Activity: Autoregressive Processes

Suppose $\{Z_t\}$ is white noise with mean zero and variance σ^2. We have seen that a process $\{X_t\}$ is said to be a moving average process of order q, denoted MA(q), if

$$X_t = \beta_0 Z_t + \beta_1 Z_{t-1} + \cdots + \beta_q Z_{t-q}$$

for some constants $\beta_0, \beta_1, \ldots, \beta_q$, with usually $\beta_0 = 1$. This activity concerns a special case of the model above with $q \to \infty$, $\beta_0 = 1$ and $\beta_i = \alpha^i$ for $i \geq 1$, specifically

$$X_t = Z_t + \alpha Z_{t-1} + \alpha^2 Z_{t-2} + \cdots \tag{1}$$

1. What condition on α is necessary for the right–hand side of Equation (1) to be well-defined?

2. Show that $X_t = \alpha X_{t-1} + Z_t$.

3. We call $\{X_t\}$ an *autoregressive process* of order 1, denoted AR(1) . Each value in the series is modelled as the previous value plus noise. Find $\mathrm{E}(X_t)$.

4. Find the variance of X_t. (Remember if $|r| < 1$, $\sum_{k=0}^{\infty} ar^k = a/(1-r)$.) How does this relate to your answer to Question 1?

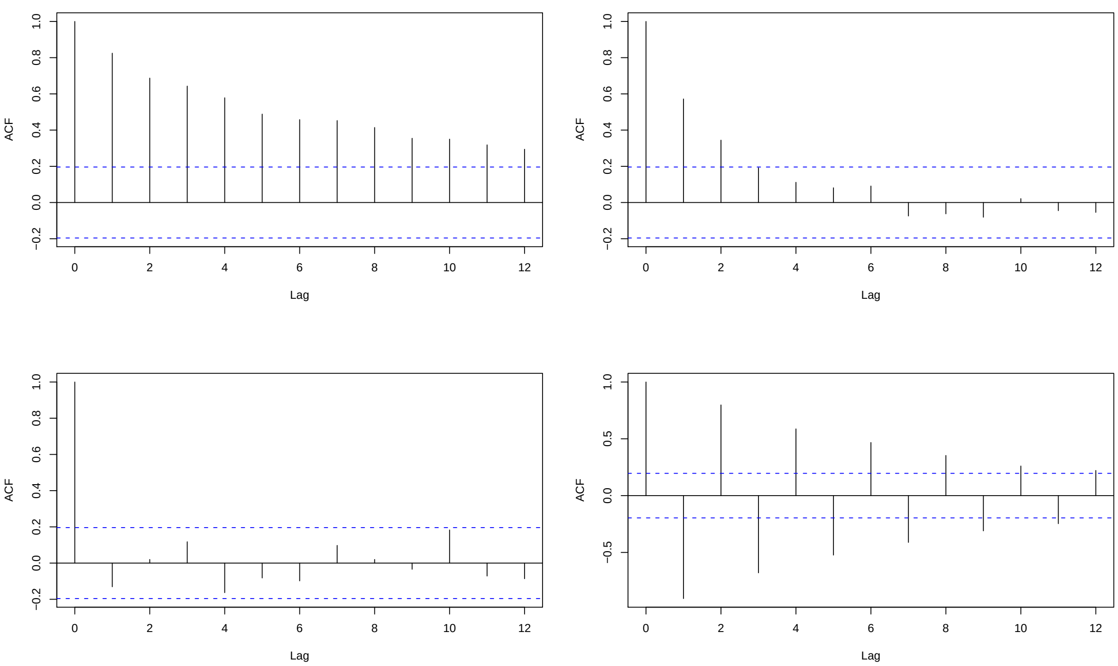
5. If $E(X_t) = 0$ for all t, recall the acvf of X_t at lag h is

   $$\gamma(h) = E(X_t X_{t+h}).$$

   By substituting Equation (1) into the above, compute $\gamma(h)$ for $h \geq 0$. The identity given in Question 4 may again be useful.

6. Is $\{X_t\}$ stationary?

7. Four AR(1) processes are generated with $\alpha = -0.9, 0, 0.5$ and 0.9. Identify which autocorrelation plot corresponds to each value of α.



ACF
Lag