ENG1467 - OTIMIZAÇÃO - PUC-Rio, 2022.1

Lista 3: Simplex Parte 2

06/05/2022

Professor: Alexandre Street Aluno: Thiago Novaes

O código completo desenvolvido durante essa atividade pode ser encontrado em https://github.com/Thiago-NovaesB/MestradoPuc.jl/tree/main/Programa%C3%A7%C3%A3o%20Linear/Lista%203/Simplex

1 Implementação

Para a implementação do algoritmo, foi criado o pacote Simplex.jl:

```
module Simplex

include("types.jl")
include("utils.jl")
include("log.jl")
include("solver.jl")

export create, solve

end # module
```

O arquivo types.jl contém estruturas em julia para guardar as entradas e saídas:

```
struct Input
       A:: Matrix { }
       b:: Vector { }
       c:: Vector { }
       base :: Vector { Int }
       nbase :: Vector { Int }
       tol::Float64
       max_iter::Int
       verbose::Int
10 end
11
12 struct Output
       x:: Vector { Float64 }
13
       z::Float64
14
       termination status::Int
15
       base:: Vector { Int }
17
       nbase :: Vector { Int }
18 end
```

O arquivo *utils.jl* contém funções para tratar entradas e saídas:

```
function create (A:: Matrix {}, b:: Vector {}, c:: Vector {}, base:: Vector {Int},
                   nbase::Vector{Int}; tol::Float64 = 1E-6,
                   max_iter::Int = 1000, verbose::Int = 1)
          input = Simplex. Input (A, b, c, base, nbase, tol, max_iter, verbose)
      return input
6
  end
  function write_output(input::Input, termination_status::Int, d::Vector{})
      A = input.A
10
      b = input.b
11
      c = input.c
      base = input.base
      nbase = input.nbase
14
      B = view(A,:,base)
      x = B \setminus b
17
      z = c[base]'x
      x_{opt} = zeros(length(c))
19
      x_{opt}[base] = x
20
      if termination_status == 2
          output = Simplex.Output(d, Inf, termination_status, base, nbase)
          last_log(input, termination_status, base, nbase, z, d)
24
          output = Simplex.Output(x_opt, z, termination_status, base, nbase)
25
          last_log(input, termination_status, base, nbase, z, x_opt)
26
27
      end
      return output
28
29 end
```

O arquivo *log.jl* contém as funções para escrita de log:

```
function init_log(input::Simplex.Input)
      if input.verbose == 1
          var = length(input.c)
3
          con = length(input.b)
4
          println ("-----Inicio do algoritmo Simplex --
     )
          println ("O problema possui $var variaveis e $con restrições")
      end
 end
8
  function iteration_log(input::Simplex.Input, iter::Int, base::Vector{Int},
     nbase:: Vector{Int}, i::Int, j::Int, z::Float64, x::Vector{Float64},
     red_cost :: Vector { Float64 })
      if input. verbose == 1
11
          println("-----
                                     -----Iteração $iter
12
          println("Base: $base")
          println ("Não-Base: $nbase")
14
          println("Deixa a base: $i")
15
          println("Entra na base: $j")
16
```

```
println ("Função objetivo: $z")
17
           println("Variaveis: $x")
18
           println("Custo reduzido: $red_cost")
19
      end
20
  end
21
22
  function last_log(input::Simplex.Input, termination_status::Int, base::Vector{
      Int }, nbase :: Vector{Int}, z :: Float64, x :: Vector{Float64})
      if input.verbose == 1
24
           println("-----
                                  ----Fim do algoritmo----
25
           if termination_status == 1
26
               println("Status: Optimal")
               println("Base: $base")
               println ("Não-Base: $nbase")
               println ("Função objetivo: $z")
30
               println("Variaveis: $x")
           e1se
               println("Status: Unbound")
               println("Base: $base")
34
               println("Não-Base: $nbase")
35
               println ("Função objetivo: Inf")
               println ("Direção extrema: $x")
           end
38
      end
39
40 end
```

O arquivo solver.jl contém o algortimo simplex:

```
function solve (input :: Simplex . Input)
      termination_status = 0
      iter = 0
      max_iter = input.max_iter
      d = []
      init_log(input)
      while termination_status == 0 && iter < max_iter</pre>
           termination_status, iter, d = Simplex.iterate(input, iter)
10
      output = write_output(input, termination_status, d)
11
      return output
  end
13
14
  function iterate (input::Simplex.Input, iter::Int)
15
16
      iter += 1
17
      A = input.A
18
      b = input.b
19
      c = input.c
20
      base = input.base
21
      nbase = input.nbase
22
      tol = input.tol
```

```
B = view(A,:,base)
      N = view(A,:,nbase)
25
      xB = B \setminus b
26
      y = B' \setminus c[base]
27
      red_cost = c[nbase] - N'*y
28
      val , j = findmax(red_cost)
29
30
      if val <= tol
           return 1, iter, [] #optimal
31
      d = zeros(length(c))
33
      d_base = B \setminus N[:,j]
34
35
      d[base] = - d_base
      d[nbase[j]] = 1
36
      d_base = max.(d_base, 0)
37
      r = xB ./ d_base
38
      val, i = findmin(r)
      if val == Inf
40
           return 2, iter, d #unbounded
      end
42
      z = c[base]'xB
43
      x_{opt} = zeros(length(c))
44
      x_{opt}[base] = xB
      iteration_log(input, iter, base, nbase, base[i], nbase[j], z, x_opt,
      base[i], nbase[j] = nbase[j], base[i]
47
       return 0, iter, d #max iteration
48
49 end
```

2 Testes

2.1 Caso com solução ótima

Este caso foi mostrado em aula como sendo um exemplo pequeno de um problema de maximização com solução.

```
A = [2 1 1 0; 1 2 0 1]

b = [4, 4]

c = [4, 3, 0, 0]

base = [3, 4]

nbase = [1, 2]

input = Simplex.create(A, b, c, base, nbase)

output = Simplex.solve(input)
```

2.2 Caso ilimitado

Este caso foi mostrado em aula como sendo um exemplo pequeno de um problema de maximização ilimitado.

```
1 A = [-2 1 1 0; 1 -1 0 1]

2 b = [2, 2]

3 c = [1, 1, 0, 0]

4 base = [3, 4]

5 nbase = [1, 2]

6 input = Simplex.create(A, b, c, base, nbase)

7 output = Simplex.solve(input)
```

2.3 Caso 10 variáveis originais e 10 restrições

Este caso foi criado de forma aleatória usando seed=123.

```
Random.seed!(123)

A_prime = rand(1:20,10,10)

A = hcat(A_prime, Matrix(I,10,10))

c = zeros(20)

c[1:10] = rand(1:10,10)

b = rand(1:30,10)

nbase = collect(1:10)

base = collect(11:20)

input = Simplex.create(A, b, c, base, nbase)

output = Simplex.solve(input)
```

2.4 Caso 100 variáveis originais e 100 restrições

Este caso foi criado de forma aleatória usando seed=123.

```
Random.seed!(123)

A_prime = rand(1:200,100,100)

A = hcat(A_prime, Matrix(I,100,100))

c = zeros(200)

c[1:100] = rand(1:10,100)

b = rand(1:30,100)

nbase = collect(1:100)

base = collect(101:200)

input = Simplex.create(A, b, c, base, nbase)

output = Simplex.solve(input)
```

3 Resultados

3.1 Caso com solução ótima

```
6 Deixa a base: 3
7 Entra na base: 1
8 Função objetivo: 0.0
9 Variaveis: [0.0, 0.0, 4.0, 4.0]
10 Custo reduzido: [4.0, 3.0]
11 -----Iteração 2-----
12 Base: [1, 4]
Não-Base: [3, 2]
14 Deixa a base: 4
15 Entra na base: 2
16 Função objetivo: 8.0
17 Variaveis: [2.0, 0.0, 0.0, 2.0]
18 Custo reduzido: [-2.0, 1.0]
19 -----Fim do algoritmo -----
20 Status: Optimal
21 Base: [1, 2]
22 Não-Base: [3, 4]
23 Função objetivo: 9.3333333333333334
24 Variaveis: [1.3333333333333335, 1.3333333333333333, 0.0, 0.0]
```

3.2 Caso ilimitado

```
-----Inicio do algoritmo Simplex -----
2 O problema possui 4 variaveis e 2 restrições
3 -----Iteração 1-----
4 Base: [3, 4]
5 Não-Base: [1, 2]
6 Deixa a base: 4
7 Entra na base: 1
8 Função objetivo: 0.0
9 Variaveis: [0.0, 0.0, 2.0, 2.0]
10 Custo reduzido: [1.0, 1.0]
----Fim do algoritmo-----
12 Status: Unbound
13 Base: [3, 1]
14 Não-Base: [4, 2]
15 Função objetivo: Inf
16 Direção extrema: [1.0, 1.0, 1.0, 0.0]
```

