# **Robotic Manipulator Documentation**

Release 1.0

**Thiago Souto** 

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## FORWARD\_KINEMATICS MODULE

#### class Forward\_Kinematics.ForwardKinematics(\*\*kwargs)

Bases: object

Definition: This class generates Homogeneous transform matrices, although it uses a symbolic approach that can be used to multiply any matrix and obtain the translation or rotation.

sympy.cos and sympy.sin: cos and sin for sympy

sympy.simplify: SymPy has dozens of functions to perform various kinds of simplification. simplify() attempts to apply all of these functions in an intelligent way to arrive at the simplest form of an expression.

Returns: It returns Rotation and translation matrices.

Obs: \*\*kwargs (keyword arguments) are used to facilitate the identification of the parameters, so initiate the object

#### rot\_x (alpha)

Definition: Receives an alpha angle and returns the rotation matrix for the given angle at the X axis.

**Parameters alpha** (string) – Rotation Angle around the X axis

Returns: The Rotational Matrix at the X axis by an *given* angle

#### rot\_z (theta)

Definition: Receives an theta angle and returns the rotation matrix for the given angle at the Z axis.

Parameters theta (string) - Rotation Angle around the Z axis

Returns: The Rotational Matrix at the Z axis by an given angle

#### trans $\mathbf{x}(a)$

Definition: Translates the matrix a given amount a on the X axis by Defining a 4x4 identity matrix with a as the (1,4) element.

**Parameters a** (string) – Distance translated on the X-axis

Returns: The Translation Matrix on the X axis by a given distance

#### $trans_z(d)$

Definition: Translate the matrix a given amount d on the Z axis. by Defining a matrix T 4x4 identity matrix with d (3,4) element position.

Parameters d (string) - Distance translated on the Z-axis

Returns: The Translation Matrix on the Z axis by a given distance

#### Forward Kinematics.main()

Assessment 02 Robotic manipulator design - Forward Kinematics.

Forward\_Kinematics.printM(expr, num\_digits)

