

Activity: Moving Average Processes

The i.i.d. sequence $\{Z_t\}$ has mean zero and variance σ^2 . Suppose we define the stochastic process $\{X_t\}$ by

$$X_t = Z_t + 0.3Z_{t-1} + 0.2Z_{t-2} + 0.1Z_{t-3}.$$

1. The model X_t is an $\text{MA}(q)$ process for which value of q ?
2. Find $E(X_t)$.
3. Find $\text{Var}(X_t)$.
4. Find $\text{Cov}(X_t, X_{t+1})$.
5. Find $\text{Cov}(X_t, X_{t+2})$.
6. Find $\text{Cov}(X_t, X_{t+3})$.
7. What is $\text{Cov}(X_t, X_{t+k})$ when $k > 3$?
8. Is X_t stationary?
9. What is $\text{acf } \rho(k)$ for X_t ?
10. Suppose $E(Z_t) = 3$ in the above. What, if anything, would change?