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State-feedback stabilisation for stochastic non-holonomic mobile robots with uncertain visual servoing parameters

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The stabilising problem of stochastic non-holonomic mobile robots with uncertain parameters based on visual servoing is addressed in this paper. The model of non-holonomic mobile robots based on visual servoing is extended to the stochastic case, where their forward velocity and angular velocity are both subject to some stochastic disturbances. Based on backstepping technique, state-feedback stabilising controllers are designed for stochastic non-holonomic mobile robots. A switching control strategy for the original system is presented. The proposed controllers guarantee that the closed-loop system is asymptotically stabilised at the zero equilibrium point in probability.

Keywords: stochastic non-holonomic mobile robots; stabilisation; backstepping; visual servoing; switching control strategy

1. Introduction

In the past decades, the control of non-holonomic systems has been widely pursued. From the results of Brockett, the non-holonomic system cannot be stabilised at a single equilibrium point by any static smooth pure state-feedback controller (Brockett, 1983). To solve this problem, lots of novel approaches have been proposed such as discontinuous feedback control (Ge, Wang, & Lee, 2003; Jiang, 1996; Wang & Wei, 2007), smooth time-varying feedback controller (Tian & Li, 2002) and the method of linear matrix inequalities (LMI) (Cao, 2011). There is much attention devoted to the control of non-holonomic mobile robots since the control of non-holonomic mobile robots plays an important role in that of non-holonomic systems. The non-holonomic mobile robots were classified into four types, which were characterised by generic structures of the model equations (Campion, Bastin, & D'Andréa-Novel, 1996). Hespanha, Liberzon, and Morse (1999) introduced the mobile robot with parametric uncertainties, which was further discussed (Jiang, 2000; Xi, Feng, Jiang, & Cheng, 2000). Based on the visual servoing model in Liang and Wang (2011), the stabilisation and tracking problems of non-holonomic mobile robots and dynamic mobile robots based on the visual servoing were addressed (Wang, Mei, Liang, & Jia, 2010; Yang & Wang, 2011; Yang, Wang, & Jing, 2013). But all the above articles discussed the non-holonomic systems in the deterministic case, which was not considered stochastic disturbance. However, the signals of the configuration for the robots and their goals are sometimes affected by stochastic factors.

In recent years, stochastic non-linear systems have received much attention (Ding, Wang, Dong, & Shu, 2012; Ding, Wang, Hu, & Shu, 2013; Dong, Wang, Ho, & Gao, 2010, 2011; Hu, Wang, & Gao, 2011, 2012; Shen, Wang, Shu, & Wei, 2009; Shen, Wang, Liang, & Liu, 2011), especially stochastic control when backstepping designs were first introduced (Krstic & Deng, 1998). These papers can be classified into two cases. The first case is that the problem of stabilisation was discussed in Zhao, Yu, and Wu (2011), Zhang, Wang, Qiu, and Chen (2012), Zhang, Wang, and Chen (2012a), Gao and Yuan (2012), Gao, Yuan, and Wu (2012) when the first equation of the non-holonomic systems is ordinary differential equation and the others are stochastic. Based on the models in Hespanha et al. (1999), a non-holonomic mobile robot with kinematic unknown parameters, whose linear velocity is subject to stochastic disturbance was introduced (Zhao et al., 2011). Exponential stabilisation of similar non-holonomic mobile robots subject to stochastic disturbance was studied (Shang & Meng, 2012). The second case is that all equations of the non-holonomic systems are stochastic. The problem of stabilisation was discussed in Wang, Gao, and Li (2006), Liu and Wu (2011), Zhang, Wang, and Chen (2012b), Zhang, Wang, Chen, Yang, and Du (2013). An integrator backstepping controller was proposed to deal with exponential stabilisation for non-holonomic mobile robots with stochastic disturbance dependent of the heading angle (Wu & Liu, 2012). However, to our knowledge, the problems of stabilisation for non-holonomic mobile robots with unknown parameters based on visual servoing, whose linear velocity

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and angular velocity are subject to some stochastic disturbances simultaneously, have not been reported in literatures. This situation is more likely to occur in practical engineering. So, there exists a natural problem which is how to extend the models in Liang and Wang (2011) to their stochastic counterpart and design state-feedback stabilising controllers for stochastic non-holonomic mobile robots with uncertain parameters based on visual servoing.

The main idea of this paper is to design state-feedback stabilising controllers for stochastic non-holonomic mobile robots with unknown visual servoing parameters, which is highlighted as follows:

(1) Based on the models in Liang and Wang (2011), we extend the non-holonomic mobile robots with unknown parameters based on visual servoing to the stochastic case. Stabilising controllers are designed for stochastic non-holonomic mobile robots with unknown parameters by state-feedback and backstepping techniques.
(2) A switching control strategy for the original system is presented. It guarantees that the closed-loop system is asymptotically stabilised at the zero equilibrium point in probability.

The paper is organised as follows. Section 1 begins with the mathematical preliminaries. In Section 2, the state-feedback backstepping controllers are designed. In Section 3, a switching control strategy for the original system is discussed. Finally, a simulation example is given to show the effectiveness of the controllers in Section 4.

2. Preliminaries and problem formulation

2.1. Preliminaries

Consider the following stochastic non-linear system:

$$\mathrm{d}x = f(x)\mathrm{d}t + g(x)\mathrm{d}B, \quad x(t_0) \in \mathbb{R}^n, \tag{1}$$

where $x \in \mathbb{R}^n$ is the state, the Borel measurable functions $f : \mathbb{R}^n \to \mathbb{R}^n$ and $g : \mathbb{R}^n \to \mathbb{R}^{n\times r}$ are locally Lipschitz in x, and $B \in \mathbb{R}^r$ is an r-dimensional independent standard Wiener process defined on the complete probability space $(\Omega, \mathcal{F}, P)$.

The following definitions and lemmas will be used in the paper.

