

Modern Digital Synchronization Techniques for Reliable Communication

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Abstract—A brief description about the de implementation of Modern Digital Synchronizat niques for reliable communication.

1. TIME OFFSET: GARDNER TED

Let the m th sample in the r th received time slot be

$$Y_k(m) = X_k + V_k(m), \quad k = 1, \dots, N, m = 1, \dots, M. \quad (1.1)$$

where X_k is the transmitted symbol in the k th time slot and $V_k(m) \sim \mathcal{N}(0, \sigma^2)$. The decision variable for the k th symbol is [2]

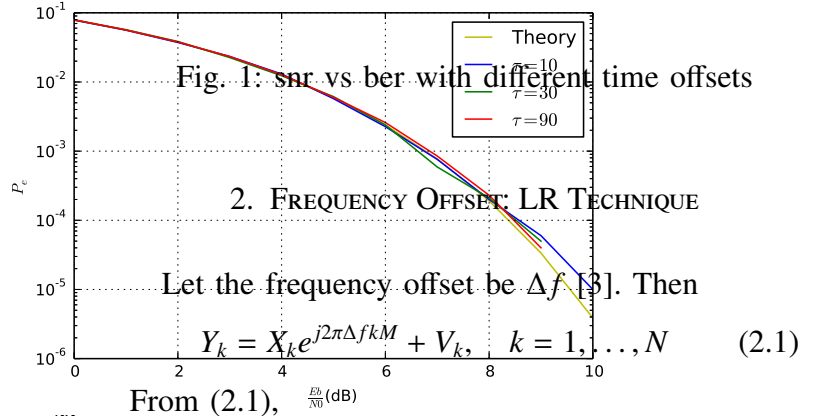
$$U_k = Y_{k-1} \left(\frac{M}{2} \right) [Y_k(M) - Y_{k-1}(M)] \quad (1.2)$$

A. Plots

The codes for generating the plots are available at

Fig. 1 shows the variation of the bit error rate respect to the snr with different timing offsets. Δf when the SNR = 10 dB.

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2. FREQUENCY OFFSET: LR TECHNIQUE

Let the frequency offset be Δf [3]. Then

$$Y_k = X_k e^{j2\pi\Delta f k M} + V_k, \quad k = 1, \dots, N \quad (2.1)$$

From (2.1), $\frac{E_b}{N_0}$ (dB)

$$Y_k X_k^* = |X_k|^2 e^{j2\pi\Delta f k M} + X_k^* V_k \quad (2.2)$$

$$\Rightarrow r_k = e^{j2\pi\Delta f k M} + \bar{V}_k \quad (2.3)$$

where

$$r_k = Y_k X_k^*, \bar{V}_k = X_k^* V_k, |X_k|^2 = 1 \quad (2.4)$$

The autocorrelation can be calculated as

$$R(k) \triangleq \frac{1}{N-k} \sum_{i=k+1}^N r_i r_{i-k}^*, \quad 1 \leq k \leq N-1 \quad (2.5)$$

Where N is the length of the received signal. For large centre frequency, the following yields a good approximation for frequency offset upto 40 MHz.

$$\Delta \hat{f} \approx \frac{1}{2\pi M} \frac{\sum_{k=1}^P \text{Im}(R(k))}{\sum_{k=1}^P k \text{Re}(R(k))}, \quad P \Delta f M \ll 1 \quad (2.6)$$

where P is the number of pilot symbols.

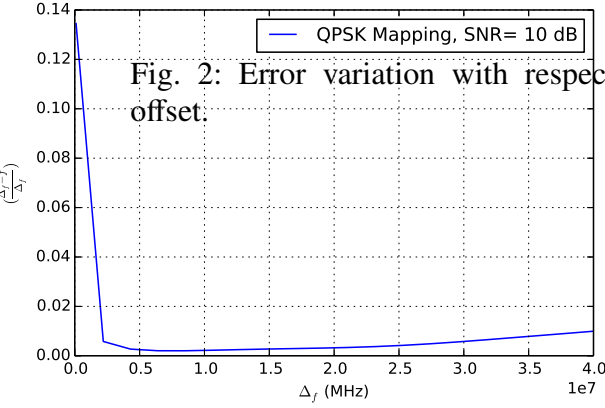


Fig. 2: Error variation with respect to frequency offset.

