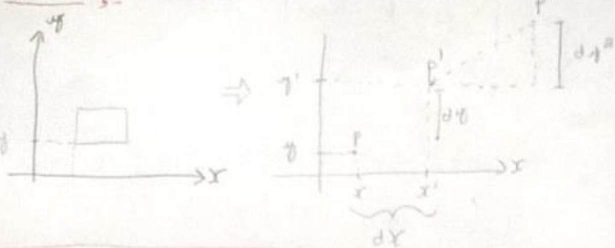
Computação Gráfica

Wednesday, July 22, 2015 10:31 AM

TRANSFORMAÇÕES GEOMÉTRICAS

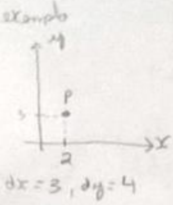
TRANSLAÇÃO



$$x' = x + dx \qquad y' = y + dy \qquad \begin{bmatrix} x' \\ y' \end{bmatrix} = \begin{bmatrix} x \\ y \end{bmatrix} + \begin{bmatrix} dx \\ dy \end{bmatrix}$$

$$P' = T(P)$$

exemplo



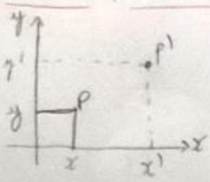
$dx = 3,\ dy = 4$

$$\begin{bmatrix} x' \\ y' \end{bmatrix} = \begin{bmatrix} x \\ y \end{bmatrix} + \begin{bmatrix} dx \\ dy \end{bmatrix}$$

$$\begin{bmatrix} x' \\ y' \end{bmatrix} = \begin{bmatrix} 2 \\ 3 \end{bmatrix} + \begin{bmatrix} 3 \\ 4 \end{bmatrix}$$

$$\begin{bmatrix} x' \\ y' \end{bmatrix} = \begin{bmatrix} 5 \\ 7 \end{bmatrix}$$

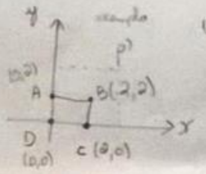
Mudança de Escala



$$x' = S_x \cdot X \qquad y' = S_y \cdot Y \qquad \begin{bmatrix} x' \\ y' \end{bmatrix} = \begin{bmatrix} S_x & 0 \\ 0 & S_y \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix}$$

$$P' = S \cdot P$$

exemplo



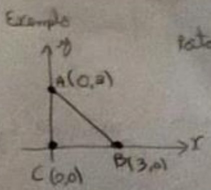
Queremos dobrar seu tamanho $S_x = 2;\ S_y = 2$

$$\begin{bmatrix} x' \\ y' \end{bmatrix} = \begin{bmatrix} 2 & 0 \\ 0 & 2 \end{bmatrix} \cdot \begin{array}{c} \begin{array}{cccc} A & B & C & D \end{array} \\ \begin{bmatrix} 0 & 2 & 2 & 0 \\ 2 & 2 & 0 & 0 \end{bmatrix} \end{array} = \begin{array}{c} \begin{array}{cccc} A' & B' & C' & D' \end{array} \\ \begin{bmatrix} 0 & 4 & 4 & 0 \\ 4 & 4 & 0 & 0 \end{bmatrix} \end{array}$$

ROTAÇÃO

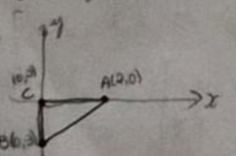
$$\begin{bmatrix} x' \\ y' \end{bmatrix} = \begin{bmatrix} \cos\theta & -\operatorname{sen}\theta \\ \operatorname{sen}\theta & \cos\theta \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} \qquad P = R_\theta^{\circ} P$$

Exemplo



Rotacionar ABC de $-90°$

$$\begin{bmatrix} x' \\ y' \end{bmatrix} = \begin{bmatrix} \cos\overset{-90°}{\theta} & -\operatorname{sen}\overset{-90°}{\theta} \\ \operatorname{sen}\overset{-90°}{\theta} & \cos\overset{-90°}{\theta} \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} \Rightarrow \begin{bmatrix} x' \\ y' \end{bmatrix} \begin{bmatrix} 0 & 1 \\ -1 & 0 \end{bmatrix} \cdot \begin{array}{c} \begin{array}{ccc} A & B & C \end{array} \\ \begin{bmatrix} 0 & 3 & 0 \\ 2 & 0 & 0 \end{bmatrix} \end{array} = \begin{array}{c} \begin{array}{ccc} A' & B' & C' \end{array} \\ \begin{bmatrix} 2 & 0 & 0 \\ 0 & -3 & 0 \end{bmatrix} \end{array}$$