Definition 1 (Krstic & Deng, 1998)**:** For any given $V(x) \in \mathcal{C}^2$, associated with stochastic system (1), the differential operator $\mathcal{L}$ is defined as follows:

$$\mathcal{L}V(x) = \frac{\partial V}{\partial x} f(x) + \frac{1}{2}\mathrm{Tr}\left\{g^T(x)\frac{\partial^2 V}{\partial x^2} g(x)\right\}.$$

Definition 2 (Deng, Krstić, & Williams, 2001)**:** The equilibrium $x = 0$ of system (1) is

- globally stable in probability if for $\forall\varepsilon > 0$, there exists a class $\mathcal{K}$ function $\gamma(\cdot)$ such that

$$P\{|x(t)| < \gamma(|x(t_0)|)\} \geq 1 - \varepsilon, \quad \forall t \geq 0, \quad x(t_0) \in \mathbb{R}^n \setminus \{0\};$$

- globally asymptotically stable in probability if it is globally stable in probability and

$$P\left\{\lim_{t\to\infty} |x(t)| = 0\right\} = 1, \quad \forall x(t_0) \in \mathbb{R}^n.$$

Definition 3 (Khas'minskii, 1980)**:** A stochastic process $x(t)$ is said to be bounded in probability if the random variable $|x(t)|$ is bounded in probability uniformly in t, that is,

$$\lim_{R\to\infty} \sup_{t>t_0} P\{|x(t)| > R\} = 0.$$

Lemma 1 (Deng et al., 2001)**:** *Considering the stochastic system* (1)*, if there exist a* $\mathcal{C}^2$ *function* $V(x)$*, class* $\mathcal{K}_\infty$ *functions* $\alpha_1(\cdot)$ *and* $\alpha_2(\cdot)$*, constants* $c_1 > 0$*,* $c_2 \geq 0$*, and a non-negative function* $W(x)$ *such that*

$$\alpha_1(|x|) \leq V(x) \leq \alpha_2(|x|),$$
$$\mathcal{L}V(x) = \frac{\partial V}{\partial x} f + \frac{1}{2}\mathrm{Tr}\left\{g^T\frac{\partial^2 V}{\partial x^2} g\right\} \leq -c_1 W(x) + c_2,$$

then

(1) *for Equation* (1)*, there exists an almost surely unique solution on* $[t_0, \infty)$ *for each* $x(t_0) \in \mathbb{R}^n$*;*
(2) *when* $c_2 = 0$*,* $f(0) = 0$*,* $g(0) = 0$ *and* $W(x)$ *is continuous, then the equilibrium* $x = 0$ *is globally stable in probability and* $P\{\lim_{t\to\infty} W(x(t)) = 0\} = 1$ *for* $\forall x(t_0) \in \mathbb{R}^n$.

Lemma 2 (Lin & Qian, 2002)**:** *Let* x *and* y *be real variables. Then, for any positive integers m, n and any real number* $\varepsilon > 0$*, the following inequality holds:*

$$|x|^m|y|^n \leq \frac{m}{m+n}\varepsilon|x|^{m+n} + \frac{n}{m+n}\varepsilon^{-\frac{m}{n}}|y|^{m+n}. \tag{2}$$

2.2. Problem formulation

The posture kinematic model for the non-holonomic wheeled mobile robot of Type (2,0) can be described by the following differential equations (Campion et al., 1996):

$$\begin{cases} \dot{x} = v\cos\theta, \\ \dot{y} = v\sin\theta, \\ \dot{\theta} = \omega, \end{cases} \tag{3}$$

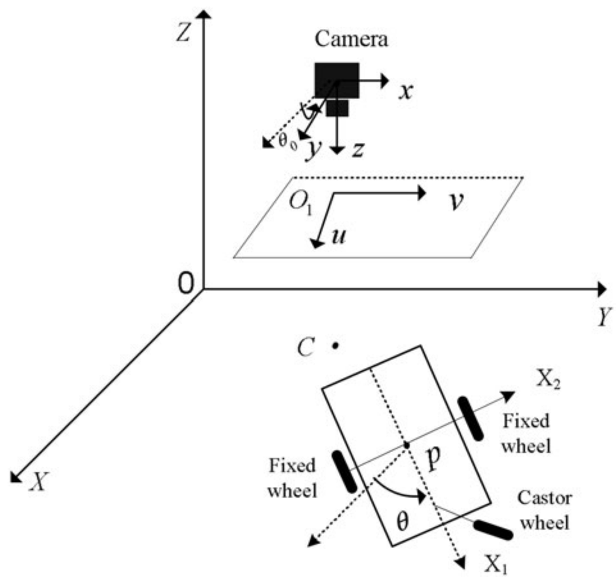

Figure 1. Non-holonomic wheeled mobile robot under a fixed camera.

where (x, y) is the position of the mass centre of the robot moving in the plane, v is the forward velocity while ω is the angular velocity of the robot.

As shown in Figure 1, we consider that the movement of the mobile robot can be measured by using a pinhole camera fixed to the ceiling. Assume that the camera plane, the image plane and the robot plane are parallel. There are four coordinate frames, namely the inertial frame $X - Y - Z$, the camera frame $x - y - z$, the image frame $u - o_1 - v$ and the attached robot frame $X_1 - P - X_2$. Point C is the crossing point between the optical axis of the camera and $X - Y$ plane. Its coordinate relative to $X - Y$ plane is (c_x, c_y). The coordinate of the original point of the camera frame with respect to the image frame is defined by (O_{c_1}, O_{c_2}). (x, y) is the coordinate of the mass centre P of the robot with respect to $X - Y$ plane. Suppose that x_m, y_m is the coordinate of (x, y) relative to the image frame.

The pinhole camera model of Type (2,0) in Liang and Wang (2011) can be expressed as

$$\begin{cases} \dot{\theta} &= \omega, \\ \dot{x}_c &= \alpha_1 v \cos(\theta - \theta_0), \\ \dot{y}_c &= \alpha_2 v \sin(\theta - \theta_0), \end{cases} \tag{4}$$

where α_1 and α_2 are positive constants, which are dependent on the depth information, focal length and scalar factors along the u axis and v axis, respectively, θ_0 denotes the angle between the u axis and the X axis with a positive anticlockwise orientation. Here, α_1, α_2 and θ_0 are called camera parameters also.

We assume that the forward velocity v and the angular velocity ω are subject to some stochastic disturbances. Based on the similar methods in Mao (1997, pp. 1–2), the forward velocity v and angular velocity ω with stochastic disturbances can be expressed as follows:

$$\begin{cases} \omega = \omega_1(\theta) + \omega_2(\theta)\dot{B}(t), \\ v = v_1(x_c, y_c, \theta) + v_2(x_c, y_c, \theta)\dot{B}(t), \end{cases} \tag{5}$$

where $\dot{B}(t)$ is the derivative of a Brownian motion $B(t)$.

