# CSC 261/461
# Database Systems

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# Functional Dependencies

## Functional Dependency

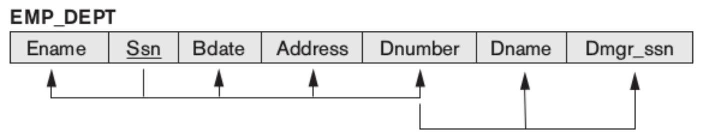
- $X \rightarrow Y$ holds if whenever two tuples have the same value for X, they must have the same value for Y
- For any two tuples t1 and t2 in any relation instance r(R): If $t1[X] = t2[X]$, then $t1[Y] = t2[Y]$
- $X \rightarrow Y$ in $R$ specifies a constraint on all relation instances $r(R)$
- FDs are derived from the real-world constraints on the attributes

# Functional Dependencies

## Inferred FDs

If we denote by $F$ the set of FDs that are specified on $R$.

- An FD $X \rightarrow Y$ is `inferred` from a set of dependencies $F$ specified on $R$ if $X \rightarrow Y$ holds in every legal relation state $r$ of $R$.
- Given a set of FDs $F$, we can infer additional FDs that hold whenever the FDs in $F$ hold.



# Functional Dependencies

## Armstrong's inference rules

- IR1. (Reflexive) If $Y \subseteq X$, then $X \rightarrow Y$
- IR2. (Augmentation) If $X \rightarrow Y$, then $XZ \rightarrow YZ$
- IR3. (Transitive) If $X \rightarrow Y$ and $Y \rightarrow Z$, then $X \rightarrow Z$
- IR1, IR2, IR3 form a `sound` and `complete` set of inference rules

# Functional Dependencies

## Other Inference Rules

- Decomposition: If $X \rightarrow YZ$, then $X \rightarrow Y$ and $X \rightarrow Z$
- Union: If $X \rightarrow Y$ and $X \rightarrow Z$, then $X \rightarrow YZ$
- Pseudotransitivity: If $X \rightarrow Y$ and $WY \rightarrow Z$, then $WX \rightarrow Z$
- The last three inference rules, as well as any other inference rules, can be deduced from IR1, IR2, and IR3 (completeness property)

# Functional Dependencies

- Closure of a set $F$ of FDs is the set $F^+$ of all FDs that can be inferred from $F$
- Closure of a set of attributes $X$ with respect to $F$ is the set $X^+$ of all attributes that are functionally determined by $X$
- $X^+$ can be calculated by repeatedly applying IR1, IR2, IR3 using the FDs in F.

# Functional Dependencies

## Algorithm for $X^+$

- **Input**: A set $F$ of FDs on a relation schema $R$, and a set of attributes $X$, which is a subset of $R$.

  $X^+ := X$
  repeat
  $\quad oldX^+ := X^+$
  for each functional dependency $Y \to Z$ in $F$ do
  $\quad$ if $Y \subseteq X^+$ then $X^+ := X^+ \cup Z$
  until $(X^+ = oldX^+)$;

# Functional Dependencies

## Example

Consider the following relation schema about classes held at a university in a given academic year.

- CLASS ( Classid, Course#, Instr_name, Credit_hrs, Text, Publisher, Classroom, Capacity).
- Let F, the set of functional dependencies for the above relation include:
  1. FD1: Classid $\rightarrow$ {Course#, Instr_name, Credit_hrs, Text, Publisher, Classroom, Capacity}
  2. FD2: Course# $\rightarrow$ Credit_hrs
  3. FD3: {Course#, Instr_name} $\rightarrow$ {Text, Classroom}
  4. FD4: Text $\rightarrow$ Publisher
  5. FD5: Classroom $\rightarrow$ Capacity

# Functional Dependencies

## Equivalent Sets

Two sets of FDs F and G are `equivalent` if:

- Every FD in F can be inferred from G, and
- Every FD in G can be inferred from F

Hence, F and G are equivalent if $F^+ = G^+$

Covers:

- F `covers` G if every FD in G can be inferred from F (if $G^+ \subseteq F^+$)
- F and G are equivalent if F covers G and G covers F

# Functional Dependencies

## Minimal Set of FDs

- A set of FDs is minimal if it satisfies the following conditions
  - Every dependency in F has a single attribute for its RHS.
  - We cannot remove any dependency from F and have a set of dependencies that is equivalent to F.
  - We cannot replace any FD $X \rightarrow A$ in F with a dependency $Y \rightarrow A$, where $Y \subset X$ and still have a set of FDs that is equivalent to F.

# Functional Dependencies

## Minimal Set of FDs

- A set of FDs is minimal if it satisfies the following conditions
  - A minimal set of FDs is a set of FDs in a standard or `canonical form` with no redundancies.
  - Condition 1 just represents every FD in a canonical form with a single attribute on the RHS.
  - Conditions 2 and 3 ensure there are no redundancies in the FDs either by having redundant attributes on the LHS of a dependency (Condition 2) or by having a dependency that can be inferred from the remaining FDs in F (Condition 3).
- A `minimal cover` of a set of FDs E is a minimal set of FDs that is equivalent to E.