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RADIOMETRY AND PHOTOMETRY

VERLab Computer Vision and Robotics

DCC DEPARTAMENTO DE CIÊNCIA DA COMPUTAÇÃO

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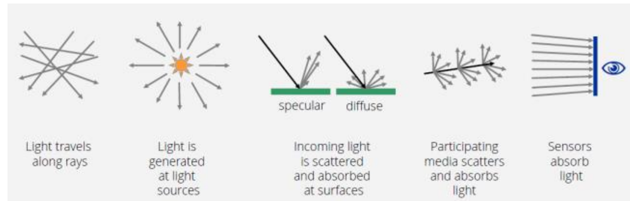
1

Illumination



Matthias Teschner, University of Freiburg

2

Illumination

- The goal of rendering algorithms is to create images that accurately represent the appearance of objects in scenes
  - For every pixel, the algorithm must find the objects that are visible at that pixels and display their appearance to the user
- Understanding the nature of light and how it scatter in the environment is crucial to correctly simulate **illumination**

3

3

Models of Light

- **Ray optics**: models light as independent rays that travel in different optical media according to a set of gemetrical rules
  - Common used in Computer Graphics
- **Wave optics**: models light as electromagnetic waves
  - Interference and diffraction
- **Electromagnetic optics**
  - Polarization and dispersion
- **Photon optics**: explains the interaction of light and matter

4

4

Assumptions of Ray optics

- Light travels in straight lines
  - No diffraction
- Light travels instantaneously through a medium
  - Infinite speed
- Light is not influenced by external factors, such as
  - Gravity
  - Magnetic fields

5

5

Radiometry

- The goal of an illumination algorithm is to compute the steady-state distribution of light energy in a scene
- Radiometry: The science of measuring light
- Light: A particular kind of electromagnetic radiation (of a frequency that can be detected by the human eye)

6

6

Radiant Flux or Power

- Light "flows" through space, and so radiant flux is the total radiant power emitted from a source or received by a surface.
  - Commonly "time rate of flow of radiant energy"

$$\boxed{\Phi = \frac{dQ}{dt}}$$

  - Q is radiant energy and t is time
  - measure in joules per second or Watts

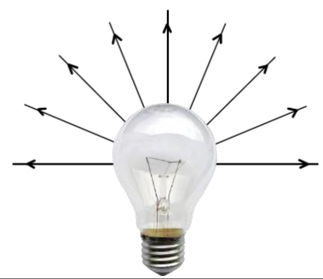
7

7

Radiant Flux or Power

- We can say
  - A ligth source emits 50 Watts of radiant power
  - 20 Watts of radiant power is incident on a table

$\Phi = \frac{dQ}{dt}$



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8

Radiant Flux

- The flow of light through space is often represented by geometrical rays of light such as those used in computer graphics ray tracing.

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Radiant Flux Density

- Radiant flux density is the power per unit area at a point on a surface.
  - The flux can be arriving at the surface, it is referred to as **Irradiance (E,** french: *éclairage*, meaning lighting/illumination**)**.
  - The flux can leave from any direction above the surface, as indicated by the rays, it is called **Radiant exitance (M) or Radiosity.**
  - It is measured in watts per square meter.

$\hookrightarrow W/m^2$

10

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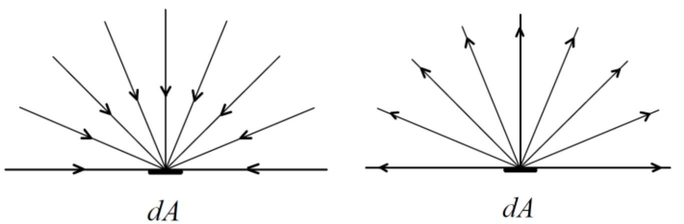
10

Irradiance and Radiance Exitance



Irradiance Radiance Exitance

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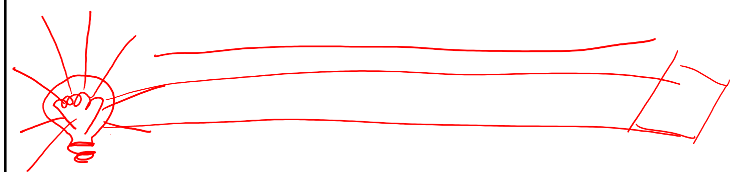
Radiant Flux Density

- Directional light source
  - Useful approximation when the distance to the light is much larger than the extent of room from the light bulb



