Implementation of Low Density Parity Check (LDPC) codes

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***Abstract*—A brief description about the design and implementataion of LDPC codes using (7,4) Hamming parity check matrix.**

1. Introduction

Let the Channel model be,

$$Y_k = X_k + V_k, \quad k = 0, \dots, 6 \tag{1.1}$$

where X_k is the transmitted symbol in the kth time slot using the BPSK modulation and $V_k(m) \sim \mathcal{N}\left(0, \sigma^2\right)$.

2. Encoding

LDPC codes are popular linear block codes with closest shannon limt channel capacity [1]. As an example Lets take (7,4) Hamming parity check matrix.

$$H = \begin{bmatrix} 1 & 1 & 1 & 0 & 1 & 0 & 0 \\ 1 & 0 & 1 & 1 & 0 & 1 & 0 \\ 1 & 1 & 0 & 1 & 0 & 0 & 1 \end{bmatrix} \tag{2.1}$$

Above H matrix having the parameters, information bits $k = 4$ i.e $m = m_0, \dots, m_3$, parity bits are $m = n - k = 3$ i.e $p = [p_0, p_1, p_2]$ and code word length $n = 7$ i.e $c = [m\ p]$. Encoding can be carried

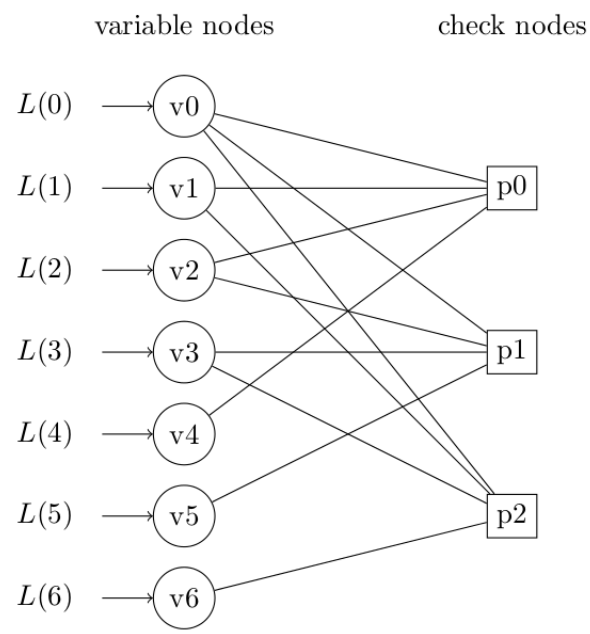

Fig. 1: Tanner Graph Representation for (7,4) Hamming parity check matrix

out by using

$$H \times c^T = 0 \tag{2.2}$$

$$\begin{bmatrix} 1 & 1 & 1 & 0 & 1 & 0 & 0 \\ 1 & 0 & 1 & 1 & 0 & 1 & 0 \\ 1 & 1 & 0 & 1 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} m_0 \\ m_1 \\ m_2 \\ m_3 \\ p_0 \\ p_1 \\ p_2 \end{bmatrix} = 0 \tag{2.3}$$

solving we get

$$p_0 = m_0 \oplus m_1 \oplus m_2 \tag{2.4}$$

$$p_1 = m_0 \oplus m_2 \oplus m_3 \tag{2.5}$$

$$p_2 = m_0 \oplus m_1 \oplus m_3 \tag{2.6}$$

This is called Systematic Encoding.i.e Encoder will

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ensures information bits followed by parity bits.

3. Decoding

A. Useful Calculations for proceeding LDPC Decoding

1) Calculation of Input Channel Log Likelihood Ratio LLR

$$L(x_j) = \log\left(\frac{Pr(x_j = 1|y)}{Pr(x_j = -1|y)}\right) \quad X = 1 - 2c \tag{3.1}$$

$$= \log\left(\frac{f(y|x_j = 1)Pr(x_j = 1)}{f(y|x_j = -1)Pr(x_j = -1)}\right) \tag{3.2}$$

$$= \log\left(\frac{\frac{1}{\sqrt{2\pi\sigma^2}}e^{\frac{-(y_j-1)^2}{2\sigma^2}}}{\frac{1}{\sqrt{2\pi\sigma^2}}e^{\frac{-(y_j+1)^2}{2\sigma^2}}}\right) \tag{3.3}$$

$$= \log\left(e^{\frac{2y_j}{\sigma^2}}\right) \tag{3.4}$$

$$L(x_j) = \frac{2y_j}{\sigma^2} \tag{3.5}$$

2) Check Node Operation :
Lets assume that we have initilized all LLR values to variable nodes and we sent to check nodes. V_j represents all the variable nodes which are connected to j^{th} check node. Using the min-sum approximation [2], the message from j^{th} check node to i^{th} variable node given by,

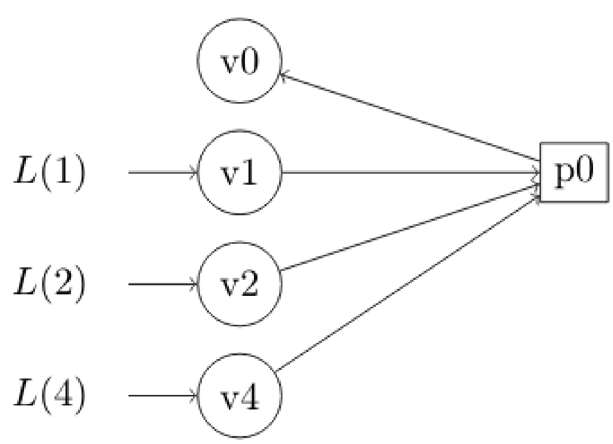

Fig. 2: Check node operation

$$L(r_{j=0,i=0}) = sign(L(1)) \times sign(L(2)) \times sign(L(4)) \tag{3.6}$$

$$\times \min(|L(1)|, |L(2)|, |L(4)|)$$
$$= \left[\prod_{k\in V_j\backslash i} sign(L(q_{kj}))\right] \min_{k\in V_j\backslash i} |L(q_{kj})| \tag{3.7}$$

