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MODELING REFLECTANCE

Computer Vision and Robotics
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Interaction of Light with Surfaces

- Light energy emitted into a scene interacts with the different objects in the scene by
  - Getting reflected
  - Transmitted at surface boundaries
  - Absorbed
  - Dissipated

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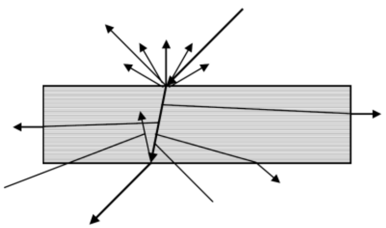
Interaction of Light with Surfaces

- When light interacts with matter, a complicated light-matter dynamic occurs.



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Interaction of Light with Surfaces

- This interaction depends on the physical characteristics of the light as well as the physical composition and characteristics of the matter
  - e.g., a rough opaque surface such as sandpaper will reflect light differently than a smooth reflective surface such as a mirror.
- The reflectance properties of a surface affect the appearance of the object

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Interaction of Light with Surfaces

- Because light is a form of energy, conservation of energy tells us that
  - *Light incident at surface = light reflected + light absorbed + light transmitted*
- Opaque materials:
  - the majority of incident light is transformed into reflected light and absorbed light.

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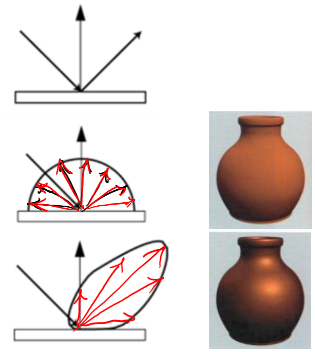
Examples of Reflections

- Ideal Specular
  - Mirror
- Ideal Diffuse
  - Matte
- Specular
  - Glossy



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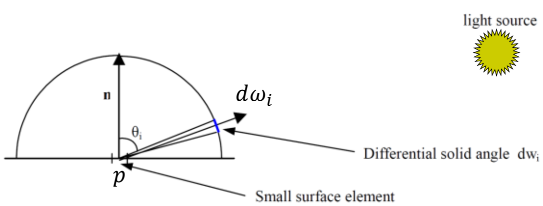
Reflectance

- Irradiance at p is:

$$dE(p,\omega_i) = L_i(p,\omega_i)cos\theta_i d\omega_i$$



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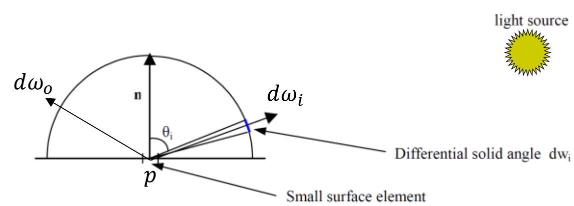
Reflectance

- Reflected differential radiance is proportional to the irradiance:

$$dL_o(p,\omega_o) \propto dE(p,\omega_i)$$



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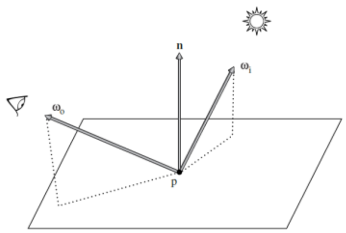
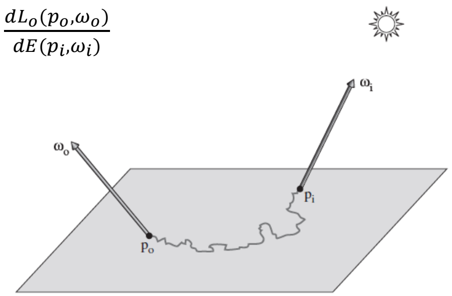
BRDF

- *Bidirectional Reflectance Distribution Function* (BRDF) describes how much light is *reflected* when light makes contact with a certain material at a point p
  - $f_r = \frac{dL_o(p,\omega_o)}{dE(p,\omega_i)}$
  - ω_i is the incoming light direction
  - ω_o is the outgoing view diretion



