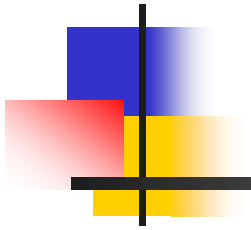


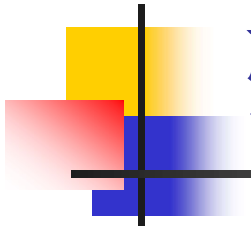
# 计算机科学导论

张家琳

中国科学院计算技术研究所

zhangjialin@ict.ac.cn

2018-4-13



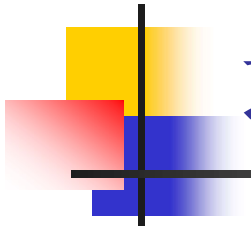
# 张家琳

- 中国科学院计算技术研究所，副研究员
  - 前瞻研究实验室，算法与复杂性课题组，http://theory.ict.ac.cn/cn/

- 研究方向：
  - 算法与复杂性：社会网络、计算博弈论、在线算法、近似算法、组合优化等
  - 量子计算

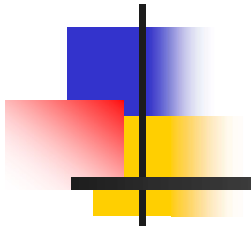
# 逻辑思维

- 逻辑——①思维的规律、规则：这个想法似乎不合逻辑。②研究思维规律的科学，即逻辑学。③客观事物的规律：历史的逻辑。④观点，主张。多用于贬义：霸权主义的逻辑。（新华字典）
- 广义：泛指规律、道理
- 狭义：逻辑学
  - 哲学（古希腊）、数学（19世纪）
  - 计算机科学、语言学、心理学.....



# 逻辑思维主要内容

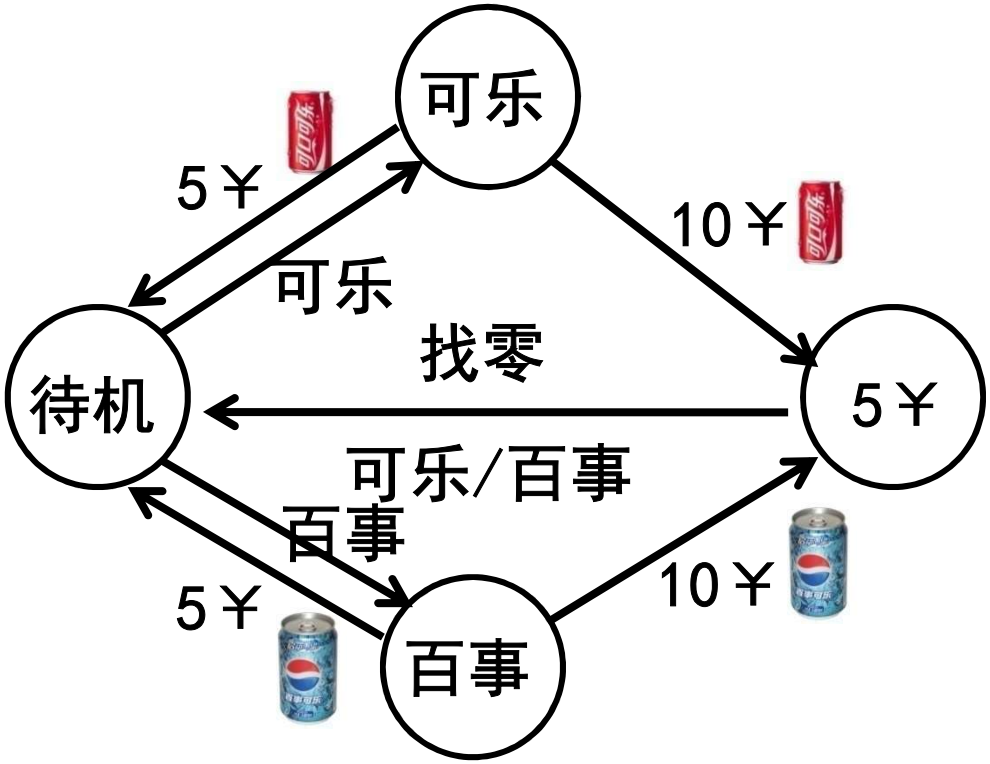
- 逻辑学基础
  - 布尔逻辑、真值表
  - 合取范式、析取范式
  - 谓词逻辑
  - 公理系统
- 图灵机模型
- 后续课程
  - 离散数学、数理逻辑、理论计算机基础.....