3.3 Caso 10 variáveis originais e 10 restrições

```
8 Função objetivo: 0.0
21.0, 22.0, 1.0, 14.0, 1.0, 2.0, 2.0]
10 Custo reduzido: [6.0, 1.0, 6.0, 2.0, 7.0, 10.0, 3.0, 10.0, 7.0, 6.0]
11 -----Iteração 2-----
12 Base: [11, 12, 13, 14, 15, 6, 17, 18, 19, 20]
13 Não-Base: [1, 2, 3, 4, 5, 16, 7, 8, 9, 10]
14 Deixa a base: 18
15 Entra na base: 3
16 Função objetivo: 0.5263157894736842
17 Variaveis: [0.0, 0.0, 0.0, 0.0, 0.0, 0.05263157894736842, 0.0, 0.0, 0.0, 0.0,
     8.105263157894736, 8.736842105263158, 23.05263157894737,
     20.57894736842105, 21.263157894736842, 0.0, 13.210526315789474,
     0.26315789473684215, 1.0526315789473686, 1.5263157894736843]
18 Custo reduzido: [1.7894736842105257, -0.5789473684210527, 4.421052631578947,
     -0.6315789473684212, -3.0, -0.5263157894736842, -5.947368421052632,
     3.1578947368421044, 0.1578947368421053, 3.894736842105263]
 -----Iteração 3-----
20 Base: [11, 12, 13, 14, 15, 6, 17, 3, 19, 20]
21 Não-Base: [1, 2, 18, 4, 5, 16, 7, 8, 9, 10]
22 Deixa a base: 20
23 Entra na base: 8
24 Função objetivo: 0.7692307692307693
25 Variaveis: [0.0, 0.0, 0.05494505494505496, 0.0, 0.0, 0.04395604395604395, 0.0,
      0.0\,,\;\;0.0\,,\;\;0.0\,,\;\;7.9230769230769225\,,\;\;7.681318681318681\,,\;\;22.21978021978022\,,\;\;
     20.373626373626376, 20.945054945054945, 0.0, 13.230769230769232, 0.0,
     0.6593406593406594, 0.7802197802197801
26 Custo reduzido: [-10.307692307692307, -9.615384615384615, -0.923076923076923,
     1.8461538461538463, -3.9230769230769234, 0.15384615384615372,
     -10.076923076923077, 11.076923076923075, 3.461538461538462,
     -9.999999999999999
27 -----Iteração 4-----
28 Base: [11, 12, 13, 14, 15, 6, 17, 3, 19, 8]
29 Não-Base: [1, 2, 18, 4, 5, 16, 7, 20, 9, 10]
30 Deixa a base: 6
31 Entra na base: 1
32 Função objetivo: 1.0298210735586482
33 Variaveis: [0.0, 0.0, 0.09708416169648774, 0.0, 0.0, 0.02120609675281644, 0.0,
      0.02352551358515573, 0.0, 0.0, 8.009940357852884, 6.505301524188205,
     21.470841616964876, 19.96852220013254, 20.550033134526174, 0.0,
     13.276010603048377, 0.0, 0.3416169648774024, 0.0
34 Custo reduzido: [1.0298210735586508, -1.874751491053675, 0.02385685884691868,
     -6.250497017892645, -3.978131212723653, -0.3856858846918489,
     -8.495029821073558, -0.33399602385685884, -0.20874751491053622,
     -0.45725646123260155
35 -----Fim do algoritmo ------
36 Status: Optimal
37 Base: [11, 12, 13, 14, 15, 1, 17, 3, 19, 8]
38 Não-Base: [6, 2, 18, 4, 5, 16, 7, 20, 9, 10]
39 Função objetivo: 1.052132701421801
```

40 Variaveis: [0.021665538253215984, 0.0, 0.0775220040622884, 0.0, 0.0, 0.0, 0.0, 0.0, 0.0, 0.04570074475287746, 0.0, 0.0, 8.205145565335139, 6.321259309410968, 21.437711577522006, 19.794515910629656, 20.410291130670277, 0.0, 13.41198375084631, 0.0, 0.3706838185511171, 0.0]