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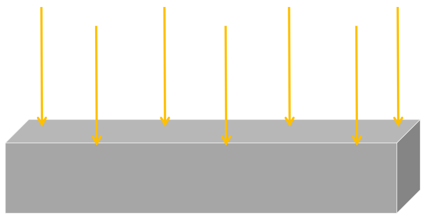
Radiant Flux Density

- Irradiance can be used to measure the illumination of an ideal directional light
  - $\mathrm{E} = 0.3\ \mathrm{W/m^2}$



13

13

Irradiance and Radiance Exitance

- Irradiance is defined as:

$$E = d\Phi/dA$$

  - where Φ is the radiance flux **arriving** at the point and dA the differential area surronding the point
- Radiance Exitance:

$$M = d\Phi/dA$$

  - where Φ is the radiance flux **leaving** the point

14

14

Irradiance

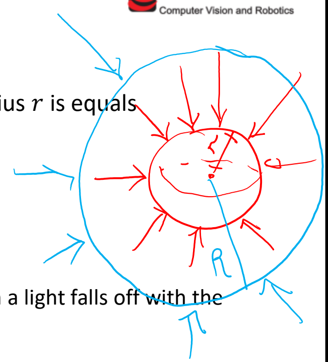
$R = 2r$

- Irradiance for a sphere that has radius r is equals
  - Total area of a sphere: $4\pi r^2$

$E_R = \frac{\Phi}{4\pi\, 4r^2}$

$$E = \frac{\Phi}{4\pi r^2}$$

$E_R = \frac{1}{4} E_r$



  - The amount of energy received from a light falls off with the squared distance from the light

$E_R = \frac{\Phi}{4\pi R^2}$ $E_r = \frac{\Phi}{4\pi r^2}$

15

15

Irradiance and Radiance Exitance

- The radiant flux density can be measured anywhere in three-dimensional space.
  - surface of physical objects
  - in the space between them (e.g., in air or a vacuum),
  - inside transparent media such as water and glass.
- Surface can be real or imaginary (i.e., a mathematical plane).

16

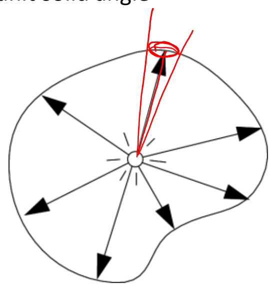
16

Radiant Intensity

- The radiant intensity is the power per unit solid angle emanating from a point source:

$$I = \frac{d\Phi}{d\omega}$$

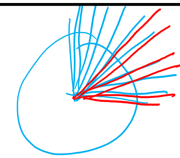
- Solid what?



17

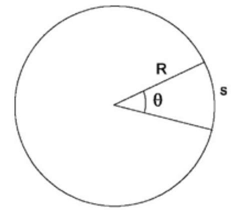
17

Solid Angle



- First of all, what is an angle?

$$\theta \equiv \frac{s}{R} \text{ radians}$$



- Unit circle has 2π radians
- An angle is a set of directions in a plane

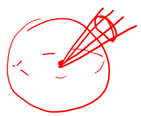
$\delta s = 2\pi r$

$\theta = \frac{2\pi r}{r} \rightsquigarrow 2\pi$ rad.anos

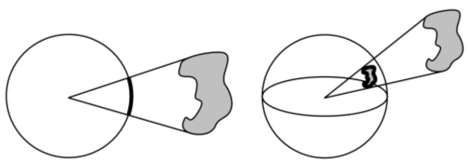
18

18

Solid Angle



- A solid angle is a three-dimensional extension of the concept in 2D
  - A continous sef of directions



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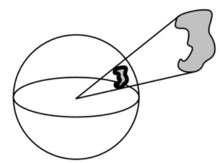
19

Solid Angles

- The *solid angle* represents the angular "size" of a beam as well as the direction
  - It represents both a direction and an infinitesimal area on the unit sphere



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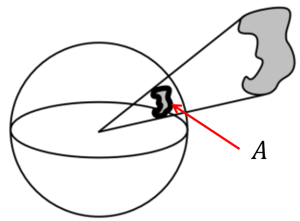
20

