ECE 440 HW#3, Kefu Zhu, kzhu6

Question 7

(a)

accept_Offer.m

```
function [acceptedX,n] = accept_Offer(J,K,L)
    % Create a list of random permutation of numbers from 1 to J
    offer = randperm(J);
    % The first Kth offers are rejected
    rejectOffer = offer(1:K);
    % sort the rejectOffer in ascending order
    rejectOffer_sorted = sort(rejectOffer);
    % Select the L-th best offer
    bestL = rejectOffer_sorted(L);
    for i = K+1:J
        % If found Xi such that Xi < X0, end the loop
        if (offer(i) < bestL)
            % Record the accepted offer Xi and the value of i
            acceptedX = offer(i);
            n = i;
            return;
        end
    end
    % If the loop did not end early, I will accept the last offer J
    acceptedX = offer(J);
    n = J;
end
```

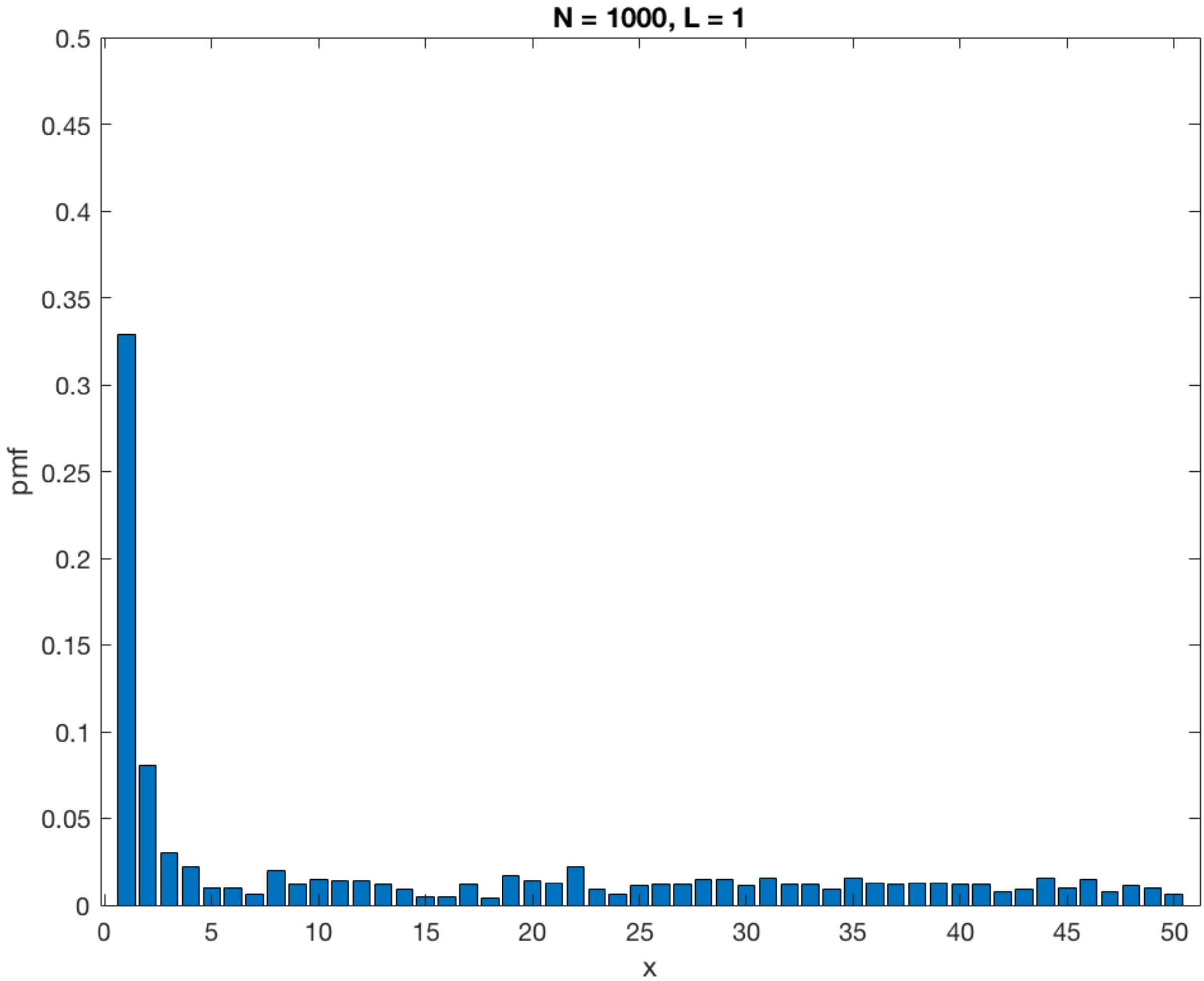
(b)

