MATHS FOR DATA SCIENCE (ASSIGNMENT 1)

1.

(i) $A \cap B$ is the intersection so we need elements which are in both sets. We test elements of A to see if they ~~bi~~ are in B.

- Firstly, all elements of A are rational numbers ($x \in \mathbb{Q}$ for all $x \in A$)
- But an element $y \in B$ satisfies $\frac{y-1}{2} \in \mathbb{Z}$. As of this table below, only 1 & 3 satisfy the condition.

y	$\frac{y-1}{2}$
0	$-\frac{1}{2}$
1	0
$\frac{1}{2}$	$-\frac{1}{4}$
2	$\frac{1}{2}$
3	1

Hence, $A \cap B = \{1, 3\}$.

$\boxed{\text{Ans: } A \cap B = \{1,3\}}$

(ii) $C \setminus B$ contains elements of C which are not in B. An element of B satisfies $\frac{y-1}{2} = n$ for some $n \in \mathbb{Z}$. This means that $y - 1 = 2n$

$\Leftrightarrow y = 2n + 1$. for some $n \in \mathbb{Z}$.

So, elements of B are odd integers.

The such elements in $(1, 5]$ are 3 and 5.

Therefore, $C \setminus B = (1,3) \cap (3,5)$

$\boxed{\text{Ans: } C \setminus B = (1,3) \cap (3,5)}$

(iii) Firstly, we find $C \cap B$, which is the intersection of B & C. With (ii), we can conclude that:

$C \cap B = \{3, 5\}$.

So we need to find $A \cup \{3,5\}$, which is the union of A and $\{3,5\}$. Notice that 3 is already in A, so $A \cup \{3,5\} = \{0, \frac{1}{2}, 1, 2, 3, 5\}$.

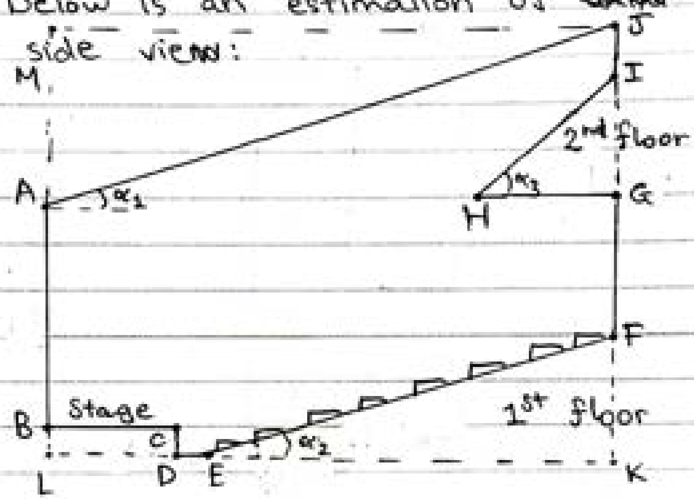
$\boxed{\text{Ans: } A \cup (C \cap B) = \{0, \frac{1}{2}, 1, 2, 3, 5\}}$

2. The statement says
"For all rational numbers x, there exists an ~~a~~ integer y such that $x = 3y - 1$".

This statement is definitely false, as for $x = \frac{1}{2}$, $y = \frac{x+1}{3} = \frac{1}{2}$ which ~~do~~ is not an ~~be~~ integer.

$\boxed{\text{Ans: False}}$

3. Below is an estimation of ~~what~~ Scott Theatre in side view:



Let's denote the points like above.

- AB is ~2.5 times the height of the door, which is ~ 2m

$\Rightarrow$ AB = $2 \times 2.5 = 5$ (m).

- Let's say $\alpha_1 = 30°$, $\alpha_2 = 15°$, $\alpha_3 = 45°$.
- Because the 1st floor has 19 rows of chairs, each row is ~ 60 cm (0.6 m) apart.

$\Rightarrow$ EK = $0.6 \times 19 = 11.4$ (m).