Substituting Equation (5) into Equation (4), the system (4) can be transformed into

$$\begin{cases} \mathrm{d}\theta &= \omega_1 \mathrm{d}t + \omega_2 \mathrm{d}B, \\ \mathrm{d}x_c &= \alpha_1 v_1 \cos(\theta - \theta_0)\mathrm{d}t + \alpha_1 v_2 \cos(\theta - \theta_0)\mathrm{d}B, \\ \mathrm{d}y_c &= \alpha_2 v_1 \sin(\theta - \theta_0)\mathrm{d}t + \alpha_2 v_2 \sin(\theta - \theta_0)\mathrm{d}B. \end{cases} \tag{6}$$

Assumption 1: *For systems* (6)*, we assume that* $\theta_0 \geq 0$, $\alpha_1 = \alpha_2 = \alpha$ *are unknown camera parameters.*

Remark 1: $\alpha_1 = \alpha_2 = \alpha$ in Assumption 1 means that the scalar factor along u axis is the same as the one along v axis since some charge-coupled device (CCD) cameras are designed like this.

Assumption 2: *For function* v_1*, there exists a smooth function* $\overline{v}_1$*, such that* $v_1 = \theta\overline{v}_1$.

For system (6), if Assumption 1 holds, we introduce the following state and input transformations:

$$\begin{cases} x_0 = \theta, \ u_0 = \omega_1, \ u = \overline{v}_1, \\ x_1 = x_c \sin\theta - y_c \cos\theta, \\ x_2 = x_c \cos\theta + y_c \sin\theta, \end{cases} \tag{7}$$

and it is easy to see

$$\mathrm{d}x_0 = u_0 \mathrm{d}t + \omega_2(x_0)\mathrm{d}B, \tag{8.1}$$

$$\left.\begin{aligned} \mathrm{d}x_1 &= x_2 u_0 \mathrm{d}t + \left(\alpha_1^* x_0 u - \frac{1}{2}x_1\omega_2^2 + \alpha_2^* v_2\omega_2\right)\mathrm{d}t + (x_2\omega_2 + \alpha_1^* v_2)\mathrm{d}B, \\ \mathrm{d}x_2 &= \alpha_2^* x_0 u \mathrm{d}t - \left(x_1 u_0 + \frac{1}{2}x_2\omega_2^2 + \alpha_1^* v_2\omega_2\right)\mathrm{d}t + (\alpha_2^* v_2 - x_1\omega_2)\mathrm{d}B, \end{aligned}\right\} \tag{8.2}$$

where $\alpha_1^* = \alpha\sin\theta_0$ and $\alpha_2^* = \alpha\cos\theta_0$.

Assumption 3: *For systems* (8)*, if there exist known positive constants* α_{1min}*,* α_{1max}*,* α_{2min} *and* α_{2max}*, then* $0 < \alpha_{1\min} \leq \alpha_1^* \leq \alpha_{1\max}$, $0 < \alpha_{2\min} \leq \alpha_2^* \leq \alpha_{2\max}$.

Remark 2: Since $\varphi_i(\cdot)$, $i = 1, 2$ in Wu and Liu (2012) are only functions of θ, system (10) in Wu and Liu (2012) is the special case of system (8) in this paper. Because unknown camera parameter α and $v_2(\cdot)$ are functions of θ, x_c, y_c, the design of controllers for system (8) in this paper will be more difficult.

Remark 3: For system (8), the variable x_2 appears in the term $x_2\omega_2\mathrm{d}B$ in the first equation of (8.2); this is different from the upper triangular structure by using traditional stochastic backstepping technique in Deng et al. (2001), Zhao et al. (2011), Zhang et al. (2012a, 2013).

3. Controllers design

In this section, we will design a state-feedback controller such that all the signals in the closed-loop system are regulated to the origin in probability. For this end, the following assumptions are imposed on system (6).

Assumption 4: *For the smooth function $\omega_2(\theta)$, there exists a known positive constant m_1, such that*

$$\omega_2(\theta) = m_1\theta.$$

Assumption 5: *For smooth function $v_2(x_c, y_c, \theta)$, there exists a known non-negative constant m_2 such that*

$$|v_2(x_c, y_c, \theta)| \leq m_2\theta|x_c\cos\theta + y_c\sin\theta|.$$

In the following two sections, we will consider the problem of stabilisation for system (8) *under the condition of $x_0(t_0) \neq 0$. As for the case of $x_0(t_0) = 0$, it will be discussed in Section* 3.

3.1. The first state stabilisation

Let us consider the first subsystem of stochastic non-holonomic non-linear system (8.1)

$$\mathrm{d}x_0 = u_0\mathrm{d}t + \omega_2(x_0)\mathrm{d}B.$$

In order to guarantee that x_0 converges to zero, one can take u_0 as follows:

$$u_0 = -\eta_0 x_0, \quad \eta_0 = \lambda + \frac{3}{2}m_1^2, \tag{9}$$

where $\lambda > 0$ is a design parameter.

We employ a Lyapunov function of the form

$$V_0(x_0) = \frac{1}{4}x_0^4. \tag{10}$$

From Equations (8.1), (9) and (10) and Assumption 1, one can obtain

$$\mathcal{L}V_0 \leq x_0^3 u_0 + \frac{3}{2}x_0^2\omega_2^2 \leq -\lambda x_0^4. \tag{11}$$

Theorem 1: *If Assumptions* 4 *holds and one can choose positive constants m_1, λ and the controller u_0 as Equation* (9)*, then*

1. *the closed-loop subsystem composed of Equations* (8.1) *and* (9) *has an almost surely unique solution on $[t_0, \infty)$ for $\forall x_0(t_0)$;*
2. *the equilibrium $x_0 = 0$ of the closed-loop subsystem composed of Equations* (8.1) *and* (9) *is globally asymptotically stable in probability.*

Proof: Choosing Lyapunov function as Equation (10), by Equation (11), $\lambda > 0$ and Lemma 1, the equilibrium $x_0 = 0$ of the closed-loop subsystem containing Equations (8.1) and (9) is globally stable in probability and for $\forall x_0(t_0) \neq 0$, $P\{\lim_{t\to\infty}(|x_0(t)| = 0)\} = 1$. From Definition 2, it is easy to obtain that the equilibrium $x_0 = 0$ of the closed-loop subsystem composed of Equations (8.1) and (9) is globally asymptotically stable in probability. □

Remark 4: From Theorem 1, one can conclude that the state x_0 is bounded in probability, that is, there exists a positive constant m_3, such that the following equality holds:

$$\lim_{m_3\to\infty}\sup_{t>t_0} P\{|x_0(t)| > m_3\} = 0.$$

Substituting Equation (9) into the subsystem (8.1), one gets

$$\mathrm{d}x_0 = -\eta_0 x_0\mathrm{d}t + m_1 x_0\mathrm{d}B. \tag{12}$$

Proposition 1: *For initial state $x_0(t_0) \neq 0$, the solution of Equation* (12) *will never reach zero, which avoids the uncontrollability of the subsystem* (8.2).

