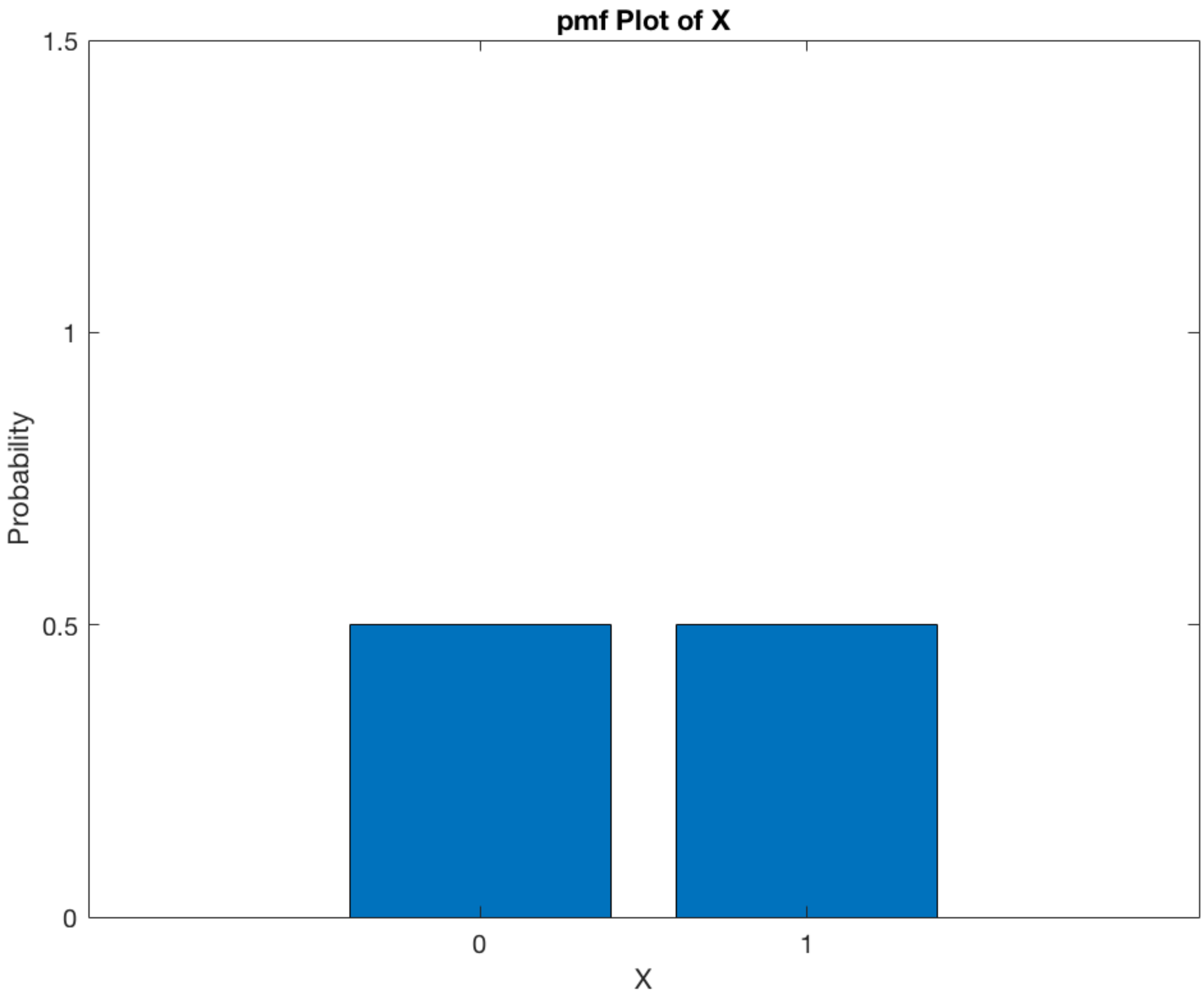
