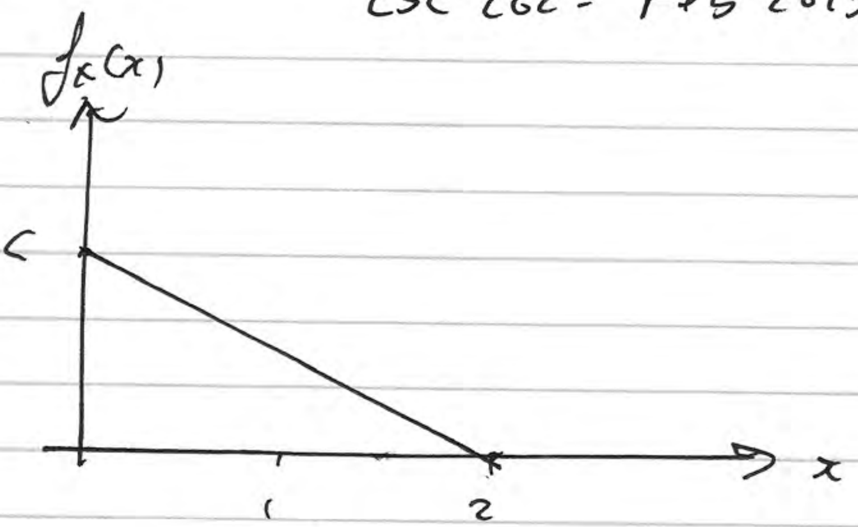
# MIDTERM PRACTICE PROBLEMS

CSC 262 - Feb 2015

Q 1




$$f_X(x) = \begin{cases} \frac{c(1-x)}{2} & x \in [0,2] \\ 0 & \text{ow}. \end{cases}$$

a) for triangle $A = \frac{1}{2}$ Base × Height.

$$\int f_X(x)\,dx = 1 = \frac{1}{2}\, c \times 2$$

$\therefore\; c = 1.$

b) $F(x) = 0 \quad , x \leq 0$

$F(x) = 1 \quad , x \geq 2$.

for $x \in (0, 2)$

$$F(x) = \int_0^x 1 - u/2 \, du = u - u^2/4 \Big|_0^x$$

$$= x - x^2/4.$$

$$F(x) = \begin{cases} 0 & ; x \leq 0 \\ x - x^2/4 & ; x \in (0,2) \\ 1 & ; x \geq 2. \end{cases}$$

c) $$E[X] = \int_{-\infty}^{\infty} x f_X(x) \, dx$$

$$= \int_0^2 x(1 - x/2) \, dx$$

$$= x^2/2 - x^3/6 \Big|_0^2$$

$$= 4/2 - 8/6 = 2 - 4/3 = \boxed{2/3}$$

Q2 $X = \#H$, $X \sim bin(3, p)$.

a) A toss settles the matter if $X \in \{1,2\}$.

$$P(X \in \{1,2\}) = \binom{3}{1} p(1-p)^2 + \binom{3}{2} p^2(1-p)$$
$$= 3p(1-p)^2 + 3p^2(1-p)$$
$$= 3p(1-p).$$

$Y = \#$ tosses to reach decision.

$Y \sim geom(3p(1-p))$

$$P(Y \le n) = 1 - (1-3p(1-p))^n$$

b) Find minimum $n$ for which $P(Y \le n) \geq 0.95$

Equivalently, find minimum $n$ for which.

$$P(Y > n) \le 0.05$$
or $$(1-3pq)^n \le 0.05$$
or $$n \log 1/4 \le \log 0.05$$
or $$n \geq 2.16$$

$\therefore \boxed{n = 3}$

Q3. $f_X(x) = \begin{cases} 1 & ; x \in (-1/2, 1/2) \\ 0 & ; \text{o.w.} \end{cases}$

$$E[|X|] = \int_0^{1/2} x\,dx + \int_{-1/2}^{0} -x\,dx$$

$$= 2\int_0^{1/2} x\,dx \quad \text{by symmetry.}$$

$$= 2 \left.\frac{x^2}{2}\right|_0^{1/2} = 2 \cdot \tfrac{1}{8} = 1/4.$$

$$E(|X|^2) = 2\int_0^{1/2} x^2\,dx = 2 \left.\frac{x^3}{3}\right|_0^{1/2} = 2 \cdot \frac{1}{24} = \frac{1}{12}.$$

$$Var(|X|) = \frac{1}{12} - (1/4)^2 = \frac{1}{12} - \frac{1}{16}$$

$$= \frac{1}{48}.$$

Q4. $R = \{ 3^{rd}$ drawn ball is red $\}$

$BB = \{$discarded balls are both blue$\}$.

