Modern Synchronization Techniques for Reliable Communication

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1. Time Offset: Gardner TED

Let the mth sample in the rth received time slot be

$$Y_k(m) = X_k + V_k(m), \quad k = 1, \dots, N, m = 1,$$

where X_k is the transmitted symbol in the slot and $V_k(m) \sim \mathcal{N}\left(0, \sigma^2\right)$. The decision for the kth symbol is

$$U_k = Y_{k-1}\left(\frac{M}{2}\right)\left[Y_k\left(M\right) - Y_{k-1}\left(M\right)\right] \tag{1.2}$$

A. Plots

2. Frequency Offset: LR Technique

Let the frequency offset be Δf [1]. Then

$$Y_k = X_k e^{j2\pi\Delta f k M} + V_k, \quad k = 1, \dots, N \tag{2.1}$$

From (2.1),

$$Y_k X_k^* = |X_k|^2 e^{j2\pi\Delta f k M} + X_k^* V_k \tag{2.2}$$

$$\implies r_k = e^{j2\pi\Delta f k M} + \bar{V}_k \tag{2.3}$$

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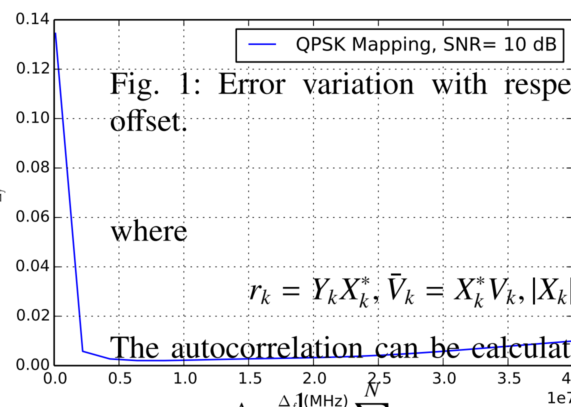

Fig. 1: Error variation with respect to frequency offset.

where

$$r_k = Y_k X_k^*, \bar{V}_k = X_k^* V_k, |X_k|^2 = 1 \tag{2.4}$$

The autocorrelation can be calculated as

$$R(k) \triangleq \frac{1}{N-k} \sum_{i=k+1}^{N} r_i r_{i-k}^*, 1 \le k \le N-1 \tag{2.5}$$

Where N is the length of the received signal. For large centre frequency, the following yields a good approximation for frequency offset upto 40 MHz.

$$\Delta\hat{f} \approx \frac{1}{2\pi M} \frac{\sum_{k=1}^{P} \operatorname{Im}(R(k))}{\sum_{k=1}^{P} k \operatorname{Re}(R(k))}, \quad P\Delta f M << 1 \tag{2.6}$$

where P is the number of pilot symbols.

A. Plots

The number of pilot symbols is $P = 18$. The codes for generating the plots are available at

Fig. 1 shows the variation of the error in the offset estimate with respect to the offset Δf when the SNR = 10 dB. Similarly Fig. **??** shows the variation of the error with respect to the SNR for $\Delta f = 5MHz$.

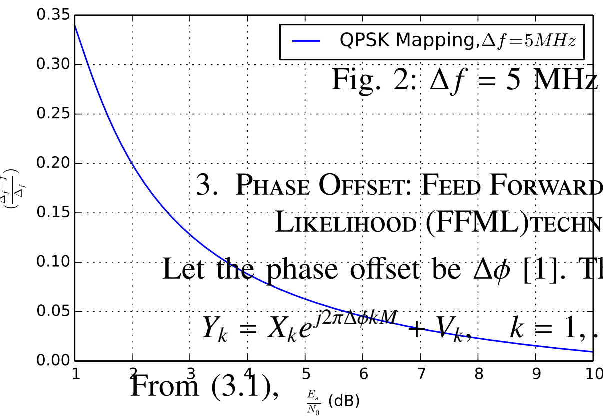

Fig. 2: Δf = 5 MHz

3. Phase Offset: Feed Forward Maximum Likelihood (FFML)technique

Let the phase offset be $\Delta\phi$ [1]. Then

$$Y_k = X_k e^{j2\pi\Delta\phi kM} + V_k, \quad k = 1, \ldots, N \tag{3.1}$$

From (3.1),

$$Y_k X_k^* = |X_k|^2 \, e^{j2\pi\Delta\phi kM} + X_k^* V_k \tag{3.2}$$

$$\implies r_k = e^{j2\pi\Delta\phi kM} + \bar{V}_k \tag{3.3}$$

where

$$r_k = Y_k X_k^*, \bar{V}_k = X_k^* V_k, |X_k|^2 = 1 \tag{3.4}$$

$\hat{\phi}$ can be written as:

$$\hat{\phi}_k = arg(r_k) \tag{3.5}$$

This equation gives the final estimation of phase

$$\hat{\theta}_f^{(p)}(l) = \hat{\theta}_f^{(p)}(l-1) + \alpha SAW[\hat{\theta}_f^{(p)}(l) - \hat{\theta}_f^{(p)}(l-1)] \tag{3.6}$$

Where SAW is a saw tooth non-linearity and $\alpha <= 1$

A. *Plots*

References

[1] M. Luise and R. Reggiannini:'Carrier frequency recovery in all-digital modems for burst mode transmissions,' IEEE Trans. Commun., vol. 43, no. 2/3/4, pp. 1169-1178, Feb/Mar/Apr 1995.

[2] U. Mengali and A. N. D'Andrea:'synchronization Techniques for Digital Receivers,' New York: Plenum, 1997.