Ensemble Learning Models

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What is Ensemble Learning?

A spam email filter

- Design an email filter that can distinguish spam from non-spam.
- How can we approach this problem with ML?
 - 1. Gathering as many example as possible of both spam and non-spam emails.
 - 2. Train a classifier using these examples and their labels.
 - 3. Take the learned classifier, or prediction rule, and use it to filter your mail.
 - 4. The goal is to train a classifier that makes the most accurate predictions possible on new test examples.
- But, building a highly accurate classifier is a difficult task. (You still get spam, right?!)





A spam email filter



We could probably come up with many quick rules of thumb. These could be only moderately accurate. Can you think of an example for this situation?

 An example could be "if the subject line contains 'buy now' then classify as spam."

• This certainly doesn't cover all spams, but it will be significantly better than random guess.

Weak Learner:

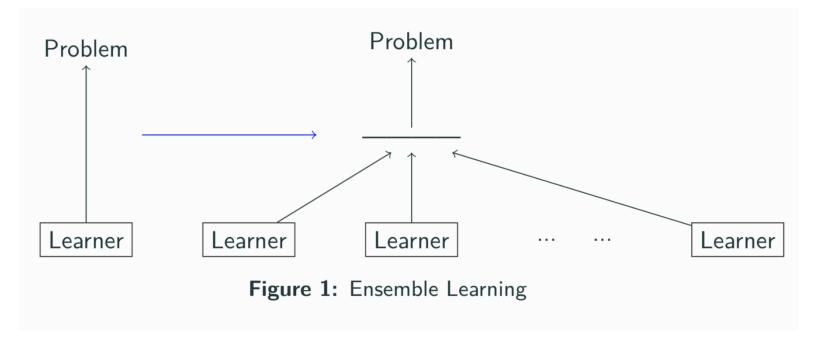
The learner performance is slightly higher than random guess. For example, 50% accuracy in binary classification task.

Ensemble Learning

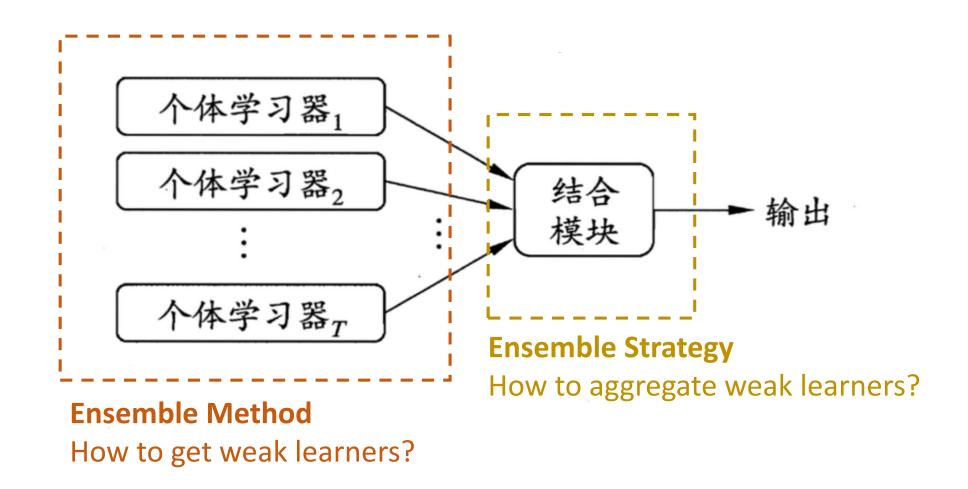
Weak Learner: label examples better than random guess.

Strong Learner: arbitrarily well-correlated with the true classification.

 Can a set of weak learners create a single strong learner (Kearns and Valiant 1988, 1989)?



Ensemble Learning



Is Ensemble Learning Better?

Is Ensemble Learning Better?

 Kearns and Valiant proposed a question "Whether the notions of strong and weak learnability are equivalent"

 Robert Schapire gave an affirmative answer in a 1990 paper to the question of [Kearns and Valiant 1989].



Robert Schapire

Is Ensemble Learning Better?

 Many experiments show that Ensemble Learning is much better. For example, Hansen and Salamon, TPAMI90.

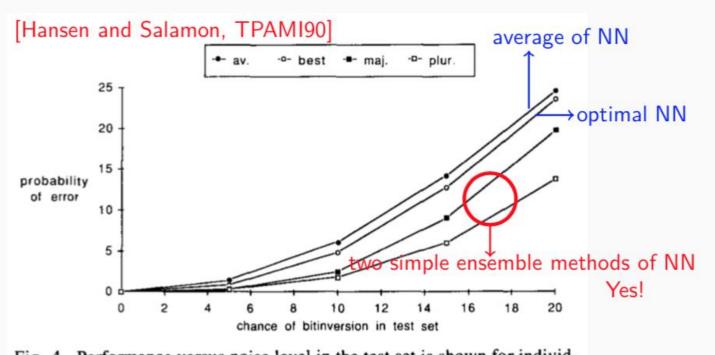


Fig. 4. Performance versus noise level in the test set is shown for individual and for consensus decisions. Data displayed shows the average and the best network, as well as collective decisions using majority and plurality for seven networks trained on individual training sets.

