ECE 440, HW#6, Kefu Zhu

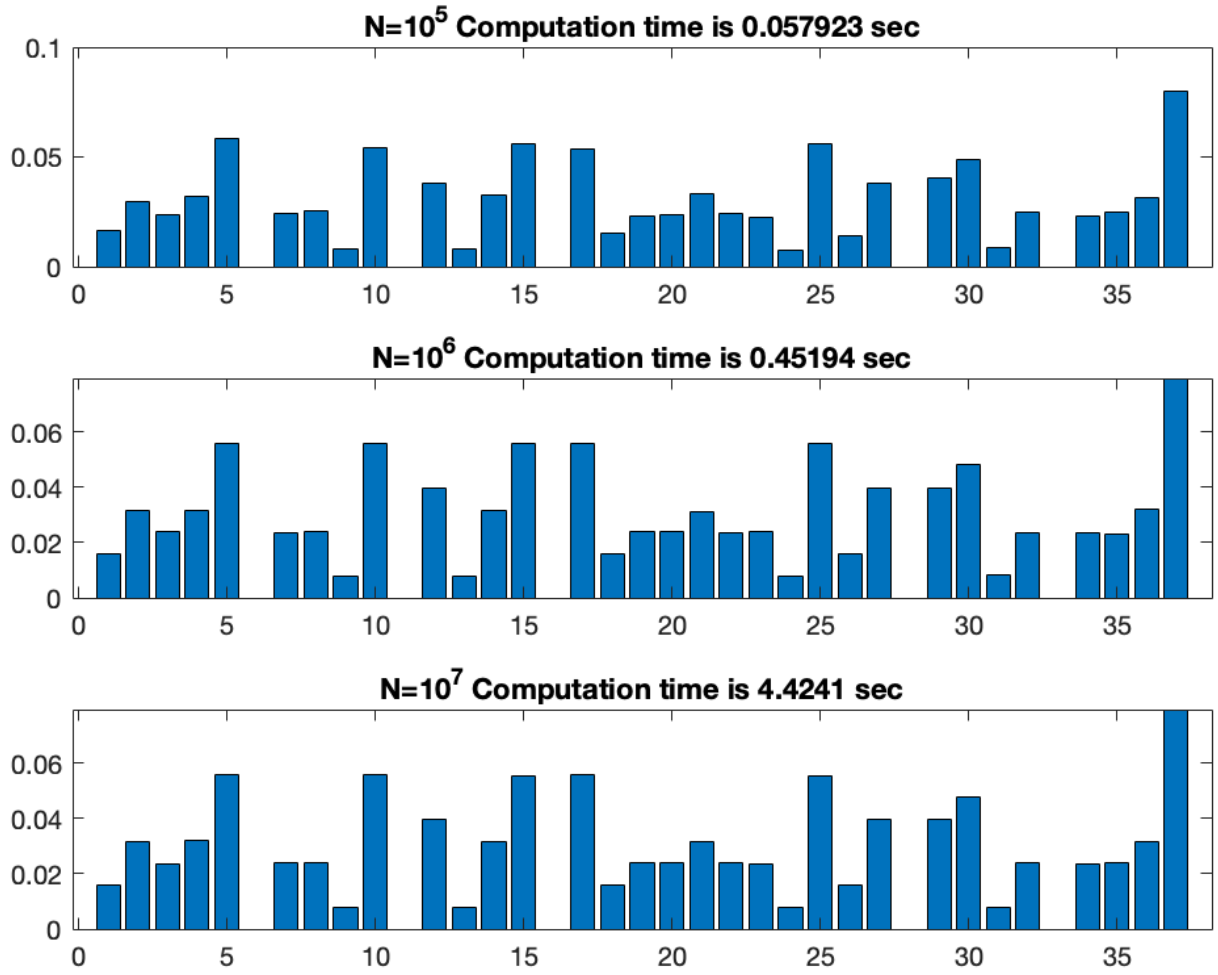
Question 4

(B)