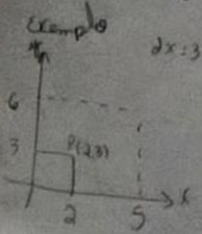


TRANSFORMAÇÕES COM COORDENADAS HOMOGÊNEAS

TRANSLAÇÃO:

$$\begin{bmatrix} x' \\ y' \\ w' \end{bmatrix} = \begin{bmatrix} 1 & 0 & dx \\ 0 & 1 & dy \\ 0 & 0 & 1 \end{bmatrix} \cdot \begin{bmatrix} x \\ y \\ w \end{bmatrix} \quad \therefore \quad \begin{bmatrix} x' \\ y' \\ w' \end{bmatrix} = \begin{bmatrix} 1 & 0 & dx \\ 0 & 1 & dy \\ 0 & 0 & 1 \end{bmatrix} \cdot \begin{bmatrix} x \\ y \\ 1 \end{bmatrix}$$

Exemplo



$dx = 3,\ dy = 3$ ANTES

$$\begin{bmatrix} x' \\ y' \end{bmatrix} = \begin{bmatrix} x \\ y \end{bmatrix} + \begin{bmatrix} dx \\ dy \end{bmatrix}$$

$$\begin{bmatrix} x' \\ y' \end{bmatrix} = \begin{bmatrix} 2 \\ 3 \end{bmatrix} + \begin{bmatrix} 3 \\ 3 \end{bmatrix}$$

$$\begin{bmatrix} x' \\ y' \end{bmatrix} = \begin{bmatrix} 5 \\ 6 \end{bmatrix} \Rightarrow (5,6)$$

\# Em Coordenadas Homogêneas

$$\begin{bmatrix} x' \\ y' \\ w' \end{bmatrix} = \begin{bmatrix} 1 & 0 & dx \\ 0 & 1 & dy \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \\ w \end{bmatrix}$$

$$\begin{bmatrix} x' \\ y' \\ w' \end{bmatrix} = \begin{bmatrix} 1 & 0 & 3 \\ 0 & 1 & 3 \\ 0 & 0 & 1 \end{bmatrix} \cdot \begin{bmatrix} 2 \\ 3 \\ 1 \end{bmatrix}$$

$$\begin{bmatrix} x' \\ y' \\ w' \end{bmatrix} = \begin{bmatrix} 5 \\ 6 \\ 1 \end{bmatrix} \Rightarrow P' = \begin{bmatrix} 5 \\ 6 \\ 1 \end{bmatrix}$$

Coordenadas Cartesianas

$$\frac{5}{1};\ \frac{6}{1};\ \frac{1}{1}$$

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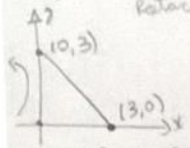
DESLOCAMENTO

ROTAÇÃO

$$\begin{bmatrix}x'\\y'\\w'\end{bmatrix} = \begin{bmatrix}\cos\theta & -\text{sen}\,\theta & 0\\ \text{sen}\,\theta & \cos\theta & 0\\ 0 & 0 & 1\end{bmatrix}\cdot\begin{bmatrix}x\\y\\w\end{bmatrix} \qquad P' = R^{o}_{\theta}\cdot P$$

Exemplo

Rotacionar o triângulo $\theta = 90°$ $\quad P' = R^{o}_{\theta}\cdot P$



$$\begin{bmatrix}x'\\y'\\w'\end{bmatrix} = \begin{bmatrix}\cos 90° & -\text{sen}\,90° & 0\\ \text{sen}\,90° & \cos 90° & 0\\ 0 & 0 & 1\end{bmatrix}\cdot\begin{bmatrix}3\\0\\1\end{bmatrix} = \begin{bmatrix}0 & -1 & 0\\ 1 & 0 & 0\\ 0 & 0 & 1\end{bmatrix}\cdot\begin{bmatrix}3\\0\\1\end{bmatrix} = \begin{bmatrix}0\\3\\1\end{bmatrix}$$

$$\begin{bmatrix}x'\\y'\\w'\end{bmatrix} = \begin{bmatrix}0\\3\\1\end{bmatrix} \rightarrow \frac{0}{1}, \frac{3}{1}; \frac{1}{1}$$

