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Robust control laws for wheeled mobile robots

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The paper deals with feedback control of wheeled mobile robots. The proposed controller is based on a tracking scheme where the real robot tracks a fictitious reference one with equivalent kinematical properties. A solution to the parking problem is derived from the basic tracking scheme by considering a reference vehicle which converges to the desired configuration. The goal achievement is analysed by the stability of the zero equilibrium point of the tracking state error. Simulation as well as experimental results illustrate these controllers' designs. Robustness with respect to errors in the state estimation is investigated. It is defined by the existence of a compact attractive domain around the zero error. A practical computation of such a domain is a result of this paper.

1. Introduction

Common mobile robots are car-like or cart-like vehicles whose kinematics exhibit non-holonomic constraints. In opposition to holonomic open chain manipulators where stabilization around a fixed configuration is simpler than tracking a desired trajectory, the non-holonomy constraint of a vehicle implies that tracking control is easier than regulation around a fixed reference (Samson 1993). It has been shown that the problems encountered stabilizing the mobile robot at a given configuration can be avoided if the control is designed to track a fictitious (or real) robot with the same non-holonomy constraints and which converges to the desired configuration (Kanayama *et al.* 1990, Samson and Aït-Abderrahim 1990, Canudas and Sordalen 1991, and Canudas *et al.* 1994).

Another fundamental difference between manipulators and mobile robots lies in the configuration estimation which is used for feedback. The attitude of the end effector of a manipulator robot can be obtained from the joints angular encoders, and it is defined with respect to a stationary frame at the base of the arm. In opposition to this basic localization procedure, it is well known that a mobile robot configuration is not observable from measurements of wheel speeds.

Exteroceptive measurements must then be processed to update the vehicle localization to limit the error estimation drift caused by use of proprioceptive information. This is the price to pay for dealing with the obvious fact that a vehicle has no fixed point in the world frame in opposition to a manipulator which is tied to its basis.

Estimation of robot localization has received much interest (Hamel *et al.* 1993, Leonard and Durrant-White 1991, Preciado *et al.* 1991), but the fact is that localization errors should not be neglected in any case.

In this paper the robustness of mobile robot feedback tracking control law with respect to configuration and curvature estimation errors is addressed. A new robust dynamic controller is proposed here. It is an extension of that formerly given by Hamel *et al.* (1994) and it better takes into account the structural dynamics of car-like vehicles. Its robustness is defined with respect to the configuration estimation errors and is expressed by the definition of a bounded attraction domain around the nominal configuration followed when feedback of the true state is performed. This domain is expressed using Lyapunov analysis. The paper is organized as follows: some preliminary definitions are given in the next section. Robustness analysis is investigated in § 3. A robust dynamic state feedback tracking controller is proposed in § 4. Its extension for the parking problem is described in § 5. Some simulation results are given in § 6, and in § 7 experimental results are given. Finally some conclusions are given in § 8.

2. Problem statement

The robustness of tracking control naturally stems from the statement of both the tracking control (§ 2.2) and the vehicle kinematics (§ 2.1).

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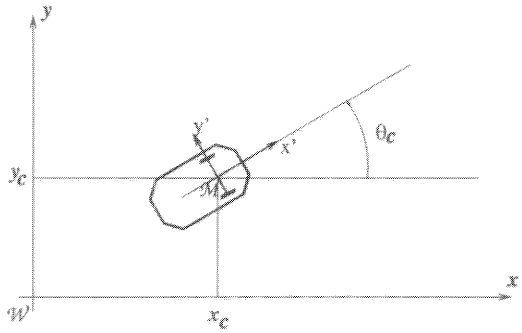


Figure 1. Vehicle configuration.

2.1. *Vehicle kinematics*

The mobile robot that we study is represented in Fig. 1. Its configuration is described by a vector $P_c = (x_c, y_c, \theta_c)^T$ composed of the vehicle orientation θ_c and the coordinates (x_c, y_c) in a world frame $\mathscr{W}$ of a robot characteristic point C.

The kinematics of such a robot are described by

$$\dot{P}_c(t) = \begin{cases} \dot{x}_c = v_c \cos\theta_c, \\ \dot{y}_c = v_c \sin\theta_c, \\ \dot{\theta}_c = \omega_c = v_c \chi_c, \end{cases} \tag{1}$$

$$\dot{\chi}_c = \vartheta_c. \tag{2}$$

The control inputs are $(v_c(t), \vartheta_c(t))^T$, and the curvature is a state component (2). The motions then naturally exhibit a continuous curvature property, thus complying with structural dynamic requirements.

2.2. *Tracking control*

The tracking control problem is represented in Fig. 2, and consists of tracking a fictitious kinematically equivalent robot. Let $P_r = (x_r, y_r, \theta_r)^T$ be the configuration of the reference robot (triangular), and let $(v_r(t), \vartheta_r(t))^T$ be its translational speed and curvature derivative. Under the hypothesis that the reference robot never stops, its configuration $P_r(t)$ constitutes the moving set point for $P_c(t)$. Consider the error configuration vector P_e with respect to the mobile reference frame $\mathscr{M}$:

$$\left.\begin{aligned} P_e &= P_e^{\mathscr{M}} = TP_e^{\mathscr{W}} = T(P_r - P_c), \\ T &= \begin{pmatrix} \cos\theta_c & \sin\theta_c & 0 \\ -\sin\theta_c & \cos\theta_c & 0 \\ 0 & 0 & 1 \end{pmatrix}. \end{aligned}\right\} \tag{3}$$

Under the hypothesis that the reference trajectory curvature is continuous and bounded, the curvature error is introduced as a new state. The configuration and curvature errors evolution, as presented by Kanayama *et al.* (1990), is then defined by (4), where the motion is parametrized with respect to the reference trajectory:

$$\left.\begin{aligned} \begin{pmatrix} \dot{x}_e \\ \dot{y}_e \\ \dot{\theta}_e \end{pmatrix} &= \begin{pmatrix} v_r \chi_c^r y_e - v_r v_c + v_r \cos\theta_e \\ -v_r \chi_c^r x_e + v_r \sin\theta_e \\ v_r \chi_e \end{pmatrix}, \\ \dot{\chi}_e &= \vartheta_r - \vartheta_c, \end{aligned}\right\} \tag{4}$$

where $\chi_e = \chi_r - \chi_c^r$, χ_r is the curvature of the reference trajectory $(\dot{\theta}_r/v_r)$, χ_c^r is the curvature related to the reference trajectory $(\dot{\theta}_c/v_r)$, and v_c is the relative velocity (v_c/v_r).

v_r is factor common to the right-hand side of the state equation (4), emphasizing its role as time versus length parametrization. In addition, $(v_r v_c, \vartheta_c)^T$ are considered as the vehicle control inputs of the tracking error process (4).

