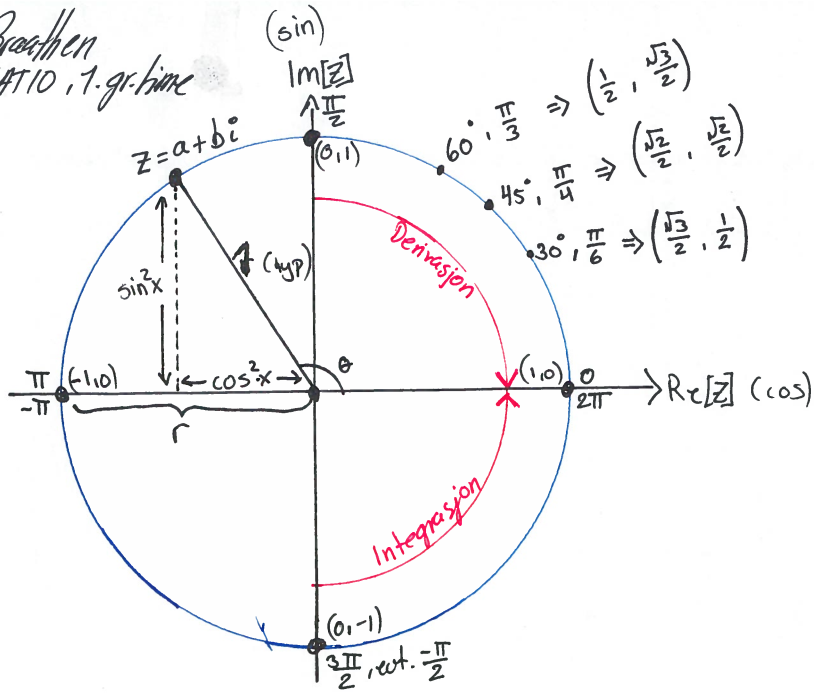
1 Christian Braathen
MAT10, 1.gr.time

I



II Huskeregel: Soh Cah Toa :

$$\sin = \frac{\text{opposite}}{\text{hypotenus}}$$

$$\cos = \frac{\text{adjacent}}{\text{hypotenus}}$$

$$\tan = \frac{\text{opposite}}{\text{adjacent}} = \frac{\frac{\text{opposite}}{\text{hypotenus}}}{\frac{\text{adjacent}}{\text{hypotenus}}} = \frac{\sin}{\cos}$$

III

$$\sin(a \pm b) = \underbrace{\sin a}_{\text{V.S.}} \cdot \underbrace{\cos b}_{\cos} \pm \underbrace{\cos a}_{\text{motsatt av V.S.}} \cdot \underbrace{\sin b}_{\sin}$$

$$\cos(a \pm b) = \cos a \cdot \cos b \mp \sin a \cdot \sin b$$

IV

$$f(x) = a \cdot \begin{Bmatrix} \cos \\ \sin \end{Bmatrix} \cdot \big(k(x-c)\big) + d \quad ,$$

a : amplitude
K : # perioder i $[0; 2\pi] \Rightarrow p = \frac{2\pi}{K}$ (periodelengde)
c : faseforskyvning
d : likevektslinja

V

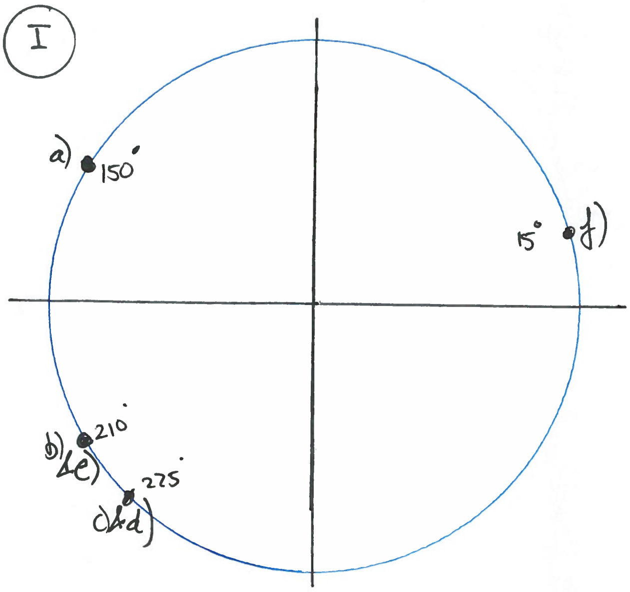
$$x + iy = \underbrace{r(\cos\theta + i \cdot \sin\theta)}_{\text{polarform}} = \underbrace{re^{i\theta}}_{\text{eksponentielform}}$$

VI

$$Z_K = \sqrt[n]{r} \cdot \left(\cos\left[\frac{\theta + 2\pi \cdot k}{n}\right] + i \cdot \sin\left[\frac{\theta + 2\pi \cdot k}{n}\right]\right)$$

Oppgave 6

Tegn tom sirkel for å indikere hvor "x" er.



a) $\sin\left(\pi - \frac{\pi}{6}\right)$ | III

$= \sin(\pi)\cdot\cos\left(\frac{\pi}{6}\right) - \cos(\pi)\cdot\sin\left(\frac{\pi}{6}\right)$ | I

$= 0\cdot\frac{\sqrt{3}}{2} - (-1)\cdot\frac{1}{2}$ | Trekker sammen

$$\underline{\underline{\sin\left(\pi - \frac{\pi}{6}\right) = \frac{1}{2}}}$$

b) $\cos\left(\pi + \frac{\pi}{6}\right)$ | III

$= \cos(\pi)\cdot\cos\left(\frac{\pi}{6}\right) - \sin(\pi)\cdot\sin\left(\frac{\pi}{6}\right)$ | I