#### 1.1 Python Program

```
import numpy as np
   import sympy as sympy
2
   from sympy import *
   class ForwardKinematics:
       Definition: This class generates Homogeneous transform matrices, although it uses,
   →a symbolic approach
       that can be used to multiply any matrix and obtain the translation or rotation.
10
11
       sympy.cos and sympy.sin: cos and sin for sympy
12
13
       sympy.simplify: SymPy has dozens of functions to perform various kinds of,
14
    \hookrightarrow simplification.
       simplify() attempts to apply all of these functions
15
       in an intelligent way to arrive at the simplest form of an expression.
17
       Returns: It returns Rotation and translation matrices.
18
19
       Obs: **kwargs (keyword arguments) are used to facilitate the identification of
20
   →the parameters, so initiate the
       object
21
22
       np.set_printoptions(precision=3, suppress=True)
23
24
       sympy.init_printing(use_unicode=True, num_columns=400)
25
26
       def __init__(self, **kwargs):
27
28
           Initializes the Object.
30
           self._x_angle = kwargs['x_angle'] if 'x_angle' in kwargs else 'alpha_i-1'
31
           self._x_dist = kwargs['x_dist'] if 'x_dist' in kwargs else 'a_i-1'
32
           self._y_angle = kwargs['y_angle'] if 'y_angle' in kwargs else '0'
33
           self._y_dist = kwargs['y_dist'] if 'y_dist' in kwargs else '0'
34
           self._z_angle = kwargs['z_angle'] if 'z_angle' in kwargs else 'theta_i'
35
           self._z_dist = kwargs['z_dist'] if 'z_dist' in kwargs else 'd_i'
36
37
       def trans_x(self, a):
38
            n n n
39
           Definition: Translates the matrix a given amount `a` on the *X* axis by
40
    → Defining a 4x4 identity
           matrix with `a` as the (1,4) element.
41
42
            :type a: string
43
            :param a: Distance translated on the X-axis
44
45
           Returns: The Translation Matrix on the *X* axis by a given distance
46
47
           self._x_dist = a
48
49
           t_x = sympy.Matrix([[1, 0, 0, self._x_dist],
50
                                 [0, 1, 0, 0],
```

```
[0, 0, 1, 0],
52
                                   [0, 0, 0, 1]])
53
54
            t_x = t_x.evalf()
55
56
            return t_x
57
58
        def trans_z(self, d):
59
60
            Definition: Translate the matrix a given amount `d` on the *Z* axis. by
61
    → Defining a matrix T 4x4 identity
            matrix with *d* (3,4) element position.
62
63
            :type d: string
64
            :param d: Distance translated on the Z-axis
65
66
            Returns: The Translation Matrix on the *Z* axis by a given distance
67
68
            self._z_dist = d
69
70
            t_z = sympy.Matrix([[1, 0, 0, 0],
71
                                   [0, 1, 0, 0],
72.
                                   [0, 0, 1, self._z_dist],
73
                                   [0, 0, 0, 1]])
74
75
76
            t_z = t_z.evalf()
77
            return t_z
78
79
        def rot_x(self, alpha):
80
81
82
            Definition: Receives an alpha angle and returns the rotation matrix for the
    ⇒given angle at the *X* axis.
83
            :type alpha: string
84
            :param alpha: Rotation Angle around the X axis
85
86
            Returns: The Rotational Matrix at the X axis by an *given* angle
87
88
            self._x_angle = alpha
89
90
            r_x = sympy.Matrix([[1, 0, 0, 0],
91
                                   [0, sympy.cos(self._x_angle), -sympy.sin(self._x_angle),_
92
    → 0],
                                   [0, sympy.sin(self._x_angle), sympy.cos(self._x_angle),
    \hookrightarrow 0]
                                   [0, 0, 0, 1]])
94
95
            r_x = r_x.evalf()
96
97
            return r_x
        def rot_z(self, theta):
100
101
            Definition: Receives an theta angle and returns the rotation matrix for the
102
    \rightarrowgiven angle at the *Z* axis.
103
```

```
:type theta: string
104
                                                                         :param theta: Rotation Angle around the Z axis
105
106
                                                                        Returns: The Rotational Matrix at the Z axis by an *given* angle
 107
 108
                                                                        self._z_angle = theta
109
110
                                                                        r_z = sympy.Matrix([[sympy.cos(self._z_angle), -sympy.sin(self._z_angle), 0,,,
111
                         \hookrightarrow 01,
                                                                                                                                                                                                     [sympy.sin(self._z_angle), sympy.cos(self._z_angle), 0,_
112
                         \hookrightarrow0],
                                                                                                                                                                                                      [0, 0, 1, 0],
113
114
                                                                                                                                                                                                      [0, 0, 0, 1]])
115