BRDF vs BSDF

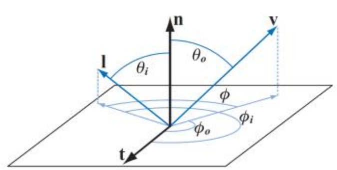
- The term *Bidirectional Scattering Distribution Function (BSDF)* is used to denote the reflection and transparent parts together
  - $f_s = \frac{dL_o(p_o,\omega_o)}{dE(p_i,\omega_i)}$



BRDF in spherical coordinates

- A general BRDF in functional notation can be written as (*position-invariant* or *shift-invariant)*

$$f_r(\theta_i, \phi_i, \theta_o, \phi_o, \lambda)$$



Position-invariant BRDF

- We will not include the spatial position as a parameter to the function.
  - We assume that the reflectance properties of a material do not vary with spatial position.
  - Only valid for homogenous materials
- We can include the positional variance using a detail texture

BRDF and color

- BRDF do depend on the wavelength or color channel under consideration
  - The value of the BRDF function must be determined separately for each color channel (i.e., R, G, and B separately)

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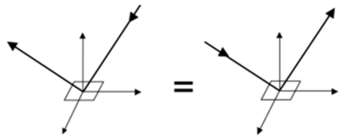
Properties of the BRDF

- *Helmholtz reciprocity*: input and output angles can be switched and the function will not change:
  - It means that reversing the direction of light does not change the amount of light that gets reflected

$$f_r(\theta_i, \phi_i, \theta_o, \phi_o) = f_r(\theta_o, \phi_o, \theta_i, \phi_i)$$



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BRDF characteristics

- In pratice, reciprocity is violated in rendering
- But it is useful tool for determining if a BRDF is physically plausible

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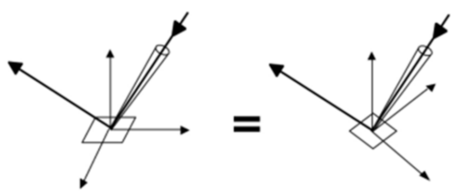
Isotropic BRDFs

- Invariant with respect to rotation of the surface around the surface normal vector
  - smooth plastics have isotropic BRDFs



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Anisotropy BRDFs

- Anisotropy refers to BRDFs that describe reflectance properties that do exhibit change with respect to rotation of the surface around the surface normal vector
  - brushed metal and hair.
- Most real-world BRDFs are anisotropic to some degree
  - But most real-world BRDFs are probably more isotropic than anisotropic

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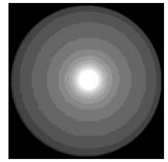
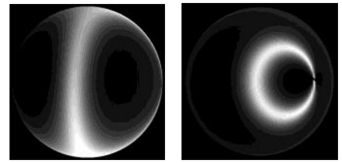
Isotropic vs Anisotropy



Isotropic



Anisotropic

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Isotropic vs Anisotropy

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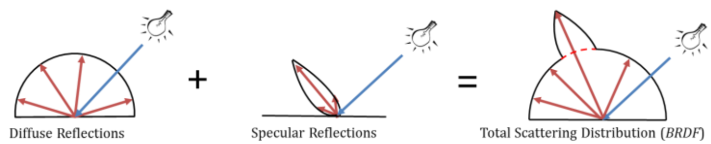
BRDF characteristics

- *Linearity:* The value of the BRDF for a specific indicent direction is not dependent on the possible presence of irradiance along other incident angles



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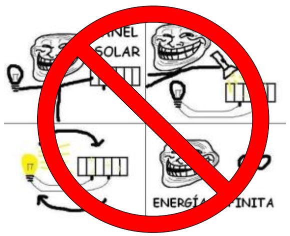
BRDF characteristics

- *Conservation of energy:* a surface cannot reflect more than 100% of incoming light energy.



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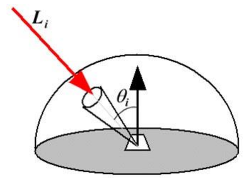
Computing total flux

- Total flux is equal to

$$\Phi = \int_{H^2} L_i(dA \cos\theta)\, dw$$

  - where H^2 is the entire upper hemisphere
  - dA for all directions



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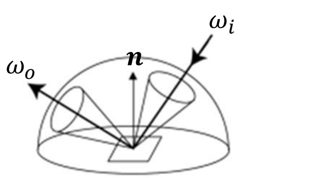
BRDF characteristics

