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Coordination-free Collaborative Replication based on Operational Transformation

Masato Takeichi

University of Tokyo
takeichi@acm.org

Abstract. We introduce Coordination-free Collaborative Replication (CCR), a new method for maintaining consistency across replicas in distributed systems without requiring explicit coordination messages. CCR automates conflict resolution, contrasting with traditional data sharing systems that typically involve centralized update management or predefined consistency rules.
Operational Transformation (OT), commonly used in collaborative editing, ensures consistency by transforming operations while maintaining document integrity across replicas. However, OT assumes server-based coordination, which is unsuitable for modern, decentralized Peer-to-Peer (P2P) systems.
Conflict-free Replicated Data Type (CRDT), like Two-Phase Sets (2P-Sets), guarantees eventual consistency by allowing commutative and associative operations but often result in counterintuitive behaviors, such as failing to re-add an item to a shopping cart once removed.
In contrast, CCR employs a more intuitive approach to replication. It allows for straightforward updates and conflict resolution based on the current data state, enhancing clarity and usability compared to CRDTs. Furthermore, CCR addresses inefficiencies in messaging by developing a versatile protocol based on data stream confluence, thus providing a more efficient and practical solution for collaborative data sharing in distributed systems.
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1 Introduction

We propose a novel idea of *Coordination-free Collaborative Replication* (CCR) that guarantees the consistency of replicas under individual updates in distributed systems. It makes automatic conflict resolution without any explicit message exchange for coordination.

Operational Transformation [1,12,13] is a technology to guarantee consistency when updating common data in collaborative data sharing. For example, we put a replica of a text document at each site for editing. Each site sends the editing operations done on the local replica to the server. The server transmits operations to other clients to keep all the replicas the same in distributed environments. Each site appropriately applies the editing operations sent from the

server to its local replica. For the transformation of text editing, the basic operations of inserting and deleting character strings and moving the cursor keeps the consistency that all replicated documents are the same when the editing operation is completed. Various practical document processing systems have been developed and used in this way. A conflict may occur when character strings are inserted concurrently to the same point at multiple sites. To resolve this kind of conflict, we may take one of the insertions results are made valid, or both are made effective by shifting either one.

While this approach is extendable to other structured documents, OT-based data sharing usually assumes that the server manages updates of clients. However, this is not the case for recent applications of distributed systems. We should be more generous and let ourselves to make our replication to the server-less configuration for Peer-to-Peer applications suitable for the *local-first* applications [5]. We explore the feasibility of defining OT for such systems and propose a *Collaborative Replication* technique.

CRDT [11,10,17] is another method for ensuring consistency when data is updated by an operation on the replica placed at sites. The goal of the CRDT is to ensure eventual consistency, where the replicas converge when updates terminate. For that purpose, we make data closed under commutative and associative operations. There, the operations on the data are limited to satisfy the monotonic condition which is not always clear from our intuitive aims of updating.

A shopping cart implemented using Two-Phase Set (2P-Set) CRDT [10] is an example. It accepts operations (Add x) to add item x into the cart and (Rem x) to remove item x from the cart. 2P-Set consists of a pair (A, R) that keeps added items in A and removed items in R, which grow every time these operations are performed as $A \cup \{x\}$ and $R \cup \{x\}$. The items in the cart can be calculated as a set difference $A \setminus R$. This implementation fails the item x added again when x has once been added and then removed. Can this shopping cart be acceptable, or is this implementation intuitive?

In contrast to this CRDT shopping cart, Collaborative Replication implements the cart C to which x is added as $C \cup \{x\}$ if $x \notin C$, and from which x is removed as $C \setminus \{x\}$ if $x \in C$. It is straightforward to specify how the cart C is updated. It enables us to add x once having been removed. Note here that these operations refer to the current status of C for checking the effectiveness of updates on C called *Set with Effectful Operations* [15]. It is much more understandable than CRDT implementation.

Another point to note in updating common data in a distributed environment is how to eliminate inefficiency due to messaging in the distributed system. We have to study more about protocols for avoiding such inefficiency as well as resolving conflicts related to updating. We develop a concise but versatile protocol based on the property of the *confluence* of data stream updated by operations. It might be what we come to think of if we know it.

Combined these, we call this novel approach *Coordination-free* protocol for *Collaborative Replication*.

2 Collaborative Replication

The problem of conflicts caused by concurrent updates at different sites can be solved by either making adjustments in advance so that no concurrent updates start, or by choosing one of the concurrent updates.

The elimination of conflicting data updates involves a temporary shutdown of other sites other than the one updating. When choosing one of the concurrent updates, a slight difference in the update timing may cause consistency problems.

In contrast, there are proposals to set certain restrictions on the nature of the data to be shared, i.e., the operations to update the data, so that conflicts themselves cannot occur or can be resolved to make the result in a deterministic way. CRDT (Conflict-free Replicated Data Type) and CCR (Coordination-free Collaborative Replication) aims in this direction.

Shattering the CALM for Collaborative Data Sharing CRDTs and CCR are within the same framework of collaboration in how updates performed and how they reflected as shared data. CCR has aspects that are more concise and versatile than the structure of CRDTs, and that is the focus of this discussion.

While coordination is a "killer" of performance in distributed systems, no coordination may suffer from the consistency of distributed data. The CALM (Consistency As Logical Monotonicity) theorem brings about a solution to the question [3]: "What is the family of problems that can be consistently computed in a distributed fashion without coordination, and what lies outside that family?" as "A program has a consistent, coordination-free distributed implementation if and only if it is monotonic."

CALM leverages static analysis to certify the state-based convergence properties provided by CRDTs which provide a framework for monotonic programming patterns.

However, monotonicity is not the only golden rule for coordination-free data sharing in distributed systems! A binary operation is commutative if the order of its operands makes no difference to the result. The confluence property as applied to program operations is a generalization of commutativity.

- An operation is confluent if it produces the same outputs for any nondeterministic ordering of a set of inputs.
- If the outputs of one confluent operation are consumed by another confluent operator as inputs, the resulting composite operation is confluent.
- Hence, if we restrict ourselves to building programs by composing confluent operations, our programs are confluent by construction, despite orderings of messages or execution steps within and across sites of distributed systems.

Although confluent operations are the basic constructs of monotonic systems, they can do more than that if collaboration is utilized for establishing confluence of components of distributed systems.

Thus, we can shatter the restricted CALM for monotonicity to open up more flexible CCR than CRDTs.

Operational Transformation is Disliked in Replication, but ... Compared with the CRDT approach, Operational Transformation originally proposed in 1989 has been kept away from replicated data in server-less systems since most algorithms were proved to be wrong [14,7].

In OT, every updating process is a sequence of a patch of operations transmitted between sites. Received operations in each site are transformed to include the local operations performed since the last common data. They are then applied locally to get the new data.

