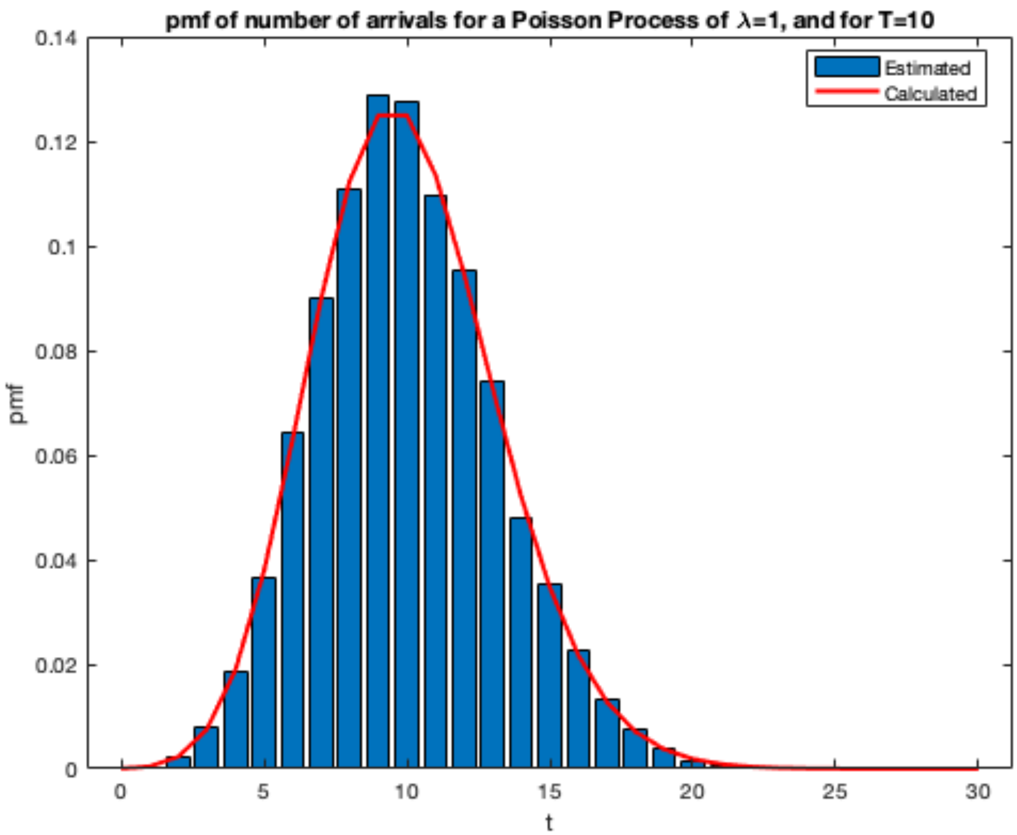
ECE 440, HW#7, Kefu Zhu

Question 7

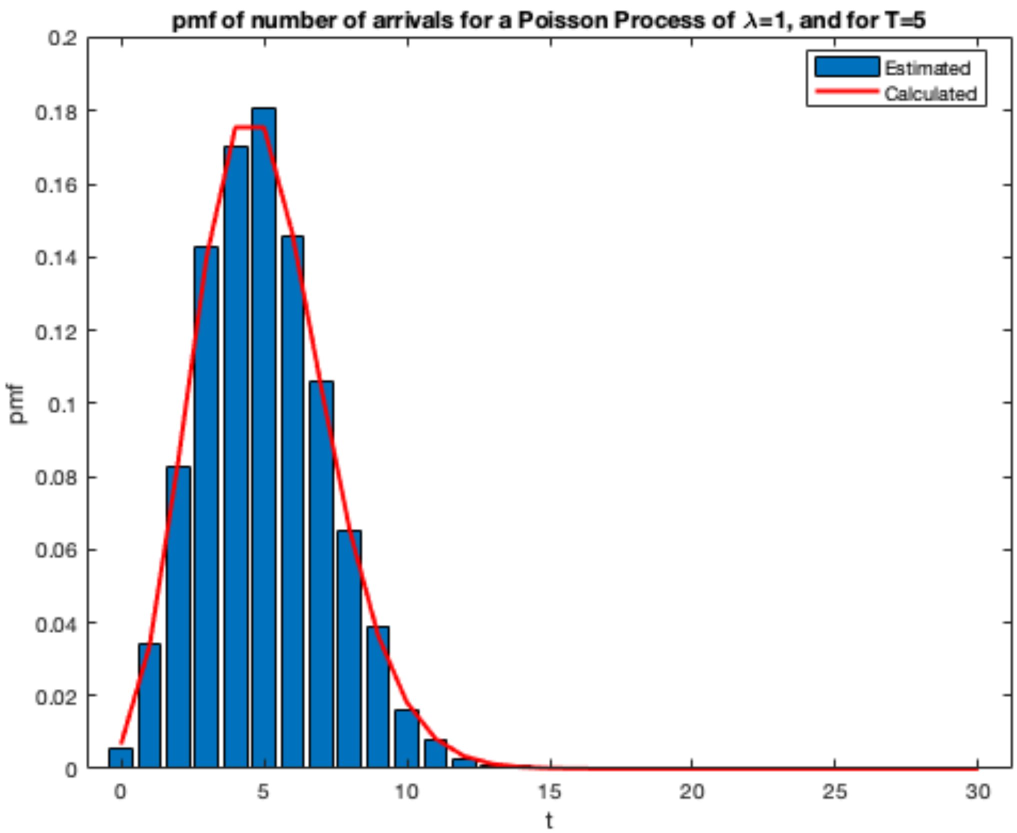
(a)

```
clc; clear all; close all

% Set parameters
T=10;
lambda= 1;
nr_experiments=10^4;
n=10^3;
h=T/n;
p = lambda*h;

% Generate arrivals for all times and experiments
arrival = binornd(1,p,n,nr_experiments);
```

```
% Compare with Poisson pmfs
x=0:30;
pdf_approx = hist(sum(arrival),x)/nr_experiments;
bar(x,pdf_approx)
hold on
plot(x,poisspdf(x,lambda*T),'r','Linewidth',2)
xlabel('t')
ylabel('pmf')
title('pmf of number of arrivals for a Poisson Process of \lambda=1, and for T=10')
legend('Estimated','Calculated','Location','Best')
```




```
figure
pdf_approx = hist(sum(arrival(1:n/2,:)),x)/nr_experiments;
bar(x,pdf_approx)
hold on
plot(x,poisspdf(x,lambda*T/2),'r','Linewidth',2)
xlabel('t')
ylabel('pmf')
title('pmf of number of arrivals for a Poisson Process of \lambda=1, and for T=5')
legend('Estimated','Calculated','Location','Best')
```



