Trabalho II – Métodos Numéricos II – 2017.1

Nome: Thiago de Sousa Garcia – 374204

Gauss Legendre – Grau 4

Gauss Legendre

– Grau 4

$$P_4(x) = \frac{1}{2^4 \cdot 4!} \cdot \frac{d^4}{dx^4} \cdot (x^2-1)^4$$

temos que $\frac{d^4}{dx^4}(x^2-1) = 48(35x^4 - 30x^2 + 3)$

daí, temos:

$$P_4(x) = \frac{1}{384} \cdot 48(35x^4 - 30x^2 + 3)$$

$$= \frac{35x^4 - 30x^2 + 3}{8}$$

daí,

$x_1 = 0{,}33998$

$x_2 = -0{,}33998$

$x_3 = 0{,}86114$

$x_4 = -0{,}86114.$

Daí temos:

$$L_1(x) = \frac{(x + 0{,}33998)}{(0{,}33998 + 0{,}33998)} \cdot \frac{(x - 0{,}86114)}{(0{,}33998 - 0{,}86114)} \cdot \frac{(x + 0{,}86114)}{(0{,}33998 + 0{,}86114)}$$

$$L_1(x) = \frac{(x + 0{,}33998)}{0{,}67996} \cdot \frac{(x - 0{,}86114)}{-0{,}52116} \cdot \frac{(x + 0{,}86114)}{1{,}20112}$$

$$w_1 = \int_{-1}^{1} L_1(x) \cdot dx \cong 0{,}65214$$

$$L_2(x) = \frac{(x - 0{,}33998)}{(-0{,}33998 - 0{,}33998)} \cdot \frac{(x - 0{,}86114)}{(-0{,}33998 - 0{,}86114)} \cdot \frac{(x + 0{,}86114)}{(-0{,}33998 + 0{,}8\ldots}$$

$$L_2(x) = \frac{(x-0,33998)}{-0,67996} \cdot \frac{(x-0,86114)}{-1,20112} \cdot \frac{(x+0,86114)}{0,52116}$$

$$w_2 = \int_{-1}^{1} L_2(x) \cong 0,65214.$$

$$L_3(x) = \frac{(x-0,33998)}{0,86114-0,33998} \cdot \frac{(x+0,33998)}{0,86114+0,33998} \cdot \frac{(x+0,86114)}{0,86114+0,86114}.$$

$$L_3(x) = \frac{(x-0,33998)}{0,52116} \cdot \frac{(x+0,33998)}{1,20112} \cdot \frac{(x+0,86114)}{1,72228}.$$

$$w_3 = \int_{-1}^{1} L_3(x) \cong 0,3478.$$

$$L_4(x) = \frac{(x-0,33998)}{-0,86114-0,33998} \cdot \frac{(x+0,33998)}{-0,86114+0,33998} \cdot \frac{(x-0,86114)}{-0,86114-0,86114}.$$

$$L_4(x) = \frac{(x-0,33998)}{-1,20112} \cdot \frac{(x+0,33998)}{-0,52116} \cdot \frac{(x-0,86114)}{-1,72228}$$

$$w_4 = \int_{-1}^{1} L_4(x) \cong 0,3478.$$

Daí, temos que

$$I = \frac{b-a}{2} \cdot \Big[f(x(n_1)) \cdot 0,65214 + f(x(n_2)) \cdot 0,65212 + f(x(n_3))\, 0,3478 + f(x(n_4)) \cdot 0,3478 \Big]$$

onde $x(n) = \frac{a+b}{2} + \frac{b-a}{2} \cdot n$

onde n_i são as raízes do polinômio de Legendre