For high-level operations like server-less text editing such transformation has too many edge cases which are difficult to produce in the same state. However, it is not in many cases of selected operations dealt with by CRDTs. Or more than that, we should notice that OT generates "new operations" from the already performed local operations for making the states confluent and hence leads to more flexible "collaborative" replication. It is in contrast with CRDT where the predefined common operations are used in both sites.

Our CCR is a new challenge to exploit OT-based data sharing. It comes from the Tutorial Blog[1] "Realtime collaborative editor. Algebraic properties of the problem" by Yuras Shumovich, March 5, 2016 which explains concisely the idea and pitfalls of Collaborative Editor (RTCE) implementations based on operational transformation. Although it assumes that the system uses a server for maintaining the state of clients, it works in a server-less environment, too.

OT-based Collaborative Replication is very lightweight without extra metadata essential in CRDTs, and hence enable composing replicated data from simple data as the popular abstract data definitions using data constructors, e.g., tuples, maps, etc. It is what we claim our CCR to exploit a novel scheme for consistent data sharing.

3 Collaborative Replication based on Operational Transformation

3.1 Updating Operations and Operational Transformation

Updating Operations Updating operations on the replica are defined for manipulating the replica to share the data according to the user's intention.

- The set O of operations on data $D \in \mathcal{D}$ is a *monoid* $(O, !, \odot)$ with the identity operation "!" with several basic operations as generators and $\odot :: O \times O \to O$.
- Updating operator $p \in O$ applied to D is generically written as $D \odot p$ using a binary postfix operator symbol $\odot :: \mathcal{D} \times O \to \mathcal{D}$, which is overloaded with the monoid operation. Thus, $D \odot p \odot q = (D \odot p) \odot q = D \odot (p \odot q)$.
- The sequence of updating operators $p_1, p_2, \cdots$ is called *patch* and written as $\langle p_1, p_2, \cdots \rangle$, which is of course an operator in O.
- The equality on $\mathcal{D}$ of data $D \odot p$ and $D \odot q$ both of which are D applied updating operators p and q respectively defines the equivalence relation $p \leftrightarrow_D q$

[1] http://blog.haskell-exists.com/yuras/posts/realtime-collaborative-editor.html

on O. This defines the equality of operators $p =_D q$. Note that these relations depend on D to which the operators are applied.

In practical situations, basic operations p and q are defined on the data $D \in \mathcal{D}$ to be replicated. For example, basic operators for text editing are $(\mathsf{Ins}\ k\ t) \in O$ for inserting string t at position k into the replicated text, and $(\mathsf{Del}\ k\ n) \in O$ for deleting n characters at k from the replica. These operations are usually "generators" of the monoid applied to the replicated data. Successive application of these generator operators to the data can be expressed using the application of the patch of these operators, e.g., $\langle \mathsf{Ins}\ k_1\ t_1, \mathsf{Del}\ k_2\ n_2, \mathsf{Ins}\ k_3\ t_3 \rangle$ which is a representation of the element $(\mathsf{Ins}\ k_1\ t_1 \odot \mathsf{Del}\ k_2\ n_2 \odot \mathsf{Ins}\ k_3\ t_3) \in O$ of the monoid $(O, !, \odot)$. Note that the element of the monoid may have several patch representations.

Operational Transformation Operational Transformation $T_D :: O \times O \to O \times O$ takes operations p and q on D and gives a pair of operations p' and q' by $(p', q') = T_D(p, q)$.

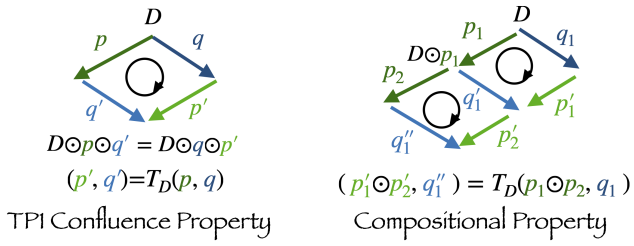

Fig. 1. Operational Transformation for Replication

Our OT for replication is assumed to have the following properties:

Local Confluence Property The transformation T_D takes operations p and q on D and produces $(p', q') = T_D(p, q)$ which makes $D \odot p \odot q' = D \odot q \odot p'$. That is, p and q on the same D are made into the common confluent state by T_D. This property is called "TP1-Confluence" which is the basis of "TP2-Confluence" for global integration stability described below.

Minimal Property If $(p', q') = T_D(p, q)$ holds, there is no operations p'', q'', $r_p \neq !$ and $r_q \neq !$ satisfying $(p'', q'') = T_D(p, q)$ where $p' = p'' \odot r_p$ and $q' = q'' \odot r_q$. That is, no extra operations are introduced by T_D.

Symmetric Property If $(p', q') = T_D(p, q)$ and $(q'', p'') = T_D(q, p)$, then $p' = p''$ and $q' = q''$ hold.

Compositional Property If $(p'_1, q'_1) = T_D(p_1, q_1)$ and $(p'_2, q''_1) = T_{D \odot p_1}(p_2, q'_1)$, $T_D(p_1 \odot p_2, q_1)$ gives $(p'_1 \odot p'_2, q''_1)$.

These properties are the most fundamental and more are required for advanced purposes as described in the next section. Before going there, we review the presumption of this property.

– In case that p and q have their inverse p^{-1} and q^{-1}, that is the operation monoid $(O, !, \odot)$ is a semigroup, we may define T_D as $T_D(p, q) = (q^{-1}, p^{-1})$ which always makes D into the confluent data D. However, no one would like to use this transformation.
– The Symmetric Property is natural for server-less P2P replication where there is no dedicated site for the server to control the update propagation and all the sites in the system are on an equal footing with others. One of the transformations for text editing claimed to be correct is actually non-symmetric because it is aimed solely at the environment of the server-clients system.
– The Compositional Property is very natural for the transformation is defined for the basic operations and extends it to accept any operations of the monoid $(O, !, \odot)$.

3.2 Building up Confluence Property of Operational Transformation

Operational transformation $T_D :: O \times O \to O \times O$ for two sites P and Q gives $(p_q, q_p) = T_D(p, q)$ for operations p and q respectively, and fulfills *TP1-Confluence* $p \odot q_p \leftrightarrow_D q \odot p_q$, which we write as $p \#_D q$. When D is clear from the context, $\#_D$ may be written simply as $\#$.

Idempotence of Confluence Relation When P and Q share a common updating operation p applied to the same data D to get $D \odot p$, it is natural to assume that the confluent state is $D \odot p$ and hence $p \#_D p = p$.

