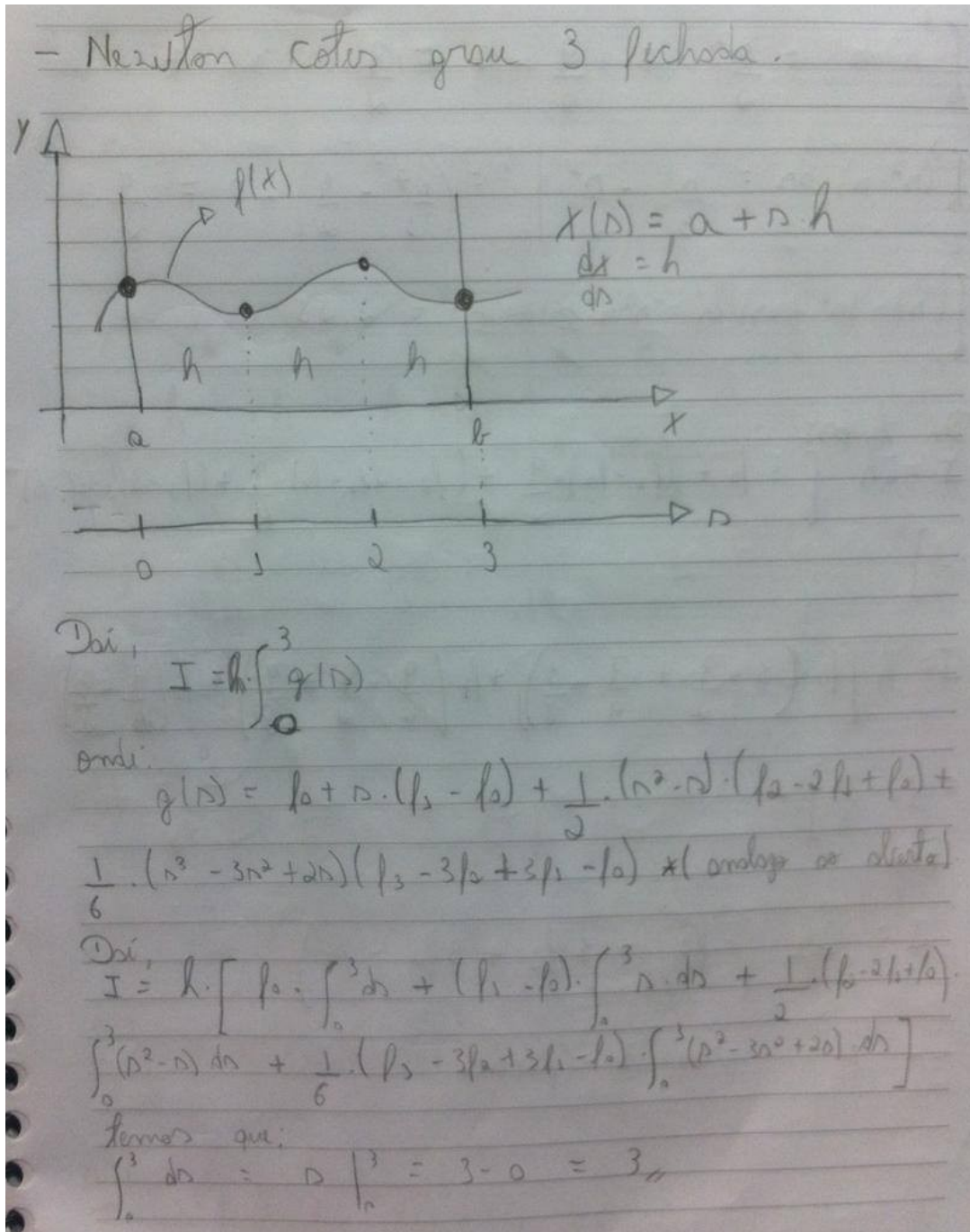


Trabalho I – MNII-2017.1

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Desenvolvimento

Grau 3 – Fechado



$$\int_0^3 \Delta \, d\Delta = \frac{\Delta^2}{2} \Big|_0^3 = \frac{9}{2} - 0 = \frac{9}{2}$$

