UNIVERSITY OF BRITISH COLUMBIA
Department of Statistics
Stat 443: Time Series and Forecasting
Assignment 2: Time Series Models

The assignment is due on **Thursday, February 13** at **9:00 pm**.

- Submit your assignment online in the **pdf format** under module "Assignments".
- This assignment can be hand-written and scanned or typeset (e.g., using either LaTeX or R Markdown).
- When answering the questions, writing down the final answers will not be sufficient to receive full marks. Please explain your steps and show all calculations unless otherwise specified.
- Please make sure your submission is clear and neat. It is the student's responsibility that the submitted file is in good order (e.g., not corrupted and is what you intend to submit).
- **Late submission penalty**: 1% per hour or fraction of an hour. (In the event of technical issues with submission, you can email your assignment to the instructor to get a time stamp but submit it on canvas as soon as possible to make it available for grading.)

Let $\{Z_t\}_{t\in\mathbb{Z}}$ denote a white noise process with mean zero and variance σ^2.

1. Consider the stationary second-order AR process $\{X_t\}_{t\in\mathbb{Z}}$ with $X_t = 0.1X_{t-1} + 0.2X_{t-2} + Z_t$.

   (a) Derive the Yule-Walker equations and find the autocorrelation function of $\{X_t\}$.

   (b) Assume $\sigma^2 = 1$. Use set.seed(2025) and arima.sim() function to simulate 10,000 observations from the AR(2) process defined above, and plot the sample acf for the first 15 lags along with the theoretical acf obtained in part(a). Do not use the ARMAacf() function. Include your R code.

2. Consider the ARMA$(1, 2)$ process $X_t = 0.5X_{t-1} + Z_t - 0.8Z_{t-1} + 0.8Z_{t-2}$.

   (a) Check whether the process is stationary and invertible.

   (b) Write the above ARMA$(1, 2)$ process as a pure MA process.

   (c) Find the acf of $\{X_t\}$.

   (d) Write the above ARMA$(1, 2)$ process as a pure AR process.

3. Consider the ARMA$(1, 2)$ process given by

$$X_t - aX_{t-1} = Z_t - bZ_{t-2}, \quad a, b \in \mathbb{R}\backslash\{0\}.$$

   (a) Under which conditions is the process $\{X_t\}$ stationary but **NOT** invertible?

   (b) Assuming $\sigma^2 = 1, a = \frac{1}{3}$, and $b = 2$, compute $\gamma(0), \gamma(1)$, and $\gamma(2)$, where $\gamma(\cdot)$ is the acvf of $\{X_t\}$. Then find the values of $\rho(1)$ and $\rho(2)$, where $\rho(\cdot)$ is the acf of $\{X_t\}$.

4. Show that SARIMA$(1, 2, 1) \times (0, 1, 2)_3$ can be written as an ARMA(p, q) process. Specify the values of order parameters p and q.