$= (-1)\cdot\frac{\sqrt{3}}{2} - 0\cdot\frac{\sqrt{3}}{2}$ | Trekker sammen

$$\underline{\underline{\cos\left(\pi + \frac{\pi}{6}\right) = \frac{-\sqrt{3}}{2}}}$$

c) $\sin\left(\frac{-3\pi}{4}\right)$ | $\sin(x) = \sin(x + 2\pi\cdot n)$ "sin er 2pi-periodisk"

$= \sin\left(\frac{-3\pi}{4} + 2\pi\cdot n\right)$ | $\left(\frac{-3\pi}{4} + 2\pi\cdot n\right) \in [0;2\pi] \Rightarrow n = 1$

$= \sin\left(\frac{-3\pi}{4} + 2\pi\right)$ | Felles nevner

$= \sin\left(\frac{-3\pi + 8\pi}{4}\right)$ | Trekk sammen / Σ

$= \sin\left(\frac{5\pi}{4}\right)$ | $\frac{5}{4}\pi = \pi + \frac{1}{4}\pi$

$= \sin\left(\pi + \frac{1}{4}\pi\right)$ | III

$= \sin(\pi)\cdot\cos\left(\frac{1}{4}\pi\right) + \cos(\pi)\cdot\sin\left(\frac{1}{4}\pi\right)$ | I

$= 0\cdot\frac{\sqrt{2}}{2} + (-1)\cdot\frac{\sqrt{2}}{2}$ | Trekker sammen / Σ

$$\underline{\underline{= \frac{-\sqrt{2}}{2}}}$$

d) $\cos\left(\frac{5}{4}\pi\right)$ | $\frac{5\pi}{4} = \pi + \frac{1}{4}\pi$

$= \cos\left(\pi + \frac{1}{4}\pi\right)$ | III

$= \cos(\pi) \cdot \cos\left(\frac{1}{4}\pi\right) - \sin(\pi) \cdot \sin\left(\frac{1}{4}\pi\right)$ | I

$= (-1) \cdot \frac{\sqrt{2}}{2} - 0 \cdot \frac{\sqrt{2}}{2}$ | Σ

$= -\frac{\sqrt{2}}{2}$

e) $\tan\left(\frac{7\pi}{6}\right)$ | II

$= \frac{\sin\left(\frac{7\pi}{6}\right)}{\cos\left(\frac{7\pi}{6}\right)}$ | $\frac{7\pi}{6} = \pi + \frac{\pi}{6}$. Bruker III

$= \frac{\sin(\pi) \cdot \cos\left(\frac{\pi}{6}\right) + \cos(\pi) \cdot \sin\left(\frac{\pi}{6}\right)}{\cos(\pi) \cdot \cos\left(\frac{\pi}{6}\right) - \sin(\pi) \cdot \sin\left(\frac{\pi}{6}\right)}$ | I

$= \frac{(-1) \cdot \frac{1}{2}}{(-1) \cdot \frac{\sqrt{3}}{2}}$ | Utleder

$= \frac{1}{\sqrt{3}}$, evt $\frac{\sqrt{3}}{3}$ siden det ser penere ut å ha kvadratrot i telleren.

f) 4

$\sin\left(\frac{\pi}{12}\right)$ | "Må finne en kombinasjon av det vi har i (I). $\frac{\pi}{12}$ er $15°$, så vi kan jo si $45°-30°$." Setter inn

$= \sin\left(\frac{\pi}{4} - \frac{\pi}{6}\right)$ | (III)

$= \sin\left(\frac{\pi}{4}\right)\cdot\cos\left(\frac{\pi}{6}\right) - \cos\left(\frac{\pi}{4}\right)\cdot\sin\left(\frac{\pi}{6}\right)$ | (I)

$= \frac{\sqrt{2}}{2}\cdot\frac{\sqrt{3}}{2} - \frac{\sqrt{2}}{2}\cdot\frac{1}{2}$ | Σ ~~og felles nevner.~~

$$= \frac{\sqrt{6}-\sqrt{2}}{4}$$

(B.1.) Oppgave 9

$\sin(x+y)\cdot\sin(x-y)$ | (III)

$= (\overset{i)}{\sin x\cdot\cos y} + \overset{ii)}{\cos x\cdot\sin y})\cdot(\overset{iii)}{\sin x\cdot\cos y} - \overset{iv)}{\cos x\cdot\sin y})$ | Utleder

$= \overset{i)}{(\sin x\cdot\cos y)}\cdot\overset{iii)}{(\sin x\cdot\cos y)} - \overset{i)}{(\sin x\cdot\cos y)}\cdot\overset{iv)}{(\cos x\cdot\sin y)} - \overset{ii}{(\cos x\cdot\sin y)}\cdot\overset{iv)}{(\cos x\cdot\sin y)}$
$+ \overset{iii)}{(\sin x\cdot\cos y)}\cdot\overset{ii)}{(\cos x\cdot\sin y)}$ | Σ og Utleder

$= \overset{i)\cdot iii)}{(\sin^2 x\cdot\cos^2 y)} - \overset{ii)\cdot iv)}{(\cos^2 x\cdot\sin^2 y)}$ | Fra (I) har vi $1 = \sin^2 x + \cos^2 x$ | Flytter
$\cos^2 x = 1 - \sin^2 x$

