Data Mining:

Concepts and Techniques

(3rd ed.)

— Chapter 6 —

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Another Big Name

named **Royal** Society of **Canada** Fellow in 2019

Misc.

- Eager to start the course project?
  - Idea
  - Data & other resources
  - References!
  - Plan for a team of *at most* two (names attached to tasks)
- Wait …
  - there is a small project #1 for everyone on frequent itemset mining
  - Midterm: review class 10/24(?), exam 10/29, solution November
  - **Guest lectures (10/22 and 10/24, away in conference)**

Chapter 6: Mining Frequent Patterns, Association and Correlations: Basic Concepts and Methods

- Basic Concepts
- Frequent Itemset Mining Methods
- Which Patterns Are Interesting?—Pattern Evaluation Methods
- Summary

What Is Frequent Pattern Analysis?

- Frequent pattern: a pattern (a set of items, subsequences, substructures, etc.) that occurs frequently in a data set
- First proposed by Agrawal, Imielinski, and Swami [AIS93] in the context of frequent itemsets and association rule mining
- Motivation: Finding inherent *regularities* in data
  - What products were often purchased *together*?— Beer and diapers?!
  - What are the *subsequent* purchases after buying a PC?
  - What kinds of DNA are sensitive to this new drug?
  - Can we automatically classify web documents?

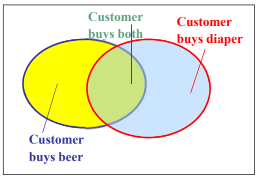
- Applications
  - Basket data analysis, cross-marketing, catalog design, sale campaign analysis, Web log (click stream) analysis, and DNA sequence analysis.

Why Is Freq. Pattern Mining Important?

- Freq. pattern: An intrinsic and important property of datasets
- **Foundation** for many essential data mining tasks
  - Association, correlation (#, linear), and causality analysis
  - Sequential, structural (e.g., sub-graph) patterns (why?)
  - Pattern analysis in spatiotemporal, geospatial, *multimedia*, time-series, and stream data
  - Classification: discriminative, frequent pattern analysis
  - Cluster analysis: frequent pattern-based clustering
  - Data warehousing: *iceberg cube* and *cube-gradient*
  - Semantic data compression: *fascicles*
  - Broad applications

Basic Concepts: Frequent Patterns

Tid	Items bought
10	Beer, Nuts, Diaper
20	Beer, Coffee, Diaper
30	Beer, Diaper, Eggs
40	Nuts, Eggs, Milk
50	Nuts, Coffee, Diaper, Eggs, Milk



- itemset: A set of one or more items
- k-itemset X = $\{x_1, \ldots, x_k\}$
- *(absolute) support*, or, *support count* of X: Frequency or occurrence of an itemset X
- *(relative) support*, *s*, is the fraction of transactions that contains X (i.e., the probability that a transaction contains X)
- An itemset X is *frequent* if X's support is no less than a *minsup* threshold

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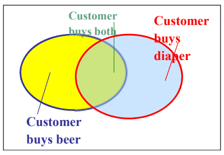
Basic Concepts: Association Rules

Tid	Items bought
10	Beer, Nuts, Diaper
20	Beer, Coffee, Diaper
30	Beer, Diaper, Eggs
40	Nuts, Eggs, Milk
50	Nuts, Coffee, Diaper, Eggs, Milk



- Find all the rules $X \rightarrow Y$ with minimum support and confidence
  - support, *s*, probability that a transaction contains $X \cup Y$
  - confidence, *c*, conditional probability that a transaction having X also contains *Y*

Let minsup = 50%, minconf = 50%

Freq. Pat.: Beer:3, Nuts:3, Diaper:4, Eggs:3, {Beer, Diaper}:3

- Association rules: (many more!)
  - *Beer* → *Diaper* (**60%,** 100%)
  - *Diaper* → *Beer* (**60%,** 75%)

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Closed Patterns and Max-Patterns

- A **long** pattern contains a combinatorial number of sub-patterns, e.g., $\{a_1, \ldots, a_{100}\}$ contains $\binom{100}{1} + \binom{100}{2} + \ldots + \binom{100}{100} = 2^{100} - 1$ ~= $1.27*10^{30}$ sub-patterns!
- Solution: *Mine closed patterns and max-patterns instead*
- An itemset X is closed if X is *frequent* and there exists *no* ***super-pattern*** $Y \supset X$, *with the* ***same*** *support* as X (proposed by Pasquier, et al. @ ICDT'99)
- An itemset X is a max-pattern if X is frequent and there exists no **frequent super-pattern** $Y \supset X$ (proposed by Bayardo @ SIGMOD'98)
- Closed pattern is a **lossless** compression of freq. patterns
  - Reducing the # of patterns and rules

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Closed Patterns and Max-Patterns

- Exercise. DB = $\{<a_1, \ldots, a_{100}>, < a_1, \ldots, a_{50}>\}$
  - Min_sup = 1.
- What is the set of closed itemset?
  - $<a_1, \ldots, a_{100}>$: **1** (the support is 1)
  - $< a_1, \ldots, a_{50}>$: **2** (the support is 2 – more support)
- What is the set of max-pattern?
  - $<a_1, \ldots, a_{100}>$: 1 (the support is 1)
- What is the set of all patterns?
  - Huge!!

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Computational Complexity of Frequent Itemset Mining

- How many itemsets can potentially be generated in the worst case?
  - The number of frequent itemsets to be generated is **sensitive** to the minsup threshold
  - When minsup is low, there exist potentially an exponential number of frequent itemsets
  - The **worst case**: M^N where M: # distinct items, and N: max length of transactions
- The worst case probability vs. the ***expected*** probability
  - Ex. Suppose Walmart has 10^4 kinds of products
    - The chance to pick up one product 10^{-4}
    - The chance to pick up a particular set of 10 products: $\sim 10^{-40}$
    - What is the chance this particular set of 10 products to be frequent - 10^3 times in 10^9 transactions?