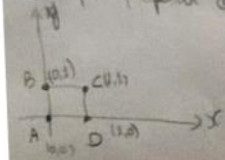
$x' = 0$
$y' = 3$

Mudança de Escala

$$P' = S(S_x, S_y)\cdot P$$

$$\begin{bmatrix}x'\\y'\\w'\end{bmatrix} = \begin{bmatrix}S_x & 0 & 0\\ 0 & S_y & 0\\ 0 & 0 & 1\end{bmatrix}\cdot\begin{bmatrix}x\\y\\1\end{bmatrix}$$

Exemplo, para o tamanho $S_x = 2$, $S_y = 3$



$$\begin{bmatrix}x'\\y'\\w'\end{bmatrix} = \begin{bmatrix}2 & 0 & 0\\ 0 & 2 & 0\\ 0 & 0 & 1\end{bmatrix}\cdot \begin{array}{cccc} A & B & C & D \\ \hline \end{array}\begin{bmatrix}0 & 0 & 1 & 1\\ 0 & 1 & 1 & 0\\ 1 & 1 & 1 & 1\end{bmatrix} \Rightarrow \begin{bmatrix}x'\\y'\\w'\end{bmatrix} = \begin{array}{cccc} A' & B' & C' & D' \\ \hline \end{array}\begin{bmatrix}0 & 0 & 2 & 2\\ 0 & 3 & 3 & 0\\ 1 & 1 & 1 & 1\end{bmatrix}$$

* TRANSFORMAÇÃO FORA DA ORIGEM

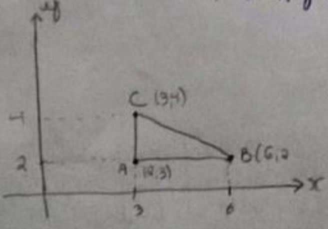
$$\begin{bmatrix}1 & 0 & dx\\ 0 & 1 & dy\\ 0 & 0 & 1\end{bmatrix}$$

1º Transladar o objeto para a origem $T(-x_A, -y_A)$

2º Rotação $R^{o}_{90°}$ $\begin{bmatrix}\cos\theta & -\text{sen}\,\theta & 0\\ \text{sen}\,\theta & \cos\theta & 0\\ 0 & 0 & 1\end{bmatrix} = \begin{bmatrix}0 & -1 & 0\\ 1 & 0 & 0\\ 0 & 0 & 1\end{bmatrix}$

* mudança de Escala (S_{xA}, S_{yA}) $\begin{bmatrix}S_x & 0 & 0\\ 0 & S_y & 0\\ 0 & 0 & 1\end{bmatrix}$

3º Translação de volta $T(x_A, y_A)$ $\begin{bmatrix}1 & 0 & dx\\ 0 & 1 & dy\\ 0 & 0 & 1\end{bmatrix}$



$$\underbrace{\begin{bmatrix}1 & 0 & 3\\ 0 & 1 & 2\\ 0 & 0 & 1\end{bmatrix}}_{\text{TRANS VOLTA}}\cdot\underbrace{\begin{bmatrix}0 & -1 & 0\\ 1 & 0 & 0\\ 0 & 0 & 1\end{bmatrix}}_{\text{ROTAÇÃO } R^{o}_{90}}\cdot\underbrace{\begin{bmatrix}1 & 0 & -3\\ 0 & 1 & -2\\ 0 & 0 & 1\end{bmatrix}}_{\text{TRAS ORIGEM}}\cdot\begin{array}{ccc} A & B & C \\ \hline \end{array}\begin{bmatrix}3 & 6 & 3\\ 2 & 2 & 4\\ 1 & 1 & 1\end{bmatrix}$$

