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Full paper

Variable structure control for a wheeled mobile robot

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Abstract—This paper is concerned with a robust control for wheeled mobile robots. Mobile robots equipped with undeformable wheels are referred to as 'wheeled mobile robots' and constitute a typical example of non-holonomic systems, where the standard control algorithms developed for robotic manipulators without constraints are no longer applicable. It is shown using the formulation of a dynamic feedback linearization (DFL) methodology that a robust sliding mode controller is an efficient design tool to take into account stabilization and tracking control problems. Compared with previous studies based on DFL, the proposed method shows improvement of the trajectory tracking and stabilization process. The robustness is guaranteed in the presence of parameter uncertainty or unmodeled dynamics by the robust sliding mode control technique. Simulation results along with the conclusions drawn are discussed.

Keywords: Wheeled mobile robots; dynamic feedback linearization; variable structure control; trajectory tracking.

1. INTRODUCTION

The problem of motion planning and control of wheeled mobile robots (wmRs) has attracted the interest of a lot of researchers [1]. Due to their non-holonomic properties, restricted mobility and their relevance in applications, the trajectory tracking of those systems has been a challenging class of control problem. Furthermore, the linear control method cannot be employed and, thus, non-linear control has been studied extensively [2–5]. Among those methods, variable structure control (VSC) emerges as a robust approach in different applications and has been successfully applied to control problems as diverse as automatic flight control, control of electrical motors, regulation in chemical processes, helicopter stability augmentation, space systems and robotics. One particular type of VCS system (VSCS) is the so-called sliding mode control (SMC) methodology. In SMC, the VSCS is designed to

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drive and then constrain the system state to lie within a neighborhood of the switching function. Its two main advantages are that the dynamic behavior of the system may be tailored by the particular choice of a switching function and the closed-loop response becomes totally insensitive to a particular class of uncertainty.

Early works in the area of variable structure systems (VSS) and related SMCs were mostly in the Russian literature and these ideas have appeared in English in a survey paper [6]. Subsequently, important theoretical contributions have been reported along with various VSC algorithms [7–9]. Successfully used for the different control problems and applications. Most research and investigations were for the sliding phase, and recent work has mainly involved the design of new sliding (switching) manifolds for sliding motion [10–14]. Indeed, after Fliess's Generalized Observability Canonical Form for designing the sliding surface based on the system inputs and outputs and their time derivatives, Bartolini *et al.* developed and still work on the new concept of the 'second-order sliding mode'. In this approach, the first derivative of the control signal is used as the control instead of the actual signal; extensive works conducted in this direction can be found in Refs [12, 13] and also in our previous work [15]. In addition, new design approaches have used the new concept of the sliding sector [14]. These last two recently introduced concepts are simulating discussion and inviting further research [16]. These issues of switching plane design are reviewed in Zinober [11].

The robustness of SMC still remains an attractive subject for research investigations. In Ref. [17], Lu and Spurgeon considered issues of robustness of static SMC whose design was based on affine non-linear state models in a regular form. It is essentially based on the works of DeCarlo *et al.* [18, 19]. A main feature of this approach is that it provides static feedback and chattering appears if discontinuous switching is adopted. The second area is the dynamic SMC, which is based on differential input–output systems [3, 20–23]. The main characteristic of this approach is that it naturally provides dynamic feedback control and thus reducing will be filtered out [24]. SMC design can be formulated following a dynamic feedback linearization (DFL) methodology, which is an extension of the basic feedback linearization technique, as will be derived later. The resulting technique is abbreviated as DFL-SMC. Recently, various techniques of robust non-linear control have been applied to non-holonomic systems [25–28]. In Ref. [25], the backstepping technique combined with time varyiance has been applied to control non-holonomic systems with unknown parameters. Shim *et al.* [26] proposed SMC for tracking desired trajectory with bounded errors of position and velocity. In Ref. [27], a robust multivariable SMC controller solution is proposed for tracking problems of mobile robots. They exploited the differential flatness of the non-linear kinematics and dynamics of the mobile robot for multivariable sliding controller synthesis. In Ref. [28], the authors propose a SMC by representing the kinematics equation of a mobile robot as two-dimensional polar coordinates which improve the trajectory tracking performance and reduce the constraints on the posture of the robot.

This article is concerned with presenting and developing the design approach of the robust SMC using the formulation of a DFL methodology. This resulting technique will be applied to a WMR type (2, 0). The advantage of following this methodology is the fact that feedback linearization is a technique that can be used for both stabilization and tracking control problems, and single-input as well as multiple-input systems. It has been successfully applied to a number of practical non-linear control problems both as an analytic tool and a simplifying design procedure. These concepts are detailed in the following sections. The derivation of the SMC using the formulation of DFL is presented in Section 2. Section 3 deals with the application of DFL-SMC to a WMR of type (2, 0) with simulation results. Section 4 contains the conclusions.

