

Investment

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Chapter One: The Investment Environment

I. Real assets versus financial assets

1. The material wealth of a society is determined ultimately by the productive capacity of its economy, which is a function of the **real assets** of the economy: the land, buildings, knowledge, and machines that are used to produce goods and the workers whose skills are necessary to use those resources.
2. **Financial assets**, like stocks or bonds, contribute to the productive capacity of the economy indirectly, because they allow for separation of the ownership and management of the firm and facilitate the transfer of funds to enterprise with attractive investment opportunities. Financial assets are claims to the income generated by real assets.
3. **Real vs. Financial assets:**
 - a. Real assets produce goods and services, whereas financial assets define the allocation of income or wealth among investors.
 - b. They are distinguished operationally by the balance sheets of individuals and firms in the economy. Whereas **real assets appear only on the asset side** of the balance sheet, **financial assets always appear on both sides of the balance sheet**. Your financial claim on a firm is an asset, but the firm's issuance of that claim is the firm's liability. When we aggregate over all balance sheets, financial assets will cancel out, leaving only the sum of real assets as the net wealth of the aggregate economy.
 - c. Financial assets are created and destroyed in the ordinary course of doing business. E.g. when a loan is paid off, both the creditor's claim and the debtor's obligation cease to exist. In contrast, real assets are destroyed only by accident or by wearing out over time.

II. Financial markets and the economy

1. **Smoothing consumption**: "Store" (e.g. by stocks or bonds) your wealth in financial assets in high earnings periods, sell these assets to provide funds for your consumption in low earnings periods (say, after retirement).
2. **Allocation of risk**: virtually all real assets involve some risk (so do financial assets). If a person is uncertain about the future of GM, he can choose to buy GM's stock if he is more risk-tolerant, or he can buy GM's bonds, if he is more conservative.
3. **Separation of ownership and management**: Let professional managers manage the firm. Owners can easily sell the stocks of the firm if they don't like the incumbent management team or "police" the managers through board of directors ("stick") or use compensation plans tie the income of managers to the success of the firm ("carrot"). In some cases, other firms may acquire the firm if they observe the firm is underperforming (market discipline).

III. Clients of the financial system

1. **Household sector:**
 - a. **Tax concerns**: people in different tax brackets need different financial assets with different tax characteristics.

- b. **Risk concerns:** Differences in risk tolerance create demand for assets with a variety of risk-return combination.
2. **Business sector:** business is more concerned about how to finance their investments, through debt or equity either privately or publicly.

Business issuing securities to the public have several objectives. **First**, they want to get the best price possible for their securities. **Second**, they want to market the issues to the public at the lowest possible cost. This has two implications. First, business may hire “specialists” to market their securities. Second, most business will prefer to issue fairly simple securities that require the least extensive incremental analysis and, correspondingly, are the least expensive to arrange.

Such a **demand for simplicity or uniformity by business-sector issuers is likely to be at odds with the household sector’s demand for a wide variety of risk-specific securities.** This mismatch of objectives gives rise to an industry of middlemen who act as intermediaries between the two sectors, specializing in transforming simple securities into complex issues that suit particular market niches.

3. **Government sector:** Governments cannot sell equity shares. They are restricted to borrowing to raise funds when tax revenues are not sufficient to cover expenditures. A special role of the government is in regulating the financial environment.

IV. **The environment responds to clientele demands:** The smallness of households creates a market niche for financial intermediaries, mutual funds, and investment companies. Economies of scale and specialization are factors supporting the investment banking industry.

V. **Markets and market structure**

1. **Direct search market:** buyers and sellers must seek each other out directly.
2. **Brokered market:** e.g. real estate market, primary market and block transactions.
3. **Dealer markets:** dealers trade assets for their own accounts. Their profit margin is the “bid-asked” spread.
4. **Auction market:** all transactors in a good converge at one place to bid on or offer a good. If all participants converge, they can arrive at mutually agreeable prices and thus save the bid-asked spread.

VI. **Ongoing trends**

1. **Globalization:**
U.S. investors can participate in foreign investment opportunities in several ways:
 - a. purchase foreign securities using American Depositary Receipts (ADRs), which are domestically traded securities that represent claims to shares of foreign stocks.
 - b. purchase foreign securities that are offered in dollars.
 - c. Buy mutual funds that invest internationally.
 - d. buy derivative securities with payoffs that depend on prices in foreign security markets.

2. **Securitization**: the biggest asset-backed securities are for credit card debt, car loans, home equity loans, student loans and debt of firms. Pools of loans typically are aggregated into pass-through securities. The transformation of these pools into standardized securities enables issuers to deal in a volume large enough that they can bypass intermediaries.
3. **Financial engineering**: the process of bundling and unbundling of an asset.

Chapter Two: Financial Instruments

- I. **Financial markets** are segmented into **money markets and capital markets**.
 1. **Money market instruments** (they are called cash equivalents, or just cash for short) include short-term, marketable, liquid, low-risk debt securities.
 2. **Capital markets** include longer-term and riskier securities. We subdivide the capital market into **four segments: longer-term bond markets, equity markets, and the derivative markets for options and futures.**
- II. **Money Market:**
 1. **T-bills**: Investors buy the bills at a discount from the stated maturity value and get the face value at the bill's maturity. T-bills with initial maturities of 91 days or 182 days are issued weekly. Offerings of 52-week bills are made monthly. Sales of bills are conducted via auction, at which investors can submit competitive or noncompetitive bids. T-bills sell in minimum denominations of only \$10,000. The income earned on T-bills is **tax-free**.
 2. **CD**: certificates of deposit is a time deposit with a bank. CDs issued in denominations greater than \$100,000 are usually negotiable. Short-term CDs are highly marketable.
 3. **CP**: commercial paper, large companies often issue their own short-term **unsecured** debt notes rather than borrow directly from banks. Very often, CP is backed by a bank line of credit, which gives the borrower access to cash that can be used to pay off the paper at maturity. CP maturities range up to 270 days. Usually, it is issued in multiples of \$100,000. So, small investors can invest in CP only indirectly, via money market mutual funds.
 4. **Bankers' acceptances**: starts as an order to a bank by a bank's customer to pay a sum of money at a future date, typically within six months. At this stage, it is similar to a postdated check. When the bank endorses the order for payment as "accepted", it assumes responsibility for ultimate payment to the holder of the acceptance. They are considered very safe assets because **traders can substitute the bank's credit standing for their own**.

5. **Eurodollars:** are dollar-denominated deposits at foreign banks or foreign branches of American banks. These banks **escape regulation by FED**.
6. **Repos:** Dealers in government securities use Repos as a form of short-term, usually overnight, borrowing. The dealer thus takes out a one-day loan from the investor, and the securities serve as collateral.
7. **Federal funds:** Banks maintain deposits of their own at FED. Funds in the bank's reserve account are called federal funds
8. **LIBOR: London Interbank Offered Rate** is the rate at which large banks in London are willing to lend money among themselves. This rate, which is quoted on dollar-denominated loans, has become the **premier short-term rate** quoted in the European money market, and it serves as a reference rate for a wide range of transactions.

III. **Bond market (fixed income capital market):** it is composed of **longer-term** borrowing instruments than those that trade in the money market, including **Treasury notes and bonds**, corporate bonds, municipal bonds, mortgage securities, and federal agency debt.

1. **Treasury notes and bonds:** T-note maturities range up to 10 years, whereas bonds are issued with maturities ranging from 10 to 30 years. Both are issued in denominations of \$1000 or more. Both make semiannual interest payments called coupon payments. The only major distinction between T-notes and T-bonds is that **T-bonds may be callable during a given period**, usually the last five years of the bond's life.
2. **International bonds:**
 1. **Eurobond** is a bond denominated in a currency other than that of the country in which it is issued. Eg, a dollar-denominated bond sold in Britain would be called a Eurodollar bond.
 2. **Foreign bonds:** bonds issued and denominated in the currency of a country other than the one in which the issuer is primarily located. A Yankee bond is a dollar-denominated bond sold in the US by a non-US issuer. Samurai bonds are yen denominated bonds sold within Japan.
3. **Municipal bonds** are issued by state and local governments. They are similar to Treasury and corporate bonds except that their interest income is exempt from federal income taxation.
 1. two types of municipal bonds: general obligation bonds, which are backed by the "full faith and credit" of the issuer, and revenue bonds, which are issued to finance particular projects and are backed either by the revenues from that project or by the particular municipal agency operating the project..
 2. the key feature of municipal bonds is "**Tax-exempt status**". Because investors pay neither federal nor state taxes on the interest proceeds, they are willing to accept lower yields on

these securities. These lower yields represent a huge savings to state and local governments.

3. **Equivalent taxable yield:** the rate that a taxable bond must offer to match the after-tax yield on the tax-free municipal.

$$R(1-t) = r_m \rightarrow \text{municipal bond yield.}$$

$$R = r_m / (1 - t)$$

4. **Corporate bonds:** they often come with options attached.
 1. Callable bonds give the firm the option to repurchase the bond from the holder at a stipulated call price.
 2. Convertible bonds give the bondholder the option to convert each bond into a stipulated number of shares of stock.
5. **Mortgages and Mortgage-backed securities** are either an ownership claim in a pool of mortgages or an obligation that is secured by such a pool. Mortgage lenders originate loans and then sell packages of these loans in the secondary market. Specifically, they sell their claim to the cash inflows from the mortgages as those loans are paid off. The mortgage originator continues to serve the loan, collecting principal and interest payments, and passes these payments along to the purchaser of the mortgage. These securities are called **pass-throughs**. Although pass-through securities often guarantee payment of interest and principal, they do not guarantee the rate of return. Intestors can be hurt in years when interest rates drop significantly. This is because homeowners usually have an option to prepay, or pay ahead of schedule, the remaining principal outstanding on their mortgages.

IV. **Equity securities:**

1. **Common stock:** also known as equity securities or equities, represent ownership shares in a corporation. Each share of common stock entitles its owner to one vote on any matters of corporate governance that are put to a vote at the corporation's annual meeting and to a share in the financial benefits of ownership.
2. **Characteristics of Common stock:**
 1. **Residual claim:** stockholders are the last in line of all those who have a claim on the assets and income of the corporation. In a liquidation of the firm's assets the shareholders have a claim to what is left after all other claimants such as the tax authorities, employees, suppliers, bondholders, and other creditors have been paid. For a firm not in liquidation, shareholders have claim to the part of operating income left over after interest and taxes have been paid. Management can either pay this residual as cash dividends to shareholders or reinvest it in the business to increase the value of the shares.
 2. **Limited liability:** the most shareholders can lose in the event of failure of the corporation is their original investment.
3. **Preferred stock:** has features similar to both equity and debt.

- a.** Like a bond, it promises to pay to its holder a fixed amount of income each year and it does not convey voting power regarding the management of the firm.
- b.** It is an equity investment, however. The firm retains discretion to make the dividend payments to the preferred stockholders; it has no contractual obligation to make those dividends. Instead, preferred dividends are usually cumulative; that is, unpaid dividends cumulative and must be paid in full before any dividends may be paid to holders of common stock. In contrast, the firm does have a **contractual obligation** to make the interest payments on the debt. Failure to make these payments sets off corporate bankruptcy proceedings.
- c.** Unlike interest payment of bonds, dividends on preferred stock are not considered tax-deductible expenses to the firm. This reduces their attractiveness as a source of capital to issuing firms. However, there is an offsetting tax advantage to preferred stock. **When one corporation buys the preferred stock of another corporation, it pays taxes on only 30% of the dividends received.** Given this tax rule, it is not surprising that most preferred stock is held by corporations.
- d.** Preferred stock rarely gives its holders full voting privileges in the firm. However, if the preferred dividend is skipped, the preferred stockholders will then be provided some voting power.

V. Stock and bond market indexes:

1. Dow Jones Averages: **Price-weighted average**, measures the return (excluding dividends) on a portfolio that holds one share of each stock. It gives higher-priced shares more weight in determining performance of the index. Divisor, d now is .146, instead of 30, because of splits or dividends. The averaging procedure is adjusted whenever a stock splits or pays a stock dividend of more than 10%, or there is a company changes.
2. S&P 500: **market-value-weighted index**. The split irrelevant to the performance of the index.
3. **Equally weighted indexes**: an averaging technique, by placing equal weight on each return, corresponds to an implicit portfolio strategy that **places equal dollar values on each stock**. This is in contrast to both price weighting (which requires equal numbers of shares of each stock) and market value weighting (which requires investments in proportion to outstanding value). Unlike price- or market-value-weighted indexes, equally weighted indexes do not correspond to buy-and-hold portfolio strategies.

VI. Derivative markets:

1. Derivatives are instruments that provide payoffs that depend on the values of other assets such as commodity prices, bond and stock prices, or market index values.
2. **Options:** a call (put) option gives its holder the right to purchase (sell) an asset for a specified price, called the **exercise or strike price**, on or before a specified expiration date. The prices of call (put) options decrease (increase) as the exercise price increases. The purchase price of option is called the premium.
3. **Futures contracts:** call for delivery of an asset at a specified delivery or maturity date for an agreed-upon price, called the futures price, to be paid at contract maturity. The long position is held by the trader who commits to purchasing the asset on the delivery date. The trader who takes the short position commits to delivering the asset at contract maturity.

Chapter Three: How Securities Are Traded

I. **Where securities are traded**

Purchase and sale of already-issued securities take place in the secondary markets, which consist of (1) national and local securities exchanges, (2) the over-the-counter market, and (3) direct trading between two parties.

1. Exchanges:

- a. There are several stock exchanges in USA. Two of these, the NYSE and the American Stock Exchange (Amex), are national in scope. The others, such as the Boston and Pacific exchanges, are regional exchanges, which primarily list firms located in a particular geographic area.
- b. An exchange provides a facility for its members to trade securities, and **only members of the exchange can trade there**.
- c. The majority of memberships, or seats, are commission broker seats, most of which are owned by the large full-service brokerage firms. The seat entitles the firm to place one of its brokers on the floor of the exchange where he or she can execute trades. The exchange member charges investors for executing trades on their behalf.
- d. NYSE is by far the largest single exchange. It accounts for about 85-90% of the trading of that takes place on U.S. **stock exchanges** (not include OTC).

While most common stocks are traded on the exchanges, most bonds and other fixed-income securities are not. Corporate bonds are traded both on the exchanges and over the counter, but most federal and municipal government bonds are traded over the counter.

2. Over-the-counter market:

- a. Roughly 35,000 issues are traded on the over-the-counter (OTC) market and any security may be traded there.
- b. But the **OTC market is not a formal exchange**. There are **no membership requirements** for trading, nor are there listing requirements for securities (although

there are requirements to be listed on Nasdaq, the computer-linked network for trading of OTC securities).

- c. In the OTC market, thousands of **brokers** register with the SEC as **dealers** in OTC securities. Security dealers quote prices at which they are willing to buy or sell securities. A broker can execute a trade by contacting a dealer listing an attractive quote.
- d. Before 1971, all OTC quotation of stock were recorded manually and published daily. In 1971 the National Association of Securities Dealers Automated Quotation System, or Nasdaq, began to offer immediate info on a computer-linked system of bid and asked prices for stocks offered by various dealers. A broker who receives a buy or sell order from an investor can examine all current quotes, contact the dealer with the best quote, and execute the trade.
- e. This system, now called the Nasdaq stock market, is divided into two sectors, the Nasdaq National Market System (comprising about 3000 companies) and the Nasdaq SmallCap Market (comprised of about 850 smaller companies). The National Market securities must meet more stringent listing requirements and trade in a more liquid market.

II. Some useful definitions (<http://www.investopedia.com>):

1. **Stockbroker**

- a. An agent that charges a fee or commission for executing buy and sell orders submitted by an investor.
- b. The firm that acts as an agent for a customer, charging the customer a commission for its services.

It used to be that only the wealthy could afford a broker and have access to the stock market. With the internet came the explosion of discount brokers that let you trade at a smaller fee, but don't provide personalized advice. Because of discount brokers, nearly anybody can afford to invest in the market now.

2. **Dealer**

- a. An individual or firm willing to buy or sell securities for their own account.
 - b. One who purchases goods or services for resale to consumers.
- A dealer differs from an agent in that a dealer acts as a principal in a transaction. That is, a dealer takes ownership of assets and is exposed to inventory risk, while an agent only facilitates a transaction on behalf of a client.

3. **Broker-Dealer**

A person or firm in the business of buying and selling securities operating as both a broker and dealer depending on the transaction.

Technically, a broker is only an agent who executes orders on behalf of clients, whereas a dealer acts as a principal and trades for his or her own account. Because **most brokerages act as both brokers and principals, the term broker-dealer is commonly used to describe them.**

4. **Specialist**

A person on the trading floor of certain exchanges who holds an inventory of particular stocks. The specialist is responsible for managing limit trades, but does not make information on outstanding limit orders available to other traders. There is usually one specialist for each stock traded on the NYSE, except for lower volume stocks

A specialist “makes a market” in the shares of one or more firms. This task may require the specialist to act as either a broker or dealer. The specialist’s role as a broker is simply to execute the orders of other brokers. Specialists may also buy or sell shares of stock for their own portfolios. When no other broker can be found to take the other side of a trade, specialists will do so even if it means they must buy for or sell from their own accounts.

5. **Market Maker**

A broker-dealer firm that accepts the risk of holding a particular number of shares of a particular security in order to facilitate trading in that security. Each market maker competes for customer order flow by displaying buy and sell quotations for a guaranteed number of shares. Once an order is received, the market maker immediately sells from its own inventory or seeks an offsetting order. This process takes place in mere seconds. The Nasdaq is the prime example of an operation of market makers. There are over 500 member firms that act as Nasdaq market makers, who keep the financial markets running efficiently because they are willing to quote both bid and offer prices for an asset.

6. **What is the difference between a Nasdaq market maker and a NYSE specialist?**

Specialists working on the NYSE have **four roles** to fulfill in order to ensure steadily flowing markets:

Auctioneer - because the NYSE is an auction market, bids and asks are competitively forwarded by investors. These bids and asks must be posted for the entire market to view such that the best price is always maintained. It is the job of the specialist to ensure that all bids and asks are reported in an accurate and timely manner, that all marketable trades are executed and that order is maintained on the floor.

Agent – the specialist also accepts limit orders relayed by investors through brokers or electronic trading. It is the responsibility of the specialist to ensure that the order is transacted appropriately on behalf of others, using the same fiduciary care as the brokers themselves.

Catalyst – as the specialists are in direct contact with the bidders and sellers of particular securities, it is their responsibility that enough interest exists for a particular stock such that a reasonable market exists. This is carried out by specialists seeking out recently active investors in cases where the bid and asks can't be matched.

Principal – in the instance where a market imbalance occurs between the demand and supply of a certain security, the market maker must make adjustments by purchasing and selling out of his/her own inventory to equalize the market.

Market makers working on the Nasdaq exchange are actually not at the exchange.

Rather, they are large investment companies that participate in the purchase and sale of actual securities. These market makers maintain inventories and buy and sell stocks from their personal inventories to individual customers and other dealers.

Each security on the Nasdaq market generally has more than one market maker, and open displays of competition among them facilitates competitive prices; as a result, individual

investors generally will get the best price. As this competition is evident in the limited spreads between posted bids and asks, the market makers on the Nasdaq will in some instances act very much like the specialists on the NYSE.

So, what's the main difference between a specialist and a market maker? Not much anymore.

Investment Companies

1. Investment Companies are financial intermediaries that collect funds from individual investors and invest those funds in a potentially wide range of securities or other asset.
2. Types: Unit investment trusts and managed investment companies (either closed-end or open-end). **Open-end companies are called mutual funds.**
3. Unit investment trusts (unmanaged): invested in a portfolio that is fixed for the life of the fund. Most unit trusts hold fixed-income securities and expire at their maturity. 90% of all unit trusts are invested in fixed-income portfolios, and about 90% of fixed-income unit trusts are invested in tax-exempt debt.
4. $NVA = (\text{asset-liabilities}) / \text{share outstanding}$
5. Mutual funds (open-end investment companies): account for about 90% of investment company assets.

WEBSITES:

www.finpipe.com

information on equity securities

www.nasdaq.com

www.nyse.com

www.bloomberg.com

information on bond and market rates

www.investinginbonds.com

glossaries on financial terms

www.bondsonline.com/docs/bondprofessor-glossary.html

www.investorwords.com

info on derivative securities

www.cboe.com/education

www.commoditytrader.com

Chapter Six: Risk and risk aversion

- I. **The investment process consists of two broad tasks.** One task is **security and market analysis**, by which we assess the risk and expected-return attributes of the entire set of possible investment vehicles. The second task is the **formation of an optimal portfolio of assets**. This task involves the determination of the best risk-return opportunities available from feasible investment portfolios and the choice of the best portfolio from the feasible set. The formal analysis of investments with the latter task is called portfolio theory. This chapter introduces **three themes in portfolio theory, all centering on risk**.
- The first** is the basic tenet that **investors avoid risk and demand a reward for engaging in risky investments**. The reward is taken as a risk premium, the difference between the expected rate of return and that available on alternative risk-free investments.
 - The second theme** allows us to **quantify investors' personal trade-offs between portfolio risk and expected return**. To do this we introduce the utility function, which assumes that investors can assign a welfare or "utility" score to any investment portfolio depending on its risk and return.
 - The third theme** is that **we cannot evaluate the risk of an asset separate from the portfolio of which it is a part**; that is, the proper way to measure the risk of an individual asset is to assess its impact on the volatility of the entire portfolio of investments. Taking this approach, we find that seemingly risky securities may be portfolio stabilizers and actually low-risk assets.

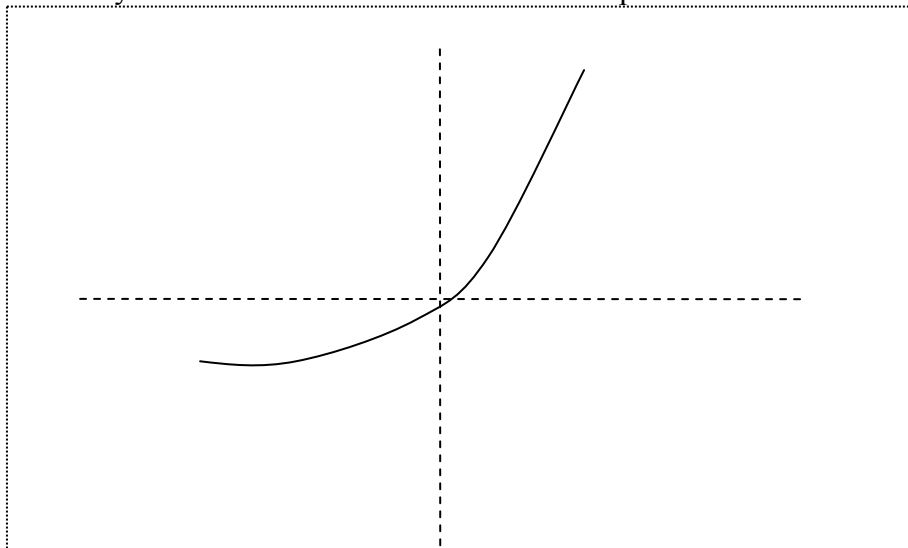
II. Risk and risk aversion

- Risk:** The chance that an investment's actual return will be different than expected. This includes the possibility of losing some or all of the original investment. It is usually measured using the historical returns or average returns for a specific investment. Higher risk means a greater opportunity for high returns... and a higher potential for loss.
- Risk Premium:** The extra return that a risky investment provides over the risk free rate to compensate for the risk of the investment. **A higher rate of return is required to entice investors into a riskier investment.**
- Risk Averse:** Describes an investor who, when faced with two investments with a similar expected return (but different risks), will prefer the one with the lower risk. **A risk averse person dislikes risk.**
- Utility:** Assume each investor can assign a welfare, or utility, score to competing investment portfolios based on the expected return and risk of those portfolios. The utility score may be viewed as a means of ranking portfolios. Portfolios receive higher utility scores for higher expected returns and lower scores for higher volatility. Many particular "scoring" systems or utility functions are legitimate. We choose the one that assigns a portfolio with expected return $E(r)$ and variance of returns σ^2 the following utility score:

$$U = E(r) - 0.005A\sigma^2$$
 where U is the utility value and A is an index of the investor's risk aversion.
- Certainty equivalent rate** of a portfolio is the rate that risk-free investments

would need to offer with certainty to be considered equally attractive as the risky portfolio. A portfolio is desirable only if its certainty equivalent return exceeds that of the risk-free alternative. A sufficient risk-averse investor may assign any risky portfolio, even one with a positive risk premium, a certainty equivalent return that is below the risk-free rate, which will cause the investor to reject the portfolio.

