

Time independent Schrodinger Equation in one spatial Dimension.

$$\frac{2m}{\hbar}(V-E) \Psi = \frac{d^2}{dx^2} \Psi$$

for $0 < x < l \Rightarrow V = 0$; $(V - E) = -E$.

$$\frac{2m}{\hbar}(-E) \Psi = \frac{d^2}{dx^2} \Psi$$

Solution to this differential equation:

$$\Psi(x) = c_1 \sin\left(\frac{\sqrt{2mE}}{\hbar} x\right) + c_2 \cos\left(\frac{\sqrt{2mE}}{\hbar} x\right)$$

Boundary Conditions:

$$x = 0 \Rightarrow \Psi = 0; x = l \Rightarrow \Psi = 0$$

By substituting the boundary conditions:

$$\Psi(x) = c_1 \sin\left(\frac{n\pi}{l} x\right)$$

By the normalization Condition:

$$\int_0^l \left[c_1 \sin\left(\frac{n\pi}{l} x\right) \right]^2 dx = 1 \Rightarrow c_1 = \sqrt{\frac{2}{l}}$$

Solution of the schrodinger equation to the infinite square box problem:

$$\Psi(x) = \sqrt{\frac{2}{l}} \sin\left(\frac{n\pi}{l} x\right)$$