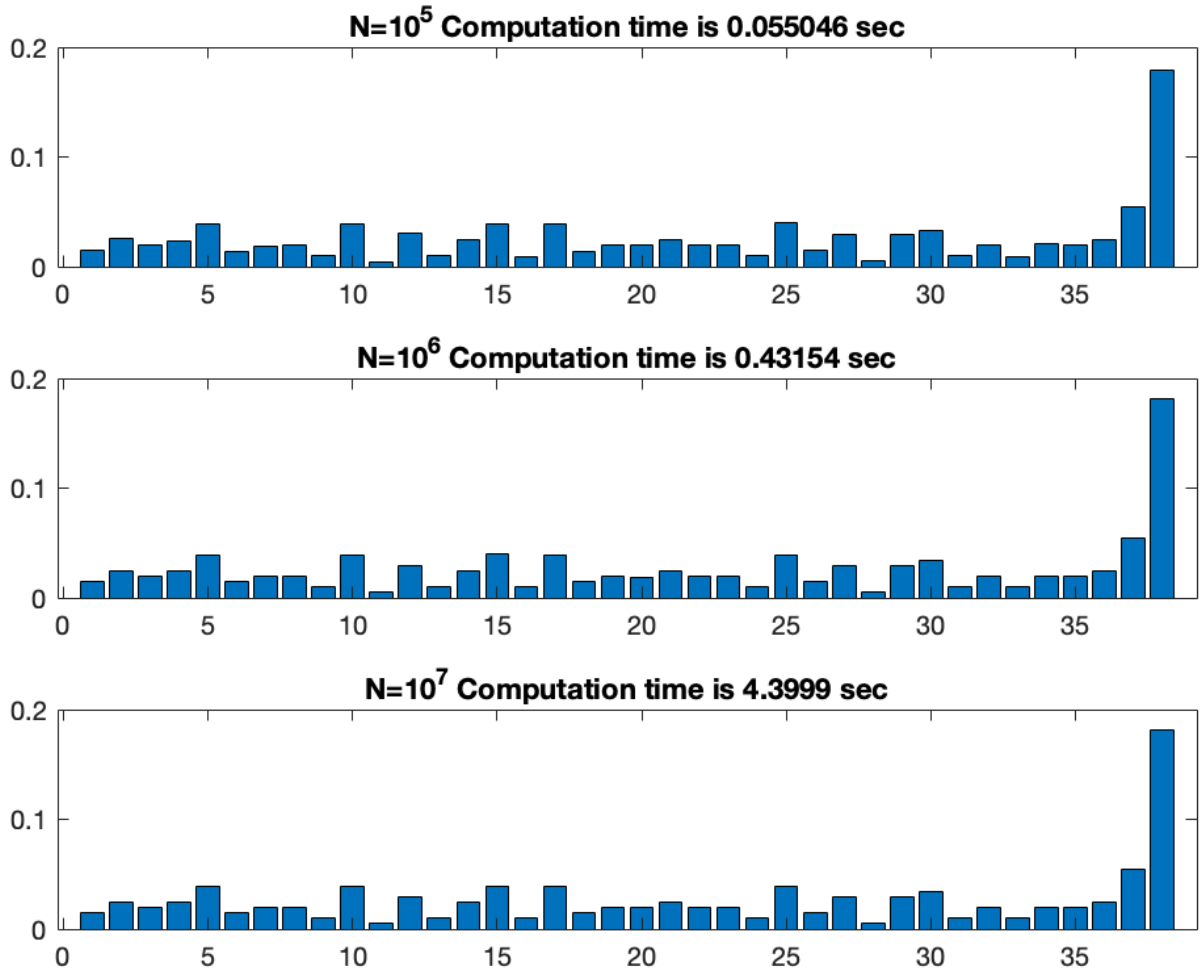
ranks_by_random_walk

```
function ranks=ranks_by_random_walk(graph,N,initial_state)

    % Total number of states
    J=min(size(graph));
    % Number of neighbors for each state
    n_neighbors = sum(graph);

    k=max(n_neighbors);
    Neighbors=zeros(J,k);

    for i=1:J
        temp=find(graph(i,:));
        Neighbors(i,1:length(temp))=temp;
    end

    i=initial_state;
    nr_visits=zeros(J,1);

    for n=1:N
        nr_visits(i)=nr_visits(i)+1;
        i=Neighbors(i,randi(n_neighbors(i)));
    end

    ranks=nr_visits/N;
end
```

For $A_0 = 1$, experiment with different number of students (N)



Add an extra node (professor) which is connected to all other nodes to make the MC independent on the initial state. The ranks result are shown below



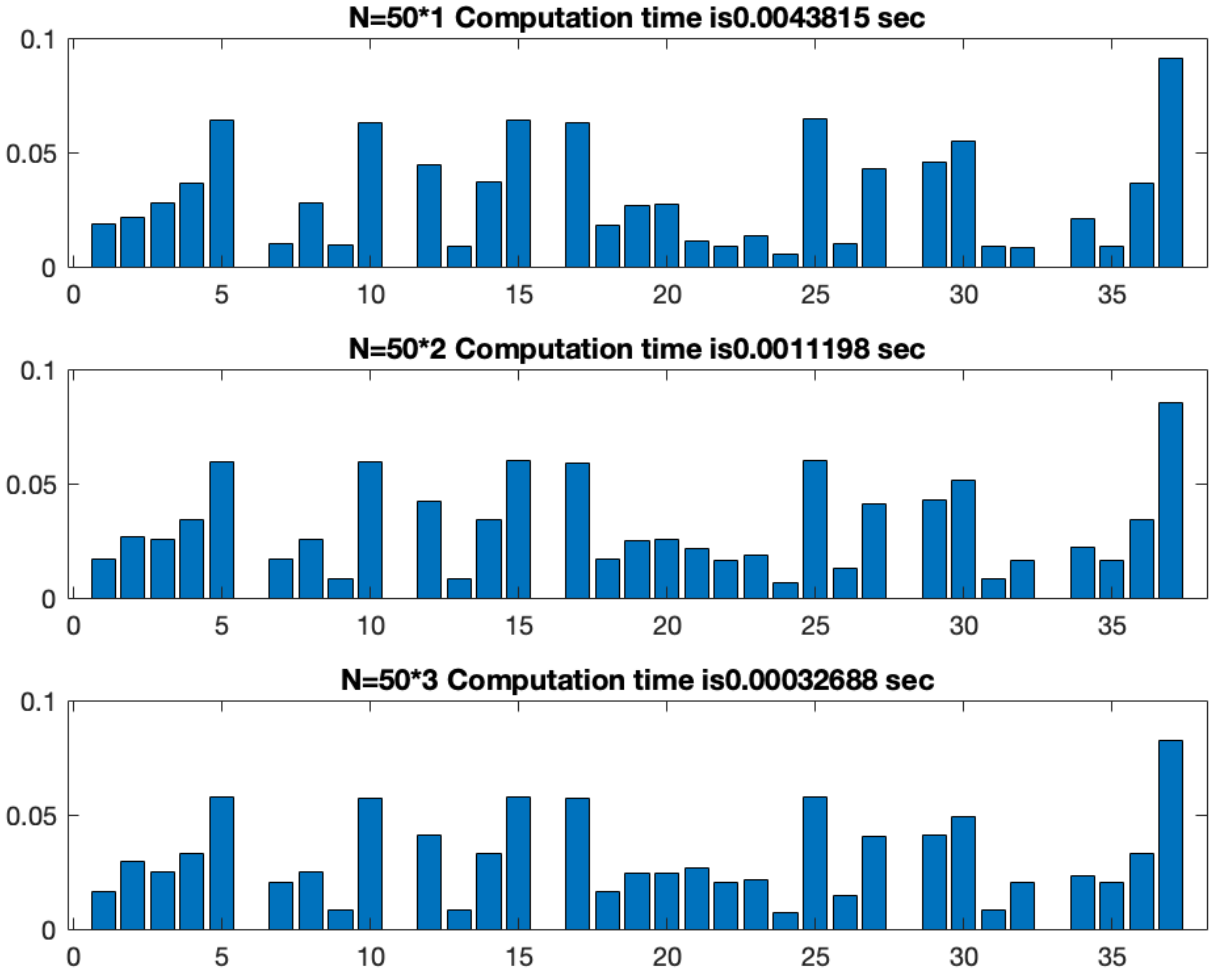
(D)

