CSC 449, HW#1, Kefu Zhu

Problem 3

1. Derive the form of the 3D structure tensor from the sum of squared dierences (SSD) error function

$E(u, v, w) = \sum_{x,y,t \in W} [I(x+u, y+v, t+w) - I(x, y, t)]^2$

$\approx_{Taylor\ approximation} \sum_{x,y,t \in W} [I(x, y, t) + u \cdot I_x + v \cdot I_y + w \cdot I_t - I(x, y, t)]^2$

$\approx \sum_{x,y,t \in W} [u \cdot I_x + v \cdot I_y + w \cdot I_t]^2$

$\approx \sum_{x,y,t \in W} [(u\ v\ w) \cdot \begin{pmatrix} I_x \\ I_y \\ I_t \end{pmatrix}]^2$

$\approx \sum_{x,y,t \in W} (u\ v\ w) \cdot \begin{pmatrix} I_x \\ I_y \\ I_t \end{pmatrix} \cdot (I_x\ I_y\ I_t) \cdot \begin{pmatrix} u \\ v \\ w \end{pmatrix}$

$\approx (u\ v\ w) \cdot M \cdot \begin{pmatrix} u \\ v \\ w \end{pmatrix}$

where $M = \begin{pmatrix} \sum I_x^2 & \sum I_x I_y & \sum I_x I_t \\ \sum I_x I_y & \sum I_y^2 & \sum I_y I_t \\ \sum I_x I_t & \sum I_y I_t & \sum I_t^2 \end{pmatrix}$

Define the eigenvalues of the M matrix to be $\lambda_1, \lambda_2, \lambda_3$, where λ_1, λ_2 represent change in horizontal and vertical directions, and λ_3 represent change in time

The criterion to extract "3D corners" is to have large values for all $\lambda_1, \lambda_2, \lambda_3$

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The variation among three eigenvalues $\lambda_1, \lambda_2, \lambda_3$ can be summarized as below

- Both λ_1, λ_2 are small $\rightarrow$ flat region
  - λ_3 is small, does not change with time
  - λ_3 is large, changes with time
- (small λ_1 , large λ_2) or (large λ_1 , small λ_2) $\rightarrow$ edge
  - λ_3 is small, does not change with time
  - λ_3 is large, changes with time
- Both λ_1, λ_2 are large $\rightarrow$ corner
  - λ_3 is small, does not change with time
  - λ_3 is large, changes with time