6. **Risk neutral investors judge risky prospects solely by their expected return.**
The level of risk is irrelevant to the risk-neutral investor, meaning that there is no penalization for risk. For this investor a portfolio's certainty equivalent rate is simply its expected return.
7. **A risk lover** is willing to engage in fair games and gambles; this investor adjusts the expected return upward to take into account the "fun" of confronting the prospect's risk. Risk lovers will always take a fair game because their upward adjustment of utility for risk gives the fair game a certainty equivalent that exceeds the alternative of the risk-free investment.
8. **Indifference curve**: the curve that connects all portfolio points with the same utility value in the mean-standard deviation plan.



III. Key statistical concepts

1. **Expected return:**

- a. Definition: a probability-weighted average of an asset's return in all states (scenarios).

- b. Formula:
$$E(R) = \sum_{i=1}^n P_i R_i$$

Where

i: states of nature

P_i: probability of that the event will happen in state i.

R_i: the return of the asset if the event happens in state i.

- c. Example: suppose an asset is priced at \$100. There are three possible outcomes in one year: good, so-so and bad.

Three possible outcomes (i):		<u>Good</u>	<u>Soso</u>	<u>Bad</u>
Probability (Pi)	25%	50%	25%	
Prices	\$ 120	\$ 110	90	
Return (Ri)	20%	10%	-10%	

How do you calculate the average of a probability distribution? Simply take the probability of each possible return outcome and multiply it by the return outcome itself.

$$E(R) = (0.5)(0.1) + (0.25)(0.2) + (0.25)(-0.1) = 0.075 = 7.5\%$$

Although this is what you expect the return to be, there is no guarantee that it will be the actual return.

- d. the **rate of return on a portfolio** is a weighted average of the return of each asset comprising the portfolio, with portfolio proportions as weights. This implies that the **expected return on a portfolio** is a weighted average of the expected return on each component asset.

For example, a portfolio with value of \$200, which consists of \$100 stock A and \$100 stock B. $E(r_A) = 20\%$, $E(r_B) = 10\%$,

$$E(\text{expected return of portfolio}) = (100/200) \cdot 20\% + (100/200) \cdot 10\% = 15\%$$

2. **Variance (the second central moment):**

- a. Definition: A measure of the dispersion of a set of data points around their mean value. It is the expected value of the squared deviations from the expected return.

- b. Formula: $\sigma^2 = \text{Var}(R) = \sum_{i=1}^n P_i [R_i - E(R)]^2$ Variance measures the variability (volatility) from an average. **Volatility is a measure of risk, so this statistic can help determine the risk an investor might take on when purchasing a specific security.**

- c. Example: $\text{Var}(R) = 0.25(0.2-0.075)^2 + 0.5(0.1-0.075)^2 + 0.25(-0.1-0.075)^2 = 1.19\%$

3. **Standard deviation:**

- a. **Definition: The square root of the variance.** A measure of the dispersion of a set of data from its mean. The more spread apart the data is, the higher the deviation. In finance, standard deviation is applied to the annual rate of return of an investment to measure the investment's volatility (risk).

- b. Formula: $\sigma = \sqrt{\text{Var}(R)}$

- c. Example: Standard deviation = $\sqrt{\text{Var}(R)} = \sqrt{1.19\%} = 10.90\%$

4. **Covariance:**

- a. **Definition: A measure of the degree to which returns on two risky assets move in tandem.** A positive covariance means that asset returns move together. A negative covariance means returns vary inversely.

- b. Formula: $\text{Cov}(R_A, R_B) = \sum_{i=1}^n P_i [R_{iA} - E(R_A)][R_{iB} - E(R_B)]$, where

A: Asset A.

B: Asset B.

If you owned one asset that had a high covariance with another asset that you didn't own, then you would receive very little increased diversification by adding the

second asset. Of course, the opposite is true as well, adding assets with low covariance to your portfolio lowers overall portfolio risk.

c. example: suppose two risky assets A and B:

Three possible outcomes (i):	<u>Good</u>	<u>Soso</u>	<u>Bad</u>
Probability (P _i)	25%	50%	25%
Return of Asset A (R _{iA})	20%	10%	-10%
Return of Asset B (R _{iB})	30%	0%	-30%

$$E(R_A) = 7.5\%$$

$$E(R_B) = 0\%$$

$$\text{Cov}(R_A, R_B) = 0.25(0.2-0.075)(0.3-0) + 0.5(0.1-0.075)(0-0) + 0.25(-0.1-0.075)(-0.3-0) = 2.25\%$$

5. **Portfolio's variance:** when two risky assets with variance var1 and var2, respectively, are combined into a portfolio with portfolio weights w1 and w2, respectively, the portfolio variance σ_p^2 is given by

$$\sigma_p^2 = w_1^2 \sigma_1^2 + w_2^2 \sigma_2^2 + 2w_1 w_2 \text{cov}(r_1, r_1)$$

A positive covariance increases portfolio variance, and a negative covariance acts to reduce portfolio variance.

If var1 = var2 = - covariance and w1 and w2, then portfolio's variance is zero (prove it!)

6. **Correlation coefficient: a statistical measure of how two securities move in relation to each other. It ranges between -1 and +1.**

a. Formula: $\text{corr}(R_A, R_B) = \frac{\text{Cov}(R_A, R_B)}{\sigma_A \sigma_B}$

- b. **Perfect positive correlation** (a correlation co-efficient of +1) implies that as one security moves, either up or down, the other security will move in lockstep, in the same direction. Alternatively, **perfect negative correlation** means that if one security moves in either direction the security that is perfectly negatively correlated will move by an equal amount in the opposite direction. **If the correlation is 0,** the movements of the securities is said to have no correlation, it is completely random. If one security moves up or down there is as good a chance that the other will move either up or down, the way in which they move is totally random.

In real life however you likely will not find perfectly correlated securities, rather you will find securities with some degree of correlation. For example, the performance of two stocks within the same industry is strongly positively correlated although it may not be exactly +1.

Correlation V.S. Covariance:

Basically, Correlation is better than covariance:

1 -- Because correlation removes the effect of the variance of the variables, it provides a standardized, absolute measure of the strength of the relationship, bounded by -1.0 and 1.0. This is good because it makes it

possible to compare any correlation to any other correlation and see which is stronger. You cannot do this with covariance.

2 -- The squared correlation (r^2) is a measure of how much of the variance in one variable is explained by the other variable. This measure, the coefficient of determination, ranges from 0.0 to 1.0. You cannot do this with covariance.

7. Skewness (the third central moment): A statistical term used to describe a situation's asymmetry in relation to a normal distribution. A positive skew describes a distribution favoring the right tail, whereas a negative skew describes a distribution favoring the left tail.

Formula:
$$M_3 = \sum_{i=1}^n P_i [R_i - E(R)]^3$$

8. Kurtosis: A statistical measure used to describe the distribution of observed data around the mean. Used generally in the statistical field, it describes trends in charts. A high kurtosis portrays a chart with fat tails and a low even distribution, whereas a low kurtosis portrays a chart with skinny tails and a distribution concentrated towards the mean. It is sometimes referred to as the "volatility of volatility."

IV. Appendix A

1. **Variance** (the second central moment) does not provide a full description of risk. Consider the two probability distributions for rates of return on a portfolio, A and B. A and B are probability distributions with identical expected values and variance. However, A is characterized by more likely but small losses and less likely but extreme gains. This pattern is reversed in B. the difference is important. **When we talk about risk, we really mean "bad surprises."** The bad surprises in A, although they are more likely, are small (and limited) in magnitude. The bad surprises in B, are more likely to be extreme. A risk-averse investor will prefer A to B. We use skewness to describe this asymmetry.

$$M_3 = \sum_{i=1}^n P_i [R_i - E(R)]^3$$

Cubing the deviations from the expected value preserves their signs, which allows us to distinguish good from bad surprises. Because this procedure gives greater weight to larger deviations, it causes the "long tail" of the distribution to dominate the measure of skewness. Thus the skewness of the distribution will be positive for a right-skewed distribution such as A and negative for a left-skewed distribution such as B.

2. To summarize, **the first moment (expected value) represents the reward. The second and higher central moments characterize the uncertainty of the reward. All the even moments (variance, M4, etc.) represent the likelihood of extreme values.** Larger values for these moments indicates greater uncertainty. **The odd moments (M3, M5, etc.) represent measures of asymmetry.** Positive numbers are associated with positive skewness and hence are desirable.
3. We can write the utility value derived from the probability distribution as:

$$U = E(r) - b_0\sigma^2 + b_1M_3 - b_2M_4 + b_3M_5 - \dots$$

Where the importance of the terms lessens as we proceed to higher moments. Notice that the “good” (odd) moments have positive coefficients, whereas the “bad” (even) moments have minus signs in front of the coefficients.

4. Samuelson (1970, Review of Economic Studies, 37) proves that in many important circumstances:
 - a. The importance of all moments beyond the variance is much smaller than that of the expected value and variance. In other words, disregarding moments higher than the variance will not affect portfolio choice.
 - b. The variance is as important as the mean to investor welfare.

Samuelson’s proof is the major theoretical justification for mean-variance

analysis. Under the conditions of this proof mean and variance are equally important, and we can overlook all other moments without harm.

5. **Modern portfolio theory, for the most part, assumes that asset returns are normally distributed, because the normal distribution can be described completely by its mean and variance, consistent with mean-variance analysis.**

Chapter Seven: Capital Allocation between the Risky asset and the risk-free Asset

- I. Most institutional investors follow a top-down approach. Capital allocation and asset allocation decisions will be made at a high organizational level, with the choice of the specific securities to hold within each asset class delegated to particular portfolio managers.
 1. **Capital allocation decision** is the choice of the proportion of the overall portfolio to place in safe but low-return money market securities versus risky but higher return securities like stocks.
 2. **Asset allocation decision** describes the distribution of risky investments across broad asset classes – stocks, bonds, real estate, foreign assets, and so on.
 3. **Security selection decision** describes the choice of which particular securities to hold within each asset class.

II. **The risk-free asset**

1. **Treasury bills:** Their short-term nature makes their values insensitive to interest rate fluctuation. Moreover, inflation uncertainty over the course of a few weeks, or even months, is negligible compared with the uncertainty of stock market returns.
2. In practice, most investors use a broader range of money market instruments as a risk-free asset. All the money market instruments are virtually free of interest rate risk because of their short maturities and are fairly safe in terms of default or credit risk, e.g. bank certificates of deposits (CDs), and commercial paper (CP)

III. **Portfolio of one risky asset and one risk-free asset**

1. Assumptions:

- Suppose a risky portfolio, P, consists of different risky assets, which are held in fixed proportions. In this case, we can actually treat P as one single risky asset.
- We allocate y proportion of our investment budget to P. The remaining proportion, $1 - y$, is to be invested in the risk-free asset, F.
- r_p , the return of P, its expected return $E(r_p)$, and its standard deviation by σ_p .
- R_f , the rate of return on the risk-free asset. Of course, the standard deviation of F is 0 and $r_f = E(r_f)$.

2. return on the complete portfolio, C is r_c where

$$r_c = yr_p + (1-y)r_f$$

$$E(r_c) = yE(r_p) + (1-y)r_f = r_f + y(E(r_p) - r_f) \quad (7.1)$$

The base return for any portfolio is the risk-free rate. In addition, the portfolio is expected to earn a risk premium that depends on the risk premium of the risky portfolio, $E(r_p) - r_f$, and the investor's position in the risky asset, y.

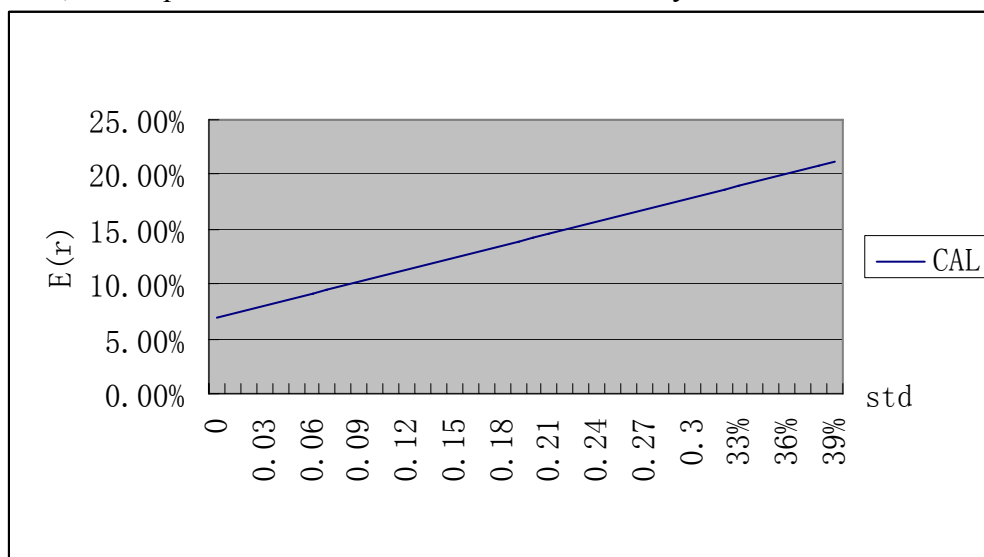
$$3. \text{ Standard deviation of C, } \sigma_c = y \sigma_p \quad (7.2)$$

$$4. \quad E(r_c) = r_f + y[E(r_p) - r_f] = r_f + \frac{\sigma_c}{\sigma_p}[E(r_p) - r_f] \quad (7.3)$$

Equation 7.3 describes the expected return – standard deviation trade-off. The **expected return of the complete portfolio as a function of its standard deviation is a straight line**, with intercept r_f and slope as follows:

$$S = \frac{E(r_p) - r_f}{\sigma_p} = \frac{E(r_c) - r_f}{\sigma_c}$$

The straight line in the following figure is called the **capital allocation line (CAL)**. It depicts all the risk-return combinations available to investors. The slope of the CAL, denoted S, equals the increase in the expected return of the complete portfolio per unit of additional standard deviation – in other words, incremental return per incremental risk. Thus, the slope is also called the reward-to-variability ratio.



5. the range of y – y can be greater than 1 (short risk-free asset (leveraged position in the

risky asset)).

- a. If investors can borrow at the risk-free rate, the CAL is the same as before.
- b. If investors must have to borrow at a higher rate, the CAL is kinked at $y=1$. When $y < 1$, investors are lending at r_f ; when $y > 1$, the investors are borrowing at $r > r_f$.

IV. **Risk tolerance and asset allocation**

1. The investor confronting the CAL now must choose one optimal portfolio, C, from the set of feasible choices. This choice entails a trade-off between risk and return.
Individual investor differences in risk aversion imply that, given an identical opportunity set (that is, a risk-free rate and a reward-to-variability ratio), different investors will choose different positions in the risky asset. In particular, the more risk-averse investors will choose to hold less of the risky asset and more of the risk-free asset.

2. The utility that an investor derives from a portfolio can be described by:

$$U = E(r) - 0.005A \sigma^2. \text{----- (7.4)}$$

3. By equations 7.1 and 7.2, we maximize the investor's utility:

$$\underset{y}{\text{Max}} U = E(r_C) - 0.005A\sigma_C^2$$

, Solve this maximization problem: we get the optimal

$$= r_f + y[E(r_P) - r_f] - 0.005Ay^2\sigma_P^2$$

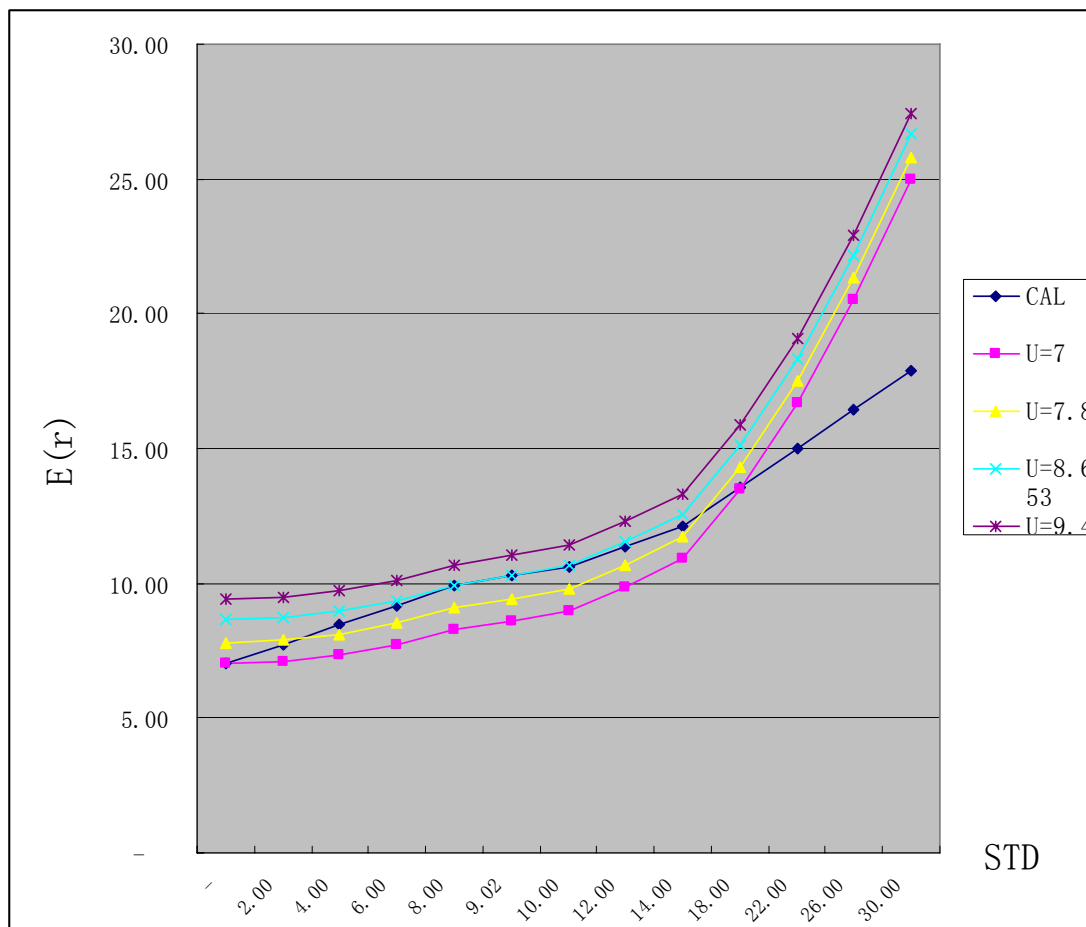
position for risk-averse investors in the risky asset, y^* , as follows:

$$y^* = \frac{E(r_P) - r_f}{0.01A\sigma_P^2} \text{----- (7.5)}$$

This solution shows that the **optimal position in the risky asset is inversely proportional to the level of risk aversion and the level of risk (as measured by the variance) and directly proportional to the risk premium offered by the risky asset.**

4. A graphical way of presenting this decision problem is to use **indifference curve analysis**.

- a. an indifference curve is a graph in the expected return-standard deviation plane of all points that result in a given level of utility. The curve displays the investor's required trade-off between expected return and standard deviation.
- b. The more risk-averse investor has steeper indifference curves than the less risk-averse investor. Steeper curves mean that the investor requires a greater increase in expected return to compensate for an increase in portfolio risk.
- c. Higher indifference curves correspond to higher levels of utility. The investor thus attempts to find the complete portfolio on the highest possible indifference curve.



- V. Passive strategies: the capital Market Line: A passive investment strategy disregards security analysis, targeting instead the risk-free asset and a broad portfolio of risky assets such as the S&P 500 stock portfolio.

Chapter Eight: Optimal Risky Portfolios:

I. Diversification and Portfolio risk:

1. Diversification can reduce portfolio risk.
2. **The risk that remains even after extensive diversification is called market risk,** risk that is attributable to marketwide risk sources. Such risk is also called systematic risk or nondiversifiable risk.
3. **A risk that can be eliminated by diversification is called unique risk, firm-specific risk, nonsystematic risk or diversifiable risk.**

II. Portfolios of two risky assets:

Consider a portfolio comprised of two risky assets: D, long-term bond and E, stock. A proportion of w_D is invested in the bond and the remainder, $1 - w_D = w_E$, is invested in the stock. The rate of return on this portfolio, r_p , will be $r_p = w_D r_D + w_E r_E$ and the expected return on the portfolio is:

$$E(r_p) = w_D E(r_D) + w_E E(r_E) \text{ ----- (8.1)}$$

The portfolio variance is: $\sigma_p^2 = w_D^2 \sigma_D^2 + w_E^2 \sigma_E^2 + 2w_D w_E \text{Cov}(r_D, r_E) \text{ ----- (8.2)}$

- Equation 8.2 reveals that **variance is reduced if the covariance term is negative**. However, **even if the covariance term is positive, the portfolio standard deviation still is less than the weighted average of the individual security standard deviations**, unless the two securities are perfectly positively correlated.

Proof:

Since $Cov(r_D, r_E) = \rho_{DE} \sigma_D \sigma_E$, thus $\sigma_p^2 = w_D^2 \sigma_D^2 + w_E^2 \sigma_E^2 + 2w_D w_E \rho_{DE} \sigma_D \sigma_E$

because $\rho_{DE} \leq 1 \rightarrow \sigma_p^2 \leq w_D^2 \sigma_D^2 + w_E^2 \sigma_E^2 + 2w_D w_E \sigma_D \sigma_E = (w_D \sigma_D + w_E \sigma_E)^2$, Thus

$$\sigma_p \leq w_D \sigma_D + w_E \sigma_E$$

□

In words, the standard deviation of the portfolio with perfect positive correlation is just the weighted average of the component standard deviations. In all other cases, the correlation coefficient is less than 1, making the portfolio standard deviation less than the weighted average of the component standard deviations.

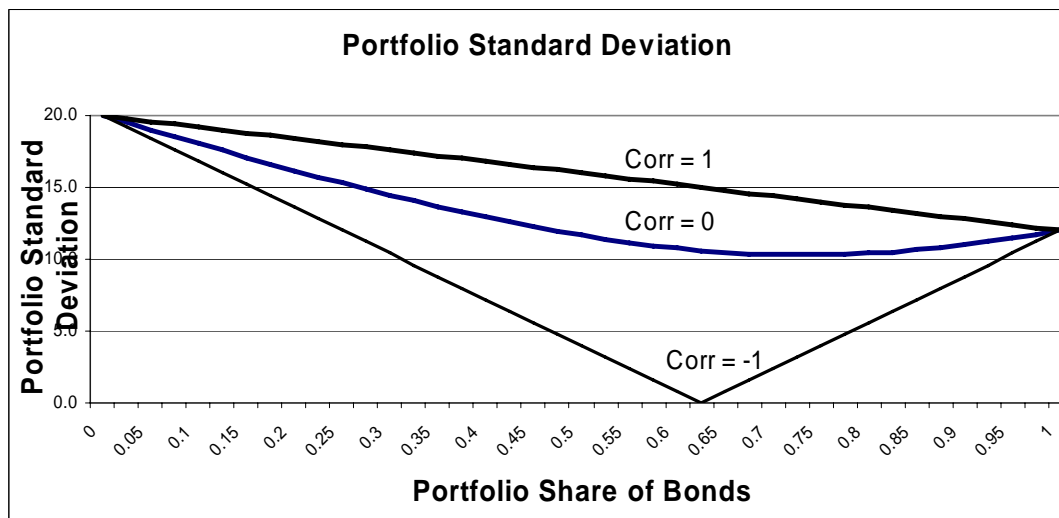
- Because the portfolio's expected return is the weighted average of its component expected returns, whereas its standard deviation is less than the weighted average of the component standard deviations, portfolios of less than perfectly correlated assets always offer better risk-return opportunities than the individual component securities on their own. The lower the correlation between the assets, the greater the gain in efficiency.**

- The lowest possible standard deviation = 0. Let correlation = -1, the equation 8.5 simplifies to

$$\sigma_p^2 = (w_D \sigma_D - w_E \sigma_E)^2, \text{ let } w_D \sigma_D - w_E \sigma_E = 0 \rightarrow w_D = \frac{\sigma_E}{\sigma_D + \sigma_E}, w_E = \frac{\sigma_D}{\sigma_D + \sigma_E} = 1 - w_D$$

- Minimum-variance portfolio: Maximize equation 8.2 relative to w_D (replace w_E with

$$1 - w_D), \text{ we can get minimum-variance portfolio. } w_{\min}(D) = \frac{\sigma_E^2 - \text{cov}(r_D, r_E)}{\sigma_D^2 + \sigma_E^2 - 2 \text{cov}(r_D, r_E)}$$



5. **Expected return – standard deviation:** By equation 8.1 expected return, $E(r_p)$ has one-for-one relationship with w_D , substitute $E(r_p)$ for w_D in equation 8.2, we can get the relationship between σ_p^2 and $E(r_p)$
6. **Portfolio opportunity set:** points satisfy the relationship between expected return and standard deviation.

III. Asset Allocation with stocks, bonds and bills

Now consider three assets: two risky assets: bond (D) and stock (E) and one risk-free asset: T-bill (T).