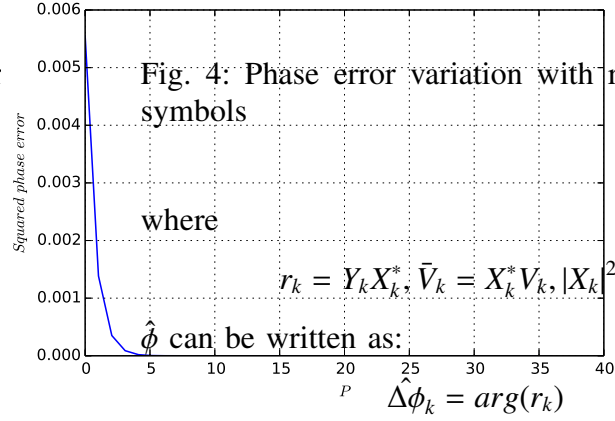


Fig. 4: Phase error variation with respect to pilot symbols

where

$$r_k = Y_k X_k^*, \bar{V}_k = X_k^* V_k, |X_k|^2 = 1 \quad (3.4)$$

$\hat{\phi}$ can be written as:

$$\Delta \hat{\phi}_k = \arg(r_k) \quad (3.5)$$

This equation gives the final estimation of phase

$$\Delta \hat{\phi}_f^{(p)}(l) = \Delta \hat{\phi}_f^{(p)}(l-1) + \alpha SAW[\Delta \hat{\phi}_f^{(p)}(l) - \Delta \hat{\phi}_f^{(p)}(l-1)] \quad (3.6)$$

$$SAW[\phi] = [\phi]_{-\pi}^{\pi} \quad (3.7)$$

Where SAW is sawtooth non-linearity and $\alpha \leq 1$.

l counts the number of pilot-based estimates.

$\Delta \hat{\phi}_f^{(p)}(l)$ is the final unwrapped pilot estimate.

A. Plots

The codes for generating the plots are available at

Fig. 4 shows the variation of the phase error in the offset estimate with respect to the pilot symbols. Δf when the SNR = 10 dB. Similarly Fig. 5 shows the variation of the error with respect to the SNR for pilot symbols $P = 18$.

REFERENCES

- [1] U. Mengali and A. N. D'Andrea: 'synchronization Techniques for Digital Receivers,' New York: Plenum, 1997.
- [2] F. M. Gardner: 'A BPSK/QPSK timing-error detector for sampled receivers,' IEEE TRANSACTIONS ON COMMUNICATIONS, VOL.COM-34,NO.5,MAY 1986

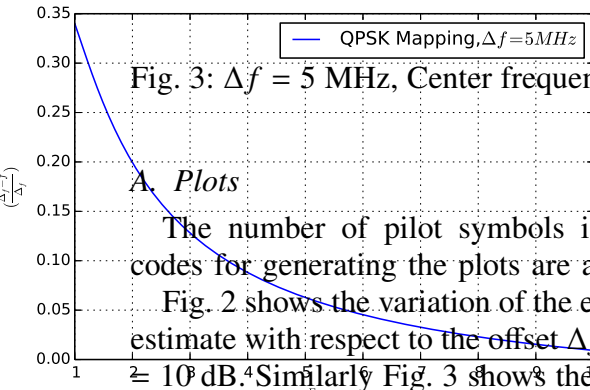


Fig. 3: $\Delta f = 5$ MHz, Center frequency $f_c = 25$ GHz

A. Plots

The number of pilot symbols is $P = 18$. The codes for generating the plots are available at

Fig. 2 shows the variation of the error in the offset estimate with respect to the offset Δf when the SNR = 10 dB. Similarly Fig. 3 shows the variation of the error with respect to the SNR for $\Delta f = 5$ MHz.

3. PHASE OFFSET: FEED FORWARD MAXIMUM LIKELIHOOD (FF-ML) TECHNIQUE

Let the phase offset be $\Delta \phi$ [4]. Then

$$Y_k = X_k e^{j\Delta \phi} + V_k, \quad k = 1, \dots, N \quad (3.1)$$

From (3.1),

$$Y_k X_k^* = |X_k|^2 e^{j\Delta \phi} + X_k^* V_k \quad (3.2)$$

$$\Rightarrow r_k = e^{j\Delta \phi} + \bar{V}_k \quad (3.3)$$