3.4 Caso 100 variáveis originais e 100 restrições

Este caso foi criado de forma aleatória usando seed=123.

```
-----Inicio do algoritmo Simplex-
2 O problema possui 200 variaveis e 100 restrições
                                                      -----Iteração 1-----
4 Base: [101, 102, 103, 104, 105, 106, 107, 108, 109, 110, 111, 112, 113, 114,
                 115, 116, 117, 118, 119, 120, 121, 122, 123, 124, 125, 126, 127, 128, 129,
                    130, 131, 132, 133, 134, 135, 136, 137, 138, 139, 140, 141, 142, 143,
                 144, 145, 146, 147, 148, 149, 150, 151, 152, 153, 154, 155, 156, 157, 158,
                   159, 160, 161, 162, 163, 164, 165, 166, 167, 168, 169, 170, 171, 172,
                 173, 174, 175, 176, 177, 178, 179, 180, 181, 182, 183, 184, 185, 186, 187.
                    188, 189, 190, 191, 192, 193, 194, 195, 196, 197, 198, 199, 200]
5 Não-Base: [1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19,
                 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 37,
                 38, 39, 40, 41, 42, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52, 53, 54, 55,
                 56, 57, 58, 59, 60, 61, 62, 63, 64, 65, 66, 67, 68, 69, 70, 71, 72, 73,
                 74, 75, 76, 77, 78, 79, 80, 81, 82, 83, 84, 85, 86, 87, 88, 89, 90, 91,
                92, 93, 94, 95, 96, 97, 98, 99, 100]
6 Deixa a base: 124
7 Entra na base: 3
8 Função objetivo: 0.0
0.0\,,\;\;0.0\,,\;\;0.0\,,\;\;0.0\,,\;\;0.0\,,\;\;0.0\,,\;\;0.0\,,\;\;0.0\,,\;\;0.0\,,\;\;0.0\,,\;\;0.0\,,\;\;0.0\,,\;\;0.0\,,\;\;0.0\,,\;\;0.0\,,\;\;0.0\,,\;\;0.0\,,\;\;0.0\,,\;\;0.0\,,\;\;0.0\,,\;\;0.0\,,\;\;0.0\,,\;\;0.0\,,\;\;0.0\,,\;\;0.0\,,\;\;0.0\,,\;\;0.0\,,\;\;0.0\,,\;\;0.0\,,\;\;0.0\,,\;\;0.0\,,\;\;0.0\,,\;\;0.0\,,\;\;0.0\,,\;\;0.0\,,\;\;0.0\,,\;\;0.0\,,\;\;0.0\,,\;\;0.0\,,\;\;0.0\,,\;\;0.0\,,\;\;0.0\,,\;\;0.0\,,\;\;0.0\,,\;\;0.0\,,\;\;0.0\,,\;\;0.0\,,\;\;0.0\,,\;\;0.0\,,\;\;0.0\,,\;\;0.0\,,\;\;0.0\,,\;\;0.0\,,\;\;0.0\,,\;\;0.0\,,\;\;0.0\,,\;\;0.0\,,\;\;0.0\,,\;\;0.0\,,\;\;0.0\,,\;\;0.0\,,\;\;0.0\,,\;\;0.0\,,\;\;0.0\,,\;\;0.0\,,\;\;0.0\,,\;\;0.0\,,\;\;0.0\,,\;\;0.0\,,\;\;0.0\,,\;\;0.0\,,\;\;0.0\,,\;\;0.0\,,\;\;0.0\,,\;\;0.0\,,\;\;0.0\,,\;\;0.0\,,\;\;0.0\,,\;\;0.0\,,\;\;0.0\,,\;\;0.0\,,\;\;0.0\,,\;\;0.0\,,\;\;0.0\,,\;\;0.0\,,\;\;0.0\,,\;\;0.0\,,\;\;0.0\,,\;\;0.0\,,\;\;0.0\,,\;\;0.0\,,\;\;0.0\,,\;\;0.0\,,\;\;0.0\,,\;\;0.0\,,\;\;0.0\,,\;\;0.0\,,\;\;0.0\,,\;\;0.0\,,\;\;0.0\,,\;\;0.0\,,\;\;0.0\,,\;\;0.0\,,\;\;0.0\,,\;\;0.0\,,\;\;0.0\,,\;\;0.0\,,\;\;0.0\,,\;\;0.0\,,\;\;0.0\,,\;\;0.0\,,\;\;0.0\,,\;\;0.0\,,\;\;0.0\,,\;\;0.0\,,\;\;0.0\,,\;\;0.0\,,\;\;0.0\,,\;\;0.0\,,\;\;0.0\,,\;\;0.0\,,\;\;0.0\,,\;\;0.0\,,\;\;0.0\,,\;\;0.0\,,\;\;0.0\,,\;\;0.0\,,\;\;0.0\,,\;\;0.0\,,\;\;0.0\,,\;\;0.0\,,\;\;0.0\,,\;\;0.0\,,\;\;0.0\,,\;\;0.0\,,\;\;0.0\,,\;\;0.0\,,\;\;0.0\,,\;\;0.0\,,\;\;0.0\,,\;\;0.0\,,\;\;0.0\,,\;\;0.0\,,\;\;0.0\,,\;\;0.0\,,\;\;0.0\,,\;\;0.0\,,\;\;0.0\,,\;\;0.0\,,\;\;0.0\,,\;\;0.0\,,\;\;0.0\,,\;\;0.0\,,\;\;0.0\,,\;\;0.0\,,\;\;0.0\,,\;\;0.0\,,\;\;0.0\,,\;\;0.0\,,\;\;0.0\,,\;\;0.0\,,\;\;0.0\,,\;\;0.0\,,\;\;0.0\,,\;\;0.0\,,\;\;0.0\,,\;\;0.0\,,\;\;0.0\,,\;\;0.0\,,\;\;0.0\,,\;\;0.0\,,\;\;0.0\,,\;\;0.0\,,\;\;0.0\,,\;\;0.0\,,\;\;0.0\,,\;\;0.0\,,\;\;0.0\,,\;\;0.0\,,\;\;0.0\,,\;\;0.0\,,\;\;0.0\,,\;\;0.0\,,\;\;0.0\,,\;\;0.0\,,\;\;0.0\,,\;\;0.0\,,\;\;0.0\,,\;\;0.0\,,\;\;0.0\,,\;\;0.0\,,\;\;0.0\,,\;\;0.0\,,\;\;0.0\,,\;\;0.0\,,\;\;0.0\,,\;\;0.0\,,\;\;0.0\,,\;\;0.0\,,\;\;0.0\,,\;\;0.0\,,\;\;0.0\,,\;\;0.0\,,\;\;0.0\,,\;\;0.0\,,\;\;0.0\,,\;\;0.0\,,\;\;0.0\,,\;\;0.0\,,\;\;0.0\,,\;\;0.0\,,\;\;0.0\,,\;\;0.0\,,\;\;0.0\,,\;\;0.0\,,\;\;0.0\,,\;\;0.0\,,\;\;0.0\,,\;\;0.0\,,\;\;0.0\,,\;\;0.0\,,\;\;0.0\,,\;\;0.0\,,\;\;0.0\,,\;\;0.0\,,\;\;0.0\,,\;\;0.0\,,\;\;0.0\,,\;0.0\,,\;0.0\,,\;0.0\,,\;0.0\,,\;0.0\,,\;0.0\,,\;0.0\,,\;0.0\,,\;0.0\,,\;0.0\,,\;0.0\,,\;0.0\,,\;0.0\,,\;0.0\,,\;0.0\,,\;0.0\,,\;0.0\,,\;0.0\,,\;0.0\,,\;0.0\,,\;0.0\,,\;0.0\,,\;0.0\,,\;0.0\,,\;0.0\,,\;0.0\,,\;0.0\,,\;0.0\,,\;0.0\,,\;0.0\,,\;0.0\,,\;0.0\,,\;0.0\,,\;0.0\,,\;0.0\,,\;0.0\,,\;0.0\,,\;0.0\,,\;0.0\,,\;0.0\,,\;0.0\,,\;0.0\,,\;0.0\,,\;0.0\,,\;0.0\,,\;0.0\,,\;0.0\,,\;0.0\,,\;0.0\,,\;0.0\,,\;0.0\,,\;0.0\,,\;0.0\,,\;0.0\,,\;0.0\,,\;0.0\,,\;0.0\,,\;0.0\,,\;0.0\,,\;0.0\,,\;0.0\,,\;0.0\,,\;0.0\,,\;0.0\,,\;0.0\,,\;0.0\,,\;0.0\,,\;0.0\,,\;0.0\,,\;0.0\,,\;0.0\,,\;0.0\,,\;0.0\,,\;0.0\,,\;0.0\,,