$= \sin^2 x\cdot(1-\sin^2 y) - (1-\sin^2 x)\cdot\sin^2 y$ | Utleder

$= \sin^2 x - \sin^2 x\cdot\sin^2 y - \sin^2 y$
$-(-\sin^2 x - \sin^2 y)$ | Σ

$$= \sin^2 x - \sin^2 y$$

(B.1) oppgave 10

a) $f(x) = \sin(2x)$

$f(x) = 1 \cdot \sin(2(x-0)) + 0$

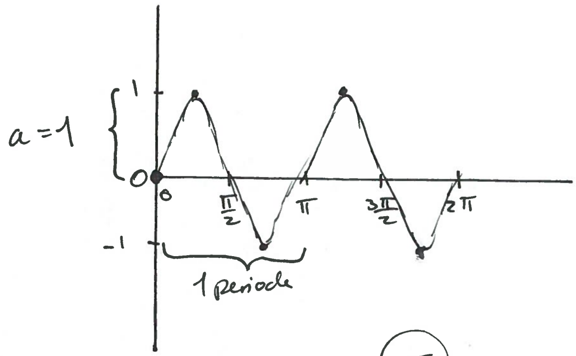
IV

$a = 1$

$K = 2 \Rightarrow p = \frac{2\pi}{2} = \pi$

$c = 0$

$d = 0$



b) $g(x) = 3\sin\left(\frac{x}{2}\right)$

$g(x) = 3\sin\left(\frac{1}{2}(x-0)\right) + 0$

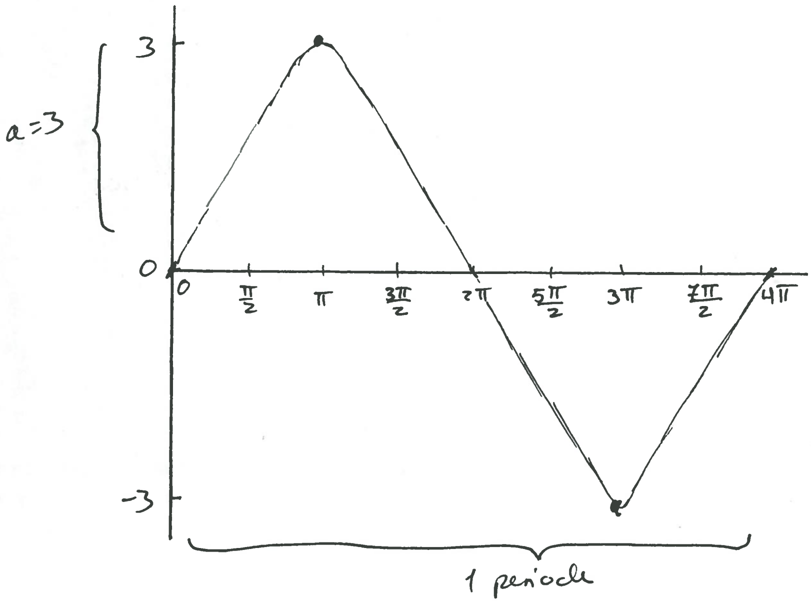
IV

$a = 3$

$K = \frac{1}{2} \Rightarrow p = \frac{2\pi}{\frac{1}{2}} = 4\pi$

$c = 0$

$d = 0$



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c) $h(x) = 2\sin(3x+4)+2$

$h(x) = 2\sin\left(3\left[x - \left(\frac{-4}{3}\right)\right]\right)+2$

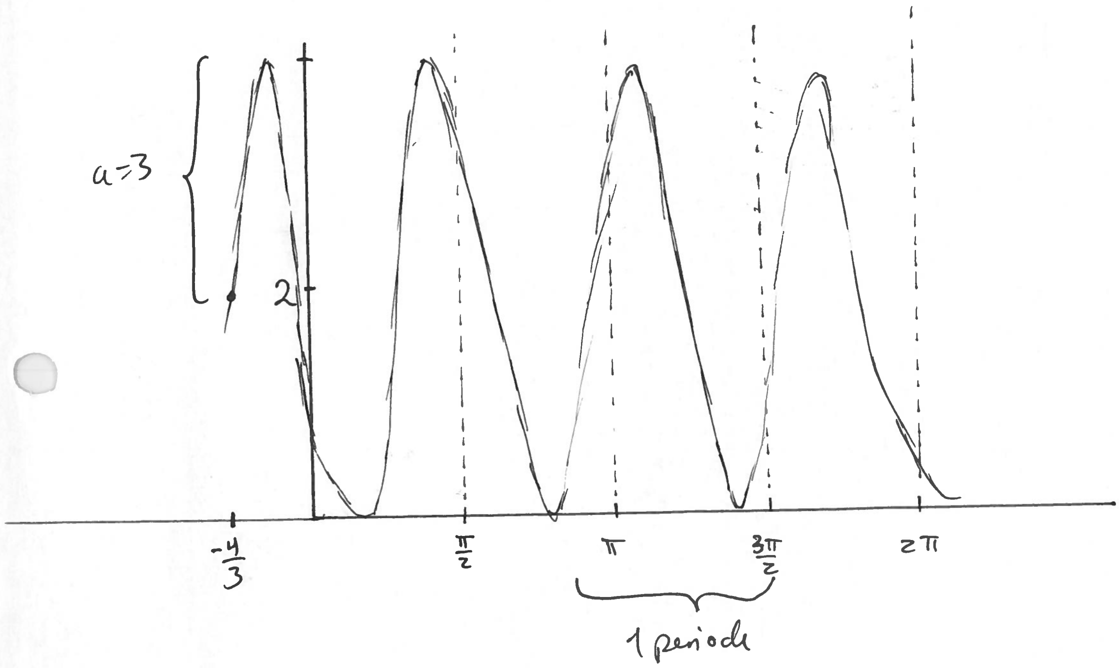
IV

$a=2$

$K=3 \Rightarrow p = \frac{2\pi}{3}$

$c=-4/3$ → Kurven, slik den normalt skjærer i $x=0$, gjør det nå i $\frac{-4}{3}$ istedet.