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Sendo Cossim Tomas

$$\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 2 \\ 0 & 0 & 1 \end{bmatrix} \cdot \begin{bmatrix} \sqrt{2}/2 & -\sqrt{2}/2 & 0 \\ \sqrt{2}/2 & \sqrt{2}/2 & 0 \\ 0 & 0 & 1 \end{bmatrix} \cdot \begin{bmatrix} 1 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & 0 \end{bmatrix} \cdot \begin{bmatrix} \sqrt{2}/2 & \sqrt{2}/2 & 0 \\ -\sqrt{2}/2 & \sqrt{2}/2 & 0 \\ 0 & 0 & 1 \end{bmatrix} \cdot \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & -2 \\ 0 & 0 & 1 \end{bmatrix} \cdot \begin{bmatrix} 2 & 4 & 4 \\ 7 & 7 & 9 \\ 1 & 1 & 1 \end{bmatrix}$$

Podemos Calcular Direto em função de M

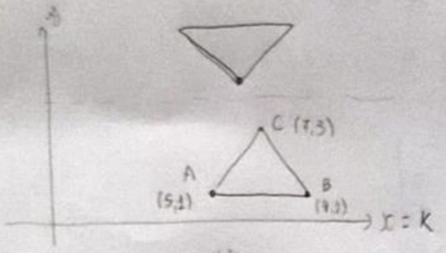
$$M.AB = \begin{bmatrix} \frac{1-m^2}{m^2+1} & \frac{2m}{m^2+1} & \frac{-2bm}{m^2+1} \\ \frac{2m}{m^2+1} & \frac{m^2-1}{m^2+1} & \frac{2b}{m^2+1} \\ 0 & 0 & 1 \end{bmatrix} \Rightarrow M_{obs} . M_{AB}$$

Em Relação a X

$b = K$

Exercício

$k = 4$



$m : 0$

$K = 4$

$$M_{AB}: \begin{bmatrix} \overset{1}{\frac{1-m^2}{m^2+1}} & \overset{0}{\frac{2m}{m^2+1}} & \overset{0}{\frac{-2bm}{m^2+1}} \\ \frac{2m}{m^2+1} & \frac{m^2-1}{m^2+1} & \frac{2b}{m^2+1} \\ 0 & 0 & 1 \end{bmatrix}$$

M obs PONTO

$$\begin{bmatrix} 1 & 0 & 0 \\ 0 & -1 & 8 \\ 0 & 0 & 1 \end{bmatrix} \cdot \begin{bmatrix} 5 & 9 & 7 \\ 1 & 1 & 3 \\ 1 & 1 & 1 \end{bmatrix} = \begin{bmatrix} 5 & 9 & 7 \\ 7 & 7 & 5 \\ 1 & 1 & 1 \end{bmatrix}$$

Se não der = 1 tem que dividir tudo por essa LINHA, ou seja passar para coordenadas cartesianas)

Paralelo a y $\begin{bmatrix} -1 & 0 & 2k \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$ Paralelo a X $\begin{bmatrix} 1 & 0 & 0 \\ 0 & -1 & 2k \\ 0 & 0 & 1 \end{bmatrix}$

* Definição de Computação Gráfica (POSTILA)

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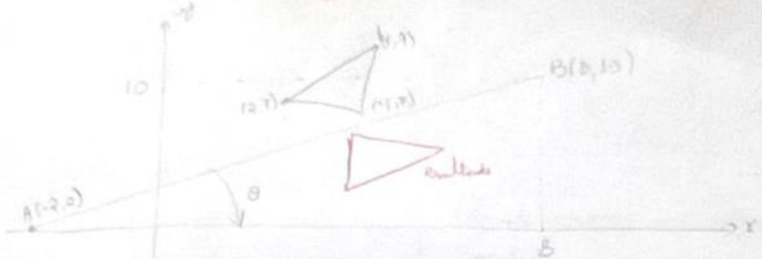
Espelhamento de um objeto em relação a um eixo



	30°	45°	60°
SEN	$\frac{1}{2}$	$\frac{\sqrt{2}}{2}$	$\frac{\sqrt{3}}{2}$
COS	$\frac{\sqrt{3}}{2}$	$\frac{\sqrt{2}}{2}$	$\frac{1}{2}$
TG	$\frac{\sqrt{3}}{3}$	1	$\sqrt{3}$

1º Translação para origem $T(0,-b)$ $\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & -b \\ 0 & 0 & 1 \end{bmatrix}$

2º Girar $-\theta$ graus $R^{\circ}_{-\theta}$ $\begin{bmatrix} \cos\theta & +\text{sen}\,\theta & 0 \\ -\text{sen}\,\theta & \cos\theta & 0 \\ 0 & 0 & 1 \end{bmatrix}$