                                                                        r_z = r_z.evalf()
116
117
                                                                        return r_z
118
119
120
                         # def printM(expr, num_digits):
121
                                                           return expr.xreplace({n.evalf(): n if type(n) == int else Float(n, num_digits)_
122
                         →for n in expr.atoms(Number)})
123
                      def printM(expr, num_digits):
124
                                              return expr.xreplace({n.evalf(): round(n, num_digits) for n in expr.atoms(Number)}
125
126
127
                      def main():
128
129
130
                                              Assessment 02 Robotic manipulator design - Forward Kinematics.
131
                                              a1 = ForwardKinematics()
                                                                                                                                                                                                                                                # Rx(a_i-1)
132
                                              a2 = ForwardKinematics()
                                                                                                                                                                                                                                                # Dx(a_i-1)
133
                                              a3 = ForwardKinematics()
                                                                                                                                                                                                                                                # Dz(d i)
134
                                              a4 = ForwardKinematics()
                                                                                                                                                                                                                                                # Rz(theta_i)
135
136
                                              print('Matrix t_0_1:')
137
138
                                              t_0_1 = (a1.rot_x('0')) * (a2.trans_x('0')) * (a3.trans_z('11')) * (a4.rot_z('0')) * (a4.rot_z('0')) * (a4.rot_z('0')) * (a4.rot_z('0')) * (a5.trans_z('0')) * (a5.t
                          \rightarrow 'theta_1'))
                                              print(sympy.pretty(t_0_1))
139
140
                                              print('\nMatrix t_1_2:')
141
                                              t_1_2 = (a1.rot_x('alpha_1')) * (a2.trans_x('0')) * (a3.trans_z('0')) * (a4.rot_z('0')) * (a4.rot_z('0')) * (a4.rot_z('0')) * (a5.trans_z('0')) 
142

    'theta_2'))

                                              t_1_2_subs = t_1_2.subs('alpha_1', np.deg2rad(90.00))
143
                                              print(sympy.pretty(printM(t_1_2_subs, 3)))
144
145
                                              print('\nMatrix t_2_3:')
146
                                              t_2_3 = (a1.rot_x('0')) * (a2.trans_x('12')) * (a3.trans_z('0')) * (a4.rot_z('0')) * (a4.rot_z('0'))
147
                          \rightarrow 'theta_3'))
                                              print(sympy.pretty(t_2_3))
148
149
                                              print('\nMatrix t 3 4:')
150
                                               t_3_4 = (a1.rot_x('0')) * (a2.trans_x('13')) * (a3.trans_z('0')) * (a4.rot_z('13')) * (a3.trans_z('13')) * (a3.trans_z('13')) * (a4.rot_z('13')) * (a3.trans_z('13')) * (a3.tra
151
                         → 'theta_4'))
152
                                              print(sympy.pretty(t_3_4))
```

```
153
        print('\nMatrix t_4_5:')
154
        t_4_5 = (a1.rot_x('0')) * (a2.trans_x('14')) * (a3.trans_z('0')) * (a4.rot_z('0'))
155
        print(sympy.pretty(t_4_5))
156
157
        t_0_5 = \text{sympy.simplify}(t_0_1 * t_1_2 * t_2_3 * t_3_4 * t_4_5)
158
        print('\nMatrix T_0_5: with substitutions Round')
159
        print(sympy.pretty(sympy.simplify(printM(t_0_5.subs('alpha_1', np.deq2rad(90.00)),
160
    → 3))))
       t_0_5 subs = t_0_5.subs([('alpha_1', np.deg2rad(90.00)), ('11', 230), ('12', 500),
161
    print('\nMatrix T_0_5: with substitutions Round')
162
163
        print(sympy.pretty(sympy.simplify(printM(t_0_5_subs, 3))))
164
        t_1_5 = sympy.simplify(t_1_2 * t_2_3 * t_3_4 * t_4_5)
165
        print('\nMatrix T_1_5:')
166
       print(sympy.pretty(printM(t_1_5.subs('alpha_1', np.deg2rad(90.00)), 3)))
167
168
        print('\nMatrix T_1_5: for theta_1 = 0 ')
169
        print(sympy.pretty(printM(t_1_5.subs([('alpha_1', np.deg2rad(90.00)), ('theta_1',...
170
    \rightarrownp.deg2rad(0.00))]), 3)))
171
        # Calculations for the Inverse kinematics problem
172
173
        t_1_4 = sympy.simplify(t_1_5 * t_4_5.inv())
174
175
        print('\nMatrix T_1_4:')
        print(sympy.pretty(printM(t_1_4.subs('alpha_1', np.deg2rad(90.00)), 3)))
176
177
        t_3_5 = sympy.simplify(t_1_5 * t_1_2.inv() * t_2_3.inv())
178
        print('\nMatrix t_3_5:')
179
        print(sympy.pretty(printM(t_3_5.subs('alpha_1', np.deg2rad(90.00)), 3)))
180
181
182
   if __name__ == '__main__':
183
        main()
184
```