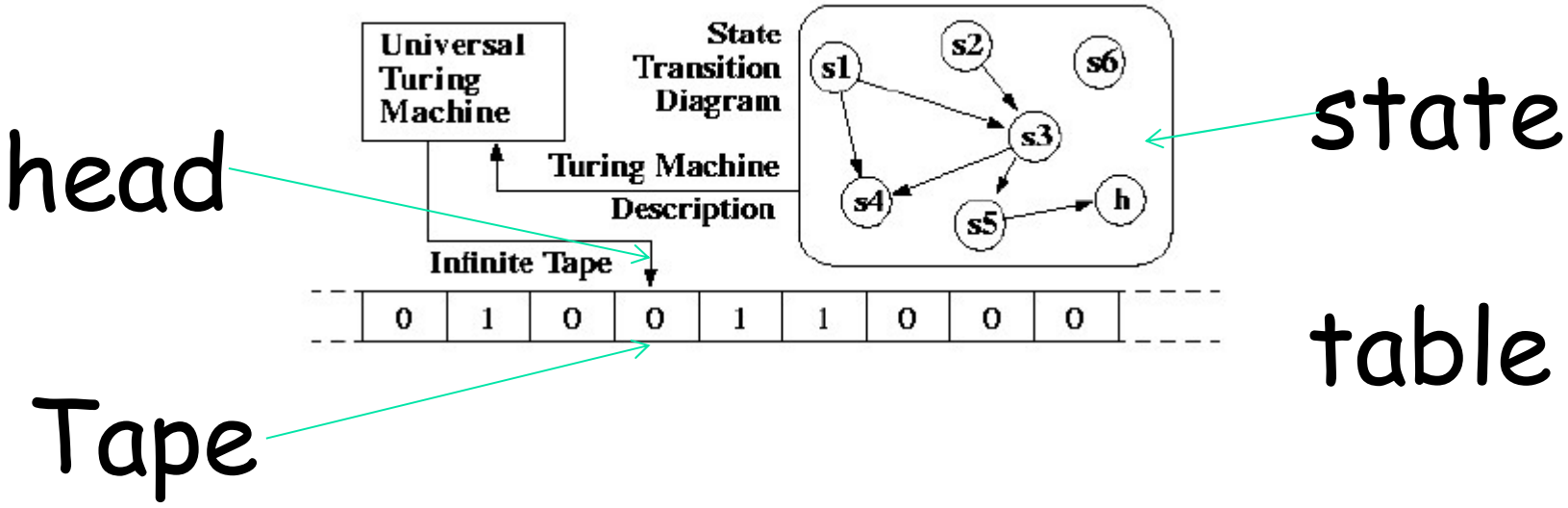
# 图灵机模型

# 自动售卖机

## DFA：确定性有穷自动机



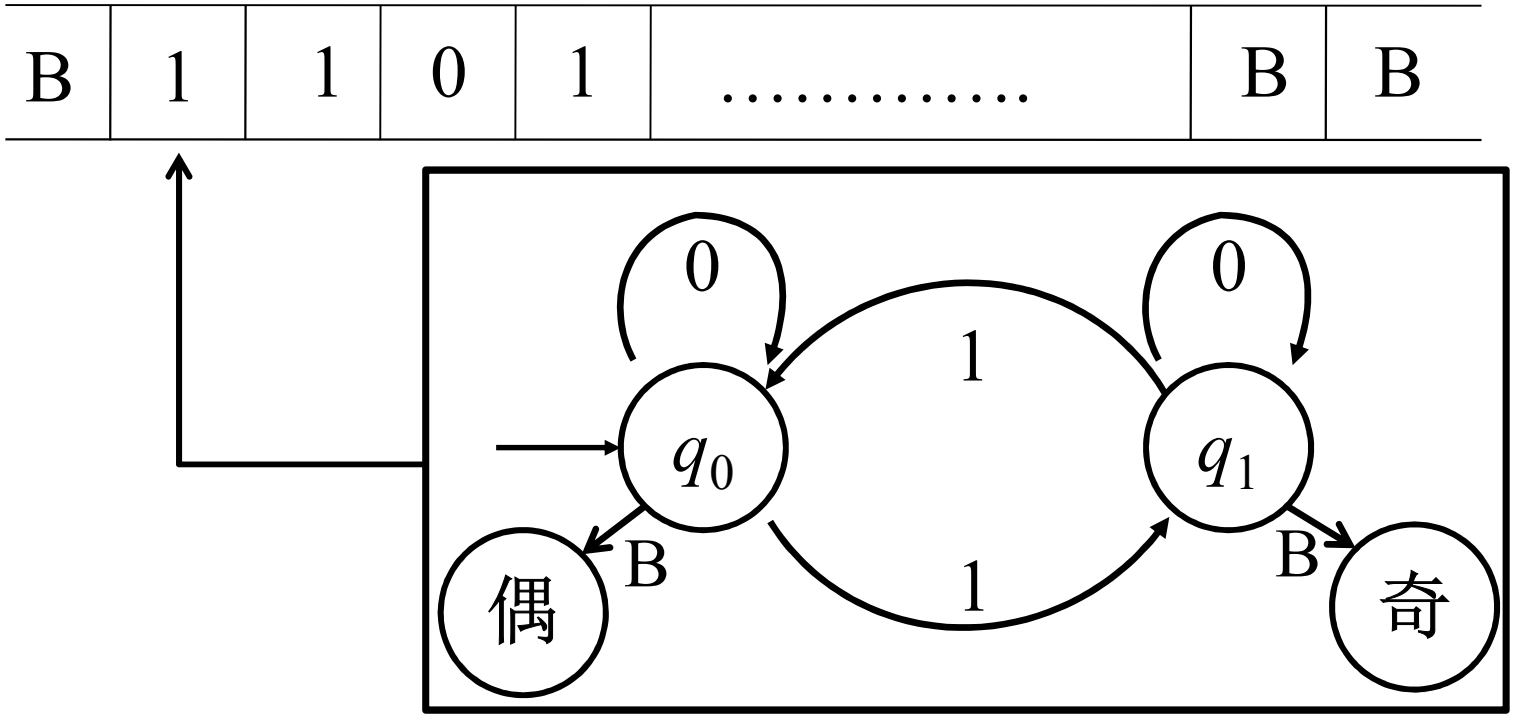
# 图灵机(Turing Machine)




...an unlimited memory capacity obtained in the form of an infinite tape marked out into squares, on each of which a symbol could be printed. At any moment there is one symbol in the machine; it is called the scanned symbol. The machine can alter the scanned symbol and its behavior is in part determined by that symbol, but the symbols on the tape elsewhere do not affect the behavior of the machine. However, the tape can be moved back and forth through the machine, this being one of the elementary operations of the machine. Any symbol on the tape may therefore eventually have an innings. (by Turing 1948)

