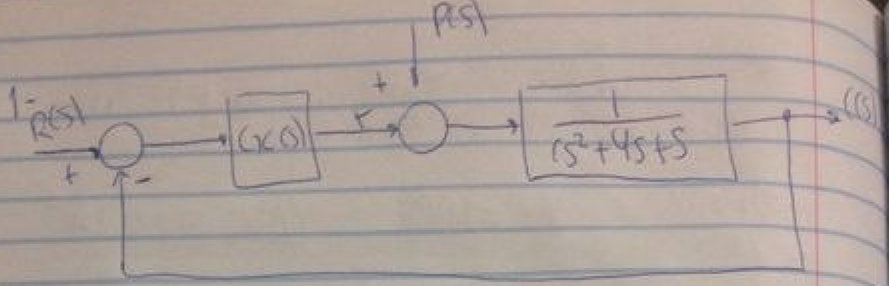


$M_p = 12\%$ PID zero duplo.

$t_p = 0{,}77s$

A) $G_c(s)$

$$G(s) = \frac{1}{s^2 + 4s + 5}$$

polos: $s = -2 \pm j1$

$$G_c(s) = K_p\left(1 + \frac{1}{sT_i} + sT_D\right)$$

$$G_c(s) = \frac{K_p}{sT_i}\left(T_i T_D s^2 + T_i s + 1\right)$$

$$G_c(s) = \frac{K_p}{T_i} \cdot \frac{1}{s} \, \cancel{T_i} T_D \left(s^2 + \frac{s}{T_D} + \frac{1}{T_i T_D}\right)$$

$$G_c(s) = \frac{K_P T_D}{s}\left(s^2 + \frac{s}{T_D} + \frac{1}{T_I T_D}\right)$$

polo duplo $\Delta = 0$

$$\frac{1}{T_D^2} - \frac{4}{T_I T_D} = 0$$

$$\frac{1}{T_D} = \frac{4}{T_I} \Rightarrow \boxed{T_I = 4T_D}$$

$$s = -\frac{1}{T_D} \cdot \frac{1}{2} = -\frac{1}{2T_D}$$

Assim:

$$\boxed{G_c(s) = \frac{K_D T_D}{s}\left(s + \frac{1}{2T_D}\right)^2}$$

$$t_p = 0{,}77s = \frac{\pi}{w_d} \Rightarrow w_d = 4{,}08 \text{ rad/s}$$

$$\xi = \sqrt{\frac{(\ln(M_p))^2}{\pi^2 + (\ln(M_p))^2}} = 0{,}56$$

$$w_n = \frac{w_d}{\sqrt{1-\xi^2}} = 4{,}9$$

$$\boxed{s = -2{,}75 \pm j\,4{,}08}$$

$$G(s) = K \frac{(s+z)^2}{s}$$

$K = K_P T_D$

$z = \frac{1}{2T_D}$

$$\boxed{FTMA = K \frac{(s+z)^2}{s} \cdot \frac{1}{(s+2-j)(s+2+j)}}$$

condição de fase

$\angle FTMA = 180° \pm n360°$

$s = -2{,}75 + j\,4{,}08$

$2 \cdot \angle (z-2{,}75) + j\,4{,}08 - \angle -2{,}75 + j\,4{,}08$

$- \angle -0{,}75 + j\,3{,}08 - \angle -0{,}75 + j\,5{,}08$

$= 180 \pm n360°$

$2 \operatorname{arctg} \frac{4{,}08}{(z-2{,}75)} - 180 + 56{,}02 - 180 + 76{,}31$

$-180 + 81{,}6 = 180 \pm n360°$

$2 \operatorname{arctg} \frac{4{,}08}{z-2{,}75} = 326{,}07 + 180 \pm 360\,n$

$\cancel{+23{,}39}$ $+13$

$n=1$

$\operatorname{arctg} \frac{4{,}08}{z-2{,}75} = \cancel{33{,}93}$ $73{,}03$

157,91

326,07

$\frac{4{,}08}{2-2{,}75} = \cancel{0{,}305}$ $3{,}28$

$\boxed{Z = 4}$

condicao

$K \cdot 0{,}2273 = 1$

$\boxed{K = 4{,}4}$

$4{,}4 = K_p T_D$

$4 = \frac{1}{2T_D}$

$T_D = \frac{1}{8}$ $T_I = \frac{1}{2}$

$K_p = \cancel{0{,}55}$ $4{,}4 \cdot 8$

$\boxed{PID = 35{,}2 \left(1 + \frac{2}{s} + \frac{s}{8}\right)}$

B) forneça a expressão no domínio do tempo para o sinal de saída c(t) em regime estacionário como resposta para uma entrada.