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Chapter 5: Mining Frequent Patterns, Association and Correlations: Basic Concepts and Methods

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Scalable Frequent Itemset Mining Methods

- Apriori: A Candidate Generation-and-Test Approach
- Improving the Efficiency of Apriori
- FPGrowth: A Frequent Pattern-Growth Approach
- ECLAT: Frequent Pattern Mining with Vertical Data Format

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The Downward Closure Property and Scalable Mining Methods

- The downward closure property of frequent patterns
  - Any subset of a frequent itemset must be frequent
  - If **{beer, diaper, nuts}** is frequent, so is **{beer, diaper}**
  - i.e., every transaction having {beer, diaper, nuts} also contains {beer, diaper}
- Scalable mining methods: Three major approaches
  - Apriori (Agrawal & Srikant@VLDB'94)
  - Freq. pattern growth (**FPgrowth—Han, Pei & Yin** @SIGMOD'00)
  - Vertical data format approach (Charm—Zaki & Hsiao @SDM'02)

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Apriori: A Candidate Generation & Test Approach

- Apriori pruning principle: If there is any itemset which is infrequent, its superset should not be generated/tested! (Agrawal & Srikant @VLDB'94, Mannila, et al. @ KDD' 94)
- Method:
  - Initially, scan DB once to get frequent 1-itemset
  - Generate length (k+1) candidate itemsets from length k frequent itemsets
  - Test the candidates against DB
  - Terminate when no frequent or candidate set can be generated

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The Apriori Algorithm—An Example

Database TDB

Tid	Items
10	A, C, D
20	B, C, E
30	A, B, C, E
40	B, E

Sup_{min} = 2

1st scan → C_1

Itemset	sup
{A}	2
{B}	3
{C}	3
{D}	1
{E}	3

→ L_1

Itemset	sup
{A}	2
{B}	3
{C}	3
{E}	3

C_2

Itemset
{A, B}
{A, C}
{A, E}
{B, C}
{B, E}
{C, E}

2nd scan → C_2

Itemset	sup
{A, B}	1
{A, C}	2
{A, E}	1
{B, C}	2
{B, E}	3
{C, E}	2

→ L_2

Itemset	sup
{A, C}	2
{B, C}	2
{B, E}	3
{C, E}	2

C_3

Itemset
{B, C, E}

3rd scan → L_3

Itemset	sup
{B, C, E}	2

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The Apriori Algorithm (Pseudo-Code)

C_k: Candidate itemset of size k
L_k : frequent itemset of size k

L_1 = {frequent items};
for (k = 1; L_k !=∅; k++) **do begin**
  C_{k+1} = **candidates** generated from L_k;
  for each transaction t in database do
    increment the count of all candidates in C_{k+1} that are contained in t
  L_{k+1} = candidates in C_{k+1} with min_support
  end
return $\cup_k L_k$;

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Implementation of Apriori

- How to generate candidates?
  - Step 1: self-joining L_k
  - Step 2: pruning (redundant & infrequent itemsets)
- Example of Candidate-generation
  - L_3={*abc, abd, acd, ace, bcd*}
  - Self-joining: L_3*L_3
    - *abcd* from *abc* and *abd*
    - *acde* from *acd* and *ace*
  - Pruning:
    - *acde* is removed because *ade* is not in L_3
  - C_4 = {*abcd*}

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How to Count Supports of Candidates?

- Why is counting supports of candidates a problem?
  - The total number of candidates can be very huge
  - One transaction may contain many candidates
- Method:
  - Candidate itemsets are stored in a *hash-tree*
  - *Leaf* node of hash-tree contains a list of itemsets **and counts**
  - *Interior* node contains a hash table
  - *Subset function*: finds all the candidates contained in a transaction

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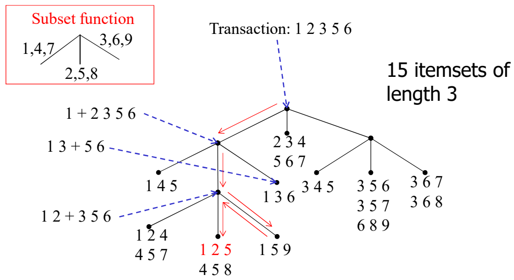
Counting Supports of Candidates Using Hash Tree



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Candidate Generation: An SQL Implementation

- **SQL** Implementation of candidate generation
  - Suppose the items in L_{k-1} **are listed in an <u>order</u>**
  - Step 1: self-joining L_{k-1}

    insert into C_k

    select $p.item_1, p.item_2, \ldots, p.item_{k-1}, q.item_{k-1}$

    from $L_{k-1}\ p, L_{k-1}\ q$

    where $p.item_1=q.item_1, \ldots, p.item_{k-2}=q.item_{k-2}, p.item_{k-1} < q.item_{k-1}$
  - Step 2: pruning

    forall ***itemsets c in*** C_k do

    forall ***(k-1)-subsets s of c*** do

    if *(s is not in* L_{k-1}*)* **then delete** *c* **from** C_k
- Use object-relational extensions like UDFs, BLOBs, and Table functions for efficient implementation [See: S. Sarawagi, S. Thomas, and R. Agrawal. Integrating association rule mining with relational database systems: Alternatives and implications. SIGMOD'98]

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Further Improvement of the Apriori Method

- Major computational challenges
  - Multiple scans of transaction database
  - Huge number of candidates
  - Tedious workload of support counting for candidates
- Improving Apriori: general ideas
  - **Reduce passes** of transaction database scans
  - **Shrink number** of candidates
  - **Facilitate support counting** of candidates

Project #1

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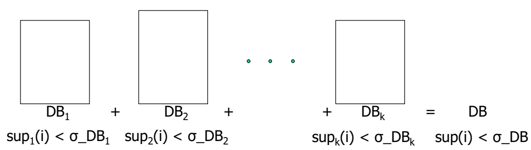
Partition: Scan Database Only Twice

- Any itemset that is potentially frequent in DB must be frequent in at least one of the partitions of DB
  - Scan 1: partition database and find **local** frequent patterns
  - Scan 2: <u>consolidate</u> **global** frequent patterns
- A. Savasere, E. Omiecinski and S. Navathe, *VLDB'95*



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DHP: Reduce the Number of Candidates

- A *k*-itemset whose corresponding hashing bucket count is below the threshold cannot be frequent
  - Candidates: a, b, c, d, e
  - **Hash entries**
    - {ab, ad, ae}
    - {bd, be, de}
    - …
  - Frequent 1-itemset: a, b, d, e
  - *ab* is not a candidate 2-itemset if the **sum** of count of {*ab*, ad, ae} is below support threshold
- J. Park, M. Chen, and P. Yu. An effective hash-based algorithm for mining association rules. *SIGMOD'95* [DHP: direct hashing & pruning]

count	itemsets
35	{ab, ad, ae}
88	{bd, be, de}
. . .	. . .
102	{yz, qs, wt}

Hash Table

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Sampling for Frequent Patterns