1. **Optimal risky portfolio:** the objective is to find the weights w_D and w_E that result in the highest slope of the CAL (i.e., the weights that result in the risky portfolio with the highest reward-to-variability ratio). Therefore, the objective is to maximize the slope of the CAL for any possible portfolio, P.

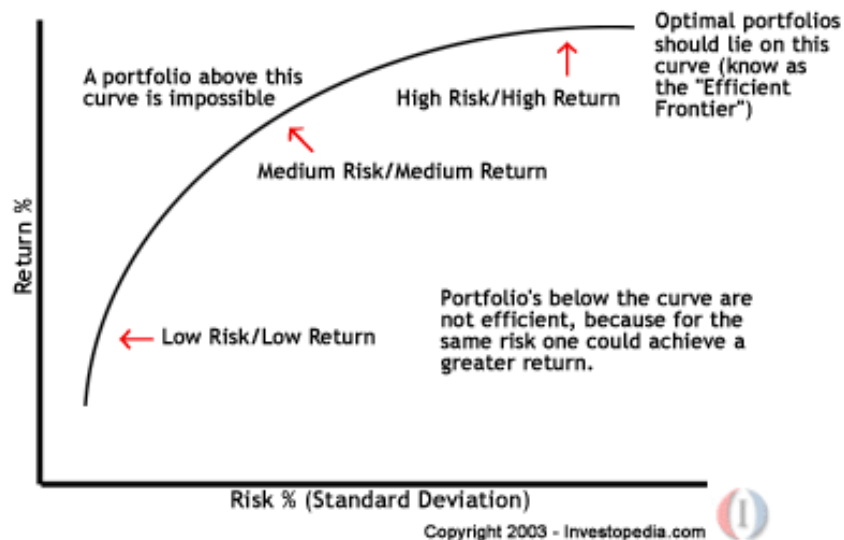
$$\text{Max } S = \frac{E(r_p) - r_f}{\sigma_p}, \text{ subject to equation 8.1, 8.2 and } 1 - w_D = w_E \rightarrow \text{get optimal risky}$$

portfolio.

2. Optimal complete portfolio: use equation 7.5 to get y – the proportion taken in risky portfolio P.
3. Steps we followed to arrive at the complete portfolio:
 - a. Specify the return characteristics of all securities (expected returns, variances, covariances).
 - b. Establish the risky portfolio:
 - (1) Calculate the optimal risk portfolio, P.
 - (2) calculate the properties of Portfolio P using the weights determined in step (a) and equations 8.1 and 8.2
 - c. allocate funds between the risky portfolio and the risk-free asset:
 - (1) calculate the fraction of the complete portfolio allocated to Portfolio P (the risky portfolio) and to T-bills (the risk-free asset)
 - (2) Calculate the share of the complete portfolio invested in each asset and in T-bills.

IV. The Markowitz Portfolio Selection Model

1. **Minimum-variance frontier** of risky assets: it is a graph of the lowest possible variance that can be attained for a given portfolio expected return.
2. **Efficient frontier:** the part of the frontier that lies above the global minimum-variance portfolio. For any portfolio on the lower portion of the minimum-variance frontier, there is a portfolio with the same standard deviation and a greater expected return positioned directly above it. Hence the bottom part of the minimum-variance frontier is inefficient.



3. **add in the risk-free asset:** the CAL that is supported by the optimal portfolio, P, is tangent to the efficient frontier. This CAL dominates all alternative feasible lines.
4. **Finally, the individual investor chooses the appropriate mix between the optimal risky portfolio P and T-bills.**
5. **Harry Markowitz's** model is precisely step one of portfolio management: the identification of the efficient set of portfolios, or the efficient frontier of risky assets. The principal idea behind the frontier set of risky portfolios is that, for any risk level, we are interested only in that portfolio with the highest expected return. Alternatively, the frontier is the set of portfolios that minimize the variance for any target expected return.
6. **The optimal risky portfolio is the same for all investors!!!**
7. **Separation property:** the portfolio choice problem may be separated into two independent tasks. The first task, determination of the optimal risky portfolio – the best risky portfolio is the same for all investors, regardless of risk aversion. The second task, however, allocation of the complete portfolio to risk-free asset versus the risky portfolio, depends on personal preference. Here the investor is the decision maker. This analysis suggests that a limited number of portfolios may be sufficient to serve the demands of a wide range of investors. This is the theoretical basis of the mutual fund industry.
8. **Asset allocation and security selection:** the theories of security selection and asset allocation are identical. Both all for the construction of an efficient frontier, and the choice of a particular portfolio from along that frontier. In reality, top management of an investment firm continually updates the asset allocation of the organization, adjusting the investment budget allotted to each asset-class portfolio. The next level managers then optimize the security selection of each asset-class portfolio independently.

Chapter Nine: The Capital Asset Pricing Model

- I. The capital asset pricing model, CAPM is a centerpiece of modern financial economics. The model gives a precise prediction of the relationship that we should observe between the risk of an asset and its expected return. This relationship serves two vital functions. First, it provides a benchmark rate of return for evaluating possible investments. Second, the model helps us to make an educated return on assets that have not yet been traded in the marketplace.

Intuition: Risk-averse investors measure the risk of the optimal risky portfolio by its variance. In this world, we would expect the reward, or the risk premium on individual assets, to depend on the contribution of the individual asset to the risk of the portfolio. The beta of a stock measures the stock's contribution to the variance of the market portfolio. Hence we expect, for any asset or portfolio, the required risk premium to be a function of beta.

II. The Capital Asset Pricing Model:

1. Assumptions:

A1. Investors are price-takers, in that act as though security prices are unaffected by their own trades.

A2. All investors plan for one identical holding period. This behavior is myopic in that it ignores everything that might happen after the end of the single-period horizon.

A3. Investments are limited to a universe of publicly traded financial assets, such as stocks and bonds, and to risk-free borrowing or lending arrangement. This assumption rules out investment in nontraded assets such education, private enterprises, etc. It is assumed also that investors may borrow or lend any amount at a fixed, risk-free rate.

A4. Investors pay no taxes on returns and no transaction costs on trades in securities.

A5. All investors are rational mean-variance optimizers, meaning that they all use the Markowitz portfolio selection model.

A6. Homogeneous expectations: all investors have the same beliefs concerning returns, variances, and covariances. But all investors may have different aversion to risk. All investors use the same expected returns and covariance matrix of security returns to generate the efficient frontier and the unique optimal risky portfolio.

2. Results under these assumptions:

- All investors will choose to hold the same **market portfolio** (M), which is a market-value-weighted portfolio of all existing securities
- Not only will the market portfolio be on the efficient frontier, but it also will be the tangency portfolio to the optimal **capital allocation line (CAL)** derived by each and every investor. As a result, the **capital market line (CML)**, the line from the risk-free rate through the market portfolio, M, is also the best attainable capital allocation line. All investors hold M as their optimal risky portfolio, differing only in the amount invested in it versus in the risk-free asset.
- The risk premium on the market portfolio will be proportional to its risk and the degree of risk aversion of the representative investor.
- The risk premium on individual assets will be proportional the risk premium on the market portfolio, M, and the beta coefficient of the security relative to the market

portfolio. Beta measure the extent to which returns on the stock and the market move together. Beta is defined as $\beta_i = \frac{Cov(r_i, r_M)}{\sigma_M^2}$

And the risk premium on individual securities is

$$E(r_i) - r_f = \frac{Cov(r_i, r_M)}{\sigma_M^2} [E(r_M) - r_f] = \beta_i [E(r_M) - r_f]$$

3. Why do all investors hold the market portfolio?
 - a. When we aggregate the portfolios of all investors, lending and borrowing will cancel out, and the value of the aggregate risky portfolio will equal the entire wealth of the economy. This is the market portfolio, M. The proportion of each stock in this portfolio equals the market value of the stock (price per share times number of shares outstanding) divided by the sum of the market value of all stocks.
 - b. If all investors use identical Markowitz analysis (A5) applied to the same universe of securities (A3) for the same time horizon (A2) and use the same input list (A6), they all must arrive at the same determination of the optimal risky portfolio, the portfolio on the efficient frontier identified by the tangency line from risk-free asset to that frontier.
 - c. Suppose that the optimal portfolio of investors does not include stock AB. When all investors avoid AB, the demand is zero, and AB's price takes a free fall. As AB stock gets progressively cheaper, it becomes ever more attractive and other stocks look relative less attractive. Ultimately, AB reaches a price where it is attractive enough to include in the optimal stock portfolio. Thus, all assets have to be included in the market portfolio.

4. The risk premium of the market portfolio:

- a. Recall the each individual investor chooses a proportion y, allocated to the optimal portfolio M, such that: $y = \frac{E(r_M) - r_f}{.001A\sigma_M^2}$

(9.1)

- b. In the simplified CAPM economy, risk-free investments involve borrowing and lending among investors. This means that net borrowing and lending across all investors must be zero, and in consequence the average position in the risky portfolio is 100%, or y = 1, Setting y = 1 in 9.1 and rearranging, we get that the risk premium on the market portfolio is related to its variance by the average degree of risk aversion:

$$E(r_M) - r_f = 0.01 \times A \sigma_M^2$$

5. Expected returns on individual securities:

- a. The CAPM is built on the insight that the appropriate risk premium on an asset will be determined by its contribution to the risk of investors' overall portfolios. Portfolio risk is what matters to investors and is what governs the risk premiums they demand.
- b. Suppose, we want to gauge the portfolio risk of GM stock. We measure the contribution to the risk of the overall portfolio from holding GM stock by its covariance with the market portfolio.

Portfolio	W ₁	W ₂	W _{GM}	W _n
-----------	----------------	----------------	-----------------	----------------

Weights						
W_1	$\text{Cov}(r_1, r_1)$	$\text{Cov}(r_1, r_2)$...	$\text{Cov}(r_1, r_{GM})$...	$\text{Cov}(r_1, r_n)$
W_2	$\text{Cov}(r_2, r_1)$	$\text{Cov}(r_2, r_2)$...	$\text{Cov}(r_2, r_{GM})$...	$\text{Cov}(r_2, r_n)$
\vdots	\vdots	\vdots	\vdots	\vdots	\vdots	\vdots
W_{GM}	$\text{Cov}(r_{GM}, r_1)$	$\text{Cov}(r_{GM}, r_2)$...	$\text{Cov}(r_{GM}, r_{GM})$...	$\text{Cov}(r_{GM}, r_n)$
\vdots	\vdots	\vdots	\vdots	\vdots	\vdots	\vdots
W_n	$\text{Cov}(r_n, r_1)$	$\text{Cov}(r_n, r_2)$...	$\text{Cov}(r_n, r_{GM})$...	$\text{Cov}(r_n, r_n)$

The contribution of GM's stock to the variance of the market portfolio is

$$W_{GM} [W_1 \text{Cov}(r_{GM}, r_1) + W_2 \text{Cov}(r_{GM}, r_2) + \dots + W_{GM} \text{Cov}(r_{GM}, r_{GM}) + W_n \text{Cov}(r_{GM}, r_n)]$$

Therefore, GM's contribution to variance = $W_{GM} \text{Cov}(r_{GM}, r_M)$

- c. If the covariance between GM and the rest of the market is negative, then GM makes a "negative contribution" to portfolio risk: by providing returns that move inversely with the rest of the market, GM stabilizes the return on the overall portfolio. If the covariance is positive, GM makes a positive contribution to overall portfolio risk because its returns amplify swings in the rest of the portfolio.

- d. The reward-to-risk ratio for investments in GM can be expressed as:

GM's contribution to risk premium / GM's contribution to variance =

$$\frac{W_{GM} [E(r_{GM}) - r_f]}{W_{GM} \text{Cov}(r_{GM}, r_M)} = \frac{E(r_{GM}) - r_f}{\text{Cov}(r_{GM}, r_M)}$$

- e. The market portfolio is the tangency portfolio. The reward-to-risk ratio for investment in the market portfolio is:

$$\text{Market risk premium / market variance} = \frac{E(r_M) - r_f}{\sigma_M^2} \quad \text{-----} \quad (9.5)$$

Notice that for components of the efficient portfolio, such as shares of GM, we measure risk as the contribution to portfolio variance. In contrast, for the efficient portfolio itself, its variance is the appropriate measure of risk.

- f. A basic principle of equilibrium is that all investments should offer the same reward-to-risk ratio. If the ratio were better for one investment than another, investors would rearrange their portfolios, tilting toward the alternative with the better trade-off and shying away from the other. Such activity would impart pressure on security prices until the ratios were equalized. Therefore:

$$\frac{E(r_M) - r_f}{\sigma_M^2} = \frac{E(r_{GM}) - r_f}{\text{Cov}(r_{GM}, r_M)}$$

- g. CAPM – expected return-beta relationship:

$$E(r_i) - r_f = \frac{\text{Cov}(r_i, r_M)}{\sigma_M^2} [E(r_M) - r_f] = \beta_i [E(r_M) - r_f]$$

An expected return consists of two components: the risk-free rate, which is compensation for the time value of money, and a risk premium, determined by multiplying a benchmark risk premium (i.e., the risk premium offered by the market portfolio) times the relative measure of risk, beta.

- h. if the expected return-beta relationship holds for any individual asset, it must hold for any combination of assets. Let $E(r_p) = \sum_k w_k E(r_k)$, and $\beta_p = \sum_k w_k \beta_k$ is the portfolio

beta, then CAPM has to hold for a portfolio: $E(r_p) - r_f = \beta_p[E(r_M) - r_f]$

- i. In a rational market investors receive high expected returns only if they are willing to bear risk.
6. The security market line:
 - a. The expected return-beta relationship can be portrayed graphically as the **security market line (SML)**.
 - b. **SML VS. CML**: the **CML** graphs the risk premiums of efficient portfolios as a function of **portfolio standard deviation**. This is appropriate because standard deviation is a valid measure of risk for efficiently diversified portfolios that are candidates for an investor's overall portfolio. The **SML** graphs individual asset risk premiums as a function of asset risk. The relevant measure of risk for individual asset held as parts of well-diversified portfolios is not the asset's standard deviation or variance; it is, instead, the contribution of the asset to the portfolio variance, which we measure by the **asset's beta**. The SML is valid for both efficient portfolios and individual assets.
 - c. CAPM can be **used in the money-management industry**: suppose that the SML relation is used as a benchmark to assess the fair expected return on a risky asset. The difference between the expected returns indicated in SML and the actual return is called the stock's **alpha**. Alpha is one measure of professional money managers' performance.
 - d. CAPM can be **used in capital budgeting decisions**: for a firm considering a new project, the CAPM can provide the required rate of return that the project needs to yield, based on its beta, to be acceptable to investors. Managers can use the CAPM to obtain this cutoff internal rate of return (IRR) for the project.

III. Extensions of the CAPM – let's relax some of the assumptions:

1. Investors cannot borrow at a common risk-free rate: then they may choose risky portfolios from the entire set of efficient frontier portfolios according to how much risk they choose to bear. The market is no longer the common optimal portfolio. In fact, with investors choosing different portfolios, it is no longer obvious whether the market portfolio, which is the aggregate of all investors' portfolios, will even be on the efficient frontier. If the market portfolio is no longer mean-variance efficient, then the expected return-beta relationship of the CAPM will no longer characterize market equilibrium.
2. Blacel's model of capital market equilibrium with restricted borrowing:
 - a. any portfolio constructed by combining efficient portfolios is itself on the efficient frontier.
 - b. Every portfolio on the efficient frontier has a "companion" portfolio on the bottom half of the minimum-variance frontier with which it is uncorrelated. Because the portfolios are uncorrelated, the companion portfolio is referred to as the zero-beta portfolio of the efficient portfolio.
 - c. The expected return of any asset can be expressed as an exact, linear function of the expected return on any two frontier portfolios.

$$E(r_i) = E(r_Q) + [E(r_P) - E(r_Q)] \frac{\text{Cov}(r_i, r_P) - \text{cov}(r_P, r_Q)}{\sigma_P^2 - \text{cov}(r_P, r_Q)}, \text{ where, P and Q are two}$$

efficient portfolios.

- d. If there is no risk-free asset, the risk-free rate can be replaced by the zero-beta portfolio's expected rate of return:

$$E(r_i) = E(r_{Z(M)}) + \beta_i [E(r_M) - E(r_{Z(M)})]$$

IV. The CAPM and liquidity

1. Liquidity refers to the cost and ease with which an asset can be converted into cash, that is, sold. Several studies show that liquidity plays an important role in explaining rates of return on financial assets.
2. Chordia, Roll, and Subrahmanyam find commonality across stocks in the variable cost of liquidity: quoted spreads, quoted depth, and effective spread covary with the market and industrywide liquidity. Hence, liquidity risk is systematic and therefore difficult to diversify.
3. Investors prefer more liquid assets with lower transaction costs, so that all else equal, relatively illiquid assets trade at lower prices, or, that the expected return on illiquid assets must be higher. Therefore, an **illiquidity premium** must be impounded into the price of each asset.
4. when there is a large number of assets with any combination of beta and liquidity costs C_i , the expected return is bid up to reflect this undesired property according to:

$$E(r_i) - r_f = \beta_i [E(r_M) - r_f] + f(c_i)$$

Chapter Ten: Index Models:

- I. **Markowitz procedure** requires a huge number of estimates of covariances between all pairs of available securities and also a huge optimization program. This is burdensome and sometimes, impossible. If we plan to analyze n stocks, we have to estimate n (estimates of expected returns) + n (estimates of variances) + $(n^2 - n)/2$ (estimates of covariances) = $(n^2 + 3n)/2$. If $n = 50$, we will estimate 1325 estimates. If $n = 3,000$, we need more than 4.5 million estimates!!! We need to find one way out of this:

II. A single-index security market:

1. Assumptions:

A1: we summarize all relevant economic factors by one macroeconomic indicator and it moves the security market as a whole.

A2: beyond this common effect, all remaining uncertainty in stock returns is firm specific – there is no other source of correlation between securities.

A3: firm-specific events would include new inventions, deaths of key employees, and other factors that affect the fortune of the individual firm without affecting the broad economy in a measurable way.

2. The model:

- a. the holding-period return on security I is:
- $r_i = E(r_i) + m_i + e_i$
- , ----- (10.1)

where $E(r_i)$ is the expected return on the security as of the beginning of the holding period, m_i is the impact of unanticipated macro events on I's return during the period, and e_i is the impact of unanticipated firm-specific events.

- b. Both
- m_i and e_i have zero expected values
- because each represents the impact of unanticipated events, which by definition must average out to zero.

- c.
- Single-factor model
- : Different firms have different sensitivities to macroeconomic events. Thus if we denote the unanticipated components of the macro factor by
- F
- , and denote the responsiveness of security I to macro events by beta,
- β_i
- , then the macro

component of security I is $m_i = \beta_i F$,

$$r_i = E(r_i) + \beta_i F + e_i \text{ ----- (10.2)}$$

- d. If we future assume that the rate of return on a broad index of securities such as the S&P 500 is a valid proxy for the common macro factor, then we get a
- single-index model
- because it uses the market index to proxy for the common or systematic factor.

3. Decomposition of the return on i: We can separate the actual or realized return on I into macro (systematic) and micro (firm-specific) components in a manner similar to that in equation 10.2. We write the rate of return on each security as a sum of three components:

- a. α_i : The stock's expected return if the market is neutral, that is, if the market's excess return, $r_M - r_f$, is zero;
- b. $\beta_i(r_M - r_f)$: The component of return due to movements in the overall market: is I's responsiveness to market movements.
- c. e_i : the unexpected component due to unexpected events that are relevant only to I (firm specific)

$$r_i - r_f = \alpha_i + \beta_i(r_M - r_f) + e_i$$

4. Excess return: let us denote excess returns over the risk-free rate by capital R, then
- $R_i = r_i - r_f$

$$R_i = \alpha_i + \beta_i R_M + e_i \text{ ----- (10.3)}$$

5. risk of security I:

Variance of return (excess return) of i: $\sigma_i^2 = \beta_i^2 \sigma_M^2 + \sigma^2(e_i)$ (the covariance between R_M and e_i is zero because e_i is defined as firm specific, that is, independent of movements in the market. The variance has two components: the uncertainty of the common macroeconomic factor ($\beta_i^2 \sigma_M^2$) and the firm-specific uncertainty ($\sigma^2(e_i)$).

6. Covariance between returns on two stocks:

$$\text{cov}(R_i, R_j) = \text{cov}(\alpha_i + \beta_i R_M + e_i, \alpha_j + \beta_j R_M + e_j)$$

Since α_i and α_j are constants, their covariance with any variables are zero. Further, the firm-specific terms (e_i, e_j) are assumed uncorrelated with the market and with each other. Therefore, the only source of covariance in the returns between the two stocks derives from their common dependence on the common factor, R_M .

$$\text{cov}(R_i, R_j) = \text{cov}(\beta_i R_M, \beta_j R_M) = \beta_i \beta_j \sigma_M^2$$

$$\text{cov}(R_i, R_M) = \text{cov}(\beta_i R_M + e_i, R_M) = \beta_i \sigma_M^2$$

$$\beta_i = \frac{\text{cov}(R_i, R_M)}{\sigma_M^2}$$

7. The magic of simplification: if we plan to analyze n stocks, we have to estimate n (estimates of expected returns) + n (estimates of sensitivity coefficients, β_i) + n (estimates of firm-specific variances $\text{var}(e_i)$) + 1 estimate for the variance of the common macroeconomic factor, $\sigma_M^2 = 3n + 1$. If $n = 50$, we will estimate 151 estimates.
8. The cost of the model: the classification of uncertainty into a simple dichotomy – macro versus micro risk – oversimplifies sources of real-world uncertainty and misses some important sources of dependence in stock returns. For example, this dichotomy rules out industry events, events that may affect many firms within an industry without substantially affecting the broad macroeconomy.
9. The index model and diversification:

Let $R_p = \sum_{i=1}^n w_i R_i$, and assume $w_i = 1/n$, plug 10.3 into it:

$$R_p = \frac{1}{n} \sum_{i=1}^n (\alpha_i + \beta_i R_M + e_i)$$

$$\sum w_i = 1$$

$$r_p = E(r_p) + \beta_p F + e_p$$

$$\beta_p = \sum w_i \beta_i$$

Let $\alpha_p = \frac{1}{n} \sum_{i=1}^n \alpha_i$, $\beta_p = \frac{1}{n} \sum_{i=1}^n \beta_i$, $e_p = \frac{1}{n} \sum_{i=1}^n e_i$, hence the portfolio's variance is

$$\sigma_p^2 = \beta_p^2 \sigma_M^2 + \sigma^2(e_p)$$

- ◆ The systematic risk depends on portfolio beta and variance of market portfolio and will persist regardless of the extent of portfolio diversification. No matter how many stocks are held, their common exposure to the market will be reflected in portfolio systematic risk.
- ◆ The nonsystematic risk is $\sigma^2(e_p)$. Because e_i s are independent with zero expected values, the law of averages can be applied to conclude that as more and more stocks are added to the portfolio, the firm-specific components tend to cancel out, resulting in ever-smaller nonmarket risk. Such risk is thus termed **diversifiable**.

III. The CAPM and the index model:

1. The CAPM is a statement about ex ante or expected returns, whereas in practice all we can observe directly are ex post or realized return. To make the leap from expected to realized returns, we can employ the index model:

$$R_i = \alpha_i + \beta_i R_M + e_i$$

$$2. \text{ since } \text{cov}(R_i, R_M) = \text{cov}(\beta_i R_M + e_i, R_M) = \beta_i \sigma_M^2 \rightarrow$$

$$\beta_i = \frac{\text{cov}(R_i, R_M)}{\sigma_M^2} \rightarrow \text{the index model beta coefficient turns out to be the same beta as that}$$

of the CAPM expected return-beta relationship, except that we replace the (theoretical) market portfolio of the CAPM with the well-specified and observable market index.

3. The index model and the expected return-beta relationship:

Since $r_i - r_f = \alpha_i + \beta_i(r_M - r_f) + e_i$, if the index M represents the true market portfolio, we can take the expectation of each side of the equation to show that the index model specification is:

$$E(r_i) - r_f = \alpha_i + \beta_i[E(r_M) - r_f], \text{ compare this with CAPM:}$$

$$E(r_i) - r_f = \beta_i[E(r_M) - r_f]$$

$$E(r_P) = r_f + \beta_P[E(r_M) - r_f] \rightarrow \text{CAPM predicts that alpha should be zero for all}$$

assets. The alpha of a stock is its expected return in excess of (or below) the fair expected return as predicted by the CAPM. If the stock is fairly priced, its alpha must be zero. The CAPM states that the expected value of alpha is zero for all securities, whereas the index model representation of the CAPM holds that the realized value of alpha should averaged out to zero for a sample of historical observed returns. The sample alphas should be unpredictable, that is, independent from one sample period to the next.