$d=2$



Oppgave 12

a) $a = 2$

$c = 0$ for sinuskurve

$p = 8\pi \Rightarrow k = \frac{2\pi}{p} = \frac{1}{4}$

$d = 0$

IV gir $f(x) = 2\sin\left(\frac{1}{4}(x-0)\right) + 0$

$$\underline{\underline{f(x) = 2\sin\left(\frac{x}{4}\right)}}$$

b) $a = 1$

$c = 0$ for cosinuskurve

$p = 2\pi \Rightarrow k = \frac{2\pi}{p} = 1$

$d = 2$

IV gir $g(x) = 1\cdot\cos\left(1(x-0)\right) + 2$

$$\underline{\underline{g(x) = \cos(x) + 2}}$$

c) $a = |2e^{-x/\pi}| = 2e^{-x/\pi}$

$c = 0$ for cosinuskurve

$p = 2\pi \Rightarrow k = \frac{2\pi}{p} = 1$

$d = 0$

IV gir $h(x) = 2e^{-x/\pi}\cdot\cos\left(1(x-0)\right) + 0$

$$\underline{\underline{h(x) = 2e^{-x/\pi}\cdot\cos(x)}}$$

B.2 oppgave 1

a) $y = \sin\left(\frac{1}{2}x\right)$ $\Big|$ $u = \frac{1}{2}x \Rightarrow \frac{du}{dx} = \frac{1}{2}$

$\cdot \frac{d}{dx}$

$$\frac{dy}{dx} = \frac{d}{dx}\left[\sin\left(\cancel{\tfrac{1}{2}}u\right)\right] \quad \Big| \quad \frac{d}{dx} = \frac{du}{dx} \cdot \frac{d}{du}$$

$$\frac{dy}{dx} = \frac{du}{dx} \cdot \frac{d}{du}\left[\sin(u)\right] \quad \Big| \quad \text{I} \text{ og setter inn}$$

$$\frac{dy}{dx} = \frac{1}{2} \cdot \cos\left(\frac{1}{2}x\right)$$

b) $y = x\cos x$ $\Big|$ $\cdot \frac{d}{dx}$

$$\frac{dy}{dx} = \frac{d}{dx}\left[x\cos x\right] \quad \Big| \quad \text{Produktregelen}$$

$$\frac{dy}{dx} = \frac{d}{dx}[x] \cdot \cos x + x \cdot \frac{d}{dx}[\cos x] \quad \Big| \quad \text{I} \text{ og utleder}$$

$$\frac{dy}{dx} = \cos x - x \cdot \sin x$$

c) $y = \tan^2$

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c) $y = \tan x^2$

$u = x^2 \Rightarrow \frac{du}{dx} = 2x$

$\cdot \frac{d}{dx}$

$\frac{dy}{dx} = \frac{d}{dx}[\tan u]$

$\frac{d}{dx} = \frac{du}{dx} \cdot \frac{d}{du}$

$\frac{dy}{dx} = \frac{du}{dx} \cdot \frac{d}{du}[\tan u]$

$\frac{d}{du}[\tan u] = \frac{d}{du}\left[\frac{\sin u}{\cos u}\right]$ | Kvotientregelen

$= \frac{\frac{d}{du}[\sin u] \cdot \cos u - \sin u \cdot \frac{d}{du}[\cos u]}{\cos^2 u}$ | Utleder vha I

$= \frac{\cos^2 u + \sin^2 u}{\cos^2 u}$ | $\cos^2 u + \sin^2 u = 1$, ref I

$= \frac{1}{\cos^2 u}$ Setter inn, $u = x^2$

$$\frac{dy}{dx} = 2x \cdot \frac{1}{\cos^2(x^2)}$$

d) $y = e^{2x} \cdot \cos x$ | $\cdot \frac{d}{dx}$

$\frac{dy}{dx} = \frac{d}{dx}\left[e^{2x} \cdot \cos x\right]$ | Produktregelen

$\frac{dy}{dx} = \frac{d}{dx}\left[e^{2x}\right] \cdot \cos x + \cancel{\frac{d}{dx}} e^{2x} \cdot \frac{d}{dx}\left[\cos x\right]$ | Utleder og vha I