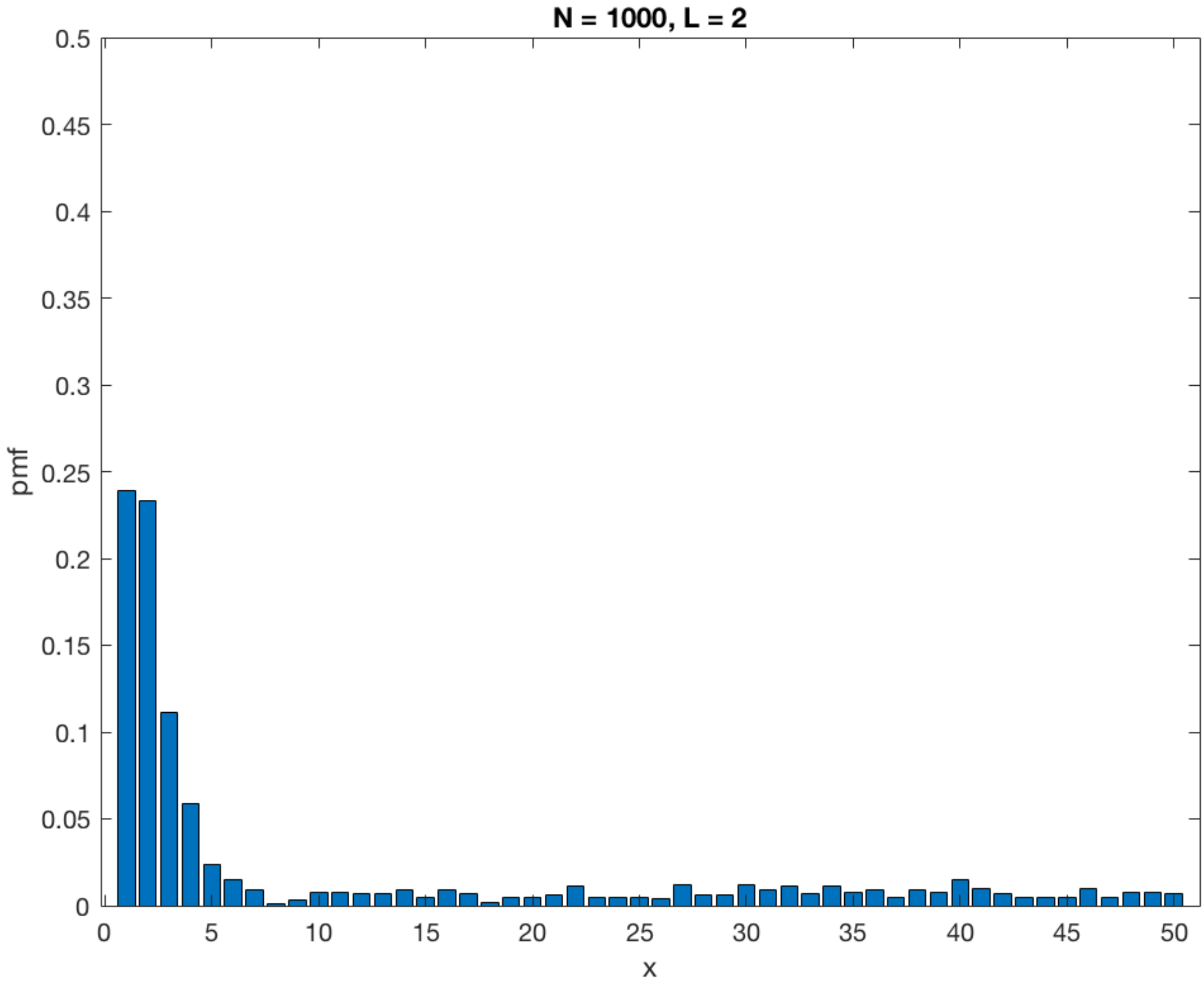
pmf_of_rank.m

```
% Plot the pmf of ranks
function [] = pmf_of_rank(J,K,L,N)
    % Initialization
    accepted_rank = zeros(1,N);
    % Record the accepted rank value, the return time N is useless here
    for i=1:N
        [accepted_rank(i), tmp] = accept_Offer(J,K,L);
    end


    [freq, bin] = hist(accepted_rank,J);
    pmfList=freq/N;
    bar(1:J,pmfList);
    xlabel('x');ylabel('pmf');title('N = ' + string(N) + ', L = ' + string(L)); ylin
end
```