sistema linear

"sup"

$$E = R - C$$

$$C = G_c G E$$

$$E = R - G_c G E$$

$$\boxed{E = \frac{R}{1 + G_c G}}$$

$$e_\infty = \lim_{s \to 0} s \frac{1}{s} \frac{1}{1 + G_c G} \cdot S$$

$$e_\infty = \frac{S}{1 + \frac{K}{S}} = 0\%$$

$$C = [P + G_c E] G$$

$$E = -C$$

$$-E = PG + GG_c E$$

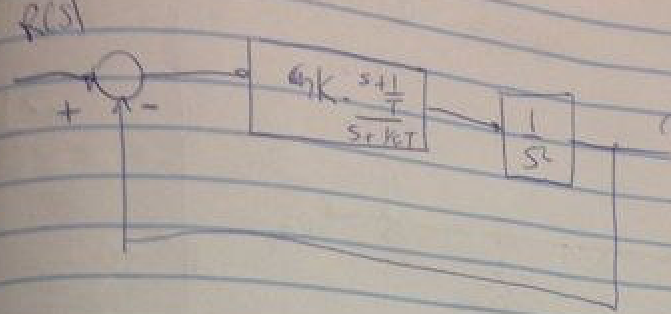
$$E = \frac{-GP}{1 + GG_c}$$

$$e_\infty = \lim_{s \to 0} -\cancel{s} \frac{3}{\cancel{s}} \cdot \frac{\frac{1}{(s^2+4s+5)}}{1 + \frac{K(s+2)^2}{s(s^2+4s+5)}}$$

$$e_\infty = 0$$

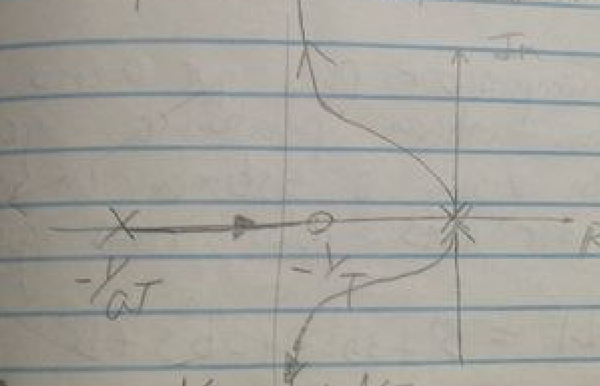
$$\boxed{C = 5 + 0 + 0 = 5}$$

3)



A) Para estabilizar o sistema que tipo de compensador de fase (avanço ou atraso) deve ser usado? Justifique sua resposta.

Supondo Avanço de fase.



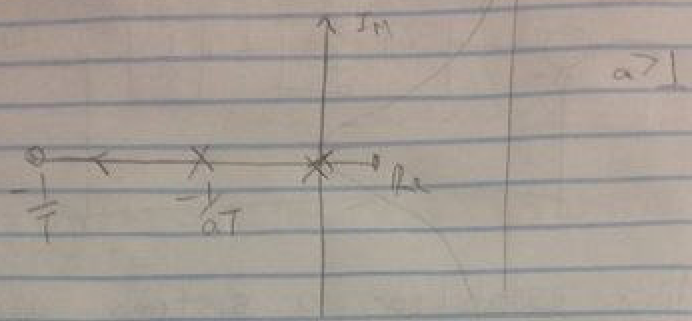
$0 < a < 1$

$$\sigma_c = \frac{-1/aT + 1/T}{3-1} = \frac{-1}{2aT} + \frac{1}{T2}$$

$$= \frac{a-1}{2aT} < 0$$

$$\phi_1 = \frac{180}{2} = 90^\circ \quad \phi_2 = \frac{3.180}{2} = \frac{180}{2} = 90^\circ$$

Supondo atraso de fase



$$\delta c = \frac{-\frac{1}{aT} + \frac{1}{T}}{2} = \frac{a-1}{2aT}$$

B) Projetar o compensador para que o erro estacionário resultante de uma entrada parabólica seja de $2{,}083\ s^{-2}$. Além disso, o sistema em malha fechada deve ter três polos reais iguais.