$$\frac{dy}{dx} = 2e^{2x}\cos x - e^{2x}\sin x$$

Komplekse #1

$z = 1+2i$
$w = 3+4i$

a) $z + w$ | setter inn

$= (1+2i) + (3+4i)$ | Σ

$\underline{\underline{= 4+6i}}$

b) zw | setter inn

$= (1+2i)(3+4i)$ | Utleder

$= 3 + 4i + 6i + 8i^2$ | $i^2 = -1$ og Σ

$= (3-8) + 10i$ | Σ

$\underline{\underline{= -5 + 10i}}$

c) $\frac{z}{w}$ | setter inn

$= \frac{(1+2i)}{(3+4i)}$ | ~~Utleder~~ $\cdot \frac{(3-4i)}{(3-4i)}$

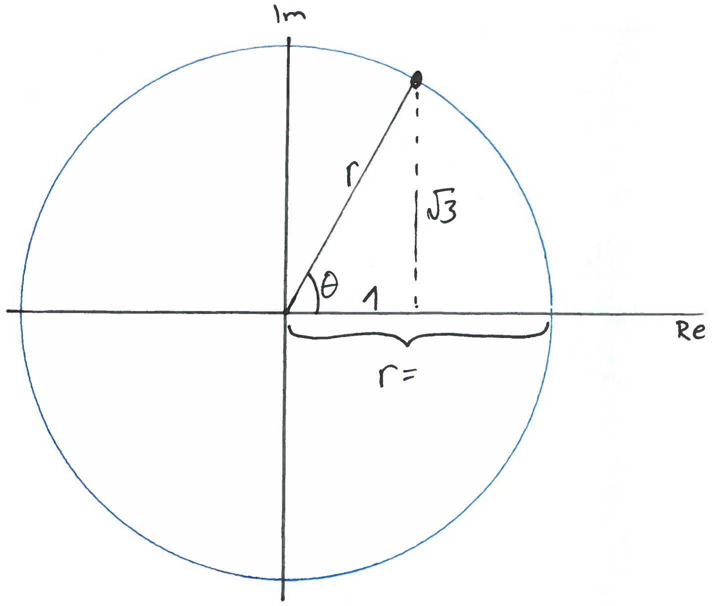
$= \frac{(1+2i)(3-4i)}{(3+4i)(3-4i)}$ | Utleder

$= \frac{3-4i+6i-8i^2}{9-12i+12i-16i^2}$ | $i^2=-1$. Σ

$= \frac{(3+8)+2i}{9+16}$ | Σ

$\underline{\underline{= \frac{11}{25} + \frac{2}{25}i}}$

Komplekse #2



a)

$Z = 1 + \sqrt{3}\,i$

$\cos\theta = \frac{adj}{hyp} \quad | \cdot hyp$

$hyp \cdot \cos\theta = adj \quad \left| \begin{array}{l} hyp = r \\ adj = 1 \end{array} \right.$

$r \cdot \cos\theta = 1$

$Z = r\cos\theta + \sqrt{3}\,i$

$\sin\theta = \frac{opp}{hyp} \quad | \cdot hyp$

$hyp \cdot \sin\theta = opp \quad \left| \begin{array}{l} hyp = r \\ opp = \sqrt{3} \end{array} \right.$

$r\sin\theta = \sqrt{3}$

$Z = r\cos\theta + i \cdot r\cos\theta \quad |$ faktoriserer

$Z = r(\cos\theta + i \cdot \sin\theta)$

$\theta = \arg[Z] \equiv \arctan\left(\frac{\sqrt{3}}{1}\right)$

$\theta = \frac{1}{3}\pi + \pi \cdot k, \quad k \in \mathbb{N}_0 \quad | k = 0$

$\theta = \frac{1}{3}\pi$

$Z = r\left(\cos\left(\frac{1}{3}\pi\right) + i \cdot \sin\left(\frac{1}{3}\pi\right)\right) \quad | \quad r = \sqrt{1^2 + (\sqrt{3})^2} = \sqrt{4} = 2, \; r \in [0, \infty\rangle$

$\underline{Z = 2\left(\cos\left(\frac{1}{3}\pi\right) + i \cdot \sin\left(\frac{1}{3}\pi\right)\right)}$ | "Dette er polarform, men er vanlig å omskrive til eksponentialform også."

$e^{i\theta} = \cos\theta + i \cdot \sin\theta$

$\underline{\underline{Z = 2e^{i\frac{1}{3}\pi}}}$

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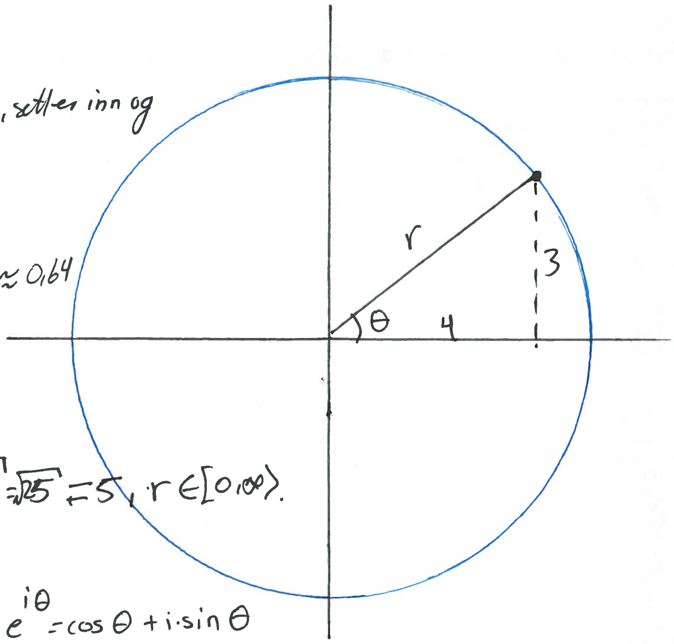
b)

$Z = 4 + 3i$

$\cos\theta = \frac{4}{r} \Leftrightarrow r\cos\theta = 4$

$\sin\theta = \frac{3}{r} \Leftrightarrow r\sin\theta = 3$, setter inn og faktoriserer



$Z = r(\cos\theta + i\cdot\sin\theta)$ | $\theta = \arg[z] = \arctan\left(\frac{3}{4}\right) \approx 0{,}64$

$Z = r\left(\cos\left[\arctan\left(\frac{3}{4}\right)\right] + i\cdot\sin\left[\arctan\left(\frac{3}{4}\right)\right]\right)$ | $r = \sqrt{4^2+3^2} = \sqrt{25} = 5,\ r \in [0, \infty\rangle.$