Figure 2: Ensemble vs. Single of the Best

Table 1: single learners vs. Ensemble learner

| Learner test1test2test3 | | | | Learner | test1test2test3 | | | Learner | test1test2test3 | | |
|-------------------------|-----------|-----------|-----------|--------------|-----------------|-----------|---|-------------------|-----------------|-----------|-----------|
| h1 | $\sqrt{}$ | $\sqrt{}$ | × | h1 | | | × | h1 | $\sqrt{}$ | × | × |
| h2 | \times | $\sqrt{}$ | $\sqrt{}$ | h2 | $\sqrt{}$ | | × | h2 | \times | $\sqrt{}$ | × |
| h3 | $\sqrt{}$ | \times | $\sqrt{}$ | h3 | $\sqrt{}$ | $\sqrt{}$ | × | h3 | \times | \times | $\sqrt{}$ |
| Ensemble $\sqrt{}$ | | | | Ensemb | le√ | | × | $Ensemble \times$ | | × | × |
| (a) better | | | | (b) no sense | | | | (c) worse | | | |

- Single learners should be different and better than random.
- Is it right, theoretically?

- Label $y \in \{1, -1\}$
- True classifier: *f*
- T base classifiers: independent and error of each one is ϵ , i.e.,

$$P(h_i(x) \neq f(x)) = \epsilon.$$

Ensemble classifier:

$$H(x) = sign(\sum_{i=1}^{T} h_i(x)).$$

Error of Ensemble Learner:

semble Learner:
$$P(H(x) \neq f(x)) = \sum_{k=0}^{\lfloor T/2 \rfloor} {T \choose k} (1 - \epsilon)^k \epsilon^{T-k}$$

$$\leq \exp(-\frac{1}{2}T(1 - 2\epsilon)^2)$$

• Error:

$$\epsilon \downarrow$$
, P(H(x) $\neq f(x)$) \downarrow

• Number:

$$T \uparrow, P(H(x) \neq f(x)) \downarrow$$

• $good(\epsilon \downarrow)$ v.s. independence

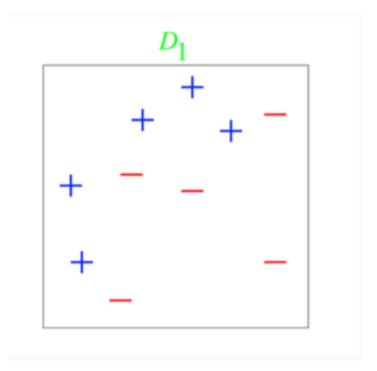
Boosting

Boosting

- Idea: given a weak learner, run it multiple times on (reweighted) training data, then let learned classifiers vote
- On each iteration t:
 - Weight each training example by how incorrectly it was classified
 - Learn a weak hypothesis h_t
 - A strength for this hypothesis α_t
- Final classifier:

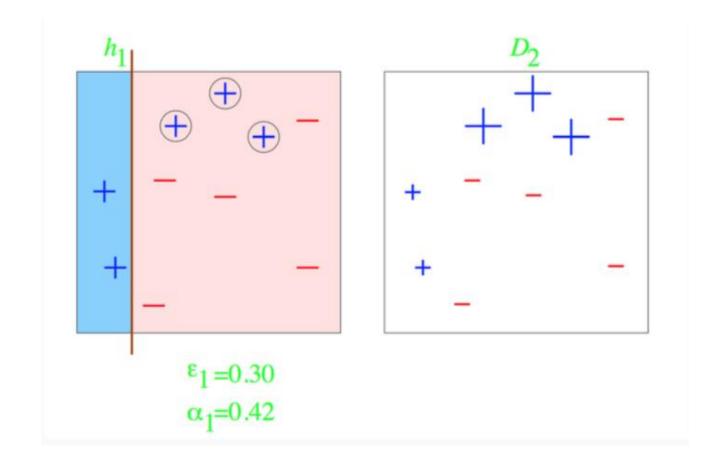
$$H(x) = sign\left(\sum \alpha_t h_t(x)\right)$$

• Toy Example:

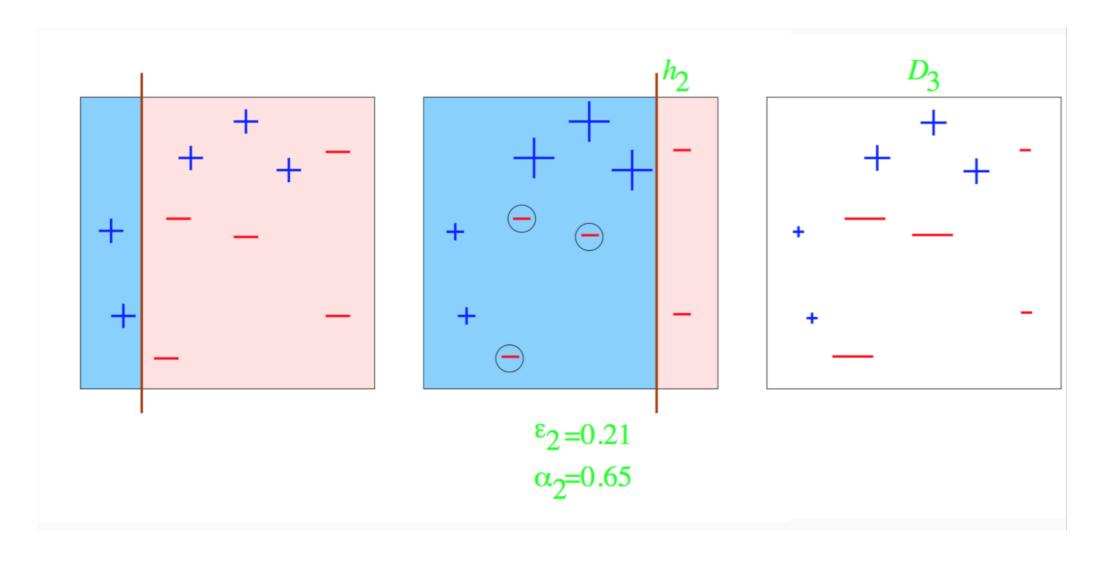


• weak classifiers: vertical or horizontal half-planes

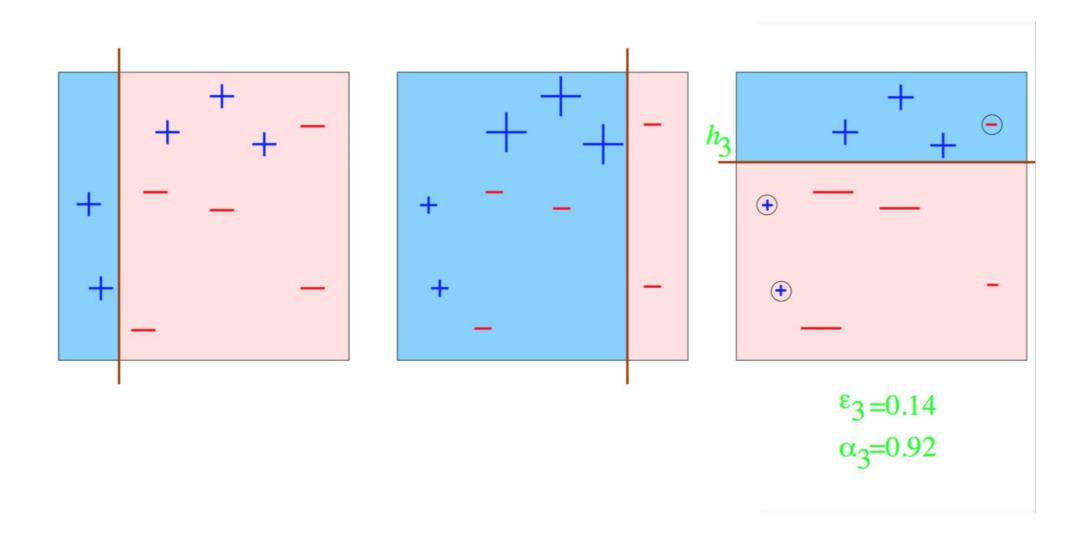
• Round 1:



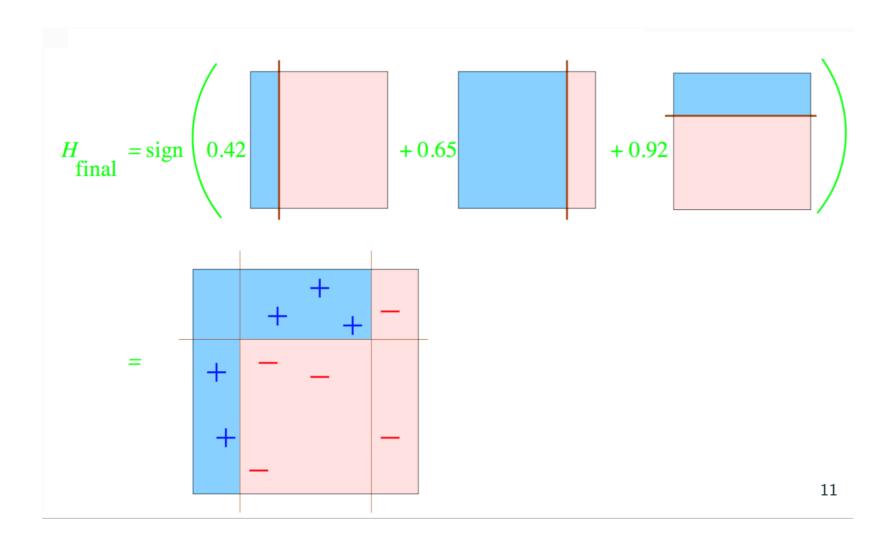
• Round 2:



• Round 3:



• Final Classifier:



Many Variations

AdaBoost.M1, AdaBoost.MR,

FilterBoost, GentleBoost,

GradientBoost, MadaBoost,

LogitBoost, LPBoost,

MultiBoost, RealBoost,

RobustBoost, ...

Key Questions in Boosting

- ?
- How should the distribution be chosen each round?

- How should the weak rules be combined into a single rule?
- How many weak classifiers should we learn?
- What is the criterion that we are optimizing?

- AdaBoost[Freund and Schapire, 1997] is a typical Boosting algorithm.
- Input:
 - *N* examples $\{(x^1, y^1), ..., (x^N, y^N)\}$
 - A weak base learner $h = h(x; \theta)$
- Final classifier is the sign of

$$H(x) = \alpha_1 h(x; \theta) + \dots + \alpha_m h(x; \theta_m)$$

Exponential Loss:

$$\ell_{exp}(H) = \sum_{i=1}^{N} \exp(-y^{i}H(x^{i}))$$

• How to weak learner weight α_t ?

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minimize the exponential loss $\ell_{exp}(H)$

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- How to weak learner weight α_t ?

 minimize the exponential loss $\ell_{exp}(H)$
- How to update the distribution in each round? derived from exponential loss