$$\int_0^3 (\Delta^2 - \Delta) \, d\Delta = \left. \frac{\Delta^3}{3} - \frac{\Delta^2}{2} \right|_0^3 = \left(\frac{27}{3} - \frac{9}{2} \right) - 0 = \frac{9}{2}$$

$$\int_0^3 (\Delta^3 - 3\Delta^2 + 2\Delta) \, d\Delta = \left. \frac{\Delta^4}{4} - \Delta^3 + \Delta^2 \right|_0^3 = 2.25$$

Bei Trapez:

$$I = h \cdot \left[3l_0 + (l_1 - l_0) \cdot \frac{9}{2} + (l_2 - 2l_1 + l_0) \cdot \frac{9}{4} + (l_3 - 3l_2 + 3l_1 - l_0) \cdot \frac{3}{8} \right]$$

$$I = h \cdot \left[l_0 \cdot \left(3 - \frac{9}{2} + \frac{9}{4} - \frac{3}{8} \right) + l_1 \cdot \left(\frac{9}{2} - \frac{9}{2} + \frac{9}{8} \right) + l_2 \cdot \left(\frac{9}{4} - \frac{9}{8} \right) + l_3 \cdot \frac{3}{8} \right]$$

Logo:

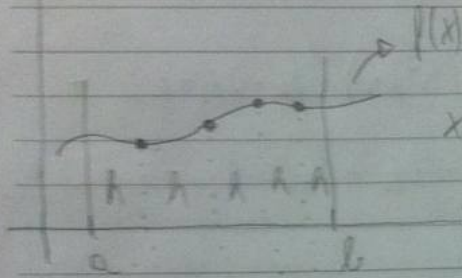
$$I = \frac{3h}{8} \cdot (l_0 + 3l_1 + 3l_2 - l_3)$$

-- lim grau 3 - Pecher

Grau 3 – Aberto

- Newton - cotas aberta de grau 3. aberta.

Δ



$$x(n) = a + h + n \cdot h$$

$$\frac{dx}{dn} = h$$

Δ

x

Δ

$$I = h \int_{-1}^4 g(n) dn$$

-1 0 1 2 3 4

$$g(n) = \sum_{k=0}^3 \binom{n}{k} \Delta^k f_0$$

$$k=0 \rightarrow f_0$$

$$k=1 \rightarrow n \cdot (f_1 - f_0)$$

$$k=2 \rightarrow \frac{1}{2} \cdot (n^2 - n) \cdot (f_0 - 2f_1 + f_2)$$

$$k=3 \rightarrow \frac{1}{6} \cdot (n^3 - 3n^2 + 2n) \cdot \Delta^3 f_0$$

$$\frac{n!}{3! (n-3)!} \cdot \Delta^3 f_0 \rightarrow \frac{n \cdot (n-1) \cdot (n-2) \cdot (n-3)!}{6 \cdot (n-3)!} \cdot \Delta^3 f_0$$

$$\rightarrow \frac{n^3 - 3n^2 + 2n}{6} \cdot \Delta^3 f_0$$

$$\Delta^3 f_0 = \Delta^2 f_1 - \Delta^2 f_0$$

$$= (\Delta^1 f_2 - \Delta^1 f_1) - (\Delta^1 f_1 - \Delta^1 f_0)$$

Endomult

$$\begin{aligned}
 & ((l_2 - l_0) - (l_2 - l_1)) - ((l_2 - l_1) - (l_1 - l_0)) \\
 & (l_2 - l_0 - l_2 + l_1) - (l_2 - l_1 - l_1 + l_0) \\
 & (l_1 - 2l_2 + l_1) - (l_2 - 2l_1 + l_0) \\
 & (l_1 - 2l_2 + l_1) - l_2 + 2l_1 - l_0 \\
 & (l_1 - 3l_2 + 3l_1 - l_0)
 \end{aligned}$$