Proof: It is easy to see that the above equality is the special case of (2.6) in Lemma 2.3 (Mao, 1997, p. 93). Therefore, we have the following equality which will be the solution of Equation (12):

$$x_0(t) = x_0(t_0)\exp\left\{\int_{t_0}^{t}\left(-\eta_0 - \frac{1}{2}m_1^2\right)\mathrm{d}s + \int_{t_0}^{t} m_1\mathrm{d}\omega\right\}.$$

From the above expression of $x_0(t)$ and $x_0(t_0) \neq 0$, we conclude that Proposition 1 holds at the time interval $t \in (t_0, +\infty)$. □

In the following section (3.2), the other states x_1 and x_2 will be regulated to the origin in probability by the design of the control input u.

3.2. Other states stabilisation

In order to design a smooth adaptive state-feedback controller, the following state-input scaling discontinuous transformation is needed:

$$z_1 = \frac{x_1}{x_0}, \quad z_2 = x_2. \tag{13}$$

Remark 5: For the initial state $x_0(t_0) \neq 0$, from Proposition 1, one can obtain that the transformation (13) is meaningful.

Under the new z-coordinate, the subsystem (8.2) is transformed into

$$\begin{cases} \mathrm{d}z_1 = \Big\{ -\eta_0 z_2 + \alpha_1^* u - \dfrac{1}{2} z_1 \omega_2^2 + \eta_0 z_1 + z_1 \dfrac{\omega_2^2}{x_0^2} \\ \qquad - z_2 \dfrac{\omega_2^2}{x_0^2} - \alpha_1^* \dfrac{v_2 \omega_2}{x_0^2} + \alpha_2^* \dfrac{\omega_2}{x_0} v_2 \Big\} \mathrm{d}t \\ \qquad + \Big\{ z_2 \dfrac{\omega_2}{x_0} + \alpha_1^* \dfrac{v_2}{x_0} - z_1 \dfrac{\omega_2}{x_0} \Big\} \mathrm{d}B, \\ \mathrm{d}z_2 = \alpha_2^* x_0 u \mathrm{d}t - \left(x_1 u_0 + \dfrac{1}{2} x_2 \omega_2^2 + \alpha_1^* v_2 \omega_2 \right) \mathrm{d}t \\ \qquad + (\alpha_2^* v_2 - x_1 \omega_2) \mathrm{d}B. \end{cases} \tag{14}$$

To invoke the backstepping method, the error variables ε_1 and ε_2 are given by

$$\varepsilon_1 = z_1, \quad \varepsilon_2 = z_2 - x_0 \alpha_1(\varepsilon_1). \tag{15}$$

Step 1. Define the first Lyapunov candidate function

$$V_1 = \frac{1}{4} \varepsilon_1^4. \tag{16}$$

From Equations (14)– (16) and Itô formula, one has

$$\begin{aligned} \mathcal{L}V_1 \le \varepsilon_1^3 \Big\{ & -\eta_0 z_2 + \alpha_1^* u - \frac{1}{2} z_1 \omega_2^2 + \eta_0 z_1 + z_1 \frac{\omega_2^2}{x_0^2} \\ & - z_2 \frac{\omega_2^2}{x_0^2} - \alpha_1^* \frac{v_2 \omega_2}{x_0^2} + \alpha_2^* \frac{\omega_2}{x_0} v_2 \Big\} \\ & + \frac{9}{2} \varepsilon_1^2 \Big\{ z_2^2 \frac{\omega_2^2}{x_0^2} + z_1^2 \frac{\omega_2^2}{x_0^2} + \frac{(\alpha_1^* v_2)^2}{x_0^2} \Big\}. \end{aligned} \tag{17}$$

The virtual control can be chosen as

$$\alpha_1(\varepsilon_1) = c_1 \varepsilon_1, \tag{18}$$

where c_1 is a positive constant, which will be chosen later. From Equation (17), Lemma 2 and simple operation, we have the following inequalities:

$$\begin{aligned} & -\eta_0 z_2 \varepsilon_1^3 \le \eta_0 \Big\{ \frac{3d}{4} \varepsilon_1^4 + \frac{1}{4d^3} \varepsilon_2^4 \Big\} - \eta_0 c_1 \varepsilon_1^4, \\ & -\frac{1}{2} z_1 \omega_2^2 \varepsilon_1^3 \le \frac{1}{2} m_1^2 m_3^2 \varepsilon_1^4, \ \eta_0 z_1 \varepsilon_1^3 \le \eta_0 \varepsilon_1^4, \ z_1 \frac{\omega_2^2}{x_0^2} \varepsilon_1^3 \le m_1^2 \varepsilon_1^4, \\ & -z_2 \frac{\omega_2^2}{x_0^2} \varepsilon_1^3 \le \frac{1}{4} \varepsilon_1^4 + \frac{3}{4} m_1^{\frac{8}{3}} \varepsilon_2^4 + c_1 m_1^2 \varepsilon_1^4, \\ & -\alpha_1^* \frac{v_2 \omega_2}{x_0} \varepsilon_1^3 \le \Big\{ \frac{1}{4} \varepsilon_1^4 + \frac{3}{4} (\alpha_{1\max} m_1 m_2 m_3)^{\frac{4}{3}} \varepsilon_2^4 \Big\} \\ & \quad + \alpha_{1\max} m_1 m_2 m_3 c_1 \varepsilon_1^4, \\ & \alpha_2^* \frac{v_2}{x_0} \omega_2 \varepsilon_1^3 \cos(2x_0) \le \Big\{ \frac{1}{4} \varepsilon_1^4 + \frac{3}{4} (\alpha_{2\max} m_1 m_2 m_3)^{\frac{4}{3}} \varepsilon_2^4 \Big\} \\ & \quad + \alpha_{2\max} m_1 m_2 m_3 c_1 \varepsilon_1^4, \\ & \frac{9}{2} z_2^2 \frac{\omega_2^2}{x_0^2} \varepsilon_1^3 \le \frac{9}{2} \varepsilon_1^4 + \frac{9}{2} m_1^4 \varepsilon_2^4 + 9 m_1^2 c_1^2 \varepsilon_1^4, \\ & \frac{9}{2} z_1^2 \frac{\omega_2^2}{x_0^2} \varepsilon_1^3 \le \frac{9}{2} m_1^2 \varepsilon_1^4, \\ & \frac{9}{2} (\alpha_1^*)^2 z_2^2 \varepsilon_1^2 \le \frac{9}{2} \varepsilon_1^4 + \frac{9}{2} \alpha_{1\max}^4 \varepsilon_2^4 + 9 (c_1 \alpha_{1\max})^2 \varepsilon_1^4, \end{aligned}$$