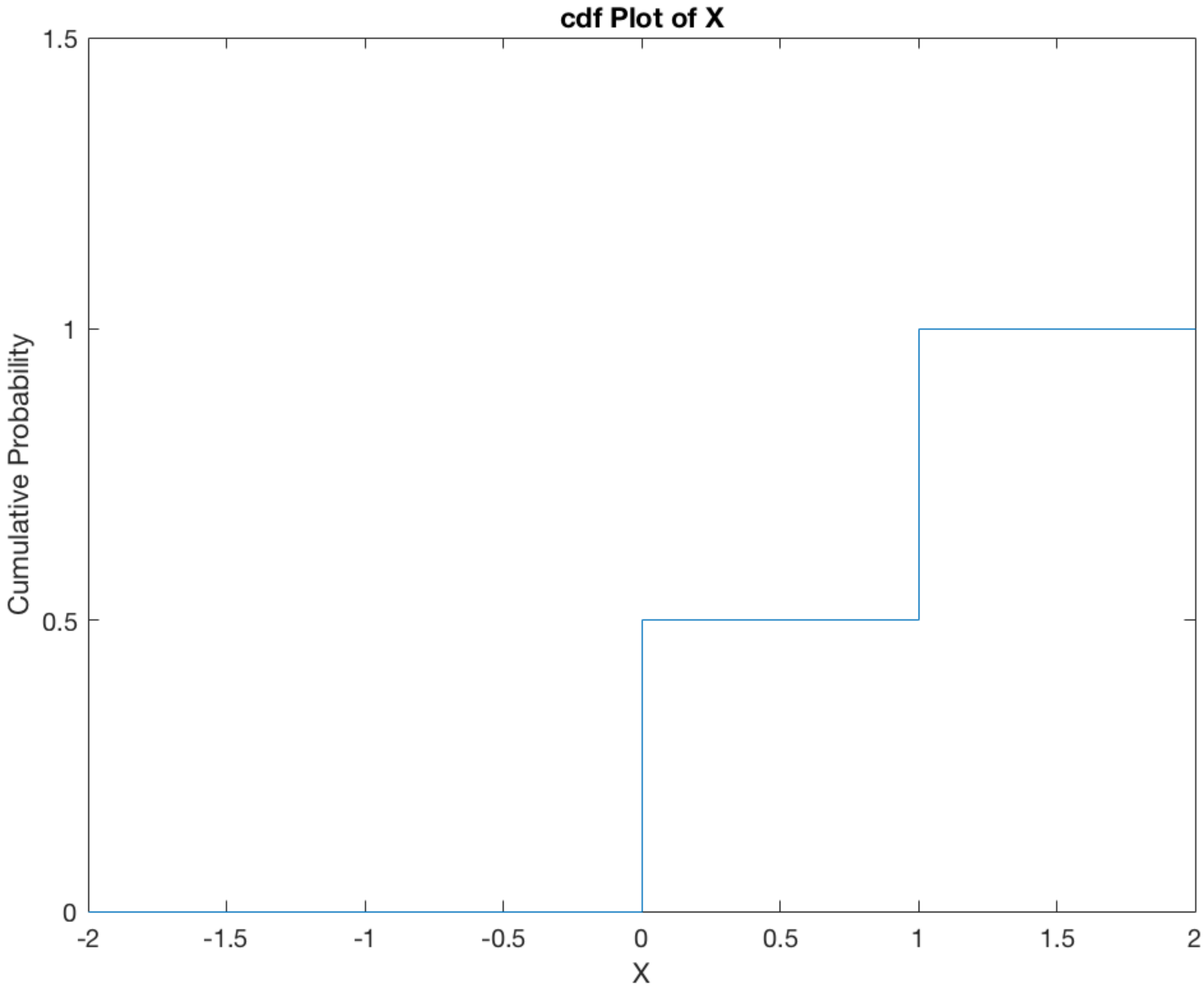
ECE 440 HW#2, Kefu Zhu, kzhu6