Dai

$$\begin{aligned}
 g(n) &= l_0 + n \cdot (l_1 - l_0) + \frac{1}{2} \cdot (n^2 - n) \cdot (l_2 - 2l_1 + l_0) + \\
 & \frac{1}{6} \cdot (n^3 - 3n^2 + 2n) \cdot (l_3 - 3l_2 + 3l_1 - l_0)
 \end{aligned}$$

$$I = h \int_{-1}^1 g(n) \cdot dn$$

$$\begin{aligned}
 I &= h \left[l_0 \int_{-1}^1 dn + (l_1 - l_0) \cdot \int_{-1}^1 n \cdot dn + \frac{1}{2} (l_2 - 2l_1 + l_0) \int_{-1}^1 n^2 - n \cdot dn \right. \\
 & \left. + \frac{1}{6} (l_3 - 3l_2 + 3l_1 - l_0) \cdot \int_{-1}^1 n^3 - 3n^2 + 2n \cdot dn \right]
 \end{aligned}$$

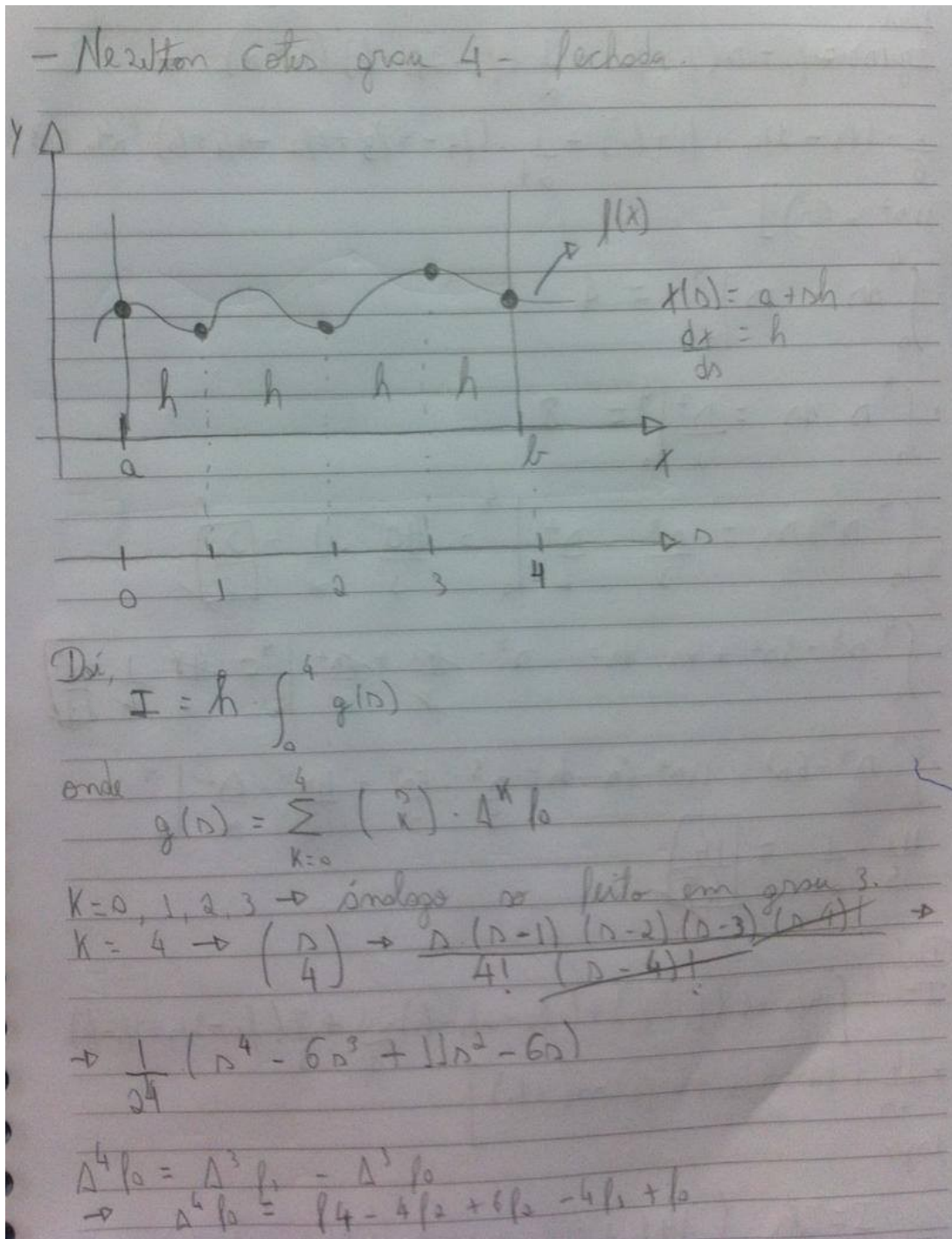
$$\begin{aligned}
 I &= h \left[l_0 \cdot 5 + (l_1 - l_0) \cdot \frac{15}{2} + \frac{85}{12} \cdot (l_2 - 2l_1 + l_0) + \right. \\
 & \left. \frac{55}{24} \cdot (l_3 - 3l_2 + 3l_1 - l_0) \right]
 \end{aligned}$$