Although this conforms to the Minimality of T_D, it is subtle to define $T_D(p, p) = (!, !)$ that implies $p \#_D p = p$. As mentioned in Section 1 in the text editing application, a conflict may occur when strings x and y are inserted to the same point k by operations $p = (\mathsf{Ins}\ k\ x)$ and $q = (\mathsf{Ins}\ k\ y)$. To resolve the conflict, we may reasonably make both insertions effective by shifting q's k to $k' = k + length(x)$ to define $T_D(\mathsf{Ins}\ k\ x, \mathsf{Ins}\ k\ y) = (\mathsf{Ins}\ k\ x, \mathsf{Ins}\ k'\ y)$. However, if x and y are equal, this contradicts $T_D(p, p) = (!, !)$ and $p \#_D p = p$.

What happens there? So far, we have never been concerned about the independence of the operations applied in different sites: above p and q look the same for $x = y$ in spite of their independence. Hereafter to make the independence explicit by attaching the site ID $\$i$ in the operation as $(\mathsf{Ins}\ \$i\ k\ x)$ for the internal representation of the operation, while the external representation remains as $(\mathsf{Ins}\ k\ x)$.

Commutativity of Confluence Relation It is straightforward that the confluent relation $\#_D$ is commutative, that is, $p \#_D q = q \#_D p$ for operations p and q on the same data D.

Associativity of Confluence Relation Then, how about the confluence relation for three sites P, Q, and R? Consider updates by three sites implemented

by applying the transformation T_D in two steps. We first apply replication of P with another site Q or R, and then apply next with the rest of R or Q. Thus,

- $P \to Q \to R$: P having updated D into $D \odot p$ receives Q's update q and computes $(q_p, _) = T_D(q, p)$ to produce $D \odot p \odot q_p$, and then receives R's update r and computes $(r_{p \odot q_p}, _) = T_D(r, p \odot q_p)$ to get $D \odot (p \odot q_p) \odot r_{p \odot q_p} = D \odot ((p \#_D q) \#_D r)$.
- $P \to R \to Q$: P having updated D into $D \odot p$ receives R's update r and computes $(r_p, _) = T_D(r, p)$ to produce $D \odot p \odot r_p$, and then receives Q's update q and computes $(q_{p \odot r_p}, _) = T_D(q, p \odot r_p)$ to get $D \odot (p \odot r_p) \odot q_{p \odot r_p} = D \odot ((p \#_D r) \#_D q)$.

Here, we could say that it is necessary to assure $(p \#_D q) \#_D r = (p \#_D r) \#_D q$ for each site to lead its data to the confluent state. Similarly, other cases of permutations of P, Q, and R, namely $Q \to R \to P$, $Q \to P \to R$, $R \to P \to Q$, and $R \to Q \to P$.

To establish the confluence property of T_D applied in two steps for updates on the same data D by three sites, the pairwise equivalent relations need to hold regardless of the order of applications. That is, $(p \#_D q) \#_D r = (p \#_D r) \#_D q$, $(q \#_D r) \#_D p = (q \#_D p) \#_D r$, and $(r \#_D p) \#_D q = (r \#_D q) \#_D p$ hold.

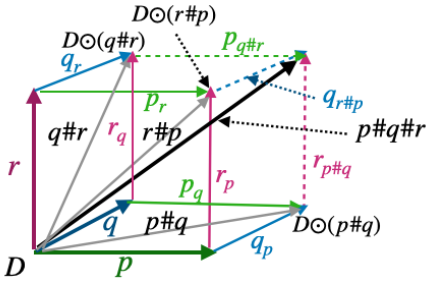

Fig. 2. TP2 Confluence Property

Therefore, we assume that our OT for collaborative replication must satisfy the associativity property of the confluence relation. That is, for operations p, q and r on the same data,

$$(p \#_D q) \#_D r = p \#_D (q \#_D r)$$

holds, and these may be written as $p \#_D q \#_D r$ without parentheses.

TP2-Confluence Property The above confluence property is a case of *TP2-Confluence Property* widely known in the OT community [8,9,2]. This requires a confluence of the transformation for more than three sites in general. That is, it should hold for more than four, not just for three.

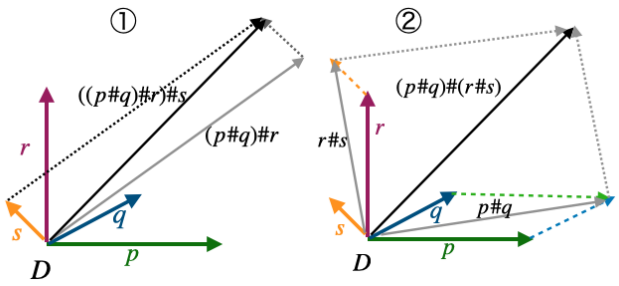

Fig. 3. Confluence of Four Sites

However, it is ready to extend the above story to the cases of more than four sites.

Updates of four sites are implemented by three applications of T_D; the last step of applications may take a fresh operation s with the confluent operation $p\#_D q\#_D r$ of the three, or take the confluent operations $p\#_D q$ and $r\#_D s$ of the two. The confluence property of two-step applications up to three sites is assured as above. Therefore, updates of four sites are made confluent. This inductively concludes that the associativity and commutativity properties of the confluence relation assure the general confluence of any number of sites. This is what the TP2-Confluence Property holds.

Algebraic Structure of Confluence Property

Identity element $p\#_D ! = !\#_D p = p$
Idempotence $p\#_D p = p$, which comes from the Minimal Property of T_D
Commutativity $p\#_D q = q\#_D p$, which comes from TP1-Property of T_D
Associativity $(p\#_D q)\#_D r = p\#_D (q\#_D r)$, which is for TP2-Property of T_D

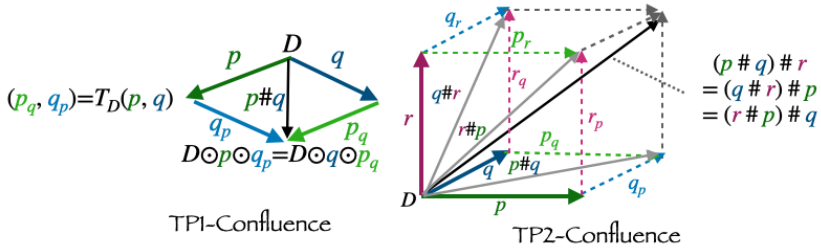

Fig. 4. TP1+TP2 Confluence Property

The above discussion suggests us to put the TP2-Confluence Property into practical coordination-free replication by sending updated operations on the common replicated data in arbitrary order to others.

4 Replication Protocol for Coordination-free Collaborative Replication

Replication Protocol of Operation-based CRDT In Operation-based CRDT (CmRDT), the exchange of updated operations for replication is performed by Reliable Causal Broadcast (RCB), while in State-based CRDT (CvRDT), by merging data states (merge).