where $d > 0$ is a design parameter. Substituting the above inequalities and Equation (18) into Equation (17), it is easy to see

$$\begin{aligned} \mathcal{L}V_1 \le \Big\{ & -\eta_0 c_1 e + \left(1 + \frac{3d}{4}\right) \eta_0 - \frac{c_1 \eta_0 (1-e)}{2} - \frac{c_1 \eta_0 (1-e)}{2} + \frac{39 + 4\alpha_{1\max} m_1 m_2 m_3 c_1}{4} \\ & + \frac{4\alpha_{2\max} m_1 m_2 m_3 c_1 + 2m_1^2 m_3^2 + 4m_1^2 + 4c_1 m_1^2 + 36 m_1^2 c_1^2 + 18 m_1^2}{4} \Big\} \varepsilon_1^4 \\ & + \Big\{ \frac{\eta_0}{4d^3} + \frac{3}{4} (\alpha_{1\max} m_1 m_2 m_3)^{\frac{4}{3}} + \frac{3}{4} (\alpha_{2\max} m_1 m_2 m_3)^{\frac{4}{3}} + \frac{3}{4} m_1^{\frac{8}{3}} + \frac{9}{2} m_1^4 + \frac{3}{4} (\alpha_{1\max} m_1)^{\frac{4}{3}} + \frac{9}{2} \alpha_{1\max}^4 \Big\} \varepsilon_2^4 + \alpha_1^* u \varepsilon_1^3, \end{aligned} \tag{19}$$

where e is a design parameter and $0 < e < 1$. If we select parameters d, e and c_1 to satisfy

$$c_1 = \alpha_{2\max}/\alpha_{1\min}, \ 4ec_1 \ge 4 + 3d,$$

we have

$$\begin{aligned} \mathcal{L}V_1 \le -\frac{c_1 \eta_0 (1-e)}{2} \varepsilon_1^4 + \Big\{ & \frac{\eta_0}{4d^3} + \frac{3}{4} (\alpha_{1\max} m_1 m_2 m_3)^{\frac{4}{3}} \\ & + \frac{3}{4} (\alpha_{2\max} m_1 m_2 m_3)^{\frac{4}{3}} + \frac{3}{4} m_1^{\frac{8}{3}} + \frac{9}{2} m_1^4 \\ & + \frac{3}{4} (\alpha_{1\max} m_1)^{\frac{4}{3}} + \frac{9}{2} \alpha_{1\max}^4 \Big\} \varepsilon_2^4 + \alpha_1^* u \varepsilon_1^3. \end{aligned} \tag{20}$$

Step 2. From Equations (14), (15) and (20) and Itô formula, one gets

$$
\begin{aligned}
\mathrm{d}\varepsilon_2 = \Bigg\{ & (\alpha_2^* - c_1\alpha_1^*)ux_0 - z_1u_0 - \frac{1}{2}z_2\omega_2^2 - \alpha_1^*\frac{v_2}{x_0} \\
& + c_1\eta_0 z_2 x_0 + \frac{1}{2}c_1 z_1 x_0\omega_2^2 - c_1\eta_0 z_1 x_0 - c_1 z_1 x_0\frac{\omega_2^2}{x_0^2} \\
& + c_1 z_2 x_0\frac{\omega_2^2}{x_0^2} + c_1\alpha_1^* x_0\frac{v_2\omega_2}{x_0^2} - c_1\alpha_2^* x_0 v_2\frac{\omega_2}{x_0}\Bigg\}\mathrm{d}t \\
& + \big\{\alpha_2^* v_2 - z_2\omega_2 \\
& - c_1\alpha_1^* v_2 - c_1 z_2\omega_2 + c_1 z_1\omega_2\big\}\mathrm{d}B. \qquad (21)
\end{aligned}
$$

Define the second Lyapunov candidate function

$$
V_2 = V_1 + \frac{1}{4}\varepsilon_2^4. \qquad (22)
$$

From Equations (21) and (22) and Itô formula, one can obtain

$$
\begin{aligned}
\mathcal{L}V_2 \le & -\frac{c_1\eta_0(1-e)}{2}\varepsilon_1^4 + \Bigg\{\frac{\eta_0}{4d^3} + \frac{3}{4}(\alpha_{1\max}m_1m_2m_3)^{\frac{4}{3}} \\
& + \frac{3}{4}(\alpha_{2\max}m_1m_2m_3)^{\frac{4}{3}} + \frac{3}{4}m_1^{\frac{8}{3}} + \frac{9}{2}m_1^4 \\
& + \frac{3}{4}(\alpha_{1\max}m_1)^{\frac{4}{3}} + \frac{9}{2}\alpha_{1\max}^4\Bigg\}\varepsilon_2^4 + \alpha_1^* u\varepsilon_1^3 \\
& + \frac{1}{\alpha_3^*}x_0\varepsilon_2^3\Bigg\{u + \alpha_3^* z_1\eta_0 - \frac{1}{2}\alpha_3^* m_1 z_2\omega_2 \\
& - \alpha_1^*\alpha_3^* m_1m_2z_2 + c_1\eta_0\alpha_3^* z_2 + \frac{1}{2}\alpha_3^* m_1c_1z_1\omega_2 \\
& - \alpha_3^* c_1\eta_0 z_1 - \alpha_3^* c_1 m_1^2 z_1 + \alpha_3^* c_1 m_1^2 z_2 + \alpha_1^*\alpha_3^* c_1 m_1 z_2 \\
& - \alpha_2^*\alpha_3^* c_1 m_1 x_0 z_2\Bigg\} + 9\frac{1}{\alpha_3^*}x_0\varepsilon_2^2\Bigg\{(\alpha_2^*)^2\alpha_3^* m_1^2 x_0 z_2^2 \\
& + \alpha_3^* m_1 z_2^2\omega_2 + (c_1\alpha_1^*)^2\alpha_3^* m_1^2 z_2 + c_1^2\alpha_3^* m_1 z_2^2\omega_2 \\
& + c_1^2\alpha_3^* m_1 z_1^2\omega_2\Bigg\}, \qquad (23)
\end{aligned}
$$