- Select a *sample* of original database, mine frequent patterns within *sample* using Apriori (with a lower thresh.)
- Scan *the whole database* once to verify frequent itemsets found in *sample*, only *borders* of closure of the found frequent patterns are checked
  - Example: check *abcd* instead of *ab, ac, …, etc.*
- Scan database again to find missed frequent patterns, stop when further scans cannot be afforded
- H. Toivonen. Sampling large databases for association rules. In *VLDB'96*

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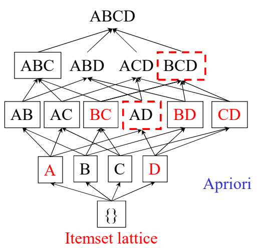
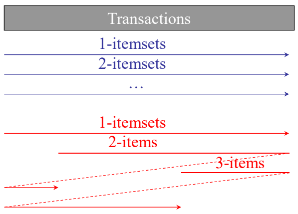
DIC: Reduce Number of Scans



- Once both A and D are determined frequent, the counting of AD begins
- Once **all** length-2 subsets of BCD are determined frequent, the counting of BCD begins (see different triggering points in time)



S. Brin, R. Motwani, J. Ullman, and S. Tsur. **Dynamic itemset counting** and implication rules for market basket data. *SIGMOD'97*

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Pattern-Growth Approach: Mining Frequent Patterns <u>Without</u> Candidate Generation

- Bottlenecks of the Apriori approach
  - **Breadth-first** (i.e., level-wise) search
  - Candidate generation and test
    - Often generates a huge number of candidates
- The FPGrowth Approach (J. Han, J. Pei, and Y. Yin, SIGMOD' 00)
  - **Depth-first** search
  - <u>Avoid explicit candidate generation</u>
- Major philosophy: **Grow long patterns from short ones using local frequent items only**
  - "abc" is a frequent pattern
  - Get all transactions having "abc", i.e., **project** DB on abc: DB|abc
  - "d" is a local frequent item in DB|abc → <u>abc</u>d is a frequent pattern

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Construct FP-tree from a Transaction Database

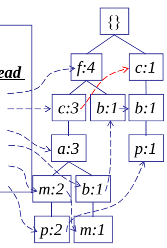
TID	*Items bought*	*(ordered) frequent items*
100	{*f, a, c, d, g, i, m, p*}	{*f, c, a, m, p*}
200	{*a, b, c, f, l, m, o*}	{*f, c, a, b, m*}
300	{*b, f, h, j, o, w*}	{*f, b*}
400	{*b, c, k, s, p*}	{*c, b, p*}
500	{*a, f, c, e, l, p, m, n*}	{*f, c, a, m, p*}

min_support = 3

1. Scan DB once, find frequent 1-itemset (single item pattern)
2. **Sort** frequent items in frequency **descending** order, f-list
3. Scan DB again, construct FP-tree

Header Table

Item	*frequency*	*head*
f	4	
c	4	
a	3	
b	3	
m	3	
p	3	



F-list = f-c-a-b-m-p

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Partition Patterns and Databases

- Frequent patterns can be partitioned into **subsets** according to f-list
  - F-list = f-c-a-b-m-p
  - Patterns containing p
  - Patterns having m but no p
  - …
  - Patterns having c but no a nor b, m, p
  - Pattern f
- **Completeness** and **non-redundancy (compactness)**

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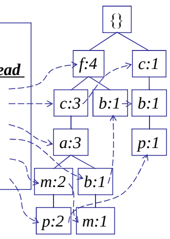
Find Patterns Having P From P-conditional Database

- Starting at the frequent item header table in the FP-tree
- Traverse the FP-tree by following the link of each frequent item *x*
- Accumulate all of *transformed prefix paths* of item *x* to form *x*'s conditional pattern base

Header Table

Item	frequency	head
f	4	
c	4	
a	3	
b	3	
m	3	
p	3	



***Conditional* pattern bases**

item	cond. pattern base
c	f:3
a	fc:3
b	fca:1, f:1, c:1
m	fca:2, fcab:1
p	fcam:2, cb:1

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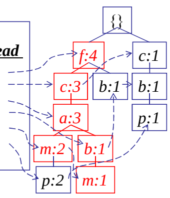
From Conditional Pattern-bases to Conditional FP-trees

- For each pattern-base
  - Accumulate the count for each item in the base
  - Construct the FP-tree for the frequent items of the pattern base

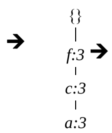
Min-support = 3

Header Table

Item	frequency	head
f	4	
c	4	
a	3	
b	3	
m	3	
p	3	



***m-conditional* pattern base:**
fca:2, fcab:1



All frequent patterns relate to *m*
m,
fm, cm, am,
fcm, fam, cam,
fcam

***m-conditional* FP-tree (from red part)**

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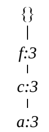
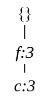
Recursion: Mining Each Conditional FP-tree



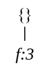
m-conditional FP-tree

Cond. pattern base of "am": (fc:3)



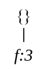
am-conditional FP-tree

Cond. pattern base of "cm": (f:3)



cm-conditional FP-tree

Cond. pattern base of "cam": (f:3)



cam-conditional FP-tree

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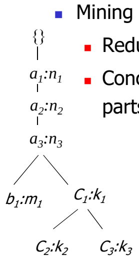
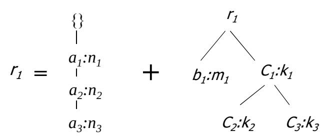
A Special Case: Single Prefix Path in FP-tree

- Suppose a (conditional) FP-tree T has a shared single prefix-path P
- Mining can be decomposed into two parts
  - Reduction of the single prefix path into one node
  - Concatenation of the mining results of the two parts





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Benefits of the FP-tree Structure

- Completeness
  - Preserve complete information for frequent pattern mining
  - Never break a long pattern of any transaction
- Compactness
  - Reduce irrelevant info—infrequent items are *gone*
  - Items in frequency *descending* order: the more frequently occurring, the more likely to be shared
  - Never be larger than the original database (not counting node-links and the *count* field)

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The Frequent Pattern Growth Mining Method

- Idea: Frequent pattern growth
  - *Recursively* grow frequent patterns by pattern and database partition
- Method
  - For each frequent item, construct its conditional pattern-base, and then its conditional FP-tree
  - *Repeat* the process on each newly created conditional FP-tree
  - **Until** the resulting FP-tree is *empty*, or it contains only *one path* — single path will generate all the combinations of its sub-paths, each of which is a frequent pattern