2. DFL-SMC FOR MOBILE ROBOTS

2.1. DFL

Feedback linearization can be used as a non-linear design method [16, 22, 23, 29, 30] based on the central idea of first algebraically transforming the original system dynamics from a non-linear form into a linear one (fully or partially) and then use linear techniques (e.g. pole placement) to complete the control design.

To perform input–output linearization by output feedback, we consider, in the neighborhood Ω_i of a point x_0, the multi-input multi-output (MIMO) non-linear square system (i.e. systems with the same number of inputs and outputs) of the form:

$$\begin{aligned} \dot{x} &= f(x) + G(x).u = f(x) + \sum_{i=1}^{m} u_i.g_i(x), \\ y &= h(x), \end{aligned} \tag{1}$$

where x is the $n \times 1$ state vector, u is the $m \times 1$ control input vector (of components u_i), y is the $m \times 1$ vector of the system outputs (of components y_i), f and h are smooth vector fields, and G is a $n \times m$ matrix whose columns are smooth vector fields g_i on R^n.

This multivariable non-linear system has a vector relative degree $(r_1, r_2, \dots, r_m)$ at point x_0, where r_i is the r_i times of differentiation of the output of the system y_i to generate an explicit relationship between the output y_i and the input u_i. The scalar $r = r_1 + r_2 + \cdots + r_m$ is called the total relative degree of the system at x_0.

Since the concepts of single input single output systems can be extended to MIMO systems, the relative degree r_i is defined as:

$$\begin{aligned} &L_{g_j}.L_f^k.h_i(x) = 0 \quad &&\text{for all } 1 \leqslant j \leqslant m,\ 1 \leqslant i \leqslant m,\ k \prec r_i - 1, \\ &L_{g_j}.L_f^{r_i-1}.h_i(x) \neq 0 \quad &&\text{for all } 1 \leqslant j \leqslant m,\ 1 \leqslant i \leqslant m, \end{aligned}$$

and where $L_f h$ denotes the Lie derivative of the function h along the vector field f and is given by:

$$L_f h = \left(\frac{\partial h}{\partial x}\right) f = \sum_{i=1}^{n} \frac{\partial h}{\partial x_i} f_i(x). \tag{2}$$

Thus, (1) can be written as [23]:

$$y_i^{(r_i)} = L_f^{r_i} h_i + \sum_{j=1}^{m} L_{g_j} L_f^{r_i - 1} h_i u_j, \tag{3}$$

with $L_{g_j} L_f^{r_i-1} h_i(x) \neq 0$ for at least one j, in the neighborhood Ω_i of the input x_0.

For each y_i, (2) will be written as follows:

$$\begin{bmatrix} y_1^{(r_1)} \\ \cdots \\ \cdots \\ y_m^{(r_m)} \end{bmatrix} = \begin{bmatrix} L_f^{r_1} h_1(x) \\ \cdots \\ \cdots \\ L_f^{r_m} h_m(x) \end{bmatrix} + \xi(x).u, \tag{4}$$

where $\xi(x)$ is the $m \times m$ matrix:

$$\xi(x) = \begin{bmatrix} L_{g_1} L_f^{r_1-1} h_1 & \cdots & \cdots & L_{g_m} L_f^{r_1-1} h_1 \\ \cdots & \cdots & \cdots & \cdots \\ \cdots & \cdots & \cdots & \cdots \\ L_{g_1} L_f^{r_m-1} h_m & \cdots & \cdots & L_{g_m} L_f^{r_m-1} h_m \end{bmatrix}.$$

If $\xi(x)$ is invertible over the region Ω, then the input transformation:

$$u = \xi^{-1} \begin{bmatrix} v_1 - L_f^{r_1}.h_1 \\ \cdots \\ \cdots \\ v_m - L_f^{r_m}.h_m \end{bmatrix}, \tag{5}$$

yields m equations of the simple form:

$$y_i^{r_i} = v. \tag{6}$$

The above input–output linearization can be achieved only when the decoupling matrix ξ is invertible in the region Ω. However, when ξ is singular, a dynamic extension which involves choosing some new inputs as the derivatives of some of the original system inputs is used in a way that the corresponding matrix ξ is invertible. The control system is designed based on the new set of inputs and the actual system inputs are then computed by integration. Thus, the actual control law contains an integrator which gives rise to the property of dynamism, yielding a 'dynamic controller'. Therefore, redefining the inputs of the system performs a dynamic extension of the original system to be feedback linearized. If the matrix ξ is still singular, the procedure can be repeated — amounting to adding more integrators at the system input.

2.2. SMC

The principle of SMC consists of finding a variable structure control to first drive and then constrain the system state to lie within a neighborhood of the switching function. Thus, we consider the following non-linear MIMO system:

$$\dot{x} = f(x,t) + g(x,t).u, \tag{7}$$

where $x \in R^n$ and $u \in R^m$. The basic SMC problem consists of designing:

(i) A set of m switching functions, $s(x)$, representing the design specifications.

(ii) A variable structure control function $u(x,t)$:

$$u(x,t) = \begin{cases} u^+(x,t) > 0 & \text{when } s(x) > 0 \\ u^-(x,t) < 0 & \text{when } s(x) < 0. \end{cases} \tag{8}$$

Such that the switching surface $s(x) = 0$ is reached in finite time.

