Midterm Practice Problems 2 - CSC 262 - Feb 2015

1. A random variable X possesses the following density function for some constant c:

$$f_X(x) = \begin{cases} c(1-x/2) & ; \quad x \in [0,2] \\ 0 & ; \quad ow \end{cases} .$$

   (a) Determine c.

   (b) Determine the CDF of X.

   (c) Determine $E[X]$.

2. Three people have same winning number for a lottery. They decide, rather than to share the prize, to play a winner-take-all game. Each tosses a coin. If one outcome is different from the other two, that person wins the entire prize. If all three outcomes are the same, the toss is repeated, until a winner is determined. Let Y be the number of tosses needed to determine the winner.

   (a) Suppose each coin has the same probability p of landing Heads. Derive an expression for $P(Y \leq n)$ as a function of n and p.

   (b) Suppose $p = 1/2$. What is the minimum value of n for which the probability that the game requires no more than n tosses is at least 0.95?

3. Suppose $X \sim unif(-1/2, 1/2)$. Determine $E[|X|]$ and $var[|X|]$, where $|X|$ is the absolute value of X.

4. A box contains 7 red and 13 blue balls. Two balls are selected at random and discarded without being seen. A third ball is selected and is observed to be red. Determine the probability that the first two balls were blue, given that the third ball was red.

5. A test for Hepatitis-C is developed. The test is administered to a test group of 47 individuals known to have Hepatitis-C. Of this group 45 test positive. The test is also administered to a control group of 120 subjects known to be free of Hepatitis-C. Of these, 5 test positive. Suppose that the test is used clinically in a population estimated to have an incidence of Hepatitis-C of 0.4%. Estimate the sensitivity, specificity, positive predictive value (PPV) and negative predictive value (NPV).

6. A genotype found in a blood sample has a population frequency of 1/1000. A genotype from a suspect is extracted. Suppose that due to imperfect typing procedures if the suspect's blood is the same as that of the sample then the probability that the genotype is incorrectly typed is 0.01. Assume that this will be the only type of error. Let A be the event that the suspect's blood is the same as that of the sample, and let E be the event that the suspect's observed genotype matches that of the sample. Give the posterior odds of A given evidence of the form E in terms of the prior odds of A. Give the posterior odds of A^c given evidence of the form E^c in terms of the prior odds of A^c.

7. A random sample of size $n = 10$ from a normally distributed population with mean 101.2 and standard deviation 2.4 is collected. What is the probability that the sample mean is within 1 of the true mean?

8. Suppose for a certain test for HIV status the probability of testing positive for an HIV-positive subject is 0.98, and that the probability of testing positive for an HIV-negative subject is 0.07. In Canada, an estimate of the proportion of the population with HIV (in 1992) is 0.00229. Give the PPV and NPV.

9. The wheel of a car in a certain parking spot is marked with chalk. Suppose that the position of the chalk mark is recorded as a number from 1 to 12, corresponding to the face of a clock. It is noted that 3 hours later (in a parking zone with a 2 hour limit) the same car occupies the parking spot. Let

$$\begin{aligned} A &= \{ \text{ The car has remained stationary for at least 3 hours } \} \\ E &= \{ \text{ Chalk mark position has remained the same } \} \end{aligned}$$

   Give $Odds(A \mid E)$ and $Odds(A \mid E^c)$. Assume that if the car has been driven, the chalk mark is equally likely to be in any of the 12 positions when it returns to the parking space.