Weight of Weak Learner: α_t

1.
$$\frac{\partial \ell_{exp}(H|D)}{\partial H(x)}$$
:
$$\frac{\partial \ell_{exp}(H|D)}{\partial H(x)} = -e^{-H(x)}P(f(x) = 1|x) + e^{H(x)}P(f(x) = -1|x) = 0$$
2.
$$H(x)$$
:
$$I_{f(x)} = \frac{1}{2} P(f(x) = 1|x)$$

$$H(x) = \frac{1}{2} \ln \frac{P(f(x) = 1|x)}{P(f(x) = -1|x)}$$

3. sign(H(x)):

$$sign(H(x)) = sign\left(\frac{1}{2}ln\frac{P(f(x) = 1|x)}{P(f(x) = -1|x)}\right)$$

$$= \begin{cases} 1, & if \ P(f(x) = 1|x) > P(f(x) = -1|x) \\ -1, & if \ P(f(x) = 1|x) < P(f(x) = -1|x) \end{cases}$$

$$= \arg\max_{y \in \{-1,1\}} P(f(x) = y|x)$$

Weight of Weak Learner: α_t

- α_t minimizes $\ell_{exp}(\alpha_t h_t | \mathcal{D}_t)$
- $\ell_{exp}(\alpha_t h_t | \mathcal{D}_t)$:

$$\begin{aligned} \ell_{exp}(\alpha_t h_t | \mathcal{D}_t) &= \mathbb{E}_{x \sim \mathcal{D}_t} \left[e^{-f(x)\alpha_t h_t(x)} \right] \\ &= \mathbb{E}_{x \sim \mathcal{D}_t} \left[e^{-\alpha_t} \mathbb{I} \left(f(x) = h_t(x) \right) + e^{\alpha_t} \mathbb{I} \left(f(x) \neq h_t(x) \right) \right] \\ &= e^{-\alpha_t} P_{x \sim \mathcal{D}_t} \left(f(x) = h_t(x) \right) + e^{\alpha_t} P_{x \sim \mathcal{D}_t} \left(f(x) \neq h_t(x) \right) \\ &= e^{-\alpha_t} (1 - \epsilon_t) + e^{\alpha_t} \epsilon_t \end{aligned}$$

where $\epsilon_t = P_{x \sim \mathcal{D}_t}(f(x) \neq h_t(x))$.

•
$$\frac{\partial \ell_{exp}(\alpha_t h_t | \mathcal{D}_t)}{\partial \alpha_t}$$
:

$$\frac{\partial \ell_{exp}(\alpha_t h_t | \mathcal{D}_t)}{\partial \alpha_t} = -e^{-\alpha_t} (1 - \epsilon_t) + e^{\alpha_t} \epsilon_t = 0$$

$$\alpha_t = \frac{1}{2} \ln(\frac{1 - \epsilon_t}{\epsilon_t})$$

Update the distribution:

• h_{t+1} can minimize the loss $\ell_{exp}(H_t + h_{t+1}|\mathcal{D})$

•
$$\ell_{exp}(H_t + h_{t+1}|\mathcal{D})$$
:
• $\ell_{exp}(H_t + h_{t+1}|\mathcal{D}) = \mathbb{E}_{x \sim \mathcal{D}} \left[e^{-f(x)(H_t(x) + h_{t+1}(x))} \right]$
= $\mathbb{E}_{x \sim \mathcal{D}} \left[e^{-f(x)H_t(x)} e^{-f(x)h_{t+1}(x)} \right]$

$$\simeq \mathbb{E}_{x \sim \mathcal{D}} \left[e^{-f(x)H_t(x)} \left(1 - f(x)h_{t+1}(x) + \frac{f^2(x)h_{t+1}^2(x)}{2} \right) \right]$$

= $\mathbb{E}_{x \sim \mathcal{D}} \left[e^{-f(x)H_t(x)} (1 - f(x)h_{t+1}(x) + \frac{1}{2}) \right]$

•
$$h_{t+1}$$
:
 $h_{t+1} = \arg\min_{h} \ell_{exp}(H_t + h|\mathcal{D})$
 $= \arg\max_{h} \mathbb{E}_{x \sim \mathcal{D}} \left[e^{-f(x)H_t(x)} f(x)h(x) \right]$
 $= \arg\max_{h} \mathbb{E}_{x \sim \mathcal{D}} \left[\frac{e^{-f(x)H_t(x)}}{\mathbb{E}_{x \sim \mathcal{D}} \left[e^{-f(x)H_t(x)} \right]} f(x)h(x) \right]$

• h_{t+1} :

$$h_{t+1} = \arg\max_{h} \mathbb{E}_{x \sim \mathcal{D}} \left[\frac{e^{-f(x)H_t(x)}}{\mathbb{E}_{x \sim \mathcal{D}} [e^{-f(x)H_t(x)}]} f(x)h(x) \right]$$

• Let
$$\mathcal{D}_{t+1}(x) = \frac{\mathcal{D}(x)e^{-f(x)H_t(x)}}{\mathbb{E}_{x\sim\mathcal{D}}[e^{-f(x)H_t(x)}]}$$

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$$= \frac{\mathcal{D}(x)e^{-f(x)H_t(x)}}{\mathbb{E}_{x\sim\mathcal{D}}[e^{-f(x)H_t(x)}]}$$

$$= \frac{\mathcal{D}(x)e^{-f(x)H_{t-1}(x)}e^{-f(x)\alpha_th_t(x)}}{\mathbb{E}_{x\sim\mathcal{D}}[e^{-f(x)H_t(x)}]}$$

$$= \frac{\mathcal{D}(x)e^{-f(x)H_{t-1}(x)}e^{-f(x)\alpha_th_t(x)}}{\mathbb{E}_{x\sim\mathcal{D}}[e^{-f(x)H_t(x)}]}$$

$$= \mathcal{D}_t(x)e^{-f(x)\alpha_th_t(x)} \frac{\mathbb{E}_{x\sim\mathcal{D}}[e^{-f(x)H_t(x)}]}{\mathbb{E}_{x\sim\mathcal{D}}[e^{-f(x)H_t(x)}]}$$

where $\frac{\mathbb{E}_{x \sim \mathcal{D}}[e^{-f(x)H_{t-1}(x)}]}{\mathbb{E}_{x \sim \mathcal{D}}[e^{-f(x)H_{t}(x)}]}$ is the normalization factor.