problem7b.m

```
clear all;

% Set the value of J, K, L, N
J=50; K=30; L=1; N = 1000;

figure()
pmf_of_rank(J,K,L,N)

% Set the value of J, K, L
J=50; K=30; L=2; N = 1000;

figure()
pmf_of_rank(J,K,L,N)
```


N = 1000, L = 1
pmf
x
0.5
0.45
0.4
0.35
0.3
0.25
0.2
0.15
0.1
0.05
0
0
5
10
15
20
25
30
35
40
45
50



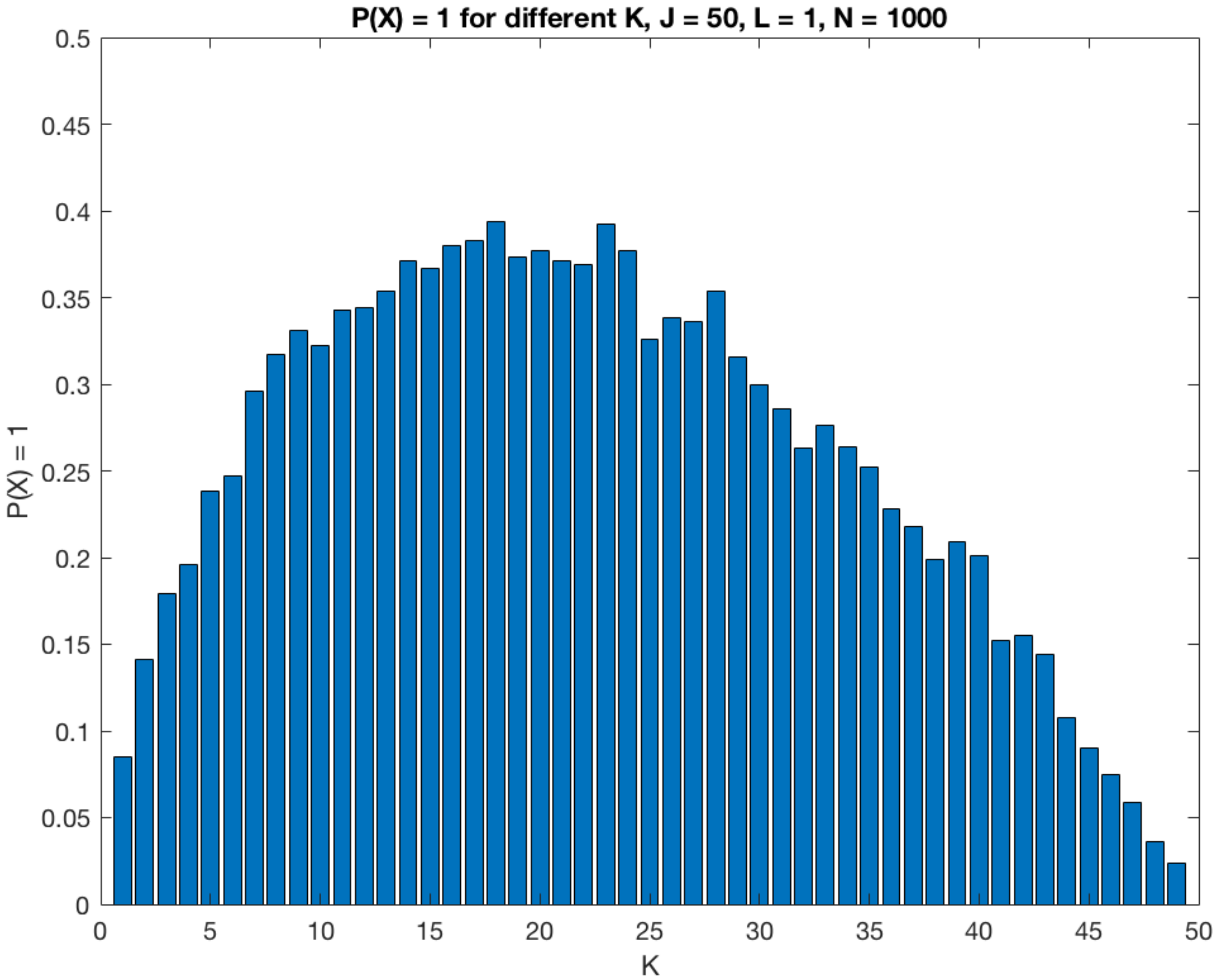
(c)