RCB for exchanging updates must realize that every local update at each site is eventually reflected at other sites. Local updates at a site must be sent to the other site again in the event of a disconnection or system outage after a local update. According to the CmRDT definition, when a site sends a message to other sites, there is no confirmation of its arrival by the other sites, and each site can operate even when disconnected.

Replication Protocol of OT-based Collaboration Although OT-based replication shares some similarities with CmRDT, its protocol does not necessarily follow RCB.

OT-based collaborative data sharing also realizes reliable replication by broadcast, where all the local updates eventually arrive on all sites. The causality of operation events is not built into the protocol, but it is handled in the definition of OT with information accompanying the update operation. In OT-based replication, each site receives a sequence of operations to be applied to each site by converting a sequence of operations (patch), and each site applies them in a cooperative manner (collaboration) to achieve replication

In CmRDT, operations are defined independently from the data state, but in OT-based replication, operations are defined on the data state. Unlike CmRDT operations, update operations in OT-based replication are not necessarily commutative, so the result of individual replication may depend on the order in which operations are applied. The OT's compositionality guarantees that replication is possible for three or more sites by applying a combination of OTs that define a two-party replication, and also guarantees the Coordination-free Collaborative Replication.

Asynchronous Messaging for Collaborative Replication
Collaborative replication in distributed sites can be possible by exchange messages of their updated information. A simple way to do this would be like this:

- When sites P and Q perform operations p and q independently and concurrently to get their states $D \odot p$ and $D \odot q$ respectively, computing $(p_q, q_p) = T_D(p, q)$ from p and q brings both states into the confluent state $D \odot p \odot q_p = D \odot q \odot p_q$.
- But this is not practical because P and Q cannot share both p and q at the same time. Either of P or Q, say P first sends p to Q and then Q receives it and computes p_q by $(p_q, _) = T_D(p, q)$ to update the state into $D \odot q \odot p_q$.

– For P to update its state to $D \odot p \odot q_p$, it requires Q to send q just when Q receives p from P. If no other sites exist, P merely waits for Q's reply of q. Otherwise, for P to get Q's reply without the intervention of other's updates, P and Q need to keep step update exchange between them.
– To do this, P blocks communication with other sites except Q until it receives Q's reply.

Such control to synchronize messages to wait messages of the particular site needs to spread blocking commands globally across the distributed sites of the system. It causes a great loss of efficiency and needs complex concurrency control.

On the contrary, our CCR scheme is simple enough to send an update information at any time without any synchronizing control, that is, asynchronous messaging in a way that each site sends updating information at any time and receives information from any site at any time. It relies on the algebraic structure of the confluence property of our Operational Transformation.

Our coordination-free asynchronous replication proceeds as follows:

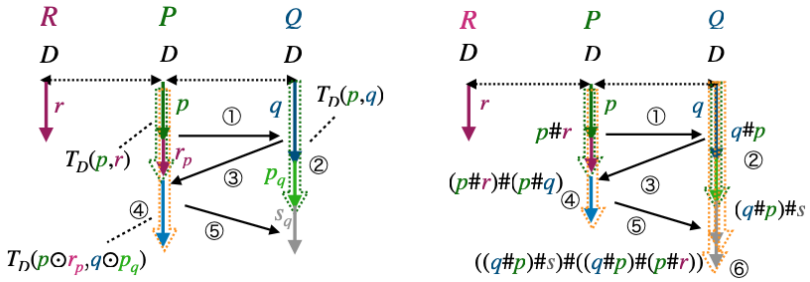

Fig. 5. Asynchronous Messaging in Collaborative Replication

① P sends p to Q and other sites at its convenient time.
② Q computes $(p_q, q_p) = T_D(p, q)$ to get the operation $q \odot p_q$ that makes D into the confluent state $D \odot q \odot p_q$. Note that $q \odot p_q$ is a representation of $q\#_D p = p\#_D q$ which stands for $q \odot p_q \leftrightarrow_D p \odot q_p$ determined by $T_D(p, q)$. Q keeps $q \odot p_q$ for its local representation of $q\#_D p$.
③ Then, Q sends $q\#_D p$ to other sites including P. Note that Q actually broadcasts the representation $q \odot p_q$ of the operation $q\#_D p$.
④ When P receives $q \odot p_q$ sent from Q after its state D has been updated to $D \odot p \odot r_p$ by some r from R, P makes $p \odot r_p$ and $q \odot p_q$ confluent into $(p \odot r_p)\#_D(q \odot p_q)$ by computing $T_D(p \odot r_p, q \odot p_q)$.
⑤ P broadcasts this to other sites. Here, $p \odot r_p$ is a representation of $p\#_D r$ and $q \odot p_q$ is a representation of $q\#_D p = p\#_D q$, and then $(p \odot r_p)\#_D(q \odot p_q)$ is equivalent to $(p\#_D r)\#_D(p\#_D q) = p\#_D(r\#_D q)$.
⑥ In the same manner, when Q has reflected S's update s as $(q\#_D p)\#_D s$ and receives $(p\#_D r)\#_D(p\#_D q)$ and computes a representation of the new operation confluent to $((q\#_D p)\#_D s)\#_D((p\#_D r)\#_D(p\#_D q))$ which is equivalent

to $(q\#_D p)\#_D(s\#_D(p\#_D r))$. This can be read as "$Q$'s operation q is first made confluent with p and then with $s\#_D(p\#_D r)$, which has been made confluent with s and $p\#_D r$, and so on.

We can manipulate the confluence relation according to the algebraic property. Note that the representation of the confluent operation $p\#_D q$ differs site by site depending on the OT application.

Eventual Consistency of Collaborative Replication As we have seen above, CCR assures that every update on the local replica of a site is sent to other sites to produce the new confluent operations in each site. Every time updates occur in the system, they continue the replication process of making a new confluent state. When no updates are in the system, the replicated data in the sites is the same across the system. Thus, the confluent replication leads to the eventual consistency.

But when does the replication process terminate? If R has not operated r in step ④, P makes the confluent operation $p\#_D(q \odot p_q)$ or $p\#_D(q\#_D p)$ which is reduced to $p\#_D q$ of which representaion is $p \odot q_p$. This implies that if S has neither done s in ⑥ then Q computes $T_D(q \odot p_q, p \odot q_p)$ which is $(!, !)$ by the compositional property. Note that in this context, $p \odot q$ stands for the operation sequence $\langle p, q\rangle$. The above observation shows that the replication process should terminate when OT produces the identity operation.

Moreover, this implies that the update propagation over the circular connection safely terminates.

Incremental Messaging in Collaborative Replication Up to now, we have not been concerned about the problem of efficiency about the size of messages. It costs too much if updated operations of each site are always sent from the beginning of replication. We can reduce the message size between connected sites by keeping the indices of the last message of the partner site.