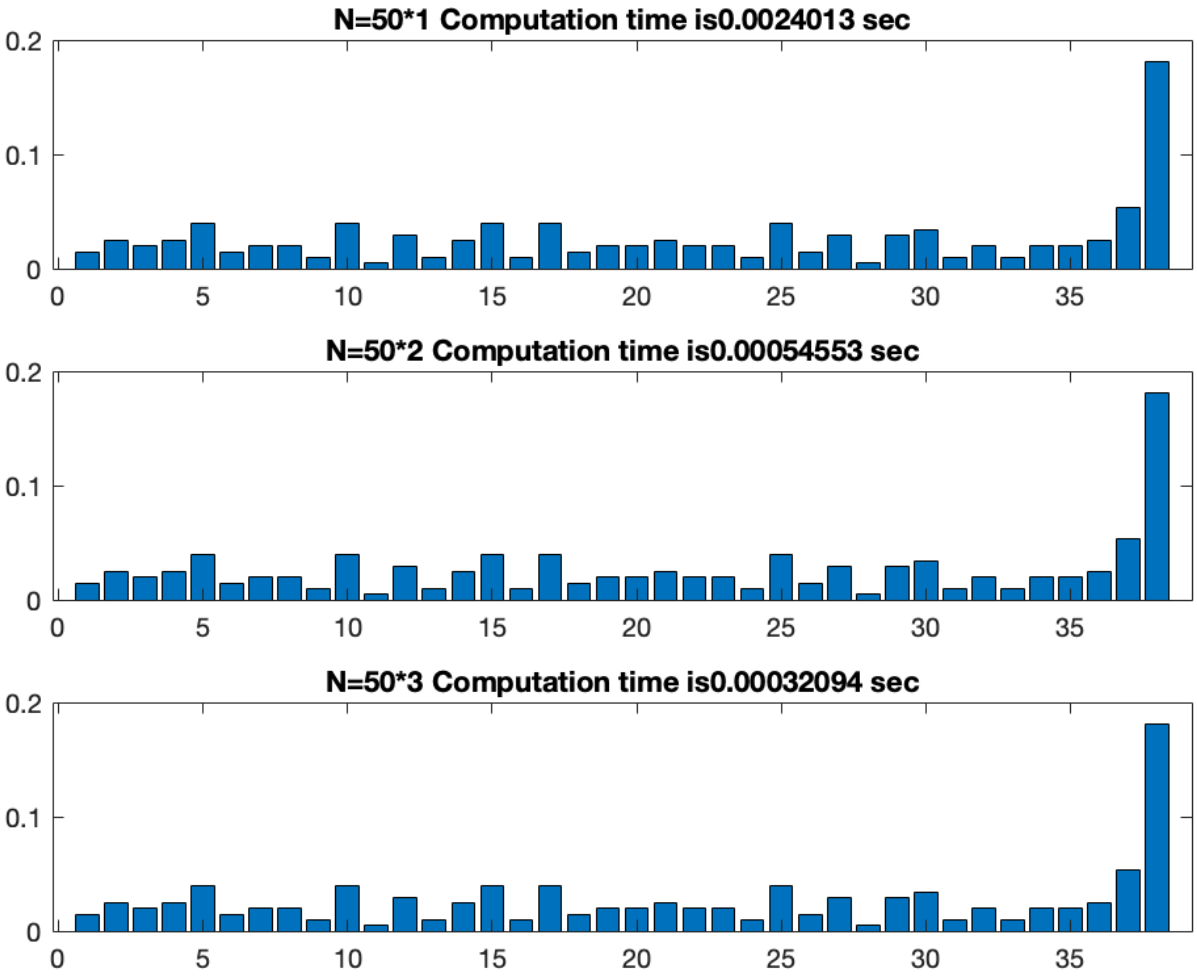
Since we are limiting the initial start is at A_0 , then the rank of state i can be calculated as

$$r_i(A_0) = lim_{n\to\infty} \frac{1}{n} \sum_{m=1}^{n} I(A_m = i) = lim_{n\to\infty} P^n_{A_0,i} = lim_{n\to\infty} p_i(n)$$