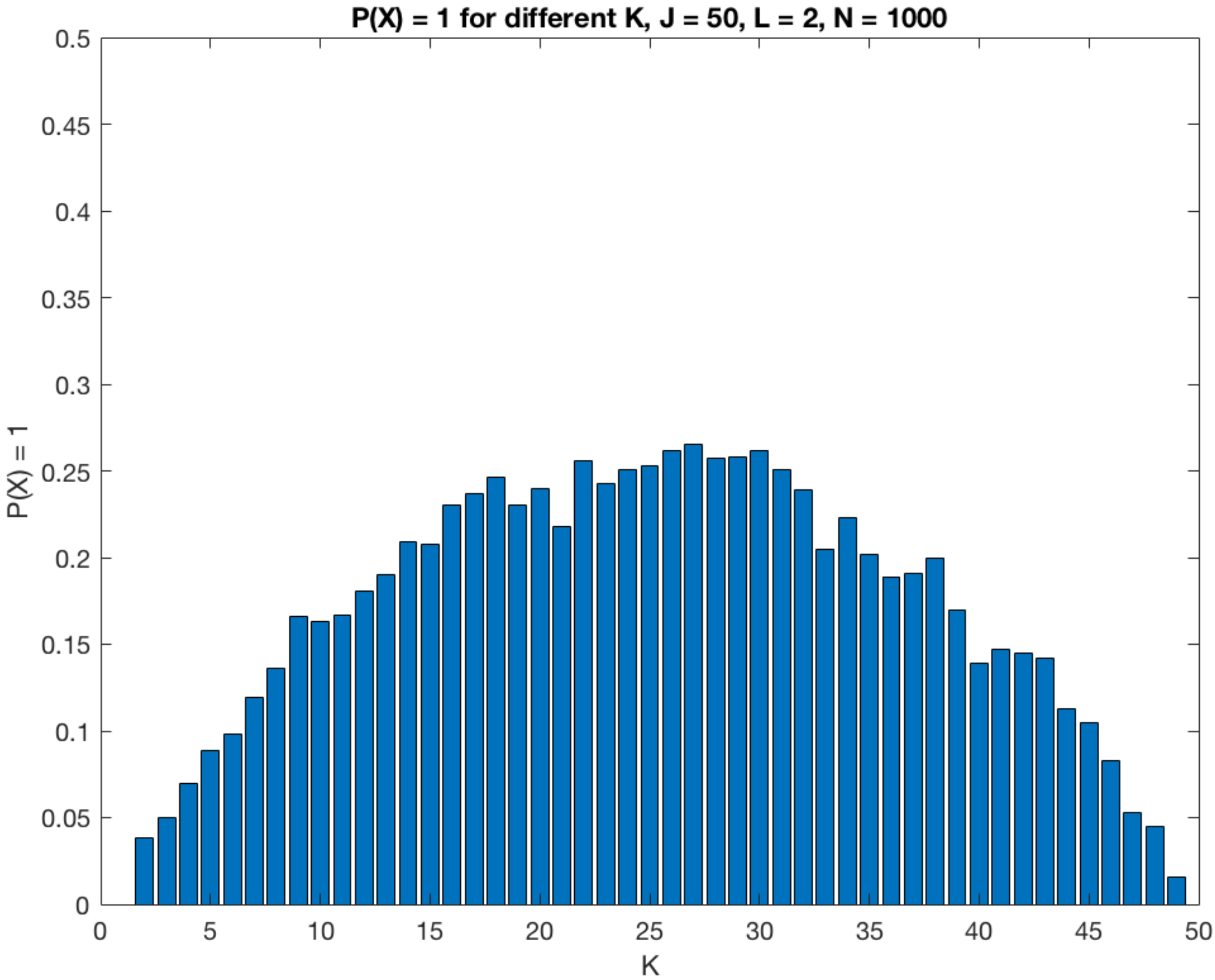
prob_1_versus_K.m

```
function []=prob_1_versus_K(J,L)
    N=1000;
    kList = L:J-1;
    prob_of_rank_1 = zeros(1,J-L);
    kIndex = 1;
    for K=kList
        % Initialization
        accepted_rank = zeros(1,N);
        % Record the accepted rank value, the return time N is useless here
        for i=1:N
            [accepte_rank(i),tmp] = accept_Offer(J,K,L);
        end
        [freq, bin] = hist(accepte_rank,J);
        pmf_vector = freq/N;
        prob_of_rank_1(1,kIndex) = pmf_vector(1,1);
        kIndex = kIndex + 1;
    end
    % Plot histogram
    bar(kList,prob_of_rank_1)
    xlabel('K');ylabel('P(X) = 1');
    title('P(X) = 1 for different K, J = ' + string(J) + ', L = ' + string(L) + ', N
    axis([0,J,0,0.5])
end
```