Note that the indices vary partner-wise of the connection. It is an easy way of reducing the cost of messaging.

5 Coordination-free Collaborative Replication in Practice

5.1 CCR Agent

An experimental but being straightly put into practical use Agent for Coordination-free Collaborative Replication is built for confirming the principles and assumptions. It is written in Haskell in several hundred lines of code using standard libraries.

The CCR Agent consists of modules, *Agent*, *AgentMain*, and *Replica*. The module *Agent* does Collaborative Replication for the replica defined in the module *Replica* with coordination-free asynchronous messaging with other Agents. The *AgentMain* module provides a REPL (Read-Eval-Print-Loop) for the user

asks the Agent to make or break WebSocket connections with other Agents and asks to put update operations on the local replica. These modules are independent of replica data type.

The *Replica* module defines the type of the replica data, the operations on the replica, and the transformation. Some examples follow.

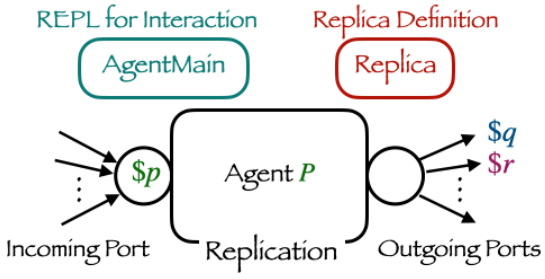

Fig. 6. Configuration of CCR Agent of 3 Modules

5.2 Realtime Collaborative Editing

As mentioned above RTCE pushed me into resolving to extend OT-based data sharing to server-less collaborative replication. An operational transformation satisfying our requirement is shown in Fig.7.

While the algorithm should be verified by formal proofs, it has been tested by more than a thousand combinations of randomly generated operations at three sites. The formal proof is left to our further subject.

5.3 Basic Replica Data

Counter Operations are (Incr $\$i$ n) for increment $D :: Int$ by n and (Decr $\$i$ x) for decrement D by n. The parameter $\$i$ is the ID $(= 0, 1, 2, \cdots)$ of the site in which the operation is issued. See the discussion on the Idempotence of Confluence in Section 3.2. The equality of operations is decided by comparing all the parameters, i.e., $\$i$ and n.

Definitions of the replica and the operations with transformation for the Int Counter are in Fig.8.

The *PN-Counter* CRDT implements the counter replica by collecting local amounts of separated counters for increments and decrements. It is much more elaborate than the CCR Counter.

LWW Register The LWW (Last-Write-Win) Register is a typical CRDT that uses metadata for maintaining the causality of events. The updated data is associated with the timestamp of the event that is compared when concurrent updates occur. The LWW policy chooses the most recent one of these events by

comparing the timestamps. While the timestamp may be the wall clock time, some logical clock is preferred to avoid the clock skew problem. However, the logical clock time is ordered in partial rather than total in general, and some rules of conflict resolution are required.

In the demonstration of LWW_String in CCR, the result strings by concurrent updates are kept as a set of strings, each of which is the candidate of the winner.

Operations are (Write $\$i$ x) for replacing the set of string D by string x. When there are no concurrent updates but one, D becomes a singleton set. Definitions of the replica and operations with transformations for an LWW of strings are in Fig.9.

ESet In Section 1, we have seen the shopping cart example implemented by the set with effectful operations. Operations are (Add $\$i$ x) for adding a element x into the Set replica D if $x \notin D$ and (Rem $\$i$ x) for removing a element x from D if $x \in D$. The internal parameter $\$i$ is the ID $(= 0, 1, 2, \cdots)$ of the site in which the operation is issued. The equality of operations is determined by comparing all the parameters, i.e., $\$i$ and x.

Definitions of the replica and operations with transformations for a Set of strings are shown in Fig.10.

Mergeable Replicated Queue Mergeable Replicated Queue is an example that demonstrates a framework using the relational specification of the replicated data [4].

Operations are (Enq $\$i$ x) for enqueuing an element x into the queue replica D and (Deq $\$i$) for dequeuing an element x from D if it exists. The queue is represented by a pair of lists (deq, enq) for the dequeued elements deq and for the enqueued elements enq. When concurrent enqueue operations occur, both elements are put into enq in the same way as insertion operations at the same location in RTCE.

Definitions of the replica and operations with transformations for a Set of strings are in Fig.11.

Integer register supporting add and mult operators M. Weidner proposes *Semidirect Product* for CRDT [16] that makes concurrent updates by addition Add and multiplication Mult operations confluent. This rule supersedes the ordinal distributive law for retaining the monotonicity of CRDT.

CCR transforms (Add m) and (Mult n) into (Add $m \times n$, Mult n) to make D become the confluent $(D + m) \times n = (D \times n) + (m \times n)$, which illustrates the semidirect product law using operational transformation.

Definitions of the replica and operations with transformations for a Set of strings are in Fig.12.

5.4 Composite Replica Data

The construction of replicated replicas of composite data is a remarkable feature of CCR. It is as concise as the construction of composite data structures using data constructors for tuples and maps.

Algebraic Replicated Data Types proposed in [6] gives an example of a $SocialMedia$ replica for a local-first social media application used by a group of friends in a P2P network to share the quadruples $SocialPost$ of $messages$, $comments$, $likes$, and $dislikes$. The $SocialMedia$ replica is the map of $SocialPost$ records.

While the ARDT approach assumes CRDTs with an intricate composition of the replicated data and operations, CCR deals with this in constructive programming.

Tuple of Replica Data The $SocialPost$ quadruple of $messages$, $comments$, $likes$, and $dislikes$ are in Fig.13 and Fig.14

Map of Replica Data The $SocialMedia$ map of the $SocialPost$ quadruples are in Fig.15 and. Fig.16

6 Remarks

We have discussed *Coordination-free Collaborative Replication* (CCR) with several examples of construction of composite replica types. It may bring us the possibility of replacing the position of CRDTs with CCR, which makes replication easier in most application areas of CRDTs.

We might have described in a top-down style that introduces mathematical laws of the relational operation $\#_D$ for the confluence property and then tries to find concrete representations of updating operations. However, we have followed the process of coming up with ideas for explanation.

As mentioned in the RTCE example of CCR Agent, we have to develop some formal method that gives the proof of TP2-Confluence property for the OT of given updating operations.

Also, the cost model of CCR implementation is required for evaluating the efficiency with benchmarking.