- Conservation of energy

$$\frac{d\Phi_o}{d\Phi_i} = \frac{\int_{\Omega_o} L_o(\omega_o)\cos\theta_o d\omega_o}{\int_{\Omega_i} L_i(\omega_i)\cos\theta_i d\omega_i} \le 1$$



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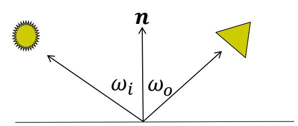
Reflection Equation

$f_r(\omega_o, \omega_i) = \frac{dL_o(\omega_o)}{L_i \cos\theta\, d\omega_i}$

- $f_r(\omega_o, \omega_i) = \frac{dL_o(\omega_o)}{dE(\omega_i)}$, $dE(\omega_i) = \frac{d\Phi}{dA} = L_i \cos\theta\, d\omega_i$
- Lets considere one ray with direction ω_i
  - $dL_o(\omega_o) = \underbrace{f_r(\omega_o, \omega_i)}_{\text{BRDF}}\, L_i(\omega_i)(\omega_i \cdot \boldsymbol{n})$

$\cos\theta$



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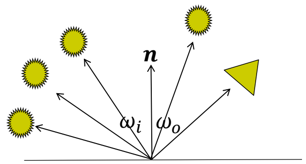
Reflection Equation

- $L_o(\omega_o) = \sum_i f_r(\omega_o, \omega_i)\, L_i(\omega_i)(\omega_i \cdot \boldsymbol{n})$



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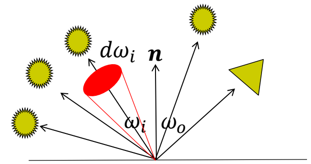
Reflection Equation

$$L_o(\omega_o) = \int_{\Omega} f_r(\omega_o, \omega_i)\, L_i(\omega_i)(\omega_i \cdot \boldsymbol{n}) d\omega_i$$



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Conservation of Energy

- For each one unit of light energy that arrives at a point, no more than one unit of light energy can be reflected in total to all possible outgoing directions

$$\int_{\Omega} f_r(\omega_o, \omega_i)\, (\omega_i \cdot \boldsymbol{n}) d\omega_i \leq 1$$

  - Ω indicates integral over a hemisphere of all directions.

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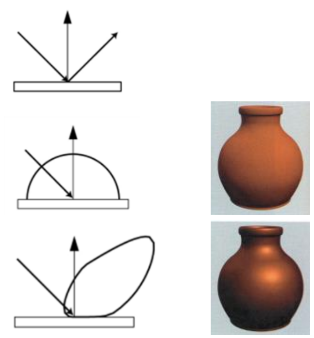
BRDF examples

- Depending on the nature of the BRDF, the material will appear
  - Mirror
  - Diffuse
  - Glossy



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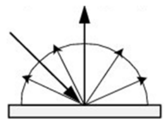
Ideal Diffuse BRDF

- Let $f_r(\omega_o, \omega_i) = \mathrm{k}_d$ and assume BRDF reflects a fraction ρ of the incoming light
  - $L_o(\omega_o) = \int_\Omega f_r(\omega_o, \omega_i)\, L_i(\omega_i)(\omega_i \cdot \boldsymbol{n}) d\omega_i = \rho L_i$
- The quantity ρ is known as the albedo of the surface



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Ideal Diffuse BRDF

- $\int_\Omega f_r(\omega_o, \omega_i)\, L_i(\omega_i)(\omega_i \cdot \boldsymbol{n}) d\omega_i = \rho L_i$
- $\int_\Omega k_d\, L_i(\omega_i)(\omega_i \cdot \boldsymbol{n}) d\omega_i = \rho L_i$
- $k_d \int_\Omega L_i(\omega_i)(\omega_i \cdot \boldsymbol{n}) d\omega_i = \rho L_i$
- $k_d E = \rho L_i$

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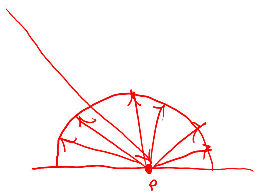
Ideal Diffuse BRDF

