(2,0)$ . The variable node 0 to check node 0 i.e  $R(0,0)$  can be defined as ,

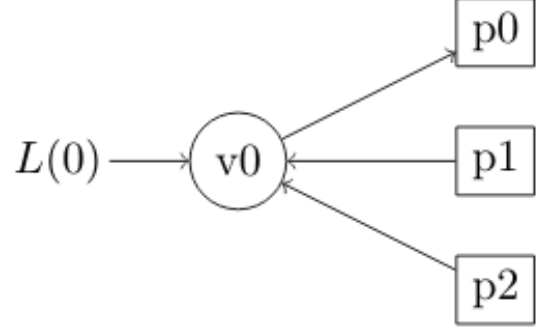


Fig. 3: Variable node operation

$$L(q_{i=0,j=0}) = \log \left( \frac{Pr(x_j = 1|y_0, y_1, y_2)}{Pr(x_j = -1|y_0, y_1, y_2)} \right) \quad X = 1 - 2c \quad (3.6)$$

$$= \log \left( \frac{f(y_0, y_1, y_2|x_j = 1)Pr(x_j = 1)}{f(y_0, y_1, y_2|x_j = -1)Pr(x_j = -1)} \right) \quad (3.7)$$

$$= \log \left( \frac{\frac{1}{\sqrt{2\pi\sigma^2}} e^{-\frac{(y_0-1)^2}{2\sigma^2}} \frac{1}{\sqrt{2\pi\sigma^2}} e^{-\frac{(y_1-1)^2}{2\sigma^2}} \frac{1}{\sqrt{2\pi\sigma^2}} e^{-\frac{(y_2-1)^2}{2\sigma^2}}}{\frac{1}{\sqrt{2\pi\sigma^2}} e^{-\frac{(y_0+1)^2}{2\sigma^2}} \frac{1}{\sqrt{2\pi\sigma^2}} e^{-\frac{(y_1+1)^2}{2\sigma^2}} \frac{1}{\sqrt{2\pi\sigma^2}} e^{-\frac{(y_2+1)^2}{2\sigma^2}}} \right) \quad (3.8)$$

$$= \log \left( e^{\frac{2(y_0+y_1+y_2)}{\sigma^2}} \right) \quad (3.9)$$

$$L(q_{i=0,j=0}) = \frac{2(y_0 + y_1 + y_2)}{\sigma^2} = L(x_i) + \sum_{k \in C_i \setminus j} L(r_{ki}) \quad (3.10)$$

#### B. Message Passing Algorithm using min-sum Approximation

Transmitted frames = N, Total number of bits =  $N \times 7$  and Total number of information bits =  $N \times 4$ . For Each Frame,

- 1) Initialize  $L(q_{ij})$  using (3.5) for all  $i, j$  for which  $h_{ij} = 1$  with channel LLR's.
- 2) Update  $\{L(r_{ji})\}$  using .
- 3) Update  $\{L(q_{ji})\}$  using (3.10).
- 4) Update  $\{L(V_i)\}$  using,

$$L(V_i) = L(x_i) + \sum_{k \in C_i} L(r_{ki}) \quad i = 0, \dots, 6. \quad (3.11)$$

5) Proceed to step 2.

After maximum specified iterations,

Decoding can be done using,

$$\hat{c}_i = \begin{cases} 1 & L(V_i) < 0 \\ 0 & \text{else} \end{cases} \quad (3.12)$$

#### 4. RESULTS

For frames N=10000. Fig .4 Shows the Com-

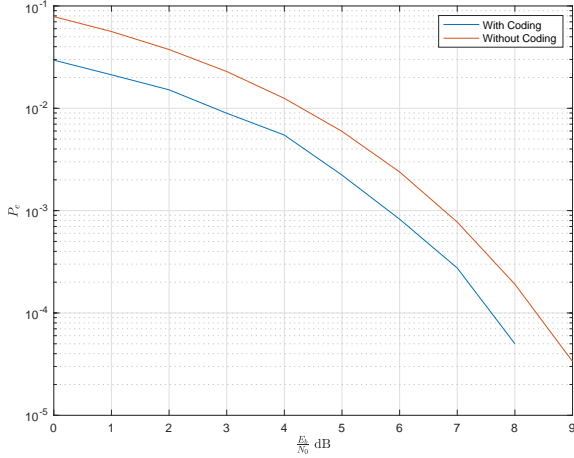


Fig. 4: SNR vs BER curves using LDPC channel coding and no channel coding

parison of Probability error with channel coding and without channel coding. Since the parity check matrix taken was not much sparse, we are not getting near shannon limit performance. For Good sparse matrix i.e number of entries in  $H \ll m \times n$

#### REFERENCES

- [1] A. Morello and V. Mignone, "DVB-S2X: The New Extensions to the Second Generation DVB Satellite Standard DVB-S2," *Int. J. Satell. Commun. Netw.*, vol. 34, no. 3, pp. 323–325, May 2016. [Online]. Available: <https://doi.org/10.1002/sat.1167>
- [2] Y. Zhang and W. E. Ryan, "Toward low LDPC-code floors: a case study," *IEEE Transactions on Communications*, vol. 57, no. 6, pp. 1566–1573, June 2009. [Online]. Available: <https://doi.org/10.1109/TCOMM.2009.06.0700712>