The reason that the limit can be calculated as such for any initial state is that if the initial state is in some other classes which are not A_0 , then $r_i(A_0) = 0$ because state i is never visited

Below is the computation of ranks


N=50*1 Computation time is0.0043815 sec
0.1
0.05
0
0
5
10
15
20
25
30
35
N=50*2 Computation time is0.0011198 sec
0.1
0.05
0
0
5
10
15
20
25
30
35
N=50*3 Computation time is0.00032688 sec
0.1
0.05
0
0
5
10
15
20
25
30
35



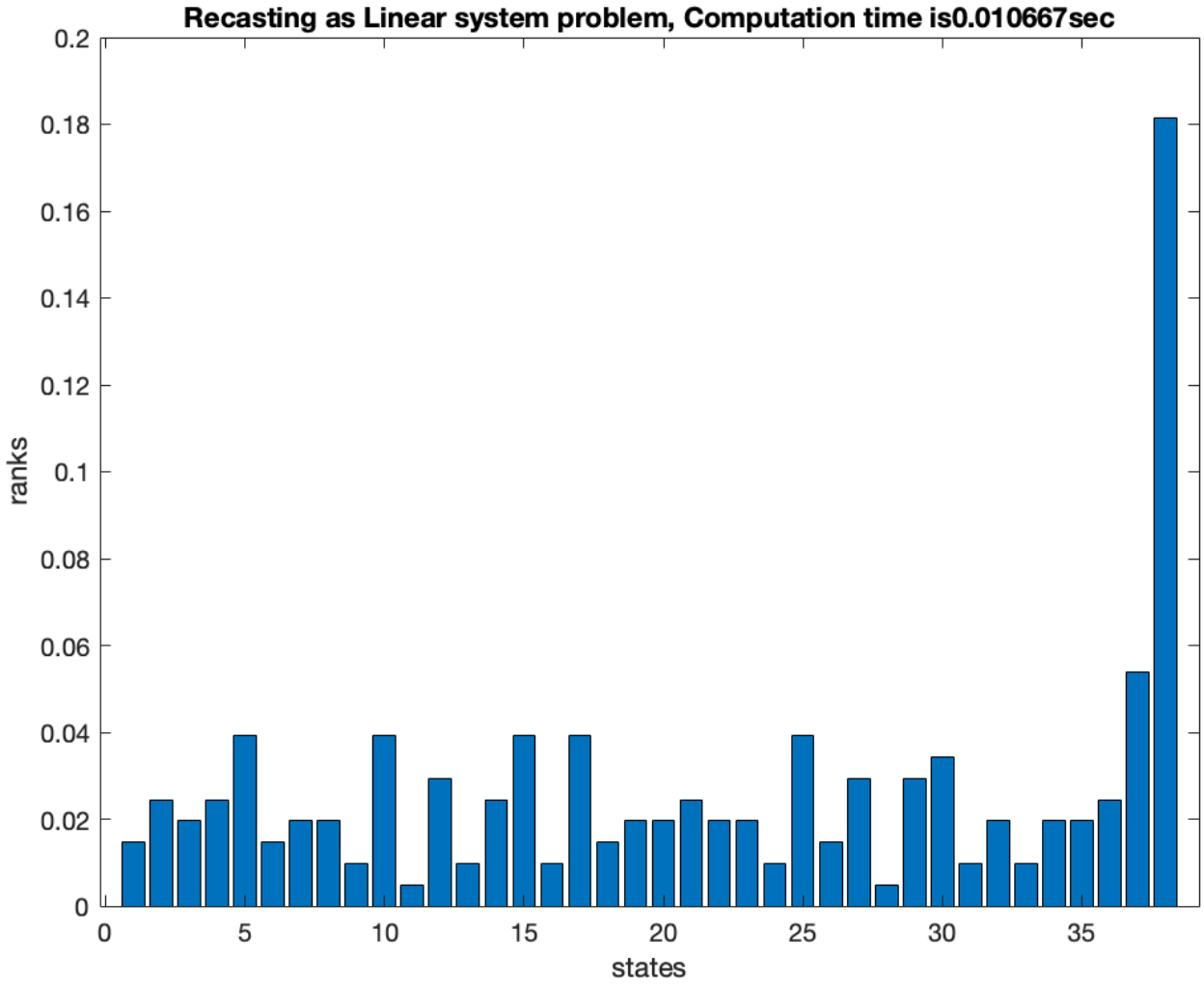
(E)

Since we are focusing on the modified graph again now, similar to part D, we can compute the rank for state i as such:

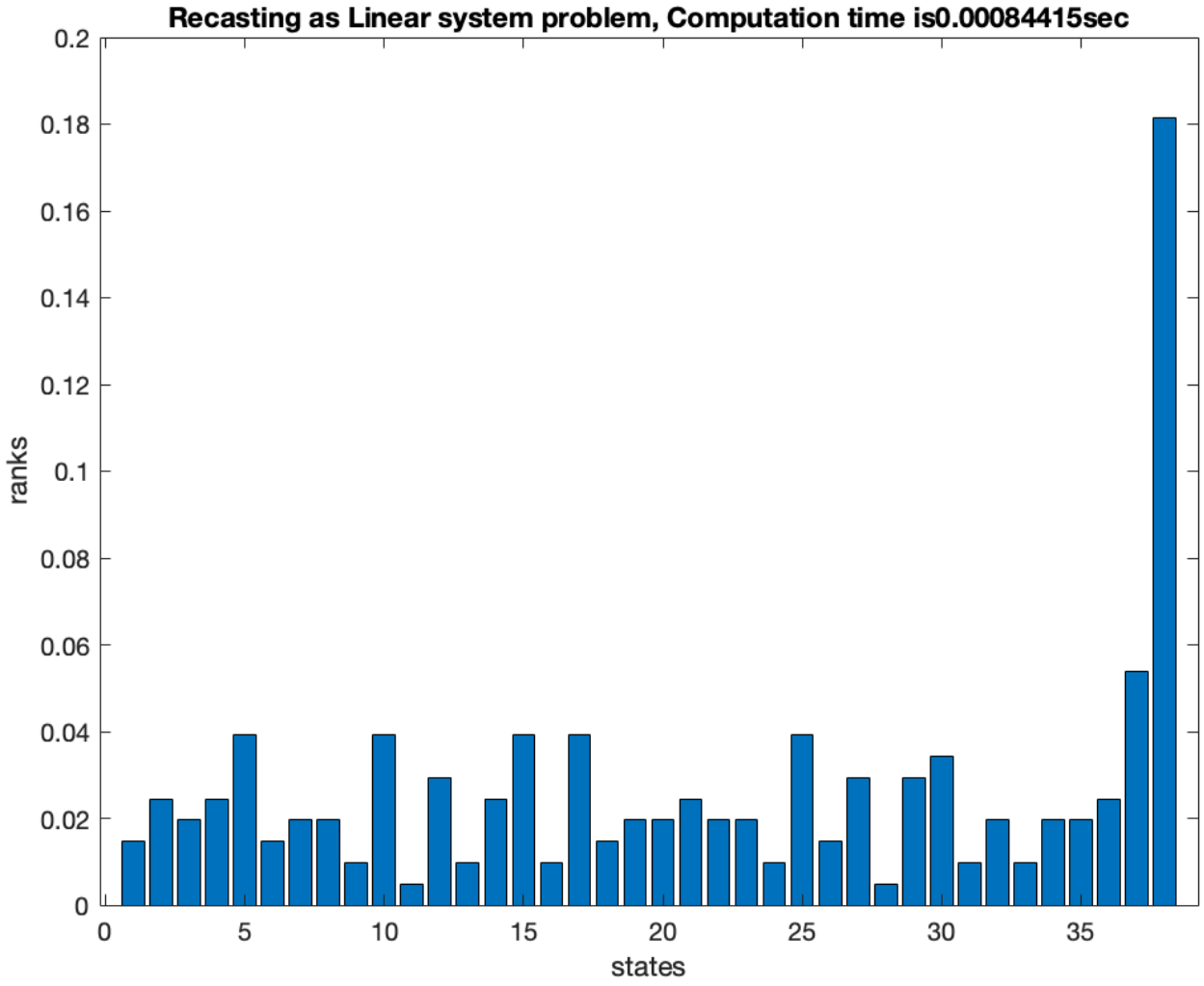
$r_i(A_0) = lim_{n\to\infty} \frac{1}{n} \sum_{m=1}^{n} I(A_m = i) = \pi_i$

Because of ergodicity, π is the unique solution of a system of linear equations as below

$\binom{I-P^T}{1^T}\pi = \binom{0}{1}$, where I is the $J \times J$ identity matrix and 0 is the $(J+1) \times 1$ array


Recasting as Linear system problem, Computation time is0.010667sec
ranks
states
0
0.02
0.04
0.06
0.08
0.1
0.12
0.14
0.16
0.18
0.2
0
5
10
15
20
25
30
35

(F)



(G)

(1) Random Walk

The random walk approach is secure in the way that rank or graph topology information is not shared between nodes. For a node to determine its own rank, it only needs to know its outgoing neighborhood $n(i)$ and the state of a global synchronization clock (yielding the value of n). This also means that implementation can be distributed.

(2) Probability Update

Similar to the random walk in that implementation can be distributed and it is fairly secure (but less secure than the random walk algorithm, since probability updates require exchanging current ranks with neighbors). Probability update approach converges to the true ranks significantly faster than the random walk approach, which is very efficient when facing large Markov Chain.

Because probability update approach uses matrix multiplications, it can leverage a lot of useful algorithms for fast matrix computation.