Thus, the meaning of the above statements is (i) that the switching surface, $s(x) = 0$, is selected to represent the desired dynamics of the plant (this surface is of lower order then the given system) and (ii) a relay-type control law is designed to attract the state trajectories and ensure their convergence to the switching surface, by switching to a particular control structure as the system state crosses the discontinuity surface in state place. Sliding mode takes place in the closed-loop system when the state is confined to the switching surface and cannot leave it for the remainder of the motion. In this way, the overall system is asymptotically stable and follows the desired dynamics.

3. DFL-SMC

As shown above, a DFL is provided to the non-linear system as a first stage of the control design. This results, as shown in Fig. 1, in an inner loop with non-linear dynamics and an outer loop corresponding to linear dynamics. Indeed, a second stage of the control design can be performed now by applying a robust sliding mode control technique to the outer loop inputs, v.

These two stages, then, establish the previously nominated DFL-SMC approach.

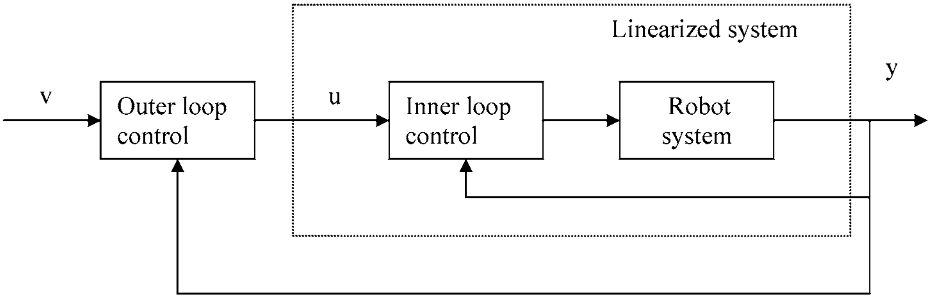

Figure 1. 'Inner loop/outer loop' realization.

3.1. DFL-SMC methodology to a type (2, 0) WMR

In this section, following the control design strategy presented in the precedent section, we explicitly develop a robust law to control a type (2, 0) WMR for the purposes of stabilization and trajectory tracking. This simulation work is the first issue to be implemented on the real WMR shown in Fig. 2.

A control design for a type (2, 0) WMR is tackled following the DFL-SMC methodology. Indeed, the DFL approach copes well with the structural properties of a type (2, 0) WMR; in particular, since the non-linear models of this system are coupled along with the decoupling procedures resulting in the serious problem of invertibility of the decoupling matrix, this makes a dynamically extended feedback linearization a fundamental requirement.

We consider a type (2, 0) WMR. The posture dynamic model of this robot with three posture state variables to be controlled using only two control inputs is given by the following five established equations:

$$\begin{aligned} \dot{x} &= -\eta_1 \sin\theta, \\ \dot{y} &= \eta_1 \cos\theta, \\ \dot{\theta} &= \eta_2, \\ \dot{\eta}_1 &= u_1, \\ \dot{\eta}_2 &= u_2, \end{aligned} \tag{9}$$

where x, y are the position coordinates on the plane of the center of mass of the robot, θ is the heading angle, η_1 is the instantaneous velocity in the heading direction and η_2 is the instantaneous angular velocity of the coordinates fixed to the body of the robot. u_1, u_2 represent, respectively, linear acceleration or angular accelerations (equivalently, pushing forces and rotation torques). Thus, the state vector $X = [xy\theta\eta_1\eta_2]^T$ and the input vector $U = [u_1u_2]^T$.

Then, our control objective, for these mobile robots, is to make the three posture variables x, y and θ tend to their desired values x_d, y_d and θ_d, respectively, i.e. to make the errors: $\tilde{x} = x_d - x$, $\tilde{y} = y_d - y$ and $\tilde{\theta} = \theta_d - \theta$ tend to zero.

Figure 2. A photograph of our WMR.

Two cases may be of interest. When the desired values are given constant values corresponding to target points in the plane of motion that would be equilibrium configurations for the robot, e.g. from an initial configuration to a final one, the control problem becomes that of stabilization, which is not trivial. Alternatively, when the desired values are moving trajectories such as line or circles, trajectory tracking would be the goal of the control.

3.2. Control law synthesis

Let us choose the system output, Z, in such a way to control the whole robot posture, i.e. $Z = [xy\theta]^T$, where the output variables are:

$$\begin{aligned} z_1 &= x, \\ z_2 &= y, \\ z_3 &= \theta. \end{aligned} \tag{10}$$

Then the first derivatives of the output-variables are:

$$\begin{aligned} \dot{z}_1 &= -\eta_1 \sin\theta, \\ \dot{z}_2 &= \eta_1 \cos\theta, \\ \dot{z}_3 &= \eta_2. \end{aligned} \tag{11}$$

Deriving the outputs twice yields the acceleration-level derivatives:

$$\begin{aligned} \ddot{z}_1 &= -\eta_1\eta_2 \cos\theta - u_1 \sin\theta, \\ \ddot{z}_2 &= -\eta_1\eta_2 \sin\theta + u_1 \cos\theta, \\ \ddot{z}_3 &= \dot{\eta}_2. \end{aligned} \tag{12}$$

Now if we consider the partial relative degrees for the three output variables in the equations above, then they are, respectively: $r_1 = 2, r_2 = 2, r_3 = 2$. Thus the corresponding decoupling matrix is singular since the system state is of an order: 5.

