

# Modern Digital Synchronization Techniques for Reliable Communication

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**Abstract**—A brief description about the de implementation of Modern Digital Synchronizat niques for reliable communication.

### 1. TIME OFFSET: GARDNER TED

Let the  $m$ th sample in the  $r$ th received time slot be

$$Y_k(m) = X_k + V_k(m), \quad k = 1, \dots, N, m = 1,$$

where  $X_k$  is the transmitted symbol in the  $k$ th time slot and  $V_k(m) \sim \mathcal{N}(0, \sigma^2)$ . The decision variable for the  $k$ th symbol is [2]

$$U_k = \frac{1}{N} \sum_{i=1}^N Y_{k-i} \left( \frac{M}{2} \right) (Y_{k-i+1}(M) - Y_{k-i}(M)) \quad (1.2) \quad \text{where}$$

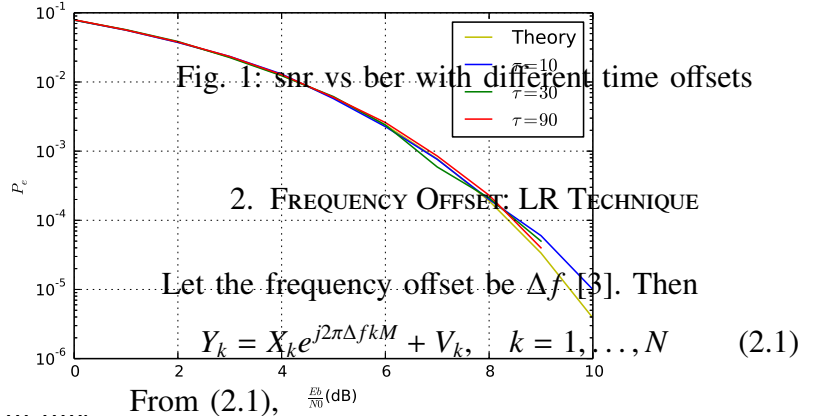
Where  $N$  is the previous symbol knowledge for averaging.

#### A. Plots

The codes for generating the plots are available at

Fig. 1 shows the variation of the bit error rate respect to the snr with different timing offsets.  $\Delta f$  when the SNR = 10 dB.

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From (2.1),  $\frac{E_b}{N_0}$  (dB)

$$Y_k X_k^* = |X_k|^2 e^{j2\pi\Delta f k M} + X_k^* V_k \quad (2.2)$$

$$\Rightarrow r_k = e^{j2\pi\Delta f k M} + \bar{V}_k \quad (2.3)$$

$$r_k = Y_k X_k^*, \bar{V}_k = X_k^* V_k, |X_k|^2 = 1 \quad (2.4)$$

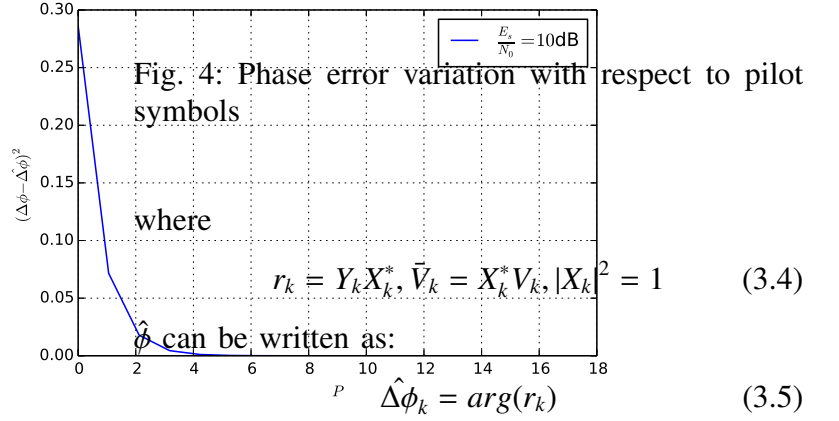
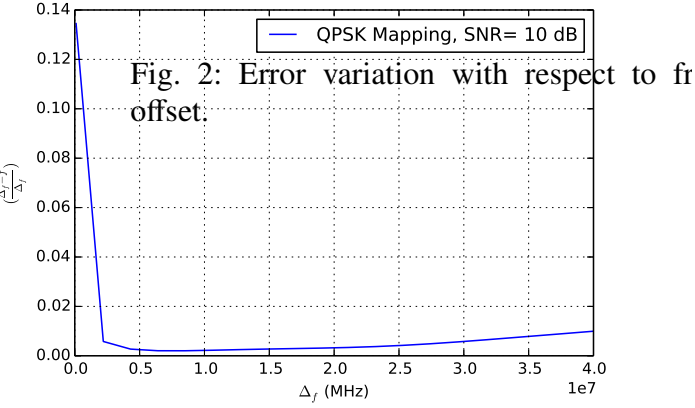
The autocorrelation can be calculated as