$\Rightarrow$ FK = $11.4 \times \tan(15°) \approx 3$ (m).

- Let's say $\begin{cases} CD = DE = 0.6 \text{ (m)} \\ BC = 3 \text{ (m)} \end{cases}$
- Because the 2nd floor has 7 rows of chairs, each row is ~ 60 cm (0.6 m) apart

$\Rightarrow$ HG = $0.6 \times 7 = 4.2$ (m).

$\Rightarrow$ IG = $4.2 \times \tan(45°) = 4.2$ (m)

- We have: LK = BC + DE + EK

$= 3 + 0.6 + 11.4$

$= 15$ (m)

- And MJ = LK = 15 (m)

$\Rightarrow$ MA = ~~AB~~ MJ $\times \tan(\alpha_1) = 15 \times \tan(30) \approx 8.6$ (m)

$\Rightarrow ML = MA + AB + CD$
$= 8.6 + 5 + 0.6$
$= 14.2$ (m)

With all the estimated data above, we can calculate the area of the side view of Scott Theate:

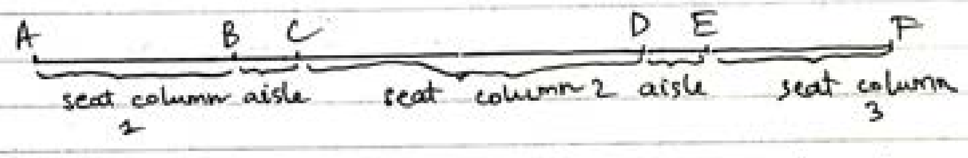
$$S = S_{MJKL} - S_{AMJ} - S_{HIG} - S_{EFK} - S_{BCDL}$$

$$= ML \times LK - \frac{AM \times LK}{2} - \frac{HG \times IG}{2} - \frac{EK \times FK}{2} - BC \times CD$$

$$= 14.2 \times 15 - \frac{8.6 \times 15}{2} - \frac{4.2 \times 4.2}{2} - \frac{11.4 \times 3}{2} - 3 \times 0.6$$

$$= 120.78\ (m^2). \quad (1)$$

Now we will calculate the width of the theatre. Here is an estimation of the width of theatre.



- Seat column 1 & 3 have 7 chairs, each chair is ~ 50cm (0.5m) apart.

~~⇒ the length of column 1~~

$\Rightarrow AB = EF = 0.5 \times 7 = 3.5$ (m).

- seat column 2 have 11 chairs, each chair is ~ 50cm (0.5) apart.

$\Rightarrow CD = 0.5 \times 11 = 5.5$ (m)

- let's say the aisle is 80cm (0.8 m) apart.

⇒ ~~AF~~ the width of the theatre is:

$$L = AB + BC + CD + DE + EF$$
$$= 3.5 + 0.8 + 5.5 + 0.8 + 3.5$$
$$= 14.1\ (m). \quad (2)$$

With (1) & (2), we can calculate the volume of Scott Theatre:

$$V = S.L = 120.78 \times 14.1 \approx 1703\ (m^3).$$

Let's say a tennis ball has a diameter of 10cm

⇒ The radius of the ball is

$$R = 10/2 = 5\ (cm)$$
$$= 0.05\ (m)$$

⇒ The volume of a tennis ball is:

$$V_{ball} = 0.05^3 \cdot \frac{4}{3} \cdot \pi \approx 0.0005236\ (m^3)$$

So Scott Theatre can fit:

$$1703 : 0.0005236 \approx 3,252,483\ (balls)$$

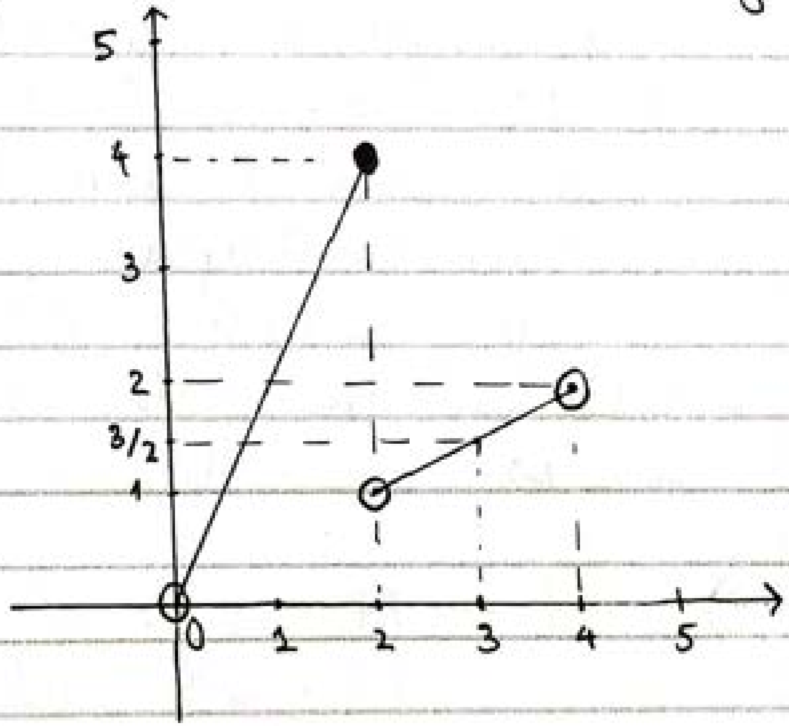
We will times this result by a variable $\alpha = 0.9$ This is because of loss of space between balls, the volume of chairs, and other factors.

Therefore, the estimation of ~~Scott Theatre~~ the number of tennis balls that can fit into Scott Theatre is:

$$3,252,483 \times 0.9 = 2,927,235\ (balls)$$

$\boxed{\text{Ans: } 2,927,235 \text{ tennis balls}}$

4. Firstly, we will draw out the graph of f:



(i) From the graph above, we can see that the range of f is $(0, 4]$.

$\boxed{\text{Ans}: (0, 4]}$

(ii) We have $f \circ f(3) = f(f(3))$.

So we need to find $f(3)$ first, which is $\frac{3}{2}$ according the graph above.

And now we ~~we~~ will find $f\left(\frac{3}{2}\right)$.

Notice that, $\frac{3}{2} \in (0,2]$.

$\Rightarrow f\left(\frac{3}{2}\right) = 2 \times \frac{3}{2} = 3$.

Therefore, $f(f(3)) = f\left(\frac{3}{2}\right) = 3$.

$\boxed{\text{Ans: } 3}$

(iii) If we look at the graph, we can deduce that the only solution to $f(x)=2$ is $x=1$

We will now prove that $x=1$ is the only solution.

Indeed, from the definition we know that the domain of f is $(0,4)$.

We will divide this into 2 cases:

- Case 1: $x \in (0,2]$ and $f(x)=2$.

According to the definition, $f(x)=2x$.

$\Rightarrow 2 = f(x) = 2x$

$\Rightarrow x = 1$.

By replacing 1 into the function, we can conclude that $x=1$ is indeed a solution

- Case 2: $x \in (2,4)$ and $f(x)=2$

According to the definition, $f(x) = \frac{x}{2}$.

$\Rightarrow 2 = f(x) = \frac{x}{2}$

$\Rightarrow x = 4$. (which does not satisfy as $x \in (2,4)$)

Overall, we can ~~so~~ conclude that $x=1$ is the only solution to $f(x)=2$

$\boxed{\text{Ans: } x=1}$

(iv) The inverve funtion of f does not exist since ~~for $f(x)=$~~ $f(1) = f(3) = \frac{3}{2}$.

Because of that, if $f^{-1}(x)$ does exists, the function $f^{-1}\left(\frac{3}{2}\right)$ must be both 1 & 3 (contradiction)

Hence, the inverse function of f does not exist.

$\boxed{\text{Ans: No}}$