## 1.2 Output

```
Matrix t_0_1:
1.0cos(\theta_1) -1.0sin(\theta_1)
                            0
1.0sin(\theta_1) 1.0cos(\theta_1)
                             0
                                    0
     0
                     0
                              1.0 1.011
                                     1.0
Matrix t_1_2:
1.0\cos(\theta_2) -1.0\sin(\theta_2)
                            0
                     0
                                -1.0 0
1.0sin(\theta_2) 1.0cos(\theta_2)
                             0.0
                                   0
```

(continues on next page)

```
0 0 1.0
Matrix t_2_3:
1.0cos(\theta_3) -1.0sin(\theta_3) 0 1.01<sub>2</sub>
1.0\sin(\theta_3) 1.0\cos(\theta_3)
                                                                                                                     0
                 0
                                                                              0 1.0 0
                                                                              0
                                                                                                                      0 1.0
                 0
Matrix t_3_4:
1.0cos(\theta_4) -1.0sin(\theta_4) 0 1.0l<sub>3</sub>
1.0\sin(\theta_4) 1.0\cos(\theta_4) 0 0
                       0 0 1.0 0
                 0 0 1.0
Matrix t_4_5:
1.0 0 0 1.014
  0 1.0 0
                                                                           0
   0 0 1.0 0
                0 0 1.0
Matrix T_0_5: with substitutions Round
1.0\cos(\theta_1)\cos(\theta_2 + \theta_3 + \theta_4) - 1.0\sin(\theta_2 + \theta_3 + \theta_4)\cos(\theta_1) - 1.0\sin(\theta_1) - 1.0(1_2\cos(\theta_2) + 1_1)\cos(\theta_1) - 1.0(1_2\cos(\theta_2) + 1_2)\cos(\theta_1) - 1.0(1_2\cos(\theta_2) + 1_2)\cos(\theta_1) - 1.0(1_2\cos(\theta_2) + 1_2)\cos(\theta_2) - 
  \rightarrow 1<sub>3</sub>cos (\theta_2 + \theta_3) + 1_4cos (\theta_2 + \theta_3 + \theta_4)) cos (\theta_1)
1.0sin(\theta_1)cos(\theta_2 + \theta_3 + \theta_4) -1.0sin(\theta_1)sin(\theta_2 + \theta_3 + \theta_4) -1.0cos(\theta_1) 1.0(1<sub>2</sub>cos(\theta_2) +...
  \rightarrow 1<sub>3</sub>cos(\theta_2 + \theta_3) + 1<sub>4</sub>cos(\theta_2 + \theta_3 + \theta_4))sin(\theta_1)
            1.0sin(\theta_2 + \theta_3 + \theta_4) 1.0cos(\theta_2 + \theta_3 + \theta_4)
                                                                                                                                                                                                                                                                                                                                                                1.01_1 + 1.
  \rightarrow01<sub>2</sub>sin(\theta_2) + 1.01<sub>3</sub>sin(\theta_2 + \theta_3) + 1.01<sub>4</sub>sin(\theta_2 + \theta_3 + \theta_4)
                                                                    0
                                                                                                                                                                                                                       0
                                                                                                                                                                                                                                                                                                                                    0
                                                                                                                                  1.0
Matrix T_0_5: with substitutions Round
```