We proceed, then, by augmenting the system state in (9) from five to six variables by redefining the system inputs. That is, we introduce a new state variable which is the old input u_1 and we define a new fictive input w_1 which is the derivative of u_1, such that:

$$\dot{u}_1 = w_1. \tag{13}$$

In this case, the augmented state vector and the redefined input vector are defined, respectively, as:

$$\begin{aligned} \overline{X} &= [x \;\; y \;\; \theta \;\; \eta_1 \;\; \eta_2 \;\; u_1]^T, \\ \overline{U} &= [u_2 \;\; w_1]^T. \end{aligned}$$

Then, having in mind that u_1 is a state variable, we can derive once more the system output variables. This yields variables of the jerk-level (third order), given by the following equations:

$$\begin{aligned}
\dddot{z}_1 &= \eta_2^2\eta_1 \sin\theta - 2\eta_2 u_1 \cos\theta - \eta_1 u_2 \cos\theta + w_1 \sin\theta,\\
\dddot{z} &= -\eta_2^2\eta_1 \cos\theta - 2\eta_2 u_1 \sin\theta - \eta_1 u_2 \sin\theta + w_1 \cos\theta,\\
\dddot{z}_3 &= \ddot{\eta}_2.
\end{aligned} \tag{14}$$

At this stage, the partial relative degrees are $r_1 = 3, r_2 = 3, r_3 = 2$. For r_1 and r_2, here, both inputs appear in the output equations. Thus, when considering only the first two components of the output vector, z, the total relative degree is $r = r_1 + r_2 = 6$, which is exactly equal to the number of state variables. Therefore, a full output feedback linearization could be obtained in this case. Furthermore, the third output variable of Z could be also adjusted.

In vectorized form, system (14) becomes:

$$\begin{bmatrix} \dddot{z}_1 \\ \dddot{z}_2 \end{bmatrix} = \begin{bmatrix} \eta_2^2\eta_1 \sin\theta - 2\eta_2 u_1 \cos\theta \\ -\eta_2^2\eta_1 \cos\theta - 2\eta_2 u_1 \sin\theta \end{bmatrix} + \begin{bmatrix} -\eta_1 \cos\theta & \sin\theta \\ -\eta_1 \sin\theta & \cos\theta \end{bmatrix} \cdot \begin{bmatrix} u_2 \\ w_1 \end{bmatrix}. \tag{15}$$

We set:

$$\overline{F} = \begin{bmatrix} \eta_2^2\eta_1 \sin\theta - 2\eta_2 u_1 \cos\theta \\ -\eta_2^2\eta_1 \cos\theta - 2\eta_2 u_1 \sin\theta \end{bmatrix} \quad \text{and} \quad \overline{G} = \begin{bmatrix} -\eta_1 \cos\theta & \sin\theta \\ -\eta_1 \sin\theta & \cos\theta \end{bmatrix}.$$

Now, we define v_1 and v_2 as the auxiliary input variables that set the desired linear dynamics of the two system output variables, z_1 and z_2, through their third-order derivatives as follows:

$$\begin{cases} \dddot{z}_1 = v_1 \\ \dddot{z}_2 = v_2. \end{cases}$$

Let V represent the vector of the auxiliary outer-loop control inputs, i.e.

$$V = \begin{bmatrix} v_1 \\ v_2 \end{bmatrix}.$$

Then, from (15), the inner loop non-linear dynamics of the system can be assigned by the redefined input vector $\overline{U} = \begin{bmatrix} u_2 \\ w_1 \end{bmatrix}$ that is determined in terms of V and given by:

$$\overline{U} = (\overline{G})^{-1}.\left[V - \overline{F}(\overline{X})\right], \tag{16}$$

where the decoupling matrix $\overline{G}$ should verify the non-singularity condition expressed as:

$$\mathrm{Det}(\overline{G}) \neq 0,$$

which is true if and only if $\eta_1 \neq 0$, since $\mathrm{Det}(\overline{G}) = -\eta_1^2(\sin\theta)^2 - \eta_1^2(\cos\theta)^2 = -\eta_1^2$. That is, the decoupling matrix is non-singular whenever the (longitudinal) velocity is ensured to be different from zero. In other words, when the robot is moving and does not encounter any rest configuration. In this case, the augmented system is said to be fully dynamically input–output feedback linearizable. Here the term dynamically indicates that we control the derivative of the input or, alternatively, the input is delayed.

