Activity: Estimation of seasonal effects using smoothing

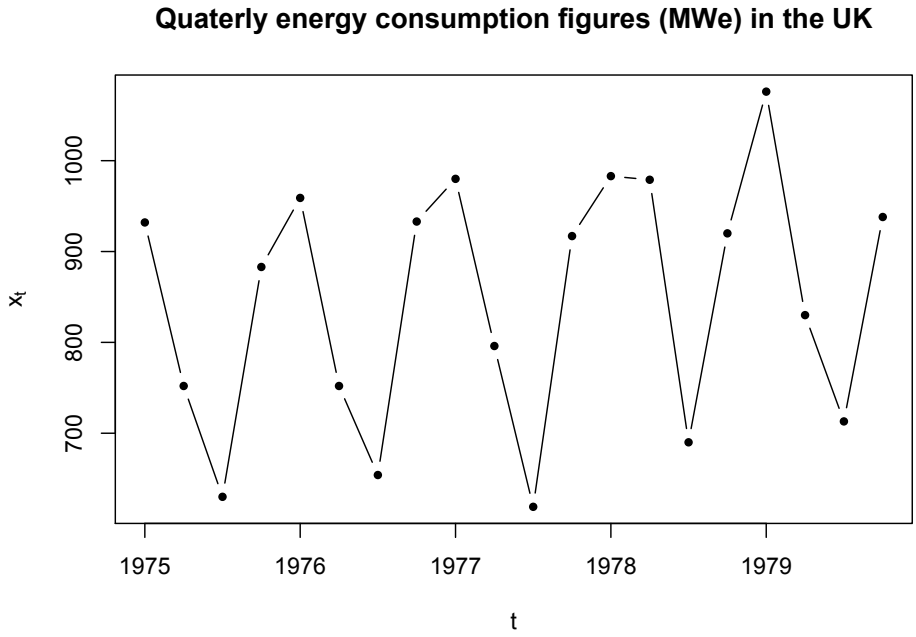
Recall: Estimation of seasonal effects using smoothing

- The (centred) moving average series can be taken as the **trend estimate**, denoted $\{\hat{m}_t\}$.
- The additive seasonal effect at time t is then $\hat{s}_t = x_t - \hat{m}_t$.
- Averaging these estimates for each period (e.g., month, quarter) gives estimates of the seasonal effects, denoted $\widehat{S}_j$ $(j = 1, \ldots, p)$.
- An adjustment can be made to make seasonal effects add up to zero.
- $\{x_t - \widehat{S}_j\}$ for index t corresponding to seasonal period j is then a **seasonally adjusted series** (under the additive model).
- Under the multiplicative model, subtraction is simply replaced by division.

The following data are the quarterly energy consumption figures (in MWe) in the UK for the years 1975–1979, where y_t is the moving average of order 4 of original series $\{x_t\}$ and $\hat{m}_t$ is the moving average of order 2 of series $\{y_t\}$.

Year	Quarter	t	x_t	y_t	$\hat{m}_t$	$x_t - \hat{m}_t$
1975	1	1	932			
1975	2	2	752			
1975	3	3	630	*	803	*
1975	4	4	883	*	*	77
1976	1	5	959	806	809	150
1976	2	6	752	812	818	-66
1976	3	7	654	824	827	-173
1976	4	8	933	830	835	98
1977	1	9	980	841	836	144
1977	2	10	796	*	*	-34
1977	3	11	619	*	828	-209
1977	4	12	917	829	852	65
1978	1	13	983	874	883	100
1978	2	14	979	892	893	86
1978	3	15	690	893	905	-215
1978	4	16	920	916	898	22
1979	1	17	1076	*	*	*
1979	2	18	830	*	*	*
1979	3	19	713	*		
1979	4	20	938			

1. Below is a plot of the time series. Comment on what you observe.



2. Find the numbers indicated by "$*$" in the above table.

3. Assuming an additive seasonal effect and making use of the filtered series, estimate the adjusted seasonal indices $\hat{S}_1$, $\hat{S}_2$, $\hat{S}_3$, $\hat{S}_4$.

4. Why would the method you applied in 3. be preferable here to the method first applied to the Lake of the Woods data that does not use smoothing?

5. When the filtered data are regressed against index t, the fitted line is

$$T_t = 776.18 + 6.98t, \qquad t = 1, 2, \ldots.$$

Using this, forecast the energy consumption for the first two quarters of year 1980.