Question 1

$$p(x) = \begin{cases} \frac{1}{2} - 0 = 0.5, \ x = 0 \\ 1 - \frac{1}{2} = 0.5, \ x = 1 \end{cases}$$

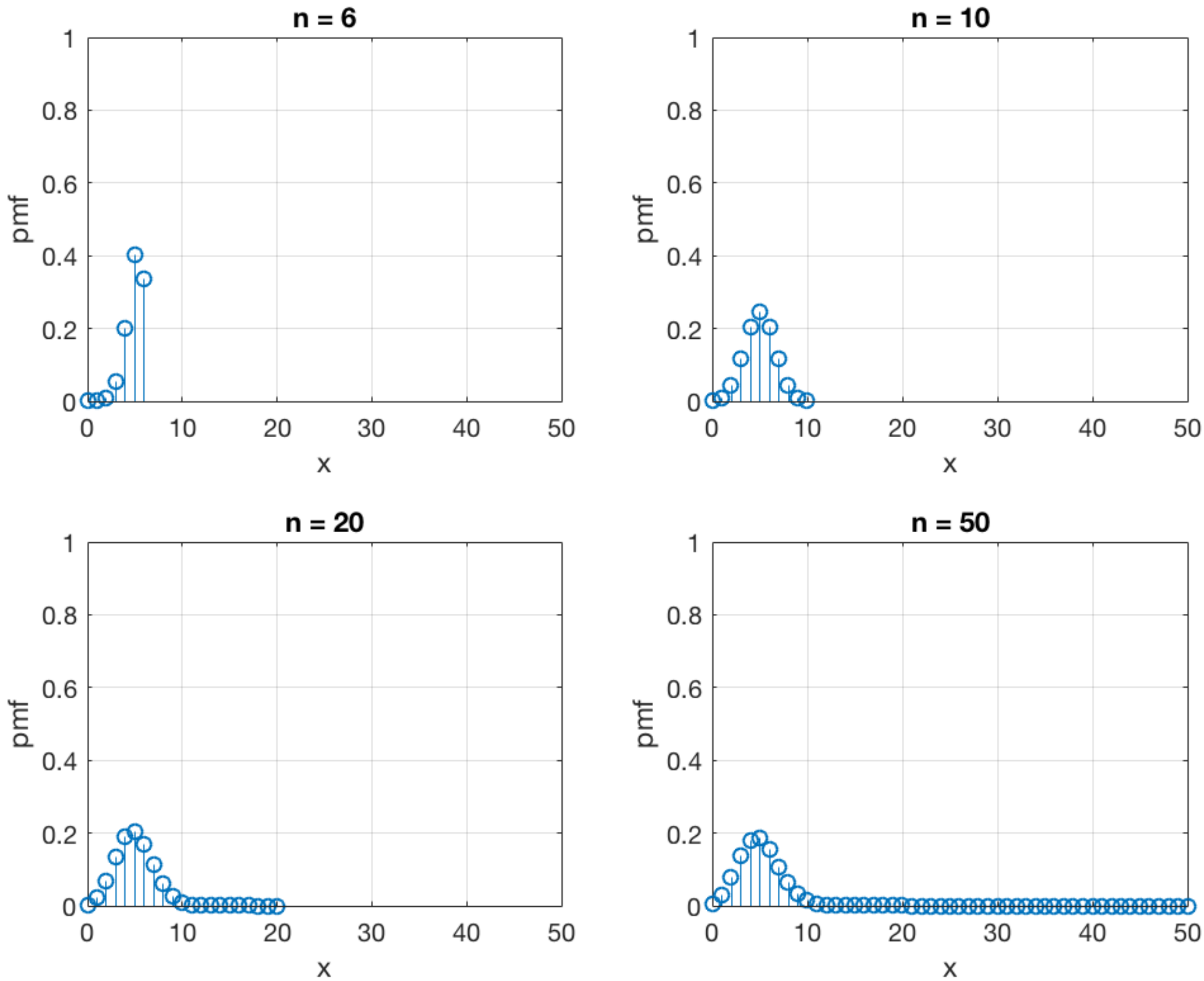


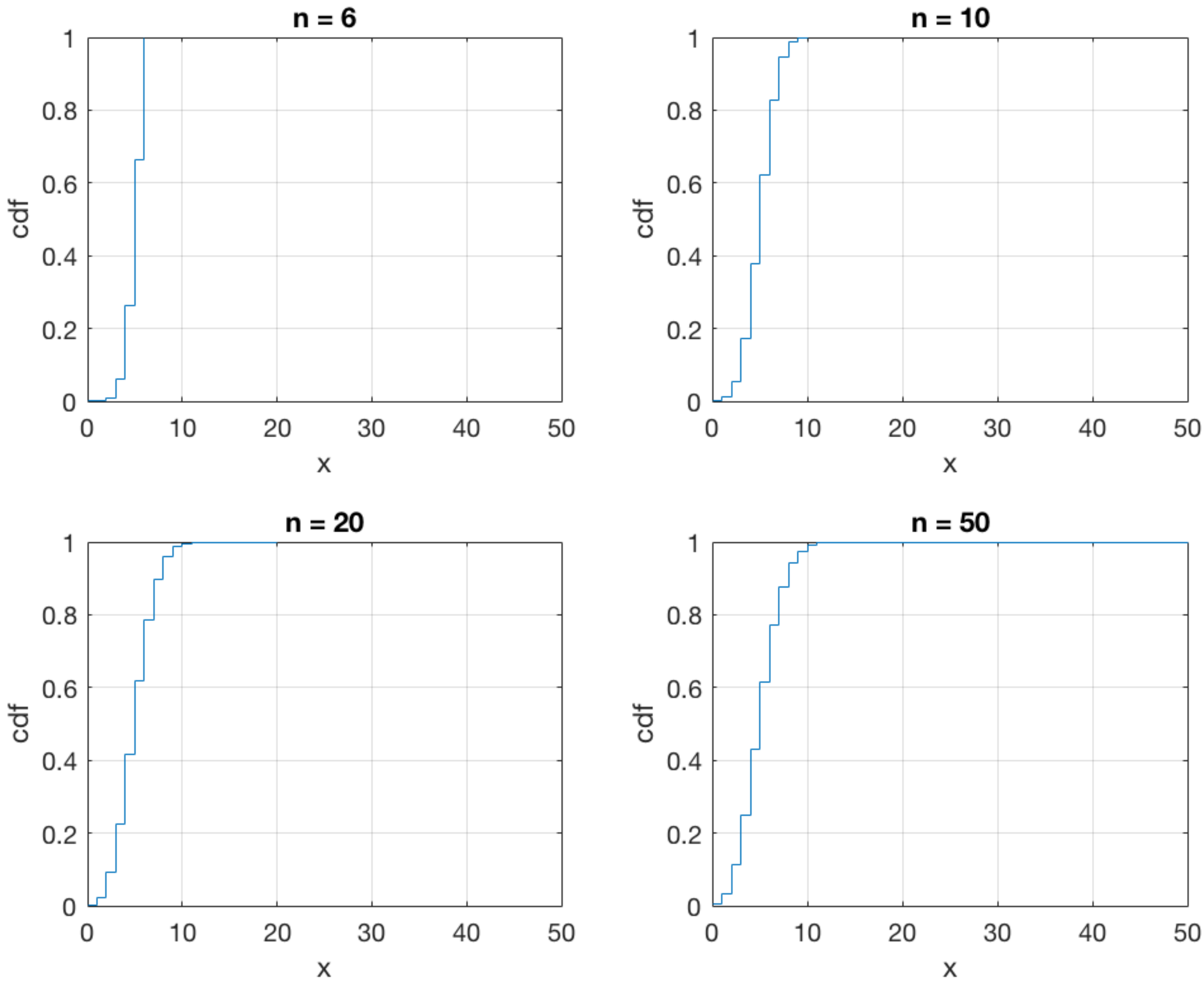


cdf Plot of X
1.5
1
0.5
0
Cumulative Probability
-2
-1.5
-1
-0.5
0
0.5
1
1.5
2
X

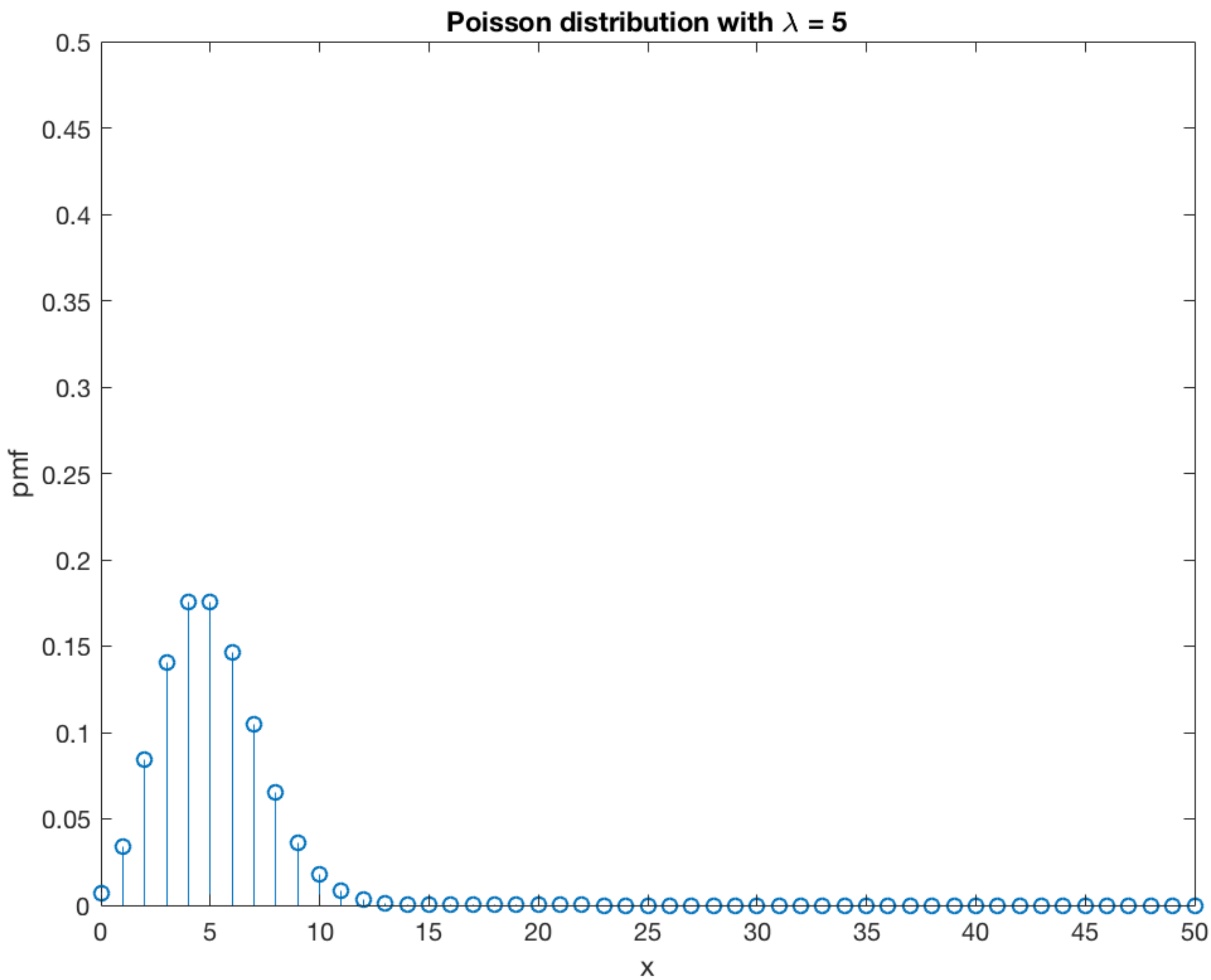