(3) System of Linear Equations

Solving the system of linear equations eliminates the problem of slow convergence, because (in theory) it does require an iterative procedure. In practice, be aware that Matlab's routines to solve linear systems are iterative. As a disadvantage, solving the linear system requires having the whole network topology centrally available to carry out the computations, which is the least secure and robust. The worst case complexity of solving a linear system is cubic in the input size (J), so for moderate-to-large graphs this approach could become infeasible.

(4) Eigenvalue

Very similar to the system of linear equations in terms of advantages and disadvantages

Appendix

problem4b_1.m

```
clc;close all;clear all;

% collaboration_matrix
graph=[0,0,0,0,0,0,0,0,1,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0;
0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,1,1,0,1,0,0,0,0,0,0,0,0,0,0,0;
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0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0;
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0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0;
0,1,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,1,0,0,0,0,0,1,0,0;
0,0,0,0,0,0,1,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0;
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0,0,0,1,1,0,0,0,0,1,0,1,0,0,1,0,1,0,0,0,0,0,0,0,1,0,0,0,1,1,0,0,0,0,1,0];

graph=graph+graph';
graph=graph>0;
graph=graph*1;
n_neighbors = sum(graph);

random_walk = graph;
for k=1:min(size(graph))
    if n_neighbors(k)>0
        random_walk(k,:)=graph(k,:)/n_neighbors(k);
    end
end

J=min(size(graph));

figure
for j=1:3
    N=10^(4+j);
    tic;
    ranks=ranks_by_random_walk(graph,N,1);
    t=toc;
    subplot(3,1,j)
    bar(1:J,ranks)
    title('N=10^' + string(j+4) + ' Computation time is ' + string(t) + ' sec')
end
```

problem4b_2.m

```
clc;close all;clear all;

% collaboration_matrix
graph=[0,0,0,0,0,0,0,0,1,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0;
0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,1,1,0,1,0,0,0,0,0,0,0,0,0,0,0;
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0,0,0,0,0,0,0,0,0,0,0,1,0,0,0,0,1,0,1,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0;
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0,0,0,1,1,0,0,0,0,1,0,1,0,0,1,0,1,0,0,0,0,0,0,0,1,0,0,0,1,1,0,0,0,0,0,1,0];

graph=graph+graph';
graph=graph>0;
graph=graph*1;
```

```
n_neighbors = sum(graph);

random_walk = graph;
for k=1:min(size(graph))
    if n_neighbors(k)>0
        random_walk(k,:)=graph(k,:)/n_neighbors(k);
    end
end

% Add professor
J=min(size(graph));
graph=[graph ones(J,1);ones(1,J) 0];

J=min(size(graph));

figure
for j=1:3
    N=10^(4+j);
    tic;
    ranks=ranks_by_random_walk(graph,N,1);
    t=toc;
    subplot(3,1,j)
    bar(1:J,ranks)
    title('N=10^' + string(j+4) + ' Computation time is ' + string(t) + ' sec')
end
```

problem4d_1.m

```
clc;close all;clear all;

% collaboration_matrix
graph=[0,0,0,0,0,0,0,0,1,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0;
0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,1,1,0,1,0,0,0,0,0,0,0,0,0,0,0,0;
0,0,0,0,0,0,0,0,0,0,0,0,0,1,1,0,0,0,0,0,0,0,0,1,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0;
0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0;
1,0,0,1,0,0,0,0,0,1,0,0,1,0,0,0,0,0,0,1,0,0,0,0,0,0,0,0,0,1,0,0,0,0,0,1;
0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,1,0,0,0,0,0,0,0,0,0,0,0,0,0,0,1,0,0,0,0;
0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,1,0,0;
0,0,0,0,0,0,0,0,1,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,1,0,0,0,0,1,0,0,0,0,0,0,0;
1,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0;
0,0,0,1,1,0,0,0,0,0,0,0,0,0,0,0,0,1,1,0,1,0,0,0,0,0,0,1,0,0,0,0,0,0,0,0,0,0,1;
0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0;
0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,1,0,0,0,0,0,0,0,1,0,0,0,0,0,0,0,0,0,0;
0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0;
```

```
0,0,1,0,0,0,0,0,0,0,0,0,0,1,0,0,0,0,0,0,0,0,1,0,0,0,1,0,0,0,0,0,0,0;
0,0,1,0,0,0,0,0,0,0,0,0,1,1,0,0,0,0,0,0,0,0,1,0,0,0,0,0,0,0,0,0,0,0;
0,0,0,0,0,1,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0;
0,0,0,0,0,0,0,0,1,0,1,0,0,1,0,0,1,1,0,0,0,0,0,0,0,1,0,0,0,0,0,0,0,1;
0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0;
0,0,0,0,0,0,0,0,0,0,1,0,0,0,0,1,0,0,0,0,0,0,0,0,1,0,0,0,0,0,0,0,0,0;
0,0,0,1,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0;
0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,1,1,0,0,0,0,0,0,0,0,1,0,0,1,0,0;
0,0,0,0,0,0,1,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,1,0,0,0,0,0;
0,0,0,0,0,0,1,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,1,0,0,0,0,0;
0,1,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0;
0,0,1,0,0,0,0,0,0,0,0,0,1,1,0,0,0,0,0,0,0,0,0,0,0,0,1,1,0,0,0,0,0,0,0,0;
0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,1,0,0,0,0,0,0,0,0,0,0,0;
0,0,0,0,0,0,0,0,0,0,1,0,0,0,0,1,0,1,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0;
0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0;
0,0,0,0,0,0,0,0,0,0,0,1,0,0,0,0,0,0,0,0,0,0,0,1,0,0,0,1,0,0,0,0,0,0,0,0;
0,0,0,0,0,0,0,0,0,0,0,1,0,0,0,0,0,0,0,0,0,0,0,1,0,0,0,1,0,0,0,0,0,0,1,0;
0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0;
0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,1,1,0,0,0,0,0,0,0,0,1,0,0,0;
0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0;
0,1,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,1,0,0,0,0,0,1,0,0,0;
0,0,0,0,0,0,1,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0;
0,0,0,0,0,0,0,0,0,0,0,1,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,1,1,0,0,0,0,0,1;
0,0,0,1,1,0,0,0,0,1,0,1,0,0,1,0,1,0,0,0,0,0,0,0,1,0,0,0,1,1,0,0,0,0,1,0];

graph=graph+graph';
graph=graph>0;
graph=graph*1;
n_neighbors = sum(graph);

random_walk = graph;
for k=1:min(size(graph))
    if n_neighbors(k)>0
        random_walk(k,:)=graph(k,:)/n_neighbors(k);
    end
end

J=min(size(graph));
init_dist(1:J) = 1/J;

figure
for j=1:3
    N=50*j;
    tic;
    ranks=ranks_by_probability_update(graph,N,init_dist');
```

```
    t=toc;
    subplot(3,1,j)
    bar(1:J,ranks)
    title(strcat('N=50*', string(j), ' Computation time is ', string(t), ' sec'))
end
```