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Scaling FP-growth by Database Projection

- What if FP-tree cannot fit in memory?
  - DB **projection**
- First partition a database into a set of projected DBs (see next slide)
- Then construct and mine FP-tree for each projected DB
- Parallel projection vs. partition projection techniques
  - Parallel projection
    - Project the DB in parallel for *each* frequent item
    - Parallel projection is space costly
    - *All* the partitions can be processed in parallel
  - Partition projection
    - Partition the DB based on the **ordered** frequent items
    - Passing the *unprocessed parts* to the subsequent partitions

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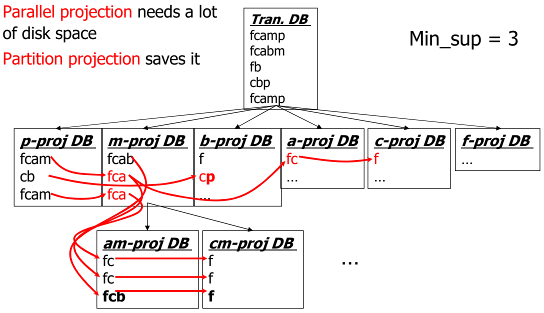
Partition-Based Projection

- Parallel projection needs a lot of disk space
- Partition projection saves it



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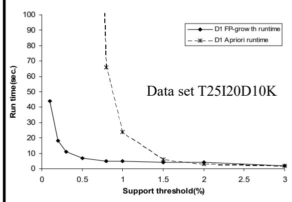
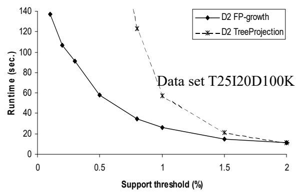
Performance of FPGrowth in Large Datasets



FP-Growth vs. Apriori



FP-Growth vs. Tree-Projection

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Advantages of the Pattern Growth Approach

- Divide-and-conquer:
  - Decompose both the mining task and DB according to the frequent patterns obtained so far
  - Lead to focused search of smaller databases (cond. patt. base)
- Other factors
  - No candidate generation, no candidate test
  - Compressed database: FP-tree structure
  - No repeated scan of entire database
  - Basic ops: counting *local* freq items and building sub FP-tree, no pattern search and matching
- A good open-source implementation and *refinement* of FPGrowth
  - FPGrowth+ (Grahne and J. Zhu, FIMI'03)

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Further Improvements of Mining Methods

- AFOPT (Liu, et al. @ KDD'03)
  - A "push-right" method for mining condensed frequent pattern (CFP) tree
- Carpenter (Pan, et al. @ KDD'03)
  - Mine data sets with small rows but numerous columns (***large p*** *small n problem*)
  - Construct a row-enumeration tree for efficient mining
- FPgrowth+ (Grahne and Zhu, FIMI'03)
  - Efficiently Using **Prefix-Trees** in Mining Frequent Itemsets, Proc. ICDM'03 Int. **Workshop** on Frequent Itemset Mining Implementations (FIMI'03), Melbourne, FL, Nov. 2003
- TD-Close (Liu, et al, SDM'06)

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Extension of Pattern Growth Mining Methodology

- Mining closed frequent itemsets and max-patterns
  - CLOSET (DMKD'00), FPclose, and FPMax (Grahne & Zhu, Fimi'03)
- Mining **sequential patterns**
  - **PrefixSpan** (ICDE'01), CloSpan (SDM'03), BIDE (ICDE'04)
- Mining **graph patterns**
  - gSpan (ICDM'02), CloseGraph (KDD'03)
- **Constraint-based** mining of frequent patterns
  - Convertible constraints (ICDE'01), gPrune (PAKDD'03)
- Computing iceberg data cubes with complex measures
  - H-tree, H-cubing, and Star-cubing (SIGMOD'01, VLDB'03)
- **Pattern-growth-based Clustering**
  - MaPle (Pei, et al., ICDM'03)
- **Pattern-Growth-Based Classification**
  - Mining frequent and *discriminative* patterns (Cheng, et al, ICDE'07)

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Scalable Frequent Itemset Mining Methods

- Apriori: A Candidate Generation-and-Test Approach
- Improving the Efficiency of Apriori
- FPGrowth: A Frequent Pattern-Growth Approach
- ECLAT: Frequent Pattern Mining with Vertical Data Format
- Mining Close Frequent Patterns and Maxpatterns

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ECLAT: Mining by Exploring Vertical Data Format

- Vertical format: t(AB) = $\{T_{11}, T_{25}, \ldots\}$, e.g. **Table** 6.3
  - tid-list: list of trans.-ids containing an itemset (*index lookup table*)
- Deriving frequent patterns based on vertical intersections
  - t(X) = t(Y): X and Y always happen together
  - t(X) $\subset$ t(Y): transactions having X always have Y
- Using diffset to accelerate mining
  - Only keep track of differences of tids
  - t(X) = $\{T_1, T_2, T_3\}$, t(XY) = $\{T_1, T_3\}$
  - Diffset (XY, X) = $\{T_2\}$
- Eclat (Zaki et al. @KDD'97)
- Mining Closed patterns using vertical format: **CHARM** (**Zaki** & Hsiao@SDM'02)

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Scalable Frequent Itemset Mining Methods

- Apriori: A Candidate Generation-and-Test Approach
- Improving the Efficiency of Apriori
- FPGrowth: A Frequent Pattern-Growth Approach
- ECLAT: Frequent Pattern Mining with Vertical Data Format
- Mining Close Frequent Patterns and Maxpatterns

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Mining Frequent Closed Patterns: CLOSET

- Flist: list of all frequent items in support **ascending** order
  - Flist: d-a-f-e-c
- Divide search space
  - Patterns having d
  - Patterns having d but no a, etc.
- Find *frequent closed pattern* recursively
  - Every transaction having d also has *cfa* → *cfad* is a frequent closed pattern (superset)
- J. Pei, J. Han & R. Mao. "CLOSET: An Efficient Algorithm for Mining Frequent Closed Itemsets", DMKD'00.