where $\alpha_3^* = \frac{1}{\alpha_2^* - c_1\alpha_1^*}$.

Proposition 2: *For positive numbers c_1, α_1^*, α_2^* and α_3^*, if Assumption 3 holds, then there exists a positive number α_{3max}, such that $\alpha_3^* \le \alpha_{3\max}$, where*

$$
\alpha_{3\max} = \frac{1}{\alpha_{2\min} - c_1\alpha_{1\max}}\mathrm{sgn}\left(\frac{1}{\alpha_{2\min} - c_1\alpha_{1\max}}\right),
$$

and sgn$(\cdot)$ is a sign function.

By Lemma 2, Proposition 2 and simple calculations, we have the following inequalities:

$$
\begin{aligned}
& \left(\alpha_3^* z_1\eta_0 + \frac{1}{2}\alpha_3^* m_1 z_1\omega_2 - c_1\alpha_3^*\eta_0 z_1 - c_1 m_1^2\alpha_3^* z_1\right)\varepsilon_2^3 \\
& \le \frac{1}{4}\varepsilon_1^4 + \frac{3}{4}\Bigg\{\eta_0\alpha_{3\max} + \frac{1}{2}m_1^2m_3\alpha_{3\max} + c_1\eta_0\alpha_{3\max} \\
& \quad + c_1 m_1^2\alpha_{3\max}\Bigg\}^{\frac{4}{3}}\varepsilon_2^4, \\
& \left(-\frac{1}{2}\alpha_3^* m_1 z_2\omega_2 - \alpha_1^*\alpha_3^* m_1m_2z_2 + c_1\eta_0\alpha_3^* z_2\right)\varepsilon_2^3 \\
& \le \frac{1}{4}\varepsilon_1^4 + \Bigg\{\left(\frac{1}{2}m_1^2m_3\alpha_{3\max} + m_1m_2\alpha_{1\max}\alpha_{3\max}\right. \\
& \quad \left. + c_1\eta_0\alpha_{3\max}\right) + \frac{3}{4}(c_1\alpha_{3\max})^{\frac{4}{3}}\left(\frac{1}{2}m_1^2m_3 + m_1m_2\alpha_{1\max}\right. \\
& \quad \left. + c_1\eta_0\right)^{\frac{4}{3}}\Bigg\}\varepsilon_2^4, \\
& 9\big((\alpha_2^*)^2\alpha_3^* m_1^2 x_0 z_2^2 + \alpha_3^* m_1 z_2^2\omega_2 + (c_1\alpha_1^*)^2\alpha_3^* m_1^2 z_2 \\
& \quad + c_1^2\alpha_3^* m_1 z_2^2\omega_2\big)\varepsilon_2^3 \\
& \le 9\varepsilon_1^4 + 9\Bigg\{\alpha_{3\max}\big(\alpha_{2\max}^2 m_1^2m_3 + m_1^2m_3 + c_1^2\alpha_{1\max}^2 m_1^2 \\
& \quad + c_1^2m_1^2m_3\big) + c_1^2\alpha_{3\max}^2\big(\alpha_{2\max}^2 m_1^2m_3 + m_1^2m_3 \\
& \quad + c_1^2\alpha_{1\max}^2 m_1^2 + c_1^2m_1^2m_3\big)^2\Bigg\}\varepsilon_2^4, \\
& 9c_1^2\alpha_3^* m_1 z_1^2\omega_2\varepsilon_2^3 \le \frac{9}{2}\varepsilon_1^4 + \frac{9}{2}\alpha_{3\max}^2 c_1^4 m_1^4 m_3^2\varepsilon_2^4.
\end{aligned}
$$

Substituting the above inequalities into Equation (23), adding and subtracting the term $c_2\varepsilon_2^4$ on the right-hand side of Equation (23), we have

$$
\begin{aligned}
\mathcal{L}V_2 \le & -\left\{\frac{c_1\eta_0(1-e)}{2} - \frac{56}{4}\right\}\varepsilon_1^4 - c_2\varepsilon_2^4 + \frac{1}{\alpha_3^*}x_0H_1\varepsilon_2^4 \\
& + \frac{1}{\alpha_3^*}x_0\varepsilon_2^3 u + \alpha_1^* u\varepsilon_1^3, \qquad (24)
\end{aligned}
$$

where

$$
\begin{aligned}
H_1 = & \frac{\eta_0}{4d^3} + \frac{3}{4}(\alpha_{1\max}m_1m_2m_3)^{\frac{4}{3}} + \frac{3}{4}(\alpha_{2\max}m_1m_2m_3)^{\frac{4}{3}} \\
& + \frac{3}{4}m_1^{\frac{8}{3}} + \frac{9}{2}m_1^4 + \frac{3}{4}(\alpha_{1\max}m_1)^{\frac{4}{3}} + \frac{9}{2}\alpha_{1\max}^4 \\
& + \frac{3}{4}\Bigg(\eta_0\alpha_{3\max} + \frac{1}{2}m_1^2m_3\alpha_{3\max} + c_1\eta_0\alpha_{3\max} \\
& + c_1m_1^2\alpha_{3\max}\Bigg)^{\frac{4}{3}} + \frac{1}{2}m_1^2m_3\alpha_{3\max} + m_1m_2\alpha_{1\max}\alpha_{3\max}
\end{aligned}
$$