$\underline{Z = 5\left(\cos\left[\arctan\left(\frac{3}{4}\right)\right] + i\cdot\sin\left[\arctan\left(\frac{3}{4}\right)\right]\right)}$ | Bonus: $e^{i\theta} = \cos\theta + i\cdot\sin\theta$

$\underline{Z = 5e^{i\cdot\arctan\left(\frac{3}{4}\right)}}$

Oppgave 3

$$e^{i(u+v)} = e^{iu} \cdot e^{iv} \qquad \Big| \; e^{i\theta} = \cos\theta + i\cdot\sin\theta$$

$$\cos(u+v) + i\cdot\sin(u+v) = (\cos u + i\cdot\sin u)\cdot(\cos v + i\cdot\sin v) \qquad \Big| \text{ Utleder H.S.}$$

$$\cos(u+v) + i\cdot\sin(u+v) = \cos u\cdot\cos v + i\cdot\cos u\cdot\sin v + i\cos v\cdot\sin u + i^2\cdot\sin u\cdot\sin v \qquad \Big| \; i^2=-1 \text{ og faktoriserer}$$

$$\cos(u+v) + i\cdot\sin(u+v) = (\cos u\cdot\cos v - \sin u\cdot\sin v) + i\,(\cos u\cdot\sin v + \cos v\cdot\sin u) \qquad \Big| \; \begin{array}{l} Re[VS] = Re[HS] \\ Im[VS] = Im[HS] \end{array}$$

Re: $\cos(u+v) = \cos u\cdot\cos v - \sin u\cdot\sin v$

Im: $\sin(u+v) = \cos u\cdot\sin v + \cos v\cdot\sin u$

Oppgave 4

$$\left(\cos\theta + i\cdot\sin\theta\right)^n = \cos(n\theta) + i\cdot\sin(n\theta)$$ | Fra relasjonene ser vi at $n=2$ og $\theta = x$

$$(\cos x + i\cdot\sin x)^2 = \cos(2x) + i\cdot\sin(2x)$$ | Utleder V.S.

$$\cos^2 x + 2\cdot i\cdot\cos x\cdot\sin x + i^2\cdot\sin^2 x = \cos(2x) + i\cdot\sin(2x)$$ | $i^2 = -1$

$$(\cos^2 x - \sin^2 x) + 2i\cos x\sin x = \cos(2x) + i\cdot\sin(2x)$$ | $Re[VS] = Re[HS]$, $Im[VS] = Im[HS]$

Re : $\cos^2 x - \sin^2 x = \cos(2x)$

Im : $2\cos x\sin x = \sin(2x)$

Kompleks #5

Generelt:

$Z = a + bi$ | men dette er egentlig $z^n = a + bi$, her med $n = 1$

$z^n = a + bi$ | V

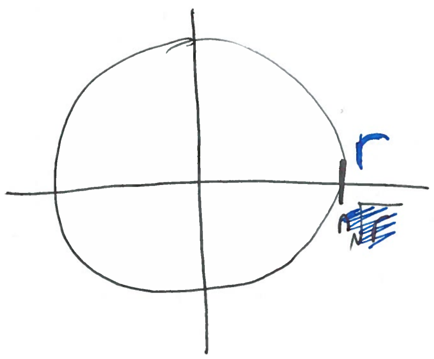
$z^n = r(\cos\theta + i \cdot \sin\theta)$ | men cos og sin er to-pi-periodisk, så
$\cos\theta = \cos(\theta + 2\pi \cdot k)$, $k \in \mathbb{Z}$ ↳ heltall
og tilsvarende for $\sin\theta$.

$Z_k^n = r\left(\cos[\theta + 2\pi \cdot k] + i \cdot \sin[\theta + 2\pi \cdot k]\right)$ | $\sqrt[n]{VS} = \sqrt[n]{HS}$

Setter nå på en indeks her

$Z_k = \sqrt[n]{r} \cdot \left(\cos[\theta + 2\pi k] + i \cdot \sin[\theta + 2\pi \cdot k]\right)^{1/n}$ | Fra oppg. 4
$(\cos\theta + i \cdot \sin\theta)^n = \cos(n\theta) + i \cdot \sin(n\theta)$
Obs at n her har $\frac{1}{n}$, så det blir $\cos\left(\frac{\theta}{n}\right) + i\sin\left(\frac{\theta}{n}\right)$.

$Z_k = \sqrt[n]{r} \cdot \left(\cos\left[\frac{\theta + 2\pi \cdot k}{n}\right] + i \cdot \sin\left[\frac{\theta + 2\pi \cdot k}{n}\right]\right)$ | Finn z for ~~hver rot k, k~~ alle k røtter, $k \in \mathbb{N}_0$.



⇒ fire løsninger:
f.x $z = -1 - \sqrt{3}i \Rightarrow r = \sqrt{1^2 + \sqrt{3}^2} = 2$

$z^3 = 1$

"Skriv alltid både reell og imaginær enhet når du jobber med komplekse tall."