- Since
  - $E = \frac{d\Phi}{dA} = \frac{\pi L_i dA}{dA} = \pi L_i$
- Then
  - $k_d E = \rho L_i$
  - $k_d(\pi L_i) = \rho L_i$



albedo

$$k_d = \frac{\rho}{\pi}$$

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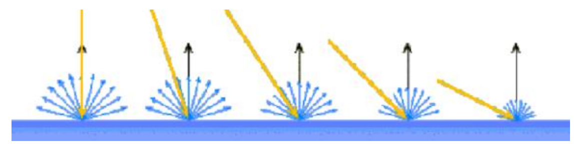
Ideal Diffuse BRDF

Irradiância

BRDF

- Lambert's cosine law:
  - $L_o(w_o) = \int_\Omega f_r(\omega_o, \omega_i)\, L_i(\omega_i)(\omega_i \cdot \boldsymbol{n}) d\omega_i = \rho L_i$
  - $\rho L_i = k_d E = \frac{\rho}{\pi} E$
  - $L_o = \frac{\rho}{\pi} E = \frac{\rho}{\pi}\left(L_i(\omega_i)\boxed{(\omega \cdot \boldsymbol{n})}\right) = \frac{\rho}{\pi} L_i(\omega_i) \cos\theta$



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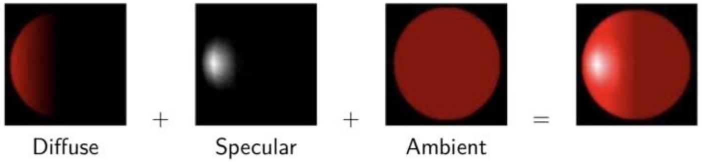
Phong Lighting Model



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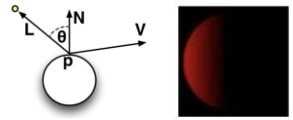
Diffuse term

- $I_d = I_{in}k_d(\boldsymbol{L}.\boldsymbol{N}) = I_{in}k_d \cos\theta$
  - I_{in}: Light intensity
  - θ: Angle between normal vector and direction to light source
  - k_d Diffuse reflectivity
- No dependence on the angle between the **direction** to the **camera (V)** and the **surface normal (N)**.



Diffuse

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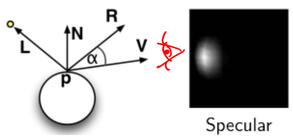
Specular term

- $I_s = I_{in}k_s(\boldsymbol{R}.\boldsymbol{V})^n = I_{in}k_s \cos^n \alpha$
  - I_{in}: Light intensity
  - α : Angle between reflection vector and direction to camera
  - k_s: Specular reflectivity
  - n: Specular intensity



Specular

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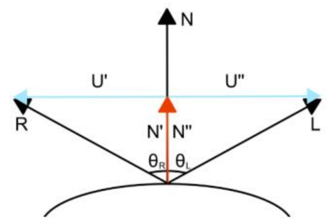
Reflected vector

- $\Theta_R = \Theta_L$
- $R.N = L.N$ e $U' = -U''$
- $U'' = L - N'' = L - (L.N)N$
- $R - (R.N)N = -(L - (L.N)N)$
- $R.N = L.N$ então
  - $R - (L.N)N = -(L - (L.N)N)$
  - $R = (L.N)N - L + (L.N)N$
  - $R = 2(L.N)N - L$



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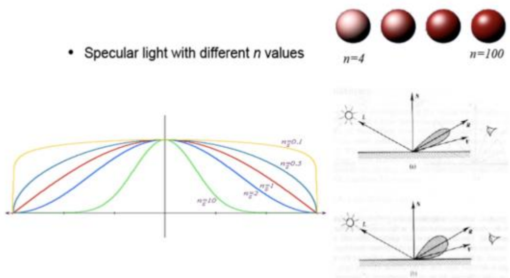
Specular intensity

- Specular light with different *n* values



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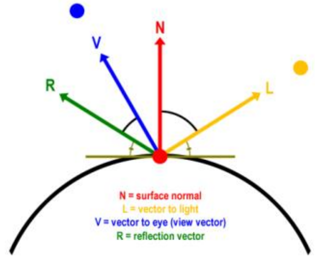
Phong Lighting Model

- $I_{out} = I_{in}(k_d(\boldsymbol{l}.\boldsymbol{n}) + k_s(\boldsymbol{r}.\boldsymbol{v})^n)$
  - k_d contribution of diffuse component
  - k_s contribution of specular component
  - n controls the width of the specular highlight