• Iteration equation:

$$\mathcal{D}_{t+1}(x) \propto \mathcal{D}_t(x) e^{-\alpha_t f(x) h_t(x)}$$

Input: Trainset $D = (x^1, y^1), (x^2, y^2), ..., (x^m, y^m);$ Algorithm to obtain the base learner \mathcal{L} ; Turns of training T;

Process:

1.
$$\mathcal{D}_1(x) = 1/m$$

2. **for**
$$t = 1, 2, ..., T$$
 do

3.
$$h_t = \mathcal{L}(D, \mathcal{D}_t);$$

4.
$$\epsilon_t = P_{x \sim \mathcal{D}_t}(h_t(x) \neq f(x))$$

5. if
$$\epsilon_t > 0.5$$
 then Break

6.
$$\alpha_t = \frac{1}{2} ln \left(\frac{1 - \epsilon_t}{\epsilon_{t,\epsilon}} \right) \ge 0$$

7.
$$\mathcal{D}_{t+1} = \frac{\mathcal{D}_t}{Z_t} \times \begin{cases} \exp(-\alpha_t) \le 1, & \text{if } h_t(x) = f(x) \\ \exp(\alpha_t) \ge 1, & \text{if } h_t(x) \ne f(x) \end{cases} = \frac{\mathcal{D}_t(x)\exp(-\alpha_t f(x)h_t(x))}{Z_t}$$

8. end for

Output:
$$H(x) = sign(\sum_{t=1}^{T} \alpha_t h_t(x))$$

- 给定如表所示的训练数据
- 假设弱分类器由x < v 或 x > v产生,其阈值v使该分类器在训练数据集上分类误差率最低
- 试用AdaBoost算法学习一个强分类器

| 序号 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 |
|----------|---|---|---|----|----|----|---|---|----|----|
| x | 0 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 |
| <u>y</u> | 1 | 1 | 1 | -1 | -1 | -1 | 1 | 1 | _1 | -1 |

解: 初始化数据权值分布

$$D_1 = (w_{11}, w_{12}, ..., w_{110})$$

$$w_{1i} = 0.1, i = 1, 2, ..., 10$$

| 序号 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 |
|----------|---|---|---|----|----|----|---|---|---|----|
| x | 0 | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 |
| <u>y</u> | 1 | 1 | 1 | -1 | -1 | -1 | 1 | 1 | 1 | -1 |

- 对m =1,
- (a) 在权值分布为 D_1 的训练数据上,阈值v取2.5时分类误差率最低,故基本分类器为 $G_1(x) = \begin{cases} 1, & x < 2.5 \\ -1, & x > 2.5 \end{cases}$
 - (b) $G_1(x)$ 在训练数据集上的误差率 $e_i = P(G_1(x_i) \neq y_i) = 0.3$
 - (c) 计算 $G_1(x)$ 的系数: $\alpha_i = \frac{1}{2} \log \frac{1-e_i}{e_i} = 0.4236$
 - (d) 更新训练数据的权值分布:

$$D_2 = (w_{21}, \dots, w_{2i}, \dots, w_{210})$$

$$w_{2i} = \frac{w_{1i}}{Z_1} \exp(-\alpha_1 y_i G_1(x_i)), \quad i = 1, 2, \dots, 10$$

$$D_2 = (0.0715, 0.0715, 0.0715, 0.0715, 0.0715, 0.0715, 0.0715, 0.1666, 0.1666, 0.1666, 0.0715)$$

$$f_1(x) = 0.4236G_1(x)$$

分类器 $sign[f_1(x)]$ 在训练数据集上有3个误分类点.

- 对m =2,
- (a) 在权值分布为 D_2 的训练数据上,阈值v是8.5时分类误差率最低,基本分类器为 $G_2(x) = \begin{cases} 1, & x < 8.5 \\ -1, & x > 8.5 \end{cases}$
 - (b) $G_2(x)$ 在训练数据集上的误差率 $e_2 = 0.2143$
 - (c) 计算 $\alpha_2 = 0.6496$
 - (d) 更新训练数据的权值分布:

$$D_3 = (0.0455, 0.0455, 0.0455, 0.1667, 0.1667, 0.1667, 0.1060, 0.1060, 0.1060, 0.0455)$$

 $f_2(x) = 0.4236G_1(x) + 0.6496G_2(x)$

分类器 $sign[f_2(x)]$ 在训练数据集上有3个误分类点.

- 对m =3,
- (a) 在权值分布为 D_3 的训练数据上,阈值v是5.5时分类误差率最低,基本分类器为 $G_3(x) = \begin{cases} 1, & x > 5.5 \\ -1, & x < 5.5 \end{cases}$
 - (b) $G_3(x)$ 在训练数据集上的误差率 $e_3 = 0.1820$
 - (c) 计算 $\alpha_3 = 0.7514$
 - (d) 更新训练数据的权值分布:

$$D_4 = (0.125, 0.125, 0.125, 0.102, 0.102, 0.102, 0.065, 0.065, 0.065, 0.125)$$

于是得到:

$$f_3(x) = 0.4236G_1(x) + 0.6496G_2(x) + 0.7514G_3(x)$$

分类器 $sign[f_3(x)]$ 在训练数据集上误分类点个数为0.