(continues on next page)

```
1.0\cos(\theta_1)\cos(\theta_2 + \theta_3 + \theta_4) - 1.0\sin(\theta_2 + \theta_3 + \theta_4)\cos(\theta_1) - 1.0\sin(\theta_1) (500.0cos(\theta_2) + ...
\rightarrow500.0cos(\theta_2 + \theta_3) + 180.0cos(\theta_2 + \theta_3 + \theta_4))cos(\theta_1)
1.0 \sin{(\theta_1)} \cos{(\theta_2 + \theta_3 + \theta_4)} - 1.0 \sin{(\theta_1)} \sin{(\theta_2 + \theta_3 + \theta_4)} - 1.0 \cos{(\theta_1)}  (500.0 \cos(\theta_2) + \(\theta_3\)
\rightarrow500.0cos(\theta_2 + \theta_3) + 180.0cos(\theta_2 + \theta_3 + \theta_4))sin(\theta_1)
     1.0\sin(\theta_2 + \theta_3 + \theta_4) 1.0\cos(\theta_2 + \theta_3 + \theta_4)
                                                                                                               0
                                                                                                                                  500.0\sin(\theta_2)
\rightarrow+ 500.0sin(\theta_2 + \theta_3) + 180.0sin(\theta_2 + \theta_3 + \theta_4) + 230.0
                        0
                                                                                0
                                                                                                                        0
                                              1.0
Matrix T_1_5:
1.0\cos{(\theta_2 + \theta_3 + \theta_4)} - 1.0\sin{(\theta_2 + \theta_3 + \theta_4)} \qquad 0 \qquad 1.01_2\cos{(\theta_2)} + 1.01_3\cos{(\theta_2 + \theta_3)} + 1.
\hookrightarrow 01_4\cos(\theta_2 + \theta_3 + \theta_4)
               0
                                                        0
                                                                                   -1.0
                                                                                                                                                       0_
1.0\sin(\theta_2 + \theta_3 + \theta_4) 1.0\cos(\theta_2 + \theta_3 + \theta_4) 0.0 1.0l_2\sin(\theta_2) + 1.0l_3\sin(\theta_2 + \theta_3) + 1.
\hookrightarrow 01<sub>4</sub>sin(\theta_2 + \theta_3 + \theta_4)
               0
                                                        0
                                                                                    Ω
                                                                                                                                                     1.
→ 0
Matrix T_1_5: for theta_1 = 0
1.0\cos{(\theta_2 + \theta_3 + \theta_4)} - 1.0\sin{(\theta_2 + \theta_3 + \theta_4)} \qquad 0 \qquad 1.01_2\cos{(\theta_2)} + 1.01_3\cos{(\theta_2 + \theta_3)} + 1.
\hookrightarrow 01_4 \cos(\theta_2 + \theta_3 + \theta_4)
                                                                                                                                                       0_
                                                                                    -1.0
1.0sin(\theta_2 + \theta_3 + \theta_4) 1.0cos(\theta_2 + \theta_3 + \theta_4) 0.0 1.0l<sub>2</sub>sin(\theta_2) + 1.0l<sub>3</sub>sin(\theta_2 + \theta_3) + 1.
\hookrightarrow 01_4 \sin(\theta_2 + \theta_3 + \theta_4)
                0
                                                        0
                                                                                    0
                                                                                                                                                     1.
\hookrightarrow 0
Matrix T_1_5: for theta_1 = 0
1.0\cos(\theta_2 + \theta_3 + \theta_4) - 1.0\sin(\theta_2 + \theta_3 + \theta_4) = 0 \qquad 1.01\cos(\theta_2) + 1.01\cos(\theta_2 + \theta_3) + 1.
\rightarrow 01_4\cos(\theta_2 + \theta_3 + \theta_4)
                                                                                                                                  (continues on next page)
```

```
0
                                                                                      -1.0
1.0sin(\theta_2 + \theta_3 + \theta_4) 1.0cos(\theta_2 + \theta_3 + \theta_4) 0.0 1.0l<sub>2</sub>sin(\theta_2) + 1.0l<sub>3</sub>sin(\theta_2 + \theta_3) + 1.
\hookrightarrow 01_4 \sin(\theta_2 + \theta_3 + \theta_4)
                  0
                                                          0
                                                                                                                                                        1.
→0
Matrix T_1_4:
1.0\cos(\theta_2 + \theta_3 + \theta_4) - 1.0\sin(\theta_2 + \theta_3 + \theta_4) \qquad 0 \qquad 1.01_2\cos(\theta_2) + 1.01_3\cos(\theta_2 + \theta_3)
                 0
                                                          0
                                                                                     -1.0
1.0 \sin{(\theta_2 + \theta_3 + \theta_4)} - 1.0 \cos{(\theta_2 + \theta_3 + \theta_4)} - 0.0 - 1.0 l_2 \sin{(\theta_2)} + 1.0 l_3 \sin{(\theta_2 + \theta_3)}
                0
                                                         0
                                                                                     0
                                                                                                                              1.0
Matrix t_3_5:
1.0\cos(\theta_3)\cos(\theta_3 + \theta_4) 1.0\sin(\theta_3)\cos(\theta_3 + \theta_4) -\sin(\theta_3 + \theta_4) 1.0l_2\cos(\theta_2) -\ldots
\Rightarrow 1_2 \cos(\theta_3) \cos(\theta_3 + \theta_4) + 1.01_3 \cos(\theta_2 + \theta_3) + 1.01_4 \cos(\theta_2 + \theta_3 + \theta_4)
              -\sin(\theta_3)
                                                         1.0\cos(\theta_3)
                                                                                                       0.0
                                      1.01_2 \sin(\theta_3)
1.0sin(\theta_3 + \theta_4)cos(\theta_3) 1.0sin(\theta_3)sin(\theta_3 + \theta_4) 1.0cos(\theta_3 + \theta_4) 1.0l<sub>2</sub>sin(\theta_2) - l<sub>2</sub>sin(\theta_3 + \theta_4)
\rightarrow \theta_4) cos (\theta_3) + 1.01<sub>3</sub>sin (\theta_2 + \theta_3) + 1.01<sub>4</sub>sin (\theta_2 + \theta_3 + \theta_4)
                    0
                                                                                                            0
                                                                    0
                                                     1.0
```