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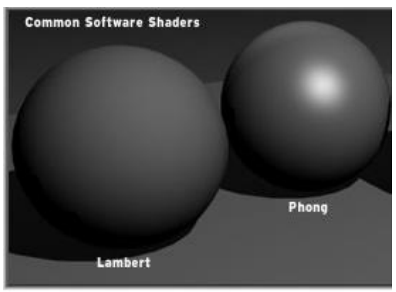
Phong vs Diffuse Reflection



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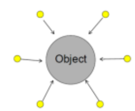
Ambient term

- Points on the surface that are not directly lit by the light will not be illuminated
  - Environmental light: Light reflected or scattered from other objects in the scene
  - Precise simulation of this is very hard!
- Simple approximation
  - $I_{out} = K_a I_a$



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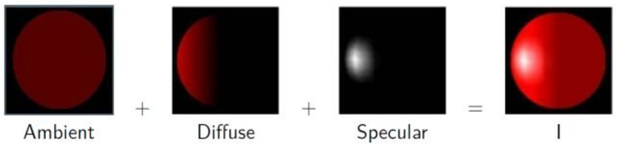
Combined lighting models

- $I_{out} = I_a k_a + I_p(k_d \cos\theta + k_s \cos^n \alpha)$



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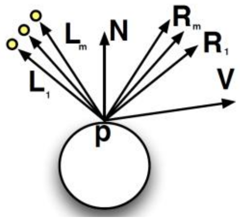
Multiple light sources

- $I_{out} = I_a k_a + \sum_l I_l(k_d \cos\theta + k_s \cos^n \alpha)$



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Color

L.N $(R.N)^n$

- $I_{out,R} = I_{a,R} k_{a,R} + I_{p,R}(k_{d,R} \cos\theta + k_{s,R} \cos^n \alpha)$
- $I_{out,G} = I_{a,G} k_{a,G} + I_{p,G}(k_{d,G} \cos\theta + k_{s,G} \cos^n \alpha)$
- $I_{out,B} = I_{a,B} k_{a,B} + I_{p,B}(k_{d,B} \cos\theta + k_{s,B} \cos^n \alpha)$

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Phong Lighting Model

- We can rewrite Phong model as
  - $L_o = L_i(k_d(\boldsymbol{l}.\boldsymbol{n}) + k_s(\boldsymbol{r}.\boldsymbol{v})^n)$
  - Let $g(\boldsymbol{l},\boldsymbol{v}) = k_d(\boldsymbol{l}.\boldsymbol{n}) + k_s(\boldsymbol{r}.\boldsymbol{v})^n$ then

$$L_o = L_i g(\boldsymbol{l},\boldsymbol{v})$$

  - $\boldsymbol{l}$ and $\boldsymbol{v}$ correspond to incoming direction ω_i and outgoing direction ω_o respectivelly

$$L_o = g(\omega_i, \omega_o) L_i$$

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Phong Lighting Model

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- $L_o = g(\omega_i, \omega_o) L_i$
- $L_o = \frac{\cos\theta_i d\omega_i}{\cos\theta_i d\omega_i} g(\omega_i, \omega_o) L_i$
- $L_o = \frac{g(\omega_i,\omega_o)}{\cos\theta_i d\omega_i} \cos\theta_i \, d\omega_i L_i$
- $L_o = \frac{g(\omega_i,\omega_o)}{\cos\theta_i d\omega_i} \boxed{L_i \cos\theta_i \, d\omega_i}$
- This feels like a general BRDF lighting:

$$f_r(\omega_i, \omega_o) = \frac{g(\omega_i,\omega_o)}{\cos\theta_i d\omega_i} = \frac{k_d(\mathbf{l}.\mathbf{n}) + k_s(\mathbf{r}.\mathbf{v})^n}{\cos\theta_i d\omega_i}$$

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Phong Lighting Model

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- Phong model is a computational convenient method to analytically approximate the reflectance properties of a small set of materials
- It is not actually physically plausible because:
  - It is not necessarily energy conserving
  - nor reciprocal.
  - Why?