$$R(k) \triangleq \frac{1}{N-k} \sum_{i=k+1}^N r_i r_{i-k}^*, \quad 1 \leq k \leq N-1 \quad (2.5)$$

Where  $N$  is the length of the received signal. For large centre frequency, the following yields a good approximation for frequency offset upto 40 MHz.

$$\Delta \hat{f} \approx \frac{1}{2\pi M} \frac{\sum_{k=1}^P \text{Im}(R(k))}{\sum_{k=1}^P k \text{Re}(R(k))}, \quad P \Delta f M \ll 1 \quad (2.6)$$

where  $P$  is the number of pilot symbols.



This equation gives the final estimation of phase

$$\hat{\Delta\phi}_f^{(p)}(l) = \hat{\Delta\phi}_f^{(p)}(l-1) + \alpha SAW[\hat{\Delta\phi}^{(p)}(l) - \hat{\Delta\phi}_f^{(p)}(l-1)] \quad (3.6)$$

$$SAW[\phi] = [\phi]_{-\pi}^{\pi} \quad (3.7)$$

Where  $SAW$  is sawtooth non-linearity and  $\alpha \leq 1$ .  $l$  counts the number of pilot-based estimates.  $\hat{\Delta\phi}_f^{(p)}(l)$  is the final unwrapped pilot estimate.

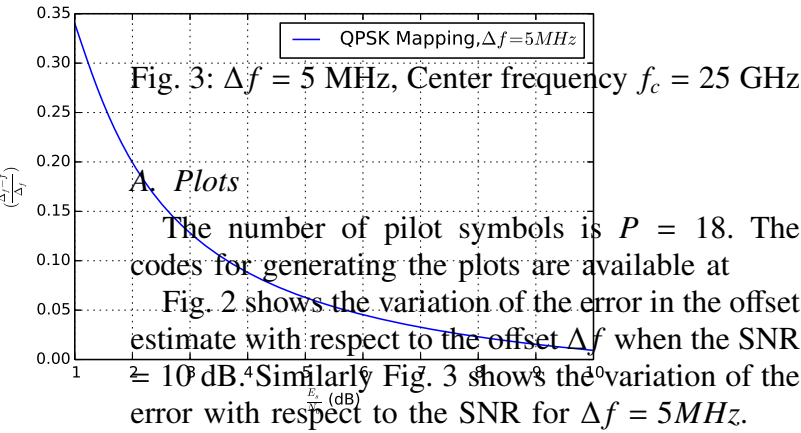
#### A. Plots

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Fig. 4 shows the variation of the phase error in the offset estimate with respect to the pilot symbols.  $\Delta f$  when the SNR = 10 dB. Similarly Fig. 5 shows the variation of the error with respect to the SNR for pilot symbols  $P = 18$ .

#### REFERENCES

- [1] U. Mengali and A. N. D'Andrea: 'synchronization Techniques for Digital Receivers,' New York: Plenum, 1997.
- [2] F. M. Gardner: 'A BPSK/QPSK timing-error detector for sampled receivers,' IEEE TRANSACTIONS ON COMMUNICATIONS, VOL.COM-34,NO.5,MAY 1986



### 3. PHASE OFFSET: FEED FORWARD MAXIMUM LIKELIHOOD (FF-ML) TECHNIQUE

Let the phase offset be  $\Delta\phi$  [4]. Then

$$Y_k = X_k e^{j\Delta\phi} + V_k, \quad k = 1, \dots, N \quad (3.1)$$

From (3.1),

$$Y_k X_k^* = |X_k|^2 e^{j\Delta\phi} + X_k^* V_k \quad (3.2)$$

$$\Rightarrow r_k = e^{j\Delta\phi} + \bar{V}_k \quad (3.3)$$

