Activity: MA Representations

This activity should help you appreciate how to obtain the MA representation of an ARMA model, and also why that representation can be useful in forecasting. We will consider the model

$$X_t = 0.7X_{t-1} + Z_t - 0.3Z_{t-1},$$

where $\{Z_t\}$ is white noise with variance 0.4.

1. Express X_t as a pure MA process. That is, write X_t as

   $$\left(1 + \psi_1 B + \psi_2 B^2 + \cdots\right) Z_t$$

   for some constants $\psi_1, \psi_2 \ldots$.

2. Write down a general formula for ψ_j in this case.

3. Suppose we are interested in forecasting the value X_{N+3} based on values up to time N. Write down an expression for the *forecast error*,

   $$e_N(3) := X_{N+3} - \hat{X}_N(3).$$

4. Find $E\left(e_N(3)\right)$ and $\mathrm{Var}(e_N(3))$.

5. Assume now that $Z_t \sim N\left(0, 0.4\right)$ for all t. Write down a 95% confidence interval for your forecast of X_{N+3}. (More properly this should be referred to as a *prediction interval.)*

6. As the lead time increases, what happens to the prediction interval for $\hat{x}_N\left(l\right)$?