IV. The industry versions of the index model: Practitioners routinely estimate the index model using total rather excess returns. This makes their estimate of alpha equal to

$$\alpha + r_f(1 - \beta).$$

Betas show a tendency to evolve toward 1 over time. Beta forecasting rules attempt to predict this drift. Moreover, other financial variables can be used to help forecast betas.

V. Index models and tracking portfolios:

1. suppose a portfolio manager believes a portfolio P, has a positive alpha with a beta of 1.4:

but she wants to eliminate systematic risk – because if the market as a whole turns down, she still could lose money on her investment (which has a large positive beta) even if she is correct about the portfolio that it is underpriced.

$$R_P = .04 + 1.4R_{S\&P500} + e_P$$

2. she can construct tracking portfolio (T): T is designed to match the systematic component of P's return. This means that T must have the same beta on the index portfolio as P and as little nonsystematic risk as possible. T includes positions of 1.4 in the S&P 500 and -0.4 in T-bills. Because T is constructed from the index and bills, it has an alpha value of zero.

3. She can long P and short T. The combined portfolio, C, provides a return per dollar

of:

$$R_C = R_p - R_T = (.04 + 1.4R_{S\&P500} + e_p) - 1.4R_{S\&P500} = .04 + e_p$$

While this portfolio is still risky (due to the residual risk), the systematic risk has been eliminated.

4. This long-short strategy is characteristics of the activity of many hedge funds. Hedge fund managers identify an underpriced security and then try to attain a “pure play” on the perceived underpricing. They hedge out all extraneous risk, focusing the bet only on the perceived “alpha.” Tracking funds are the vehicle used to hedge the exposures to which they do not want exposures.

Chapter Eleven: Arbitrage Pricing Theory and multifactor models of risk and return

I. Multifactor models: an overview:

1. Factor models of security returns:
 - a. the single-factor model:

$$r_i = E(r_i) + \beta_{iGDP}GDP + \beta_{iIR}IR + e_i \text{-----}$$

----- (11.1)

Where, the expected value of F is zero. F is the surprise or the deviation of the common factor from its expected value.

- b. The problem of this single factor model is that the systematic or macro factor summarized by the market return arises from a number of sources, for example, uncertainty about the business cycle, interest rates, inflation, and so on. The market return reflects both macro factors as well as the average sensitivity of firms to those factors. When we estimate a single-index regression, therefore, we implicitly impose an (incorrect) assumption that each stock has the same relative sensitivity to each risk factor. If stocks actually differ in their betas relative to the various macroeconomic factors, then lumping all systematic sources of risk into one variable such as the return on the market index will ignore the nuances that better explain individual – stock return.
- c. Therefore, a more explicit representation of systematic risk, allowing for the possibility that different stocks exhibit different sensitivities to its various components, would constitute a useful refinement of the single-factor model.
2. Multifactor models – a model allows for several factors can provide better descriptions of security returns.
 - a. Lets start with a two-factor model: suppose the two most important macroeconomic sources of risk are uncertainties surrounding the state of the business cycle and changes in interest rates.

$$r_i = E(r_i) + \beta_{iGDP}GDP + \beta_{iIR}IR + e_i \text{-----} \quad (11.2)$$

The two macro factors comprise the systematic factors in the economy. Both of these macro factors have **zero expectation**: they represent changes in these variables that have not already been anticipated. The coefficients of each factor measure the sensitivity of share returns to that factor. For this reason the coefficients are sometimes called **factor sensitivities, factor loadings, or factor betas**. e_i reflects firm-specific influences.

- b. However, a multifactor model like equation 11.2 is no more than a description of the factors that affect security returns. There is no “theory” in the equation. The question left unanswered by a factor model is where $E(r)$ comes from, in other words, what determines a security’s expected rate of return. This is where we need a theoretical model of equilibrium security returns.

II. Arbitrage Pricing theory:

1. Stephen Ross developed the arbitrage pricing theory (APT) in 1976. Ross’s APT relies on three key propositions:
 - a. security returns can be described by a factor model;
 - b. there are sufficient securities to diversify away idiosyncratic risk;
 - c. no arbitrage: well-functioning security markets do not allow for the persistence of arbitrage opportunities.
2. an **arbitrage** opportunity arises when an investor can earn riskless profits without making a net investment. The Law of one price states that if two assets are equivalent in all economically relevant respects, then they should have the same market price. The critical property of a risk-free arbitrage portfolio is that any investor, regardless of risk aversion or wealth, will want to take an infinite position in it. Because those large positions will quickly force prices up or down until the opportunity vanishes, security prices should satisfy a “no-arbitrage condition,” that is, a condition that rules out the existence of arbitrage opportunities.
3. Well-diversified portfolios:
 - a. construct an n-stock portfolio with weights w_i , $\sum w_i = 1$, then the rate of return on this portfolio is:

$$r_p = E(r_p) + \beta_p F + e_p, \text{ where, } \beta_p = \sum w_i \beta_i$$

The portfolio variance is:

$$\sigma_p^2 = \beta_p^2 \sigma_M^2 + \sigma^2(e_p)$$

$$\sigma^2(e_p) = \text{var}(\sum w_i e_i) = \sum w_i^2 \sigma^2(e_i)$$

$$\sigma^2(e_p) = \text{var}(\sum w_i e_i) = \sum w_i^2 \sigma^2(e_i) \text{ (notice that firm-specific } e_i \text{ s are uncorrelated)}$$

- b. If the portfolio were equally weighted, $w_i = 1/n$, then the nonsystematic variance would be

$$\sigma^2(e_p) = \text{var}(\sum w_i e_i) = \sum \left(\frac{1}{n}\right)^2 \sigma^2(e_i) = \frac{1}{n} \sum \frac{1}{n} \sigma^2(e_i) = \frac{\overline{\sigma^2(e_i)}}{n}$$

Where, the last term is the average value across securities of nonsystematic variance. The nonsystematic variance of the portfolio equals the average nonsystematic variance divided by n. Therefore, when the portfolio gets large in the sense that n is large, its nonsystematic variance approaches zero. This is the effect of diversification.

- c. This property is true of portfolios other than the equally weighted one. Any portfolio for which each w_i becomes consistently smaller as n gets large will satisfy the condition that the portfolio nonsystematic risk will approach zero.
- d. **Well-diversified portfolio** – the one that is diversified over a large enough number of securities with each weight, w_i , small enough that for practical purposes the

nonsystematic variance, $\sigma^2(e_p)$, is negligible.

- e. **Because the expected value of e_p for any well-diversified portfolio is zero**, and its variance also is effectively zero \rightarrow any realized value of e_p will be virtually zero. We conclude that for a well-diversified portfolio, for all practical purposes:

$$r_p = E(r_p) + \beta_p F$$

4. Betas and expected return: To preclude arbitrage opportunities, the expected return on all well-diversified portfolios must lie on the straight line from the risk-free asset in Figure 11.3. the equation of this line will dictate the expected return on all well-diversified portfolios. And risk premiums are proportional to portfolio betas.
5. One-factor security market line: consider the market index portfolio, M , as a well-diversified portfolio, we can measure the systematic factor as the unexpected return on that portfolio. Because the index portfolio must be on the line and the beta of the index portfolio is 1. therefore, $E(r_p) = r_f + \beta_p[E(r_M) - r_f]$

III. Individual assets and the APT

1. Since under no arbitrage, each well-diversified portfolio's expected excess return must be proportional to its beta, if this relationship is to be satisfied by all well-diversified portfolios, it must be satisfied by almost all individual securities.
2. Imposing the no-arbitrage condition on a single-factor security market implies maintenance of the expected return – beta relationship for all well-diversified portfolios and for all but possibly a small number of individual securities.
3. APT vs. CAPM:
 - a. APT does not require that benchmark portfolio in the SML relationship be the true market portfolio. Any well-diversified portfolio lying on the SML may serve as the benchmark portfolio. Accordingly, the APT has more flexibility than does the CAPM because problems associated with an unobservable market portfolio are not a concern.
 - b. APT provides further justification for use of the index model. Even if the index portfolio is not a precise proxy for the true market portfolio, as long as it is sufficiently well diversified, the SML relationship should still hold true according to the APT.
 - c. CAPM is derived assuming an inherently unobservable “market” portfolio. The CAPM argument rests on mean-variance efficiency; that is, if any security violates the expected return-beta relationship, then many investors (each relatively small) will tilt their portfolios so that their combined overall pressure on prices will restore an equilibrium that satisfies the relationship.
 - d. APT does not fully dominate the CAPM. The CAPM provides an unequivocal statement on the expected return-beta relationship for all securities, whereas the APT implies that this relationship holds for all but perhaps a small number of securities. Because it focuses on the no-arbitrage condition, without the further assumptions of the market or index model, the APT cannot rule out a violation of the expected return-beta relationship for any particular asset. For this, we need the CAPM assumptions and its dominance arguments.

IV. A multifactor APT

1. Establishing a multifactor APT is similar to the one-factor model. But we must

introduce the concept of a factor portfolio, which is a well-diversified portfolio constructed to have a beta of 1 on one of the factors and a beta of 0 on any other factor. We can think of fundamental analysis factor portfolio as a tracking portfolio. That is, the returns on such a portfolio track the evolution of particular sources of macroeconomic risk, but are uncorrelated with other sources of risk. Factor portfolios will serve as the benchmark portfolios for a multifactor security market line.

$$2. \quad E(r_P) = r_f + \beta_{P1}[E(r_1) - r_f] + \beta_{P2}[E(r_2) - r_f] + \dots$$

V. Where should we look for factors

1. One shortcoming of the multifactor APT is that it gives no guidance concerning the determination of the relevant risk factors or their risk premiums.
2. Two principles guide us when we specify a reasonable list of factors:
 - a. we want to limit ourselves to systematic factors with considerable ability to explain security returns.
 - b. We wish to choose factors that seem likely to be important risk factors, i.e., factors that concern investors sufficiently that they will demand meaningful risk premiums to bear exposure to those sources of risk.
3. Chen, Roll and Ross (86) choose five macroeconomic factors: % change in industrial production, % change in expected inflation, % change in unanticipated inflation, excess return of long-term corporate bonds over long-term government bonds, excess return of long-term government bonds over T-bills.

VI. **A multifactor CAPM:** is a model of the risk-return trade-off that predicts the same multidimensional security market line as the APT. the ICAPM suggests that priced risk factors will be those sources of risk that lead to significant hedging demand by a substantial fraction of investors.

Chapter Twelve: Market Efficiency and Behavioral Finance

I. Random walks and the efficient market hypothesis.

1. A forecast about favorable future performance leads instead to favorable current performance, as market participants all try to get in on the action before the price jump. More generally, any information that could be used to predict stock performance should already be reflected in stock prices. As soon as there is any information indicating that a stock is underpriced and therefore offers a profit opportunity, investors flock to buy the stock and immediately bid up its price to a fair level, where only ordinary rates of return can be expected. These “ordinary rates” are simply rates of return commensurate with the risk of the stock.
2. If prices are bid immediately to fair levels, given all available information, it must be that they increase or decrease **only in response to new information**. **New information, by definition, must be unpredictable**; if it could be predicted, then the prediction would be part of today’s information. Thus stock prices that change in response to new (unpredictable) information also must move unpredictably. **This is the essence of the argument that stock prices should follow a random walk**, that is, that price changes should be random and unpredictable.

3. **Efficient market hypothesis (EMH)**: stock prices already reflect all available information. Efficient market is one in which information is rapidly disseminated and reflected in prices.
4. Competition among these many well-backed, highly paid, aggressive analysts ensures that, as a general rule, stock prices ought to reflect available information regarding their proper levels.
5. **Three versions of the Efficient Market Hypothesis**: These versions differ by their notions of what is meant by the term “all available information.”
 - a. the **Weak-form** hypothesis: stock prices already reflect all information that can be derived by examining market trading data such as the history of past prices, trading volume, or short interest. **This version implies that trend analysis is fruitless.** Past stock price data are publicly available and virtually costless to obtain. The weak-form hypothesis holds that if such data ever conveyed reliable signals about future performance, all investors already would have learned to exploit the signals. Ultimately, the signals lose their value as they become widely known because a buy signal, for instance, would result in an immediate price increase.
 - b. The **Semistrong-form** hypothesis: all publicly available information regarding the prospects of the firm must be reflected already in the stock price. Such information includes, in addition to past prices, fundamental data on the firm’s product line, quality of management, balance sheet composition, patents held, earning forecasts, and accounting practices. If investors have access to such information from publicly available sources, one would expect it to be reflected in stock prices.
 - c. **The Strong-form hypothesis**: stock prices reflect all information relevant to the firm, even including information available only to company insiders. This version of the hypothesis is quite extreme.

II. **Implications of the EMH**

1. **Technical analysis**: is essentially the search for recurrent and predictable patterns in stock prices. Technicians believe that information is not necessary for a successful trading strategy. This is because whatever the fundamental reason for a change in stock price, if the stock price responds slowly enough; the analyst will be able to identify a trend that can be exploited during the adjustment period. The **key to successful technical analysis is a sluggish response of stock prices to fundamental supply-and-demand factors.** This prerequisite is diametrically opposed to the notion of an efficient market.

Q: whether any “successful” technical rule can sustain the test of time?

A: An astounding “no”! – Once a useful technical rule (or price pattern) is discovered, it ought to be invalidated when the mass of traders attempts to exploit it – self-destructing.

2. **Fundamental analysis**: uses earnings and dividend prospects of the firm, expectations of future interest rates, and risk evaluation of the firm to determine proper stock prices. Ultimately, it represents an attempt to determine the present discounted value of all the payments a stockholder will receive from each share of stock. If that value exceeds the stock price, the fundamental analyst would recommend purchasing the stock.

The hope of fundamental analysis is to attain insight into future performance of the firm that is not yet recognized by the rest of the market. The EMH predicts that *most* fundamental analysis is doomed to failure. There are many well-informed, well-financed firms conducting such market research, and in the face of such competition it will be difficult to uncover data not also available to other analysts. Only analysts with a unique insight will be rewarded.

Discovery of good firms does an investor no good in and of itself if the rest of the market also knows those firms are good. If the knowledge is already public, the investor will be forced to pay a high price for those firms and will not realize a superior rate of return.

The trick is not to identify firms that are good, but to find firms that are better than everyone else's estimate.

3. **Active versus passive portfolio management: Proponents of the efficient market hypothesis believe that active management is largely wasted effort and unlikely to justify the expenses incurred.** Therefore, they advocate a passive investment strategy that makes no attempt to outsmart the market. A passive strategy aims only at establishing a well-diversified portfolio of securities without attempting to find under- or overvalued stocks. **Passive management is usually characterized by a buy-and-hold strategy.** Because the efficient market theory indicates that stock prices are at fair levels, given all available information, it makes no sense to buy and sell securities frequently, which generates large brokerage fees without increasing expected performance. **One common strategy is to create an index fund.**
4. There is a role for portfolio management even in an efficient market. Investors' optimal positions will vary according to factors such as age, tax bracket, risk aversion, and employment. The role of the portfolio manager in an efficient market is to tailor the portfolio to these needs, rather than to beat the market.

III. **Event Studies**

1. An event study describes a technique of empirical financial research that enables an observer to assess the impact of a particular event on a firm's stock price.
2. For example, people may want to study the impact of dividend changes on stock prices. Since on any particular day stock prices respond to all kinds of economic news such as updated forecasts for GDP, inflation rates, interest rates, or corporate profitability, isolating the part of a stock price movement that is attributable to a dividend announcement is difficult.
3. **The common statistical approach is a marriage of efficient market theory with the index model:**

$$r_t = a + br_{Mt} + e_t,$$

where r_{Mt} is the market's rate of return during the period and e_t is the part of a security's return resulting from firm-specific events. b measures sensitivity to the market return, and a is the average rate of return the stock would realize in a period with a zero market return.

The residual, e_t , is the stock's return over and above what one would predict based on board market movements in that period, given the stock's sensitivity to the market.

Therefore, we sometimes refer to the term e_t as the **abnormal return** – the return beyond what would be predicted from market movements alone:

Predicted return: $\bar{a} + \bar{b}r_{Mt}$,

Abnormal return: $e_t = r_t - (\bar{a} + \bar{b}r_{Mt})$

Q: Is it enough? How about industry-wide movement during the period?

4. **The general strategy in event studies is to estimate the abnormal return around the date that new information about a stock is released to the market and attribute the abnormal stock performance to the new information.** One shortcoming of this approach is from “leakage” of information. Leakage occurs when information regarding a relevant event is released to a small group of investors before official public release. In this case the stock price might start to increase days or weeks before the official announcement date. Any abnormal return on the announcement date is then a poor indicator of the total impact of the information release.
5. A better indicator would be the **cumulative abnormal return**, which is the sum of all abnormal returns over the time period of interest. The cumulative abnormal return thus captures the total firm-specific stock movement for an entire period when the market might be responding to new information.

IV. **Are Markets efficient?**

After the fact there will have been at least one successful investment scheme. A doubter will call the results luck, the successful investor will call it skill. The proper test would be to see whether the successful investors can repeat their performance in another period.

1. **Weak-form tests: patterns in stock returns.**
 - a. **Could speculators find trends in past prices that would enable them to earn abnormal profits?** This is essentially a test of the efficacy of technical analysis.
 - b. **Measurement:** serial correlation of stock market returns. It refers to the tendency for stock returns to be related to past returns. **Positive serial correlation** means that positive returns tend to follow positive returns (a momentum type of property). **Negative serial correlation** means that positive returns tend to be followed by negative returns (a reversal or “correction” property).
 - c. **Returns over short horizons:**
 - Lo and MacKinlay (1988) examine weekly returns of NYSE stocks and find positive serial correlation over short horizons. However, the correlation coefficients of weekly returns tend to be fairly small and the evidence does not clearly suggest the existence of trading opportunities.
 - Jegadeesh and Titman (1993), in an investigation of intermediate-horizon (using 3- to 12-month holding periods) stock price behavior, they found a momentum effect in which good or bad recent performance of particular stocks continues over time. They conclude that while the performance of individual stocks is highly unpredictable, **portfolios of the best-performing stocks in the recent past appear to outperform other stocks with enough reliability to offer profit opportunities.** Thus, it appears that **there is evidence of short- to intermediate-horizon price momentum in both**

the aggregate market and cross-sectionally.**d. Returns over long horizons (over multiyear periods):**

- Fama and French (1988, JPE) and Poterba and Summers (1988, JFE) find **suggestions of pronounced negative long-term serial correlation in performance of the aggregate market.** This result has given rise to a “fads hypothesis”, which asserts that the stock market might overreact to relevant news. Such overreaction leads to positive serial correlation (momentum) over short time horizons. Subsequent correction of the overreaction leads to poor performance following good performance and vice versa. These episodes of apparent overshooting followed by correction gives the stock market the appearance of fluctuating around its fair value.

An alternative interpretation: they indicate only that the market risk premium varies over time – market is still efficient.

- DeBondt and Thaler (1985): Over long horizons, extreme performance in particular securities also tends to reverse itself. They rank order the performance of stocks over a 5-year period and then group stocks into portfolios based on investment performance, the base-period “loser” portfolio (defined as the 35 stocks with the worst investment performance) outperformed the “winner” portfolio (the top 35 stocks) by an average of 25% (cumulative return) in the following 3-year period. This reversal effect, in which losers rebound and winners fade back, suggests that the stock market overreacts to relevant news. After the overreaction is recognized, extreme investment performance is reversed.

Q: what really causes the reversal effect? Maybe the loser hires new and more competent CEO and its performance gets better under the new management team. Maybe the CEO of the winner becomes more entrenched? Or maybe it is just because the life cycle of a company. A start-up may show great growth in the first five years. After 5 years, it becomes more mature and there are no many opportunities as before... (**a project**)

e. Predictors of broad market returns:

- Fama and French (1988) show that the return on the aggregate stock market tends to be higher when the dividend/price ratio is high.
- Campbell and Shiller (1988) found that the earnings yield can predict market returns. The interpretation is difficult. On the one hand, they may imply that stock returns can be predicted, in violation of the efficient market hypothesis. More probably, however, these variables are proxying for variation in the market risk premium. For example, given a level of dividends or earnings, stock prices will be lower and dividend and earnings yields will be higher when the risk premium (and therefore the expected market return) is higher. This does not indicate a violation of market efficiency. The predictability of market returns is due to predictability in the risk premium, not in risk-adjusted abnormal returns.

2. Semistrong Tests: market anomalies

- Investigations of the efficacy of fundamental analysis ask whether publicly available information beyond the trading history of a security can be used to improve investment performance, and therefore are tests of semistrong-form market efficiency.
- The difficulty of interpreting efficient market anomalies: any test of risk-adjust returns are joint tests of the efficient market hypothesis and the risk adjustment**

procedure. Why? – anomalies = real return – risk-adjusted expected return, which comes from a risk adjustment procedure, say, CAPM. If it appears that a portfolio strategy can generate superior returns, we must then choose between rejecting the EMH and rejecting the risk adjustment technique. Usually, the risk adjustment technique is based on more-questionable assumptions than is the EMH; by opting to reject the procedure, we are left with no conclusion about market efficiency.

c. **The small-firm-in-January effect** (so-called size or small-firm effect):

- The effect was originally documented by Banz (1981, JFE). If we divide the NYSE stocks into 10 portfolios each year according to firm size, average annual returns are consistently higher on the small-firm portfolios. Even on the risk-adjusted basis, the smallest-size portfolio outperforms the largest-firm portfolio by an average 4.3% annually for the period of 1926-2000. Later studies showed that the small-firm effect occurs virtually entirely in January, in fact, in the first 2 weeks of January. The size effect is in fact a “**small-firm-in-January**” effect.
- Some believe that the January effect is tied to tax-loss selling at the end of the year. The hypothesis is that many people sell stocks that have declined in price during the previous months to realize their capital losses before the end of the tax year. Such investors do not put the proceeds from these sales back into the stock market until after the turn of the year. At that point the rush of demand for stock places an upward pressure on prices that results in the January effect.
- Arbel and Strebel (1983) gave another interpretation – “**neglected-firm effect**”: Because small firms tend to be neglected by large institutional traders, information about smaller firms is less available. This information deficiency makes smaller firms riskier investments that command higher returns.

d. **Book-to-market ratios:**

- Fama and French (1992, JF) stratified firms into 10 groups according to book-to-market equity ratios and examined the average monthly rate of return of each of the 10 groups during the period July 1963 through December 1990. The decile with the highest book-to-market ratio had an average monthly return of 1.65%, while the lowest-ratio decile averaged only .72% per month. The dramatic dependence of returns on book-to-market ratio is independent of beta, suggesting either that high book-to-market ratio firms are relatively underpriced, or that the **book-to-market ratio is serving as a proxy for a risk factor that affects equilibrium expected returns**. In fact FF found that after controlling for the size and book-to-market effects, beta seemed to have no power to explain average security returns. This finding is an important challenge to the notion of rational markets, since it seems to imply that a factor that should affect returns – systematic risk – seems not to matter, while a factor that should not matter – the book-to-market ratio – seems capable of predicting future returns.

3. **Strong-form tests: inside information:** We do not expect markets to be strong-form efficient; we regulate and limit trades based on inside information. Insiders are able to make superior profits trading in their firm’s stock.

4. Interpreting the evidence – How should we interpret the ever-growing anomalies literature?

a. Risk premiums or inefficiencies?

- ♦ The price-earnings, small-firm, market-to-book, momentum, and long-term reversal effects are currently among the most puzzling phenomena in empirical

finance. The feature that small firms, low-market-to-book firms, and recent “losers” seem to have in common is a stock price that has fallen considerably in recent months or years. Indeed, a firm can become a small firm or a low-market-to-book firm by suffering a sharp drop in price. These groups therefore may contain a relatively high proportion of distressed firms that have suffered recent difficulties. Fama and French argue that these effects can be explained as manifestations of risk premiums. They propose a **three-factor model**, in the spirit of arbitrage pricing theory. Risk is determined by the sensitivity of a stock to three factors: **(1) the market portfolio, (2) a portfolio that reflects the relative returns of small versus large firms, and (3) a portfolio that reflects the relative returns of firms with high versus low ratios of book value to market value.** While size or book-to-market ratios per se are obviously not risk factors, they perhaps might act as proxies for more fundamental determinants of risk. Fama and French argue that these patterns of returns may therefore be consistent with an efficient market in which expected returns are consistent with risk.

- ◆ Lakonishok, Shleifer, and Vishney (1995) argue that these phenomena are evidence of inefficient markets, more specifically, of systematic errors in the forecasts of stock analysts. They believe that analysts extrapolate past performance too far into the future, and therefore overprice firms with recent good performance and underprice firms with recent poor performance. Ultimately, when market participants recognize their errors, prices reverse.

b. Anomalies or Data mining?