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Flat shading

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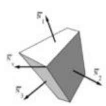
- Color is computed once for each polygon
- All pixels in a polygon are set to the same color





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Gouraud shading

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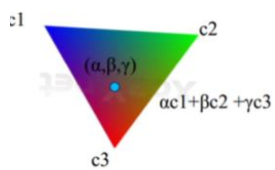
- Color is computed once per vertex using the local illumination model
- Polygons interpolate colors over their surface



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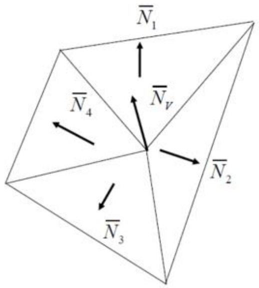
Vertex normals

- Vertex normals are found by averaging the face normals:
  - $N_v = \frac{\sum_i N_i}{\|\sum_i N_i\|}$



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Transforming vertex normals

- Recall
  - When applying a transformation M to your set of vertex, the normals are transformed as
    - $n' = (M^{-1})^T n$

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Problems with Gouraud shading

- In specular reflection the highlight can be sharp, depending on the shape of $\cos^n \alpha$
  - Gouraud shading interpolates linearly and so can make the highlight much bigger
  - Gouraud shading can miss highlights that occur in the middle of a polygon

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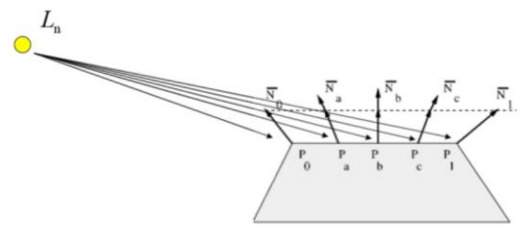
Phong shading

- Lighting computation is performed at each pixel
- Normal vectors are interpolated over the polygon



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Phong shading

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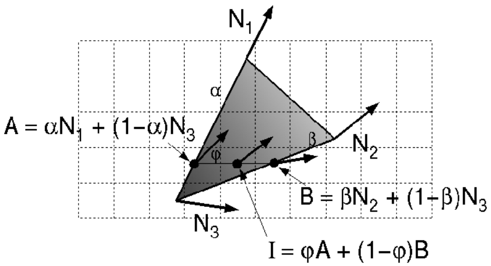
- Bilinearly interplate surface normals at vertices down and across scan lines



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Gouraud vs Phong shading

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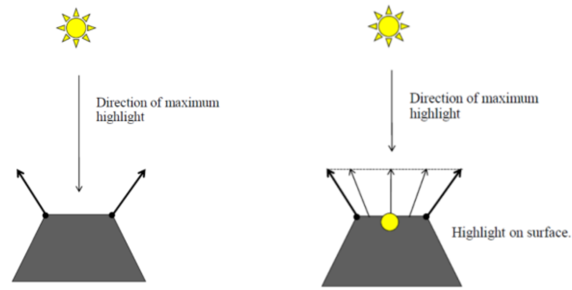

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Flat, Gouraud and Phong shading

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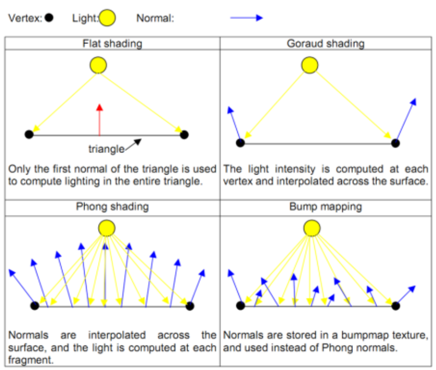

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Flat, Gouraud and Phong shading

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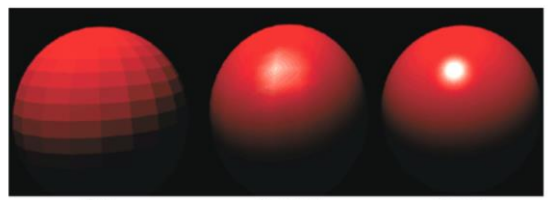
- Flat shading: one lighting calculation per **polygon**
- Gouraud shading: one lighting calculation per **vertex**
- Phong shading: one calculation per **pixel**



Flat Gouraud Phong

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Fresnel Reflectance

- An object’s surface is an interface between two different substances:
  - Air
  - Object’s substance
- The interaction of light with a planar interface between two substances follows the Fresnel Equations