3º Espelhar / refletir em relação ao eixo X. M_X

4º Girar θ graus $R^{\circ}_{+\theta}$ $\begin{bmatrix} \cos\theta & -\text{sen}\,\theta & 0 \\ \text{sen}\,\theta & \cos\theta & 0 \\ 0 & 0 & 1 \end{bmatrix}$

5º Translação de volta $T(0,b)$ $\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & b \\ 0 & 0 & 1 \end{bmatrix}$

$$T(0,b) \, . \, R^{\circ}_{\theta} \, M_X \, R^{\circ}_{-\theta} \, . \, T(0,-b) \, . \, M_{objeto}$$

$M_{obj} = \begin{bmatrix} 2 & 4 & 4 \\ 7 & 7 & 9 \\ 1 & 1 & 1 \end{bmatrix}$

$T(0,-b)$ TRANSLAÇÃO $M=1$

$m = \frac{\Delta y}{\Delta x} = \frac{10-0}{8-(-2)} = \frac{10}{10} = \boxed{1 = M}$

$T(0,-b) = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & -2 \\ 0 & 0 & 1 \end{bmatrix}$

$y = mx + b$ com ponto $(-2,0)$

$0 = -2 + b$

$\boxed{b = 2}$

$T(0,b) = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 2 \\ 0 & 0 & 1 \end{bmatrix}$

Hipotenusa $L = \sqrt{(8-(-2))^2 + (10-0)^2} = \sqrt{10^2+10^2} = L = \sqrt{200}$

$\cos\theta = \frac{10}{\sqrt{200}} = \frac{10 \cdot \sqrt{200}}{\sqrt{200} \cdot \sqrt{200}} \quad \frac{\sqrt{2}}{2} = \cos 45°$

$\text{sen}\,\theta = \frac{10}{\sqrt{200}} = \frac{\sqrt{2}}{2} = \text{sen}\,45°$

$R^{\circ}_{-\theta} \begin{bmatrix} \sqrt{2}/2 & +\sqrt{2}/2 & 0 \\ -\sqrt{2}/2 & \sqrt{2}/2 & 0 \\ 0 & 0 & 1 \end{bmatrix}$

$R^{\circ}_{\theta} \begin{bmatrix} \sqrt{2}/2 & -\sqrt{2}/2 & 0 \\ \sqrt{2}/2 & \sqrt{2}/2 & 0 \\ 0 & 0 & 1 \end{bmatrix}$

Em relação a X altera o eixo y como $m=1$

$M_X = \begin{bmatrix} 1 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & 0 \end{bmatrix}$

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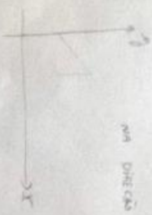
DESLOCAMENTO OU CEZALHAMENTO

NA DIREÇÃO X

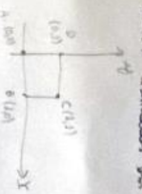
$$SH_X = \begin{bmatrix} 1 & a & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$



NA DIREÇÃO Y

$$SH_Y = \begin{bmatrix} 1 & 0 & 0 \\ b & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

Deslocar 2 unidades em X e 1,5 em Y.



$X = 2$

$$M'obj = \begin{bmatrix} 1 & 2 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{array}{c} \begin{array}{cccc} A & B & C & D \end{array} \\ \begin{bmatrix} 0 & 1 & 1 & 0 \\ 0 & 0 & 1 & 1 \\ 1 & 1 & 1 & 1 \end{bmatrix} \end{array} = \begin{array}{c} \begin{array}{cccc} A & B & C & D \end{array} \\ \begin{bmatrix} 0 & 1 & 3 & 2 \\ 0 & 0 & 1 & 1 \\ 1 & 1 & 1 & 1 \end{bmatrix} \end{array}$$

$Y = 1,5$

$$M'obj = \begin{bmatrix} 1 & 0 & 0 \\ 1,5 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \cdot \begin{bmatrix} 0 & 1 & 1 & 0 \\ 0 & 0 & 1 & 1 \\ 1 & 1 & 1 & 1 \end{bmatrix} = \begin{bmatrix} 0 & 1 & 1 & 0 \\ 0 & 1,5 & 2,5 & 1 \\ 1 & 1 & 1 & 1 \end{bmatrix}$$

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[Obtenha o Office Lens gratuitamente]