Min_sup=2

TID	Items
10	a, c, d, e, f
20	a, b, e
30	c, e, f
40	a, c, d, f
50	c, e, f

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CLOSET+: Mining Closed Itemsets by Pattern-Growth

- Itemset merging: if Y appears in every occurrence of X, then Y is merged with X
- Sub-itemset pruning: if Y ⊃ X, and sup(X) = sup(Y), X and all of X's descendants in the set enumeration tree can be pruned
- Hybrid tree projection
  - Bottom-up physical tree-projection
  - Top-down pseudo tree-projection
- Item skipping: if a *local* frequent item has the same support in several header tables at different levels, one can prune it from the header table at *higher* levels
- Efficient subset checking

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MaxMiner: Mining Max-Patterns

Tid	Items
10	A, B, C, D, E
20	B, C, D, E,
30	A, C, D, F

- 1[st] scan: find frequent items
  - A, B, C, D, E
- 2[nd] scan: find support for
  - AB, AC, AD, AE, ABCDE
  - BC, BD, BE, BCDE
  - CD, CE, CDE, DE

Potential max-patterns

- Since BCDE is a max-pattern, no need to check BCD, BDE, CDE in later scans
- R. Bayardo. Efficiently mining long patterns from databases. *SIGMOD'98*

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CHARM: Mining by Exploring Vertical Data Format

- Vertical format: t(AB) = $\{T_{11}, T_{25}, \ldots\}$
  - tid-list: list of trans.-ids containing an itemset
- Deriving closed patterns based on vertical intersections
  - t(X) = t(Y): X and Y always happen together
  - t(X) $\subset$ t(Y): transaction having X always has Y
- Using diffset to accelerate mining
  - Only keep track of differences of tids
  - t(X) = $\{T_1, T_2, T_3\}$, t(XY) = $\{T_1, T_3\}$
  - Diffset (XY, X) = $\{T_2\}$
- Eclat/MaxEclat (Zaki et al. @KDD'97), VIPER(P. Shenoy et al.@SIGMOD'00), CHARM (Zaki & Hsiao@SDM'02)

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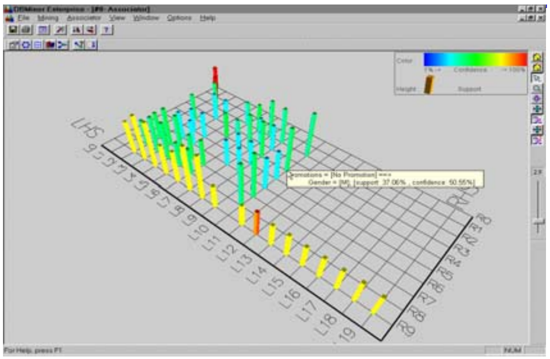
Visualization of Association Rules: Plane Graph



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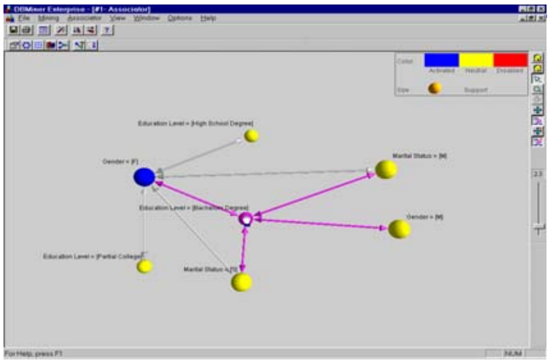
Visualization of Association Rules: Rule Graph



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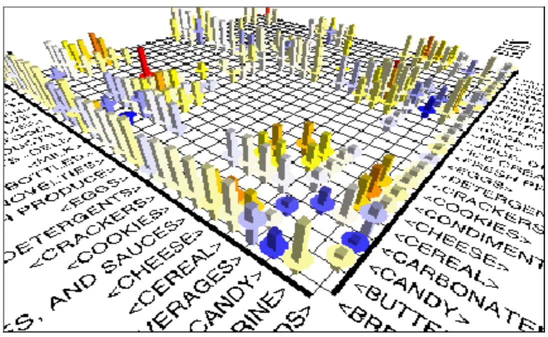
Visualization of Association Rules (SGI/MineSet 3.0)



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Chapter 6: Mining Frequent Patterns, Association and Correlations: Basic Concepts and Methods

- Basic Concepts
- Frequent Itemset Mining Methods
- Which Patterns Are Interesting?—Pattern Evaluation Methods
- Summary

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Interestingness Measure: Correlations (Lift)

- *play basketball* ⇒ *eat cereal* [40%, 66.7%] is misleading
  - The overall % of students eating cereal is **already** 75% > 66.7%.
- *play basketball* ⇒ *not eat cereal* [20%, 33.3%] is more accurate (interesting?), *although with lower support and confidence*
- Measure of dependent/correlated events: lift

$$lift = \frac{P(A \cup B)}{P(A)P(B)}$$

	Basketball	Not basketball	Sum (row)
Cereal	2000	1750	3750
Not cereal	1000	250	1250
Sum(col.)	3000	2000	5000

$$lift(B,C) = \frac{2000/5000}{3000/5000 * 3750/5000} = 0.89$$

$$lift(B,\neg C) = \frac{1000/5000}{3000/5000 * 1250/5000} = 1.33 > 0.89$$

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Are *lift* and χ^2 Good Measures of Correlation?

- "*Buy walnuts* ⇒ *buy milk* [1%, 80%]" is misleading if 85% of customers buy milk
- Support and confidence are not good to indicate correlations
- Over 20 interestingness measures have been proposed (see Tan, Kumar, Sritastava @KDD'02)
- Which are good ones?

symbol	measure	range	formula
ϕ	ϕ-coefficient	-1 ... 1	[illegible]
Q	Yule's Q	-1 ... 1	[illegible]
Y	Yule's Y	-1 ... 1	[illegible]
k	Cohen's	-1 ... 1	[illegible]
PS	Piatetsky-Shapiro's	-0.25 ... 0.25	$P(A,B) - P(A)P(B)$
F	Certainty factor	-1 ... 1	[illegible]
AV	added value	-0.5 ... 1	$\max(P(B\|A) - P(B), P(A\|B) - P(A))$
K	Klosgen's Q	-0.33 ... 0.38	$\sqrt{P(A,B)}\max(P(B\|A) - P(B), P(A\|B) - P(A))$
g	Goodman-kruskal's	0 ... 1	[illegible]
M	Mutual Information	0 ... 1	[illegible]
J	J-Measure	0 ... 1	[illegible]
G	Gini index	0 ... 1	[illegible]
s	support	0 ... 1	$P(A,B)$
c	confidence	0 ... 1	$\max(P(B\|A), P(A\|B))$
L	Laplace	0 ... 1	[illegible]
IS	Cosine	0 ... 1	$\frac{P(A,B)}{\sqrt{P(A)P(B)}}$
γ	coherence(Jaccard)	0 ... 1	[illegible]
α	all_confidence	0 ... 1	[illegible]
o	odds ratio	0 ... ∞	[illegible]
V	Conviction	0.5 ... ∞	[illegible]
λ	lift	0 ... ∞	$\frac{P(A,B)}{P(A)P(B)}$
S	Collective strength	0 ... ∞	[illegible]
χ^2	χ^2	0 ... ∞	[illegible]

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Null-Invariant Measures

Table 6: Properties of interestingness measures. Note that none of the measures satisfies all the properties.