The implication of this is that the linearization is full and system stability is totally determined externally since no internal state exists. Thus, no care is devoted to problems of zero-dynamics stability. Then, the augmented non-linear system, as linearized and decoupled, can be written into two linear subsystems given in the so-called Brunovesky form, as follows:

$$\dot{Z}_i = A_i Z_i + b_i v_i \quad \text{for} \quad i = 1, 2, \tag{17}$$

where:

$$Z_i = \begin{bmatrix} z_i \\ \dot{z}_i \\ \ddot{z}_i \end{bmatrix} \quad \text{and} \quad \dot{Z}_i = \begin{bmatrix} \dot{z}_i \\ \ddot{z}_i \\ \dddot{z}_i \end{bmatrix} \quad \text{for } i = 1, 2,$$

and:

$$A_i = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 0 & 0 & 0 \end{bmatrix} \quad b_i = \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} \quad \text{for } i = 1, 2. \tag{18}$$

As stated above, the outer loop inputs v_1 and v_2 are chosen in such a way to set the desired dynamics to the output variables z_1 and z_2 in order to achieve the control objective. In this context, the control problem is to be performed as stabilization problem of the error dynamics as to be derived in the sequel.

We define the errors in output variables and their dynamics, respectively, as:

$$e_i = \begin{bmatrix} e_{i1} \\ e_{i2} \\ e_{i3} \end{bmatrix} \quad \dot{e}_i = \begin{bmatrix} \dot{e}_{i1} \\ \dot{e}_{i2} \\ \dot{e}_{i3} \end{bmatrix} \quad \text{for } i = 1, 2,$$

where:

$$\begin{aligned} e_{i1} &= \tilde{z}_i = z_i - z_{i\mathrm{d}}, \\ e_{i2} &= \dot{e}_{i1} = \frac{\mathrm{d}}{\mathrm{d}t}\left(\tilde{z}_i\right) = \dot{z}_i - \dot{z}_{i\mathrm{d}}, \\ e_{i3} &= \dot{e}_{i2} = \frac{\mathrm{d}^2}{\mathrm{d}i^2}\left(\tilde{z}_i\right) = \ddot{z}_i - \ddot{z}_{i\mathrm{d}}, \\ \dot{e}_{i3} &= \frac{\mathrm{d}^3}{\mathrm{d}t^3}\left(\tilde{z}_i\right) = \dddot{z}_i - \dddot{z}_{i\mathrm{d}} . \end{aligned} \tag{19}$$

Here, the subscript ‘d’ indicates the desired values of the concerned variables.

We can write the original problem given by (17) in the error space as follows:

$$\dot{e}_i = A_i e_i + b_i\left(v_i - \dddot{z}_{i\mathrm{d}}\right) \quad \text{for } i = 1, 2,$$

where the objective is to stabilize the dynamics of errors e_1 and e_2 by properly choosing the inputs v_1 and v_2. In fact, as said previously, we applied the robust SMC.

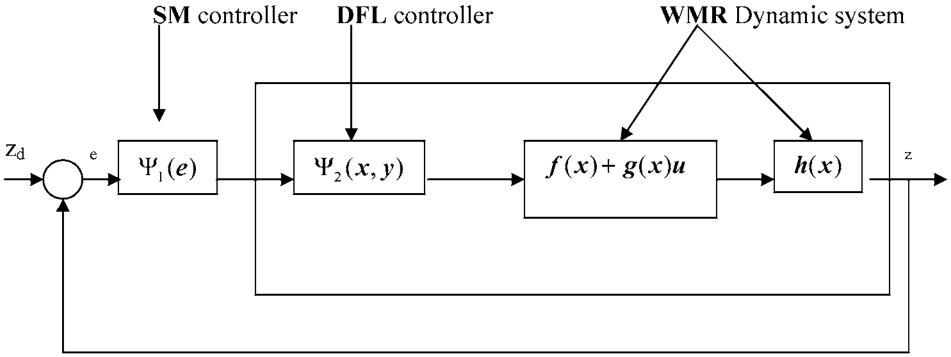

Figure 3. Input–output DFL-SMC of a type (2, 0) WMR system.

Using a SMC approach, the control inputs v_1 and v_2 can be determined for instance as a the sum of two terms — a continuous term which is the equivalent control and a discontinuous term which is a function of the Signum function. Other techniques may be applied. We define the sliding surfaces as:

$$S_i = C_i e_i \quad \text{for } i = 1, 2, \tag{20}$$

where C_i are the surface gains such that: $C_i = [c_{i1}\ c_{i2}\ c_{i3}]$ for $i = 1, 2$.

On the one hand, the surface dynamics are expressed as: $\dot{S}_i = C_i.\dot{e}_i$

$$\dot{S}_i = C_i A_i e_i + C_i b_i v_i - C_i b_i \dddot{z}_{id} \quad \text{for } i = 1, 2, \tag{21}$$

such that $C_i b_i \neq 0$ should be verified.

On the another hand, by definition, the desired surface dynamics are set as:

$$\dot{S}_i \triangleq -Q_i S_i - P_i \operatorname{sign}(S_i) \quad \text{for } i = 1, 2, \tag{22}$$

where Q_i and P_i are tuned to set the desired dynamics of the two sliding surfaces.