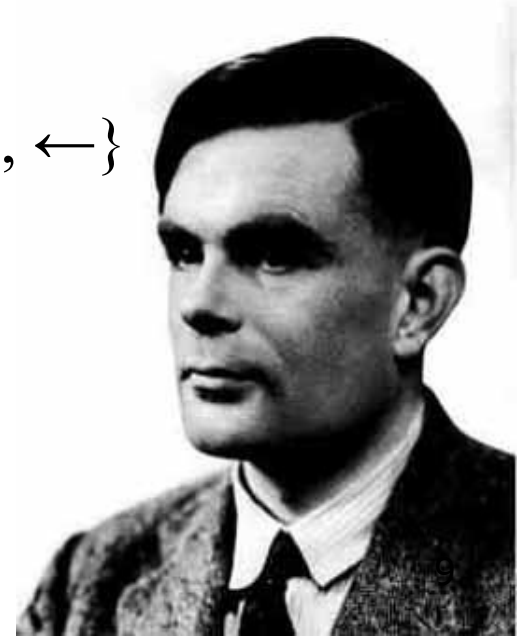
# 例1

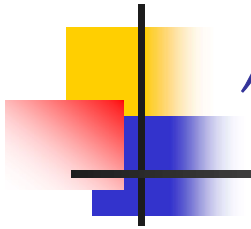
- 输入：11010011…111，判断有奇数还是偶数个1。



# 图灵机

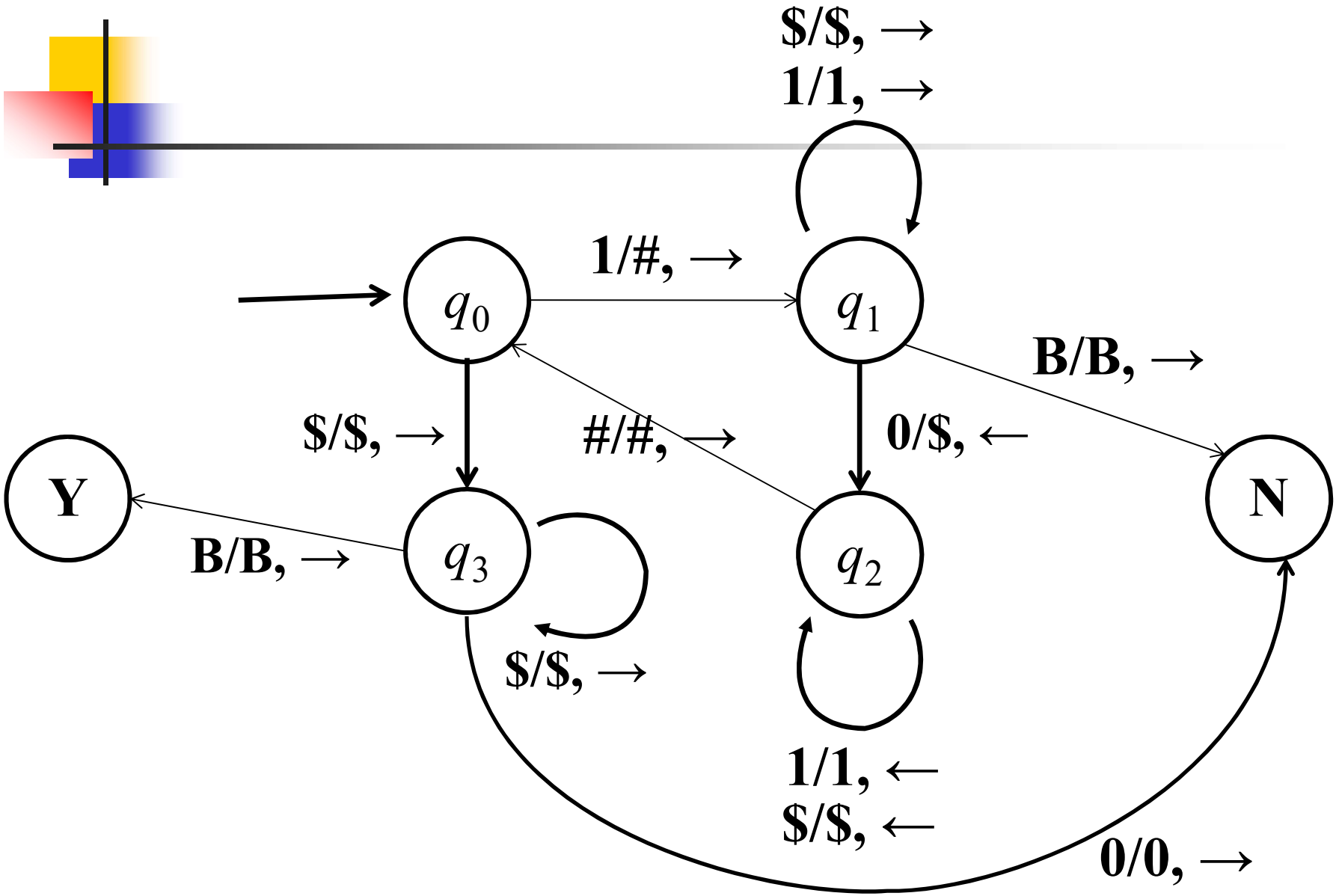
- 图灵机是一个七元组，{Q，Σ，Γ，δ，$q_0$，$q_{accept}$，$q_{reject}$}，其中 Q，Σ，Γ 都是有限集合，
  - 状态集合Q；
  - 输入字母表Σ；
  - 带字母表Γ，其中 B∈Γ；
  - 转移函数: δ：$Q\times\Gamma\rightarrow Q\times\Gamma\times\{\rightarrow,\leftarrow\}$
  - 起始状态 $q_0\in Q$；
  - 接受状态$q_{accept}$；
  - 拒绝状态$q_{reject}$。