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```
data Op
  = Ins ReplicaID Int String
  | Del ReplicaID Int Int
  | None
  deriving (Eq)

transOp :: String -> Op -> Op -> (Op, Op)
transOp d p q =
  let p'= effectfulOp p d
      q'= effectfulOp q d
  in
    if p'==q' then (None, None)
    else trans' p' q'

trans' p@(Ins i_p k_p t_p) q@(Ins i_q k_q t_q) =
  if p==q then (None,None)
  else
    if k_p==k_q then
      if t_p==t_q then -- (t_p == t_q)&&(k_p /= k_q)
        if i_p>i_q then -- same as t_p>t_q
          (Ins i_p (k_p+length t_q) t_p, Ins i_q k_q t_q)
        else
          (Ins i_p k_p t_p, Ins i_q (k_q+length t_p) t_q)
      else
        if t_p>t_q then
          (Ins i_p (k_p+length t_q) t_p, Ins i_q k_q t_q)
        else
          (Ins i_p k_p t_p, Ins i_q (k_q+length t_p) t_q)
    else
      if (k_p > k_q)
        then
          (Ins i_p (k_p+length t_q) t_p, Ins i_q k_q t_q)
        else
          (Ins i_p k_p t_p, Ins i_q (k_q+length t_p) t_q)
```

```
trans' p@(Del i_p k_p n_p) q@(Del i_q k_q n_q)
  | k_p==k_q && n_p==n_q
  = (None,None)
  | k_q >= k_p+n_p
  = (Del i_p k_p n_p, Del i_q (k_q-n_p) n_q)
  | k_p >= k_q+n_q
  = (Del i_p (k_p-n_q) n_p, Del i_q k_q n_q)
  | k_p >= k_q && k_p+n_p <= k_q+n_q
  = (Del i_p k_q 0, Del i_q k_q (n_q-n_p))
  | k_q >= k_p && k_q+n_q <= k_p+n_p
  = (Del i_p k_p (n_p-n_q), Del i_q k_p 0)
  | k_p >= k_q
  = let d = k_q+n_q-k_p
    in (Del i_p k_q (n_p-d), Del i_q k_q (n_q-d))
  | otherwise
  = let d = k_p+n_p-k_q
    in (Del i_p k_p (n_p-d), Del i_q k_p (n_q-d))

trans' (Ins i_p k_p t_p) (Del i_q k_q n_q)
  | k_p >= k_q && k_p < k_q+n_q
  = (Ins i_p k_q "", Del i_q k_q (n_q+length t_p))
  | k_p < k_q
  = (Ins i_p k_p t_p, Del i_q (k_q+length t_p) n_q)
  | otherwise
  = (Ins i_p (k_p-n_q) t_p, Del i_q k_q n_q)

trans' d@Del{} i@Ins{} =
  let (p',q') = trans' i d
  in (q',p')

trans' None q = (None,q)

trans' p None = (p,None)
```

Fig. 7. Operational Transformation for Realtime Collaborative Editing

ReplicaCOUNTER

```
type ReplicaID = Int

type RevIndex = Int

type ReplicaData = Int

data Replica = Replica
  { replicaData :: ReplicaData
  , replicaPatch :: Patch
  , replicaRev :: RevIndex  -- Revision index
  , replicaDelay :: Int
  , replicaVerbose :: Bool
  }

data Op
  = Incr ReplicaID Int  -- increment by n
  | Decr ReplicaID Int  -- decrement by n
  | None
  deriving (Eq)
```

```
effectfulOp :: Op -> ReplicaData -> Op
effectfulOp op _ = op

applyOp :: Op -> ReplicaData -> (ReplicaData,Op)
applyOp op@(Incr _ n) d = (d+n, op)
applyOp op@(Decr _ n) d = (d-n, op)
applyOp None d = (d, None)


transOp :: ReplicaData -> Op -> Op -> (Op, Op)
transOp d p q =
  if p==q then (None, None) else (p, q)

type Patch = [Op]
```

Fig. 8. Counter Replica

ReplicaLWW_String

```
import qualified Data.Set as Set
...

type ReplicaID = Int

type RevIndex = Int

type ElemType = String

type ReplicaData = Set.Set ElemType

data Replica = Replica
  { replicaData :: ReplicaData
  , replicaPatch :: Patch
  , replicaRev :: RevIndex  -- Revision index
  , replicaDelay :: Int
  , replicaVerbose :: Bool
  }

data Op
  = Write ReplicaID (Set.Set ElemType)
  | None
  deriving (Eq)
```

```
effectfulOp :: Op -> ReplicaData -> Op
effectfulOp op _ = op

applyOp :: Op -> ReplicaData -> (ReplicaData,Op)
applyOp op@(Write _ ss) d = (ss, op)
applyOp None d = (d,None)

transOp :: ReplicaData -> Op -> Op -> (Op, Op)
transOp d p@(Write i_p ss_p) q@(Write i_q ss_q) =
  let p' = effectfulOp p d
      q' = effectfulOp q d
  in
    if (p'==None)||(q'==None) then (p',q')
    else if ss_p==ss_q then (None,None)  -- Set equality
    else let ss'= Set.union ss_p ss_q
         in (Write i_p ss', Write i_q ss')
transOp d None q = (None, effectfulOp q d)
transOp d p None = (effectfulOp p d,None)

type Patch = [Op]
```

Fig. 9. Last-Write-Win Register of Strings

ReplicaESET_String

```
type ReplicaID = Int

type RevIndex = Int

type ElemType = String

type ReplicaData = [ElemType]

data Replica = Replica
  { replicaData :: ReplicaData
  , replicaPatch :: Patch
  , replicaRev :: RevIndex  -- Revision index
  , replicaDelay :: Int
  , replicaVerbose :: Bool
  }

data Op
  = Add ReplicaID ElemType
  | Rem ReplicaID ElemType
  | None
  deriving (Eq)
```

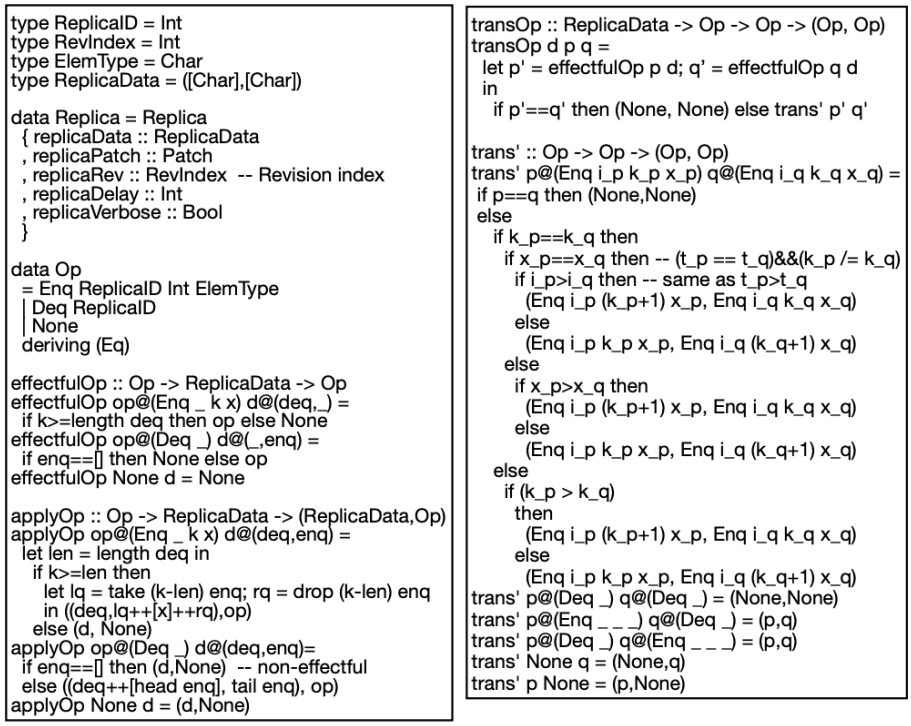
```
effectfulOp :: Op -> ReplicaData -> Op
effectfulOp op@(Add _ x) d =
  if List.elem x d then None else op
effectfulOp op@(Rem _ x) d =
  if List.elem x d then op else None
effectfulOp None d = None

