

# Modern Synchronization Techniques for Reliable Communication

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## References

### 1. TIME OFFSET: GARDNER TED

Let the  $m$ th sample in the  $r$ th received time slot be

$$Y_k(m) = X_k + V_k(m), \quad k = 1, \dots, N, m = 1,$$

where  $X_k$  is the transmitted symbol in the  $k$ th time slot and  $V_k(m) \sim \mathcal{N}(0, \sigma^2)$ . The decision variable for the  $k$ th symbol is

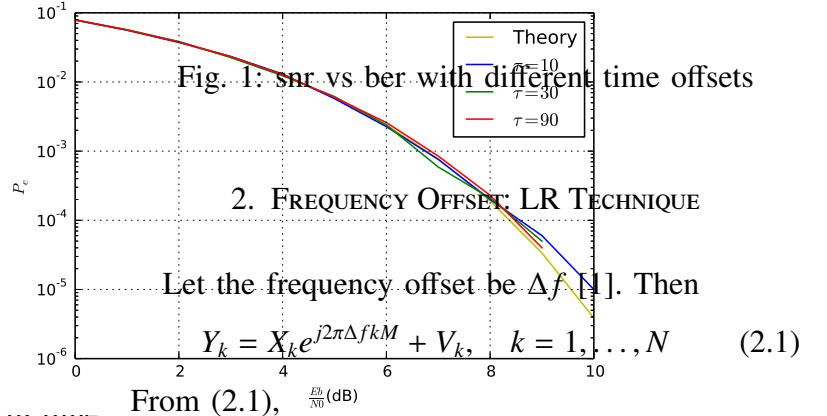
$$U_k = Y_{k-1} \left( \frac{M}{2} \right) [Y_k(M) - Y_{k-1}(M)] \quad (1.2) \quad \text{where}$$

#### A. Plots

The codes for generating the plots are available at

Fig. 1 shows the variation of the bit error rate respect to the snr with different timing offsets.  $\Delta f$  when the SNR = 10 dB. Similarly Fig. 1 shows the variation of the error with respect to the SNR .

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From (2.1),  $\frac{E_b}{N_0}$  (dB)

$$Y_k X_k^* = |X_k|^2 e^{j2\pi\Delta f k M} + X_k^* V_k \quad (2.2)$$

$$\Rightarrow r_k = e^{j2\pi\Delta f k M} + \bar{V}_k \quad (2.3)$$

$$r_k = Y_k X_k^*, \bar{V}_k = X_k^* V_k, |X_k|^2 = 1 \quad (2.4)$$

The autocorrelation can be calculated as

$$R(k) \triangleq \frac{1}{N-k} \sum_{i=k+1}^N r_i r_{i-k}^*, \quad 1 \leq k \leq N-1 \quad (2.5)$$

Where  $N$  is the length of the received signal. For large centre frequency, the following yields a good approximation for frequency offset upto 40 MHz.

$$\Delta \hat{f} \approx \frac{1}{2\pi M} \frac{\sum_{k=1}^P \text{Im}(R(k))}{\sum_{k=1}^P k \text{Re}(R(k))}, \quad P \Delta f M \ll 1 \quad (2.6)$$

where  $P$  is the number of pilot symbols.

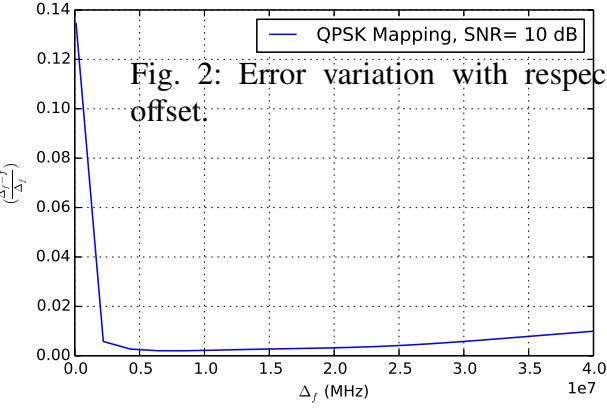


Fig. 2: Error variation with respect to frequency offset.

