MATHS FOR DATA SCIENCE (ASSIGNMENT 2)

1.

(a) We first choose 3 students for group 1, so the number of ways to choose is:

$$12C3 = \frac{12!}{3! \times 9!}$$

Then out of the 9 students remaing, the number of ways to choose 3 students for group 2 is.

$$9C3 = \frac{9!}{3! \times 6!}$$

Continue choosing 3 students for group 3 with 6 students remaining, we have the number of ways to choose is:

$$6C3 = \frac{6!}{3! \times 3!}$$

Notice that the remaining students must be in group 4. So the total number of ways to divide 12 students into 4 groups of 3 is.

$$\frac{12!}{3! \times 9!} \times \frac{9!}{3! \times 6!} \times \frac{6!}{3! \times 3!} = \frac{12!}{(3!)^4} = 369600 \text{ (ways)}$$

$\boxed{\text{Ans : 369600 ways}}$

(b) Because the 4 groups are now indistinguishable, we must divide the result from (a) by the number of permutations of the 4 groups.

We have the permutation of 4 groups is

$$4! = 24 \text{ (ways)}.$$

Hence, the number of ways to allocate 12 ~~student~~ students into 4 indistinguishable groups is:

$$369\,600 \div 24 = 15400 \text{ (ways)}$$

$\boxed{\text{Ans: 15400 ways}}$

(c) According to the problem's statement, the total number of students in all 4 groups is 10, and each group must have at least 2 students. With these requirements, there're only 2 possible cases:

- Case 1: there're 2 groups of 2, 2 groups of 3.
- Case 2: there're 3 groups of 2 and 1 group of 4.

Let's consider both cases:

Case 1: There're 2 groups of 2 and 2 groups of 3.

Let's first assume that all these groups are distinguishable. Then, with the same method used in (a), the number of ways to allocate students in these groups is:

$$10C2 \times 8C2 \times 6C3 \times 3C3$$
$$= \frac{10!}{2! \times 8!} \times \frac{8!}{2! \times 6!} \times \frac{6!}{3! \times 3!} \times \frac{3!}{3!}$$
$$= \frac{10!}{2! \times 2! \times 3! \times 3!}$$
$$= 25200 \text{ (ways)}.$$

But because groups of same size are indistinguishable, we must divide the results by the repetition of groups of the same size, which is $2! \times 2!$ (because there's 2 groups of 2 and 2 groups of 3).

So the number of ways students can form groups in this case is:

$$25200 \div (2! \times 2!) = 6300 \text{ (ways)}.$$

Case 2: There're 3 groups of 2 and 1 group of 4.

Using the same ~~method like the~~ method as the previous case, we can calculate the number of ways to allocate students into 3 groups of 2 and 1 group of 4 (groups of same size are indistinguishable), which is:

$$\frac{10C2 \times 8C2 \times 6C2 \times 4C4}{3! \times 1!}$$
$$= \frac{10!}{2! \times 8!} \times \frac{8!}{2! \times 6!} \times \frac{6!}{4! \times 2!} \div (3! \times 1!)$$
$$= \frac{10!}{2! \times 2! \times 2! \times 4! \times 3!} = \frac{3628800}{2 \times 2 \times 2 \times 24 \times 6} = 3150 \text{ (ways)}$$

With both cases, we ~~can~~ have the total number of ways to allocate the students into groups (according to the problem's requirements) is:

$$6300 + 3150 = 9450 \text{ (ways)}$$

$\boxed{\text{Ans: } 9450 \text{ ways}}$

2.

(a) Because the deck is missing all the picture cards

⇒ The remainging number of cards is:

$$4 \times (13 - 4) = 36 \text{ (cards)}$$

⇒ The number of ways to get a selection of 5 cards from the deck is: $36C5$. (1)

- Now we will calculate the number of ways to get a hand that is 4 of a kind. There're 9 ways to get 4 cards of the same rank (four twos to four tens). For the last card, there're all the cards excluding the 4 cards just picked. So there is:

$$36 - 4 = 32 \text{ (ways)}$$

to pick the last card. So combined with above, we have the number of ways to pick 5 cards and get a 4 of a kind is:

$$9 \times 32 = 288 \text{ (ways)} \ (2)$$

From (1) & (2), we have the probality of being dealt a 4 of a kind is:

$$\frac{288}{36C5} = 0.00076394194 \approx 0.00076$$

$\boxed{\text{Ans: } 0.00076}$

(b) Because there're 2 decks now, the number of cards will be: $52 \times 2 = 104$ (cards).