$$+c_1\eta_0\alpha_{3\max}+\frac{3}{4}(c_1\alpha_{3\max})^{\frac{4}{3}}\left(\frac{1}{2}m_1^2m_3+m_1m_2\alpha_{1\max}\right.$$
$$\left.+c_1\eta_0\right)^{\frac{4}{3}}+9\alpha_{3\max}\big(\alpha_{2\max}^2m_1^2m_3+m_1^2m_3+c_1^2\alpha_{1\max}^2m_1^2$$
$$+c_1^2m_1^2m_3\big)+9c_1^2\alpha_{3\max}^2\big(\alpha_{2\max}^2m_1^2m_3+m_1^2m_3$$
$$+c_1^2\alpha_{1\max}^2m_1^2+c_1^2m_1^2m_3\big)^2+\frac{9}{2}\alpha_{3\max}^2c_1^4m_1^4m_3^2.$$

One can choose the actual control law u as follows:

$$u=-H_1\varepsilon_2. \tag{25}$$

Substituting Equation (25) into Equation (24), one gets

$$\begin{aligned}\mathcal{L}V_2 &\le -\left(\frac{c_1\eta_0(1-e)}{2}-\frac{56}{4}\right)\varepsilon_1^4-c_2\varepsilon_2^4+\alpha_1^*u\varepsilon_1^3\\ &\le -\left(\frac{c_1\eta_0(1-e)}{2}-\frac{57}{4}\right)\varepsilon_1^4-\left(c_2-\frac{3}{4}(\alpha_{1\max}H_1)^{\frac{4}{3}}\right)\varepsilon_2^4.\end{aligned} \tag{26}$$

Choose the Lyapunov function as

$$V=V_0+V_2, \tag{27}$$

which together with Equations (11) and (27), gives

$$\mathcal{L}V\le-\lambda x_0^2-\bar{c}_1\varepsilon_1^4-\bar{c}_2\varepsilon_2^4, \tag{28}$$

where $\bar{c}_1=\frac{c_1\eta_0(1-e)}{2}-\frac{57}{4}$, $\bar{c}_2=c_2-\frac{3}{4}(\alpha_{1\min}H_1)^{\frac{4}{3}}$. To summarise what has been mentioned above, we have the following conclusion.

Theorem 2: *If Assumptions* 1–5 *hold, one can choose positive constants* λ, $m_i(i=1,2,3)$, α_{1min}, α_{1max}, α_{2min}, α_{2max}, c_1, c_2, $d>0$ *and* $0<e<1$ *satisfying*

$$\begin{cases} c_1=\alpha_{2\max}/\alpha_{1\min}, \quad 4ec_1\ge 4+3d,\\ \eta_0\ge\max\left\{\dfrac{39+4\alpha_{1\max}m_1m_2m_3c_1+4\alpha_{2\max}m_1m_2m_3c_1+2m_1^2m_3^2+4m_1^2+4c_1m_1^2+36m_1^2c_1^2+18m_1^2}{2c_1(1-e)},\quad \dfrac{59}{2c_1(1-e)}\right\}\\ c_2>\dfrac{3}{4}(\alpha_{1\min}H_1)^{\frac{4}{3}}\end{cases}$$

then,

(1) *the closed-loop system composed of Equations* (8.1), (9), (14) *and* (25) *has an almost surely unique solution on* $[t_0,\infty)$ *for* $\forall x_0(t_0)$, $z(t_0)$;
(2) *the equilibrium* $(x_0,z)=(0,0)$ *of the closed-loop system is globally stable in probability.*

Proof: From conditions in Theorem 2, it is easy to see that constants $\lambda>0$, $\bar{c}_1>0$ and $\bar{c}_2>0$. So, $\mathcal{L}V$ in Equation (28) becomes the same form as (3.19) in Deng et al. (2001). Using Equation (28) and Lemma 1, Theorem 2 can be proved. □

4. Switching control stability

In Section 2, the case of $x_0(t_0)\neq 0$ is discussed. We design controllers u_0 and u for system (8) as Equations (9) and (25), respectively. Now we turn to the case of $x_0(t_0)=0$. When the initial $x_0(t_0)=0$, one can choose an open-loop control $u_0=-u_0^*\neq 0$ and $u=u*$. With the similar method in Section V in Wu and Liu (2012), one gets that there exists $t_s^*>0$ such that $|x_0(t_s^*)|\neq 0$, which can drive the state x_0 away from zero in a limited time. Therefore, when $t\in[t_0,t_s^*)$, one can choose the control law $u_0=u_0^*$ and $u=u*$ in order to drive the state x_0 away from zero. After that, at the time $t=t_s^*$, we switch the control inputs u_0 and u into Equations (9) and (25) in $t\in[t_s^*,+\infty)$, respectively. Now, we give the main result of this paper.

Theorem 3: *If Assumptions* 1–5 *hold, we can apply the following switching control procedure to system* (6)*:*

(1) *When the initial state belongs to*

$$\{(\theta(t_0),x_c(t_0),y_c(t_0))\in\mathbb{R}^3|\theta(t_0)\neq 0\},$$

we design control inputs u_0 *and* u *in forms* (9) *and* (25)*, respectively;*
(2) *When the initial state belongs to*

$$\{(\theta(t_0),x_c(t_0),y_c(t_0))\in\mathbb{R}^3|\theta(t_0)=0\},$$

if $t\in[t_0,t_s^*)$*, one can choose the control law* $u_0=u_0^*\neq 0$ *and* u = u∗. *If* $t\in[t_s^*,+\infty)$*, at the time* $t=t_s^*$*, we switch the control inputs* u_0 *and* u *into Equations* (9) *and* (25)*, respectively.*

Then, for any initial conditions in the state space , system (6) will be asymptotically stabilised in probability at the equilibrium and specifically, the states are asymptotically regulated to zero in probability.