Symbol	Measure	Range	P1	P2	P3	O1	O2	O3	O3'	O4
ϕ	ϕ-coefficient	$-1 \cdots 0 \cdots 1$	Yes	Yes	Yes	Yes	No	Yes	Yes	No
λ	Goodman-Kruskal's	$0 \cdots 1$	Yes	No	No	Yes	No	No*	Yes	No
α	odds ratio	$0 \cdots 1 \cdots \infty$	Yes*	Yes	Yes	Yes	Yes	Yes*	Yes	No
Q	Yule's Q	$-1 \cdots 0 \cdots 1$	Yes	Yes	Yes	Yes	Yes	Yes	Yes	No
Y	Yule's Y	$-1 \cdots 0 \cdots 1$	Yes	Yes	Yes	Yes	Yes	Yes	Yes	No
κ	Cohen's	$-1 \cdots 0 \cdots 1$	Yes	Yes	Yes	Yes	No	No	Yes	No
M	Mutual Information	$0 \cdots 1$	Yes	Yes	Yes	No**	No	No*	Yes	No
J	J-Measure	$0 \cdots 1$	Yes	No	No	No**	No	No	No	No
G	Gini index	$0 \cdots 1$	Yes	No	No	No**	No	No*	Yes	No
s	Support	$0 \cdots 1$	No	Yes	No	Yes	No	No	No	No
c	Confidence	$0 \cdots 1$	No	Yes	No	No**	No	No	No	Yes
L	Laplace	$0 \cdots 1$	No	Yes	No	No**	No	No	No	No
V	Conviction	$0.5 \cdots 1 \cdots \infty$	No	Yes	No	No**	No	No	Yes	No
I	Interest	$0 \cdots 1 \cdots \infty$	Yes*	Yes	Yes	Yes	No	No	No	No
IS	Cosine	$0 \cdots \sqrt{P(A,B)} \cdots 1$	No	Yes	Yes	Yes	No	No	No	Yes
PS	Piatetsky-Shapiro's	$-0.25 \cdots 0 \cdots 0.25$	Yes	Yes	Yes	Yes	No	Yes	Yes	No
F	Certainty factor	$-1 \cdots 0 \cdots 1$	Yes	Yes	Yes	No**	No	No	Yes	No
AV	Added value	$-0.5 \cdots 0 \cdots 1$	Yes	Yes	Yes	No**	No	No	No	No
S	Collective strength	$0 \cdots 1 \cdots \infty$	No	Yes	Yes	Yes	No	Yes*	Yes	No
ζ	Jaccard	$0 \cdots 1$	No	Yes	Yes	Yes	No	No	No	Yes
K	Klosgen's	$(\frac{2}{\sqrt{3}} - 1)^{1/2}[2 - \sqrt{3} - \frac{1}{\sqrt{3}}] \cdots 0 \cdots \frac{2}{3\sqrt{3}}$	Yes	Yes	Yes	No**	No	No	No	No

where: P1: $O(\mathbf{M}) = 0$ if $det(\mathbf{M}) = 0$, *i.e.*, whenever A and B are statistically independent.
P2: $O(\mathbf{M_2}) > O(\mathbf{M_1})$ if $\mathbf{M_2} = \mathbf{M_1} + [k\ -k;\ -k\ k]$.
P3: $O(\mathbf{M_2}) < O(\mathbf{M_1})$ if $\mathbf{M_2} = \mathbf{M_1} + [0\ k;\ 0\ -k]$ or $\mathbf{M_2} = \mathbf{M_1} + [0\ 0;\ k\ -k]$.
O1: Property 1: Symmetry under variable permutation.
O2: Property 2: Row and Column scaling invariance.
O3: Property 3: Antisymmetry under row or column permutation.
O3': Property 4: Inversion invariance.
O4: Property 5: Null invariance.
Yes*: Yes if measure is normalized.
No*: Symmetry under row or column permutation.
No**: No unless the measure is symmetrized by taking $\max(M(A,B), M(B,A))$.

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Comparison of Interestingness Measures

- Null-(transaction) invariance is crucial for correlation analysis
- Lift and χ^2 are not null-invariant
- 5 null-invariant measures

	Milk	No Milk	Sum (row)
Coffee	m, c	~m, c	c
No Coffee	m, ~c	~m, ~c	~c
Sum(col.)	m	~m	Σ

Measure	Definition	Range	Null-Invariant
$\chi^2(a,b)$	$\sum_{i,j=0,1} \frac{(e(a_i,b_j) - o(a_i,b_j))^2}{e(a_i,b_j)}$	$[0, \infty]$	No
$Lift(a,b)$	$\frac{P(ab)}{P(a)P(b)}$	$[0, \infty]$	No
$AllConf(a,b)$	$\frac{sup(ab)}{max\{sup(a), sup(b)\}}$	$[0, 1]$	Yes
$Coherence(a,b)$	$\frac{sup(ab)}{sup(a)+sup(b)-sup(ab)}$	$[0, 1]$	Yes
$Cosine(a,b)$	$\frac{sup(ab)}{\sqrt{sup(a)sup(b)}}$	$[0, 1]$	Yes
$Kulc(a,b)$	$\frac{sup(ab)}{2}(\frac{1}{sup(a)} + \frac{1}{sup(b)})$	$[0, 1]$	Yes
$MaxConf(a,b)$	$max\{\frac{sup(ab)}{sup(a)}, \frac{sup(ab)}{sup(b)}\}$	$[0, 1]$	Yes

Table 3. Interestingness measure definitions.