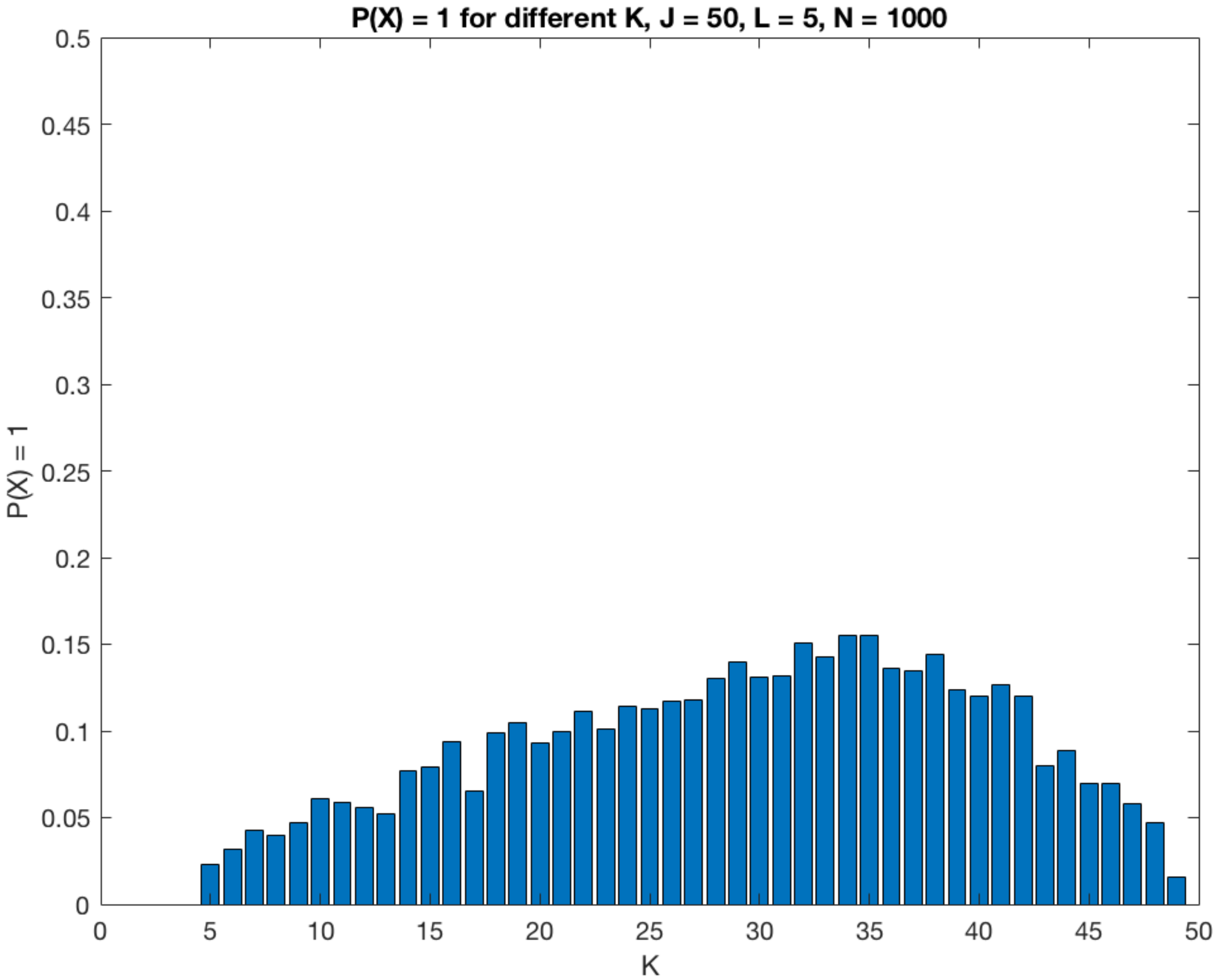
problem7c.m

```
clear all;
% fix J
J=50;
for L=[1 2 5]
    figure()
    prob_1_versus_K(J,L)
end
```