2.3. *Robustness problem statement*

Any feedback control law is given by (5) and assumes knowledge of feedback information.

$$\left.\begin{aligned} v_c &= v_c(P_e^T, \chi_e, \chi_r, \vartheta_r), \\ \vartheta_c &= \vartheta_c(P_e^T, \chi_e, \chi_r, \vartheta_r). \end{aligned}\right\} \tag{5}$$

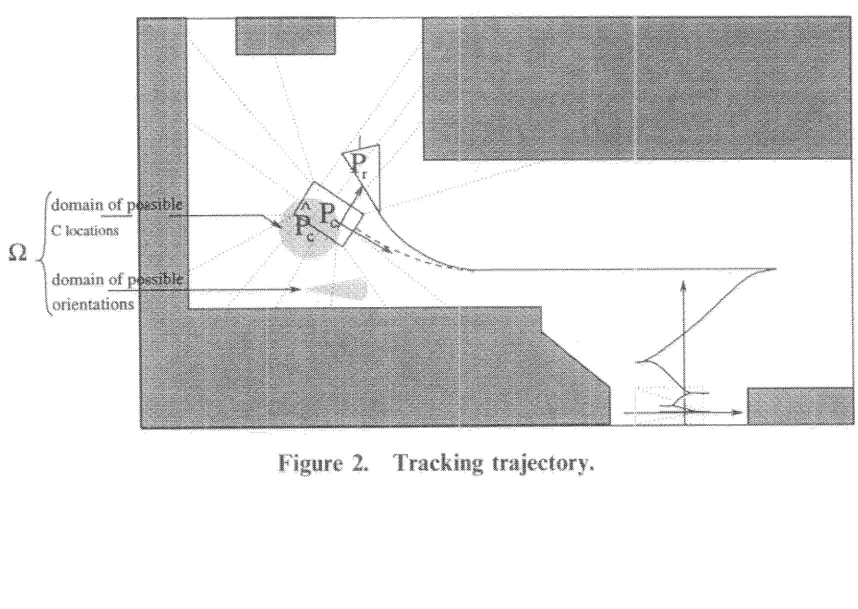


Figure 2. Tracking trajectory.

In a real situation the configuration and the curvature are not directly measurable and their estimates are used in the feedback control law instead of the true values:

$$\left.\begin{aligned}\hat{v}_c &= \vartheta_c(\hat{P}_e^T, \hat{\chi}_e, \chi_r, \vartheta_r),\\ \hat{\vartheta}_c &= \vartheta_c(\hat{P}_e^T, \hat{\chi}_e, \chi_r, \vartheta_r).\end{aligned}\right\} \tag{6}$$

Estimates $\hat{P}_e$ of P_e (3) and $\hat{\chi}_e$ of χ_e (4) are obtained from the estimates ($\hat{P}_c = P_c + \delta P_c, \hat{\chi}_c = \chi_c^r + \delta\chi_c^r$) of both the configuration and the curvature continuously updated by combining dead reckoning and exteroceptive measurements (Leonard and Durrant-White 1991, Rombaut and Meizel 1994). This estimate lies in a compact confidence domain centred on $\hat{P}_c$ (see Fig. 2) ($\hat{P}_c = P_c + \delta P_c; \delta P_c \in \Omega$).

By virtue of EKF formalism (Cheeseman and Smith 1986) or merely for computational ease (Preciado *et al.* 1991), the feasible domain Ω is desribed as an ellipsoid, a truncated cylinder or a bounding box. Under the assumption that $\delta\theta_c \ll 1$ and $\delta\chi_c^r \in \mathscr{X} = [-\delta\chi_{max}, \delta\chi_{max}]$, and by following (3)

$$\left.\begin{aligned}\hat{P}_e &= \hat{T}(P_r - \hat{P}_c) = P_e + \Delta P_e,\\ \hat{\chi}_e &= \chi_r - \hat{\chi}_c = \chi_e - \delta\chi_c^r,\end{aligned}\right\} \tag{7}$$

where, from (7) and (3)

$$\Delta P_e = \begin{cases}\Delta x_e = \delta\theta_c y_e - \delta^* x_c,\\ \Delta y_e = -\delta\theta_c x_e - \delta^* y_c,\\ \Delta\theta_e = -\delta^*\theta_c,\end{cases} \tag{8}$$

and

$$\delta^* P_c = \hat{T}\delta P_c. \tag{9}$$

It is worth noting that $\delta^* P_c$ is just a rotation of δP_c. Thus, if the feasible domain Ω is regarded as a truncated cylinder with flat circular ends containing the domain updated by EKF, the admissible domains for δP_c and δP_c^* are the same:

$$\delta P_c \in \Omega \Leftrightarrow \delta^* P_c \in \Omega. \tag{10}$$

From the control point of view, the robustness problem is stated as follows.

Consider a feedback control law (5) and use the estimate $(\hat{P}_c^T, \hat{\chi}_c^r)$ instead of the true value (P_c^T, χ_c^r). Is the equilibrium point of the system (4) with the uncertain control law (6) still stable under the assumption that the estimation error $(\delta P_c^T, \delta\chi_c^r)$ lies in an *a priori* known bounded domain $(\Omega, \mathscr{X})$? Moreover, if the stability is proven, what is the regulation precision (i.e. what is the size of the attractive domain containing $[(P_e^T, \chi_e) = 0]$)?

Technically, those questions are answered via a stability analysis by using convenient Lyapunov functions.

3. Robustness

It is now proposed to study the robustness problem, which concerns the conservation of such a stability property with respect to the error in the configuration estimation, as follows.

Determine a control law so that it is possible to choose a Lyapunov function $V(P_e^T, \chi_e)$ such that $\dot{V}(P_e^T, \chi_e)$ is a negative definite function in the nominal case.