problem4d_2.m

```
clc;close all;clear all;

% collaboration_matrix
graph=[0,0,0,0,0,0,0,0,1,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0;
0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,1,1,0,1,0,0,0,0,0,0,0,0,0,0,0;
0,0,0,0,0,0,0,0,0,0,0,0,0,1,1,0,0,0,0,0,0,0,0,1,0,0,0,0,0,0,0,0,0,0,0,0,0;
0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0;
1,0,0,1,0,0,0,0,0,1,0,0,1,0,0,0,0,0,0,1,0,0,0,0,0,0,0,0,0,1,0,0,0,0,0,1,1;
0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,1,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,1,0,0,0,0;
0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,1,0,0,0;
0,0,0,0,0,0,0,1,0,0,0,0,0,0,0,0,0,0,0,0,0,0,1,0,0,0,0,1,0,0,0,0,0,0,0,0,0;
1,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0;
0,0,0,1,1,0,0,0,0,0,0,0,0,0,0,0,1,1,0,1,0,0,0,0,0,0,1,0,0,0,0,0,0,0,0,0,1;
0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0;
0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,1,0,0,0,0,0,0,0,1,0,0,0,0,0,0,0,0,0;
0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0;
0,0,1,0,0,0,0,0,0,0,0,0,0,1,0,0,0,0,0,0,0,0,1,0,0,0,1,0,0,0,0,0,0,0,0,0,0;
0,0,1,0,0,0,0,0,0,0,0,0,1,1,0,0,0,0,0,0,0,0,1,0,0,0,0,0,0,0,0,0,0,0,0,0,0;
0,0,0,0,0,1,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0;
0,0,0,0,0,0,0,0,0,1,0,1,0,0,1,0,0,1,1,0,0,0,0,0,0,0,1,0,0,0,0,0,0,0,0,0,1;
0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0;
0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0;
0,0,0,0,0,0,0,0,0,0,0,1,0,0,0,0,1,0,0,0,0,0,0,0,0,0,1,0,0,0,0,0,0,0,0,0,0;
0,0,0,1,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0;
0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,1,1,0,0,0,0,0,0,0,0,1,0,0,1,0,0;
0,0,0,0,0,0,1,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,1,0,0,0,0,0;
0,0,0,0,0,0,1,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,1,0,0,0,0,0;
0,1,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0;
0,0,1,0,0,0,0,0,0,0,0,0,1,1,0,0,0,0,0,0,0,0,0,0,0,0,1,1,0,0,0,0,0,0,0,0,0;
0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,1,0,0,0,0,0,0,0,0,0;
0,0,0,0,0,0,0,0,0,0,0,1,0,0,0,0,1,0,1,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0;
0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0;
0,0,0,0,0,0,0,0,0,0,0,0,1,0,0,0,0,0,0,0,0,0,1,0,0,0,0,1,0,0,0,0,0,0,0,0,0;
0,0,0,0,0,0,0,0,0,0,0,0,0,1,0,0,0,0,0,0,0,0,1,0,0,0,1,0,0,0,0,0,0,1,0,0,0;
0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0;
0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,1,1,0,0,0,0,0,0,0,0,0,1,0,0,0;
0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0];
```

```
0,1,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,1,0,0,0,0,0,0,1,0,0,0;
0,0,0,0,0,0,1,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0;
0,0,0,0,0,0,0,0,0,0,0,1,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,1,1,0,0,0,0,0,0,1;
0,0,0,1,1,0,0,0,0,1,0,1,0,0,1,0,1,0,0,0,0,0,0,0,1,0,0,0,1,1,0,0,0,0,0,1,0];

graph=graph+graph';
graph=graph>0;
graph=graph*1;
n_neighbors = sum(graph);

random_walk = graph;
for k=1:min(size(graph))
    if n_neighbors(k)>0
        random_walk(k,:)=graph(k,:)/n_neighbors(k);
    end
end

% Add professor
J=min(size(graph));
graph=[graph ones(J,1);ones(1,J) 0];

J=min(size(graph));
init_dist(1:J) = 1/J;

figure
for j=1:3
    N=50*j;
    tic;
    ranks=ranks_by_probability_update(graph,N,init_dist');
    t=toc;
    subplot(3,1,j)
    bar(1:J,ranks)
    title(strcat('N=50*', string(j), ' Computation time is ', string(t), ' sec'))
end
```