3) Variable Node Operation :
Let C_i denotes all the check nodes connected to i^{th} variable node. The message from i^{th} variable node to j^{th} check node given by,

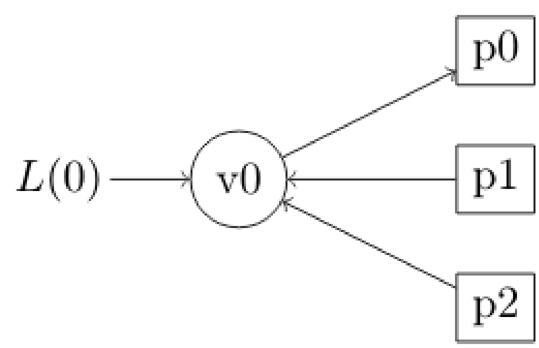

Fig. 3: Variable node operation

$$L(q_{i=0,j=0}) = \log\left(\frac{Pr(x_j = 1|y_0, y_1, y_2)}{Pr(x_j = -1|y_0, y_1, y_2)}\right) \quad X = 1 - 2c \tag{3.8}$$

$$= \log\left(\frac{f(y_0, y1, y_2|x_j = 1)Pr(x_j = 1)}{f(y_0, y_1, y_2|x_j = -1)Pr(x_j = -1)}\right) \tag{3.9}$$

$$= \log\left(\frac{\frac{1}{\sqrt{2\pi\sigma^2}}e^{\frac{-(y_0-1)^2}{2\sigma^2}}\frac{1}{\sqrt{2\pi\sigma^2}}e^{\frac{-(y_1-1)^2}{2\sigma^2}}\frac{1}{\sqrt{2\pi\sigma^2}}e^{\frac{-(y_2-1)^2}{2\sigma^2}}}{\frac{1}{\sqrt{2\pi\sigma^2}}e^{\frac{-(y_0+1)^2}{2\sigma^2}}\frac{1}{\sqrt{2\pi\sigma^2}}e^{\frac{-(y_1+1)^2}{2\sigma^2}}\frac{1}{\sqrt{2\pi\sigma^2}}e^{\frac{-(y_2+1)^2}{2\sigma^2}}}\right) \tag{3.10}$$

$$= \log\left(e^{\frac{2(y_0+y_1+y_2)}{\sigma^2}}\right) \tag{3.11}$$

$$L(q_{i=0,j=0}) = \frac{2(y_0 + y_1 + y_2)}{\sigma^2} = L(x_i) + \sum_{k\in C_i\backslash j} L(r_{ki}) \tag{3.12}$$

B. Message Passing Algorithm using min-sum Approximation

Transmitted frames = N, Total number of bits = N $\times$ 7 and Total number of information bits = N $\times$ 4. For Each Frame,

1) Initialize $L(q_{ij})$ using (3.5) for all i, j for which $h_{ij} = 1$ with channel LLR's.
2) Update $\left\{L(r_{ji})\right\}$ using (3.7).
3) Update $\left\{L(q_{ji})\right\}$ using (3.12).
4) Update $\{L(V_i)\}$ using,

$$L(V_i) = L(x_i) + \sum_{k \in C_i} L(r_{ki}) \quad i = 0, \ldots, 6. \tag{3.13}$$

5) Proceed to step 2.
   After maximum specified iterations,

Decoding can be done using,

$$\hat{c}_i = \begin{cases} 1 & L(V_i) < 0 \\ 0 & else \end{cases} \tag{3.14}$$

4. Results

For frames N=10000. Fig 4 Shows the Comparison of Probability error with channel coding and without channel coding. Since the parity check matrix taken was not much sparse, we are not getting near shannon limit performance. For Good sparse matrix i.e number of entries in H << $m \times n$

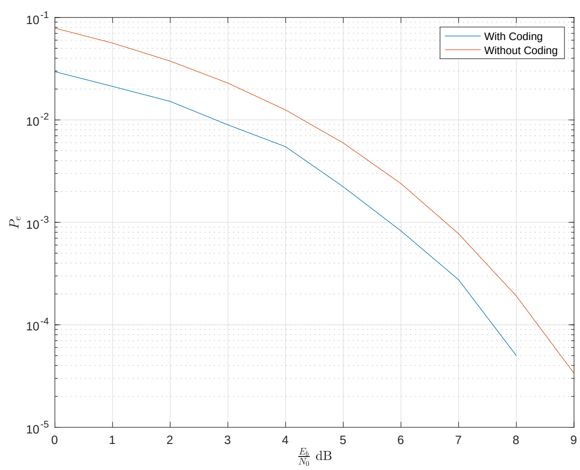

Fig. 4: SNR vs BER curves using LDPC channel coding and no channel coding

References

[1] R.Gallager, "Low-density parity check codes," *IRE Trans.Information Theory*, pp. 21–28, Jan. 1962.

[2] N. Wiberg, "Codes and Decoding on General Graphs," *Phd Dissertation,Linkoping University,Sweden*, 1996.