If the above condition is satisfied, it can then be possible to define a compact attractive domain $\mathscr{A}(\Omega, \mathscr{X})$ as a Lyapunov equipotential outside which the Lyapunov function decreases.

$$\mathscr{A}(\Omega, \mathscr{X}) = \begin{cases}(P_e^T, \chi_e) \in \mathscr{S} \subset \{\mathscr{R}^2 \times [-\pi, \pi[\times \mathscr{R}\},\\ \exists\gamma^2(\Omega, \mathscr{X}) > 0,\\ \forall(\delta P_c^T, \delta\chi_c^r) \in (\Omega, \mathscr{X}),\\ \forall(P_e^T, \chi_e) \in \mathscr{S}\backslash\mathscr{A}(\Omega, \mathscr{X}),\\ V(P_e^T, \chi_e) \geq \gamma^2(\Omega, \mathscr{X}) \Rightarrow \dot{V}(\hat{P}_e^T, \hat{\chi}_e) < 0.\end{cases} \tag{11}$$

By definition, a tracking control law of a robot is said to be robust against configuration unprecision ($\hat{P}_c = P_c + \delta P_c, \delta P_c \in \Omega$) and curvature estimation errors ($\hat{\chi}_e, \delta\chi_c^r \in \mathscr{X}$) if it is possible to define a compact attractive domain $\mathscr{A}(\Omega, \mathscr{X})$.

4. Robust dynamic state feedback tracking controller

In this section a robust control law is developed. A dynamic state feedback, stabilizing the dynamic errors (4) around zero, is proposed as

$$\left.\begin{aligned}v_c &= -\tfrac{1}{2}\chi_c^r(\mu\chi_e + \eta\,\mathrm{sign}\,(v_r)\theta_e) + k_x\,\mathrm{sign}\,(v_r)x_e + \cos\theta_e,\\ \vartheta_c &= \vartheta_r + |v_r|\left[k\eta z_e + \frac{\eta}{\mu}(1-k)\chi_e + \frac{2}{\mu}\,\mathrm{sign}\,(v_r)\sin\theta_e\right],\end{aligned}\right\} \tag{12}$$

where $z_e = y_e + \mu\chi_e + \eta\,\mathrm{sign}\,(v_r)\theta_e$; $0 < k < 1$; η, μ and $k_x > 0$.

4.1. *Stability*

In order to prove the system's stability (4) under the proposed control (12), the following Lyapunov function, whose expression respects the system kinematics constraint, is introduced:

$$V(P_e^T, \chi_e) = x_e^2 + \tfrac{1}{2}z_e^2 + \tfrac{1}{2}y_e^2 + \tfrac{1}{2}\chi_e^2 + \frac{\mu^2+2}{\mu}(1-\cos\theta_e). \tag{13}$$

The time derivative of the Lyapunov function is obtained from (4), and (12) is then

$$\dot{V}(P_c^T, \chi_c) = -|v_r|\left[2k_x x_c^2 + \mu k\eta z_c^2 + \frac{\eta}{\mu}(1-k)\chi_c^2 + \eta\theta_c \sin\theta_c\right]. \quad (14)$$

It should be noted that $\dot{V}(P_c^T, \chi_c)$ is negative definite ($\forall\theta_c \in]-\pi, \pi[$) and that

$$\{x_c = 0, \theta_c \in \{-\pi, 0, \pi\}, y_c = -\eta \operatorname{sign}(v_r)\theta_c, \chi_c = 0\}$$

are stationary points. It is easy to check using the first Lyapunov theorem that $\{x_c = 0, \theta_c = 0, y_c = 0, \chi_c = 0\}$ is the unique stable equilibrium point inside the subspace $\{\forall\theta_c \in [-\pi, \pi]\}$. Thus, the asymptotic stability can be asserted in a large positively invariant domain $\mathscr{S}$ (which can be practically computed as in Hamel *et al.* (1994); see Appendix A).

4.2. *Controller parameters*

The control law proposed (see (12)) has four parameters (k_x, k, μ, η) that should be tuned under stability constraints $k_x, \mu, \eta > 0$ and $0 < k < 1$.

A good tuning rule consists in specifying a well-damped behaviour of the error system linearized around a situation where the reference vehicle moves at a constant speed ($v_0 > 0$) along a straight path.

$$\left.\begin{aligned}
\dot{x}_c &= -v_0 k_x x_c,\\
\dot{y}_c &= v_0\theta_c,\\
\dot{\theta}_c &= v_0\chi_c,\\
\dot{\chi}_c &= -v_0\left[k\eta y_c + \left(k\eta\mu + \frac{\eta}{\mu}(1-k)\right)\chi_c + \left(k\eta^2 + \frac{2}{\mu}\right)\theta_c\right].
\end{aligned}\right\} \quad (15)$$

In this expression k_x/v_0 is the time constant of the longitudinal motion. By extension, one could say that k_x is the length constant of the longitudinal motion.

Transversal motion is governed by the following differential expression, where ($\rho = (\mathrm{d}/\mathrm{d}t)/v_0$):

$$\left(\rho^3 + \left(k\eta\mu + \frac{\eta}{\mu}(1-k)\right)\rho^2 + \left(k\eta^2 + \frac{2}{\mu}\right)\rho + k\eta\right)y_c = 0. \quad (16)$$

(k, μ, η) can then be specified by assigning desired dynamic performances via the desired characteristic polynomial (17):

$$(\rho + 1/\sigma_d)(\rho^2 + 2\zeta\omega_0\rho + \omega_0^2), \quad (17)$$

where $(\sigma_d, \zeta, \omega)$ express a desired dynamical behaviour in a natural way.