                 0.0\,,\;\;0.0\,,\;\;0.0\,,\;\;0.0\,,\;\;0.0\,,\;\;0.0\,,\;\;0.0\,,\;\;0.0\,,\;\;0.0\,,\;\;0.0\,,\;\;0.0\,,\;\;0.0\,,\;\;0.0\,,\;\;0.0\,,\;\;0.0\,,\;\;0.0\,,\;\;0.0\,,\;\;0.0\,,\;\;0.0\,,\;\;0.0\,,\;\;0.0\,,\;\;0.0\,,\;\;0.0\,,\;\;0.0\,,\;\;0.0\,,\;\;0.0\,,\;\;0.0\,,\;\;0.0\,,\;\;0.0\,,\;\;0.0\,,\;\;0.0\,,\;\;0.0\,,\;\;0.0\,,\;\;0.0\,,\;\;0.0\,,\;\;0.0\,,\;\;0.0\,,\;\;0.0\,,\;\;0.0\,,\;\;0.0\,,\;\;0.0\,,\;\;0.0\,,\;\;0.0\,,\;\;0.0\,,\;\;0.0\,,\;\;0.0\,,\;\;0.0\,,\;\;0.0\,,\;\;0.0\,,\;\;0.0\,,\;\;0.0\,,\;\;0.0\,,\;\;0.0\,,\;\;0.0\,,\;\;0.0\,,\;\;0.0\,,\;\;0.0\,,\;\;0.0\,,\;\;0.0\,,\;\;0.0\,,\;\;0.0\,,\;\;0.0\,,\;\;0.0\,,\;\;0.0\,,\;\;0.0\,,\;\;0.0\,,\;\;0.0\,,\;\;0.0\,,\;\;0.0\,,\;\;0.0\,,\;\;0.0\,,\;\;0.0\,,\;\;0.0\,,\;\;0.0\,,\;\;0.0\,,\;\;0.0\,,\;\;0.0\,,\;\;0.0\,,\;\;0.0\,,\;\;0.0\,,\;\;0.0\,,\;\;0.0\,,\;\;0.0\,,\;\;0.0\,,\;\;0.0\,,\;\;0.0\,,\;\;0.0\,,\;\;0.0\,,\;\;0.0\,,\;\;0.0\,,\;\;0.0\,,\;\;0.0\,,\;\;0.0\,,\;\;0.0\,,\;\;0.0\,,\;\;0.0\,,\;\;0.0\,,\;\;0.0\,,\;\;0.0\,,\;\;0.0\,,\;\;0.0\,,\;\;0.0\,,\;\;0.0\,,\;\;0.0\,,\;\;0.0\,,\;\;0.0\,,\;\;0.0\,,\;\;0.0\,,\;\;0.0\,,\;\;0.0\,,\;\;0.0\,,\;\;0.0\,,\;\;0.0\,,\;\;0.0\,,\;\;0.0\,,\;\;0.0\,,\;\;0.0\,,\;\;0.0\,,\;\;0.0\,,\;\;0.0\,,\;\;0.0\,,\;\;0.0\,,\;\;0.0\,,\;\;0.0\,,\;\;0.0\,,\;\;0.0\,,\;\;0.0\,,\;\;0.0\,,\;\;0.0\,,\;\;0.0\,,\;\;0.0\,,\;\;0.0\,,\;\;0.0\,,\;\;0.0\,,\;\;0.0\,,\;\;0.0\,,\;\;0.0\,,\;\;0.0\,,\;\;0.0\,,\;\;0.0\,,\;\;0.0\,,\;\;0.0\,,\;\;0.0\,,\;\;0.0\,,\;\;0.0\,,\;\;0.0\,,\;\;0.0\,,\;\;0.0\,,\;\;0.0\,,\;\;0.0\,,\;\;0.0\,,\;\;0.0\,,\;\;0.0\,,\;\;0.0\,,\;\;0.0\,,\;\;0.0\,,\;\;0.0\,,\;\;0.0\,,\;\;0.0\,,\;\;0.0\,,\;\;0.0\,,\;\;0.0\,,\;\;0.0\,,\;\;0.0\,,\;\;0.0\,,\;\;0.0\,,\;\;0.0\,,\;\;0.0\,,\;\;0.0\,,\;\;0.0\,,\;\;0.0\,,\;\;0.0\,,\;\;0.0\,,\;\;0.0\,,\;\;0.0\,,\;\;0.0\,,\;\;0.0\,,\;\;0.0\,,\;\;0.0\,,\;\;0.0\,,\;\;0.0\,,\;\;0.0\,,\;\;0.0\,,\;\;0.0\,,\;\;0.0\,,\;\;0.0\,,\;\;0.0\,,\;\;0.0\,,\;\;0.0\,,\;\;0.0\,,\;\;0.0\,,\;\;0.0\,,\;\;0.0\,,\;\;0.0\,,\;\;0.0\,,\;\;0.0\,,\;\;0.0\,,\;\;0.0\,,\;\;0.0\,,\;\;0.0\,,\;\;0.0\,,\;\;0.0\,,\;\;0.0\,,\;\;0.0\,,\;\;0.0\,,\;\;0.0\,,\;\;0.0\,,\;\;0.0\,,\;\;0.0\,,\;\;0.0\,,\;\;0.0\,,\;\;0.0\,,\;\;0.0\,,\;\;0.0\,,\;\;0.0\,,\;\;0.0\,,\;\;0.0\,,\;\;0.0\,,\;\;0.0\,,\;\;0.0\,,\;\;0.0\,,\;\;0.0\,,\;\;0.0\,,\;\;0.0\,,\;\;0.0\,,\;\;0.0\,,\;\;0.0\,,\;\;0.0\,,\;0.0\,,\;0.0\,,\;0.0\,,\;0.0\,,\;0.0\,,\;0.0\,,\;0.0\,,\;0.0\,,\;0.0\,,\;0.0\,,\;0.0\,,\;0.0\,,\;0.0\,,\;0.0\,,\;0.0\,,\;0.0\,,\;0.0\,,\;0.0\,,\;0.0\,,\;0.0\,,\;0.0\,,\;0.0\,,\;0.0\,,\;0.0\,,\;0.0\,,\;0.0\,,\;0.0\,,\;0.0\,,\;0.0\,,\;0.0\,,\;0.0\,,\;0.0\,,\;0.0\,,\;0.0\,,\;0.0\,,\;0.0\,,\;0.0\,,\;0.0\,,\;0.0\,,\;0.0\,,\;0.0\,,\;0.0\,,\;0.0\,,\;0.0\,,\;0.0\,,\;0.0\,,\;0.0\,,\;0.0\,,\;0.0\,,\;0.0\,,\;0.0\,,\;0.0\,,\;0.0\,,\;0.0\,,\;0.0\,,\;0.0\,,\;0.0\,,\;0.0\,,\;0.0\,,\;0.0\,,\;0.0\,,\;0.0\,,\;0.0\,,\;0.0\,,\;0.0\,,\;0.0\,,\;0.0\,,\;0.0\,,\;0.0\,,\;0.0\,,\;0.0\,,\;0.0\,,\;0.0\,,\;0.0\,,\;0.0\,,
                 0.0,\ 0.0,\ 0.0,\ 0.0,\ 0.0,\ 0.0,\ 0.0,\ 0.0,\ 0.0,\ 0.0,\ 0.0,\ 0.0,\ 0.0,\ 0.0,\ 0.0,\ 0.0,
                    20.0, 12.0, 7.0, 20.0, 16.0, 13.0, 9.0, 26.0, 6.0, 20.0, 10.0, 24.0, 10.0,
                     20.0, 17.0, 8.0, 11.0, 15.0, 30.0, 3.0, 5.0, 10.0, 7.0, 1.0, 19.0, 3.0,
                 7.0, 18.0, 25.0, 15.0, 26.0, 28.0, 26.0, 20.0, 30.0, 17.0, 23.0, 13.0,
                 5.0, 23.0, 4.0, 4.0, 23.0, 6.0, 16.0, 6.0, 3.0, 7.0, 8.0, 11.0, 23.0,
                 27.0, 27.0, 9.0, 24.0, 22.0, 17.0, 24.0, 1.0, 4.0, 7.0, 9.0, 7.0, 30.0,
                  16.0, 21.0, 21.0, 25.0, 9.0, 23.0, 21.0, 9.0, 25.0, 15.0, 3.0, 19.0, 10.0,
                    7.0, 19.0, 2.0, 28.0, 30.0, 30.0, 29.0, 29.0, 21.0, 8.0, 26.0, 26.0,
                  14.0, 24.0, 18.0, 3.0, 13.0, 17.0, 23.0, 22.0, 5.0, 12.0, 26.0
```

```
10 Custo reduzido: [1.0, 6.0, 10.0, 7.0, 6.0, 7.0, 6.0, 4.0, 1.0, 8.0, 4.0, 7.0,
     1.0, 4.0, 4.0, 2.0, 6.0, 6.0, 5.0, 3.0, 6.0, 10.0, 7.0, 6.0, 6.0, 8.0,
     10.0, 6.0, 10.0, 6.0, 1.0, 9.0, 4.0, 6.0, 3.0, 3.0, 10.0, 6.0, 5.0, 6.0,
     2.0,\ 2.0,\ 8.0,\ 8.0,\ 1.0,\ 7.0,\ 3.0,\ 7.0,\ 5.0,\ 8.0,\ 4.0,\ 2.0,\ 7.0,\ 8.0,
      3.0, 7.0, 8.0, 3.0, 1.0, 1.0, 7.0, 3.0, 10.0, 7.0, 10.0, 10.0, 8.0, 5.0,
     2.0, 5.0, 5.0, 10.0, 9.0, 9.0, 5.0, 1.0, 9.0, 3.0, 3.0, 1.0, 2.0, 1.0,
     7.0, 10.0, 8.0, 3.0, 6.0, 4.0, 2.0, 10.0, 7.0, 5.0, 4.0, 4.0, 4.0, 5.0,
     7.0, 3.0, 6.0
                     -----Iteração 2-----
12 Base: [101, 102, 103, 104, 105, 106, 107, 108, 109, 110, 111, 112, 113, 114,
     115, 116, 117, 118, 119, 120, 121, 122, 123, 3, 125, 126, 127, 128, 129,
     130, 131, 132, 133, 134, 135, 136, 137, 138, 139, 140, 141, 142, 143, 144,
      145, 146, 147, 148, 149, 150, 151, 152, 153, 154, 155, 156, 157, 158,
     159, 160, 161, 162, 163, 164, 165, 166, 167, 168, 169, 170, 171, 172, 173,
      174, 175, 176, 177, 178, 179, 180, 181, 182, 183, 184, 185, 186, 187,
     188, 189, 190, 191, 192, 193, 194, 195, 196, 197, 198, 199, 200]
13 Não-Base: [1, 2, 124, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18,
```