Question 7

(A)


n = 6
n = 10
n = 20
n = 50
pmf
x
0
0.2
0.4
0.6
0.8
1
0
10
20
30
40
50



(B)




```
% check the value of x such that p(x) is not too small ( p(x) < 0.05)
lambda = 5;
x = 0:10;
p_x = pdf('poiss',x,n,p);
p_x < 0.05
```

```
# Result
ans =

  1x11 logical array

   1   1   0   0   0   0   0   0   0   1   1
```

From the result, we only need to consider the MSE for $x = 2, 3, 4, 5, 6, 7, 8$

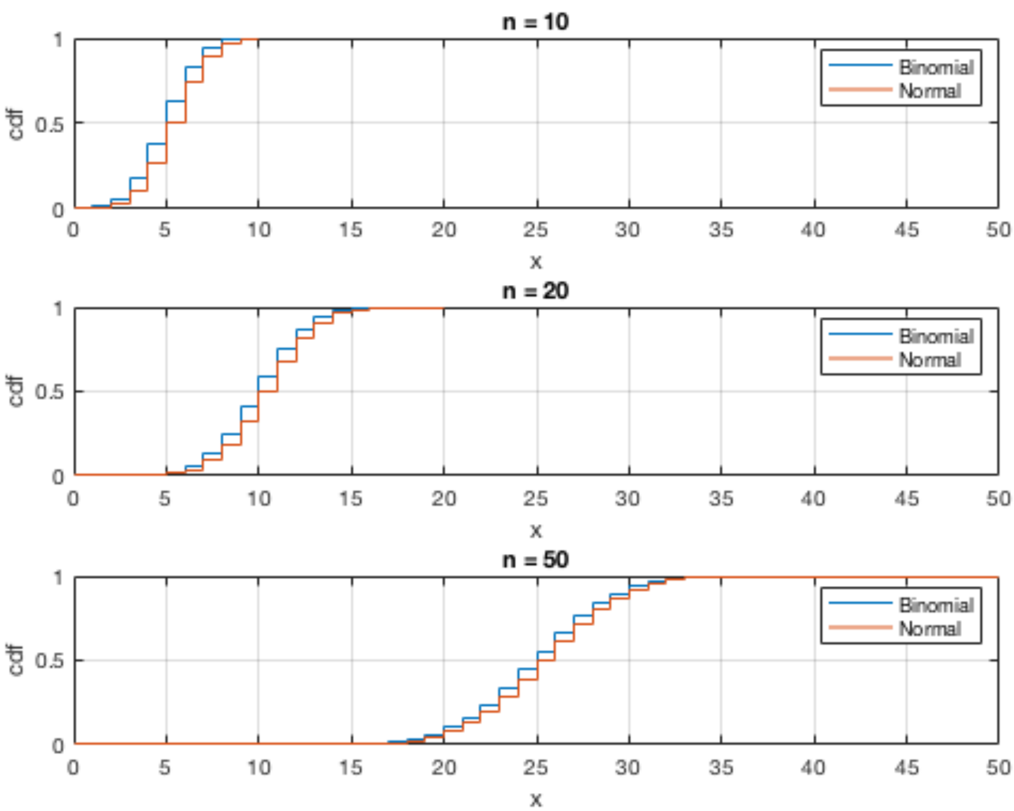
Below is the calculated MSE for $n = 6, 10, 20, 50$

```
n = 6, MSE = 0.017278
n = 10, MSE = 0.001794
n = 20, MSE = 0.000274
n = 50, MSE = 0.000035
```

