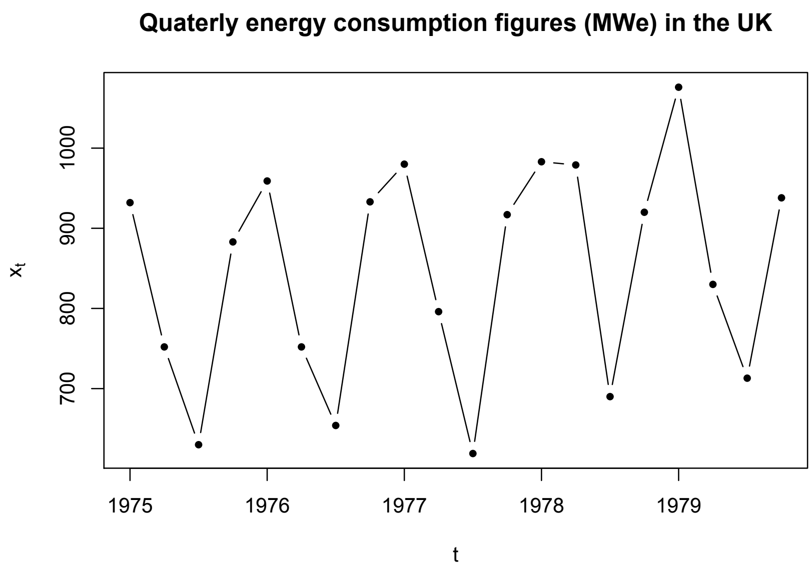
Activity solution: Estimation of seasonal effects using smoothing

The following data are the quarterly energy consumption figures (in MWe) in the UK for the years 1975–1979, where y_t is the moving average of order 4 of original series x_t and $\hat{m}_t$ is the moving average of order 2 of series y_t.

[TABLE OMITTED]

1. *The data exhibit a clear seasonal pattern with period 4, and a slight upward trend.*



2. Find the numbers indicated by “$*$” in the above table.

Year	Quarter	t	x_t	y_t	$\hat{m}_t$	$x_t - \hat{m}_t$
1975	1	1	932			
1975	2	2	752			
				799		
1975	3	3	630		803	**-173**
				806		
1975	4	4	883		**806**	77
				806		
1976	1	5	959		809	150
				812		
1976	2	6	752		818	-66
				824		
1976	3	7	654		827	-173
				830		
1976	4	8	933		835	98
				841		
1977	1	9	980		836	144
				832		
1977	2	10	796		**830**	-34
				828		
1977	3	11	619		828	-209
				829		
1977	4	12	917		852	65
				874		
1978	1	13	983		883	100
				892		
1978	2	14	979		893	86
				893		
1978	3	15	690		905	-215
				916		
1978	4	16	920		898	22
				879		
1979	1	17	1076		**882**	**194**
				885		
1979	2	18	830		**887**	**-57**
				889		
1979	3	19	713			
1979	4	20	938			

3. Assuming an additive seasonal effect and making use of the filtered series, estimate the adjusted seasonal indices S_1, S_2, S_3, S_4.

The following table illustrates the calculation:

	Q1	Q2	Q3	Q4
	$*$	$*$	-172.63	77.00
	150.00	-66.25	-173.13	97.75
	143.63	-34.00	-209.38	65.38
	99.63	86.38	-214.63	22.38
	194.12	-57.00	$*$	$*$
Mean ($\bar{s}_t$):	146.84	-17.72	-192.44	65.63

Now since $146.84 - 17.72 - 192.44 + 65.63 = 2.31$*, subtracting 0.5775(*$=2.31/4$*) from* *each term we find the adjusted seasonal indices as*

$$\begin{aligned} S_1 &= 146.26 \\ S_2 &= -18.30 \\ S_3 &= -193.02 \\ S_4 &= 65.05 \end{aligned}$$

4. Why would the method you applied in 3. be preferable here to the method first applied to the Lake of the Woods data that does not use smoothing?

   As the series has an apparent trend, the simpler method would ignore the fact that values in each quarter are tending to be higher at the end than the start.

5. When the filtered data are regressed against t, the fitted line is

   $$T_t = 776.18 + 6.98t.$$

   Using this, forecast the energy consumption for the first two quarters of year 1980.

   Extrapolating the trend and adjusting for the seasonal effect, we get

t	T_t	S_t	$\hat{x}_t$
21	922.76	146.26	1069.02
22	929.74	-18.30	911.44

   where $\hat{x}_t = T_t + S_t$, assuming the additive model.