Null-transactions w.r.t. m and c

Kulczynski measure (1927)

Null-invariant

Data set	mc	$\overline{m}c$	$m\overline{c}$	$\overline{mc}$	χ^2	$Lift$	$AllConf$	$Coherence$	$Cosine$	$Kulc$	$MaxConf$
D_1	10,000	1,000	1,000	100,000	90557	9.26	0.91	0.83	0.91	0.91	0.91
D_2	10,000	1,000	1,000	100	0	1	0.91	0.83	0.91	0.91	0.91
D_3	100	1,000	1,000	100,000	670	8.44	0.09	0.05	0.09	0.09	0.09
D_4	1,000	1,000	1,000	100,000	24740	25.75	0.5	0.33	0.5	0.5	0.5
D_5	1,000	100	10,000	100,000	8173	9.18	0.09	0.09	0.29	0.5	0.91
D_6	1,000	10	100,000	100,000	965	1.97	0.01	0.01	0.10	0.5	0.99

Table 2. Example data sets.

Subtle: They disagree

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Analysis of DBLP Coauthor Relationships

Recent DB conferences**, removing balanced associations**, low sup, etc.

ID	Author a	Author b	$sup(ab)$	$sup(a)$	$sup(b)$	$Coherence$	$Cosine$	$Kulc$
1	Hans-Peter Kriegel	Martin Ester	28	146	54	0.163 (2)	0.315 (7)	0.355 (9)
2	Michael Carey	Miron Livny	26	104	58	0.191 (1)	0.335 (4)	0.349 (10)
3	Hans-Peter Kriegel	Joerg Sander	24	146	36	0.152 (3)	0.331 (5)	0.416 (8)
4	Christos Faloutsos	Spiros Papadimitriou	20	162	26	0.119 (7)	0.308 (10)	0.446 (7)
5	Hans-Peter Kriegel	Martin Pfeifle	18	146	18	0.123 (6)	0.351 (2)	0.562 (2)
6	Hector Garcia-Molina	Wilburt Labio	16	144	18	0.110 (9)	0.314 (8)	0.500 (4)
7	Divyakant Agrawal	Wang Hsiung	16	120	16	0.133 (5)	0.365 (1)	0.567 (1)
8	Elke Rundensteiner	Murali Mani	16	104	20	0.148 (4)	0.351 (3)	0.477 (6)
9	Divyakant Agrawal	Oliver Po	12	120	12	0.100 (10)	0.316 (6)	0.550 (3)
10	Gerhard Weikum	Martin Theobald	12	106	14	0.111 (8)	0.312 (9)	0.485 (5)

Table 5. Experiment on DBLP data set.

Advisor-advisee relation: Kulc: **high**, coherence: low, cosine: middle

- Tianyi Wu, Yuguo Chen and Jiawei Han, "Association Mining in Large Databases: A Re-Examination of Its Measures", Proc. 2007 Int. Conf. Principles and Practice of Knowledge Discovery in Databases (PKDD'07), Sept. 2007

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Which Null-Invariant Measure Is Better?

- IR (Imbalance Ratio): measure the imbalance of two itemsets A and B in rule implications

$$IR(A,B) = \frac{|sup(A) - sup(B)|}{sup(A) + sup(B) - sup(A \cup B)}$$

- **Kulczynski** and **Imbalance Ratio** (IR) together present a clear picture for all the three *datasets* D_4 through D_6
  - D_4 is balanced & neutral
  - D_5 is imbalanced & neutral
  - D_6 is very imbalanced & neutral

Data	mc	$\overline{m}c$	$m\overline{c}$	$\overline{mc}$	$all\_conf.$	$max\_conf.$	$Kulc.$	$cosine$	IR
D_1	10,000	1,000	1,000	100,000	0.91	0.91	0.91	0.91	0.0
D_2	10,000	1,000	1,000	100	0.91	0.91	0.91	0.91	0.0
D_3	100	1,000	1,000	100,000	0.09	0.09	0.09	0.09	0.0
D_4	1,000	1,000	1,000	100,000	0.5	0.5	0.5	0.5	0.0
D_5	1,000	100	10,000	100,000	0.09	0.91	0.5	0.29	0.89
D_6	1,000	10	100,000	100,000	0.01	0.99	0.5	0.10	0.99

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Chapter 5: Mining Frequent Patterns, Association and Correlations: Basic Concepts and Methods

- Basic Concepts
- Frequent Itemset Mining Methods
- Which Patterns Are Interesting?—Pattern Evaluation Methods
- Summary

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Summary

- Basic concepts: association rules, support-confident framework, closed and max-patterns
- Scalable frequent pattern mining methods
  - Apriori (Candidate generation & test)
  - Projection-based (FPgrowth, CLOSET+, ...)
  - Vertical format approach (ECLAT, CHARM, ...)
- Which patterns are interesting?
  - Pattern evaluation methods

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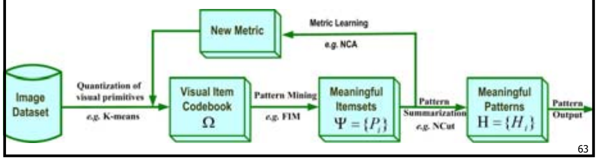
Application in Computer Vision

- Motivations: so many visual features
- itemsets: visual patterns
- Applications
  - *From Frequent Itemsets to Semantically Meaningful Visual Patterns*, Junsong Yuan, Ying Wu, Ming Yang, KDD 2007



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Example

Figure 9: Examples of meaningful itemsets from car category (6 out of 123 images). The cars are all side views, but are of different types and colors and located in various clutter backgrounds. The first row shows the original images. The second row shows their visual primitives (PCA-SIFT points), where each green circle denotes a visual primitive with corresponding location, scale and orientation. The third row shows the meaningful itemsets. Each red rectangle in the image contains a meaningful itemset (it is possible two items are located at the same position). Different colors of the items denote different semantic meanings. For example, wheels are dark red and car bodies are dark blue. The precision and recall scores of these semantic patterns are shown in Fig. 8.

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More on Vision and Learning

- **Mining Compositional Features for Boosting,** Junsong Yuan, Jiebo Luo, Ying Wu, CVPR 2008

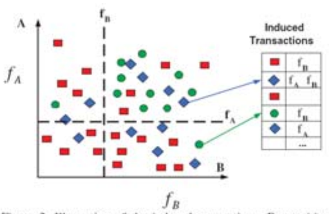

Figure 2. Illustration of the induced transaction. By partitioning the feature space into sub-regions through decision stumps f_A and f_B, we can index the training samples in terms of the sub-regions they are located. Only positive responses are considered. For example, a transaction of $T(x) = \{f_A, f_B\}$ indicates that $f_A(x) > \theta_A$ and $f_B(x) > \theta_B$.