problem4e.m

```
clc;close all;clear all;

% collaboration_matrix
graph=[0,0,0,0,0,0,0,0,1,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0;
0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,1,1,0,1,0,0,0,0,0,0,0,0,0,0,0;
0,0,0,0,0,0,0,0,0,0,0,0,0,1,1,0,0,0,0,0,0,0,0,1,0,0,0,0,0,0,0,0,0,0,0,0,0;
0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0;
```

```
1,0,0,1,0,0,0,0,0,1,0,0,1,0,0,0,0,0,0,1,0,0,0,0,0,0,0,0,0,0,1,0,0,0,0,0,1;
0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,1,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,1,0,0,0,0;
0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,1,0,0;
0,0,0,0,0,0,0,1,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,1,0,0,0,0,1,0,0,0,0,0,0,0;
1,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0;
0,0,0,1,1,0,0,0,0,0,0,0,0,0,0,1,1,0,1,0,0,0,0,0,1,0,0,0,0,0,0,0,0,0,1;
0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0;
0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,1,0,0,0,0,0,0,0,1,0,0,0,0,0,0,0,0,0;
0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0;
0,0,1,0,0,0,0,0,0,0,0,0,0,1,0,0,0,0,0,0,0,0,1,0,0,0,1,0,0,0,0,0,0,0,0;
0,0,1,0,0,0,0,0,0,0,0,0,1,1,0,0,0,0,0,0,0,0,1,0,0,0,0,0,0,0,0,0,0,0,0;
0,0,0,0,0,1,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0;
0,0,0,0,0,0,0,0,1,0,1,0,0,1,0,0,1,1,0,0,0,0,0,0,0,1,0,0,0,0,0,0,0,0,1;
0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0;
0,0,0,0,0,0,0,0,0,0,0,1,0,0,0,0,1,0,0,0,0,0,0,0,0,0,1,0,0,0,0,0,0,0,0;
0,0,0,1,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0;
0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,1,1,0,0,0,0,0,0,0,0,1,0,0,1,0,0;
0,0,0,0,0,0,1,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,1,0,0,0,0,0;
0,0,0,0,0,0,1,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0;
0,1,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0;
0,0,1,0,0,0,0,0,0,0,0,0,1,1,0,0,0,0,0,0,0,0,0,0,0,0,1,1,0,0,0,0,0,0,0;
0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,1,0,0,0,0,0,0,0,0,0,0;
0,0,0,0,0,0,0,0,0,0,1,0,0,0,0,1,0,1,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0;
0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0;
0,0,0,0,0,0,0,0,0,0,0,1,0,0,0,0,0,0,0,0,1,0,0,0,1,0,0,0,0,0,0,0,0,0,0;
0,0,0,0,0,0,0,0,0,0,0,0,1,0,0,0,0,0,0,0,0,1,0,0,0,1,0,0,0,0,0,0,1,0,0;
0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0;
0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,1,1,0,0,0,0,0,0,0,0,0,1,0,0,0;
0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0;
0,1,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,1,0,0,0,0,0,1,0,0,0;
0,0,0,0,0,0,1,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0;
0,0,0,0,0,0,0,0,0,0,0,1,0,0,0,0,0,0,0,0,0,0,0,0,0,0,0,1,1,0,0,0,0,0,1;
0,0,0,1,1,0,0,0,0,1,0,1,0,0,1,0,1,0,0,0,0,0,0,0,1,0,0,0,1,1,0,0,0,0,1,0];

graph=graph+graph';
graph=graph>0;
graph=graph*1;
n_neighbors = sum(graph);

random_walk = graph;
for k=1:min(size(graph))
    if n_neighbors(k)>0
        random_walk(k,:)=graph(k,:)/n_neighbors(k);
    end
end
```

```
% Add professor
J=min(size(graph));
graph=[graph ones(J,1);ones(1,J) 0];

J=min(size(graph));

nr_neighbors = sum(graph);
transition_probabilities = graph;
for k=1:J
    if nr_neighbors(k)>0
        transition_probabilities(k,:)=graph(k,:)/nr_neighbors(k);
    else
        transition_probabilities(k,k)=1;
    end
end

figure
tic;
pi=null(eye(J)-transition_probabilities'); % Find vector in the nullspace
ranks=pi/sum(pi); % Normalize
t=toc;
% Plot results
bar(1:J,ranks)
title(strcat('Recasting as Linear system problem,', ' Computation time is ', string(t)
xlabel('states')
ylabel('ranks')
```