Then, by equating formulas (21) and (22), the control input laws v_1 and v_2 are derived as:

$$v_i = \frac{1}{C_i b_i}\left[-Q_i S_i - P_i \operatorname{sign}(S_i) - C_i A_i e_i + C_i b_i \dddot{z}_{id}\right] \quad \text{for } i = 1, 2. \tag{23}$$

Hence, the two law resulting from the equation above define the robust auxiliary control input that is applied at the linear outerloop of the original system, as schematized in Fig. 3. The inner loop non-linear dynamics are determined by control input variables u_1 and u_2 where u_2 and w_1 are obtained first from (16), and u_1 can be obtained after that by integration of w_1, as follows from (13).

3.3. Interpretation

The appealing characteristics of the implemented method can be visualized through the presented result for different control cases and different initial conditions. Figure 4 shows the simulation results of DFL in the case of regulation. Figure 5

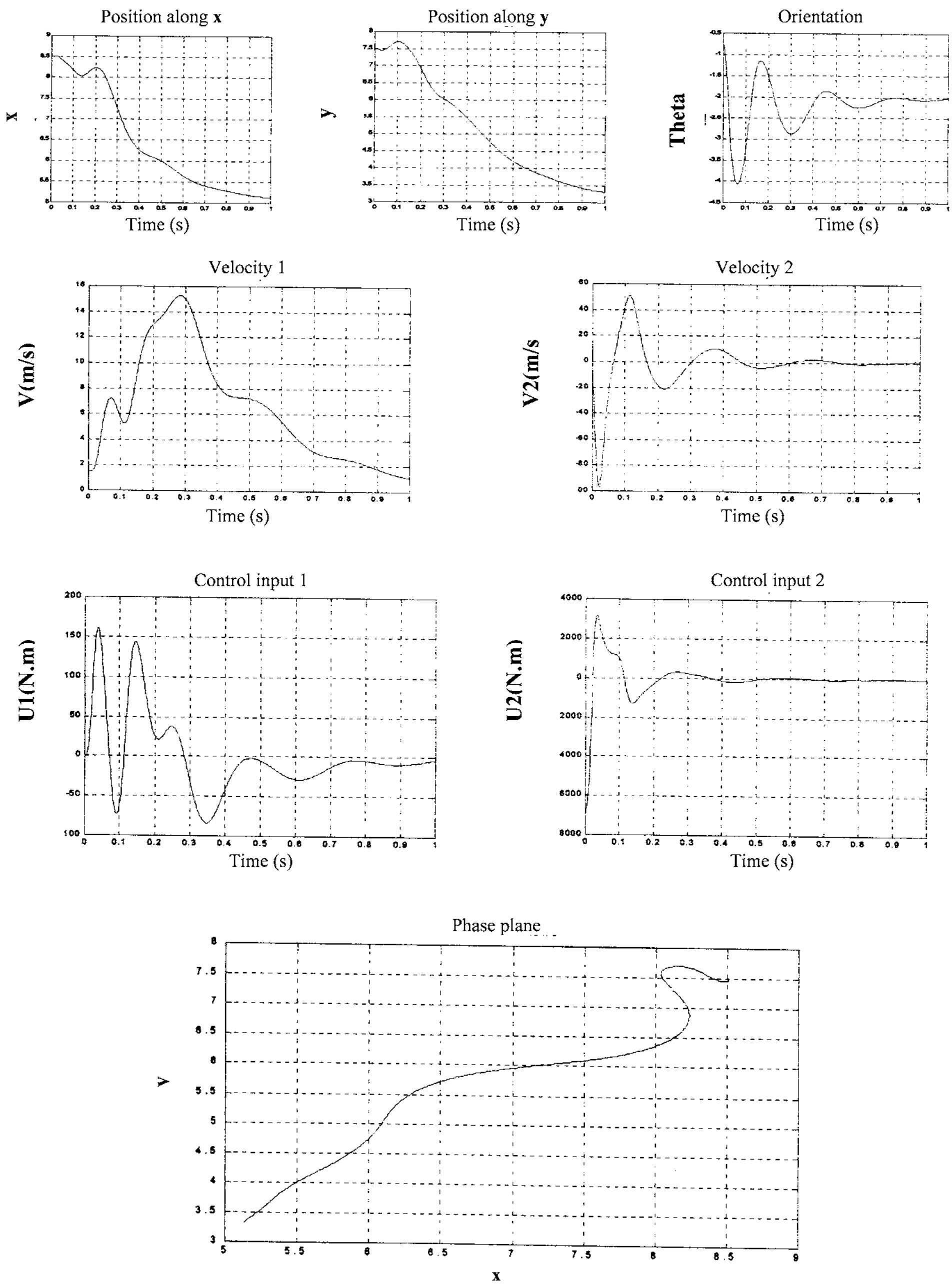

Figure 4. Results of DFL in the case of regulation.

shows the results of the DFL-SMC algorithm in the case of regulation from an initial point $Z_i = [8.5\ 7.5\ -3\pi/4]$ to a final point $Z_f = [5\ 3\ -3\pi/4]$, with $C_i = [500\ 10\ 1]$, $Q_i = 5$ and $P_i = 2.5$ for $i = 1, 2$.