4.3. *Robustness*

The controller robustness with respect to state estimation error is deduced from the negative definiteness of $\dot{V}$. From (6)–(8) and (10), the Lyapunov function derivative $\dot{V}(P_c^T, \chi_c, \delta P_c^T, \delta\chi_c^r)$ can be bounded by the following expression:

$$\dot{V} \le -|v_r|[(Q(\cdot) - C(\cdot))^T\Sigma(\cdot)(Q(\cdot) - C(\cdot)) - H(\cdot)], \quad (18)$$

where

$$Q(\cdot) = \begin{pmatrix} x_c + f(\cdot)\chi_c \\ z_c \\ \sin\theta_c + g(\cdot)x_c \\ \chi_c \end{pmatrix}, \quad C(\cdot) = \begin{pmatrix} a(\cdot) \\ b(\cdot) \\ c(\cdot) \\ d(\cdot) \end{pmatrix}, \quad (19)$$

$$\Sigma(\cdot) = \begin{pmatrix} \alpha(\cdot) & 0 & 0 & 0 \\ 0 & \beta(\cdot) & 0 & 0 \\ 0 & 0 & \kappa(\cdot) & 0 \\ 0 & 0 & 0 & \lambda(\cdot) \end{pmatrix},$$

$$H(\cdot) = C(\cdot)^T\Sigma(\cdot)C(\cdot). \quad (20)$$

Functions $(f(\cdot), g(\cdot), a(\cdot), b(\cdot), c(\cdot), d(\cdot), \alpha(\cdot), \beta(\cdot), \kappa(\cdot), \lambda(\cdot))$ are functions of χ_r, δP_c and $\delta\chi_c^r$ (see Appendix B).

The definition of a compact attractive domain $\mathscr{A}(\Omega, \mathscr{X})$ is obtained from the consideration of the Lyapunov function derivative $\dot{V}(P_c^T, \chi_c, \delta P_c^T, \delta\chi_c^r)$ (18); a global overview of this definition is the following. As $(\delta P_c^T, \delta\chi_c^r)$ belongs to a compact domain $(\Omega, \mathscr{X})$ and as the functions constituting $C(\cdot)$, $H(\cdot)$ and $\Sigma(\cdot)$ are continuous, they are thus and bounded.

Assume now that $\Sigma(\cdot)$ is a uniformly positive definite matrix for any $(\delta P_c^T, \delta\chi_c^r) \in (\Omega, \mathscr{X})$; then, for a sufficiently large (P_c^T, χ_c), $\dot{V}(P_c^T, \chi_c, \delta P_c^T, \delta\chi_c^r)$ is negative. This property can be established for any (P_c^T, χ_c) on the boundary of a compact domain $\{V(P_c^T, \chi_c) = \gamma^2(\Omega, \mathscr{X})\}$ with $\gamma^2(\Omega, \mathscr{X})$ sufficiently large, thus proving the existence of a compact attractive domain.

More technically, the determination of $\mathscr{A}(\Omega, \mathscr{X})$ is performed in two steps following the previous line. Let $\epsilon > 0$:

$$\{\forall(\delta P_c^T, \delta\chi_c^r) \in (\Omega, \mathscr{X}), \Sigma > 0\} \Leftrightarrow$$

$$\begin{cases}
\alpha \ge \epsilon \Rightarrow 4\eta(k\eta - \epsilon) \ge [k\eta^2 - 2]\delta\theta_c - \eta\delta\chi_c^r,\\
\beta \ge \epsilon \Rightarrow \mu k\eta \ge \epsilon,\\
\kappa \ge \epsilon \Rightarrow \eta \ge \epsilon,\\
\lambda \ge \epsilon \Rightarrow 4\alpha\left(\frac{\eta}{\mu}[1-k] - \epsilon\right) \ge \eta^2\delta\theta_c^2(k[\mu+1]+1)^2.
\end{cases} \quad (21)$$

These inequalities provide a limit on the achievable tracking performance with respect to the localization uncertainty. The computation of the compact attractive domain is obtained from the definition (11) and the expressions of $V(P_e^T, \chi_e)$ (13) and $\dot{V}(P_e^T, \chi_e, \delta P_c^T, \delta \chi_c^r)$.

$\mathcal{A}(\Omega)$ can also be defined by

$$\mathcal{A}(\Omega) = \begin{cases} (P_e^T, \chi_e) \in \mathcal{S} \subset \{\mathcal{R}^2 \times [-\pi, \pi[\times \mathcal{R}\}, \\ V(P_e^T, \chi_e) \le \gamma^2(\Omega, \mathcal{X}), \\ \gamma^2(\Omega, \mathcal{X}) = \\ \qquad \sup\limits_{\substack{P_e, \chi_e \\ \delta P_c, \delta\chi_c^r}} \{V(P_e^T, \chi_e) \mid \dot{V}(P_e^T, \chi_e, \delta P_c^T, \delta\chi_c^r) \ge 0\}. \end{cases} \tag{22}$$

$\mathcal{A}(\Omega, \mathcal{X})$ is then obviously computed by an optimization scheme. The following one, based upon D–K iteration used in H_∞ control, can be used.

Initialize $\{\gamma^2(\Omega, \mathcal{X}) = 0, [\delta P_c^T, \delta\chi_c^r]^* = 0, [P_e^T, \chi_e]^* \neq 0\}$.

(*a*) Maximize $\dot{V}([P_e^T, \chi_e]^*, [\delta P_c^T, \delta\chi_c^r])$ with respect to $[\delta P_c^T, \delta\chi_c^r] \in (\Omega, \mathcal{X})$; let

$$\{\delta P_c^T, \delta\chi_c^r\}^* = \mathrm{Arg}\left\{\max_{\delta P_c, \delta\chi_c^r} \dot{V}([P_e^T, \chi_e], [\delta P_c^T, \delta\chi_c^r])\right\}.$$

(*b*) Maximize $V(P_e^T, \chi_e)$ on the manifold

$$\dot{V}([P_e^T, \chi_e], [\delta P_c^T, \delta\chi_c^r]^*) = \dot{V}([P_e^T, \chi_e]^*, [\delta P_c^T, \delta\chi_c^r]^*);$$

let

$$[P_e^T, \chi_e]^* = \arg\left\{\max_{P_e, \chi_e} V(P_e^T, \chi_e) \mid \dot{V}([P_e^T, \chi_e], [\delta P_c^T, \delta\chi_c^r]^*) \ge 0\right\}.$$

(*c*) If $\dot{V}([P_e^T, \chi_e]^*, [\delta P_c^T, \delta\chi_c^r]^*) = 0$ then stop with $\gamma^2(\Omega, \mathcal{X}) = V([P_e^T, \chi_e]^*)$, or else go to (*a*).

Technically this scheme can be modified so that it can take into account the fact that the maxima of $\dot{V}([P_e^T, \chi_e]^*, [\delta P_c^T, \delta\chi_c^r])$ (step (*a*), and the one of $V(P_e^T, \chi_e)$ (step (*b*)) can be obtained for more than one argument. Non-differentiable optimization schemes can be used to take these features into account. Finally, the robustness of the tracking property of the robot (1) using the estimate feedback control law of (12) is proved by the existence of $\mathcal{A}(\Omega, \mathcal{X})$.

