

Tarefa 4 - Métodos Numéricos II - 2017.1
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$$1) J(\alpha, \beta) = \begin{bmatrix} \frac{dx}{d\alpha} & \frac{dy}{d\alpha} \\ \frac{dx}{d\beta} & \frac{dy}{d\beta} \end{bmatrix}$$

$$\frac{dx}{d\alpha} = \frac{\pi}{8} \quad \frac{dy}{d\beta} = \frac{\cos\left(\frac{\pi\alpha}{4}\right) - \sin\left(\frac{\pi\alpha}{4}\right)}{2}$$

$$\frac{dx}{d\beta} = 0.$$

Assim é necessário o cálculo de $\frac{dy}{d\alpha}$, pois
 como $\frac{dx}{d\beta}$ é 0, o determinante depende somente
 de $\frac{dx}{d\alpha}$ e $\frac{dy}{d\alpha}$.

$$2) \det(J) = \frac{dx}{d\alpha} \cdot \frac{dy}{d\beta} \\ = \frac{\pi}{8} \cdot \left(\frac{\cos\left(\frac{\pi\alpha}{4}\right) - \sin\left(\frac{\pi\alpha}{4}\right)}{2} \right)$$

$$3) V = \int_{-1}^1 \int_{-1}^1 f(x(\alpha, \beta), y(\alpha, \beta)) \cdot \left(\frac{\pi}{8} \cdot \left(\frac{\cos\left(\frac{\pi\alpha}{4}\right) - \sin\left(\frac{\pi\alpha}{4}\right)}{2} \right) \right) d\alpha d\beta$$

$$4) V = \sum_{i=1}^{Na} \sum_{j=1}^{Nb} w_i \bar{w}_j f(x(\alpha_i, \beta_j)) f(x(\alpha_i, \beta_j)) \cdot \left(\frac{\pi}{8} \cdot \left(\frac{\cos(\frac{\pi x}{4})}{2} - \text{Den}(\frac{\pi x}{4}) \right) \right)$$

onde

$$w_1 = 5/9 \quad \alpha_1 = -\sqrt{3/5}$$

$$w_2 = 8/9 \quad \alpha_2 = 0$$

$$w_3 = 5/9 \quad \alpha_3 = \sqrt{3/5}$$

$$\bar{w}_1 = 1 \quad \beta_1 = \sqrt{1/3}$$

$$\bar{w}_2 = 1 \quad \beta_2 = -\sqrt{1/3}$$