- Some anomalies have not shown much staying power after being reported in the academic literature. For example, after the small-firm effect was published in the early 1980s, it promptly disappeared for much of the rest of the decade. Similarly, the book-to-market strategy, which commanded considerable attention in the early 1990s, was ineffective for the rest of that decade.
- Still, even acknowledging the potential for data mining, there is a real puzzle to explain. Value stocks – defined by low P/E ratio, high book-to-market ratio, or depressed prices relative to historic levels – seem to have provide higher average returns than “glamour” or growth stocks.

V. A Behavioral Interpretation

1. The premise of behavioral finance is that conventional financial theory ignores how real people make decisions and that people make a difference. Many economists have come to interpret the anomalies literature as consistent with several “irrationalities” individuals exhibit when making complicated decisions. **These irrationalities stem from two main premises: first, that investors do not always process information correctly and therefore infer incorrect probability distributions about future rates of return; and second, that even given a probability distribution of returns, investors often make inconsistent or systematically suboptimal decisions.**
2. The second leg of the behavioral critique is that **in practice the actions of such arbitrageurs are limited.** Virtually everyone agrees that if prices are right, then there are no easy profit opportunities. But the reverse is not necessarily true. If behaviorists

are correct about limits to arbitrage activity, then the absence of profit opportunities does not necessarily imply that markets are efficient.

3. Information processing: Errors in information processing can lead investors to misestimate the true probabilities of possible events or associated rates or return.
 - a. **Forecasting errors:** people give too much weight to recent experience compared to prior beliefs when making forecasts and tend to make forecasts that are too extreme given the uncertainty inherent in their information. DeBondt and Thaler argue that P/E effect can be explained by earnings expectations that are too extreme. When forecasts of a firm's future earnings are high, perhaps due to favorable recent performance, they tend to be too high relative to the objective prospects of the firm. This results in a high initial P/E and poor subsequent performance when investors recognize their error.
 - b. **Overconfidence:** People tend to overestimate the precision of their beliefs or forecasts, and they tend to overestimate their abilities. In a survey, 90% of drivers in Sweden ranked themselves as better-than-average drivers. Such overconfidence may be responsible for the prevalence of active versus passive investment management.
 - c. **Conservatism:** A conservatism bias means that investors are too slow (too conservative) in updating their beliefs in response to recent evidence. This means that they might initially underreact to news about a firm, so that prices will fully reflect new information only gradually. Such a bias would give rise to momentum in stock market returns.
 - d. **Sample size neglect and representativeness:** People commonly do not take into account the size of a sample, apparently reasoning that a small sample is just as representative of a population as a large one. They may therefore infer a pattern too quickly based on a small sample and extrapolate apparent trends too far into the future. Such a pattern is consistent with overreaction and correction anomalies.
4. **Behavioral biases:** Even if information processing were perfect, individuals might make less-than-fully rational decisions using that information. These behavioral biases largely affect how investors frame questions of risk versus return and therefore make risk-return trade-offs.
 - a. **Framing:** Decisions seem to be affected by how choices are framed. Famous studies by Kahneman and Tversky (1979) find an individual may reject a bet when it is posed in terms of the risk surrounding possible gains but may accept that same bet when described in terms of the risk surrounding potential losses. In other words, individuals may act risk-averse in terms of gains but risk-seeking in terms of losses.
 - b. **Mental accounting:** it is a specific form of framing in which people segregate certain decisions. For example, an investor may take a lot of risk with one investment account but establish a very conservative position with another account that is dedicated to her child's education. Rationally, it might be better to view both accounts as part of the investor's overall portfolio with the risk-return profiles of each integrated into a unified framework.
 - c. **Regret avoidance:** Individuals who make decisions that turn out badly have more regret when that decision was more unconventional. For example, buying a blue-chip portfolio that turns down is not as painful as experiencing the same losses on an unknown start-up firm. Any losses on the blue-chip stocks can be more easily

attributed to bad luck rather than bad decision-making and cause less regret.

5. **Limits to arbitrage:** These behavioral biases would not matter for stock pricing if rational investors could fully profit from the mistakes of behavioral investors. However, behavioral advocates argue that in practice, several factors limit the ability to profit from misprice.
 - a. **Fundamental risk:** Buying a seemingly underpriced stock is hardly risk-free, since the presumed market underpricing can get worse. While price eventually should converge to intrinsic value, this may not happen until after the investor's investment horizon. For example, the investor may be a mutual fund manager who may lose clients (not to mention a job) if short-term performance is poor or a trader who may run through her capital if the market turns against her even temporarily. Risk incurred in exploiting the apparent profit opportunity presumably will limit both the activity and effectiveness of these arbitrage traders.
 - b. **Implementation costs:** exploiting overpricing can be difficult. Short selling a security entails costs; short-sellers may have to return the borrowed security on little notice, rendering the horizon of the short sale uncertain; and some investors such as pension or mutual fund managers are simply not allowed to short securities.
 - c. **Model risk:** One always has to worry that an apparent profit opportunity is more apparent than real. Perhaps you are using a faulty model to value the security, and the price actually is right.

- VI. **Mutual Fund performance:** For investors, the issue of market efficiency boils down to whether skilled investors can make consistent abnormal trading profits.
1. Between 1972 and 2001 the returns of a passive portfolio indexed to the Wilshire 5000 typically would have been better than those of the average equity fund.
 2. On the other hand, there was some evidence (admittedly inconsistent) of persistence in performance, meaning that the better managers in one period tended to be better managers in following periods.
 3. Blake, Elton, and Gruber (93) found that, on average, bond funds underperform passive fixed-income indexes by an amount roughly equal to expenses, and that there is no evidence that past performance can predict future performance. Their evidence is consistent with the hypothesis that bond managers operate in an efficient market in which performance before expenses is only as good as that of a passive index.

VII. **conclusions**

1. The performance of professional managers is broadly consistent with market efficiency. The amounts by which professional managers as a group beat or are beaten by the market fall within the margin of statistical uncertainty.
2. Markets are very efficient, but that rewards to the especially diligent, intelligent, or creative may in fact be waiting.

Chapter Fourteen: Bond prices and yields

- I. A **debt security** is a claim on a specified periodic stream of income. Debt

securities are often called fixed-income securities because they promise either a fixed stream of income or a stream of income that is determined according to a specified formula. Risk considerations are minimal as long as the issuer of the security is sufficiently creditworthy.

II. Bond Characteristics:

1. A **Bond** is a security that is issued in connection with a borrowing arrangement. The borrower issues a bond to the lender for some amount of cash; the bond is the “IOU” of the borrower. The arrangement obligates the issuer to make specified payments to the bondholder on specified dates. A typical coupon bond obligates the issuer to make semiannual payments of interest to the bondholder for the life of the bond. When the bond matures, the issuer repays the debt by paying the bondholder the bond’s **par value (face value)**. The **coupon rate** serves to determine the interest payment: The annual payment is the coupon rate times the bond’s par value.
2. The coupon rate, maturity date, and par value of the bond are part of the **bond indenture**, which is the contract between the issuer and the bondholder.
3. **Zero-coupon bonds:** bonds with no coupon payments. These bonds are issued at prices considerably below par value, and the investor’s return comes solely from the difference between issue price and the payment of par value at maturity.
4. **Accrued interest and quoted bond price:** Quoted prices are not actually the prices that investors pay for the bond. This is because the quoted price does not include the interest that accrues between coupon payment dates.

Invoice price = quoted price + accrued interest

In general, $\text{Accrued interest} = (\text{annual coupon payment}/2) * (\text{days since last coupon payment}/\text{days separating coupon payments})$.

5. Corporate bonds:
 - a. **Call provisions:** some corporate bonds are issued with call provisions allowing the issuer to repurchase the bond at a specified **call price** before the maturity date. For example, if a company issues a bond with a high coupon rate when market rates are high, and interest rates later fall, the firm might like to retire the high-coupon debt and issue new bonds at a lower coupon rate to reduce interest payments. This is called **refunding**. Callable bonds typically come with a period of call protection, an initial time during which the bonds are not callable. Such bonds are referred to as **deferred callable bonds**.
 - b. **Convertible bonds:** which give bondholders an option to exchange each bond for a specified number of shares of common stock of the firm. The conversion ratio is the number of shares for which each bond may be exchanged. The **market conversion value** is the current value of the shares for which the bonds may be exchanged. If a stock price is \$20 and the conversion ratio is 40, then the market conversion value is \$800. The **conversion premium** is the excess of the bond value over its conversion value. If bond price is \$1000, the conversion premium is \$200.
 - c. **Puttable bond:** it gives bondholder the option to extend or retire the bond at the exercise price. If the bond’s coupon rate is too low relative to current interest rates, it will be optimal not to extend.
 - d. **Floating rate bonds:** make interest payments that are tied to some measure of current

market rates. For instance, the rate might be adjusted annually to the current T-bill rate plus 2%. The major risk involved in floaters has to do with changes in the firm's financial strength. The yield spread is fixed over the life of the security. If the financial health of the firm deteriorates, then a greater yield premium would be called for than is offered by the security. In this case, the price of the bond would fall.

6. Innovation in the bond market:
 - a. **Inverse floaters:** they are similar to the floating-rate bonds, except that the coupon rate falls when the general level of interest rates rises. Investors in these bonds suffer doubly when rates rise. Not only does the PV of cash flow from the bond fall as the discount rate rises, but the level of those cash flows falls as well. Of course, investors in these bonds benefit doubly when rates fall.
 - b. **Asset-backed bonds:** Walt Disney has issued bonds with coupon rates tied to the financial performance of several of its films.
 - c. **Catastrophe bonds:** Winterthur has issued a bond whose payments depend on whether there has been a severe hailstorm in Switzerland. These bonds are a way to transfer "catastrophe risk" from the firm to the capital markets.
 - d. **Index bonds:** these make payments that are tied to a general price index or the price of a particular commodity. Mexico has issued 20-year bonds with payments that depend on the price of oil. US Treasury started issuing inflation-indexed bonds (Treasury Inflation Protected Securities (TIPs)) in January 1997.

III. Bond pricing:

1. **Bond value** = Present value of coupons + present value of par value. If we assume the interest rates are the same for each period ("flat"), then

$$\text{Bond Value} = \sum_{t=1}^T \frac{\text{coupon}}{(1+r)^t} + \frac{\text{Par value}}{(1+r)^T}$$

It can be showed that if $r = \text{coupon rate}$, then bond value = par value.

2. The bond prices **fluctuate inversely with the market interest rate**. This inverse relationship between price and yield is a central feature of fixed-income securities. Interest rate fluctuations represent the main source of risk in the fixed-income market.
3. A general rule in evaluating bond price risk is that, keeping all other factors the same, the longer the maturity of the bond, the greater the sensitivity of price to fluctuations in the interest rate.

IV. Bond yields:

1. **Current yield:** the ratio of coupon interest to current bond price.
2. **Yield to maturity:** YTM is defined as the interest rate that makes the present value of

a bond's payments equal to its price.
$$B = \sum_{t=1}^T \frac{C_t}{(1+y)^t} + \frac{P}{(1+y)^T}$$

- a. YTM is the promised yield on a bond. It is obtained if the bond is held to maturity, and if the coupon payments are invested at YTM.
- b. YTM is the internal rate of return (IRR) on an investment in the bond.
- c. It is widely accepted as a proxy for average return.
- d. Corporate bonds typically are issued at par value. This means that the underwriters of the bond issue must choose a coupon rate that very closely approximates market yields.

e. Yields annualized using simple interest are called Bond equivalent yield (annual percentage rate (APR)): also appears in **financial pages as “yield”**. This is the bill’s yield over its life, assuming that it is purchased for the asked price.

f. Compounded, or effective annual yield: accounts for compound interest.

If one earns 3% interest every 6 month, then bond equivalent yield = $3 \times 2\% = 6\%$, effective annual yield (YTM) = $(1+3\%)*(1+3\%)-1 = 6.09\%$.

3. For **premium bonds** (bonds selling above par value), coupon rate is greater than current yield, which in turn is greater than YTM. For **discount bonds**, these relationships are reversed. For bonds selling at par value, coupon rate is equal to YTM.
4. **Yield to call:** if the call price of a callable bond is less than the present value of the scheduled payments, the issuer can call the bond back from the bondholder. The yield to call is calculated just like the YTM except that the time until call replaces time until maturity, and the call price replaces the par value. This computation is sometimes called “yield to first call”, as it assumes the bond will be called as soon as the bond is first callable.

A callable premium bond is especially apt to be called if rates fall further. Therefore, it is likely to provide a lower return than could be earned on a discount bond whose potential price appreciation is not limited by the likelihood of a call. Investors in premium bonds often are more interested in the bond’s yield to call rather than YTM as a consequence, because it may appear to them that the bond will be retired at the call date.

V. Bond prices over time:

1. When the YTM is unchanged over the period, the **holding period return** on the bond will equal YTM. This is not surprising. Security returns all should be comparable on after-tax risk-adjusted basis. The bond must offer a rate of return competitive with those available on other securities.
2. However, when yields fluctuate, so will a bond’s rate of return. Unanticipated changes in market rates will result in unanticipated changes in bond returns. An increase in the bond’s yield acts to reduce its price, which means that the holding period return will be less than the initial yield. Conversely, a decline in yield will result in a holding-period return greater than the initial yield.
3. **Zero-coupon bond:** carries no coupons and provides all its return in the form of price appreciation.
 - a.** Treasury bills are examples of **short-term zero-coupon** instruments.
 - b.** **Long-term zero-coupon** bonds are commonly created from coupon-bearing notes and bonds with the help of U.S. Treasury. For example, a 10-year coupon bond would be “stripped” of its 20 semiannual coupons, and each coupon payment would be treated as a stand-alone zero-coupon bond. The maturities of these bonds would thus range from 6 months to 10 years. The final payment of principal would be treated as another stand-alone zero-coupon security. Each of the payments is now treated as an independent tradable security. The Treasury program under which coupon stripping is performed is called STRIPS (Separate Trading of Registered Interest and

Principal of Securities), and these zero-coupon securities are called **Treasury strips**.

- c. Prices of zeros as time passes: before maturity, they should sell at discounts from par, because of the time value of money. As time passes, price should approach par value. If the interest rate is constant, a zero's price will increase at exactly the rate of interest.

VI. Default risk and bond pricing

1. Bond **default risk**, usually called **credit risk**: Those rated BBB or above (S&P, Duff & Phelps, Fitch) or Baa and above (Moody's) are considered investment-grade bonds, whereas lower-rated bonds are classified as speculative-grade or junk bonds. Almost half of the bonds that were rated CCC by S&P at issue have defaulted within 10 years.
2. **Junk bonds**, also known as high-yield bonds: before 1977, almost all junk bonds were "fallen angels," that is, bonds issued by firms that originally have investment-grade ratings but that had since been downgraded. In 1977, however, firms began to issue "original-issue junk." Junk issues were a lower-cost financing alternative than borrowing from banks from firms which are not able to muster an investment-grade rating. High-yield bonds gained considerable notoriety in the 1980s when they were sued as financing vehicles in leveraged buyouts and hostile takeover attempts.
3. **Determinants of bond safety**: bond rating agencies base their quality ratings largely on an analysis of the level and trend of some of the issuer's financial ratios:
 - a. **Coverage ratios**: ratios of company earnings to fixed costs. E.g. EBIT interest coverage ratio = $\text{EBIT} / \text{interest}$. The fixed-charge coverage ratio = $\text{earnings} / \text{all fixed cash obligations}$ (including interest, lease payments and sinking fund payments).
 - b. **Leverage ratio**: debt to equity ratio.
 - c. **Liquidity ratios**: current ratio = $\text{current assets} / \text{current liabilities}$. Quick ratio = $\text{current assets excluding inventories} / \text{current liabilities}$.
 - d. **Profitability ratios**: measures of rates of return on assets or equity. $\text{ROA} = \text{EBIT} / \text{total assets}$.
 - e. **Cash flow-to-debt ratio**: $\text{cash flow} / \text{outstanding debt}$.
4. **Bond indentures**: it is the contract between the issuer and the bondholder. Part of the indenture is a set of restrictions that protect the rights of the bondholders. Such restrictions include provisions relating to collateral, sinking funds, dividend policy, and further borrowing:
 - a. **Sinking funds**: Bonds call for the payment of par value at the end of the bond's life. This payment constitutes a large cash commitment for the issuer. The firm agrees to establish a sinking fund to spread the payment burden over several years. The fund may operate in one of two ways:
 - 1). The firm may repurchase a fraction of the outstanding bonds in the open market each year.
 - 2). The firm may purchase a fraction of the outstanding bonds at a special call price associated with the sinking fund provision. The firm has an option to purchase the bonds at either the market price or the sinking fund price, whichever is lower. To allocate the

burden of the sinking fund call fairly among bondholders, the bonds chosen for the call are selected at random based on serial number.

- b. **Subordination Clauses:** restrict the amount of additional borrowing. Additional debt might be required to be subordinated in priority to existing debt; that is, in the event of bankruptcy, subordinated or junior debtholders will not be paid unless and until the prior senior debt is fully paid off.
 - c. **Dividend restrictions:** covenants also limit the dividends firms are allowed to pay. These limitations protect the bondholders because they force the firm to retain assets rather than paying them out to stockholders.
 - d. **Collateral:** some bonds are issued with specific collateral behind them. If the collateral is property, the bond is called a **mortgage bond**. If the collateral takes the form of other securities held by the firm, the bond is a collateral trust bond. In the case of equipment, the bond is called an **equipment obligation bond**. Collateralized bonds generally are considered the safest variety of corporate bonds. General debenture bonds by contrast do not provide for specific collateral; they are unsecured bonds.
5. **Default premium:** to compensate for the possibility of default, corporate bonds must offer a default premium, which is the difference between the promised yield on a corporate bond and the yield of an otherwise-identical government bond that is riskless in terms of default.

Chapter Fifteen: The Term Structure of Interest Rates

- I. In chapter 14 we assumed that the same constant interest rate is used to discount cash flows of any maturity. In reality, rates with different maturities are usually different. The relationship between time to maturity and YTM can vary dramatically from one period to another, though, usually, the longer-term securities have higher yields. In this chapter, we study the **term structure of interest rates**, the structure of interest rates for discounting cash flows of different maturities.

II. Term Structure Under Certainty

1. **Short interest rate:** the interest rate for a given time interval (usually one year). For a period of time, say, 3 years, the short interest rates for 1st year, 2nd year and 3rd year are usually different. On the contrary, the yield for a 3-year bond is the single interest rate that equates the present value of the bond's payments to the bond's price.

Year	1-year Short Interest rate	Time to maturity	Price of zeros	YTM of Zeros
0 (today)	8%	1	925.93	8%
1	10%	2	841.75	8.995%
2	11%	3	758.33	9.66%
3	11%	4	683.18	9.993%

2. **Yield curve:** a graph of YTM as a function of time to maturity.
 - a. Yield curve can be upward-sloping, flat or downward sloping.
 - b. The YTM on zeros is sometimes called the spot rate that prevails today for a

period corresponding to the maturity of the zero. The yield curve (last column of the above table) presents the spot rates for four maturities. The spot rates, or YTM, do not equal the 1-year interest rates (2nd column of the above table) for each year.

- c. By definition, the price of a two-year zero is:

$$P_2 = \frac{\$1000}{(1.08)(1.10)} = \$841.75 = \frac{\$1000}{(1+y_2)^2}$$

- Doing the math: $(1+y_2)^2 = (1.08)(1.10)$ or $(1+y_2) = [(1.08)(1.10)]^{1/2}$

- More generally: $1+y_n = [(1+r_1)(1+r_2)\dots(1+r_n)]^{1/n}$

- YTM for long bond comes from **geometric mean** of gross returns to short bonds.

3. Pricing of coupon bonds:

- a. For coupon bonds, we simply discount each payment by the spot rate (YTM) corresponding to the time until that payment. To illustrate, consider a 3-year bond with par value \$1000, paying an annual coupon rate of 8%. Using the YTM from the above table, the price of this bond is therefore:

$$P = \frac{80}{1.08} + \frac{80}{(1.08995)^2} + \frac{1080}{(1.09660)^3} = \$960.41$$

- In other words, we can treat each of the bond's payments as in effect a stand-alone zero-coupon security that can be valued independently. The total value of the bond is just the sum of the values of each of its cash flows.

- Notice that the YTM of this 3-year coupon bond is 9.58% (get it by yourself), is a bit less than that of a 3-year zero-coupon bond. This makes sense: if we think of **the coupon bond as a "portfolio" of three zeros** (corresponding to each of its three coupon payments), then its yield should be a weighted average of the three spot rates for years 1-3. Of course, the yield on the last payment will dominate, since it accounts for the overwhelming proportion of the value of the bond. But as a general rule, **yields to maturity can differ for bonds of the same maturity if their coupon rates differ**.

- b. Bond traders often distinguish between the yield curve for zero-coupon bonds and that for coupon-paying bonds. The **pure yield curve** refers to the relationship between yield to maturity and time to maturity for zeros. The **coupon-paying curve** portrays this relationship for coupon bonds.

- c. The most recently issued Treasuries are said to be **on the run**. On-the-run Treasuries have the greatest liquidity, so traders have keen interest in the on-the-run curve.

- d. **Holding period returns**: In a world with no uncertainty all bonds must offer identical rates of return over any holding period. Otherwise, at least one bond would be dominated by the others in the sense that it would offer a lower rate of return than would combinations of other bonds; no one would be willing to hold the bond, and its price would fall. In fact, despite their different yields to maturity, each bond will provide a return over the coming year equal to this year's short interest rate.

4. Forward rates:

- a. Compare two investment choices:
- buy-and-hold: invest \$100 in 3-year zero

Period 1	Period 2	Period 3

\$100 →

- \$100(1+y₃)³
 - roll-over: invest \$100 in 2-year zero. After 2 years, reinvest proceeds in 1-year zero

Period 1	Period 2	Period 3

\$100 → \$100(1+y₂)²
 \$100(1+y₂)²(1+f₃)

For some break-even value of f₃, roll-over strategy has same payoff as buy-and-hold.

- b. Forward rate “f₃”: break-even rate such that roll-over and buy-and-hold strategies have same payoff:

- f₃ solves: payoff to buy-and-hold = payoff to roll-over
 - \$100(1+y₃)³ = \$100(1+y₂)²(1+f₃)

$$\rightarrow 1 + f_3 = \frac{(1 + y_3)^3}{(1 + y_2)^2}$$

Generally, $1 + f_n = \frac{(1 + y_n)^n}{(1 + y_{n-1})^{n-1}}$

$$(1 + y_n)^n = (1 + f_n)(1 + y_{n-1})^{n-1}$$

III. Interest rate uncertainty and forward rates

1. Different rates:
 - a. The ***n*-period spot rate** is the YTM on a zero-coupon bond with a maturity of *n* periods.
 - b. The ***short rate*** of period *n* is the ***one-period*** interest rate that will prevail in period *n*.
 - c. The ***forward rate*** for period *n* is the short rate that would satisfy a “break-even condition” equating the total return on two *n*-period investment strategies.
 - d. Spot rates and forward rates are ***observable*** today, but because interest rates evolve with uncertainty, ***future short rates are not***. In the special case in which there is no uncertainty in future interest rates, the forward rate calculated from the yield curve would equal the short rate that will prevail in that period.

2. short-term investors vs. long-term investors

Under uncertainty, we don't know short rates for sure. However, we can expect what short rates will be – expected short rate (E(r_t)) of period *t*.

Let's compare two different types of investors:

- a. Short-term investors: want to make one-year investment. He can either buy one-year zero or buy two-year zero and sell at the end of the first year. If he buys one year zero, he can lock in a riskless return for one year. If he buys two-year zero, since he doesn't know the real interest rate at the end of the first year, therefore, the return of buying and holding two-year zero for one year is uncertain. If the real yield rises

(drops) at the end of first year, the bond price (the price he can sell at the end of the first year) will decrease (increase). Thus his real return is less (more) than that of one-year zero. Since investors usually are risk-averse, they would not hold the 2-year bond unless it offered an expected rate of return greater than the riskless return available on the competing 1-year bond.

For short-term investors: $(1+r_1)(1+E(r_2)) < (1+E(r_{1B}))(1+E(r_2)) = \rightarrow$
 $(1+y_2)^2 = (1+r_1)(1+f_2) \rightarrow E(r_2) < f_2$

$$E(r_2) < f_2$$

- b. Long-term investors: want to make two-year investment. He can either buy one 2-year zero or buy 1-year zero first and then reinvest the money in 1-year zero available at the end of the first year (roll-over). If he buys a 2-year zero, he can lock in a riskless return for two year. If he rolls over, since he doesn't know the real interest rate at the end of the first year, therefore, the return of buying and holding 1-year zero at the end of the first is uncertain. If the real yield rises (drops) at the end of first year, the payoff from reinvestment (the price he can sell at the end of the second year) will decrease (increase). Thus his real return is less (more) than that of 2-year zero. Since long-term investors usually are risk-averse, they would not roll over two 1-year bonds unless they offered an expected rate of total return greater than the riskless return available on the competing 2-year bond.