The size and the shape of the attractive domain $\mathcal{A}(\Omega, \mathcal{X})$ define the precision of the tracking. It can be further optimized by iteratively modifying the gains (k_x, k, μ, η) of the control law. The 3D projections of the attractive domain are displayed in Fig. 3, where the measures units on the x_e and y_e axes are graduated in cm, the θ_e-axis in rad and the χ_e-axis in cm^{-1}.

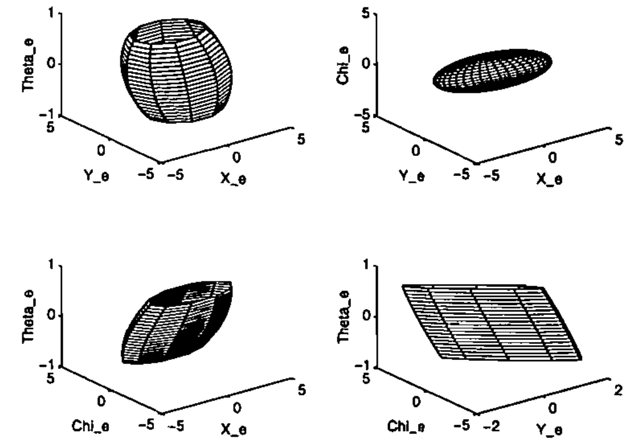


Figure 3. The attractive domain $\mathcal{A}(\Omega, \mathcal{X})$ when considering the measurement error ($|\delta x_c| \le 2$ cm, $|\delta y_c| \le 2$ cm, $|\delta\theta_c| \le 0{\cdot}04$ rad, $\delta\chi_c^r \le 0{\cdot}002\ \mathrm{cm}^{-1}$) as a uniformly distributed noise.

5. Robust parking control law

In this section a proposal is made to use the previous robust tracking control law to stabilize the vehicle in a desired parking configuration. The manoeuvre is induced by tracking the motions of a reference vehicle whose motions finally converge towards the desired configuration and roughly satisfies the geometric constraints. One assumes anyway the existence of an obstacle avoidance filtering level which prevents unexpected collisions. Assume the situation of Fig. 4, where $P_r = 0$ is the desired parking configuration, the reference vehicle motion is a periodic motion centred on the y-axis, and maintain this motion until the vehicle configuration converges towards zero (or move into the attractive domain $\mathcal{A}(\Omega, \mathcal{X})$ if $(\Omega, \mathcal{X})$ is taken into account).

Under the hypothesis that $|\theta_r| < \pi/2$, the reference trajectory parametrization can be done as follows.

$$\left.\begin{aligned} \dot{x}_r &= -k_r x_r + \sigma(P_r) f(Q, t), \\ \dot{y}_r &= v_r \sin\theta_r, \\ \dot{\theta}_r &= v_r \chi_r, \\ \dot{\chi}_r &= -|v_r|\left[k\eta z_r + \frac{\eta}{\mu}(1-k)\chi_r + \frac{2}{\mu}\operatorname{sign}(v_r)\sin\theta_r\right], \end{aligned}\right\} \tag{23}$$

where

$$v_r = [-k_r x_r + \sigma(P_r) f(Q, t)]/\cos\theta_r,$$

$$Q = (x_e, y_e, \theta_e, \chi_e^r)^T,$$

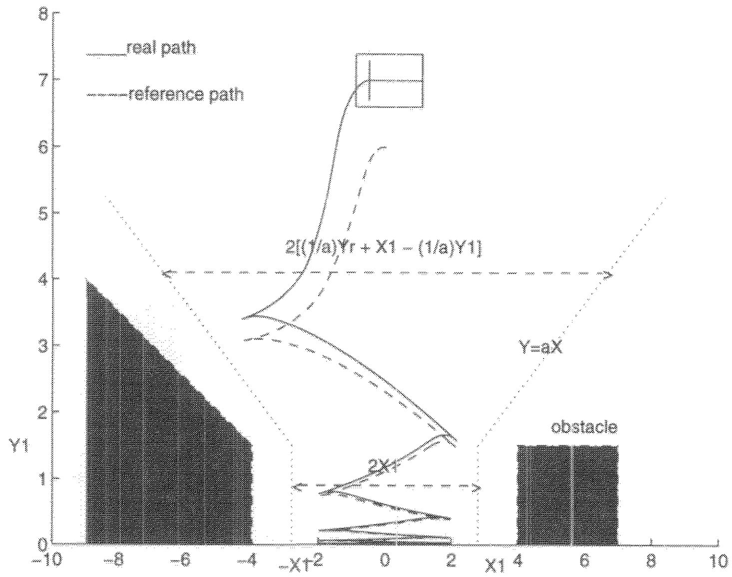


Figure 4. Parking problem.

$f(Q,t)$ is a periodic, continuous and uniformly continuous bounded function such that $f(0,t)=0$, $\forall t$; and $\partial f/\partial t$ does not tend to zero when t tends to infinity.

An example of bounded function $f(Q,t)$ satisfying the previous properties is given below. Let $\epsilon > 0$:

$$f(Q,t) = \frac{\bar{V}(Q)}{\bar{V}(Q)+\epsilon}\sin t, \tag{24}$$

where

$$\bar{V}(Q) = x_c^2 + \tfrac{1}{2}\bar{z}_c^2 + \tfrac{1}{2}y_c^2 + \tfrac{1}{2}\chi_c^{r2} + \frac{\mu^2+2}{\mu}(1-\cos\theta_c), \tag{25}$$

and $\bar{z}_c = y_c + \mu\chi_c^r + \eta\theta_c$.

$\sigma(P_r)$ is a continuous function which takes into account geometric constraints. Let us, for instance, consider the environment given by Fig. 4. In this case, assuming that $|f(Q,t)| \le 1$, $\sigma(P_r)$ can be chosen as

$$\sigma(P_r) = \begin{cases} \dfrac{1}{a}y_r + x_1 - \dfrac{1}{a}y_1, & \text{if } y_r > y_1, \\ x_1 & \text{if } 0 < y_r < y_1. \end{cases}$$

According to Canudas *et al.* (1994), the evolution equation of x_r is interpreted as an equation of stable linear system subjected to the additive periodic and bounded perturbation $f(Q,g(t))$, then state x_r associated with this equation remains periodic and bounded.