$z^3 = 1 + 0i$

$r = \sqrt{1^2 + 0^2} = 1$
$n = 3$
$\theta = \arg[z] \equiv \arctan\left(\frac{0}{1}\right) = 0$

Bruker VI og setter inn

$$Z_K = \sqrt[3]{1} \cdot \left(\cos\left[\frac{0 + 2\pi \cdot K}{3}\right] + i \cdot \sin\left[\frac{0 + 2\pi K}{3}\right]\right)$$

Renskriver

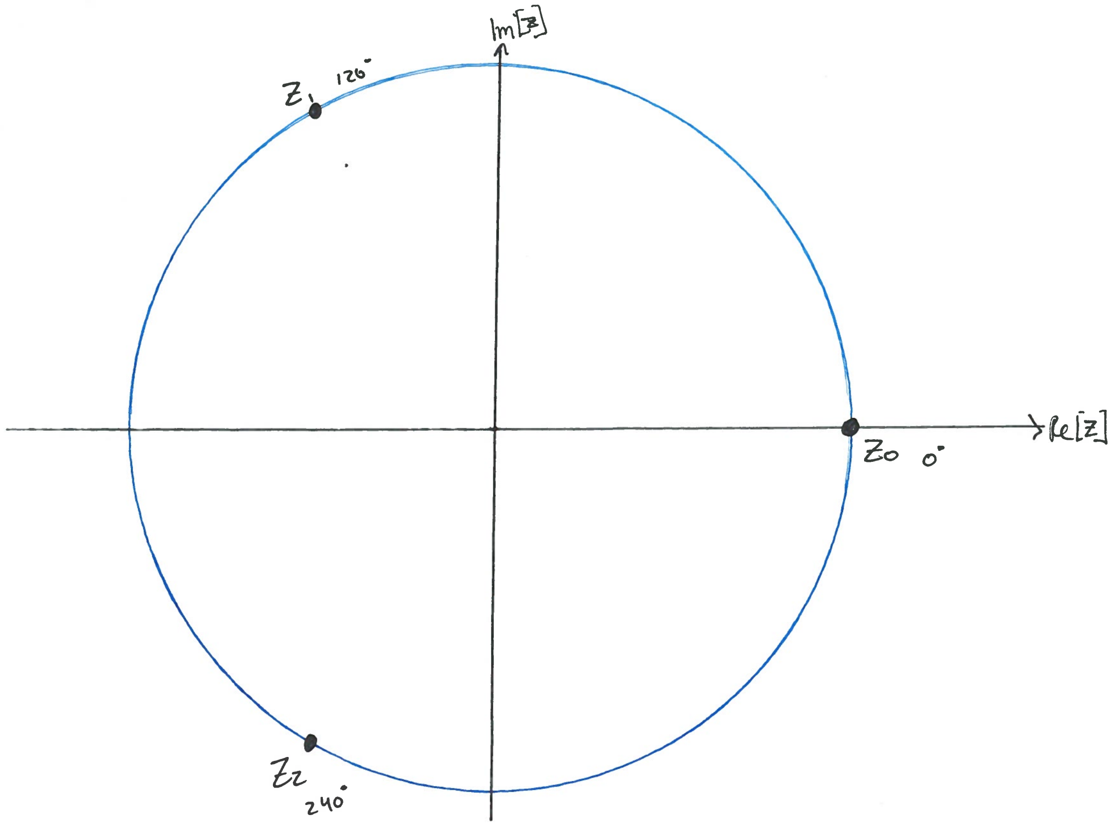
$$Z_K = \cos \frac{2\pi \cdot K}{3} + i \cdot \sin \frac{2\pi \cdot K}{3}$$

Løser for $K = \{0, 1, 2\}$ og finner også kartetisk form via I.

$$Z_0 = \cos(0) + i \cdot \sin(0) \Rightarrow Z_0 = 1 + 0i$$

$$Z_1 = \cos\left(\frac{2\pi}{3}\right) + i \cdot \sin\left(\frac{2\pi}{3}\right) \Rightarrow Z_1 = \frac{-1}{2} + \frac{\sqrt{3}}{2} i$$

$$Z_2 = \cos\left(\frac{4\pi}{3}\right) + i \cdot \sin\left(\frac{4\pi}{3}\right) \Rightarrow Z_2 = \frac{-1}{2} - \frac{\sqrt{3}}{2} i$$



b)

$z^3 = 0 + 8i$

$r = \sqrt{0^2 + 8^2} = 8$

$n = 3$

$\theta = \arg[z] \equiv \arctan\left(\frac{8}{0}\right)$ er ikke definert

derfor det er ekvivalent til. Dette fangener ikke når den reelle enheten er 0. siden den imaginære er positiv, er $\theta = \frac{\pi}{2}$

Bruker (VI) og setter inn

$$z_k = \sqrt[3]{8}\left(\cos\left[\frac{\frac{\pi}{2} + 2\pi \cdot k}{3}\right] + i \cdot \sin\left[\frac{\frac{\pi}{2} + 2\pi \cdot k}{3}\right]\right)$$

Renskriver og Løser for $k = \{0, 1, 2\}$ og finner også kartesisk form vha (I).

$$z_0 = 2 \cdot \left(\cos\frac{\pi}{6} + i \cdot \sin\frac{\pi}{6}\right) \Rightarrow z_0 = 2 \cdot \left(\frac{\sqrt{3}}{2} + i \cdot \frac{1}{2}\right) = \underline{\sqrt{3} + i}$$

$$z_1 = 2 \cdot \left(\cos\left[\frac{\frac{\pi}{2} + 2\pi}{3}\right] + i\sin\left[\frac{\frac{\pi}{2} + 2\pi}{3}\right]\right) =$$