$$\begin{aligned}
 I &= h \cdot \left[l_0 \left(\frac{55}{24} \right) + l_1 \left(-\frac{5}{24} \right) + l_2 \left(\frac{5}{24} \right) \right. \\
 & \left. + \frac{55}{24} \cdot l_3 \right]
 \end{aligned}$$

$$I = \frac{5 \cdot h}{24} (11l_0 + l_1 + l_2 + 11l_3)$$

-- lim - grau 3 diventa

Grau 4 – Fechado



Dai,

$$g(n) = f_0 + n(f_1 - f_0) + \frac{1}{2} \cdot (f_2 - 2f_1 + f_0) \cdot (n^2 - n) + \frac{1}{6} \cdot (f_3 - 3f_2 + 3f_1 - f_0) + \frac{1}{24} \cdot (f_4 - 4f_3 + 6f_2 - 4f_1 + f_0) \cdot (n^4 - 6n^3 + 11n^2 - 6n)$$

$$\cdot \int_0^4 dn = n \Big|_0^4 = 4$$

$$\cdot \int_0^4 n \cdot dn = \frac{n^2}{2} \Big|_0^4 = 8$$

$$\cdot \int_0^4 n^2 - n = \frac{n^3}{3} - \frac{n^2}{2} \Big|_0^4 = \frac{40}{3} - \frac{1}{2} = \frac{80}{3}$$

$$\cdot \int_0^4 n^3 - 3n^2 + 2n \cdot dn = \frac{n^4}{4} - \frac{3n^3}{3} + \frac{2n^2}{2} \Big|_0^4 = \frac{48}{4} - \frac{1}{3} = \frac{112}{3}$$

$$\cdot \int_0^4 n^4 - 6n^3 + 11n^2 - 6n \cdot dn = \frac{n^5}{5} - \frac{6n^4}{4} + \frac{11n^3}{3} - \frac{6n^2}{2} \Big|_0^4 =$$