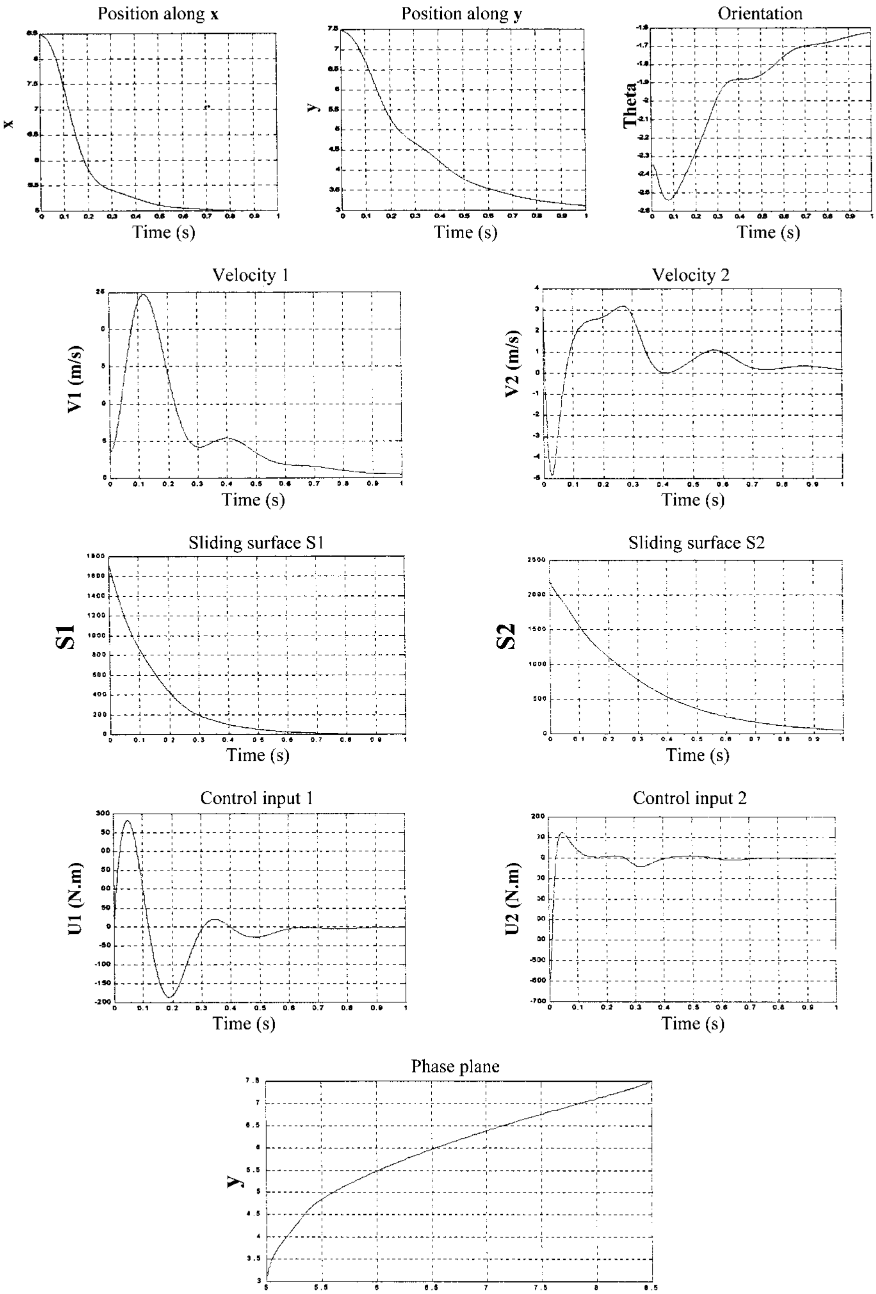

Figure 5. DFL-SMC in the case of regulation.

Figure 6 shows the results of tracking of a given trajectory as a circle, where $x^2 + y^2 = 2$ is the equation of the circle. Figures 5 and 6 show the improvement of the trajectory tracking and the stabilization process, and also good performances