(D)

To approximate normal distribution with binomial distribution with large value of n, with mean of np and variance of $np(1-p)$

The cdf of approximated normal distribution is $F(X) \approx \frac{1}{\sqrt{2\pi np(1-p)}} \int_{-\infty}^{x} e^{-\frac{(k-np)^2}{2np(1-p)}} \mathrm{d}k$



(E)

Because in the approximation for Poisson distribution, we fixed the value of λ. As $n \to \infty$, the probability $(p = \frac{\lambda}{n})$ goes to zero

However, in the approximation for Normal distribution, we only fixed the value of probability (p), which does not mean the probability goes to zero as $n \to \infty$. For example, in part(D), the value of p holds at 0.5 for any value of n

Therefore, because these two approximations have different assumptions, they are approximating different limits

Appendix

Code for Question 1

problem1.m

```
% pmf
x = [0 1];
p = [1/2 1/2];

% Plot for pmf
bar(x,p)
ylim([0,1.5]);ylabel("Probability");xlabel("X");title("pmf Plot of X");

% Generate figure in new window
figure
% cdf
X = [-2,-1,0,1,2];
F = [0,0,0.5,1,1];
stairs(X, F)
ylim([0,1.5]);ylabel("Cumulative Probability");xlabel("X");title("cdf Plot of X");
```

Code for Question 7

problem7a

```
np = 5;
nList = [6,10,20,50];

i = 1 % position for subplot

for n = nList
    p = np/n

    figure(1) % plot of pmf
    subplot(2,2,i) % position for subplot

    stem(0:n,pdf('bino',0:n,n,p));
    title('n = '+string(n)); xlabel('x'); ylabel('pmf');
    grid on; axis([0,50,0,1]); % fix the x and y axis grid

    figure(2) % plot of cdf
    subplot(2,2,i) % position for subplot
    stairs(0:n,cdf('bino',0:n,n,p));
    title('n = ' + string(n)); xlabel('x'); ylabel('cdf');
    grid on; axis([0,50,0,1]); % fix the x and y axis grid

    i = i + 1 % increment the position index
end
```

problem7b.m

```
% Parameters for poisson distribution
lambda = 5;
x = 0:50;

% Plot of poisson distribution with lambda = 5
stem(x,pdf('poiss',x,lambda));
axis([0,50,0,0.5]);
xlabel('x'); ylabel('pmf');title('Poisson distribution with \lambda = ' + string(lambdc

% check the value of x such that p(x) is not too small ( p(x) < 0.05)
clear all; close all;
lambda = 5;
x = 0:10;
p_x = pdf('poiss',x,n,p);
p_x < 0.05

% Calculate the MSE when n = 6. 10, 20, 50
clear all;
lambda = 5;
x = 2:8;
nList = [6,10,20,50];

for n = nList
    p = lambda/n;
    MSE = sum((pdf('bino',x,n,p) - pdf('poiss',x,lambda)).^2.*pdf('poiss',x,lambda));
    fprintf('n = %d, MSE = %.6f\n', n, MSE);
end
```

problem7d.m

```
clear all;
p = 0.5;
nList = [10 20 50];

index = 1;
for n = nList
    mean_normal = n*p;
    sd_normal = sqrt(n*p*(1-p));
    x=0:n;
    subplot(3,1,index);
    stairs(x,[binocdf(x,n,p)', normcdf(x,mean_normal,sd_normal)']);
    title('n = ' + string(n)); xlabel('x'); ylabel('cdf');
    legend('Binomial','Normal');
    grid on; axis([0,50,0,1]);

    % increment the position index
    index = index + 1;
end
```