Não-Base: [1, 2, 124, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 38, 39, 40, 41, 42, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52, 53, 54, 55, 56, 57, 58, 59, 60, 61, 62, 63, 64, 65, 66, 67, 68, 69, 70, 71, 72, 73, 74, 75, 76, 77, 78, 79, 80, 81, 82, 83, 84, 85, 86, 87, 88, 89, 90, 91, 92, 93, 94, 95, 96, 97, 98, 99, 100]

- 14 Deixa a base: 159
- 15 Entra na base: 64
- 16 Função objetivo: 0.05319148936170218

```
17 Variaveis: [0.0, 0.0, 0.005319148936170218, 0.0, 0.0, 0.0, 0.0, 0.0, 0.0, 0.0,
     0.0,\ 0.0,\ 0.0,\ 19.164893617021274,\ 11.154255319148936,\ 6.223404255319148,
     19.54255319148936, 15.25, 12.909574468085106, 8.803191489361701,
    25.93617021276596, 5.50531914893617, 19.329787234042552, 9.97340425531915,
     23.670212765957448, 9.223404255319148, 19.48404255319149,
    16.78191489361702, 7.079787234042552, 10.49468085106383,
    14.090425531914892, 29.170212765957444, 2.101063829787233,
    4.792553191489361, 9.053191489361701, 6.914893617021277, 0.0,
    18.47872340425532, 2.9627659574468086, 6.8936170212765955,
    17.835106382978726, 24.090425531914892, 14.085106382978722,
    25.101063829787233, 27.29255319148936, 25.111702127659576,
    19.27127659574468, 29.50531914893617, 16.654255319148938,
    22.601063829787233, 12.851063829787234, 4.835106382978723,
    22.78191489361702, 3.856382978723404, 3.4734042553191484,
    22.601063829787233, 5.861702127659574, 15.76063829787234,
    5.069148936170212, 2.99468085106383, 6.914893617021277, 7.579787234042553,
     10.053191489361701, 22.409574468085108, 26.664893617021278,
    26.50531914893617, 8.893617021276595, 23.3031914893617,
    21.196808510638295, 16.02659574468085, 23.28191489361702,
    0.015957446808509287, 3.1808510638297864, 6.297872340425531,
    8.122340425531915, 6.728723404255319, 29.095744680851066,
    15.712765957446809, 20.425531914893618, 20.03191489361702,
    24.81382978723404, 8.728723404255318, 22.819148936170212,
    20.122340425531913, 8.244680851063828, 24.27659574468085,
    14.909574468085106, 2.212765957446808, 18.840425531914892,
    9.78191489361702, 6.718085106382978, 18.882978723404253,
    1.4202127659574462, 27.601063829787233, 29.52659574468085,
    29.20744680851064, 28.925531914893618, 28.414893617021274,
    19.984042553191486, 7.127659574468084, 24.95212765957447, 25.25,
    13.191489361702127, 23.122340425531913, 17.180851063829788, 2.75,
    12.367021276595745, 16.138297872340424, 22.79255319148936,
    21.70212765957447, 4.026595744680851, 10.941489361702127,
    25.367021276595747]
```

```
-0.05319148936170213, 5.085106382978723, 0.6808510638297864,
     4.872340425531915, 1.4255319148936172, -5.042553191489361,
     -7.882978723404255, 7.095744680851064, 3.521276595744679,
     3.8085106382978724, -8.202127659574469, -0.999999999999999,
     -3.7659574468085104, -1.244680851063829, 2.436170212765957,
     0.20212765957446877, -0.957446808510638, -0.5106382978723403,
     1.4787234042553186, 1.5957446808510642, 1.5212765957446805,
     0.202127659574467, 5.680851063829788, -0.13829787234042534,
     -0.47872340425531945, 1.3191489361702136, 8.244680851063832,
     0.6808510638297873, -7.670212765957446, -0.6276595744680868,
     -0.787234042553191, -2.031914893617021, -4.765957446808511,
     -6.414893617021276, 9.095744680851064, -1.819148936170211,
     2.97872340425532, 0.255319148936171, -0.07446808510638059,
     1.3085106382978724, 6.882978723404255, -1.5212765957446788,
     -8.361702127659575, -3.4787234042553195, 0.2340425531914887,
     0.5638297872340434, 6.308510638297874, 3.404255319148936,
     -1.3085106382978733, -4.98936170212766, -8.106382978723405,
     -3.372340425531913, -0.7765957446808489, 2.1489361702127656,
     3.3297872340425534, 4.4361702127659575, -1.414893617021277,
     -8.202127659574469, 0.04255319148936154, -1.3510638297872344,
     -6.680851063829786, 9.893617021276595, 3.3829787234042548,
     -0.47872340425531945, 7.925531914893616, 3.1063829787234036,
     3.457446808510638, -3.372340425531913, -5.531914893617021,
     -1.8085106382978706, 5.053191489361701, 0.27659574468085246,
     -1.5319148936170208, 2.393617021276596, -4.744680851063831,
     -0.41489361702127425, -2.1063829787234054, -7.212765957446811,
     -4.159574468085107, 0.5106382978723405, -7.776595744680851,
     -3.2659574468085086, 0.9042553191489358, 2.042553191489361,
     -1.0957446808510642\,,\  \, -0.4893617021276597\,,\  \, -3.127659574468085\,,
     -3.319148936170212, 7.127659574468083, 3.595744680851064,
     -2.1808510638297873, -3.0212765957446805, 3.148936170212766,
     2.1382978723404245, 0.10638297872340452, 0.24468085106382986,
     -0.0319148936170226, 4.457446808510639]
              -----Iteração 3-----
20 Base: [101, 102, 103, 104, 105, 106, 107, 108, 109, 110, 111, 112, 113, 114,
     115, 116, 117, 118, 119, 120, 121, 122, 123, 3, 125, 126, 127, 128, 129,
     130, 131, 132, 133, 134, 135, 136, 137, 138, 139, 140, 141, 142, 143, 144,
      145, 146, 147, 148, 149, 150, 151, 152, 153, 154, 155, 156, 157, 158, 64,
      160, 161, 162, 163, 164, 165, 166, 167, 168, 169, 170, 171, 172, 173,
     174, 175, 176, 177, 178, 179, 180, 181, 182, 183, 184, 185, 186, 187, 188,
      189, 190, 191, 192, 193, 194, 195, 196, 197, 198, 199, 200]
21 Não-Base: [1, 2, 124, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18,
     19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36,
     37, 38, 39, 40, 41, 42, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52, 53, 54,
     55, 56, 57, 58, 59, 60, 61, 62, 63, 159, 65, 66, 67, 68, 69, 70, 71, 72,
     73, 74, 75, 76, 77, 78, 79, 80, 81, 82, 83, 84, 85, 86, 87, 88, 89, 90,
     91, 92, 93, 94, 95, 96, 97, 98, 99, 100]
22 Deixa a base: 3
23 Entra na base: 73
24 Função objetivo: 0.054369394396380576
```

```
25 Variaveis: [0.0, 0.0, 0.005317882371616791, 0.0, 0.0, 0.0, 0.0, 0.0, 0.0, 0.0,
     0.0, 0.0, 0.0, 0.0, 0.0, 0.0, 0.0, 19.151996190173822, 11.131597745852847,
     6.202158901500121, 19.528494324946422, 15.240177791888247,
    12.900547662512897, 8.800619096753712, 25.92904198745932,
    5.492816890229384, 19.32482736725137, 9.971148503849513,
    23.646479879355507, 9.202635129772204, 19.472616874355108,
    16.76613223271688, 7.076791808873721, 10.475632986744982,
    14.087427573616955, 29.16743392332725, 2.0905627430748486,
    4.777958568140329, 9.041630288118105, 6.909675371061196, 0.0,
    18.46527502182713, 2.9421779506310033, 6.888403841574728,
    17.82871656480673, 24.085641717596637, 14.065917929994445,
    25.09651559647591, 27.27593459798397, 25.096912453369317,
    19.250496071116757, 29.50472259703151, 16.64040796888642,
    22.58068100642908, 12.849789665846496, 4.820501627113263,
    22.777323597110883, 3.835701246130646, 3.4643622509722998,
    22.578895150408762\,,\;\; 5.842447813318517\,,\;\; 15.759623779665052\,,
    5.0605603619334865, 2.98872926422732, 6.913008968965791,
    7.567743471704104, 10.043058972934361, 22.39376140963569,
    26.663663782839905, 26.490435748868958, 8.873997936344155,
    23.284308278434796, 21.191404079688866, 16.008849908722915,
    23.275180569886498, 0.0, 3.1668783236764826, 6.290538931661243,
    8.106595761568379, 6.718310977061672, 29.09095959996825,
    15.71057226764029, 20.40435748868958, 20.03155012302563,
    24.796253670926266, 8.718549091197715, 22.812286689419796,
    20.112905786173506, 8.226287800619097, 24.256885467100563,
    14.901142947853005, 2.2056909278514176, 18.827367251369157,
    9.780300023811412, 6.712437495039289, 18.876101277879197,
    1.4097547424398769, 27.598063338360188, 29.51468370505596,
    29.188348281609656, 28.905667116437815, 28.408603857449002,
    19.979760298436386,\ 7.124057464878165,\ 24.944281292166053,
    25.245654417017228, 13.17977617271212, 23.109334074132864,
    17.16604492420033, 2.734463052623225, 12.3632431145329, 16.1195729819827,
    22.77236288594333, 21.683982855782205, 4.0112310500833415,
    10.92602587506945, 25.366814826573542]
```

```
26 Custo reduzido: [-11.077545836971186, -5.352885149615048, 0.01944598777680768,
      5.264147948249859, 6.468291134216999, -5.139852369235655,
     2.5052781966822755, -7.309627748233986, 0.999603143106595,
     7.518612588300659, -3.132708945154377, 5.214143979680926,
     -4.346059211048496, -8.270815144059052, -2.5354393205809984,
     -10.026748154615445, 4.128819747599014, 4.355028176839431,
     0.7560123819350739, -4.943487578379237, -7.036352091435827,
     5.838558615763155, 8.264782919279305, -4.355186919596793,
     -1.0434161441384227, 7.579728549884909, 7.8518136360028565,
     5.718231605682989, 1.1933486784665455, -0.5441701722358925,
     -4.097626795777442, 5.507262481149295, 3.0927851416779077,
     -3.5384554329708706, 1.7792681958885614, 5.4085244860703225,
     7.525597269624573, 4.872529565838558, 4.8531629494404305,