For long-term investors: $(1+r_1)(1+E(r_2)) > (1+y_2)^2 = (1+r_1)(1+f_2) \rightarrow$

$$E(r_2) > f_2$$

3. an example (where r_1 is the real short rate for period 1, r_2 is the real short rate for period 2):

Case 1: $r_1=8\%$, $r_2=10\%$					
	price at year 1	price at year 2	year 1 return	year 2 return	2-year YTM
2-year zero	108	118.8	8.00%	10%	9.00%
1-year zero	108	118.8	8%	10%	9.00%
Case 2: $r_1=8\%$, $r_2=11\%$					
	price at year 1	price at year 2	year 1 return	year 2 return	
2-year zero	107.027	118.8	7.03%	11%	9.00%
1-year zero	108	119.88	8%	11%	9.49%
Case 3: $r_1=8\%$, $r_2=9\%$					
	price at year 1	price at year 2	year 1 return	year 2 return	
2-year zero	108.9908	118.8	8.99%	9%	9.00%
1-year zero	108	117.72	8%	9%	8.50%

IV. Theories of the term structure

1. **Liquidity preference:** from the above discussion, short-term investors will be unwilling to hold long-term bonds unless the forward rate exceeds the

expected short interest rate, $E(r_2) < f_2$, whereas long-term investors will be unwilling to hold short bonds unless $E(r_2) > f_2$. In other words, both groups of investors require a premium to induce them to hold bonds with maturities different from their investment horizons. Advocates of the **liquidity preference theory** of the term structure believe that short-term investors dominate the market so that, generally speaking, the forward rate exceeds the expected short rate. The f_2 over $E(r_2)$, the **liquidity premium**, is predicted to be positive. A liquidity premium can cause the yield curve to slope upward even if no increase in short rates is anticipated.

2. Another story – **the Expectations Hypothesis**: a common version of this hypothesis states that the forward rate equals the market consensus expectation of the future short interest rate; in other words $f_2 = E(r_2)$ and liquidity premiums are zero. Because $f_2 = E(r_2)$ we can use the forward rates derived from the yield curve to infer market expectations of future short rates. The YTM would thus be determined solely by current and expected future one-period interest rates. An upward-sloping yield curve would be clear evidence that investors anticipate increases in interest rates.

V. Interpreting the term structure

$$(1 + y_n)^n = (1 + f_n)(1 + y_{n-1})^{n-1}$$

1. Mathematically, if the yield curve is rising, f_n must exceed y_{n-1} . In words, the yield curve is upward sloping at any maturity date, $n-1$, for which the forward rate for the coming period is greater than the yield at that maturity. But **what can account for that higher forward rate?**
2. there are two possible answers: since $f_n = E(r_n) + \text{liquidity premium}$, either investors **expect rising interest rates**, meaning that $E(r_n)$ is high, **or** they **require a large premium** for holding longer-term bonds. Although it is tempting to infer from a rising yield curve that investors believe that interest rates will eventually increase, this is not a valid reasoning. (Please refer to figure 15.5 on page 500)
3. Although it is true that expectations of increases in future interest rates can result in a rising yield curve, the converse is not true: **A rising yield curve does not in and of itself imply expectations of higher future interest rates. This is the heart of difficulty in drawing conclusions from the yield curve.** The effects of possible liquidity premiums confound any simple attempt to extract expectations from the term structure. But estimating the market's expectations is a crucial task, because only by comparing your own expectations to those reflected in market prices can you determine whether you are relatively bullish or bearish on interest rates.
4. One rough approach to deriving expected future spot rates is to assume that liquidity premiums are constant. However, this approach has little to recommend it for two reasons. First, it is almost impossible to obtain precise estimates of a liquidity premium. Second, there is no reason to believe that the liquidity premium should be constant.
5. Still, **very steep yield curves** are interpreted by many market professionals

as warning signs of impending rate increase. In fact, **the yield curve is a good predictor of the business cycle** as a whole, since long-term rates tend to rise in anticipation of an expansion in the economy. When the curve is steep, there is a far lower probability of a recession in the next year than when it is inverted or falling. For this reason, the yield curve is a component of the index of leading economic indicators.

6. Irregularity – why might interest rates fall? There are two factors to consider: the real rate and the inflation premium. Recall the Fisher equation:

$$1 + \text{Nominal rate} = (1 + \text{Real rate})(1 + \text{Inflation rate}),$$

Or approximately, Nominal rate \approx Real rate + inflation rate.

Therefore, an expected change in interest rates can be due to changes in either expected real rates or expected inflation rates.

7. **Term premium**: the spread between yields on long- and short-term bonds. It is generally are positive.

VI. **Measuring the term structure**

A pure yield curve could be plotted easily from a complete set of zero-coupon bonds. In practice, however, most bonds carry coupons, payable at different future times, so that yield curve estimates are often inferred from prices of coupon bonds (by using a sequence of equations of bond prices as a function of future payments and yields. Here, bond prices are dependent variables. Future payments are independent variables and yields are coefficients). Measurement of the term structure is complicated by tax issues such as tax timing options and the different tax brackets of different investors.

Chapter Sixteen: Managing Bond Portfolios

I. **Passive investment strategy vs. active investment strategy:**

1. A passive investment strategy takes market prices of securities as fairly set. Rather than attempting to beat the market by exploiting superior info or insight, passive managers act to maintain an appropriate risk-return balance given market opportunities. One special case is an immunization strategy that attempts to insulate or immunize the portfolio from interest rate risk.
2. In contrast, an active investment strategy attempts to achieve returns greater than those commensurate with the risk borne. Active managers use either interest rate forecasts to predict movements in the entire bond market or some form of intramarket analysis to identify particular sectors of the market or particular bonds that are relatively mispriced.

II. **Interest rate risk**

1. Motivation for an interest rate risk measure:
 - a. An investor would like to estimate the changes in price due to anticipated changes in interest rates.
 - b. An investor may want to hedge the risk involved in holding a fixed income instrument.
 - c. An investor may want to construct a portfolio of bonds which tracks an index in terms of risk.

- d. For comparing different bonds using a common risk measure.
2. Time to maturity:
 - a. Ignores coupon payments and does not account for different cash flow sizes.
 - b. A zero coupon and a coupon bond with same maturity do not have same interest rate risk. This can be seen from the shapes of yield curve of these bonds.
3. **Duration:** to measure how much bond prices will change if yields change. Duration can help us in constructing proper hedges, calculating estimated price changes: $\% \Delta P$ (Estimated) = $D_{Mac} * \% \Delta (1+y)$, etc.

4. **Macaulay's duration:**

- a. Definition:
$$D_{Mac} \equiv \frac{1}{P} \left[\sum_{t=1}^T \frac{t \text{Coupon}}{(1+y)^t} + \frac{T \text{Face}}{(1+y)^T} \right]$$
- b. Interpretation 1: Macaulay's duration is computed as **the weighted average** of the times to each coupon or principal payment made by the bond. **The weight is just the present value of the payment divided by the bond price.**
- c. Interpretation 2: Macaulay's duration is the fulcrum point of a seesaw with the weights on the seesaw being present value of cash flows. i.e. the point in time at which the present value of reinvested cash flows are exactly equal to the present value of the remaining future cash flows. (challenge yourself: prove it).
5. Calculation of Macaulay's duration:
 - a. the **weight**, w_t , the share of period-t cash flows in current value

$$w_t = \frac{CF_t / (1+y)^t}{\text{bond price}}$$

- b. use these weights to calculate the weighted average of the times until the receipt of each of the bond's payment, we obtain Macaulay's duration:

$$D = \sum_{t=1}^T t \times w_t$$

6. application of duration:

- a. To calculate estimated price changes:

$$\frac{\Delta P}{P} = -D \times \left[\frac{\Delta (1+y)}{1+y} \right]$$

Or, equivalently, $\% \Delta P$ (Estimated) = $D_{Mac} * \% \Delta (1+y)$

- b. **Volatility (Modified duration):** $D^* = D / (1+y)$

$$\frac{\Delta P}{P} = -D^* \Delta y$$

7. **Properties of Macaulay's Duration (remember "seesaw")**

Rule 1: Duration of a zero-coupon bond is its maturity

Rule 2: Holding maturity constant, duration varies inversely with coupon size
Bigger coupons bring higher share of near-term cash flows in current price

Rule 3: Holding coupon rate constant, duration generally increases with its maturity. Duration always increases with maturity for bonds selling at par or at a premium to par.

Rule 4: Duration varies inversely with yield-to-maturity

Larger YTM brings lower share of long-term cash flows in current price.

Rule 5: The duration of a level perpetuity is $= (1+y)/y$ (note: the maturity is infinite here!!!)

Rule 6: the duration of a coupon bond

Duration of coupon bond $= \frac{1+y}{y} - \frac{(1+y)+T(c-y)}{c[(1+y)^T - 1] + y}$, where c is the **coupon rate per**

payment period, T is the number of **payment periods** (may not equal to maturity, if a bond pays coupon more than once a year.), and y is the bond's yield per payment period.

III. Convexity

1. Estimates of %DP using duration alone can be substantially wrong:

- There's a pattern to the inaccuracy
 - Actual %DP > Estimated %DP
 - When yields rise, price declines less than estimated by duration
 - When yields fall, price declines more than estimated by duration
- Pattern called “convexity”
 - Actual %DP is convex in yield
 - Estimated %DP is linear in yield
- Convexity/curviness is a property of **second** derivatives:
 - Duration is a property of first derivatives
 - $Convexity = \frac{1}{P} \frac{d^2 P}{dy^2}$

2. Convexity allows us to improve the duration approximation for bond price changes.

$$\frac{\Delta P}{P} \cong (-D_{\text{mod}})\Delta y + \frac{Convexity}{2}\Delta y^2$$

3. Convexity is generally considered a desirable trait for investors. Bonds with greater curvature gain more in price when yields fall than they lose when yields rise.

IV. Passive bond management

1. Two types of passive management. One is an indexing strategy that attempts to replicate the performance of a given bond index. The other is immunization techniques. These two are very different in terms of risk exposure. A bond-index portfolio will have the same risk-reward profile as the bond market index to which it is tied. Immunization strategies seek to establish a virtually zero-risk profile, in which interest rate movements have no impact on the value of the firm.
2. Bond index funds: three major indexes of the broad bond market are the Salomon Smith Barney Broad Investment Grade (BIG) index, the Lehman Aggregate Bond Index, and the Merrill Lynch Domestic Master index. All three are market-value-weighted indexes of total returns. All three include government, corporate, mortgage-backed, and Yankee bonds (those are dollar-denominated, SEC-registered bonds of foreign issuers sold in the USA) in their universes. All three exclude junk bonds, convertibles, and bond with maturities less than 1 year. As time passes, and the maturity of a bond falls below 1 year, the bond is dropped from the index. Therefore, the securities used to compute bond indexes constantly change.
3. Immunization: Duration-matched assets and liabilities let the asset portfolio meet the

firm's obligations despite interest rate movements.

- a. Fixed-income investors face two offsetting types of interest rate risks: price risk and reinvestment rate risk. Increases in interest rates cause capital losses but at the same time increase the rate at which reinvested income will grow. If the portfolio duration is chosen appropriately, these two effects will cancel out exactly. When the portfolio duration is set equal to the investor's horizon date, the accumulated value of the investment fund at the horizon date will be unaffected by interest rate fluctuations. **For a horizon equal to the portfolio's duration, price risk and reinvestment risk exactly cancel out. That is, if we calculate the value of the portfolio at the horizon of duration, this value is indifferent to the small change of interest rate at that horizon. ($d(\text{value})/dr = 0$ at duration)**
 - b. As interest rates and asset durations change, we must rebalance the portfolio of fixed income assets continually to realign its duration with the duration of the obligation. Even if interest rates do not change, asset durations will change solely because of the passage of time.
 4. A portfolio duration is the weighted average of duration of each component asset, with weights proportional to the funds placed in each asset.
 5. Immunization can be an inappropriate goal in an inflationary environment. Immunization is essentially a nominal notion and makes sense only for nominal liabilities.
- V. **Active bond management:** there are two sources of potential value in active bond management. The first is interest rate forecasting. If interest rate declines are anticipated, managers will increase portfolio duration (and vice versa). The second is identification of relative mispricing within the fixed-income market. One might believe that the default premium on one particular bond is unnecessarily large and therefore that the bond is underpriced.

Chapter Eighteen: Equity Valuation Models

- I. This chapter describes the valuation models that stock market analysts use to uncover mispriced securities. The models are those used by fundamental analysts, those analysts who use information concerning the current and prospective profitability of a company to assess its fair market value.

II. **Valuation by comparables:**

1. Compare valuation ratios of the firm with those of comparable firms or industry average. E.g. P/E ratio, Price/book value per share, price/sales per share and price/cash flow per share.
2. a “floor” for the stock’s price:
 - a. **Book value:** is the net worth of a company as reported on its balance sheet. Book value reflects “history”, while market price reflects the present value of its expected future cash flows.
 - b. A better measure of a floor for the stock price is the **liquidation value** per share of the firm. This represents the amount of money that could be realized by breaking up the firm, selling its assets, repaying its debt, and distributing the remainder to the shareholders. The reasoning behind this concept is that if the market price of equity drops below the liquidation value of the firm, the firm becomes attractive as a takeover target.
3. Another measure relative to valuing a firm is the **replacement cost** of its assets less its liabilities. Some believe that the market value of the firm cannot remain for long too far above its replacement cost because if it did, competitors would try to replicate the firm. The competitive pressure of other similar firms entering the same industry would drive down the market value of all firms until they came into equality with replacement cost.

Tobin’s Q: the market value of all of the firm’s debt plus equity divided by the replacement value of the firm’s assets.

III. **Intrinsic value versus market price:**

The **intrinsic value** of a share is defined as the present value of all cash payments to the investor in the stock, including dividends as well as the proceeds from the ultimate sale of the stock, discounted at the appropriate risk-adjusted interest rate, r .

Whenever the intrinsic value of the investor’s own estimate of what the stock is really worth, exceeds (bellows) the market price, the stock is considered undervalued (overvalued) and the investor should long (short) the stock. However, it is usually very dangerous to try to “beat market”. If your estimate of the value is different from that of the market, it is probably because you have used poor dividend forecasts (not because the market mispriced the stock, though it is possible).

Therefore, the advice is “Do not use DCF valuation formulas to test whether the market is correct in its assessment of a stock’s value”.

In market equilibrium, the current market price will reflect the intrinsic value estimates of **all** market participants. This means that individual investor whose estimate

differs from the market price in effect must disagree with some or all of the market consensus estimate of expected dividends, expected sale price or required rate of return.

A common term for the market consensus value of the required rate of return, r , is the **market capitalization rate**.

IV. Price-earnings ratio:

$$1. \frac{P_0}{E_1} = \frac{1}{r} \left(1 + \frac{PVGO}{E/r} \right)$$

The ratio of PVGO to E/r is the ratio of the component of firm value due to growth opportunities to the component of value due to assets already in place (i.e. the no-growth value of the firm, E/r). When future growth opportunities dominate the estimate of total value, the firm will command a high price relative to current earnings. Thus a high P/E ratio indicates that a firm enjoys ample growth opportunities.

$$2. P_0 = \frac{EPS_1(1-pb)}{r-ROE*pb} \rightarrow \frac{P_0}{E_1} = \frac{1-pb}{r-ROE*pb}$$

a. P/E ratio increases with ROE

b. The higher the plowback rate, the higher the growth rate, but a higher plowback rate does not necessarily mean a higher P/E ratio. A higher plowback rate increases P/E only if investments undertaken by the firm offer an expected rate of return higher than the market capitalization rate. Otherwise, higher plowback hurts investors because it means more money is sunk into projects with inadequate rates of return.

3. A common Wall Street rule of thumb is that the growth rate ought to be roughly equal to the P/E ratio. In other words, the ratio of P/E to g , often called the **PEG ratio**, should be around 1.

4. It is the case that high P/E ratio stocks are almost invariably expected to show rapid earnings growth, even if the expected growth rate does not equal the P/E ratio.

5. Pitfalls in P/E analysis:

a. earnings is accounting earnings, which are influenced by somewhat arbitrary accounting rules such as the use of historical cost in depreciation and inventory valuation.

b. Earnings management is the practice of using flexibility in accounting rules to improve the apparent profitability of the firm.

c. The use of P/E ratios is related to the business cycle: we assume in our models that earnings rise at a constant rate, or, on a smooth trend line. In contrast, reported earnings can fluctuate dramatically around a trend line over the course of the business cycle.

Another way to make this point is to note that **in our models, P/E ratio is the ratio of today's price to the trend value of future earnings, EPS_1 . The P/E ratio reported in the financial pages of the newspaper, by contrast, is the ratio of price to the most recent past accounting earnings.** Current accounting earnings can differ considerably from future economic earnings. Therefore, the ratio of price to most recent earnings can vary substantially over the business cycle, as accounting earnings and the trend value of economic earnings diverge by greater and lesser amount.

d. In sum, analysts must be careful in using P/E ratios. There is no way to say P/E ratio is overly high or low without referring to the company's long-run growth prospects, as well as to current earnings per share relative to the long-run trend line.

Chapter Twenty: Option Markets: Introduction

- I. Derivatives are securities, whose prices are determined by, or “derive from”, the prices of other securities. These assets are also called **contingent claims** because their payoffs are contingent on the prices of other securities. .

The asset can be a financial asset or a commodity. It may even be an index representing a broad asset class.

◆ Users of derivative securities:

- *Hedgers*: Market participants who want to eliminate the risk of holding a position in a security.
- *Speculators*: Market participants who want to speculate on market / asset price movements.
- *Arbitragers*: Market participants who want to take advantage of any mispricings in the markets.

In addition, investors may use derivatives to increase or decrease risk exposure of their portfolios. The liquidity of some derivative contracts offer investors an attractive alternative for quickly gaining or reducing exposure to the market as a whole even if they are not modifying their overall risk.

II. Derivative markets:

1. Over the Counter

Most forward contracts and customized option contracts are traded “over the counter”.

2. Exchanges

Futures and standardized options contracts are traded on organized exchanges. Trading can be done in different formats:

- “Open outcry” in the trading pits.
- Electronic Trading: Project A, GLOBEX etc.
- Both open outcry and electronic trading (parallel sessions or at different times).

The following websites contain info on services as well as other links related to derivatives:

<http://www.numa.com/ref/exchange.htm#US>
www.stocks.about.com (link to derivatives)

Major Exchanges

Chicago Board of Trade: www.cbot.com

Chicago Board Options Exchange: <http://www.cboe.com>

Chicago Mercantile Exchange: www.cme.com

Philadelphia Board of Trade

NYME: New York Mortgage Exchange. <http://www.nyme.com/>

LIFFE: the Pan-European Exchange. www.euronext.com

International Securities Exchange: www.iseoptions.com

III. The Option contract:

1. A **call option** on a security gives its holder the right, but **not** the obligation, to purchase the security for a stated amount, K , the **exercise price** or **strike price** on or before some specified expiration date.

2. Since the call option gives the holder the right to buy the security for K, if the price of the security at the time of exercise, S, is greater than the strike price, K, the holder will exercise the option, i.e., buy the security for K and sell it for S. The difference, then, S-K, is the value of the option. Since it is a right and not an obligation, if $S < K$, holders will not exercise their right of purchase and the option will expire without being exercised. Thus, the formula for the payoff, or value, on a call option at maturity is:

$$C(S; K) = \text{Max}[S-K, 0]$$

3. The purchase price of the option is called the **premium**.
4. The **net profit** on the call is the value of the option minus the price originally paid to purchase it.
5. A **put option** on a security gives its holder the right, but **not** the obligation, to sell the security for a stated amount, K, the **exercise price** or **strike price** on or before some specified expiration date.
6. If the value of the security at maturity, S, falls short of the exercise prices, then the holder will exercise the put, purchasing the security for its market price of S and selling it to the writer, i.e., the seller of the put for the exercise price of K. Sometimes, this is called putting it to the writer. This results in a payoff of $K - S$. If on the other hand, K is below the security price, S, the holder has no incentive to exercise his/her right to sell it for K and the option will expire. Thus, the payoff of a put option at maturity is:

$$P(S; K) = \text{Max}[K - S, 0]$$

7. While the holder of an option has the right to exercise it, the seller or writer of the option has **an obligation**. Thus, if the holder of a call (put) wishes to exercise, the writer is obligated to deliver (buy) the security for the exercise price. In practice, most options are **settled in cash**, which means that rather than buying or selling the security, the cash value of the option at maturity, say absolute value of $(S - K)$, is paid by the writer to the holder.
8. An option is said to be **in the money** if it is in the interest of the holder to exercise it. Thus, a call (put) is in the money if $S > (<) K$. Similarly, options are **out of the money** if the holder does not have a payoff gain, i.e., if a call (put) has $S < (>) K$. When the price is at or near the exercise price, K, the option is said to be trading **at the money**.
9. Option trading:
 - a. Some options **trade on over-the-counter markets**. The OTC market offers the advantage that the terms of the option contract – the exercise price, maturity date, and number of shares committed – can be tailored to the needs of the traders. The costs of establishing an OTC option contract, however, are higher than for exchange-traded options.
 - b. Options contracts **traded on exchanges** are standardized by allowable maturity dates and exercise prices for each listed option. Standardization of the terms of listed option contracts means all market participants trade in a limited and uniform set of securities. This increases the depth of trading in an option, which lowers trading costs and results in a more competitive markets.
10. **American options** allow exercise on or before the exercise date. **European options** allow exercise only on the expiration date. Most traded options in the US are American in nature. Foreign currency options and CBOE stock index options are

notable exceptions.

11. **Adjustments in option contract term:** to account for a stock split, the exercise price is reduced by a factor of the split, and the number of options held is increased by that factor. A similar adjustment is made for stock dividends of more than 10%; the number of shares covered by each option is increased in proportion to the stock dividend, and the exercise price is reduced by that proportion. But, **cash dividends do not affect the terms of an option contract.**
12. The option clearing corporation (OCC): the clearinghouse for options trading. Buyers and sellers of options who agree on a price will strike a deal. At this point, the OCC steps in. The OCC places itself between the two traders, becoming the effective buyer of the option from the writer and the effective writer of the option to the buyer. All individuals, therefore, deal only with the OCC, which effectively guarantees contract performance. Option writers are required to post margin to guarantee that they can fulfill their contract obligations. When the required margin exceeds the posted margin, the writer will receive a margin call. The holder of the option need not post margin because the holder will exercise the option only if it is profitable to do so.
13. Options are traded on stocks, stock indexes, foreign currencies, fixed-income securities (interest rate options), and several futures contracts.

IV. Values of options at expiration

1. Payoff to call holder (value at expiration) = $\text{Max}[S_T - K, 0]$. The profit to the option holder is the value of the option at expiration minus the original purchase price.
2. Payoff to call writer = $\text{Min}[K - S_T, 0]$. The profit to the writer is the payoff plus the original sell price.
3. Payoff to put holder (value at expiration) = $\text{Max}[K - S_T, 0]$. The profit to the option holder is the value of the option at expiration minus the original purchase price.
4. Payoff to put writer = $\text{Min}[S_T - K, 0]$. The profit to the writer is the payoff plus the original sell price.
5. Options can be used either to lever up an investor's exposure to an asset price or to provide insurance against volatility of asset prices.

V. Option strategies

1. Protective put strategy: buy a stock and a put on the stock.
 - a. The total payoff to this portfolio = $\text{max}[K - S_T, 0] + S_T = \text{max}[K, S_T]$
 - b. The total profit to this portfolio = $\text{payoff} - (S_0 + P)$
 - c. Notice that protective put offers some insurance against stock price declines in that it limits losses. Therefore, protective put strategies provide a form of portfolio insurance. The cost of the protection is that, in the case of stock price increases, the profit is reduced by the cost of the put, which turned out to be unneeded.
2. Covered call: long a share of stock with a simultaneous short a call on that stock. The call is "covered" because the potential obligation to deliver the stock is covered by the stock held in the portfolio.
 - a. The total payoff to this portfolio = $S_T - \text{max}[S_T - K, 0] = \text{min}[K, S_T]$
 - b. The total profit to this portfolio = $\text{payoff} - S_0 + C$, where C is the premium of the call.

3. **Straddle:** A long straddle is established by buying both a call and a put on a stock, each with the same exercise price, K , and the same expiration date, T . Straddles are useful strategies for investors who believe a stock will move a lot in price but are uncertain about the direction of the move.
 - a. The total payoff to this portfolio = $\max[S_T - K, 0] + \max[K - S_T, 0]$ = absolute value of $(S_T - K)$.
 - b. The total profit to this portfolio = payoff – $C - P$
4. **Spreads:** A spread is a combination of two or more call options (or two or more puts) on the same stock with differing exercise prices or times to maturity. A money spread involves the purchase of one option and the simultaneous sale of another with a different exercise price. A time spread refers to the sale and purchase of options with differing expiration dates. E.g. long a call at K_1 and short a call at K_2 ($K_2 > K_1$), this strategy called bullish spread.