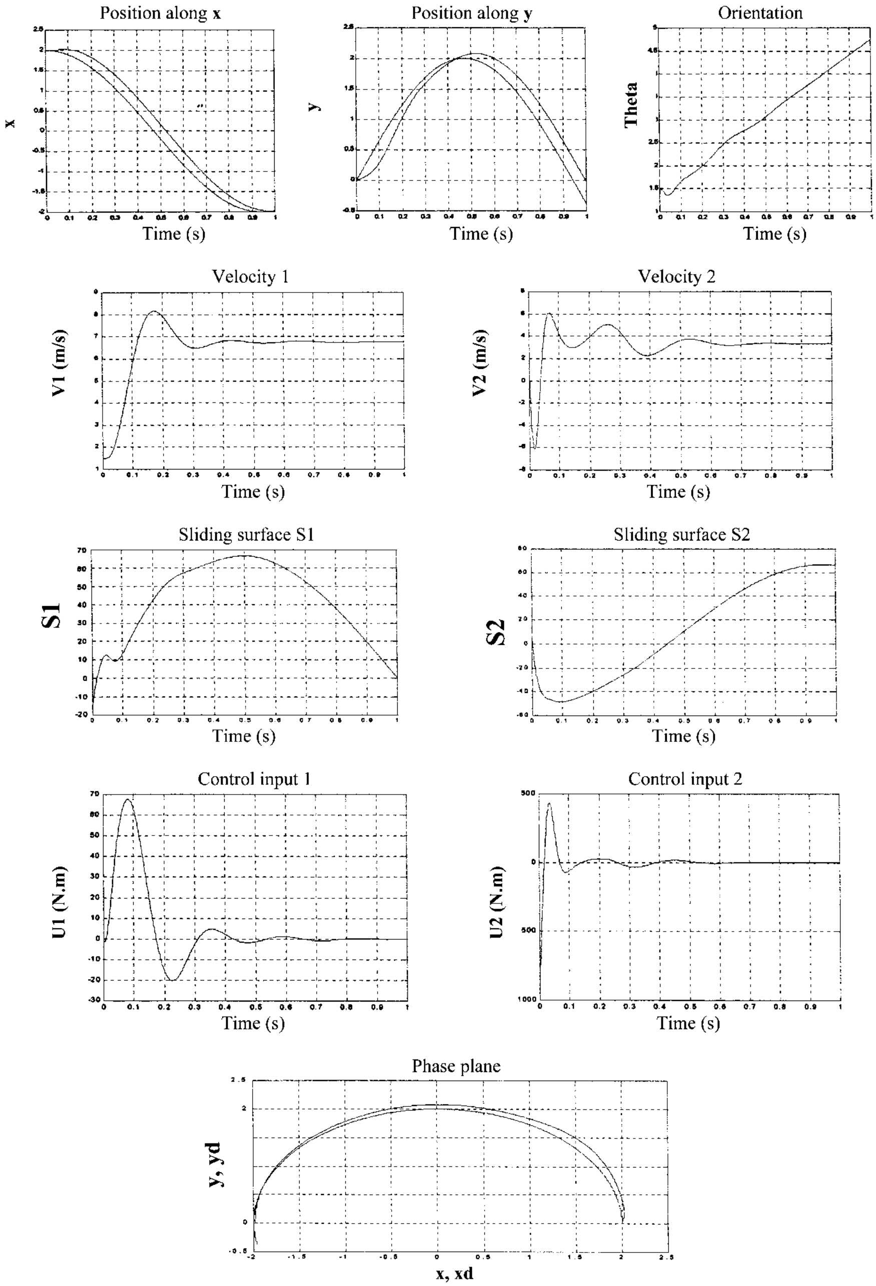

Figure 6. DFL-SMC in the case of trajectory tracking.

concerning the convergence itself, its rate, the steady state errors as well as the important issue of chattering. This latter is considerably reduced, particularly for the first control input u_1, since the control action is performed in its derivative. This manner of control is the basic idea of a large amount of works originating from the theory introduced by Fliess [21] and team about the generalized variable structure (GVS) systems of control, and its application can be found in Ref. [15].

Important errors introduced in the initial conditions show the strength of convergence in different cases. Either for stabilization to any point in the phase plane, tracking a circular trajectory from an inner point of the circle, an outer point or from a point on the circle itself. A particular remark concerns the impact of the initial orientation on trajectory tracking, which greatly influences the subsequent robot behavior and often the controller has to be adapted to this initial condition to ensure the tracking. In practice, estimation-based control may help to overcome such difficulty since empirical measures determine how the robot is to be adapted with the occurring errors. Also, only position is stabilized to the desired values and not the orientation. This shows the limitation associated with the fact of having less input than the controlled variables. Sometimes very high gains have to be supplied in order to obtain the desired global performance. In general, we can say that the results obtained with DFL-SMC are satisfactory in terms of applicability of the method to both stabilization, and tracking convergence and control smoothness for different cases.

4. CONCLUSIONS

This paper presents the main theoretical aspects of SMC and the principal design procedure using DFL along with the associated simulations of the developed control laws for the considered type (2, 0) WMR.

The sliding mode design approach consists of two main components. The first involves the design of a switching function so that the sliding motion satisfies design specifications. The second is concerned with the selection of a control law which will make the switching function attractive to the system state. Thus, DFL-SMC has allowed us to exploit the characteristic structure of the model type (2, 0) WMR. This method is applicable to both stabilization and trajectory tracking.

The effectiveness of the proposed approach is demonstrated through simulation results the different cases in this study support the application of DFL-SMC methodology. As SMC is robust against external disturbance and noise, these influences can be pursued as future research.

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