     2.490197634732915, 1.282085879831731, -8.155171045321056,
     7.891658068100642\,,\ -0.25581395348837077\,,\ -4.509167394237638\,,
     6.03285975077387, -5.068100642908167, 1.3809032462893862,
     -7.362647829192792, -4.307881577903007, -1.7360901658861785,
     6.991110405587743, -0.21049289626160927, 0.16255258353837476,
     9.437018811016745, -5.3990792920073005, -2.140011111993015,
     3.3238352250178576, 3.4329708707040223, -5.600920707992698,
     0.2427970473847112, -3.5290102389078513, -3.942614493213748,
     -0.07381538217318834, 8.027065640130168, 7.408921342963726,
     8.470116675926661, 4.474323358996745, -6.689419795221841,
     3.373521708072067, -3.0339709500754033, -1.2211286610048404,
     11.734661481069923, 5.176680688943565, 0.744582903405032,
     1.154853559806333, -4.355186919596793, 1.8863401857290247,
     -1.7027541868402256, -6.774585284546394, -9.145646479879355,
     -7.568219699976186, -3.09913485197238, -3.567108500674655, 8.970156361616,
      3.9036431462814516, -6.353520120644495, 6.674656718787203,
     -1.5877450591316782, -8.60401619176125, 10.75482181125486,
     -2.4586871973966176, -1.2326375109135643, 1.8426859274545597,
     -8.385109929359475, 0.3255020239701567, -4.50472259703151,
     1.2023176442574828, -0.39431700928645164, -7.7562504960711145]
                -----Iteração 4-----
28 Base: [101, 102, 103, 104, 105, 106, 107, 108, 109, 110, 111, 112, 113, 114,
     115, 116, 117, 118, 119, 120, 121, 122, 123, 73, 125, 126, 127, 128, 129,
     130, 131, 132, 133, 134, 135, 136, 137, 138, 139, 140, 141, 142, 143, 144,
      145, 146, 147, 148, 149, 150, 151, 152, 153, 154, 155, 156, 157, 158, 64,
      160, 161, 162, 163, 164, 165, 166, 167, 168, 169, 170, 171, 172, 173,
     174, 175, 176, 177, 178, 179, 180, 181, 182, 183, 184, 185, 186, 187, 188,
      189, 190, 191, 192, 193, 194, 195, 196, 197, 198, 199, 200]
29 Não-Base: [1, 2, 124, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18,
     19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36,
     37, 38, 39, 40, 41, 42, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52, 53, 54,
     55, 56, 57, 58, 59, 60, 61, 62, 63, 159, 65, 66, 67, 68, 69, 70, 71, 72,
     3, 74, 75, 76, 77, 78, 79, 80, 81, 82, 83, 84, 85, 86, 87, 88, 89, 90, 91,
      92, 93, 94, 95, 96, 97, 98, 99, 100]
30 Deixa a base: 180
31 Entra na base: 37
32 Função objetivo: 0.17871263640676838
```

```
0.0, 0.0, 0.0, 0.0, 0.0, 0.0, 0.0, 0.007275027676735709, 0.0, 0.0, 0.0, 0.0,
    0.0, 0.0, 0.0, 0.0, 0.0, 0.010596235963941127, 0.0, 0.0, 0.0, 0.0, 0.0, 0.0,
    0.0, 0.0, 0.0, 0.0, 0.0, 0.0, 17.31361695397755, 8.494543729242459,
    5.181875691918399, 18.84817333544204, 15.09220306816385,
    10.719911434445683, 8.575043492013286, 25.256207496441565,
    3.3533133006484355, 17.896409931994313, 7.912067056776855,
    21.453582160366924, 8.391111814012339, 17.959196583900056,
    15.386050925193741, 7.273762454531079, 8.419421160841381,
    13.669777004586436, 28.853866835362965, 1.0948916653487333,
    3.3952237861774526, 7.658548157520173, 5.85339237703622, 0.0,
    17.746797406294483, 0.9996836944488425, 5.450735410406456,
    16.388581369603042, 23.444092993832044, 13.581053297485376,
    23.82286889134905, 25.183457219674214, 23.695239601455015,
    18.21097580262534, 28.896726237545472, 15.926300806579158,
    20.095682429226642, 12.263008065791556, 3.2680689546101576,
    21.97453740313143\,,\ 1.9288312509884604\,,\ 2.327218092677531\,,
    20.537561284200546, 4.20686383046023, 14.26031946860668,
    4.603352838842326, 1.6613949074806296, 6.671674837893406,
    7.046022457694135, 8.618693658073706, 21.770836628182824,
    26.506721492962203, 25.458168590858772, 6.517475881701731,
    22.073066582318525, 20.78918235015025, 14.502767673572679,
    22.317096315040335, 0.0, 3.0706942906848043, 6.520480784437768,
    6.054246402024366, 5.278981496125262, 29.5143128261901,
    15.120037956666142, 19.68717380990037, 19.37418946702515,
    23.531235173177294, 6.414043966471619, 22.281353787758974,
    20.018662027518584, 6.65712478253994, 23.159892456112612,
    13.286098371026416, 1.4330222995413624, 16.175865886446314,
    9.506088881859878, 6.566028783805157, 16.479993673888984,
    0.7803257947177008, 27.196267594496284, 27.474458326743637,
    26.797564447256057, 27.5943381306342, 27.187252886288157,
    18.731456587063107, 5.753914281195638, 23.95824766724656,
    25.628182824608572, 11.174442511466083, 21.91032737624546,
    16.362802467183304, \ 1.7502767673572708, \ 11.94401391744425,
    13.904159417997786, 20.067847540724344, 20.579629922505145,
    3.6337181717539133, 9.79993673888977, 25.46955559070062
```

```
34 Custo reduzido: [-16.682271073857347, -23.295429384785706,
     -0.10675312351731774, 0.7822236280246697, -6.1145026095208,
     -9.863039696346672, -8.217934524750909, -28.396014550055355,
     -19.993990194527914, 5.393642258421636, -4.084769887711532,
     -2.2835679266171134, -25.959512889451208, -19.779060572513046,
     -20.724814170488692, -17.392693341768144, -4.246718329906692,
     -9.306025620749647, -13.216827455321841, -13.040645263324372,
     -17.393958563972795, -13.91902577890242, -4.715166851178239,
     -17.797248141704888, -1.6205914913806732, -11.643365491064369,
     -16.85908587695714, -5.337181717539146, -2.7336707259212396,
     -12.950656334018664, -24.46022457694132, -17.158469081132374,
     -8.19832358057884, -22.28087932943223, -16.543729242448205,
     -16.902736043017555, 5.450735410406453, -13.578522853076073,
     0.07986715166851255, -10.998260319468603, -3.6025620749644167,
     -9.53408192313775, 5.254467815910169, -22.550371659022616,
     -26.49565079867152, -18.707733670725926, -11.40218250830302,
     -13.688755337656175, -8.635774157836469, -7.845168432705991,
     -23.490589909852922, -14.329115925984503, -24.039854499446466,
     -24.17902894195793, -11.341293689704258, -7.199272497232323,
     -10.584216352996993, -4.981179819705837, -7.011861458168594,
     -27.182824608571877, -2.000948916653489, -23.0007907638779,
     -26.647319310453895, -0.0719595128894512, -0.5470504507354113,
     -17.290842954293847, 3.6058832832516208, -7.002372291633719,
     -10.041119721651114, -9.357741578364703, -27.722600031630556,
     -17.155622331171912, -23.382097105804206, -15.34366598133797,
     -23.938478570298912\,, \quad -4.9082713901628985\,, \quad -17.797248141704884\,,
     -20.185513205756763, -13.652696504823659, -30.665190574094574,
     -21.084453582160364, -10.847540724339712, -23.738257156413095,
     -27.563498339395856, -12.500395381938954, -10.072908429542942,
     -15.798038905582793, -8.678949865570145, -18.29242448204966,
     -20.908429542938478, 3.947493278507035, -10.266329274078757,
     -18.046813221572037, -14.696821129210818, -10.085086193262688,
     -3.981970583583742, -15.831092835679264, -14.617112130317885,
     -7.474458326743637, -11.055986082555748]
               -----Iteração 5-----
36 Base: [101, 102, 103, 104, 105, 106, 107, 108, 109, 110, 111, 112, 113, 114,
     115, 116, 117, 118, 119, 120, 121, 122, 123, 73, 125, 126, 127, 128, 129,
     130, 131, 132, 133, 134, 135, 136, 137, 138, 139, 140, 141, 142, 143, 144,
      145, 146, 147, 148, 149, 150, 151, 152, 153, 154, 155, 156, 157, 158, 64,
      160, 161, 162, 163, 164, 165, 166, 167, 168, 169, 170, 171, 172, 173,
     174, 175, 176, 177, 178, 179, 37, 181, 182, 183, 184, 185, 186, 187, 188,
     189, 190, 191, 192, 193, 194, 195, 196, 197, 198, 199, 200]
37 Não-Base: [1, 2, 124, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18,
     19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36,
     180, 38, 39, 40, 41, 42, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52, 53, 54,
     55, 56, 57, 58, 59, 60, 61, 62, 63, 159, 65, 66, 67, 68, 69, 70, 71, 72,
     3, 74, 75, 76, 77, 78, 79, 80, 81, 82, 83, 84, 85, 86, 87, 88, 89, 90, 91,
      92, 93, 94, 95, 96, 97, 98, 99, 100]
38 Deixa a base: 175
39 Entra na base: 91
40 Função objetivo: 0.21052525741215047
```

```
0.0, 0.0, 0.0, 0.0, 0.0, 0.0, 0.0, 0.09564275706171852, 0.0, 0.0, 0.0,
    7.879292346509386, 4.723994556329628, 18.269607770656112,
    14.568749996303463, 10.532897713528335, 8.420605747318827,
    24.712339518480036, 3.538770773806753, 17.644514248233506,
    7.782604504099029, 21.633542922134342, 7.774957696814058,
    17.552140414047802, 14.661022693197369, 6.464545464761348,
    8.356927106036245, 12.966072874446045, 28.30549672309261,
    1.059255806966177, 2.6134645082036574, 6.862381126716756,
    5.637612071652523, 0.0, 17.325277639609386, 1.0200547560846518,
    4.994022845199032, 15.526848402296949, 22.57181427881127,
    13.471458287373038, 23.575391345131905, 24.868849181822977,
    23.007740847222962, 18.22518602460061, 28.70033257017066,
    15.39666104989972, 20.162324170452422, 11.241766625994892,