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Ref: Basic Concepts of Frequent Pattern Mining

- (Association Rules) R. Agrawal, T. Imielinski, and A. Swami. Mining association rules between sets of items in large databases. SIGMOD'93
- (Max-pattern) R. J. Bayardo. Efficiently mining long patterns from databases. SIGMOD'98
- (Closed-pattern) N. Pasquier, Y. Bastide, R. Taouil, and L. Lakhal. Discovering frequent closed itemsets for association rules. ICDT'99
- (Sequential pattern) R. Agrawal and R. Srikant. Mining sequential patterns. ICDE'95

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Ref: Apriori and Its Improvements

- R. Agrawal and R. Srikant. Fast algorithms for mining association rules. VLDB'94
- H. Mannila, H. Toivonen, and A. I. Verkamo. Efficient algorithms for discovering association rules. KDD'94
- A. Savasere, E. Omiecinski, and S. Navathe. An efficient algorithm for mining association rules in large databases. VLDB'95
- J. S. Park, M. S. Chen, and P. S. Yu. An effective hash-based algorithm for mining association rules. SIGMOD'95
- H. Toivonen. Sampling large databases for association rules. VLDB'96
- S. Brin, R. Motwani, J. D. Ullman, and S. Tsur. Dynamic itemset counting and implication rules for market basket analysis. SIGMOD'97
- S. Sarawagi, S. Thomas, and R. Agrawal. Integrating association rule mining with relational database systems: Alternatives and implications. SIGMOD'98

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Ref: Depth-First, Projection-Based FP Mining

- R. Agarwal, C. Aggarwal, and V. V. V. Prasad. A tree projection algorithm for generation of frequent itemsets. J. Parallel and Distributed Computing, 2002.
- G. Grahne and J. Zhu, Efficiently Using Prefix-Trees in Mining Frequent Itemsets, Proc. FIMI'03
- B. Goethals and M. Zaki. An introduction to workshop on frequent itemset mining implementations. *Proc. ICDM'03 Int. Workshop on Frequent Itemset Mining Implementations (FIMI'03),* Melbourne, FL, Nov. 2003
- J. Han, J. Pei, and Y. Yin. Mining frequent patterns without candidate generation. SIGMOD' 00
- J. Liu, Y. Pan, K. Wang, and J. Han. Mining Frequent Item Sets by Opportunistic Projection. KDD'02
- J. Han, J. Wang, Y. Lu, and P. Tzvetkov. Mining Top-K Frequent Closed Patterns without Minimum Support. ICDM'02
- J. Wang, J. Han, and J. Pei. CLOSET+: Searching for the Best Strategies for Mining Frequent Closed Itemsets. KDD'03

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Ref: Vertical Format and Row Enumeration Methods

- M. J. Zaki, S. Parthasarathy, M. Ogihara, and W. Li. Parallel algorithm for discovery of association rules. DAMI:97.
- M. J. Zaki and C. J. Hsiao. CHARM: An Efficient Algorithm for Closed Itemset Mining, SDM'02.
- C. Bucila, J. Gehrke, D. Kifer, and W. White. DualMiner: A Dual-Pruning Algorithm for Itemsets with Constraints. KDD'02.
- F. Pan, G. Cong, A. K. H. Tung, J. Yang, and M. Zaki , CARPENTER: Finding Closed Patterns in Long Biological Datasets. KDD'03.
- H. Liu, J. Han, D. Xin, and Z. Shao, Mining Interesting Patterns from Very High Dimensional Data: A Top-Down Row Enumeration Approach, SDM'06.

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Ref: Mining Correlations and Interesting Rules

- S. Brin, R. Motwani, and C. Silverstein. Beyond market basket: Generalizing association rules to correlations. SIGMOD'97.
- M. Klemettinen, H. Mannila, P. Ronkainen, H. Toivonen, and A. I. Verkamo. Finding interesting rules from large sets of discovered association rules. CIKM'94.
- R. J. Hilderman and H. J. Hamilton. *Knowledge Discovery and Measures of Interest*. Kluwer Academic, 2001.
- C. Silverstein, S. Brin, R. Motwani, and J. Ullman. Scalable techniques for mining causal structures. VLDB'98.
- P.-N. Tan, V. Kumar, and J. Srivastava. Selecting the Right Interestingness Measure for Association Patterns. KDD'02.
- E. Omiecinski. Alternative Interest Measures for Mining Associations. TKDE'03.
- T. Wu, Y. Chen, and J. Han, "Re-Examination of Interestingness Measures in Pattern Mining: A Unified Framework", Data Mining and Knowledge Discovery, 21(3):371-397, 2010

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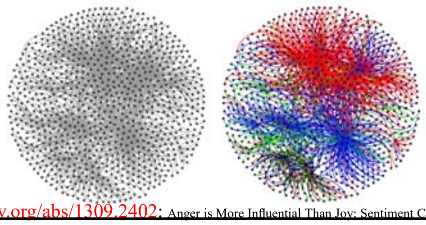
Most Influential Emotions on Social Networks

- Anger spreads faster and more broadly than joy, say computer scientists who have analysed sentiment on the Chinese Twitter-like service **Weibo**
- One well-known feature of social networks is that similar people tend to attract each other: birds of a feather flock together.
- So an interesting question is whether these similarities cause people to behave in the same way online, whether it might lead to flocking or herding behaviour, for example.



arxiv.org/abs/1309.2402: Anger is More Influential Than Joy: Sentiment Correlation in Weibo 71

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Homework Assignment #3

- Textbook (3rd Edition!)
  - 6.1, 6.3, 6.4, 6.5, 6.6, 6.11
  - Due in one week (Oct 3)
- Implementation project #1 (not HW#4)
  - 6.7 (1) & (2), plus one improvement of your choice for 6.7(1)
  - Due in two weeks (Oct 17)
  - Use the UCI Adult Census Dataset
  http://archive.ics.uci.edu/ml/datasets/Adult
  - Important note: you can use the open source code as a *reference*, but should implement the algorithms on your own. The point of the assignment is for you to know how the algorithms are implemented, not just how to run them. It would be easy to detect the latter, e.g. if more than one of you use the same code.

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