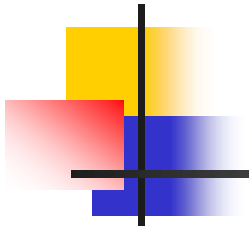




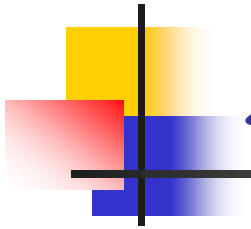
# 例2

- 输入：111…11000…00，判断1和0的个数是否相等？


$/$, →
1/1, →
1/#, →
$q_0$
$q_1$
B/B, →
$/$, →
#/#, →
0/$, ←
Y
N
B/B, →
$q_3$
$q_2$
$/$, →
1/1, ←
$/$, ←
0/0, →

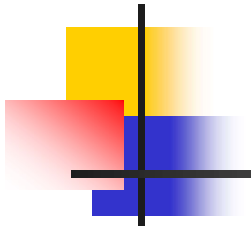


- 转移规则：
- $(q_0,1)\rightarrow(q_1,\#, R)$, $(q_0,\$)\rightarrow(q_3,\$, R)$
- $(q_1,1)\rightarrow(q_1,1, R)$, $(q_1,\$)\rightarrow(q_1,\$, R)$, $(q_1,0)\rightarrow(q_2,\$, L)$, $(q_1,B)\rightarrow (q_{reject},B, R)$
- $(q_2,1)\rightarrow(q_2,1, L)$, $(q_2,\$)\rightarrow(q_2,\$, L)$, $(q_2,\#)\rightarrow(q_0,\#, R)$
- $(q_3,\$)\rightarrow(q_2,\$, R)$, $(q_3,0)\rightarrow(q_{reject},0, R)$, $(q_3,B)\rightarrow(q_{accept},B, R)$



# 思考题

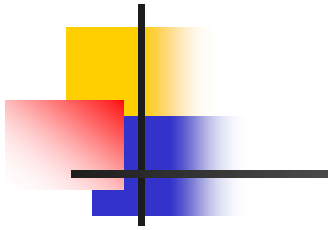
- 用图灵机判断 $1^n0^n$，需要大约$2n^2$次状态转移，是否能够做的更快？