VI. Put-call parity relationship

1. Put-call parity theorem for European options: A riskless bond that pays K for sure plus a call with an exercise price of K on an asset is equivalent to or replicated by a put with an exercise price of K and a holding of the underlying asset:

$$\frac{X}{(1+r_f)^T} \bullet (1+r_f)^T + C(S_0, X) = S_0 + P(S_0, X) \quad \frac{X}{(1+r_f)^T} + C(S_0, X) = S_0 + P(S_0, X)$$

Where S_0 is the current price of the asset and r_f is the riskless rate.

2. Proof:

At time T , payoffs the LHS and RHS:

LHS		RHS	
Riskless Bond	= $\frac{K}{(1+r_f)^T} \bullet (1+r_f)^T = K$	Underlying security	= S_T
Call	= $\max[S_T - K, 0]$	Put	= $\max[K - S_T, 0]$
If $S_T > K$	Bond + call = S_T	If $S_T > K$	Security + put = S_T
If $S_T < K$	Bond + call = K	If $S_T < K$	Security + put = K

Since LHS and RHS always provide equal payoffs at time T , then they must cost the same amount to establish. Therefore, the call-plus-bond portfolio must cost the same as the stock-plus-put portfolio.

Proved

3. For **European** call options on dividend-paying stocks, the put-call parity is:

$$C + PV(K) + PV(\text{dividends}) = P + S_0.$$

VII. Optionlike securities

1. **Callable bonds:** many corporate bonds are issued with call provisions entitling the issuer to buy bonds back at some time in the future at a specified call price. A callable bond arrangement is essentially a sale of a **straight bond** to the investor and the concurrent issuance of a **call option** by the investor to the bond-issuing firm.
2. **Convertible bonds:** they give investors the right to exchange each bond or share of preferred stock for a fixed number of shares of common stock, regardless of the market prices of the securities at the time.

- a. A bond's **conversion value** equals the value it would have if you converted it into stock immediately. Clearly, a bond must sell for at least its conversion value.
 - b. The **straight bond value**, or "**bond floor**", is the value the bond would have if it were not convertible into stock. The bond must sell for more than its straight bond value because a convertible bond has more value; it is in fact a straight bond plus a valuable call option. Therefore, the convertible bond has two lower bounds on its market price: the conversion value and the straight bond value.
 - c. When stock prices are low, the straight bond value is the effective lower bound, and the conversion option is nearly irrelevant. The convertible will trade like straight debt. When stock prices are high, the bond's price is determined by its conversion value. With conversion all but guaranteed, the bond is essentially equity in disguise.
3. Warrants: they are essentially call options issued by a firm.
- a. One difference between calls and warrants is that exercise of a warrant requires the firm to issue a new share of stock – the total number of shares outstanding increases. Also unlike options, warrants result in a cash flow to the firm when the warrant holder pays the exercise price.
 - b. Like convertible debt, warrant terms may be tailored to meet the needs of the firm. Also like convertible debt, warrants generally are protected against stock splits and dividends.
 - c. Issue of warrants and convertible securities creates the potential for an increase in outstanding shares of stock if exercise occurs. So annual reports must provide earnings per share figures under the assumption that all convertible securities and warrants are exercised. These figures are called **fully diluted earnings per share**.

VIII. **Financial Engineering**- the creation of portfolios with specified payoff patterns

- 1. LYON: first introduced by Merrill Lynch in 1985, it is a zero-coupon, convertible, callable, and puttable bond.
- 2. Index-linked CDs: these CDs pay depositors a specified fraction of the rate of return on a market index such as the S&P 500, while guaranteeing a minimum rate return should the market fall. For example, the index-linked CD may offer 70% of any market increase, but protect its holder from any market decrease by guaranteeing at least no loss.

IX. **Exotic options:**

- 1. Asian options: the payoffs are dependent on the average price of the underlying asset during at least some portion of the life of the option. E.g. ST = average price of the previous three month prices at maturity.
- 2. Barrier options: have payoffs that depends not only on some asset price at option expiration, but also on whether the underlying asset price has crossed through some "barrier." for example, a down-and-out option automatically expires worthless if and when the stock price falls below some barrier price. Similarly, down-and-in options will not provide a payoff unless the stock price does fall below some barrier at least once during the life of the option. These options also are referred to as knock-out and knock-in options.
- 3. lookback options: have payoffs that depend in part on the minimum or maximum

price of the underlying asset during the life of the option. For example, a lookback call option may provide a payoff equal to the maximum stock price during the life of the option minus the exercise price, instead of the final stock price minus the exercise price.

4. Binary options: or “bet”, options have fixed payoffs that depend on whether a condition is satisfied by the price of the underlying asset. For instance, a binary call option might pay off a fixed amount of \$100 if the stock price at maturity exceeds the exercise price.

Chapter Twenty-one: Option Valuation

I. Option valuation: introduction:

1. **Intrinsic value** and **time value** are two of the primary determinants of an option's price. Intrinsic value can be defined as the amount by which the strike price of an option is in-the-money. It is actually the portion of an option's price that is not lost due to the passage of time. The following equations will allow you to calculate the intrinsic value of call and put options:

Call Options: Intrinsic value = Underlying Stock's Current Price - Call Strike Price if the option is in the money; = 0 for out-of-the-money or at the money options
 Time Value = Call Premium - Intrinsic Value

Put Options: Intrinsic value = Put Strike Price - Underlying Stock's Current Price if the option is in the money; = 0 for out-of-the-money or at the money options
 Time Value = Put Premium - Intrinsic Value

The **intrinsic value** of an option is **not dependent on the time** left until expiration. It is simply an option's minimum value; it tells you the minimum amount an option is worth. Time value is the amount by which the price of an option exceeds its intrinsic value. Also referred to as extrinsic value, **time value decays over time**. In other words, the time value of an option is directly related to how much time an option has until expiration. The more time an option has until expiration, the greater the option's chance of ending up in-the-money. On the expiration day, all an option is worth is its intrinsic value. It's either in-the-money, or it isn't.

2. Determinants of option values:

- a. Stock price and strike price: A call option should increase in value with the stock price and decrease in value with the exercise price because the payoff to a call, if exercised, equals $S_T - K$.
- b. Call option values also increase with the volatility of the underlying stock price. Because extremely poor outcomes cannot worsen the payoff below zero, while extremely good stock outcomes can improve the option payoff with limit. This asymmetry means that volatility in the underlying stock price increases the expected payoff to the option, thereby enhancing its value.
- c. Longer time to expiration increases the value of a call option. For more distant expiration dates, there is more time for unpredictable future events to affect prices, and the range of likely stock prices increases. This has an effect similar to that of

increased volatility. Moreover, as time to expiration lengthens, the PV of the exercise price falls, thereby benefiting the call option holder and increasing the option value. As a corollary to this issue:

- d. Call option values are higher when interest rate rise (holding the stock price constant) because higher interest rates also reduce the PV of the exercise price.
- e. Dividend payout will decrease call option's value.

II. Restrictions on option values:

1. The value of a call option cannot be negative.

2. Compare two portfolios:

Portfolio A: one call option on a stock plus an amount of cash equal to $(K+D)/(1+r_f)^T$. The option's strike price is K and the expiration date is T. D is the dividend the stock will pay just before T.

Portfolio B: one stock

At time T;

Payoff to portfolio B is $S_T + D$,

Payoff to portfolio A is $\text{Max}[S_T - K, 0] + (K + D) = \text{Max}[S_T + D, K+D] \geq S_T + D$.

Since the payoff to A is greater than or equal to that to B at T, the cost of A today must be greater than or equal to the cost of B today. Therefore,

$$C + (K+D)/(1+r_f)^T = C + \text{PV}(K) + \text{PV}(D) \geq S_T$$

That is, **$C \geq S_T - \text{PV}(K) - \text{PV}(D)$** , where $\text{PV}(K)$ is the present value of the exercise price and $\text{PV}(D)$ is the present value of the dividends the stock will pay at the option's expiration.

For European Call, you can get this from put-call parity.

Recall that $C + \text{PV}(K) + \text{PV}(\text{dividends}) = P + S_0$.

Since $P > 0$, the result is obvious.

Since the holder of an American call has all the exercise opportunities open to the owner of the corresponding European call, we must have **$C_A \geq C \geq S_T - \text{PV}(K) - \text{PV}(D)$**

3. The upper bound on the call is the stock price. One would pay more than S dollars for the right to purchase a stock currently worth S dollars. Thus $C \leq S$.

4. Early exercise and dividends:

- a. A call option holder who wants to close out his position has two choices: exercise the call or sell it. If the holder exercises at time t, the call will provide a payoff of $S_t - K$. Since the option can be sold for at least $S_t - \text{PV}(K) - \text{PV}(D)$. Therefore, for an option on a non-dividend-paying stock,

$$\text{C} \geq S_t - \text{PV}(K) > S_t - K$$

It is better to sell the call, which keeps it alive, than to exercise and thereby end the option. In other words, calls on **non-dividend-paying** stocks are "worth more alive than dead."

- b. It follows that the values of otherwise identical American and European call options on stocks paying no dividends are equal, because it never pays to exercise a call option before maturity.

5. Early exercise of American puts:

it can be optimal to exercise an American put early. Suppose that you purchase a put option on a stock. Soon the firm goes bankrupt, and the stock price falls to zero.

Of course you want to exercise now, because the stock price can fall no lower. Immediate exercise gives you immediate receipt of the strike price, which can be invested to start generating income. Delay in exercise means a time-value-of-money cost. The right to exercise a put option before maturity must have value. Therefore an American put must be worth more than its European counterpart.

III. Measuring interest rates:

1. Discretely compounded interest rates

In this case compounding is done only at discrete intervals (monthly, quarterly, semi-annual, annual). Suppose that an amount A is invested for n years at an interest rate of R per annum. If the rate is compounded once per annum, the terminal value of the investment is:

$$A(1+R)^n,$$

The present value of receiving B dollars at the end of n^{th} year, invested a rate of R is:

$$B/(1+R)^n$$

If compounded m times per year, the terminal value of the investment is:

$$A(1+R/m)^{nm}$$

The corresponding present value of receiving B dollars at the end of n^{th} year is

$$B/(1+R/m)^{nm}$$

2. Continuously compounded rate

When interest rates are continuously compounded, the expression for the future value of A dollars is:

$$Ae^{Rn}$$

The corresponding present value of receiving B dollars at the end of n^{th} year is

$$Be^{-Rn}$$

IV. Binomial option pricing

Binomial model is a numerical approach to valuation of options on stocks. In this approach the stock price evolution over time is modeled with a binomial tree or lattice.

1. One-step binomial model (two-stage)

Let us consider the following scenario:

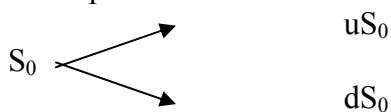
Stock price at $t = 0$:

S_0

Stock price at $t = T$ in "up" state: uS_0 ($u > 1$)

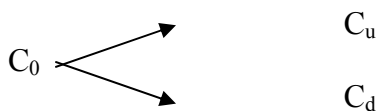
Stock price at $t = T$ in "down" state: dS_0 ($d < 1$)

Stock price tree:



Now, let us consider a call option on the stock. The payoff at $t = T$ is $C_u = \max[uS_0 - K, 0]$ in the up state, or $C_d = \max[dS_0 - K, 0]$ in the down state.

Call price tree:



Now consider a portfolio of a long position in Δ shares and a short position in one call option such that it's value remains the same both in the up and down state.

Then the value of the portfolio at $t=1$ is:

$$\Delta uS_0 - C_u = \Delta dS_0 - C_d$$

$$\rightarrow \Delta = (C_u - C_d)/(uS_0 - dS_0) \text{ ----- (1)}$$

The value of portfolio in up and down states is same, which means that this is a risk free investment. The cost of setting up the portfolio at $t=0$ is then the present value of the portfolio value at $t=T$.

$$\Delta S_0 - C_0 = (\Delta uS_0 - C_u)/(1+r_f)^T$$

If using continuously compounding, $\Delta S_0 - C_0 = e^{-r_f T} (\Delta uS_0 - C_u)$

Since $\Delta = (C_u - C_d)/(uS_0 - dS_0)$, we can get:

$$C_0 = \frac{1}{(1+r_f)^T} (pC_u + (1-p)C_d), \text{ ----- (2)}$$

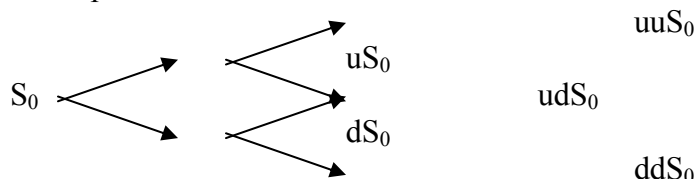
$$\text{Where, } p = \frac{(1+r_f)^T - d}{u - d}$$

2. Hedge ratio: Δ , is the ratio of the swings in the possible end-of-period values of the option and the stock (or, the first derivative of option price relative to the stock price).

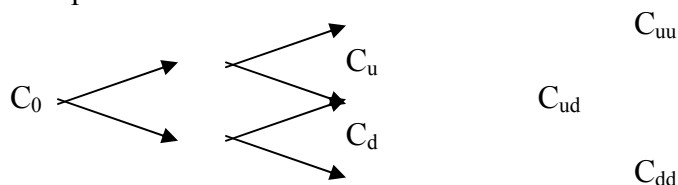
3. Two step model:

Assume the each time step is dT

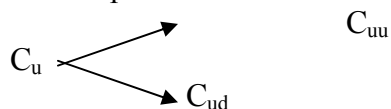
Stock price tree:



Call price tree:



Now consider the “up” and “down” states following the “up” state at the end of first time step.



Using the approach we described for one step binomial tree,

$$C_u = \frac{1}{(1+r_f)^{dT}} (pC_{uu} + (1-p)C_{ud})$$

$$C_d = \frac{1}{(1+r_f)^{dT}} (pC_{ud} + (1-p)C_{dd})$$

$$C_0 = \frac{1}{(1+r_f)^{dT}} (pC_u + (1-p)C_d), \text{ where } p = \frac{(1+r_f)^{dT} - d}{u-d}$$

Generally speaking, the value of call at node I is:

$$C_i = \frac{1}{(1+r_f)^{dT}} (pC(i+1)_u + (1-p)C(i+1)_d)$$

4. We just get the price of the call by **forming a riskless portfolio of stock and option**. On the other hand, we can get the price of the call by **forming a portfolio of stock and riskless bond that gives identical payoffs as the call**. These two approaches give identical results.

V. Black-Scholes Option valuation

In the binomial model, if we let the period-length get smaller and smaller, we obtain the Black-Merton-Scholes option pricing formula:

$$C_0 = S_0 N(d_1) - K e^{-rT} N(d_2)$$

$$P_0 = K e^{-rT} N(-d_2) - S_0 N(-d_1)$$

Where

$$d_1 = \frac{\ln(S_0 / K) + (r + \sigma^2 / 2)T}{\sigma \sqrt{T}} \quad d_1 = \frac{\ln(S_0 / K) + (r + \sigma^2 / 2)T}{\sigma \sqrt{T}}$$

$$d_2 = d_1 - \sigma \sqrt{T}$$

$$d_2 = d_1 - \sigma \sqrt{T}$$

And where

C_0 = current call option value

S_0 = Current stock price.

$N(\cdot)$ is the normal cumulative density function.

K = exercise price.

R = risk-free interest rate (the annualized continuously compounded rate on a safe asset with the same maturity as the expiration of the option, which is to be distinguished from r_f , the discrete period interest rate).

T = time to maturity of option, in years.

\ln = Natural logarithm function.

σ = Standard deviation of the annualized continuously compounded rate of return of the stock.

An interpretation of the Black-Scholes formula:

- The call is equivalent to a levered long position in the stock.
- The replicating strategy:
 - $S_0 N(d_1)$ is the amount invested in the stock.
 - $K e^{-rT} N(d_2)$ is the dollar amount borrowed
 - The option delta is $N(d_1)$

VI. Considering dividend:

The above Black-Scholes formula applies to stocks that do not pay dividends. When dividends are to be paid before the option expires, we need to adjust the formula.

1. Black calls for adjusting the stock price downward by the present value of any

dividends that are to be paid before option expiration. Therefore, we would simply replace S_0 with $S_0 - \text{PV}(\text{dividends})$ in the formula (be careful! You should replace S_0 with $S_0 - \text{PV}(\text{dividends})$ in computing d_1 and d_2 too)

2. if the underlying asset pays a dividend at a continuous rate of q , then we just replace S_0 with $S_0 e^{-qT}$.

VII. Using the Black-Scholes Formula

Hedge ratio: is the change in the price of an option for a \$1 increase in the stock price. The hedge ratio is commonly called the option's delta. Remember, if we form a portfolio by long delta shares of stock and short one call, the portfolio is risk-free (delta = 0, or "**delta neutral**"). This basic idea – delta hedging, can be used in risk management.

Reading: International Securities Exchange

The International Securities Exchange, the world's largest equity options exchange, was founded on the principle that technology fosters and infuses new efficiencies and operational innovations into securities trading. After developing an innovative market structure that integrated auction market principles into an advanced screen-based trading system, ISE launched the first fully electronic US options exchange in May 2000. ISE continually enhances its trading systems to provide investors with the best marketplace to execute their options orders.

HISTORY

ISE redefined the US options industry by introducing electronic trading, competition, and lower fees and efficient, equitable pricing, enabling instantaneous executions in tight, liquid markets.

Prior to ISE, the US options industry was markedly different. In the 1980s and 1990s, while other financial markets were embracing electronic trading, the US options exchanges remained steadfastly committed to open-outcry trading. The options exchanges also continued the practice of exclusively listing options on most blue chip stocks on only one exchange, limiting customer choice, and forcing brokerages to maintain trading operations on the floor of each exchange.

Driven by the principle that a fair market structure and technology enables better quality trading for investors, ISE's founders started in 1997 planning an exchange that would address the concerns of potential and current options investors and establish a new standard in US financial markets.

[Bill Porter](#), then-chairman of E*Trade, and his colleague, [Marty Averbuch](#), began an investigation into the costs and barriers that prevented retail investors embracing the buying and selling of options. Seeking to facilitate arrangements similar to those available for stocks in the OTC and third markets, they began to negotiate directly with the existing options exchanges to lower costs. Due to the structure under which options exchanges then operated, they were unable to obtain concessions on costs of execution. Messrs. Porter and Averbuch then engaged [David Krell](#) and [Gary Katz](#), former executives of the New York Stock Exchange's options division, to examine the feasibility of developing a new US options exchange. ISE was funded by a consortium of broker-dealers whose purchase of exchange memberships provided initial development capital.

ISE, which was launched May 26, 2000, became the first registered exchange approved by the Securities and Exchange Commission since 1973, and a member-owner of The Options Clearing Corporation.

INNOVATION & LEADERSHIP

ISE changed the fundamental nature of US options trading.

Before ISE traded its first contract, it had affected the US options industry by fostering inter-exchange competition. When ISE announced its intentions to list only those options that represented 90% of industry volumes, it prompted the multiple-listing of the most liquid options classes which dramatically improved prices for all investors and broke apart the franchises that had enabled the floor-based exchanges to exclusively list options on most blue-chip stocks.

The scalability and efficiency of ISE's business model and the absence of a trading floor and intermediaries such as floor brokers enabled the exchange to charge lower execution fees. ISE's proposed fees prompted an industry-wide scramble to lower fees. ISE's business model attracted many well-capitalized global financial institutions to join as members. These firms, that had previously viewed participation in the options market as costly and inefficient, have significantly increased the overall liquidity in the options market.

As the first fully electronic options exchange, ISE also introduced the efficiencies of electronic trading to the options industry. ISE developed patented proprietary trading technology, which resulted in tighter markets and enabled executions in 0.2 seconds. ISE also strongly advocated for electronic linkage of all options markets, which enables specialists and market makers at different exchanges to access each other's markets, thereby enabling customers to get the best price when they trade, no matter where they trade.

ISE launched on May 26, 2000. The first transaction was a purchase of 20 SBC Communications October 45 calls. The ISE introduced transparency to the options markets by insuring that displayed markets are accompanied by size. Within three months, ISE had traded more than one million options contracts. A year later, ISE had reached the 25 million contracts traded mark, and celebrated its first anniversary. By the end of 2001, ISE had traded its 50 millionth contract, completed a major trading system upgrade, and become the third-largest US options exchange.

Building on momentum, ISE experienced further growth in 2002, achieving its goal of listing equity options representing 90% of average daily trading volume in the US options industry. The exchange completed two trading system upgrades that established the foundation to offer enhanced trading capabilities. In 2003, the one-size fits all trading rule and ISEspreads went into effect, which helped provide the momentum for ISE to trade its 250-millionth contract and become the largest US equity options exchange.

In 2003, ISE implemented the technological and regulatory foundation to trade index options, which will allow ISE to expand its product base. ISE also launched the ISE Sentiment Index (ISEE) which measures opening customer transactions in call and put options, providing investors with information on the customer buying and selling activity within the world's largest equity options exchange. ISE was named Derivatives Exchange of the Year by Risk Magazine for the second time in three years in January 2004.

Market Structure

ISE's market structure integrates auction market principles into an advanced screen-based trading system.

BIN STRUCTURE

The ISE trades issues representing approximately 90% of options industry volume. The issues are divided into 10 bins.

COMPETING MARKET MAKERS

Each bin is overseen by a [Primary Market Maker \(PMM\)](#). In addition to the PMM, up to 16 [Competitive Market Makers \(CMMs\)](#) supply liquidity in each bin. Most PMMs and CMMs are operated by large, global financial institutions with significant capital base and notable trading experience.

ISE posts the best available bids and offers in each options series. Unlike other exchanges, which display the prices offered by the specialist in the options series, ISE's posted prices represent the most competitive bid and offer in the entire ISE market.

PMMs and CMMs provide continuous quotations in their bins, and the ISE market displays only the best bid and ask price in each option class. All orders at the ISE (customer, firm, and market maker) receive the same execution privileges."

ANONYMITY

Trade counter-party information is not visible to anyone in the ISE marketplace, including PMMs. Both parties of a trade receive confirmations without learning the identity of the

counter-party. Anonymous trading allows market makers and order-flow providers to post bid and ask quotations at their set prices, free from any influence of other market participants. This anonymity enables pure order executions.

PRIMARY MARKET MAKERS "PMMs"

PMMs are market makers with significant responsibilities, including overseeing the opening, providing continuous quotations in all of their assigned options classes, and handling customer orders that are not automatically executed. A PMM must purchase or lease a PMM membership entitling the member to act as PMM in one group (or "bin") of stock options. One PMM is assigned to each of the ten groups of options traded on the Exchange.

An important advantage of having a PMM assigned to each stock option is that members have a point of contact, who is responsible for maintaining orderly markets and is available to answer market questions and resolve trading related issues.

Citigroup Derivatives Markets Inc.	Bin 1
SLK-Hull Derivatives LLC	Bin 2
Adirondack Electronic Markets LLC	Bin 3
Adirondack Electronic Markets LLC	Bin 4
Citadel Derivatives Group LLC	Bin 5
UBS Securities LLC	Bin 6
Timber Hill LLC	Bin 7
Citadel Derivatives Group LLC	Bin 8
Deutsche Bank Securities Inc.	Bin 9
Morgan Stanley & Co. Incorporated	Bin 10

COMPETITIVE MARKET MAKERS "CMMs"

CMMs are market makers that add depth and liquidity to the market and are required to provide continuous quotations in at least 60% of the options classes in their assigned group. Each CMM quotes independently. A CMM must purchase or lease a CMM membership, and each membership entitles the member to enter quotations in one group of options. Up to sixteen CMMs are appointed to each of the ten groups of options traded on the Exchange.