Solid Angles

- Definition:

$$\Omega \equiv \frac{A}{R^2} \text{ steredians}$$



- Unit sphere has 4π steredians
  - Total area of a sphere: $4\pi R^2$

21

21

Radiant Intensity

- Intensity or Radiant intensity is the flux density per solid angle

$$I = \frac{d\Phi}{d\omega}$$

- Measured in Watts per steredians

22

22

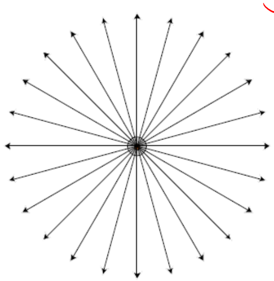
Radiant Intensity

$\frac{d\Phi}{d\omega} = I$

$d\Phi = I\, d\omega$

$\Phi = \int_{S^2} I\, d\omega$

- Isotropic Point Source (Sphere S^2)
  - $\Phi = \int_{S^2} I\, d\omega$

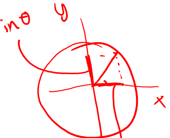


23

23

Integrals over projected solid angle

sin θ y

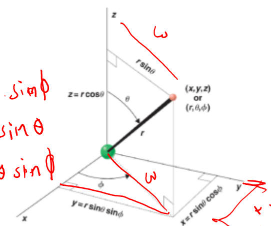


cos θ

θ φ r

- Spherical Coordinates
  - $z = r \cos\theta$
  - $x = r \sin\theta \cos\phi$
  - $y = r \sin\theta \sin\phi$

x y z



ω

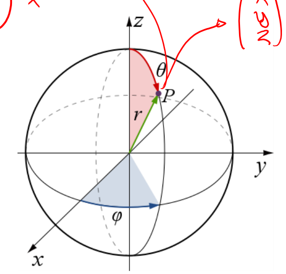
$y = \omega \sin\phi$

$\omega = r \sin\theta$

$y = r \sin\theta \sin\phi$

$x = \omega \cos\phi$

$x = r \sin\theta \cos\phi$

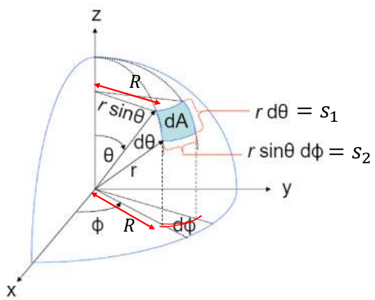


24

24

Integrals over projected solid angle

- From the angle definition:
  - $d\theta = \frac{s_1}{r}$
  - $d\phi = \frac{s_2}{R}$
  - $R = r \sin\theta$



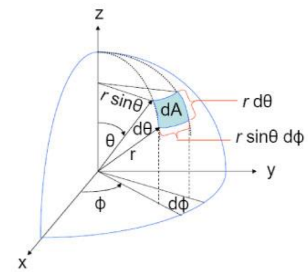
25

25

Integrals over projected solid angle

- $dA = (s_1)(s_2)$
- $dA = (r\,d\theta)(r\sin\theta\,d\phi)$

- The solid angle $d\omega$ is
  - $d\omega = \frac{dA}{r^2}$
  - $d\omega = d\theta \sin\theta d\phi$



26

26

Integrals over projected solid angle

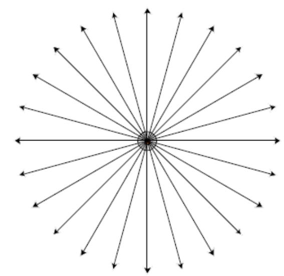
- Integrating over a sphere
  - $\Omega = \int_{S^2} d\omega$
  - $\Omega = \int_0^{\pi}\int_0^{2\pi} sin\theta d\phi d\theta$
  - $\Omega = \int_0^{\pi}[2\pi - 0]sin\theta d\theta$
  - $\Omega = 2\pi\int_0^{\pi} sin\theta d\theta = 2\pi[-cos\pi - (-cos0)]$
  - $\Omega = 4\pi$

27

27

Radiant Intensity

- Isotropic Point Source (Sphere S^2)
  - $\Phi = \int_{S^2} I\, d\omega$
  - $\Phi = 4\pi I$
  - $\mathrm{I} = \frac{\Phi}{4\pi}$



28

28

Radiance

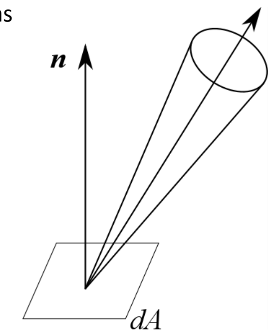
- Imagine ray of light arriving at or leaving a point on a surface in a given direction.
- Radiance is simply the infinitesimal amount of radiant flux contained in this ray.
- Radiance is probably the most important quantity in global illumination because it is the quantity that captures the appearance of objects in the scene

29

29

Radiance

- A formal definition: think of a ray as being an infinitesimally narrow cone with its apex at a point on a real or imaginary surface.