Proof: First, we consider the case that the initial state belongs to $\{(\theta(t_0),x_c(t_0),y_c(t_0))\in\mathbb{R}^3|\theta(t_0)\neq 0\}$, that is, $\{(x_0(t_0),x_1(t_0),x_2(t_0))\in\mathbb{R}^3|x_0(t_0)\neq 0\}$. From Theorem 2, for the closed-loop system composed of Equations (8.1), (9), (14) and (25), states x_0 and $z(t)$ are globally regulated to zero in probability and $P\{\lim_{t\to\infty}(|x_0(t)|+|z(t)|)=0\}=1$. This implies that states x_0 and $z(t)$ are globally

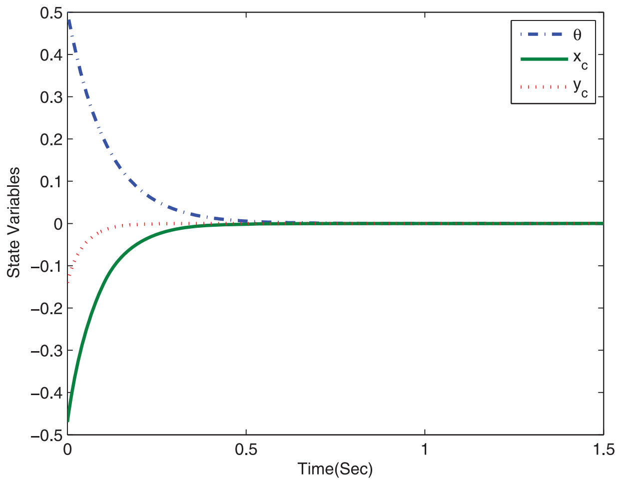

Figure 2. The responses of states θ, x_c and y_c with respect to time.

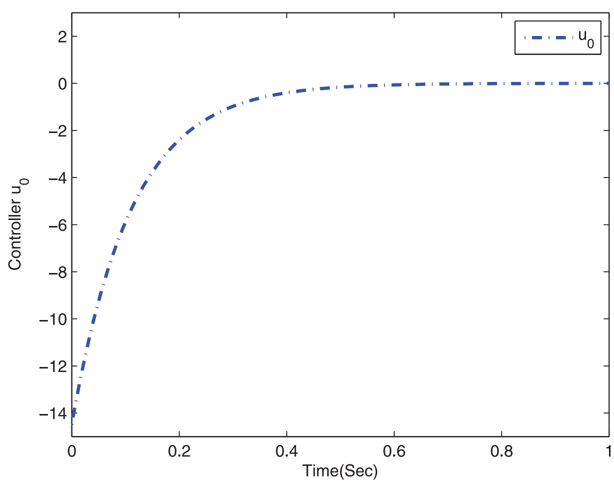

Figure 3. The responses of controllers u_0 with respect to time.

asymptotically regulated to zero in probability and bounded in probability. As a result of Equation (13), one gets that the states x_0, x_1, x_2 of closed-loop system composed of Equations (8), (9) and (25) converge to zero in probability and are all bounded in probability. This shows that the feedback laws are well defined and bounded in Ω. Therefore, the closed-loop system composed of Equations (8), (9) and (25) is globally stabilised in probability. By orthogonal transformation (7), one can obtain that the closed-loop system composed of Equations (6), (9) and (25) is globally stabilised in probability. Second, when the initial state belongs to $\{(\theta(t_0), x_c(t_0), y_c(t_0)) \in \mathbb{R}^3 | \theta(t_0) = 0\}$, we use the constant control $u_0 = u_0^* \neq 0$ in order to drive x_0 far away from the origin. Meanwhile, by application of the design procedure proposed in Section 2, we construct a controller $u = u*$, which guarantees that all the signals are bounded in probability during $[t_0, t_s^*)$. Then, in view of $x_0(t_s^*) \neq 0$,

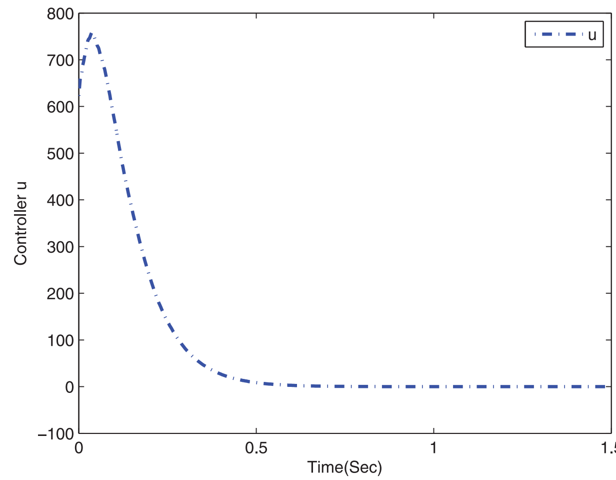

Figure 4. The responses of controllers u with respect to time.

the switching control strategy is applied to system (6) at the time instant $t_s^* > 0$. □

5. A simulation example

Consider the system (6) with $\omega_2 = 0.5\theta$ and $v_2 = 0.1\theta(x_c \cos\theta + y_c \sin\theta)$. In simulation, one can choose $e = 0.5$, $d = 1$, $c_1 = 3.6$, $c_2 = 0.2$, $\eta_0 = 29$, $\alpha = 6$, $\theta_0 = \frac{\pi}{4}$, $\alpha_{1\min} = 5$, $\alpha_{2\min} = 2.88$, $\alpha_{1\max} = 5.3$, $\alpha_{2\max} = 3.1$, $m_1 = 0.51$, $m_2 = 0.1$, $m_3 = 0.6$ and the initial values $\theta(0) = 0.5$, $x_c(0) = -0.47$, $y_c(0) = -0.14$. Figures 2 and 3 give the responses of the closed-loop system consisting of Equations (6), (9) and (26).

From Figure 2, it is easy to see that the states θ, x_c and y_c are asymptotically regulated to zero in probability in spite of the stochastic disturbances. As shown in Figures 3 and 4, the control inputs u_0 and u are convergent to a small neighbourhood of zero asymptotically.

6. Conclusions

In this paper, we extend the non-holonomic mobile robots with unknown parameters based on visual servoing to the stochastic case. State-feedback stabilising controllers are designed for stochastic non-holonomic mobile robots with unknown parameters based on visual servoing by backstepping technique. A switching control strategy for the original system is given, which guarantees that the closed-loop system is asymptotically stabilised at the zero equilibrium point in probability.

There exist some problems to be discussed. For example, when $\alpha_1 \neq \alpha_2$, how to extend Equation (4) to the stochastic case and design the controllers for the stochastic non-holonomic systems with unknown parameters based on visual servoing.

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