The convergence of (y_r, θ_r, χ_r) to zero is asserted, whereas $(x_r, v_r) \neq (0,0)$, and the stopping reference vehicle to zero is realized if and only if $Q = (x_c, y_c, \theta_c, \chi_c^r)$ tends to zero too. This is proved by means of a Lyapunov function $W(y_r, \theta_r, \chi_r)$:

$$W = \tfrac{1}{2}z_r^2 + \tfrac{1}{2}y_r^2 + \tfrac{1}{2}\chi_r^2 + \frac{\mu^2+2}{\mu}(1-\cos\theta_r), \tag{26}$$

where

$$\dot{W} = -|v_r|\eta\left[\mu k z_r^2 + \frac{1}{\mu}(1-k)\chi_r^2 + \theta_r\sin\theta_r\right]. \tag{27}$$

Proposition: *The control law* (12) *with* (v_r *and* ϑ_r) *calculated according to* (23) *asymptotically stabilizes the point* ($P_r = P_c = 0$, $\forall|\theta_c| < \pi$) *and solves the parking problem.*

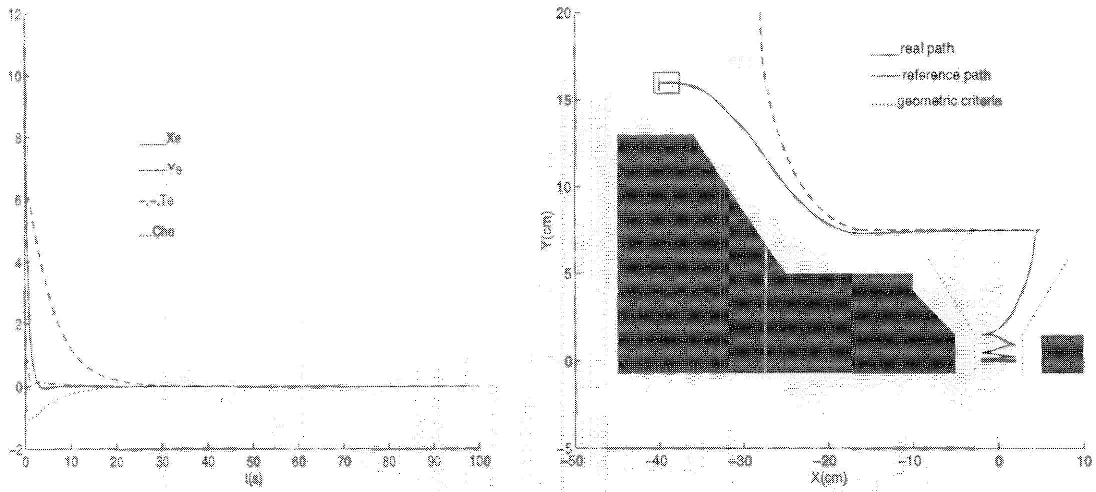
Sketch of proof: The convergence proof of real vehicle configuration towards zero is given in two steps. □

It is known that from (13) and (14) the Lyapunov function of the system (4) submitted to the control law (12) converges to zero when v_r does not tend to zero. In the same way, from (26) and (27), P_r will converge towards an orbit on the x-axis, then Q tends to zero while v_r remains non-zero.

The limit x-motion is then governed by the first line of (23), and converges towards zero as Q tends to zero.

Remark: In theory the desired configuration is reached after an unlimited number of manoeuvres. In practice this number is limited because the reference vehicle periodic motion is only maintained until $\bar{V}(Q)$ (25) becomes smaller than $\mathscr{A}(\Omega, \mathscr{X})$, i.e.

$$f(Q,t) = \begin{cases} \dfrac{\bar{V}(Q)}{\bar{V}(Q)+\epsilon}\sin t, & \text{if } \bar{V}(Q) \ge \mathscr{A}(\Omega,\mathscr{X}), \\ 0, & \text{if } 0 < \bar{V}(Q) < \mathscr{A}(\Omega,\mathscr{X}). \end{cases}$$



(a) (b)

Figure 5. (*a*) **asymptotic convergence of** (P_e^T, χ_e) **towards zero;** (*b*) **the vehicle realized path.**

6. Simulation results

In this section two sets of simulation results are shown; the controller parameters for all cases are: $k_x = 1{\cdot}5$, $\mu = 2$ and $\eta = 5$, where a well-damped case ($\zeta = 0{\cdot}75$, $k = 0{\cdot}6$) has been considered. The situation taken into account is the one in which the reference vehicle is moving along a non-smooth path, given by a concatenation of a circular arc, straight line and at the end a parking manoeuvre.

In the ideal case, the result is presented in Fig. 5, where the convergence of (P_e^T, χ_e) towards 0 when the configurations of two robots (P_r and P_c) do not coincide initially ($P_c \neq P_r$) is shown.

The last result (see Fig. 6) concerns the robustness and the regulation precision problem. The situation where the perturbed controller is simulated considering the measurement error ($|\delta x_c| \leq 2$ cm, $|\delta y_c| \leq 2$ cm, $|\delta\theta_c| \leq 0{\cdot}3$ rad, $\delta\chi_c^r \leq 0{\cdot}02\ \mathrm{cm}^{-1}$) has been considered as a uniformly distributed noise.