30

30

Solid Angles and Cones

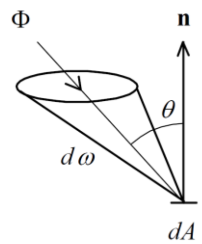
- Power per unit projected area perpendicular to the ray per unit **solid angle** in the direction of the ray

31

31

Radiance

- The ray intersects the surface at an angle θ.



32

32

Radiance

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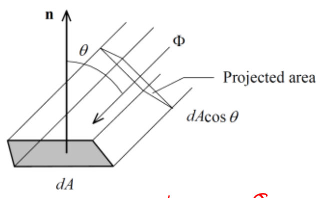
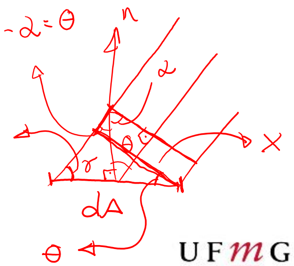
- Let dA be the differential cross-sectional area of the surface and θ is the angle between the ray and the surface normal
  - The cross-sectional area of the ray is: $dA\ \cos\theta$



$X = dA \cos\theta$

$90 - \alpha = \theta$



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Radiance

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- Radiance measures the illumination in a single ray of light. It is flux density with respect to both area and solid angle:

$$L(x, \omega) = \frac{d\Phi}{(dA\ \cos\theta)\ d\omega}$$

position $\begin{bmatrix} x \\ y \\ z \end{bmatrix}$

direction $= \begin{bmatrix} \theta \\ \phi \end{bmatrix}$

- The metric units are watts per square meter per steradian
- Radiance is a five-dimensional quantity
  - Three for position
  - Two for the direction

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Radiance

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- Radiance L is what sensors measure.
  - It is of prime importance for rendering!

- The main reason of evaluating a shading equation is to compute the radiance along a given ray.

- ***Shading*** is the process of using an equation to compute the outgoing radiance L_o along the view ray.

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Computing total flux

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- Rewriting:

$$L(x, \omega) = \frac{d\Phi}{(dA\ \cos\theta)\ d\omega}$$

$$L(x, \omega)(dA\ \cos\theta)\ d\omega = d\Phi$$

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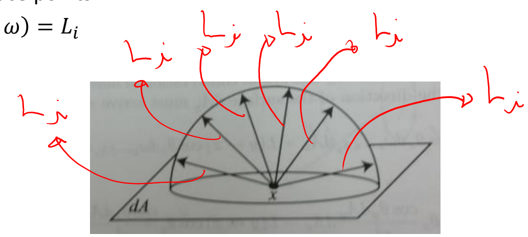
36

Computing total flux

- Let us consider a diffuse emitter
  - A diffuse emitter emits equal radiance in all directions from all its surface points
  - $L(x, \omega) = L_i$



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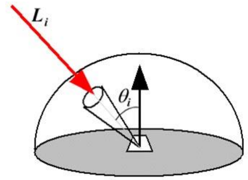
Computing total flux

$d\phi = L_i \, dA \cos\theta \, d\omega$

- Total flux is equal to

$$\Phi = \int_A \int_{H^2} L_i (dA \cos\theta) \, d\omega$$



  - where H^2 is the entire upper hemisphere
  - dA for all directions
  - A is the total area

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38

Computing total flux

- Total flux is equal to

$$d\Phi = \int_0^{\pi/2} \int_0^{2\pi} L_i (dA \cos\theta) \sin\theta \; d\phi \, d\theta$$

$$d\Phi = \int_0^{\pi/2} L_i (dA \cos\theta) \sin\theta \; (2\pi - 0) d\theta$$

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39

Computing total flux

- Total flux is equal to

$$d\Phi = \int_0^{\pi/2} 2\pi (L_i dA)(\cos\theta \sin\theta) \, d\theta$$

- Remember that

$$\frac{d(\sin^2\theta)}{d\theta} = 2 \sin\theta \cos\theta$$

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Computing total flux

- Total flux is equal to

$$d\Phi = \pi L_i dA\,(\sin^2\frac{\pi}{2} - \sin^2 0) = \pi L_i dA$$

$$\Phi = \pi L_i \int_A dA = \pi L_i A$$

41

41

Radiance and Irradiance

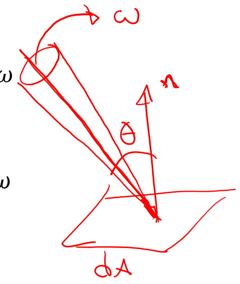
- Since the total flux is equal to (of a set of directions Ω)