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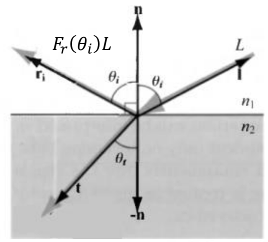
Fresnel Reflectance

- Each ray of incoming light has two directions:
  - Ideal reflection direction: r_i
  - Ideal refraction direction: t
- Fresnel reflectance $F_r(\theta_i)$ is the amount of light reflected, which depends on incoming direction r_i



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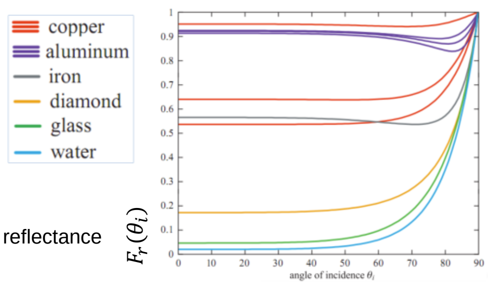
Fresnel Reflectance

- Copper and aluminum have significant variation in their reflectance over visible spectrum (It is show Red, Green, Blue)



Fresnel reflectance

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Fresnel Reflectance

- The function $F_r(\theta_i)$ has the following characteristics:
  - $\theta_i = 0$: light perpendicular to the surface, the value $F_r(\theta_i)$ is a property of the substance, i.e. specular color
  - When θ_i increases $F_r(\theta_i)$ increases and reaches 1 at $\theta_i = \frac{\pi}{2}$

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Specular Color

$F_0 = F(0°)$

Is the surface's characteristic specular color

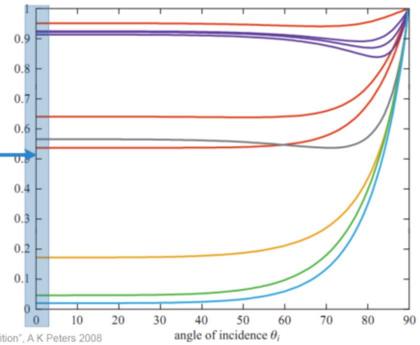

Image from "Real-Time Rendering, 3rd Edition", A K Peters 2008

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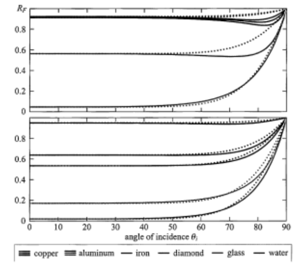
Fresnel Reflectance

- Schlick's approximation fo Fresnel reflectance:

$$F_r(\theta_i) \approx F_r(0) + \big(1 - F_r(0)\big)(1 - \cos\theta_i)^5$$



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Specular Reflection

- Since the Fresnel equation gives the fraction of light reflected $F_r(\omega_i)$:

$$L_o(\omega_o) = f_r(\omega_o, \omega_i) L_i(\omega_i) = F_r(\omega_i) L_i(\omega_i)$$

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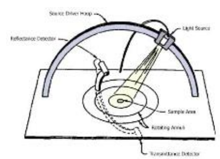
Where do BRDFs come from?

- By resampling BRDF data acquired by empirical measurements of real-world surfaces.
  - Gonioreflectometer: measures the BRDF of a material.



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Where do BRDFs come from?

- Evaluation of mathematical functions derived from analytical models.
  - Cook-Torrance model
  - Modified Phong model
  - Ward's model
  - etc.

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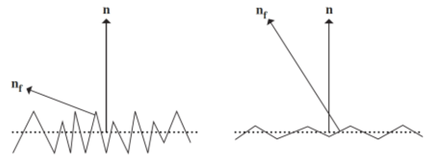
Microfacet Models

- Model the surface reflection as a collection of small microfacets
  - $\boldsymbol{n_f}$ microfacet normals
  - $\boldsymbol{n}$ surface normal



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Microfacet Models

- Main components:
  - Distribution of the facets
  - BRDF that describes how light scatters from individual microfacets
- Goal: Derive a closed-form expression giving BRDF

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Torrance-Sparrow Model

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Torrance-Sparrow Model

- Model surfaces as collections of perfectly smooth mirrored microfacets

- The surface is statistically described by a distribution function $D(\omega_h)$
  - $D(\omega_h)$ gives the probability that a microfacet has orientation ω_h
  - ω_h is called half-angle vector

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U F *m* G

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