

Conference Paper Title*

*Note: Sub-titles are not captured in Xplore and should not be used

1st Cao Hoai Sang

dept. name of organization (of Aff.)

name of organization (of Aff.)

Ho Chi Minh City, Viet Nam

21522541@gm.uit.edu.vn

2nd Nguyen Tran Gia Kiet

dept. name of organization (of Aff.)

name of organization (of Aff.)

City, Country

email address or ORCID

3rd Thi Thành Công

dept. name of organization (of Aff.)

name of organization (of Aff.)

City, Country

email address or ORCID

4th Given Name Surname

dept. name of organization (of Aff.)

name of organization (of Aff.)

City, Country

email address or ORCID

5th Given Name Surname

dept. name of organization (of Aff.)

name of organization (of Aff.)

City, Country

email address or ORCID

6th Given Name Surname

dept. name of organization (of Aff.)

name of organization (of Aff.)

City, Country

email address or ORCID

Tóm tắt nội dung—This document is a model and instructions for L^AT_EX. This and the IEEEtran.cls file define the components of your paper [title, text, heads, etc.]. *CRITICAL: Do Not Use Symbols, Special Characters, Footnotes, or Math in Paper Title or Abstract.

Index Terms—component, formatting, style, styling, insert

I. INTRODUCTION

This document is a model and instructions for L^AT_EX. Please observe the conference page limits.

II. NGHIÊN CỨU LIÊN QUAN

A. Gauss-Newton nonlinear method

Vào năm 2015, ứng dụng dự báo phục hồi sau xuất viện của Phạm Thị Hương được sử dụng trong luận văn thạc sĩ khoa học [1]. Sử dụng phương pháp Gauss-Newton phi tuyến để ước lượng giá trị nhỏ nhất của bình phương sai số. [1]

III. TÀI NGUYÊN

IV. PHƯƠNG PHÁP LUẬN

A. Gauss newton method nonlinear

1) *Least Squares*: The distance between a fitted curve and an observation is called a residual, or an error.

$$\text{Residuals} = y_i - \hat{y}_i$$

The sum of squared errors is calculated by the following equation:

$$SSE = \sum_{i=1}^n (y_i - \hat{y}_i)^2$$

Where:

- y_i is observed values
- \hat{y}_i is fitted values

Identify applicable funding agency here. If none, delete this.

2) *Newton's method*: With the function $y = y_0 e^{-kt}$ we find the minimum of SSE. we find the k value with Newton's method

$$SSE = \sum_i^n (y_i - y_0 e^{-kt_i})^2$$

$$k_{new} = k_{old} - \frac{f'(k_{old})}{f''(k_{old})}$$

Or we can explain by Hessian matrix like this:

$$\begin{pmatrix} k_{new} \\ y_{0,new} \end{pmatrix} = \begin{pmatrix} k_{old} \\ y_{0,old} \end{pmatrix} - H^{-1}G$$

Where:

$$H = \begin{bmatrix} \frac{\partial^2 f}{\partial k^2} & \frac{\partial^2 f}{\partial k \partial y_0} \\ \frac{\partial^2 f}{\partial y_0 \partial k} & \frac{\partial^2 f}{\partial y_0^2} \end{bmatrix} \quad G = \begin{bmatrix} \frac{\partial f}{\partial k} \\ \frac{\partial f}{\partial y_0} \end{bmatrix}$$

The problem with Neuton's method in nonlinear regression is the Hessian and its inverse are problematic to calculate. To solve this problem, the Gauss-Newton method instead approximates the Hessian.

We can rewrite the SSE like this:

$$SSE = \sum_i^n r_i^2 = r^T r$$

Where:

- r represents a vector with the residuals

3) *Gauss Newton method*: We differentiate SSE respect to the parameters in the model with the chain rule, we obtain the following equation.

$$\frac{\partial SSE}{\partial \beta_j} = 2 \sum_i^n r_i \cdot \frac{\partial r_i}{\partial \beta_j}$$

Then, remove two number because it will not affect the estimation of the parameters. Corresponds to the Jacobian matrix.

$$J_r = \begin{bmatrix} \frac{\partial r_1}{\partial \beta_1} & \frac{\partial r_1}{\partial \beta_2} \\ \frac{\partial r_2}{\partial \beta_1} & \frac{\partial r_2}{\partial \beta_2} \\ \vdots & \vdots \\ \frac{\partial r_n}{\partial \beta_1} & \frac{\partial r_n}{\partial \beta_2} \end{bmatrix}$$

With the following equation for the sum of squared errors (SSE):

$$SSE = \sum_{i=1}^n (y_i - y_0 e^{-kt_i})^2$$

We have:

$$\frac{\partial^2 SSE}{\partial \beta_j \partial \beta_k} = \sum_i^n \left(\frac{\partial r_i}{\partial \beta_j} \frac{\partial r_i}{\partial \beta_k} + r_i \frac{\partial^2 r_i}{\partial \beta_j \partial \beta_k} \right)$$

The main difference between Newton's method and Gauss-Newton is that the Gauss-Newton method neglects $r_i \frac{\partial^2 r_i}{\partial \beta_j \partial \beta_k}$. So that the second derivative is approximated by the following function

$$\frac{\partial^2 SSE}{\partial \beta_j \partial \beta_k} \approx \sum_i^n \left(\frac{\partial r_i}{\partial \beta_j} \frac{\partial r_i}{\partial \beta_k} \right) = J_r^T J_r$$

Using the following updating rule in Newton's method. For Gauss-Newton simply plug in the approximation for the Hessian matrix and for the gradient.

$$\begin{pmatrix} k_{\text{new}} \\ y_{0,\text{new}} \end{pmatrix} = \begin{pmatrix} k_{\text{old}} \\ y_{0,\text{old}} \end{pmatrix} - (J_r^T J_r)^{-1} J_r^T r$$

With β as a column vector with the parameters that are estimated. For a simple example where we only estimate two parameters, the equation looks like this:

$$\begin{aligned} \beta_{\text{new}} &= \beta_{\text{old}} - (J_r^T J_r)^{-1} J_r^T r(\beta_{\text{old}}) \\ \begin{pmatrix} k_{\text{new}} \\ y_{0,\text{new}} \end{pmatrix} &= \begin{pmatrix} k_{\text{old}} \\ y_{0,\text{old}} \end{pmatrix} - (J_r^T J_r)^{-1} J_r^T r \begin{pmatrix} k_{\text{old}} \\ y_{0,\text{old}} \end{pmatrix} \end{aligned}$$

TÀI LIỆU

- [1] P. T. Huong, "Linear regression, polynomial regression, and applications, master's thesis in science,"2015.