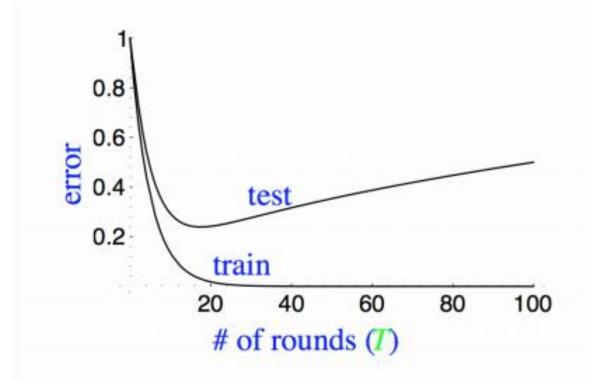
于是最终分类器为

$$G(x) = \operatorname{sign}[f_3(x)] = \operatorname{sign}[0.4236G_1(x) + 0.6496G_2(x) + 0.7514G_3(x)]$$

How Will Test Error Behave?

Test Error

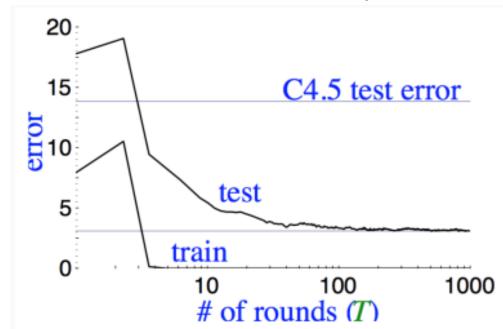
- Expect (a first guess)
 - Training error to continue to drop(or reach 0)
 - Test error increases when H_{final} becomes "too complex"
 - Occam's razor
 - Overfitting: hard to know when to stop training



Modified from the tutorial of Freund & Schapire.

Test Error

- Actually
 - Test error does not increase, even after 1000 rounds
 - Total size > 2,000,000 nodes
 - Test error continues to drop even after training error is zero!



| | # rounds | | |
|-------------|----------|-----|------|
| | 5 | 100 | 1000 |
| train error | 0.0 | 0.0 | 0.0 |
| test error | 8.4 | 3.3 | 3.1 |

Occam's razor wrongly predicts "simpler" rule is better?

Test Error - Margin Theory Explanation

Margin Theory Explanation

Based on the concept of margin, Schapire et al. [1998] proved that, given any threshold $\theta > 0$ of margin over the training data D, with probability at least $1 - \delta$, the generalization error of the ensemble $\epsilon_{\mathcal{D}} = P_{\boldsymbol{x} \sim \mathcal{D}}(f(\boldsymbol{x}) \neq H(\boldsymbol{x}))$ is bounded by

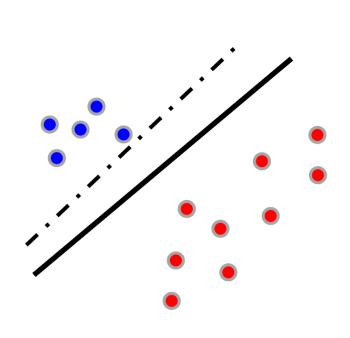
$$\epsilon_{\mathcal{D}} \leq P_{\boldsymbol{x} \sim D}(f(\boldsymbol{x})H(\boldsymbol{x}) \leq \theta) + \tilde{O}\left(\sqrt{\frac{d}{m\theta^2} + \ln\frac{1}{\delta}}\right)$$

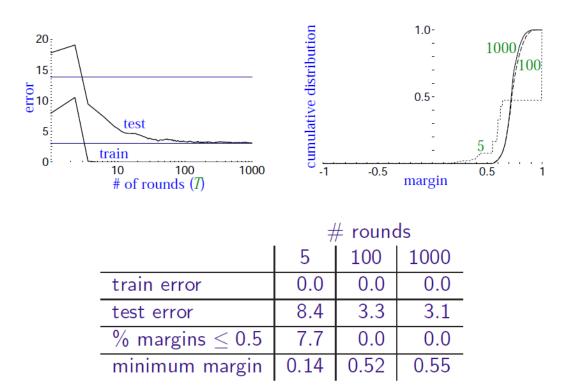
$$\leq 2^T \prod_{t=1}^T \sqrt{\epsilon_t^{1-\theta}(1 - \epsilon_t)^{1+\theta}} + \tilde{O}\left(\sqrt{\frac{d}{m\theta^2} + \ln\frac{1}{\delta}}\right)$$

 This bound implies that, when other variables are fixed, the larger the margin over the training data, the smaller the generalization error!

Test Error - Margin Theory Explanation

- Why AdaBoost tends to be resistant to overfitting? the margin theory answers:
 - It can increase the **ensemble margin** even after the training error reaches zero!





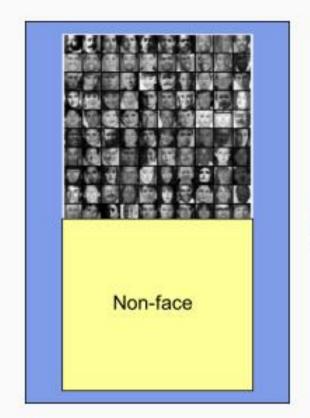
Practical Advantages of AdaBoost

- Very fast
- Simple and easy to program
- Few parameters to tune (except T)
- Flexible can combine with (m)any learning algorithm
- Little prior knowledge needed about weak learner
- Provably effective, provided can consistently find rough rules of thumb
 - → shift in mind set goal now is merely to find classifiers barely better than random guessing
- versatile
 - can use with data that is textual, numeric, discrete, etc.
 - has been extended to learning problems well beyond binary classification