=) To choose 5 cards, there are $104C5$ ways (3)

- Now we will calculate the number of ways to get a 4 of a kind. Notice that there're now 2 decks, the number of cards of the same ~~rat~~ rank is: $4 \times 2 = 8$ (cards).

So the number of ways to choose 4 cards from that rank to make a 4 of a kind is: $8C4$.

And the number of ways to choose the rank of that 4 cards is 13 (Twos to Aces).

=) The number of ways to choose 4 cards that are a 4 of a kind is: $13 \times 8C4$. (ways).

To choose the last card that doesn't make a 5 of a kind, we can choose any cards excluding the 8 cards of that rank. So the number of ways to choose the last card is:

$$104 - 8 = 96 \text{ (ways)}.$$

Combined with above, we have the number of ways to pick a poker hand that is exactly 4 of a kind is:

$$96 \times 13 \times 8C4 = 87360 \text{ (ways)} \ (4)$$

From (3) & (4), we have the probability of being dealt a 4 of a kind in this context is:

$$\frac{87360}{104C5} = 0.00094995222 \approx 0.00095$$

$\boxed{\text{Ans: } 0.00095}$

(c) The remaining numbers of cards to pick from now is:

$52 - 2 = 50$ (cards)

$\Rightarrow$ The number of ways to pick the other three cards is:

$50C3 = 19600$ (ways) (5)

- Now we will calculate the number of ways to get a 4 of a kind. Notice that the King of Hearts and Ace of clubs have been dealt alredy, so there're exactly 2 ways to get a 4 of a kind with the other 3 cards (either Ace of hearts, spaces, diamonds or King of clubs, spades, diamonds). (6).

From (5) & (6), we have the probability of being dealt a 4 of a kind in this context is:

$$\frac{2}{19600} = 0.00010204082 \approx 0.00010$$

$\boxed{\text{Ans: } 0.00010}$

3. Define: $\begin{cases} A \text{ as the event the hiker is in region } A \\ B \text{ as the event the hiker is in region } B \\ S \text{ as the event that the hiker is found after the helicopter search.} \end{cases}$

(a) According to the problem's statement, the hiker is twice as likely to be in region A as region B.

$\Rightarrow P(A) = 2 \times P(B)$ (1)

And because the hiker can only be in region A or B

$\Rightarrow P(A) + P(B) = 1$

We subtitute P(A) from (1) and get the following:

$2 \times P(B) + P(B) = 1$

$\Leftrightarrow 3 \times P(B) = 1$

$\Leftrightarrow P(B) = \frac{1}{3}$.

Hence, the probality that the hiker is in region B is $\frac{1}{3}$

$\boxed{\text{Ans: } 1/3}$

(b) From (a), we have:

$P(A) = 2 \times P(B) = \frac{2}{3}$.

$\Rightarrow$ The probability that the hiker is found after a search in region A only is:

$$P(S \mid A) \times P(A) = 60\% \times \frac{2}{3}$$

$$= 0.4$$

$\boxed{\text{Ans: } 0.4}$

(c) The probability that the hiker is in B, given that they're not found after a search in region A is:

$$P(B|(S^c|A)) = \frac{P(B)}{1-P(S|A)\times P(A)} = \frac{1/3}{1-0.6\times\frac{2}{3}}.$$

$$= \frac{1}{3-0.6\times 2} = \frac{1}{1.8}$$

$$= 0.555\ldots$$

$$= 0.\overline{5}$$

$$\boxed{\text{Ans: } 0.\overline{5}}$$

(d) The probability that they are not found after a search in both regions is:

$$P(S^c) = P(S^c|A) \times P(A) + P(S^c|B) \times P(B)$$
$$= (1 - P(S|A)) \times P(A) + (1 - P(S|B)) \times P(B)$$

$$= (1-0.6) \times \frac{2}{3} + (1-0.8) \times \frac{1}{3}$$

$$= 0.4 \times \frac{2}{3} + 0.2 \times \frac{1}{3}$$

$$= \frac{1}{3}$$

$\boxed{\text{Ans: } 1/3}$

(e) Using the Bayes' theorem, we have: the ~~pro~~ probability the hiker is in region B given that the entire park is searched and they are not found is:

$$P(B|S^c) = \frac{P(S^c|B) \times P(B)}{P(S^c)}$$

$$= \frac{(1-P(S|B)) \times P(B)}{P(S^c)}$$

Notice that $P(S^c) = \frac{1}{3}$ (from (d))

$$\Rightarrow P(B|S^c) = \frac{(1-P(S|B)) \times P(B)}{P(S^c)}$$

$$= \frac{(1-0.8) \times \frac{1}{3}}{\frac{1}{3}}$$

$$= 0.2$$

$\boxed{\text{Ans: } 0.2}$

4. Define $\begin{cases} B \text{ as the event that the player get the big payout} \\ S \text{ as the event that the player get the small payout} \\ N \text{ as the event that the player get no payout.} \end{cases}$
(in each turn)

(a) The player get a payout if he get B or S. So in order to get a payout 25% (or 0.25) of the time, we have:

$$P(B) + P(S) = 0.25 \quad (1)$$

Furthermore, in order to have an expected value of \$0.9 in each turn, we have:

$$10 \times P(B) + 2 \times P(S) + 0 \times P(N) = 0.9$$

$$(\Leftrightarrow) \quad 10 \times P(B) + 2 \times P(S) = 0.9 \quad (2)$$

From (1) & (2) we have:

$$8 \times P(B) = (10 \times P(B) + 2 \times P(S)) - 2 \times (P(B) + P(S))$$

$$= 0.9 - 2 \times 0.25$$

$$= 0.4.$$

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$\Rightarrow P(B) = 0.05$.

Subtitute that into (1), we have.

$$P(S) = 0.25 - P(B)$$
$$= 0.25 - 0.05$$
$$= 0.2.$$

$\boxed{\text{Ans: } P(B) = 0.05,\ P(S) = 0.2}$.

(b) Notice that if the player get a Big payout in any of the 3 turns, the player will get a profit. So we consider 2 cases:

- Case 1: the player gets a (B) payout in at least one of the 3 turns: We have the probability of this is:

$$1 - P(B^c)^3 = 1 - (1 - P(B))^3$$
$$= 1 - (1 - 0.05)^3$$
$$= 1 - 0.95^3$$
$$= 0.142625.$$

- Case 2: the player doesn't get any (B) payout in any of the 3 turns.

So in this case, the player will either get (S) ~~pay~~ payout or (N) payout. In order to make a profit ~~&~~ (get payout > \$3), there're 2 sub-cases for the player:

+ Sub-case 1: the player gets (S) payout in all 3 turns. The probability of this is:

$$P(S)^3 = 0.2^3$$
$$= 0.008$$

+ Sub-case 2: the player gets 2 (S) payout and 1 (N) payout after the 3 turns. Because there're 3 turns, so the (N) payout can either be in turn 1, 2, or 3. So there're 3 ways for the (N) payout to come. With that included, we have: ~~the p~~ the probability of this sub-case is:

$$3 \times P(S)^2 \times P(N) = 3 \times P(S)^2 \times (1 - P(B) - P(S))$$
$$= 3 \times 0.2^2 \times (1 - 0.05 - 0.2)$$
$$= 3 \times 0.2^2 \times 0.75$$
$$= 0.09.$$

With the sub-cases combined, we have the probability of the player making a profit in Case 2 is:

$$0.008 + 0.09 = 0.098$$

Therefore, with case 1 & 2 combined, we have the probability of the player making a profit after 3 turns is:

$$0.142625 + 0.098 = 0.240625$$

$$\boxed{\text{Ans}: 0.240625}$$