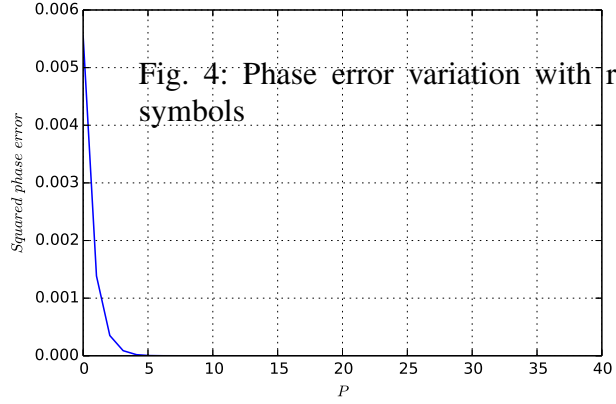


Fig. 4: Phase error variation with respect to pilot symbols

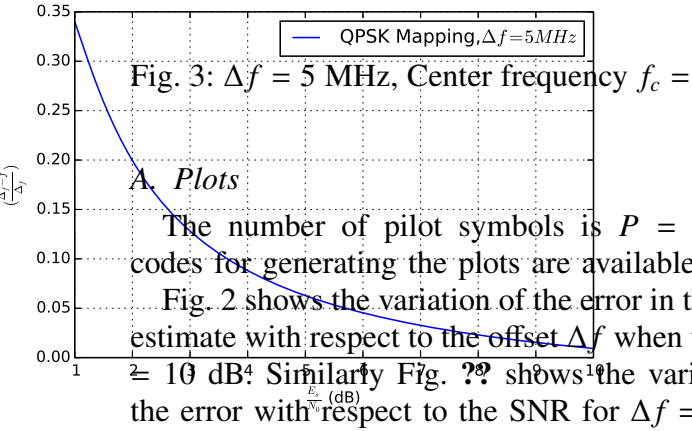


Fig. 3:  $\Delta f = 5$  MHz, Center frequency  $f_c = 2.4$  GHz

#### A. Plots

The number of pilot symbols is  $P = 18$ . The codes for generating the plots are available at [1]. Fig. 2 shows the variation of the error in the estimate with respect to the offset  $\Delta f$  when the SNR is 10 dB. Similarly Fig. 3 shows the variation of the error with respect to the SNR for  $\Delta f = 5$  MHz.

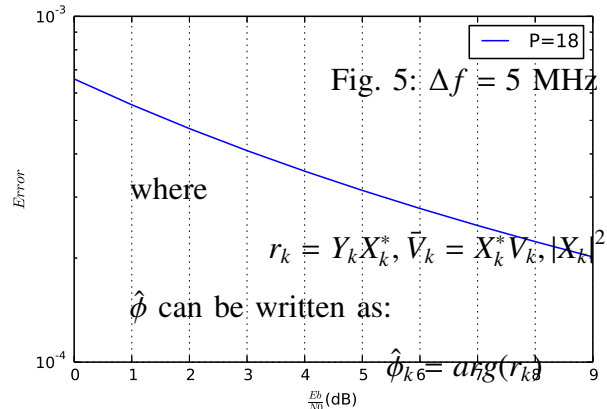


Fig. 5:  $\Delta f = 5$  MHz

where

$$r_k = Y_k X_k^*, \bar{V}_k = X_k^* V_k, |X_k|^2 = 1 \quad (3.4)$$

$\hat{\phi}$  can be written as:

$$\hat{\phi}_k = \arg(r_k) \quad (3.5)$$

This equation gives the final estimation of phase

$$\hat{\theta}_f^{(p)}(l) = \hat{\theta}_f^{(p)}(l-1) + \alpha \text{SAW}[\hat{\theta}_f^{(p)}(l) - \hat{\theta}_f^{(p)}(l-1)] \quad (3.6)$$

Where SAW is a saw tooth non-linearity and  $\alpha \leq 1$

#### A. Plots

The number of pilot symbols is  $P = 18$ . The codes for generating the plots are available at [1].

Fig. 4 shows the variation of the error in the offset estimate with respect to the offset  $\Delta f$  when the SNR is 10 dB.

### 3. PHASE OFFSET: FEED FORWARD MAXIMUM LIKELIHOOD (FFML) TECHNIQUE

Let the phase offset be  $\Delta\phi$  [1]. Then

$$Y_k = X_k e^{j\phi} + V_k, \quad k = 1, \dots, N \quad (3.1)$$

From (3.1),

$$Y_k X_k^* = |X_k|^2 e^{j\phi} + X_k^* V_k \quad (3.2)$$

$$\Rightarrow r_k = e^{j\phi} + \bar{V}_k \quad (3.3)$$

= 10 dB. Similarly Fig. 5 shows the variation of the error with respect to the SNR for  $\Delta f = 5\text{MHz}$ .

#### REFERENCES

- [1] M. Luise and R. Reggiannini: 'Carrier frequency recovery in all-digital modems for burst mode transmissions,' IEEE Trans. Commun., vol. 43, no. 2/3/4, pp. 1169-1178, Feb/Mar/Apr 1995.
- [2] U. Mengali and A. N. D'Andrea: 'synchronization Techniques for Digital Receivers,' New York: Plenum, 1997.