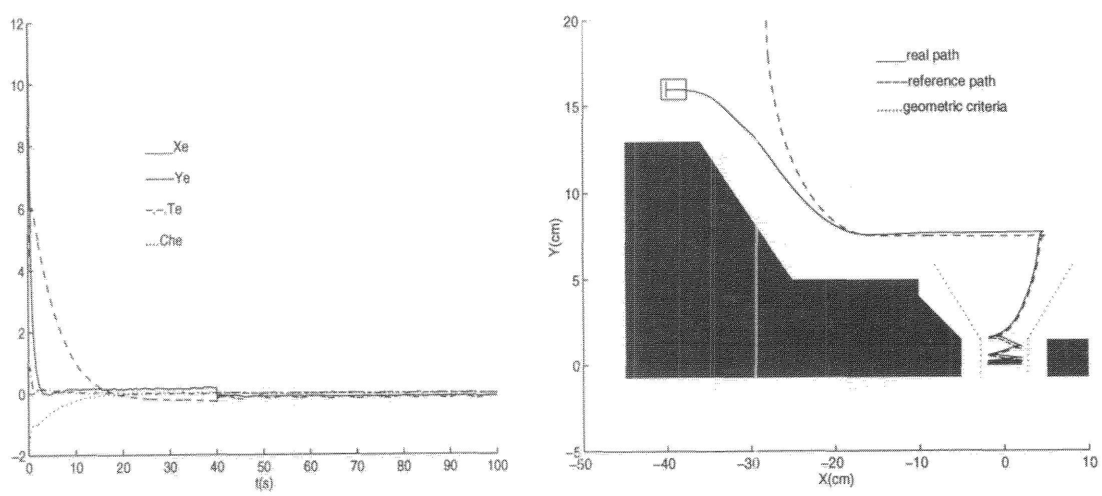
It is shown that (P_e^T, χ_e) converges towards an attractive domain $\mathscr{A}(\Omega, \mathscr{X})$ shown in Fig. 3.

7. Experimental results

The proposed controller has been implemented on a ROBUTER™ (Fig. 7) using the VxWorks Real-time kernel.

Two experiments are shown below.

The sampling time T_e of the controller has been chosen to be 0·6 s. During T_e the mobile robot's localization is continuously updated by integrating wheel rotations



(a) (b)

Figure 6. (*a*) **the perturbed controller;** (*b*) **the vehicle realized path under perturbation.**

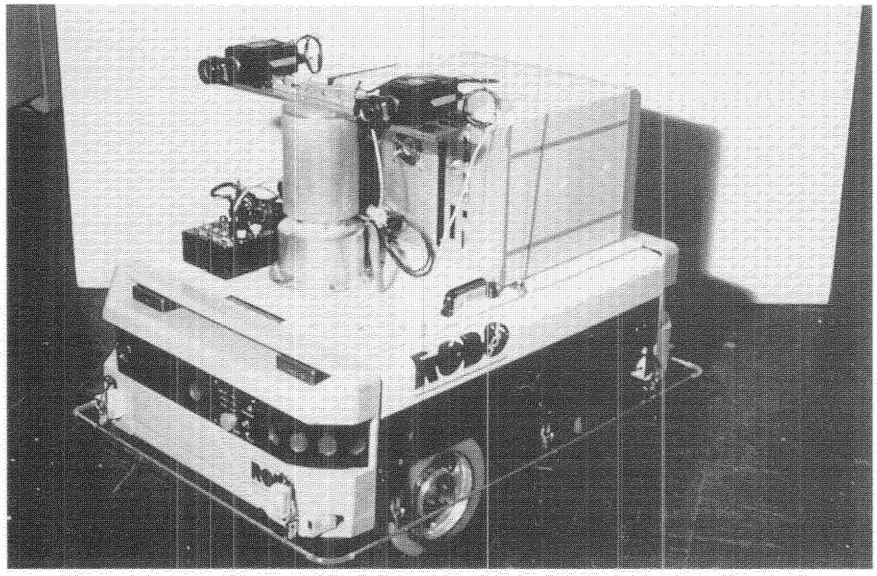


Figure 7. The ROMO SAPIENS robot.

(odometry) and, when available, by exteroceptive (telemetric) measurements. They are processed by extended Kalman filtering and they yield a configuration estimation $\hat{P}_c(t)$ with its uncertainty characterization $\Omega(t)$. The curvature estimation is updated at each period as follows:

$$\hat{\chi}_c^{t'} = \frac{\hat{\theta}_c^{t'} - \hat{\theta}_c^{t}}{v_r T_e}, \quad \text{where } (t' = t + T_e).$$

The tracked path is a sequence of Bézier curves. It is continuous up to the second order.

A constant speed of 20 cm/s was maintained for the reference vehicle along the path.

In the first experiment the vehicle has to track a reference vehicle which follows a 15 m long path. In this experiment there is an initial configuration and curvature estimation errors $(\hat{P}_e^{T}(0), \hat{\chi}_e(0)) \simeq (5\text{ cm}, \ 3\text{ cm}, \ 0\text{ rad}, \ 0\text{ cm}^{-1})$. The result is shown in Fig. 8, where the boxes represent the real robot configuration, ellipses represent the confidence domain of the position estimations, obstacles are represented by straight line segment, and finally the planned path is represented by the solid curve.

In the other experiment more complex reference trajectories are used, defined by the situation in which the reference vehicle moves along a concatenation of Bézier curves (approximately 6 cm long) ending by a path issued from a parking manoeuvre (23) (see Fig. 9). In this case, the situation with initial configuration and

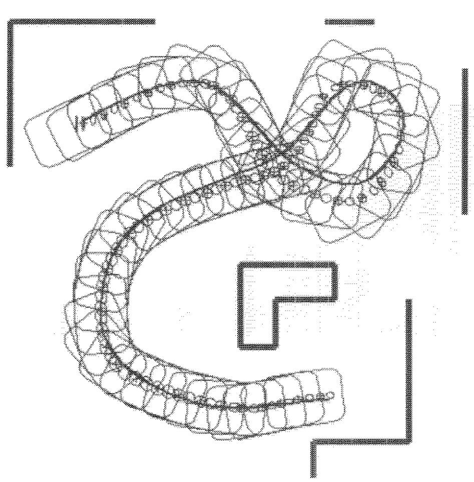


Figure 8. Experimental result. The realized trajectory for the ROBUTER^{TM}.

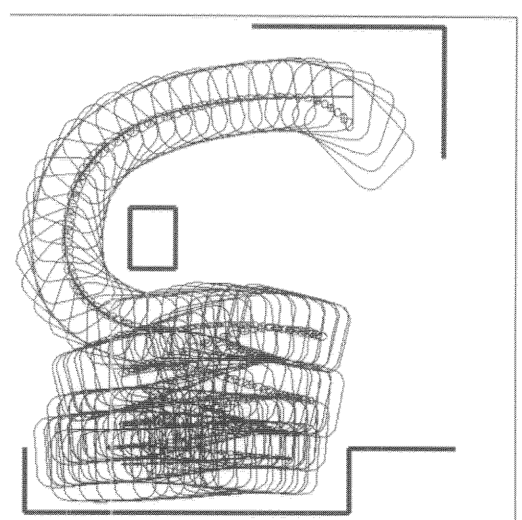


Figure 9. Experimental result. The realized tracking and parking manoeuvre for the ROBUTER^{TM}.

curvature estimation errors was considered:

$$(\hat{P}_e^T(0), \hat{\chi}_e(0)) \simeq \left(-10\ \mathrm{cm}, 70\ \mathrm{cm}, \frac{\pi}{4}\ \mathrm{rad}, 0\ \mathrm{cm}^{-1}\right).$$

For the parking manoeuvre a reference velocity dead-band $v_{r\min} = 8$ cm/s is used ($v_r = \max$ (v_r calculated according to (23), $v_{r\min}$)).