Dicas: $(s+b)^3 = s^3 + 3bs^2 + 3b^2 s + b^3$

$$e_{\infty} = \lim_{s\to 0} s E(s) = \lim_{s \to 0} \cancel{s} \cdot \frac{1}{s^3} \cdot \frac{1}{1 + \frac{K(s+\frac{1}{T})}{(s+\frac{1}{aT})}}$$

$$= \lim_{s\to 0} \frac{1}{s^2 + \frac{K(s+\frac{1}{T})}{(s+\frac{1}{aT})}} = \frac{1}{\frac{K \cdot \frac{1}{T}}{\frac{1}{Ta}}}$$

$$= \frac{1}{Ka} = 2{,}083$$

$Ka = 0{,}48$

$z = \frac{1}{T}$

$G_c(S) = K \frac{(s+z)}{(s+z/a)}$

$a = 0{,}111$

$FTMF = \frac{K(s+z)}{K(s+z) + (s+z/a) \cdot s^2}$

$FTMf = \frac{K(s+z)}{Ks + zK + s^3 + s^2 z/a}$

$: \; s^3 + \frac{z}{a} s^2 + Ks + zK$

$s^3 + \frac{1}{Ta} s^2 + Ks + \frac{K}{T}$

$s^3 + \frac{K}{T0{,}48} s^2 + Ks + \frac{K}{T}$

$3b = \frac{K}{T0{,}48}$ $\qquad b^3 = \frac{K}{T}$

$3b^2 = K$ $\qquad 3b = \frac{b^3}{0{,}48}$

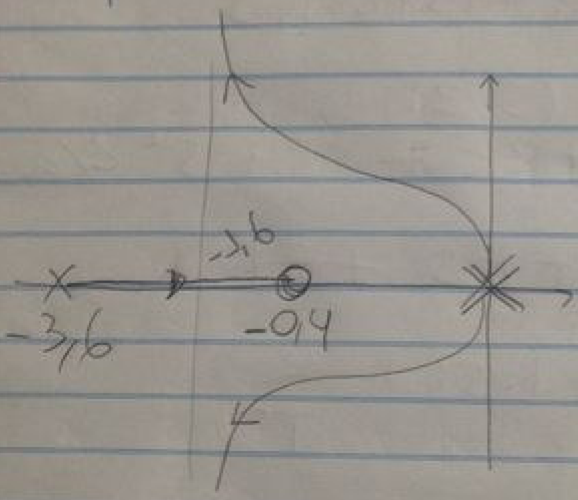
$b^2 = 3 \cdot 0{,}48$

$b = 1{,}2$

$\boxed{\begin{matrix} K = 4{,}32 \\ T = 2{,}5 \end{matrix}}$

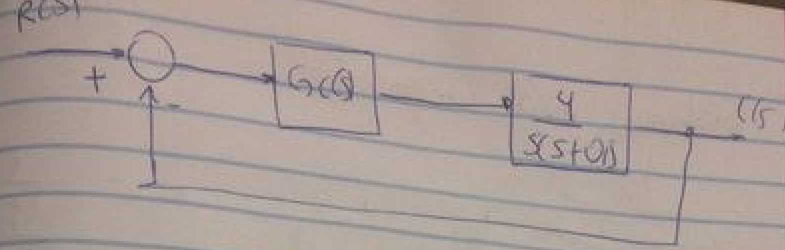
$$\boxed{G_c(s) = 4{,}32 \cdot \frac{(s+0{,}4)}{(s+3{,}6)}}$$

c) Esboce o LGR do sistema compensado.



$$s_c = \frac{-3{,}6+0{,}4}{3-1} = -1{,}6$$

2)



$G\,G_c$

$K = 100$

$$\boxed{G\,G_c = \frac{100}{s.\left(\frac{1}{10^{-2}}s+1\right)}}$$

aproximação

polos: $100s^2 + s + 100$

$s = -0{,}005 \pm j$

$$\frac{K(T_a s+1)\cdots}{s^j (T_1 s+1)\cdots(T_p s+1)}$$

$w_d = 1$

$\xi w_n = 0{,}005$

$1 = w_n \sqrt{1-\xi^2}$

$1 = \underbrace{\frac{0{,}005}{\xi}}_{} \sqrt{1-\xi^2}$

$\frac{1}{0{,}005} = \sqrt{\frac{1}{\xi^2} - 1}$

$\boxed{\xi = 5\times10^{-3}}$ $\quad w_n = 1,$

Sistema não compensado

$1/s^2$

$$e_\infty = \lim_{s\to 0} \frac{1}{s} \frac{1}{1+\frac{4}{s(s+0{,}5)}} = \frac{1}{\frac{4}{0{,}5}} = 0{,}1\text{[illegible]}$$

sistema compensado

$$e_\infty = \lim_{s\to 0} \frac{1}{s} \frac{1}{1+\frac{100}{s}} = \frac{1}{100}$$

(3) Identifique a natureza do controlador

* ganho em baixa frequência

* 2 polos a mais q/ zeros

zero: $(10^1 s+1)$

$$\boxed{G\,G_c = \frac{100\,(10s+1)\,(s+1)}{s(10^2 s+1)\,(0{,}5s+1)}}$$

$$G\,G_c = \frac{100}{s(0{,}5s+1)} \cdot \boxed{\frac{(10s+1)(s+1)}{(10^2 s+1)}}$$

É um avanço-atraso.