applyOp :: Op -> ReplicaData -> (ReplicaData,Op)
applyOp op@(Add _ x) d =
  if List.elem x d then (d,None) -- not Effectful
  else (d++[x],op)
applyOp op@(Rem _ x) d =
  if List.elem x d then (List.delete x d,op)  --effectful
  else (d,None)
applyOp None d = (d,None)

transOp :: ReplicaData -> Op -> Op -> (Op, Op)
transOp d p q =
  let p' = effectfulOp p d
      q' = effectfulOp q d
  in
    if p'==q' then (None, None) else (p',q')

type Patch = [Op]
```

Fig. 10. Set of Strings with effectful operations



```
type ReplicaID = Int
type RevIndex = Int
type ElemType = Char
type ReplicaData = ([Char],[Char])

data Replica = Replica
  { replicaData :: ReplicaData
  , replicaPatch :: Patch
  , replicaRev :: RevIndex  -- Revision index
  , replicaDelay :: Int
  , replicaVerbose :: Bool
  }

data Op
  = Enq ReplicaID Int ElemType
  | Deq ReplicaID
  | None
  deriving (Eq)

effectfulOp :: Op -> ReplicaData -> Op
effectfulOp op@(Enq _ k x) d@(deq,_) =
  if k>=length deq then op else None
effectfulOp op@(Deq _) d@(_,enq) =
  if enq==[] then None else op
effectfulOp None d = None

applyOp :: Op -> ReplicaData -> (ReplicaData,Op)
applyOp op@(Enq _ k x) d@(deq,enq) =
  let len = length deq in
    if k>=len then
      let lq = take (k-len) enq; rq = drop (k-len) enq
      in ((deq,lq++[x]++rq),op)
    else (d, None)
applyOp op@(Deq _) d@(deq,enq)=
  if enq==[] then (d,None)  -- non-effectful
  else ((deq++[head enq], tail enq), op)
applyOp None d = (d,None)
```

```
transOp :: ReplicaData -> Op -> Op -> (Op, Op)
transOp d p q =
  let p' = effectfulOp p d; q' = effectfulOp q d
  in
    if p'==q' then (None, None) else trans' p' q'

trans' :: Op -> Op -> (Op, Op)
trans' p@(Enq i_p k_p x_p) q@(Enq i_q k_q x_q) =
 if p==q then (None,None)
 else
    if k_p==k_q then
      if x_p==x_q then -- (t_p == t_q)&&(k_p /= k_q)
        if i_p>i_q then -- same as t_p>t_q
          (Enq i_p (k_p+1) x_p, Enq i_q k_q x_q)
        else
          (Enq i_p k_p x_p, Enq i_q (k_q+1) x_q)
      else
        if x_p>x_q then
          (Enq i_p (k_p+1) x_p, Enq i_q k_q x_q)
        else
          (Enq i_p k_p x_p, Enq i_q (k_q+1) x_q)
    else
      if (k_p > k_q)
        then
          (Enq i_p (k_p+1) x_p, Enq i_q k_q x_q)
        else
          (Enq i_p k_p x_p, Enq i_q (k_q+1) x_q)
trans' p@(Deq _) q@(Deq _) = (None,None)
trans' p@(Enq _ _ _) q@(Deq _) = (p,q)
trans' p@(Deq _) q@(Enq _ _ _) = (p,q)
trans' None q = (None,q)
trans' p None = (p,None)
```

Fig. 11. Mergeable Replicated Queue

```
type ReplicaID = Int

type RevIndex = Int

-- type ReplicaID = Int  -- Replica ID is 'myPort'%100

type ReplicaData = Int

data Replica = Replica
  { replicaData :: ReplicaData
  , replicaPatch :: Patch
  , replicaRev :: RevIndex  -- Revision index
  , replicaDelay :: Int
  , replicaVerbose :: Bool
  }

data Op
  = Add ReplicaID Int  -- add n
  | Mult ReplicaID Int  -- multiply n
  | None
  deriving (Eq)
```

```
effectfulOp :: Op -> ReplicaData -> Op
effectfulOp op _ = op

applyOp :: Op -> ReplicaData -> (ReplicaData,Op)
applyOp op@(Add _ n) d = (d+n, op)
applyOp op@(Mult _ n) d = (d*n, op)
applyOp None d = (d, None)

transOp :: ReplicaData -> Op -> Op -> (Op, Op)
transOp d p q =
  if p==q then (None,None) else transOp' p q

