CSC 282 - Fall 2014

http://www.cs.rochester.edu/~stefanko/Teaching/14CS282/

Name:

problem 1
problem 2
problem 3
problem 4
TOTAL (non-bonus)
problem 5 (bonus)

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1. (10 POINTS) In the KNAPSACK PROBLEM we have n items. The weight of the i-th item is $W[i]$ and the value of the i-th item is $V[i]$. Assume that the $V[i]$'s are integers and the $W[i]$'s are real numbers. Let C be the weight-capacity of the knapsack, and let M be the sum of the $V[i]$, that is, $M = \sum_{i=1}^{n} V[i]$. We would like to find a subset of the items with the maximal value where the total weight of the subset can be at most C.

   We will compute an array $K[0..M, 0..n]$, where entry $K[x, i]$ will be the minimal weight of a subset of items $1, 2, \ldots, i$ with total value greater than or equal to x. Give an expression (or a piece of code) for $K[x, i]$ in terms of some of the $K[?, i-1]$ (the question mark should be replaced by the appropriate expressions).

2. (20 POINTS) In the KNAPSACK PROBLEM REVISITED we have n items. The weight of the i-th item is $W[i]$, the value of the i-th item is $V[i]$, and the volume of the i-th item is $B[i]$. Assume that the $W[i]$'s and $B[i]$'s are integers and $V[i]$ are real numbers. Let C be the weight-capacity of the knapsack, and let D be the volume-capacity of the knapsack. We would like to find a subset of the items with maximal value where the total weight of the subset can be at most C and the total volume of the subset can be at most D.

   We will compute an array $K[0..C, 0..D, 0..n]$, where entry $K[x, y, i]$ will be the maximal value of a subset of items $1, 2, \ldots, i$ with total weight at most x and total volume at most y. Give an expression (or a piece of code) for $K[x, y, i]$ in terms of some of the $K[?, ?, i-1]$ (the question marks should be replaced by the appropriate expressions).

3. (20 POINTS) Let $a_1, \ldots, a_n$ be a sequence of numbers. We want to find the increasing subsequence of $a_1, \ldots, a_n$ with the largest sum. (For example if the input is $11, 1, 2, 3, 4, 12$ then the output is $11, 12$, a subsequence with sum 23.) We will compute a table $T[0...n]$ where $T[i]$ is the maximum sum of an increasing subsequence ending with a_i. Give an expression (or a piece of code) for $T[i]$ in terms of $a_1, \ldots, a_i$ and $T[0], T[1], \ldots, T[i-1]$.

4. (20 POINTS) We are given n positive numbers $a_1, \ldots, a_n$ and a number $k \in \{1, \ldots, n\}$. The goal is to select a subset S of the numbers with the maximal sum and such that 1) no three consecutive numbers are selected in S, AND 2) the size of S is k. We will compute a table $T[0..n, 0..k]$ where $T[x, y]$ is the maximum sum of a valid subset of $a_1, \ldots, a_x$ where the size of the subset is at most y. Give an expression (or a piece of code) for $T[x, y]$ in terms of previously computed values of T.

5. (20 BONUS POINTS) A sequence of numbers $b_1, \dots, b_k$ is convex if $2b_i \leq b_{i-1} + b_{i+1}$ for all $i \in \{2, \dots, k-1\}$. Given a sequence of numbers $a_1, \dots, a_n$ we want to find the longest convex subsequence of $a_1, \dots, a_n$. Describe the table (that is, the subproblems) and the update rule (that is, how you solve a subproblem using smaller subproblems).