No.: Date:

Pred for CI of bridge model

~~Rapt~~ CI

~~$Var(\hat{\mu}_{y|x_0}) = Var(\sigma^2$~~

Future CI,

$Var(y_{future}) = \sigma^2 + Var(x_{future}\hat{\beta})$ where $\sigma^2 = Var(\varepsilon)$

Consider law of total Variance

$$Var(x_{future}\hat{\beta}) = E(Var(x_{future}\hat{\beta} \mid x_{future})) + Var(E[x_{future}\hat{\beta} \mid x_{future}])$$

Consider the second term,

$E[x_{future}\hat{\beta} \mid x_{future}] = x_{future}E[\hat{\beta}]$ ~~$= x_{future}$~~

$Var(x_{future}E[\beta]) = E[\beta]' Var(x_{future}) E[\beta]$

Consider the first term,

$Var(x_{future}\hat{\beta} \mid x_{future}) = x_{future}' \sigma^2 (X'X)^{-1} x_{future}$

$E(x_{future}' \sigma^2 (X'X)^{-1} x_{future}) = \sigma^2 tr((X'X)^{-1}\Sigma_{future}) + E[x_{future}]' \sigma^2 (X'X)^{-1} E[x_{future}]$

$$Var(y_{future}) = \sigma^2 + E[\beta]' \Sigma_{x_{future}} E[\beta] + \sigma^2 tr((X'X)^{-1}\Sigma_{x_{future}}) + E[x_{future}]' \sigma^2 (X'X)^{-1} E[x_{future}]$$

Part CI

$Var(\hat{\mu}_{y|x_0}) = Var(x_0\hat{\beta}) = x_0' Var(\hat{\beta}) x_0 = \sigma^2 x_0'(X'X)^{-1}x_0$

For simplicity and since we use independent models to predict each component of x_{future}

$$x_{future} \sim N\left(\mu, \begin{pmatrix} Var(x_i) \\ \vdots \\ Var(x_n) \end{pmatrix}\right),\ \mu \text{ is the predicted value, } Var(x_n) = \text{mse of predicted value}$$

$$\Sigma_{x_{future}} = \begin{pmatrix} Var(x_i) & 0 & \cdots \\ 0 & & \vdots \\ \vdots & \cdots & \end{pmatrix}, \text{ should be a diagonal matrix.}$$

Can say that $x_{future}\hat{\beta}$ follows a normal distribution by CLT,

$x_{future}'\hat{\beta} = x_{future}(X'X)X'y$ thus $= \sum_{i=1}^{p}\sum_{k=1}^{n} x_{future\,i}\, y_k [(X'X)X']_{ki}$. Thus the ~~product~~ sum of many independent variable, &

By CLT

$$y_{future} \sim N\left(x_{future}'\hat{\beta},\ \sigma^2 + E[\beta]'\Sigma_{x_{future}}E[\beta] + \sigma^2 tr[(X'X)^{-1}\Sigma_{x_{future}}] + \sigma^2 E[x_{future}]'(X'X)^{-1}E[x_{future}]\right)$$