$$\frac{112}{15} \cdot \frac{1}{24} = \frac{14}{45}$$

Dai,

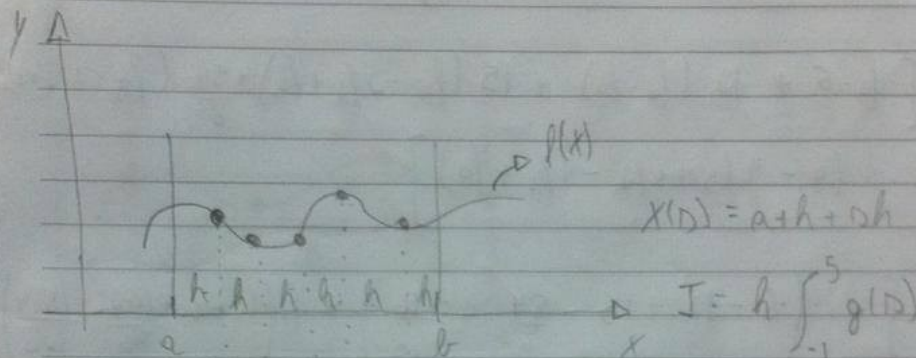
$$I = h \left[4f_0 + 8 \cdot (f_1 - f_0) + \frac{1}{3} \cdot (f_2 - 2f_1 + f_0) \cdot \frac{80}{3} + \frac{1}{3} \cdot (f_3 - 3f_2 + 3f_1 - f_0) \cdot \frac{112}{3} + \frac{14}{45} \cdot (f_4 - 4f_3 + 6f_2 - 4f_1 + f_0) \right]$$

Logo

$$I = \frac{h}{45} (14f_0 + 64f_1 + 24f_2 + 64f_3 + 14f_4)$$

Grau 4 – Aberto

- Newton cotes de grau 4 aberto



$$D \quad -1 \quad 0 \quad 1 \quad 2 \quad 3 \quad 4 \quad 5$$

$$g(D) = f_0 + D \cdot (f_1 - f_0) + \frac{1}{2} \cdot (D^2 - D) \cdot (f_2 - 2f_1 + f_0) + \frac{1}{6} \cdot (D^3 - 3D^2 + 2D) \cdot (f_3 - 3f_2 + 3f_1 - f_0) + \frac{1}{24} \cdot (D^4 - 6D^3 + 11D^2 - 6D) \cdot (f_4 - 4f_3 + 6f_2 - 4f_1 + f_0)$$

$$I = h \int_{-1}^5 g(D) dD$$

$$I = h \cdot \left[f_0 \int_{-1}^5 dD + \int_{-1}^5 D \cdot (f_1 - f_0) dD + \frac{1}{2} \int_{-1}^5 (D^2 - D) \cdot (f_2 - 2f_1 + f_0) dD + \frac{1}{6} \int_{-1}^5 (D^3 - 3D^2 + 2D) \cdot (f_3 - 3f_2 + 3f_1 - f_0) dD + \frac{1}{24} \int_{-1}^5 (D^4 - 6D^3 + 11D^2 - 6D) \cdot (f_4 - 4f_3 + 6f_2 - 4f_1 + f_0) dD \right]$$

$$\int_{-1}^5 dD = 6 \quad \int_{-1}^5 D^2 - D \cdot dD = 30$$

$$\int_{-1}^5 D \cdot dD = 12 \quad \int_{-1}^5 D^3 - 3D^2 + 2D = 54$$

Continuar

$$\int_{-1}^5 x^4 - 6x^3 + 11x^2 - 6x \cdot dx = \frac{396}{5}$$

$$I = h \left[f_0 \cdot 6 + 12 \cdot (f_1 - f_0) + 15 \cdot (f_2 - 2f_1 + f_0) + \frac{54}{6} \cdot (f_3 - 3f_2 + 3f_1 - f_0) + \frac{396}{120} \cdot (f_4 - 4f_3 + 6f_2 - 4f_1 + f_0) \right]$$

$$I = h \cdot \left[f_0 \cdot \left(6 - 12 + 15 - \frac{54}{6} + \frac{396}{120} \right) + f_1 \cdot \left(12 - 30 + \frac{162}{6} - \frac{1584}{120} \right) + f_2 \cdot \left(15 - \frac{162}{6} + \frac{2 \cdot 396}{120} \right) + f_3 \cdot \left(\frac{54}{6} - \frac{1584}{120} \right) + f_4 \cdot \left(\frac{396}{120} \right) \right]$$

$$I = 6h \cdot \left[11 \cdot f_0 - 14f_1 + 26f_2 - 14f_3 + 11f_4 \right]$$

-- lim -- grau 4 stenta.