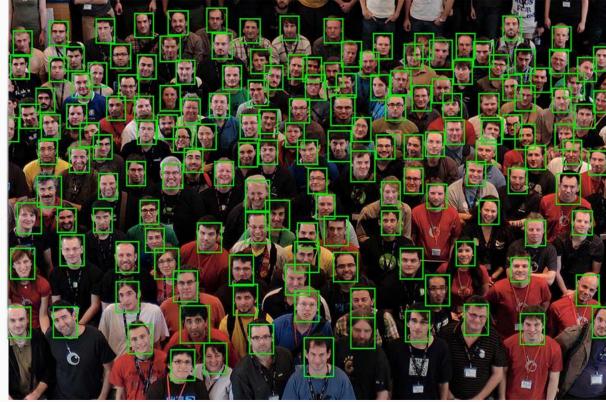
Disadvantages of AdaBoost

- performance of AdaBoost depends on data and weak learner
- consistent with theory, AdaBoost can fail if
 - weak classifiers too complex
 - overfitting
 - weak classifiers too weak ($\gamma_t \rightarrow 0$ too quickly)
 - underfitting
 - low margins → overfitting

empirically, AdaBoost seems especially susceptible to uniform noise



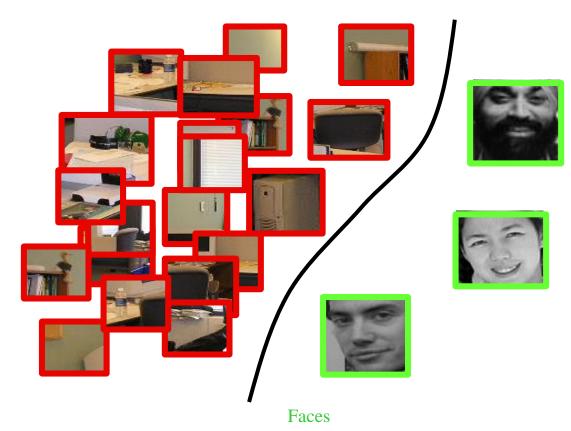




Training set

In some feature space

Background



1. hypothesize:

try all possible rectangle locations, sizes

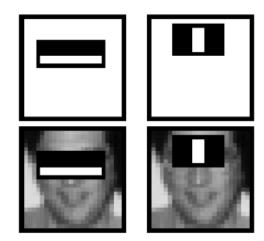
2. test:

classify if rectangle contains a face (and only the face)

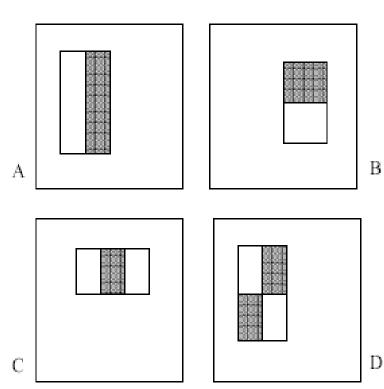
Note: 1000's more false windows then true ones.

- Rectangle filters denoted by f
- Only 4 Types of "Rectangle filters"
 (Similar to Haar wavelets Papageorgiou, et al.)

g(x) = sum(WhiteArea) - sum(BlackArea)





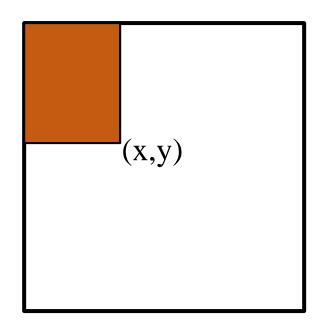


- Why rectangle features?
- Too Slow! O(n⁴)

Integral Image

• The *integral image* computes a value at each pixel (*x*,*y*) that is the sum of the pixel values above and to the left of (*x*,*y*), inclusive.

 This can quickly be computed in one pass through the image

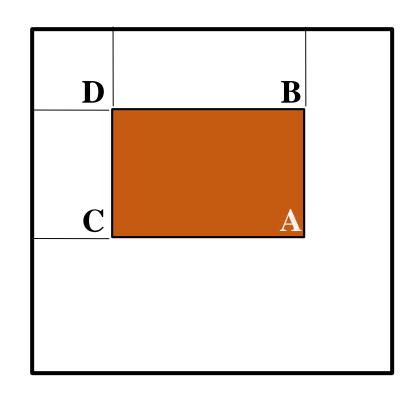


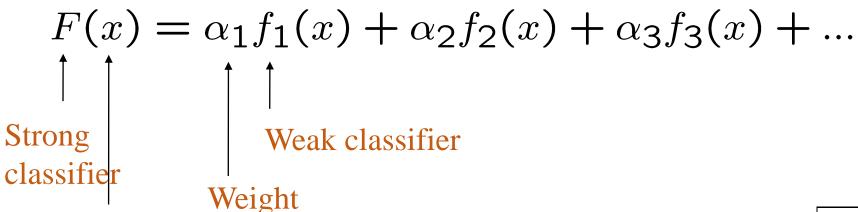
Integral Image

- Let A,B,C,D be the values of the integral image at the corners of a rectangle
- Then the sum of original image values within the rectangle can be computed:

$$sum = A - B - C + D$$

- Only 3 additions are required for any size of rectangle!
 - This is now used in many areas of computer vision





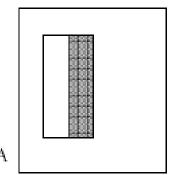


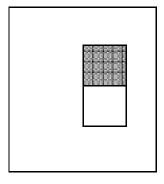
Features vector

Binary Features:
$$f_i(x) = \begin{bmatrix} 1 & \text{if } g_i(x) > \theta_i \\ -1 & \text{otherwise} \end{bmatrix}$$

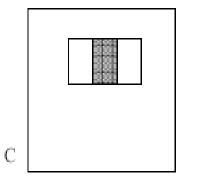
Need to:

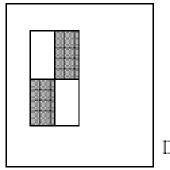
- (1) Select Features i=1..n,
- (2) Learn thresholds θ_i ,
- (3) Learn weights α_i





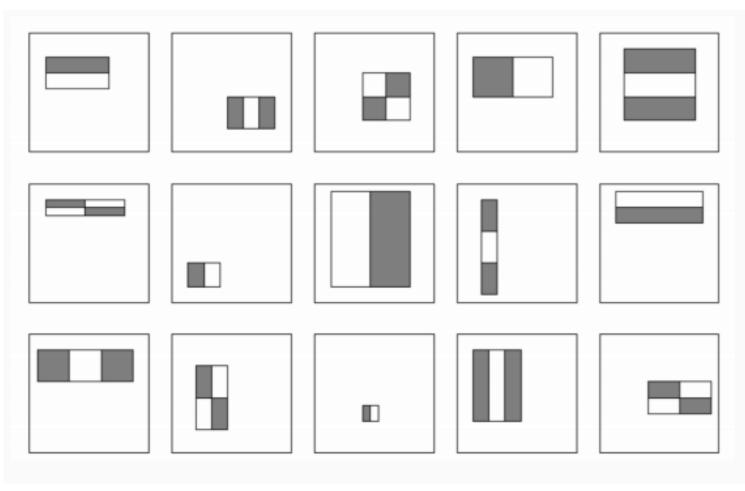
В





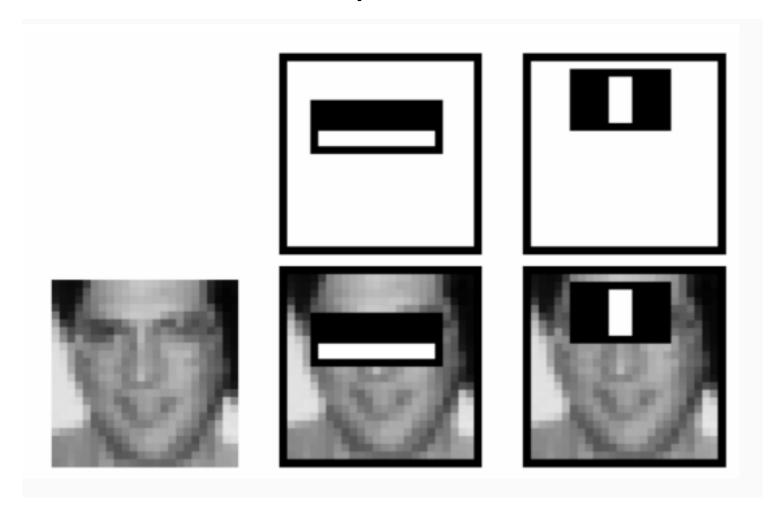
Feature Selection

The number of possible rectangle features with 24x24 region is 180,000!



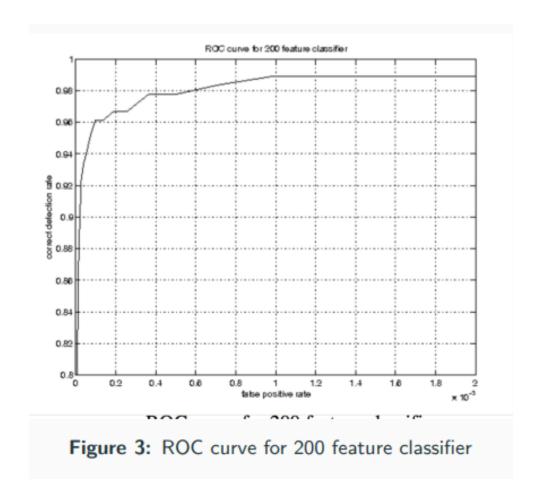
Feature Selection

Top two features selected by AdaBoost



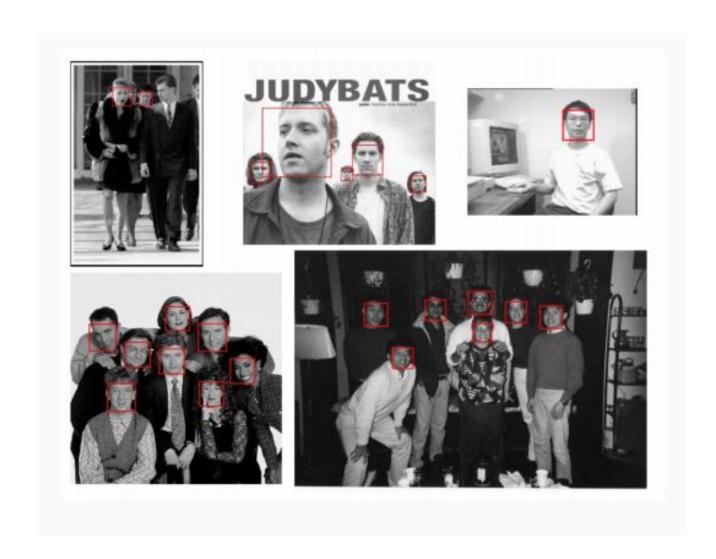
Face Detection Performance

- A classifier with 200 rectangle features was learned using AdaBoost
- detection rate of 95% with a false positive rate of 1 in 14084



Face Detection Performance

• Output of Face Detection on Test Images



Schedule

- Ensemble II (11.12)
- Clustering (11.19)
- Dimensional Reduction (11.26)
- Semi-Supervised Learning (12.3)
- Presentation (12.10)
- Presentation (12.17)
- Graphical Model (12.24)
- Reinforcement Learning (12.31)
- Exam (1.7)

Thanks!