#### INVERSE\_KINEMATICS MODULE

```
Inverse_Kinematics.disp(expr)
Displays a simplified Sympy expression.

Inverse_Kinematics.h_T(alpha, a, theta, d)
Returns a general homogeneous transform.

Inverse_Kinematics.rot_x(alpha)
Returns a homogeneous transform for just a rotation about the X axis by alpha.

Inverse_Kinematics.rot_z(theta)
Returns a homogeneous transform for just a rotation about the Z axis by theta.

Inverse_Kinematics.trans_x(a)
Returns a homogeneous transform for just a translation along the X axis by a.

Inverse_Kinematics.trans_z(d)
Returns a homogeneous transform for just a rotation along the Z axis by d.
```

## 2.1 Python Program

```
import numpy as np
   import sympy
   np.set_printoptions(precision=3, suppress=True)
   sympy.init_printing(use_unicode=True, num_columns=400)
   def disp(expr):
        """Displays a simplified Sympy expression."""
10
11
       e = sympy.simplify(expr)
12
       e = sympy.expand(e)
13
       e = e.evalf()
14
15
       print(sympy.pretty(e))
16
       return
19
20
   def rot_x(alpha):
21
        """Returns a homogeneous transform for just a rotation about the X axis by alpha."
22
23
```

```
T = sympy.Matrix([[1, 0, 0, 0],
24
                            [0, sympy.cos(alpha), -sympy.sin(alpha), 0],
25
                            [0, sympy.sin(alpha), sympy.cos(alpha), 0],
26
                            [0, 0, 0, 1]])
27
28
        return T
29
30
31
   def trans_x(a):
32
        """Returns a homogeneous transform for just a translation along the X axis by a.""
33
35
        T = sympy.Matrix([[1, 0, 0, a],
                           [0, 1, 0, 0],
36
                           [0, 0, 1, 0],
37
                            [0, 0, 0, 1]])
38
39
        return T
40
41
42
   def rot_z(theta):
43
        """Returns a homogeneous transform for just a rotation about the Z axis by theta."
44
45
        T = sympy.Matrix([[sympy.cos(theta), -sympy.sin(theta), 0, 0],
46
47
                            [sympy.sin(theta), sympy.cos(theta), 0, 0],
                            [0, 0, 1, 0],
48
                            [0, 0, 0, 1]])
49
50
        return T
51
52
53
   def trans_z(d):
54
        """Returns a homogeneous transform for just a rotation along the Z axis by d."""
55
56
        T = sympy.Matrix([[1, 0, 0, 0],
57
                           [0, 1, 0, 0],
58
                           [0, 0, 1, d],
60
                            [0, 0, 0, 1]])
61
        return T
62.
63
64
   def h_T(alpha, a, theta, d):
65
        """Returns a general homogeneous transform."""
66
67
        T = rot_x(alpha) @ trans_x(a) @ rot_z(theta) @ trans_z(d)