(b)

$N(t) = \sum_{i=1}^{t/h} N_i(h) = \sum_{i=1}^{n} N_i(h)$, where $N_i(h) \sim Bernoulli(\lambda h)$

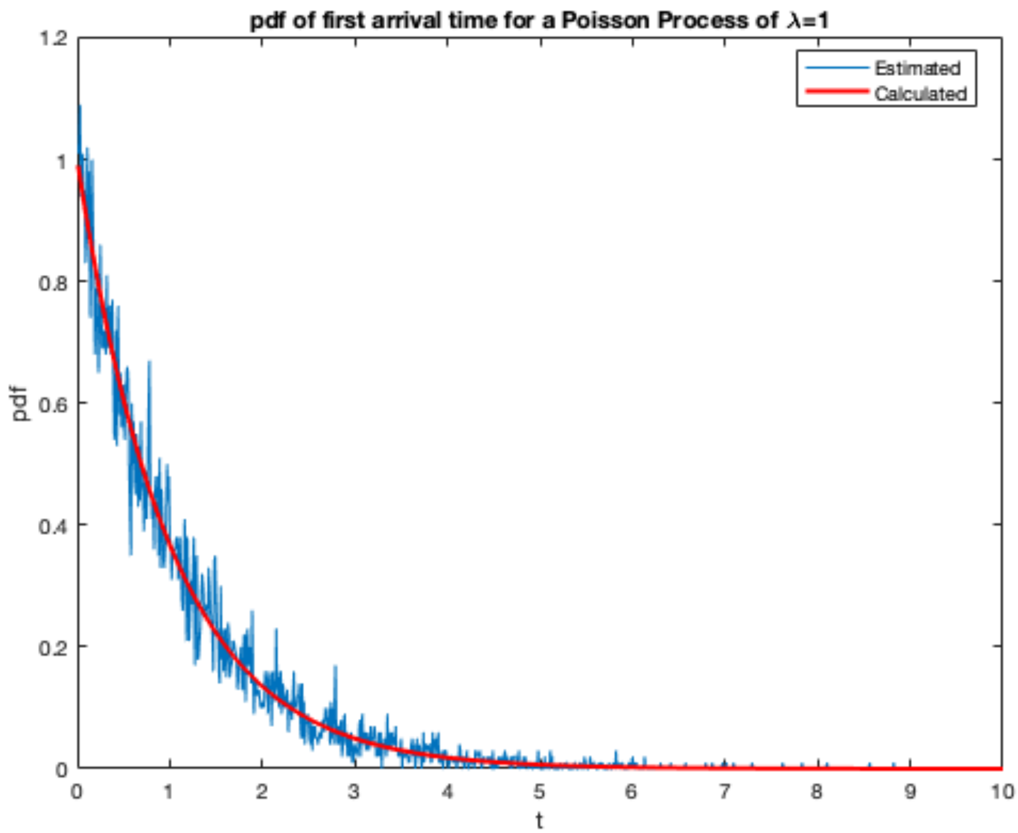
Because the sum of n i.i.d Bernoulli RV with parameter p equals to Binomial RV with parameter np, therefore $N(t) \sim Binomial(\lambda t)$ $(np = \frac{\lambda}{h} \cdot \lambda h = \lambda t)$