$$P(BB|R) = \frac{P(BB \cap R)}{P(R)}.$$

By the rule of product.

$$P(BB \cap R) = \frac{13}{20} \times \frac{12}{19} \times \frac{7}{18}$$

The $P(R) = \frac{7}{20}$

$$\therefore P(BB|R) = \frac{\frac{13}{20} \times \frac{12}{19} \cdot \frac{7}{18}}{7/20}$$

$$= \frac{13 \cdot 12}{19 \cdot 18} \approx 0.46.$$

[illegible] [illegible] [illegible] 2013

[illegible].

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| | Hep-C −ve | Hep-C +ve |
|---|---|---|
| +ve Test | 5 | 45 |
| −ve Test | 115 | 2 |

prev = 0.004

$$\text{sens} = P(\text{+ve Test} \mid \text{Hep-C +ve}) = \frac{45}{47}$$

$$\text{spec} = P(\text{−ve Test} \mid \text{Hep-C −ve}) = \frac{115}{120}$$

$$PPV = \frac{45/47 * .004}{45/47 * .004 + (1 - 115/120) * (1 - .004)}$$

$$= 0.085$$

$$NPV = \frac{115/120 * (1 - .004)}{115/120 * (1 - .004) + (1 - 45/47) * .004}$$

$$= 0.9998$$

Q 6

Ans: $A = \{\text{suspects blood same as sample}\}$

$E = \{\text{suspects genotype same as sample generally}\}$.

$$\text{odds}(A|E) = \frac{P(E|A)}{P(E|A^c)}\,\text{odds}(A)$$

$P(E|A) \approx 1 - 0.01$. [account for error].

$P(E|A^c) \approx 1/1000$ [ignore error].

$$\text{odds}(A|E) = \frac{0.99}{1/1000}\,\text{odds}(A)$$

$$= 990\ \text{odds}(A).$$

Then. $$\text{odds}(A^c|E^c) = \frac{P(E^c|A^c)}{P(E^c|A)}\,\text{odds}(A)$$

$$= \frac{1 - 1/1000}{0.01}\,\text{odds}(A^c)$$

$$= 99.9\ \text{odds}(A^c)$$

~~AB~~ If $\mu = 101.2$, $\sigma = 2.4$, $n = 10$.

Q7 $\bar{X} \sim N(101.2, \frac{2.4^2}{10})$

$P(101.2 - 1 < \bar{X} < 101.2 + 1)$

$= P(-1 < \bar{X} - \mu < 1)$

$= P\left(\frac{-1}{2.4/\sqrt{10}} \leq Z \leq \frac{1}{2.4/\sqrt{10}}\right)$, $Z \sim N(0,1)$

$= 1 - 2F_Z(-1/2.4/\sqrt{10})$

$= 1 - 2F_Z(-1.318)$

$= 0.812$

Q8

$$\text{sens} = 0.98$$
$$\text{spec} = 1 - 0.07 = 0.93$$
$$\text{prev} = 0.00229.$$

$$PPV = \frac{\text{sens} * \text{prev}}{\text{sens} * \text{prev} + (1-\text{spec}) \cdot (1-\text{prev})}$$

$$= 0.031$$

$$NPV = \frac{\text{spec}\,(1-\text{prev})}{\text{spec}\,(1-\text{prev}) + \text{[illegible]}\ \text{prev}}$$

$$= 0.99995.$$

Q9

$$P(E|A) = 1$$
$$P(E|A^c) = 1/12$$

$$ODDS(A|E) = \frac{P(E|A)}{P(E|A^c)}\, ODDS(A)$$

$$= 12\ ODDS(A)$$

$$ODDS(A|E^c) = \frac{P(E^c|A)}{P(E^c|A^c)}\, ODDS(A)$$

$$= 0 * ODDS(A)$$

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