P(X) = 1 for different K, J = 50, L = 1, N = 1000
P(X) = 1
K
0
0.05
0.1
0.15
0.2
0.25
0.3
0.35
0.4
0.45
0.5
5
10
15
20
25
30
35
40
45
50



P(X) = 1 for different K, J = 50, L = 2, N = 1000
P(X) = 1
K
0
0.05
0.1
0.15
0.2
0.25
0.3
0.35
0.4
0.45
0.5
0
5
10
15
20
25
30
35
40
45
50



(d)

As defined in the problem, $P[X = X_J]$ can be represented by the following

$P[X = X_J] = P[X_0 = 1 \cup X_0 = 2 \cap X_J = 1] = P[X_0 = 1] + P[X_0 = 2 \cap X_J = 1]$

$\because P[X_0 = 1] = \frac{K}{J}$

$\because P[X_0 = 2 \cap X_J = 1] = P[X_0 = 2, X_J = 1]$

$= P[X_0 = 2 | X_J = 1] P[X_J = 1]$

$= \frac{K}{J-1} \cdot \frac{1}{J}$

$\therefore P[X = X_J] = \frac{K}{J} + \frac{K}{J-1} \cdot \frac{1}{J}$

(e)

$P[X=1]=\sum_{n=1}^{J} P[X=1|X_n=1]P[X_n=1]=\sum_{n=1}^{J} P[X=1|X_n=1]\cdot\frac{1}{J}$

$\because$ If the first offer locates among the first K offers, it would note be accepted (Defined in the question)

$\therefore P[X=1|X_n=1]=\begin{cases}0,\ 1\le n\le K\\ P[X=1|X_n=1],\ n\ge K+1\end{cases}$

$\therefore P[X=1]=\frac{1}{J}\cdot\sum_{n=K+1}^{J} P[X=1|X_n=1]$

$\because X_n=1$ can only happens if the previous $K+1,...,n-1$ offers are all smaller than X_0, which happens with probability $\frac{K}{n-1}$

$\therefore P[X=1]=\frac{1}{J}\cdot\sum_{n=K+1}^{J}\frac{K}{n-1}=\frac{K}{J}\cdot\sum_{n=K+1}^{J}\frac{1}{n-1}$

(f)

$\sum_{n=K+1}^{J}\frac{1}{n-1}\approx\int_{K}^{J-1}\frac{1}{x}\mathrm{d}x=ln\frac{J-1}{K}$

Because in our problem set, J is far greater than 1, therefore $ln\frac{J-1}{K}\approx ln\frac{J}{K}$

So we now have $P[X=1]=\frac{K}{J}\cdot ln\frac{J}{K}$

Take the derivate respect to K to find the maximum value of K:

$\frac{\mathrm{d}}{\mathrm{d}K}P[X=1]\approx\frac{K}{J}\cdot ln(\frac{J}{K})\frac{\mathrm{d}}{\mathrm{d}K}$

$=\frac{1}{J}\cdot ln(\frac{J}{K})-\frac{K}{J}\cdot(\frac{K}{J}\cdot\frac{J}{K^2})$

$=\frac{1}{J}\cdot ln(\frac{J}{K})-\frac{1}{J}=0$

$\Rightarrow K=\frac{J}{e}$

Take $K=\frac{J}{e}$ back to the equation:

$P[X=1]=\frac{K}{J}\cdot ln\frac{J}{K}=\frac{1}{e}\cdot 1\approx 0.37$