transOp' None q = (None,q)
transOp' p None = (p,None)
transOp' p@(Add i_p n_p) q@(Add i_q n_q) =
    (p,q)
transOp' p@(Mult i_p n_p) q@(Mult i_q n_q) =
    (p,q)
transOp' (Add i_p n_p) (Mult i_q n_q) =
    (Add i_p (n_p*n_q),Mult i_q n_q)
transOp' (Mult i_p n_p) (Add i_q n_q) =
    (Mult i_p n_p,Add i_q (n_q*n_p))
```

Fig. 12. Add-Mult Replication

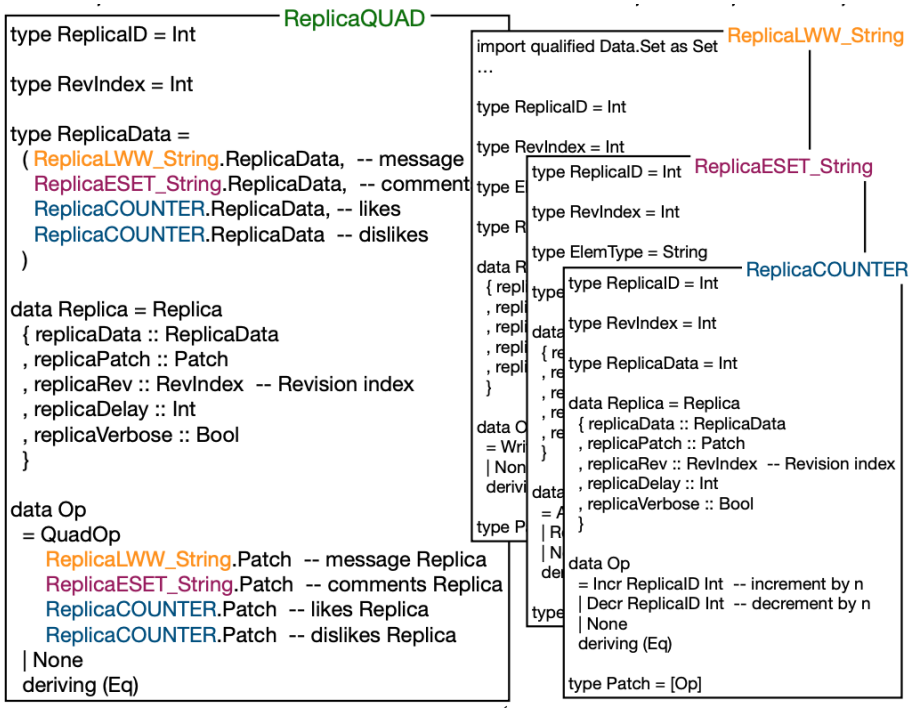


Fig. 13. SocialPost Quadruples - Composed Replica Data

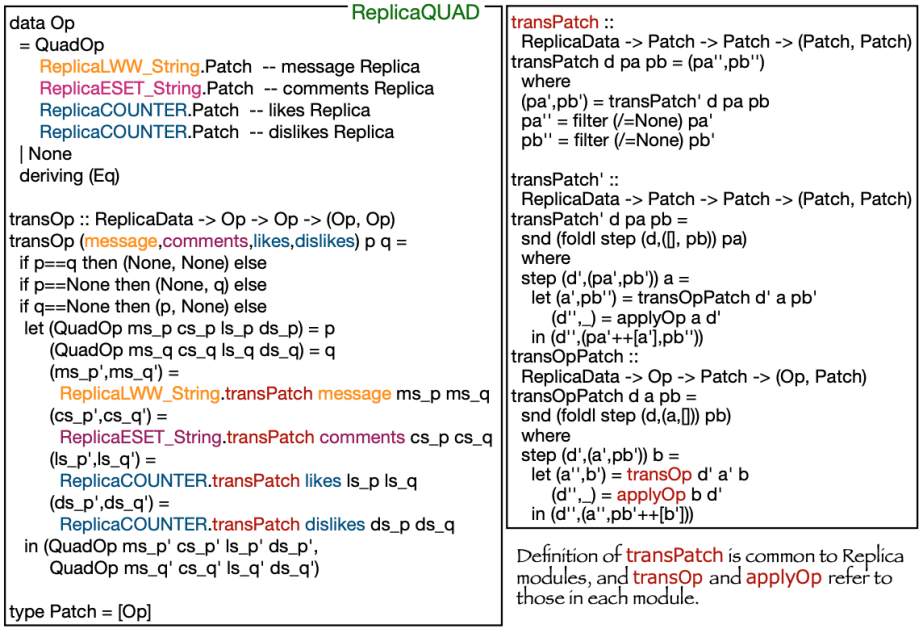


Fig. 14. SocialPost Quadruples - Operations and Transformation

ReplicaMap_QUAD

```
import qualified Data.IntMap as IntMap
...

type ReplicaID = Int

type RevIndex = Int

type ValueType = ReplicaQUAD.ReplicaData

type KeyType = Int

type ReplicaData = IntMap.IntMap ValueType

data Replica = Replica
  { replicaData :: ReplicaData
  , replicaPatch :: Patch
  , replicaRev :: RevIndex  -- Revision index
  , replicaDelay :: Int
  , replicaVerbose :: Bool
  }

data Op
  = Upd ReplicaID KeyType ReplicaQUAD.Op
  | None
  deriving (Eq)
```

ReplicaQUAD

```
type ReplicaID = Int

type RevIndex = Int

type ReplicaData =
  ( ReplicaLWW_String.ReplicaData,  -- message
    ReplicaESET_String.ReplicaData,  -- comments
    ReplicaCOUNTER.ReplicaData, -- likes
    ReplicaCOUNTER.ReplicaData  -- dislikes
  )

data Replica = Replica
  { replicaData :: ReplicaData
  , replicaPatch :: Patch
  , replicaRev :: RevIndex  -- Revision index
  , replicaDelay :: Int
  , replicaVerbose :: Bool
  }

data Op
  = QuadOp
     ReplicaLWW_String.Patch  -- message Replica
     ReplicaESET_String.Patch  -- comments Replica
     ReplicaCOUNTER.Patch  -- likes Replica
     ReplicaCOUNTER.Patch  -- dislikes Replica
  | None
  deriving (Eq)
```

Fig. 15. SocialMedia Map of SocialPost Quadruples - Composed Replica Data

ReplicaMap_QUAD

```
data Op
  = Upd ReplicaID KeyType ReplicaQUAD.Op
  | None
  deriving (Eq)

applyOp :: Op -> ReplicaData -> (ReplicaData,Op)
applyOp (Upd i k quadOp) d =
  (IntMap.insert k value' d, Upd i k quadOp')
  where
    (value',quadOp') =
      ReplicaQUAD.applyOp quadOp value
    value =
      case IntMap.lookup k d of
        Nothing ->
          ReplicaQUAD.replicaData
              ReplicaQUAD.initReplica
        Just v -> v
applyOp None d = (d,None)
```

```
transOp :: ReplicaData -> Op -> Op -> (Op, Op)
transOp _ None None = (None,None)
transOp _ None q = (None,q)
transOp _ p None = (p,None)
transOp d
  p@(Upd i_p k_p quadOp_p) q@(Upd i_q k_q quadOp_q) =
  if p==q then (None, None) else
    if k_p/=k_q then (p,q) -- p and q are independent
    else -- p and q update the same value
      let d'=
            case IntMap.lookup k_p d of
              Nothing ->
                ReplicaQUAD.replicaData
                    ReplicaQUAD.initReplica
              Just v -> v
          (quadOp_p',quadOp_q')=
            ReplicaQUAD.transOp d' quadOp_p quadOp_q
      in (Upd i_p k_p quadOp_p',Upd i_q k_q quadOp_q')

type Patch = [Op]
```

Fig. 16. SocialMedia Map of SocialPost Quadruples - Operations and Transformation