    3.228242026862304, 21.06536426281265, 1.2799959308504856,
    1.556698689532771, 20.253887131965243, 3.922686159154612,
    14.327086281938842, 4.3226356496533285, 1.0379081495956641,
    6.204682360887431, 6.275056764044334, 8.194797521367482,
    21.318577737612014, 25.50843609217334, 24.556295323257853,
    6.6825466015208494, 21.55496161218758, 20.185524828613698,
    14.230409085559199, 21.629402799078065, 0.0, 3.1066241377454418,
    5.761018493929694, 5.8567109324681, 5.077678171674349, 29.168136311777292,
     14.277620091662332, 19.400950217386033, 19.467062659579575,
    23.69291879266914, 5.944963569874348, 21.902462840916957,
    19.698064374181957, 7.004636937823044, 23.39846496065996,
    12.374022265581806, 1.1278209763474847, 16.04375282045884,
    9.024211439633165, 6.564650683031586, 16.428282126911345, 0.0,
    26.504574540254122, 27.199978589649344, 27.164239864535183,
    27.38205292141869, 26.812038413244615, 18.625289291098568,
    5.167324256212114, 23.162902604788002, 25.02796593979906,
    11.441868827889774, 21.929383103921236, 16.316493481376252,
    1.6649688219161611, 11.960311597490142, 13.954217336351293,
    20.024479364739523, 20.525227248397332, 3.0578321960809607,
    9.154420675537965, 25.46559528167919]
```

```
42 Custo reduzido: [-16.737085625433608, -21.819060061948072,
     -0.08336433496907622, -2.439687574485241, -11.402593254793214,
     -7.90641843277416, -7.1352737892949385, -23.11928463405156,
     -22.552994462289686, -0.27892718765581037, -4.487772737999411,
     -2.843686696779171, -26.551673171588583, -16.58653123338406,
     -19.91764467216434, -12.76037854919442, -5.012561724798918,
     -11.447265182570554\,,\ \ -14.054449124097825\,,\ \ -9.917337711612047\,,
     -14.731234448662772, -8.498868267790254, -8.524940988460289,
     -16.871683983224216, -0.8826115423082141, -12.764869991264337,
     -10.607551229586974, -9.661612365009812, -3.605426163566797,
     -8.969336474306697, -24.017391474067168, -15.682360887429454,
     -11.99434814058202, -20.169144136018417, -14.004319331240008,
     -13.494971820548145, -0.04076838318140066, -16.097339024622496,
     -2.996014835835248, -10.792232655989494, -4.079806194925279,
     -7.473624754623785, -2.1013916136484543, -21.840912814331453,
     -25.878660682020204, -12.877564732302606, -13.469657925289702,
     -9.72676548154659, -10.251786315237243, -9.811361207401823,
     -21.110494561356916, -16.794010543119086, -23.400571691849446,
     -20.39198271084611, -7.339315614000951, -4.410542291061298,
     -12.569078248917208, -8.252360609449777, -4.934604981987511,
     -20.437427067296515, -6.851725632862179, -18.519115243873948,
     -26.233858399511227\,, \quad -0.045624156080273426\,, \quad -5.944970075781999\,,
     -17.852689690088134, -1.4331095103949476, -6.786686073986363,
     -7.600352147038253, -10.494713950026359, -20.80850984550835,
     -15.987259658463504, -18.55671761580959, -13.816714977959759,
     -16.90461393056777, -3.9257681850469464, -20.07200206296416,
     -19.33267189938791, -13.833426880399529, -28.93809214849319,
     -19.619125830760048, -11.680758659806688, -19.677241329547282,
     -23.811050106726455, -12.002406002941864, -8.965550036048644,
     -14.080337905014929, -6.4428559515156145, -15.537341248460017,
     -16.90555847007076, 4.948913838716175, -11.63683491141023,
     -13.115208978625727, -14.408560355009634, -6.771151149386872,
     -5.319499802161271, -15.072209660563143, -11.184047396128744,
     -6.450954623659852, -11.180067554979349]
              -----Iteração 6-----
44 Base: [101, 102, 103, 104, 105, 106, 107, 108, 109, 110, 111, 112, 113, 114,
     115, 116, 117, 118, 119, 120, 121, 122, 123, 73, 125, 126, 127, 128, 129,
     130, 131, 132, 133, 134, 135, 136, 137, 138, 139, 140, 141, 142, 143, 144,
      145, 146, 147, 148, 149, 150, 151, 152, 153, 154, 155, 156, 157, 158, 64,
      160, 161, 162, 163, 164, 165, 166, 167, 168, 169, 170, 171, 172, 173,
     174, 91, 176, 177, 178, 179, 37, 181, 182, 183, 184, 185, 186, 187, 188,
     189, 190, 191, 192, 193, 194, 195, 196, 197, 198, 199, 200]
45 Não-Base: [1, 2, 124, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18,
     19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36,
     180, 38, 39, 40, 41, 42, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52, 53, 54,
     55, 56, 57, 58, 59, 60, 61, 62, 63, 159, 65, 66, 67, 68, 69, 70, 71, 72,
     3, 74, 75, 76, 77, 78, 79, 80, 81, 82, 83, 84, 85, 86, 87, 88, 89, 90,
     175, 92, 93, 94, 95, 96, 97, 98, 99, 100]
46 Deixa a base: 73
47 Entra na base: 67
48 Função objetivo: 0.27425371743406124
```

```
0.0, 0.0, 0.0, 0.0, 0.0, 0.0, 0.0, 0.016758045044751224, 0.0, 0.0, 0.0,
    0.012877261980871845, 0.0, 0.0, 0.0, 0.0, 0.0, 0.0, 0.0, 0.0, 0.0, 0.0,
    16.81043993467607, 8.637782834385392, 2.534316999252399,
    18.09502717661719, 13.601378899509205, 9.72911421364442,
    6.993732048424714, 24.470395561066987, 2.819987069031185,
    17.569691914420876, 7.3337246600714625, 21.214322934083853,
    5.392834734379608, 15.925983486738719, 14.737099804734838,
    6.37712243629382, 7.741847298469915, 13.45023750130604, 28.19161237141829,
     1.5948689329037546, 0.622011481066601, 5.86031296693814,
    4.498773947183965, 0.0, 15.983492053360367, 1.1050301336332522,
    4.466336065860715, 15.925317935276171, 21.568034856323102,
    11.081583092448295, 22.564193470426645, 24.298126476279528,
    22.65388046702215, 16.199825499046085, 27.747145981132356,
    15.350971583850946, 20.084020658810324, 11.191490040309635,
    2.0650072643676567, 19.66133586435034, 0.23743841508260796,
    0.5856534735362801\,,\ 19.508338902780913\,,\ 2.297243517822097\,,
    14.043587468788855, 3.1380906104056865, 1.2954579360886938,
    4.10960467555911, 6.093461574782439, 8.833010628618348,
    19.567985795572092, 23.46708016854909, 22.096799785783112,
    5.710802438588769, 21.226355020800952, 19.78308553050764,
    12.57951021662958, 21.987946918117753, 0.0, 2.8182257576816756,
    2.977437673059835, 7.119337470922824, 4.37003077057888,
    27.172427591868992, 11.96701242216863, 17.920983733629434,
    19.24898516840486, 22.009540090946533, 6.804080938772515,
    20.5216907743476, 18.09366966935119, 5.755464726123638,
    22.895421752012158, 12.742649267858278, 0.0, 16.242156384577733,
    6.701758938335326, 5.956901558512501, 15.548653604166866, 0.0,
    25.00471150009393, 28.190718365006486, 27.169598201295788,
    27.433031980343397, 25.669575659687123, 18.492308023829445,
    4.118592456575592, 21.59662505117444, 24.82556679922771,
    12.04444142974253, 20.945350625065707, 16.69196589245268,
    0.3238239033865222, 10.556830339724865, 14.452993884889725,
    20.626187405234763, 20.125489776887513, 1.4583485434041894,
    9.333047668978868, 24.43278457639658]
```

```
50 Custo reduzido: [-18.382496334186083, -10.893985936629377,
     -0.03190130594924516, 1.234649042955417, -9.71401092391477,
     -3.995892051815286, -9.36929043608032, -21.036809677078836,
     -20.62502713366794, 1.329716771507881, -1.3893123547896344,
     1.7759790891666603, -19.48747410483748, -18.193795274328004,
     -8.343549890887278, -11.229003727347507, -3.827593565368831,
     -4.423787104784427, -5.423663659426097, -11.051142520556485,
     -16.433359495465584, -8.460848696833004, 0.10846271145216768,
     -7.049211370543862, -7.967369853206447, -6.64525936869234,
     -5.373633713797158, -6.438553654755159, -1.9047094755360465,
     -10.552649526364203, -14.290178168658674, -8.939876870143163,
     -6.167080996470386, -10.83848954960759, -8.177479363555717,
     -11.89740340117084, -0.018667801463559543, -12.17867259665394,
     -1.9887103952444853, -8.789764181008389, -4.334885431362383,
     -14.261201518253646, -4.054749069891898, -14.436053413536992,
     -22.425145435919873, -12.213099319111063, -10.024046345439336,
     -2.890677925171481, -9.078501307440312, -11.34925830679213,
     -11.748459418135315, -4.633177876819804, -19.440039266969045,
     -7.214532041606876, -6.702575561795236, -2.1862107050734263,
     -13.660335423458637, -9.18945334721684, -5.601514277468645,
     -18.348749830769115, -4.001171785480169, -11.191487683502746,
     -20.416590699599695, -0.03549931945681449, -0.48068174684528486,
     -7.535695427400256, 2.578811082357012, -4.161651669721383,
     -12.675261074777413, -8.629356040940426, -12.391864027071765,
     -12.311790398533486, -12.962472773140284, -1.976567585874486,
     -4.580791934658356, -8.487742227075275, -16.90574949592696,
     -12.976291305978425, -13.599442880563135, -16.483987057091845,
     -10.892360887886012, -7.72878757641592, -12.834956473220686,
     -15.004130530311542, -4.836903560380236, -3.641670107109517,
     -15.413498352855349, -6.61423557825966, -13.525095874094166,
     -15.293792767603875, -0.05650582970029409, -11.04840884065414,
     -13.694279277376264, -6.288067712237154, -4.248441374043539,