Adirondack Electronic Markets LLC	Bins 1, 2, 5, 6, 10
Archelon LLC	Bins 1, 2, 3, 4, 5, 7
Bear Wagner Specialists LLC	Bins 2, 6, 7, 9
BNP Paribas Securities Corp.	Bins 1, 2, 3, 4, 5, 6, 7, 8, 9, 10
Citadel Derivatives Group LLC	Bins 1, 2, 3, 4, 6, 7, 9, 10
Citigroup Derivatives Markets Inc.	Bins 2, 3, 4, 5, 6, 7, 8, 9, 10
Credit Suisse First Boston LLC	Bins 1, 2, 3, 4, 5, 6, 7, 8, 9, 10
Deutsche Bank Securities Inc.	Bins 1, 2, 3, 4, 5, 6, 7, 8, 10
Geneva Trading LLC	Bin 8
Group One Trading, L.P.	Bins 8, 10

J.P. Morgan Securities Inc.	Bin 9
Lehman Brothers Inc.	Bins 1, 2, 3, 4, 5, 6, 7, 8, 9, 10
Merrill Lynch Professional Clearing Corporation	Bins 1, 2, 3, 4, 7, 8, 9, 10
Morgan Stanley & Co. Incorporated	Bins 1, 2, 3, 4, 5, 6, 7, 8, 9
Optiver US, LLC	Bins 3, 4, 9
SLK-Hull Derivatives LLC	Bins 1, 3, 4, 5, 6, 7, 8, 9, 10
TD Options LLC	Bins 1, 2, 3, 4, 5, 6, 7, 8, 10
Timber Hill LLC	Bins 1, 2, 3, 4, 5, 6, 8, 9, 10
UBS Securities LLC	Bins 1, 2, 3, 4, 5, 7, 8, 9, 10
Wolverine Trading, LLC	Bins 1, 2, 3, 4, 5, 6, 7, 8, 9, 10

ELECTRONIC ACCESS MEMBERS "EAMs"

EAMs are broker/dealers that represent agency and proprietary orders on the Exchange. An EAM does not purchase a membership. Rather, once approved as an ISE member, an EAM pays an access fee that permits the firm to place orders in all of the options traded on the Exchange. EAMs cannot enter quotations or otherwise engage in market making activities on the Exchange.

Chapter Twenty-two: Futures Markets

- I. A **forward contract** is a commitment to purchase at a future date (delivery or maturity date) a given amount of a commodity or an asset at a price (futures price) agreed on today.

1. Terminology and Contract Specifications

- ◆ Buyer (Long) – the party agreeing to buy the underlying instrument or commodity.
- ◆ Seller (Short) – the party agreeing to sell the underlying instrument or commodity.
- ◆ Spot Price: The current price of the underlying asset.
- ◆ Delivery price: The price at which the underlying asset is exchanged between the buyer and the seller.
- ◆ Delivery date: The date at which the delivery takes place.
- ◆ Delivery location: Place where the delivery takes place.

In addition, the amount and quality of underlying asset to be delivered may also be specified.

2. Features of forward contracts:

- ◆ Custom tailored
- ◆ Traded over the counter (not on exchanges)
- ◆ No money changes hand until maturity
- ◆ Non-trivial counter-party risk.

II. Futures contract:

1. Forward contracts have two limitations:

- a. illiquidity.
- b. Counter-party risk.

Futures contracts are designed to address these two limitations.

2. **Definition:** A futures contract is an exchange-traded, standardized, forward-like contract that is marked to market daily.

3. Features of futures contracts:

- a. Standardized contracts:
 - (1) underlying commodity or asset
 - (2) quantity
 - (3) maturity
- b. Exchange traded
- c. Guaranteed by the clearing house – no counter-party risk.
- d. Gains/losses settled daily
- e. Margin account required as collateral to cover losses.

4. Terminology and Contract Specifications

- ◆ Buyer (Long) – the party agreeing to buy the underlying instrument or commodity.
- ◆ Seller (Short) – the party agreeing to sell the underlying instrument or

commodity.

- ◆ Open Interest: The total number of contracts outstanding at any time (long and short positions are not counted separately, meaning that open interest can be defined as the number of either long or short contracts outstanding).
- ◆ Volume: The number of contracts traded in a given trading session or a day.
- ◆ Spot price: The current price of the underlying asset.
- ◆ Delivery price: The price at which the underlying asset is exchanged between the buyer and the seller.
- ◆ Delivery date: The date at which the delivery takes place.
- ◆ Delivery location: Place where the delivery takes place.
- ◆ Expiry date: The date at which the contract expires. The delivery date could be different from expiry date because a contract can have a time period during which the delivery can take place.

In addition, the amount and quality of underlying asset to be delivered are also specified. The exchanges also specify minimum price change (tick size) and price limits.

5. Clearing house

- A clearinghouse facilitates trades in futures contracts by acting as counterparty to every futures contract.
- It acts as a seller to every buyer and buyer to every seller.
- It facilitates the trading of futures contracts by making it easy for buyers and sellers to open or close their positions.
- It eliminates counter party risk.

It requires all positions to recognize profits or losses as they accrue daily.

6. Initial margin

When a contract is established, the buyer/seller is required to deposit an amount in a **margin account**. The amount that must be deposited at the time the contract is entered into is known as **initial margin**. It could be in the form of cash or treasury bills. The initial margin is usually set between 5% and 15% of the total value of the contract. Contracts written on assets with more volatile prices require higher margins.

7. Maintenance and variation margin

- A futures contract is marked to market every day. If the margin falls below a level established by an exchange (maintenance margin), then the investor gets a margin call and is expected to top up the margin account to the initial margin (not maintenance margin) level the next day. This call is required to be fulfilled with cash. Maintenance margin is somewhat lower than the initial margin.
- The variation margin is the additional money deposited by the investor.
- The purpose of various margins is to avoid default by the buyer/seller of a contract.

8. Settlement

- *Physical settlement*

Buyer takes delivery of underlying instrument or commodity from seller.

- *Cash settlement*

For contracts where physical delivery is not possible or not convenient, cash settlement is used. In this case the buyer (long) pays or receive cash from the seller (short) as the case may be.

9. Futures Contracts vs. Forward Contracts

- ◆ Futures contracts traded on organized exchanges. Forward contracts are agreements between parties.
- ◆ Futures contracts have standardized terms except for flex contracts. Forward contracts are customized contracts.
- ◆ Futures contracts marked to market on a daily basis. Forward contracts are not marked to market.
- ◆ Futures contracts can be closed at any time by offsetting transactions. This convenience does not exist for forward contracts.
- ◆ Most futures contracts are closed (offset) before contract expiration date. Almost all forward contracts are closed after the delivery of the underlying at expiration date.
- ◆ Unlike forward contracts futures contracts do not have counter party risk.

10. Futures price on the delivery date will equal the spot price of the commodity on that date. As a maturing contract calls for immediate delivery, the futures price on that day must equal the spot price – the cost of the commodity from the two competing sources is equalized in a competitive market (no arbitrage). Therefore, the futures price and the spot price must converge at maturity. This is called the **convergence property**.

Table: Operation of margins for a long position in two gold futures contracts. Each contract size is 100 ounces. The initial margin is \$2000 per contract, or \$4000 in total, and the maintenance margin is \$1500 per contract, or \$3000 in total. The contract is entered into on June 13, 2005 and closed out on June 17, 2005. the numbers in the second column except the first and the last, represent the futures prices at the close of trading.

Day	Futures price per ounce (\$)	Daily gain (loss) (\$)	Cumulative gain (loss) (\$)	Margin account balance (\$)	Margin call (\$)
	400			4000	
June 13	397	(600)	(600)	3400	
June 14	393	(800)	(1400)	2600	1400
June 15	392	(200)	(1600)	3800	
June 16	393.5	300	(1300)	4100	
June 17	396	500	(800)	4600	

Example: mini-sized Corn Futures

Contract Size
1,000 bushels.
Deliverable Grades
No. 2 Yellow at par, No. 1 yellow at 1 1/2 cents per bushel over contract price, No. 3 yellow at 1 1/2 cents per bushel under contract price.
Tick Size
1/8 cent/bushel (\$1.25 per contract).
Price Quote
Cent/bu (1,000 bu)
Contract Months
Jul, Sep, Dec, Mar, May
Last Trading Day
The business day prior to the 15th calendar day of the contract month.
Last Delivery Day
Second business day following the last trading day of the delivery month.
Trading Hours
Open Auction: 9:30 a.m. to 1:45 p.m. Central Time, Mon-Fri.
Ticker Symbols
Open Auction - YC
Daily Price Limit
20 cents/bushel or \$200 per contract.
Margin Information
Find information on margins requirements for the mini-sized Corn Futures.

<http://www.cbot.com/cbot/pub/page/0,3181,1381,00.html>

III. Investment assets vs. consumption assets

1. An **investment asset** is an asset that is held for investment purposes by significant numbers of investors. Stocks and bonds are clearly investment assets.
2. A **consumption asset** is an asset that is held primarily for consumption. It is not usually held for investment. Copper, oil, and pork bellies are examples of consumption assets.
3. We can use arbitrage arguments to determine the forward and futures prices of an investment asset from its spot price and other observable market variables. We cannot do this for the forward and futures prices of consumption assets.

IV. Assumptions and notation

1. Assumptions:
 - ◆ Markets are frictionless, there are no transaction costs, no restrictions on short sales.
 - ◆ There is no counter party risk.
 - ◆ Markets are competitive and all market participants are price takers.
 - ◆ Prices are adjusted such that there are no arbitrage opportunities.
2. Notations:
 - ◆ T : time until delivery date in a forward or futures contract (in years)
 - ◆ S_0 : Price of the asset underlying the forward or futures contract today.
 - ◆ F_0 : Forward or futures price today.
 - ◆ r : Risk-free rate of interest per annum, expressed with continuous compounding, for an investment maturing at the delivery date (i.e., in T years)

V. Forward and Futures Prices

Two ways to buy the underlying asset for date T :

Strategy 1:

- Buy one unit of underlying instrument or commodity at a price of S_0 and store it until T .

Strategy 2:

- Go long a forward contract which matures at time T at a delivery price of $F(0,T)$

Difference between buy-and-store from forward/futures:

- a. cost of buy-and-store
 - cost of storing (for commodities)
- b. benefits from buy-and-store
 - convenience yield (for commodities)
 - Dividends (for investment assets)

By arbitrage, the costs of these two approaches must equal:

Cost of strategy 1 at $t=0$ is:

$S_0 + \text{Present value of net costs of storing.}$

Cost of entering a forward contract at $t=0$ is zero but the contract holder has to pay F_0 at $t=T$, therefore, the cost of strategy 2 at $t=0$ is present value of F_0 (e.g. put present value of F_0 into a risk-free account at t_0 . At time T , get F_0 , use this amount F_0 to honor the forward contract. That is, to buy the asset at the price of F_0).

$$PV(F_0) = S_0 + PV(\text{net cost of storing})$$

Since $PV(F) = F \cdot e^{-rT}$, therefore,

$$F_0 \cdot e^{-rT} = S_0 + PV(\text{net cost of storing}),$$

Or equivalently,

$$F_0 = (S_0 + PV(\text{net cost of storing})) e^{rT}$$

Let $U = PV$ of cost of storage, $Y = PV$ of convenience benefits, $D = PV$ of dividends (or income).

$$PV(\text{net cost of storing}) = PV(\text{cost of storage} - \text{convenience benefits} - \text{dividends}) = U - Y - D$$

Therefore,

$$\mathbf{F_0 = (S_0 + U - Y - D) e^{rT}}$$

1. The underlying assets are investment assets.
Investment assets have the following features:

- ◆ No cost to store
- ◆ Dividends or interests on the underlying.

Case 1: non-dividend-paying stocks, zero-coupon bonds and gold (for gold, the cost of storage is negligible):

Since net cost of storing is zero,

$$F_0 = S_0 e^{rT} \text{----- (1)}$$

On May 16, 2005, the close price for Duke Energy is \$27.70. The settlement price of June futures contract on Duke Energy is \$27.78.

Suppose $T = 1/12 = 0.0833$ year.

From formula (1), the implied risk-free rate per annum is 3.5%

Case 2: Dividends-paying investment assets with known income— e.g. coupon-bearing bond:

Since cost of storing and convenience yield are both zero,

$$F_0 = (S_0 - PV(\text{dividends})) e^{rT}$$

e.g. Consider a long forward contract to purchase a coupon-bearing bond whose current price is \$900. Suppose that the contract matures in one year and the bond matures in five years, so that the forward contract is a contract to purchase a four-year bond in one year. Suppose that coupon payments of \$40 are expected after 6 months and 12 months, with the second coupon payment being immediately prior to the delivery date in the forward contract. We assume the six-month and one-year risk-free interest rates (continuously compounded) are 9% per annum and 10% per annum.

Here, dividends are coupon payments:

$$PV(\text{dividends}) = 40 \cdot e^{-r_1 t_1} + 40 \cdot e^{-r_2 T_2}$$

Where $r_1 = 9\%$, $t_1 = 6/12 = 0.5$, $r_2 = 10\%$, $T_2 = 1$

$$PV(\text{dividends}) = 74.4334$$

$$F_0 = (S_0 - PV(\text{dividends})) e^{r_2 T_2} = (900 - 74.4334) \cdot e^{1 \cdot 0.1} = \$912.39$$

Case 3: Dividends-paying investment assets with known yield.

Define q as the average yield per annum (continuously compounded) on an asset during the life of a forward contract. It can be shown that

$$F_0 = S_0 e^{(r-q)T}$$

◆ Stock index futures

- underlying assets are stock index – e.g. S&P 500.
- Futures settled in cash (no delivery)

A stock index can be regarded as the price of an investment asset that pays dividends. The investment asset is the portfolio of stocks underlying the index, and the dividends paid by the investment asset are the dividends that would be received by the holder of this portfolio. It is usually assumed that the dividends provide a known yield rather than a known cash income. If q is the dividend yield rate, then $F_0 = S_0 e^{(r-q)T}$

Example. Consider a 3-month futures contract on the S&P 500. Suppose that the stocks underlying the index provide a dividend yield of 1% per annum, that the current value of the index is 400, and that the continuously compounded risk-free interest rate is 6% per annum. In this case, $r = 0.06$, $S_0 = 400$, $T = 0.25$, and $q = 0.01$. Hence, $F_0 = 400 e^{(0.06 - 0.01) \cdot 0.25} = \405.03

- Since the underlying asset is a portfolio in the case of index futures, trading in the futures market is easier than trading in cash market.
- Thus, futures prices may react quicker to macro-economic news than the index itself.
- Index futures are very useful to market makers, investment bankers, stock portfolio managers:
 - a. hedging market risk in block purchases & underwriting
 - b. Creating synthetic index fund.
 - c. Portfolio insurance.

◆ Forward and futures contracts on currencies

- The underlying asset is a certain number of units of the foreign currency.
- The holder of the foreign currency can earn interest at the risk-free interest rate prevailing in the foreign country. For example, the holder can invest the currency in a foreign-denominated bond.

Define:

S_0 : the current spot price in dollars of one unit of the foreign currency.

F_0 : the forward or futures price in dollars of one unit of the foreign currency.

r_f : the value of the foreign risk-free interest rate when money is invested for time T .

r : the domestic risk-free rate when money is invested for this period of time.

$$F_0 = S_0 e^{(r-r_f)T}$$

2. The underlying assets are commodities:

- ◆ Costly to store.
- ◆ Additional benefits, convenience yield, for holding physical commodity (over holding futures)
- ◆ Not held for long-term investment, but mostly held for future use.

If the storage costs incurred at any time are proportional to the price of the commodity, they can be regarded as providing a negative yield (u). If the convenience benefit also occurred at any time are proportional to the price of the commodity, it can be regarded as providing a positive convenience yield (y).

In this case, $F_0 = S_0 e^{(r+u-y)T}$

VI. Hedging with Forwards and Futures

1. Basic principles:

- a. When a company chooses to use forward or futures markets to hedge a risk, the objective is usually to take a position that neutralizes the risk as far as possible.
- b. Short hedge: a short hedge is a hedge that involves a short position in futures or forward contract. A short hedge is appropriate when a hedger already owns an asset and expects to sell in at some time in the future. It can also be used when an asset is not owned right now but will be owned at some time in the future.

Consider, for example, a U.S. exporter who knows that he will receive euros in three months. He will realize a gain if the euro increases in value relative to dollar and will sustain a loss if the euro decreases in value. A short futures position leads to a loss if the euro increases in value and a gain if it decreases in value. It has the effect of offsetting the exporter's risk.

- c. Long hedge: it involves taking a long position in a futures or forward contract. A long hedge is appropriate when a company knows it will have to purchase a certain asset in the future and wants to lock in a price now.

2. Hedging with forwards

Hedging with forward contracts is simple, because one can tailor the contract to match maturity and size of position to be hedged.

Example 1. Suppose an oil firm has just struck oil. The manager of the firm expects that in 5 months the firm will have 1 million barrels of oil. But he is unsure of the future price of oil and would like to hedge his position.

Using forward contracts, he could hedge his position by selling forward 1 million barrels of oil. Let S_t be the spot oil price at t (in months). Then,

Position	Value in 5 months (per barrel)
Long position in oil	S_5
Short forward position	$F - S_5$
Net payoff	F

Thus, in this case you know today exactly what you will receive 5 months from now.

That is, the hedge is perfect.

Notice that if the oil price increases a lot, the firm will actually lose money because of hedging. What's wrong here???

VII. Hedging with futures

1. One problem with using forwards to hedge is that they are illiquid. Thus, if after 1 month you discover that there is no oil, then you no longer need the forward contracts. In fact, holding just the forward contracts you are now exposed to the risk of oil-price changes.

In this case, you would want to unwind your position by buying these contracts.

Given the illiquidity of forward contracts, this may be difficult and expensive.

To avoid problems with illiquid forward markets, one may prefer to use futures contracts.

Example 2. In the above example, you can sell 1 million barrels worth of futures. Suppose that the size of each futures contract is 1,000 barrels. The number of contract you want to short is $1,000,000/1,000 = 1,000$

2. Hedging interest rate risk:

If an investor is long a bond or a portfolio of bonds and desires to hedge the interest rate risk of the portfolio, then the ideal vehicle is to use futures to hedge the interest rate risk. By selling interest rate futures in the appropriate amount, the investor can hedge the interest rate risk. The number of contracts to sell is:

$$\frac{D_p * MV_p}{(D_f * MV_f)}$$

- Where, D_p = Duration of portfolio
 MV_p = Market value of portfolio
 D_f = Duration of futures contract
 MV_f = "Market value" of futures contract = $P_f * \text{Face value of futures contract}/100$
 P_f = Price of futures contract.

Duration adjustment

Usually, fixed income investors want to adjust the duration of their portfolios to reflect their changing interest rate outlook rather quickly. The cheapest and quickest way to adjust duration is by buying or selling futures. The number of contracts to buy/sell is:

$$(D_t - D_p) * MV_p / (D_f * MV_f)$$

Where, D_t = Target duration for portfolio

Example 3. We have \$10 million invested in bonds and are concerned with highly volatile interest rate over the next six months. We decide to use the 6-month T-bond futures to protect the value of the portfolio. We have

- Duration of our portfolio is 5.
- Current futures price is \$90 (for face value of \$100)
 - Each contract delivers \$100,000 face value of bonds. Therefore, $MV_f = 90 * 100,000 / 100 = \$90,000$
 - The T-bond to be delivered has a duration of 9 years.

We should short the futures:

The number of contracts to sell = $(5 * 10,000,000) / (9 * 90,000) = 61.73 \approx 62$

We can see that:

- If interest rate goes up, our portfolio's value decreases but a gain is made on the short position of futures.
- If interest rate goes down, our portfolio's value increases but a loss is made on the short position of futures.

3. Imperfect hedging

Since futures contracts are standardized, they may not perfectly match your hedging need. The following mismatches may arise when hedging with futures:

(1). Asset mismatch: the asset whose price is to be hedged may not be exactly the same the asset underlying the futures contract.

(2). Maturity mismatch: there may not exist a futures contract whose maturity is the same as that of the asset.

(3). Contract size mismatch - the hedger could only buy integer number of contracts. Example: Continuing with the example of hedging oil 5 months from now. Suppose that you can only buy futures contracts that mature either 3 months from now or 6 months from now. Then your hedge may not be perfect. Let

- S_t denotes spot price at t .
- $F_{t,T}$ denote the futures price at t with maturity at T
- Suppose that we use the 6-month futures to hedge. Ignoring marking to market, we have for each barrel:

Position	Value in 5 months (per barrel)
Long position in oil	S_5
Short futures position	$F_{0,6} - F_{5,6}$
Net payoff	$F_{0,6} + (S_5 - F_{5,6})$

Because the futures contract matures at $t = 6$, after 6 months $S_6 = F_{6,6}$.

But after only 5 months, the futures price does not equal the spot price then, S_5 . Thus, the amount that you get then will not be exactly $F_{0,6}$.

4. **Basis:** The problems with imperfect hedging give rise to “basis risk”

- a. Definition: **Basis** = spot price of asset to be hedged – futures price of contract used.
 - b. If the asset to be hedged and the asset underlying the futures contract are the same, the basis should be zero at the expiration of the contract. Hence, using futures with matching underlying and maturity gives a perfect hedge.
 - c. Prior to expiration, the basis may be positive or negative. When the spot price increases by more than the futures price, the basis increase. This is referred to as a **strengthening of the basis**. When the futures price increases by more than the spot price, the basis declines. This is referred to as a **weakening of the basis**.
5. Basis risk: a hedger often has to use instruments with basis not converging to zero on target date.
- a. Basis risk: refers to the uncertainty in the basis of a hedging instrument on the target date.
 - b. For investment assets such as currencies, stock indices, gold, and silver, the basis risk tends to be much less than for consumption commodities. The reason is that arbitrage arguments lead to a well-defined relationship between the futures price and the spot price of an investment asset. The basis risk for an investment asset arises mainly from uncertainty as to the level of the risk-free interest rate in the future
 - c. For a consumption commodity, imbalance between supply and demand and the difficulties sometimes associated with storing the commodity can lead to large variations in the convenience yield. This in turn leads to a big increase in the basis risk.
6. Choice of contract

One key factor affecting basis risk is the choice of the futures contract to be used for hedging. This choice has two components:

- a. The choice of the asset underlying the futures contract.
- b. The choice of the delivery month.

If the asset being hedged exactly matches an asset underlying a futures contract, the first choice is generally fairly easy. In other circumstances, it is necessary to carry out a careful analysis to determine which of the available futures contract has futures prices that are **most closely correlated with** the price of the asset being hedged.

The choice of the delivery month is likely to be influenced by several factors. In reality, a contract with a later delivery month (relative to the target date) is usually chosen. The reason is that futures prices are in some instances quite erratic during the delivery month. Also, a long hedger runs the risk of having to take delivery of the physical asset if the contract is held during the delivery month. Taking delivery can be expensive and inconvenient.

In generally, basis risk increases as the time difference between the hedge expiration and the delivery month increases.

Rule of thumb 1: for short-term risks, to choose a delivery month that is as close as possible to, but later than, the expiration of the hedge. Suppose delivery months are March, June, September, and December for a particular contract. For hedge expirations in December, January and February, the March contract will be chosen; for hedge expirations in March, April, and May, the June contract will be chosen; and so on.

Rule of thumb 2: On the other hand, to hedge long-term risks, the hedger can roll over short-term futures contract.

Example. It is June 20. A US firm expects to receive 50 million Japanese yen at the end of November. Yen futures contracts have delivery months of March, June, September, and December. One contract is for the delivery of 12.5 million yen. The firm shorts four December yen futures contracts on June 20. When the yen are received at the end of November, the firm close out its position. Suppose that the futures price on June 20 in cents per yen is 0.78 and that the spot and futures prices when the contract is closed out are 0.72 and 0.725, respectively.

The gain on the futures contract is $0.78 - 0.725 = 0.055$. The effective price obtained in cents per yen is the final spot price plus the gain on the futures:
 $0.72 + 0.055 = 0.775$.

7. Minimum variance hedge ratio

- a. Hedge ratio: the ratio of the size of the position taken in futures contracts to the size of the exposure
- b. With basis risk, how do we choose optimal hedge ratio?

The change in spot price, ΔS , and the change in futures price, ΔF have the regression relation:

$$\Delta S_t = a + b \Delta F_t + e_t$$

Where,

$$E[e_t] = 0$$

$$\text{Cov}[\Delta F_t, e_t]$$

$$b = \frac{\text{Cov}[\Delta S_t, \Delta F_t]}{\text{Var}[\Delta F_t]} = \rho \frac{\sigma_s}{\sigma_f}$$

Where, ρ is the coefficient of correlation between ΔS_t and ΔF_t ; σ_s is the standard deviation of ΔS_t ; σ_f is the standard deviation of ΔF_t .

c. The minimum variance hedge ratio: is the hedge ratio that gives the minimum variance for the value of the hedged position.

For an arbitrary hedge ratio h , when the hedger is long the asset and short futures, the change in the value of the hedger's position during the life of the hedge is:

$$\Delta S - h \Delta F$$

for each unit of the asset held.

the variance of hedged position is:

$$\text{Var}[\Delta S - h \Delta F] = \text{var}[a + b \Delta F + e_t - h \Delta F] = (b-h)^2 \text{Var}[\Delta F] + \text{Var}[e_t]$$

The variance is minimized with the hedge ratio:

$$H^* = b = \frac{\text{Cov}[\Delta S_t, \Delta F_t]}{\text{Var}[\Delta F_t]}$$

d. Optimal number of contracts

Define variables as follows:

N_A : Size of position being hedged (units)

Q_F : size of one futures contract (units)

N^* : Optimal number of futures contracts for hedging

The futures contracts used should have a face value of $h^* N_A$. The number of futures contracts required is therefore given by

$$N^* = h^* N_A / Q_F$$