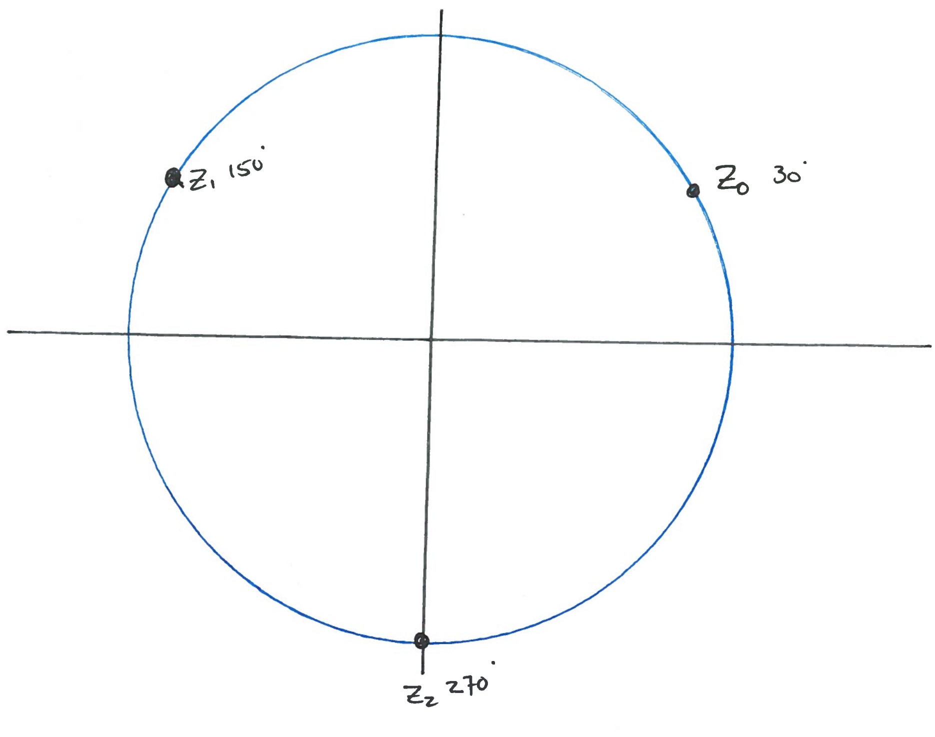
Renskriver

$$z_1 = 2 \cdot \left(\cos\frac{5}{6}\pi + i \cdot \sin\frac{5}{6}\pi\right) \Rightarrow z_1 = 2 \cdot \left(-\frac{\sqrt{3}}{2} + i \cdot \frac{1}{2}\right) = \underline{-\sqrt{3} + i}$$

$$z_2 = 2 \cdot \left(\cos\left[\frac{\frac{\pi}{2} + 4\pi}{3}\right] + i \cdot \sin\left[\frac{\frac{\pi}{2} + 4\pi}{3}\right]\right)$$

Renskriver

$$z_2 = 2 \cdot \left(\cos\frac{3\pi}{2} + i \cdot \sin\frac{3\pi}{2}\right) \Rightarrow z_2 = 2 \cdot (0 + i(-1)) = \underline{-2i}$$



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c)

$$Z^4 + 2Z^2 + 2 = 0 \qquad \Big| \quad y = z^2$$

$$y^2 + 2y + 2 = 0$$

| Med annengradsformelen får vi $y = \begin{cases} -1+i & \Rightarrow r = \sqrt{(-1)^2 + 1^2} = \sqrt{2} \\ -1-i & \overset{evt}{\Rightarrow} r = \sqrt{(-1)^2 + (-1)^2} = \sqrt{2} \end{cases}$ $\theta = \frac{-1}{4}\pi + 1\pi = \frac{3}{4}\pi$, $\theta = \frac{1}{4}\pi + \pi = \frac{5}{4}\pi$
$n = 2$.

riktig å gange opp så man kommer i riktig kvadrant.

i)

$$-1 + i = \sqrt{2}\left(\cos\left[\tfrac{3}{4}\pi + 2\pi k\right] + i \cdot \sin\left[\tfrac{3}{4}\pi + 2\pi k\right]\right) \qquad \Big| \quad -1+i = y_1 = z^2, \quad \sqrt{VS} = \sqrt{HS}$$

$$Z_k = \sqrt[4]{2}\left(\cos\left[\frac{\frac{3}{4}\pi + 2\pi k}{2}\right] + i \cdot \sin\left[\frac{\frac{3}{4}\pi + 2\pi k}{2}\right]\right) \qquad \Big| \quad \text{løser for } k = \{0, 1\}$$

$$Z_0 = \sqrt[4]{2}\left(\cos \tfrac{3}{8}\pi + i \cdot \sin \tfrac{3}{8}\pi\right)$$

$$\underline{\underline{Z_1 = \sqrt[4]{2}\left(\cos \tfrac{11}{8}\pi + i \cdot \sin \tfrac{11}{8}\pi\right)}}$$

ii)

$$-1 - i = \sqrt{2}\left(\cos\left[\tfrac{5}{4}\pi + 2\pi k\right] + i \cdot \sin\left[\tfrac{5}{4}\pi + 2\pi k\right]\right) \qquad \Big| \quad -1-i = y_2 = z^2, \quad \sqrt{VS} = \sqrt{HS}$$

$$Z_k = \sqrt[4]{2}\left(\cos\left[\frac{\frac{5}{4}\pi + 2\pi k}{2}\right] + i \cdot \sin\left[\frac{\frac{5}{4}\pi + 2\pi k}{2}\right]\right) \qquad \Big| \quad \text{løser for } k = \{0, 1\}$$

$$Z_0 = \sqrt[4]{2}\left(\cos \tfrac{5}{8}\pi + i \cdot \sin \tfrac{5}{8}\pi\right)$$

$$\underline{\underline{Z_1 = \sqrt[4]{2}\left(\cos \tfrac{13}{8}\pi + i \cdot \sin \tfrac{13}{8}\pi\right)}}$$