$$d\Phi = \int_\Omega L_i (dA \cos\theta)\, d\omega$$

- Then, Irradiance

$$\mathrm{E} = \frac{d\Phi}{dA} = \int_\Omega L_i \cos\theta\, d\omega$$

$\omega \cdot n$



42

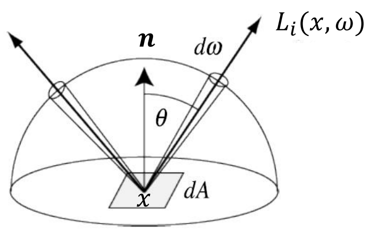
42

Radiance and Irradiance

- Irradiance at a point p with surface normal $\boldsymbol{n}$ and radiance over a set of directions Ω is

$$E(p, \boldsymbol{n}) = \int_\Omega L_i(x,\omega)(\boldsymbol{n}^T\omega)\, d\omega$$

where $L_i(x,\omega)$ is the incident radiance function at location x and direction ω



43

43

The Irradiance over the hemisphere

- Irradiance

$$E = \frac{d\Phi}{dA} = \frac{\pi L_i dA}{dA} = \pi L_i$$

- If the radiance is the same (i.e., L) from all directions:

$$E = \pi L$$

44

44

Communication

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- First test: 29/04/2019
- Final test: 19/06/2019
- New extra pratical exercise (next Wednesday – 17/04/2019)

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56

Spectral Radiance

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- Energy

$$energy \approx \int_{t_0}^{t_1} \int_{x_0}^{x_1} \int_{y_0}^{y_1} \int_{\omega \in \Omega} \int_{\lambda_0}^{\lambda_1} L(t, (x, y, 0), -\omega, \lambda)\, d\lambda\, d\omega\, dy\, dx\, dt$$

- Radiance is the quantity

$$L = \int_0^\infty L(t, P, \omega, \lambda) d\lambda$$

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57

Photometry

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- Photometry is the science of measuring visible light in units that are **weighted** according to the sensitivity of the human eye.
  - quantitative science based on a statistical model of the human visual response to light

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58

Photometry

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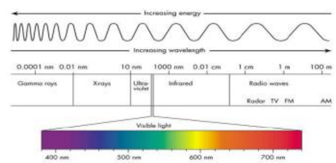
- The human visual system is complex and highly nonlinear.
  - Detector of electromagnetic radiation with wavelengths ranging from 380 to 770 nanometers (nm).



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Photometry

- The sensitivity of the human eye to light varies with wavelength.
  - A light source with a radiance of one watt/m^2-steradian of green light appears much brighter than the same source with a radiance of one watt/m^2-steradian of red or blue light.

60

60

Photometry

- In photometry, we do not measure watts of radiant energy. It measures the **subjective impression** produced by stimulating the human eye-brain visual system with radiant energy

61

61

Commission Internationale d'Eclairage (CIE)

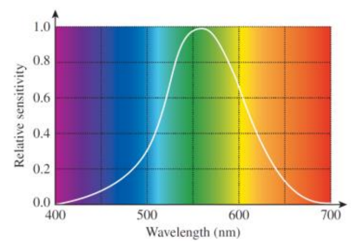
- Asked over one hundred observers to visually match the "brightness" of monochromatic light sources with different wavelengths under controlled conditions
  - photopic luminous efficiency of the human visual system as a function of wavelength.

62

62

Commission Internationale d'Eclairage (CIE)

- CIE photometric curve
  - The spectral response curve $V(\lambda)$



63

63

Luminous Intensity

- The international standard is a theoretical point source that has a luminous intensity of one candela
  - It emits monochromatic radiation with a frequency of 540 x 10^{12} Hertz (or approximately 555 nm, corresponding with the wavelength of maximum photopic luminous efficiency)
  - It has a radiant intensity (in the direction of measurement) of 1/683 watts per steradian.

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Luminous Intensity

- CIE photometric curve and the candela provides the weighting factor needed to convert between radiometric and photometric measurements
  - a monochromatic point source with a wavelength of 510 nm and a radiant intensity of 1/683 watts per steradian. The photopic luminous efficiency at 510 nm is 0.503. The source therefore has a luminous intensity of 0.503 candela

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Luminance

- Luminance is photometrically weighted radiance
  - We perceive luminance
  - It is an approximate measure of how "bright" a surface appears when we view it from a given direction.

$$L_\lambda = \int_\lambda L(\lambda)V(\lambda)d\lambda$$

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