     -5.9387943326857275, -15.172595519425116, -12.337174870047427,
     -3.273865337267309, -8.417739308686546]
                -----Iteração 7-----
52 Base: [101, 102, 103, 104, 105, 106, 107, 108, 109, 110, 111, 112, 113, 114,
     115, 116, 117, 118, 119, 120, 121, 122, 123, 67, 125, 126, 127, 128, 129,
     130, 131, 132, 133, 134, 135, 136, 137, 138, 139, 140, 141, 142, 143, 144,
      145, 146, 147, 148, 149, 150, 151, 152, 153, 154, 155, 156, 157, 158, 64,
      160, 161, 162, 163, 164, 165, 166, 167, 168, 169, 170, 171, 172, 173,
     174, 91, 176, 177, 178, 179, 37, 181, 182, 183, 184, 185, 186, 187, 188,
     189, 190, 191, 192, 193, 194, 195, 196, 197, 198, 199, 200]
53 Não-Base: [1, 2, 124, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18,
     19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36,
     180, 38, 39, 40, 41, 42, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52, 53, 54,
     55, 56, 57, 58, 59, 60, 61, 62, 63, 159, 65, 66, 73, 68, 69, 70, 71, 72,
     3, 74, 75, 76, 77, 78, 79, 80, 81, 82, 83, 84, 85, 86, 87, 88, 89, 90,
     175, 92, 93, 94, 95, 96, 97, 98, 99, 100]
54 Deixa a base: 193
55 Entra na base: 10
56 Função objetivo: 0.28003381780935194
```

```
0.014694276042041743, 0.0, 0.0, 0.0, 0.0, 0.0, 0.0, 0.0, 0.0, 0.0, 0.0,
    17.190514188295083, 8.908702175270154, 2.48289222547697,
    17.961903318653448, 13.470283111251469, 9.580465207676006,
    6.758266917837466, 24.496014492190756, 2.4873276065416845,
    17.627772801866776, 7.115844910349959, 21.095031371722886,
    5.0074627585547375, 15.957694572260381, 14.98998892911488,
    6.698696231032537, 7.673521537537595, 13.85675539861455,
    28.413584828331555, 1.4749926360892813, 0.6217727997713223,
    5.855184437015523, 4.439854290489141, 0.0, 15.713440761949165,
    1.2239256051548661, 4.513632005049909, 16.329649613693206,
    21.609125858907653, 10.850016404188214, 22.3250854703186,
    24.131518542260356, 22.784460762847328, 15.936965213257881,
    27.368134767749314, 15.207599183739454, 20.25181332124134,
    11.545111680168587, 1.6402378713870456, 19.46052651211986,
    0.2840037180658973, 0.5425245115350854, 19.262152349440683,
    2.1788096076192636\,,\ 13.837593605123203\,,\ 2.9792023940829733\,,
    1.233487778256565, 3.8517700861526003, 6.315168658935856,
    9.16536805445389, 19.309138706235764, 23.438512131277434,
    21.926035121256778, 5.569467645525166, 21.132681232680184,
    19.957560390661378, 12.393220458452351, 22.054899187312568, 0.0,
    2.552403731115544, 2.5270484849708783, 7.312122169419486,
    4.101980219088854, 26.97672973502891, 11.60001840029304,
    17.801827904548404, 19.134367119337472, 21.491863804448634,
    7.11879677907802\,,\ 20.507661754851807\,,\ 17.610277331398958\,,
    5.29296525328836, 22.54701365194804, 12.970112638479554, 0.0,
    16.11737334859357, 6.405015613513923, 5.643263785668991,
    15.292987671081084, 0.0, 25.03615653970729, 28.564165593677526,
    27.131187911430168, 27.215288324065693, 25.60278319516012,
    18.52350865950017, 4.0465262591474405, 21.644118804980273,
    24.84485808995276, 12.00216599954615, 20.64983200023035,
    16.648250452953224, 0.11571248098778072, 10.264114945001255,
    14.43525465077071, 20.541695577068857, 20.13867370196374,
    1.2031726361160169, 9.693185988160597, 24.094739272038453
```

```
58 Custo reduzido: [-19.321251847119534, -20.99739371934014,
     -0.09358353637302223, -1.290966836226593, -13.541718536790487,
     -7.2726669084151325, -11.688479895539224, -28.714189545129074,
     -27.332457636423285, 0.6530068546420589, -2.946930382764032,
     -2.3374825096959206, -28.790047345438886, -21.24738133626446,
     -18.72172994403283, -14.566166792641045, -6.709630116536033,
     -11.198218845852544, -13.121825764290477, -13.296509107097322,
     -19.04896201284248, -14.89083438157439, -6.966436608208552,
     -15.297422618602674, -5.194788869372616, -14.782725127115743,
     -15.570918678255612, -10.58027717640729, -3.687546260406881,
     -13.968734314545, -24.477849974350498, -18.645800103306094,
     -11.54905582073743, -20.567364455540094, -16.33498927540744.
     -19.51866918010888, -0.023542114578440514, -18.993192830916325,
     -3.477632444084591, -13.632398779330412, -5.608320299641896,
     -11.999119979701506, -3.1729014444103534, -24.22989838435455,
     -30.43108793997038, -20.42106471183996, -13.12557605688692,
     -10.681998787018575, -9.772827770980975, -11.512381070538655,
     -22.406092263686553, -15.816010437117331, -28.206618346598724,
     -20.418970337231876, -13.529528243293711, -3.9770244179078302,
     -15.454342218065182, -10.864446851170818, -8.619229039643798,
     -26.338747516294628, -5.334966911807609, -20.544722773915886,
     -29.62005793798143, -0.042223126801936184, -4.729854072633717,
     -19.137577578478798, -3.449149563600905, -8.679854085278727,
     -11.766803689177458, -13.040628230009228, -24.01558215430978,
     -18.695334501467126, -22.763458109093676, -13.23400945709308,
     -17.88442213415435, -8.432084944683687, -21.954053424147133,
     -22.27307736537428\,,\ \ -17.19154658093977\,,\ \ -29.016563009514783\,,
     -18.29130872111387, -10.2867199922309, -22.2770260762103,
     -26.237261861532318, -14.2626972228347, -10.146673927550673,
     -17.812511129865058, -11.304113561998676, -19.60244680181916,
     -20.04640811864428, -0.03238097515917082, -13.441181076983991,
     -18.94098834899803, -14.648749750125607, -6.195952463007297,
     -6.790827470174307, -18.543785887210056, -16.878743839126297,
     -6.825876072346354, -10.553103054249931]
               ----Fim do algoritmo----
60 Status: Optimal
 Base: [101, 102, 103, 104, 105, 106, 107, 108, 109, 110, 111, 112, 113, 114,
     115, 116, 117, 118, 119, 120, 121, 122, 123, 67, 125, 126, 127, 128, 129,
     130, 131, 132, 133, 134, 135, 136, 137, 138, 139, 140, 141, 142, 143, 144,
      145, 146, 147, 148, 149, 150, 151, 152, 153, 154, 155, 156, 157, 158, 64,
      160, 161, 162, 163, 164, 165, 166, 167, 168, 169, 170, 171, 172, 173,
     174, 91, 176, 177, 178, 179, 37, 181, 182, 183, 184, 185, 186, 187, 188,
     189, 190, 191, 192, 10, 194, 195, 196, 197, 198, 199, 200]
62 Não-Base: [1, 2, 124, 4, 5, 6, 7, 8, 9, 193, 11, 12, 13, 14, 15, 16, 17, 18,
     19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36,
     180, 38, 39, 40, 41, 42, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52, 53, 54,
     55, 56, 57, 58, 59, 60, 61, 62, 63, 159, 65, 66, 73, 68, 69, 70, 71, 72,
     3, 74, 75, 76, 77, 78, 79, 80, 81, 82, 83, 84, 85, 86, 87, 88, 89, 90,
     175, 92, 93, 94, 95, 96, 97, 98, 99, 100]
63 Função objetivo: 0.2804588299312376
```

```
0.0, 0.0, 0.004729266302564598, 0.0, 0.0, 0.0020705902255231754, 0.0,
    0.0, 0.0, 0.0, 0.0, 0.0, 0.0, 0.0, 0.014767380948691752, 0.0, 0.0, 0.0,
    0.0, 0.0, 0.0, 0.0, 0.0, 0.0, 17.14800634837941, 8.92331004279684,
    2.460376125741143, 18.007267693115693, 13.47485222316133,
    9.562256560661641, 6.677147878717352, 24.46145161345749,
    2.4536249978874403, 17.66959114586471, 7.161465583907538,
    21.055236144848216, 5.011673790944005, 15.959878698650538,
    15.00445140579506, 6.660712323122434, 7.599655315045025,
    13.82787749008871, 28.36241867948468, 1.4843887172357264,
    0.6784451362162832, 5.851834920970579, 4.445876248275907, 0.0,
    15.651102680914546, 1.1744808888860425, 4.513688120303374,
    16.387900098822332, 21.608539129951748, 10.769491444937744,
    22.327046031800805, 24.08922466271495, 22.8600345182444,
    15.816433092819539, 27.35050813474582, 15.257183487808419,
    20.248186740981154, \ 11.533742702793528, \ 1.5572928695026294,
    19.55748035427604, 0.2817460995450398, 0.5829231522128885,
    19.304649577947977, 2.2034759922393627, 13.768201395960874,
    2.942957215714324, 1.2527327460163484, 3.8208328760658365,
    6.344240917635527, 9.128649822116564, 19.275792209643036,
    23.41687771141243, 22.006371072155762, 5.550346240474677,
    21.139074399098746, 19.98726664399911, 12.378002152107127,
    22.045135688602873, 0.0, 2.488612144947306, 2.546744066757558,
    7.341973936055161, 4.064807086706504, 26.885403505670496,
    11.570106419384345\,,\ 17.72505998702788\,,\ 19.03603242015105\,,
    21.388871891118097, 7.17075124894955, 20.40745229543541,
    17.561727075334232, 5.2641348617905175, 22.497887806558282,
    13.049789683689724, 0.0, 16.05522483474873, 6.389301454715951,
    5.63408111235638, 15.243794903518689, 0.0, 25.005154493813603,
    28.562777824748125, 27.035332759749434, 27.175050472271547,
    25.631061429795707, 18.43628655592846, 4.004554447474972,
    21.662714199961865, 24.7968973000776, 11.937265614814807,
    20.545762094945502, 16.662693150914862, 0.0, 10.147941177223167,
    14.361401099128193, 20.538643497679672, 20.0289980551438,
    1.144381846583989, 9.6737490808333, 24.087673245398346
```