# 图灵机模型

- 这是一个用于计算问题的数学模型
- 核心：状态转移函数
- 注意：状态集合是有限的！

- 图灵机模型的计算能力
  - 哪些问题能用图灵机解决？

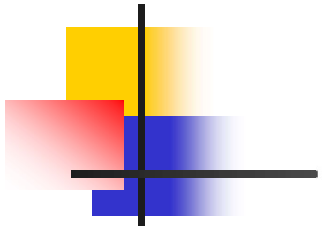


- Alonzo Church & Alan Turing:

  Church-Turing Hypothesis:

*Any reasonable attempt to model mathematically computer algorithms and their performance is bound to end up with a model of computation and associated time cost that is equivalent to Turing machines within a polynomial.*

Alan Turing
1912-1954



# ON COMPUTABLE NUMBERS, WITH AN APPLICATION TO THE ENTSCHEIDUNGSPROBLEM

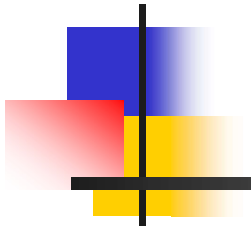
*By* A. M. TURING.

[Received 28 May, 1936.—Read 12 November, 1936.]

The "computable" numbers may be described briefly as the real numbers whose expressions as a decimal are calculable by finite means. Although the subject of this paper is ostensibly the computable *numbers*, it is almost equally easy to define and investigate computable functions of an integral variable or a real or computable variable, computable predicates, and so forth. The fundamental problems involved are, however, the same in each case, and I have chosen the computable numbers for explicit treatment as involving the least cumbrous technique. I hope shortly to give an account of the relations of the computable numbers, functions, and so forth to one another. This will include a development of the theory of functions of a real variable expressed in terms of computable numbers. According to my definition, a number is computable if its decimal can be written down by a machine.

In §§ 9, 10 I give some arguments with the intention of showing that the computable numbers include all numbers which could naturally be regarded as computable. In particular, I show that certain large classes of numbers are computable. They include, for instance, the real parts of all algebraic numbers, the real parts of the zeros of the Bessel functions, the numbers $\pi$, $e$, etc. The computable numbers do not, however, include all definable numbers, and an example is given of a definable number which is not computable.

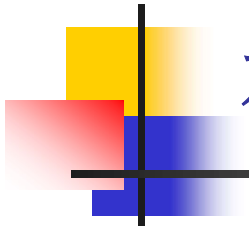
Although the class of computable numbers is so great, and in many ways similar to the class of real numbers, it is nevertheless enumerable. In § 8 I examine certain arguments which would seem to prove the contrary. By the correct application of one of these arguments, conclusions are reached which are superficially similar to those of Gödel†. These results



# 逻辑学基础

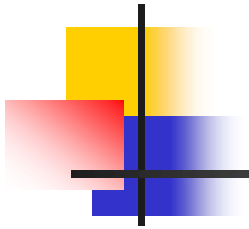
# 引入

- 用“若p则q”的形式，写出“全等三角形一定相似”的四种命题，并判断它们的真假。
  - 原命题：　若是全等三角形，则它们一定是相似三角形
  - 逆命题：　若是相似三角形，则它们一定是全等三角形
  - 否命题：　若不是全等三角形，则它们一定不是相似三角形
  - 逆否命题：若不是相似三角形，则它们一定不是全等三角形
- 原命题和逆否命题为等价命题
- 否命题和逆命题为等价命题
- 为什么？



# 布尔逻辑

- 真 T (true)，假 F (false)
  - 今天下雨。
  - $a^2 \geq 0$。
- 合取，与 $\wedge$ (conjunction, and)
  - $x \wedge y = 1$ (T) *iff* $x = y = 1$ (T)
- 析取，或 $\vee$ (disjunction, or)
  - $x \vee y = 0$ (F) *iff* $x = y = 0$ (F)



- 非 ¬ (negation, not)
  - $\neg x = 1$ (T) *iff* $x = 0$ (F)
- 蕴含 → (material implication)
  - $(x \rightarrow y) = 1$ (T) *iff* $x = 0$ (F) or $y = 1$ (T)
  - 山无陵，江水为竭，冬雷震震，夏雨雪，天地合，乃敢与君绝。
- 异或 ⊕ (exclusive or)
  - $x \oplus y = 1$ (T) *iff* $x \neq y$
  - $x + y \bmod 2$