The stopping test is given by $\bar{V}(Q) \le \mathscr{A}(\Omega, \mathscr{X}) = 5\ \mathrm{cm}^2$ (ϵ was chosen to be 10^{-3}). It is shown (Fig. 9) that the parking is realized after five manoeuvres.

8. Concluding remarks

The tracking control or path following a non-holonomic robot has been presented in this paper. We have shown the importance of the choice of a dynamic feedback control law (v_r, v_c, ϑ_c), which takes into account the robot dynamics, which guarantees the asymptotic stability of the configuration and curvature errors and presents a more realistic control solution, where the robot needs to track (to follow) a reference robot (geometric path) with a desired rotational veolocity (curvature). The stability of the proposed control is proven through the use of a convenient Lyapunov function. The tracking robustness property of the robot using the feedback control law perturbed by the configuration and curvature estimation errors is proven by the existence of a compact attractive domain. The size and the shape of the attractive domain define the tracking precision. A parking control law is deduced from the previous robust tracking control.

Experimental results on a laboratory mobile robot show a good performance of the proposed controller with an absolute error given by the attractive domain $\mathscr{A}(\Omega, \mathscr{X})$ from the desired trajectories.

Appendix A

The positively invariant domain $\mathscr{S}$ given by $V(P_e^T, \chi_e)$ must be bounded by $\theta_e = \pm\pi$. This leads us to find the relationship between the initial configuration error $(P_0^T, \chi_0) = (x_0, y_0, \theta_0, \chi_0)^T$ and (μ, η).

Let

$$x_0^2 + \tfrac{1}{2} z_0^2 + \tfrac{1}{2} y_0^2 + \tfrac{1}{2} \chi_0^2 + \frac{\mu^2 + 2}{\mu}(1 - \cos\theta_0) = V_0, \tag{A 1}$$

Since $V(P_e^T, \chi_e)$ is decreasing in $\mathscr{S}$, it follows that

$$V(P_e^T, \chi_e) = x_e^2 + \tfrac{1}{2} z_e^2 + \tfrac{1}{2} y_e^2 + \tfrac{1}{2} \chi_e^2 + \frac{\mu^2 + 2}{\mu}(1 - \cos\theta_e) \le V_0. \tag{A 2}$$

Thus

$$x_e^2 + \tfrac{1}{2} z_e^2 + \tfrac{1}{2} y_e^2 + \tfrac{1}{2} \chi_e^2 \le V_0, \tag{A 3}$$

$$\Rightarrow \theta_e \le \frac{1}{\eta}[(2V_0 - 2x_e^2 - \chi_e^2 - y_e^2)^{1/2} - y_e - \mu\chi_e].$$

Let

$$\phi = \frac{1}{\eta}[(2V_0 - 2x_e^2 - \chi_e^2 - y_e^2)^{1/2} - y_e - \mu\chi_e].$$

$\phi_{\max}$ is reached when $x_e = 0$ and $\chi_e = \mu y_e$; then

$$\left.\begin{aligned} y_e &= -\frac{(2V_0)^{1/2}}{(\mu^2+2)^{1/2}}, \\ \chi_e &= -\frac{(2V_0)^{1/2}\mu}{(\mu^2+2)^{1/2}}, \\ \phi_{\max} &= \frac{1}{\eta}\left(\frac{(2V_0)^{1/2}(\mu+2)}{(\mu^2+2)^{1/2}}\right). \end{aligned}\right\} \tag{A 4}$$

By considering that x_0, y_0 and χ_0 are bounded, from (A 1) and (A 4)

$$\lim_{\eta\to\infty} \phi_{\max} = \frac{\mu+2}{(\mu^2+2)^{1/2}}\theta_0.$$

Thus if

$$\theta_0 = \pi\frac{(\mu^2+2)^{1/2}}{\mu+2},$$

θ_e is always smaller than $\phi_{\max} = \pi$ when η tends to infinity. Finally, for any initial condition x_0, y_0, χ_0 and $\theta_0 = \pm\pi$, (μ, η) can be found in such that (P_e^T, χ_e) converges towards 0 asymptotically.

Appendix B

The expressions of the functions are:

$$f = -\frac{2\eta^2\delta\theta_c(k[\mu+1]+1)}{4k\eta^2 - ([k\eta^2 - 2]\delta\theta_c - \eta\delta\chi_c^r)^2},$$

$$g = -\frac{[k\eta^2 - 2]\delta\theta_c - \eta\delta\chi_c^r}{2\eta},$$

$$\alpha = \frac{4k\eta^2 - ([k\eta^2 - 2]\delta\theta_c - \eta\delta\chi_c^r)^2}{4\eta},$$

$$\beta = \mu k\eta,$$

$$\kappa = \eta,$$

$$\lambda = -\frac{\eta}{\mu}(1-k) - \frac{\eta^3(k[\mu+1]+1)^2\delta\theta_c^2}{4k\eta^2 - ([k\eta^2 - 2]\delta\theta_c - \eta\delta\chi_c^r)^2},$$

$$a = \frac{k_x\delta x_c - k\eta\delta\theta_c - [\delta\chi_c^r + \chi_r]\delta\psi_c}{2\alpha}, \quad b = \frac{\delta\vartheta_c}{2k\eta},$$

$$c = 0, \qquad d = \frac{\delta\vartheta_c - 2\alpha af}{2\gamma},$$

where

$$\delta\psi_c = \mu\delta\chi_c^r + \eta\delta\theta_c,$$

$$\delta\vartheta_c = -k\eta(\delta y_c + \delta\psi_c) - \frac{\eta}{\mu}(1-k)\delta\chi_c^r - \frac{2}{\mu}\delta\theta_c.$$

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