By law of rare events, because $lim_{n \to \infty} p = 0$ and np still equals to λh, $N(t) \sim Poisson(\lambda t)$

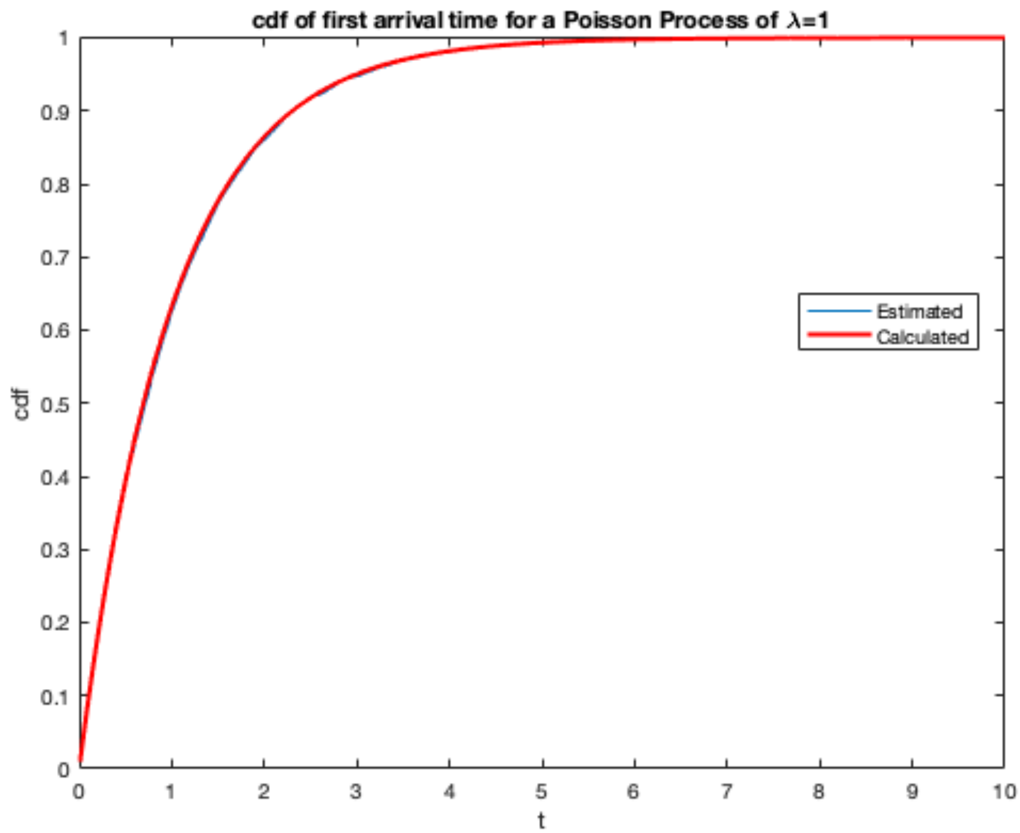
(c)

```
clc; clear all; close all;

% Set parameters
T=10;
lambda= 1;
nr_experiments=10^4;
n=1000;
h=T/n;
p = lambda*h;

% Generate arrivals for all times and experiments
arrival = binornd(1,p,n,nr_experiments);

% Compute time of first arrival
time=0;
experiment=1;
time_histogram = zeros(n,1);
while (experiment <= nr_experiments) && (time < n)
    time = time+1;
    if arrival(time, experiment)
        time_histogram(time)=time_histogram(time)+1;
        experiment = experiment+1;
        time=0;
    end
end
```

```
%Compare with exponential pdf
figure
plot((1:n)*h,time_histogram/nr_experiments/h)
hold on
plot((1:n)*h,exppdf((1:n)*h,lambda),'r','Linewidth', 2)
xlabel('t')
ylabel('pdf')
title('pdf of first arrival time for a Poisson Process of \lambda=1')
legend('Estimated','Calculated','Location','Best')
```




```
%Compare with exponential cdf
figure
plot((1:n)*h,cumsum(time_histogram/nr_experiments))
hold on
plot((1:n)*h,expcdf((1:n)*h,lambda),'r','Linewidth', 2)
xlabel('t')
ylabel('cdf')
title('cdf of first arrival time for a Poisson Process of \lambda=1')
legend('Estimated','Calculated','Location','Best')
```



(d)

We have $S_1 > t$ if and only if there are no arrivals by time $t \iff P(S_1 > t) = P(N(t) = 0)$. As shown in part B, $N(t) \sim Poisson(\lambda t)$, which has pdf of this form

$$P(N(t) = k) = e^{-\lambda t} \frac{(\lambda t)^k}{k!}$$

Plug in $k = 0$, we then have $P(S_1 > t) = P(N(t) = 0) = e^{-\lambda t}$, which is the basically saying the first arrival time follows $Exp(\lambda)$