# 请课后思考

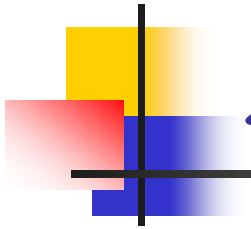
- 原命题： $x \rightarrow y$
- 逆命题： $y \rightarrow x$
- 否命题：？
- 逆否命题：？

# 真值表

| $x$ | $y$ | $x \wedge y$ | $x \vee y$ | $x \rightarrow y$ | $x \oplus y$ |
|---|---|---|---|---|---|
| 0 | 0 | 0 | 0 | 1 | 0 |
| 0 | 1 | 0 | 1 | 1 | 1 |
| 1 | 0 | 0 | 1 | 0 | 1 |
| 1 | 1 | 1 | 1 | 1 | 0 |

# 布尔函数

- 布尔函数$f:\{0,1\}^n \to \{0,1\}$
- 举例
  - $g(x_1, x_2, \cdots, x_n) = (\vee_{i=1}^{n-1} x_i) \oplus x_n$
- 两个布尔函数相同：有相同的真值表
  - 类似：$f(x,y) = x^2 - y^2$和$g(x,y) = (x+y)(x-y)$是相同的多项式

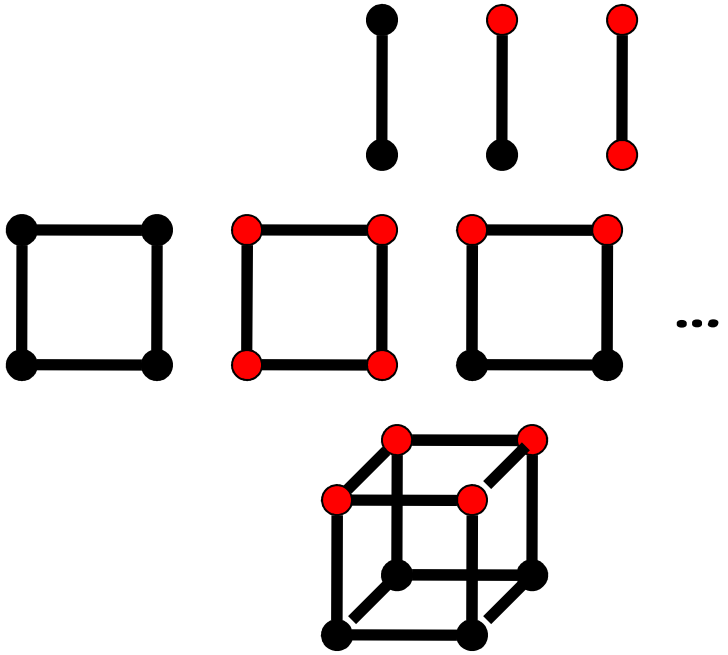


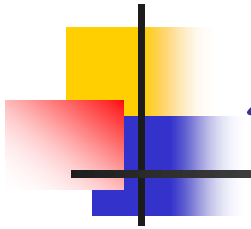
# 思考题

- $n$个变量的布尔函数有多少个？
  - $n$=0, 2个（0和1）
  - $n$=1, 4个（1，0，$x$和$\neg x$）
  - $n$=2, ?
  - $\mathbf{2^{2^n}}$

# 思考题

- $n$个变量的单调布尔函数有多少个？
  - $n$=0, 2个
  - $n$=1, 3个
  - $n$=2, 6个
  - $n$=3, ？作业！





# 总结

- 图灵机模型
- 逻辑学基础
  - 布尔逻辑、真值表
- 思考题
  - 判断串$1^n0^m$中0和1的个数是否相同，图灵机是否能做更快？
  - 单调布尔函数的个数（n=3时）

谢谢！