68
69
        return T
70
71
72
   # Forward kinematics for the given problem.
73
74
   theta_1 = np.deg2rad(0.0)
75
   T01 = h_T(0, 0, theta_1, 0)
76
   print("Matrix T01:")
77
   disp(T01)
```

```
79
   alpha_1 = np.deg2rad(90)
80
   theta_2 = np.deg2rad(0.0)
81
   T12 = h_T(alpha_1, 0, theta_2, 0)
82
    print("\nMatrix T12:")
83
   disp(T12)
85
   11 = 500
86
   theta_3 = np.deg2rad(45.0)
87
   T23 = h_T(0, 11, theta_3, 0)
88
   print("\nMatrix T23:")
   disp(T23)
92
   12 = 500
   theta_4 = np.deg2rad(45.0)
93
   T34 = h_T(0, 12, theta_4, 0)
94
   print("\nMatrix T34:")
   disp(T34)
   13 = 230
98
    theta_5 = np.deg2rad(0)
99
   T45 = h_T(0, 13, theta_5, 0)
100
   print("\nMatrix T45:")
101
   disp(T45)
102
   T05 = T01 @ T12 @ T23 @ T34 @ T45
   print("\nMatrix T05:")
105
   disp(T05)
106
107
   print("\nProblem: location x, y, z")
108
   x05 = float(T05[0, 3])
110
   y05 = float(T05[1, 3])
   z05 = float(T05[2, 3])
111
112
   print("x05: {}".format(x05))
113
   print("y05: {}".format(y05))
114
   print("z05: {}".format(z05))
115
    # Inverse kinematics for the given problem.
118
   x = 853.553
119
   y = 0.0
120
   z = 583.553
121
122
   theta_1 = np.arctan2(y, z)
123
   T01 = h_T(0, 0, theta_1, 0)
124
   print("\nMatrix T01 for given problem:")
125
   disp(T01)
126
127
   T45 = h_T(0, 230, 0, 0)
128
   print("\nMatrix T45 for given problem:")
   disp(T45)
131
   T15 = T05 @ T01.inv()
132
   print("\nMatrix T15 for given problem:")
133
   disp(T15)
134
135
```

```
T14 = T15 @ T45.inv()
136
   print("\nMatrix T14 for given problem:")
137
   disp(T14)
138
139
   x14 = float(T14[0, 3])
   y14 = float(T14[1, 3])
141
   z14 = float(T14[2, 3])
142
143
   print("x14: {}".format(x14))
144
   print("y14: {}".format(y14))
145
   print("z14: {}".format(z14))
   B = np.arctan2(z14, x14)
   c2 = (12**2 - 11**2 - x14**2 - z14**2) / (-2*11*np.sqrt(x14**2 + z14**2))
149
   s2 = np.sqrt(1 - c2**2)
150
   w = np.arctan2(s2, c2)
151
   theta_2 = B - w
152
   c3 = (x14**2 + z14**2 - 11**2 - 12**2)/(2*11*12)
154
   s3 = np.sqrt(1 - c3**2)
155
   theta_3 = np.arctan2(s3, c3)
156
157
   T35 = T34 @ T45
158
   disp(T35)
159
   x35 = float(T35[0, 3])
   y35 = float(T35[1, 3])
162
   z35 = float(T35[2, 3])
163
164
   print("x35: {}".format(x35))
165
   print("y35: {}".format(y35))
   print("z35: {}".format(z35))
168
   c4 = (x35 - 12) / 13
169
   s4 = np.sqrt(1 - c4**2)
170
   theta_4 = np.arctan2(s4, c4)
171
172
   print("\ntheta_2: {}".format(np.rad2deg(theta_2)))
   print("\ntheta_3: {}".format(np.rad2deg(theta_3)))
   print("\ntheta_4: {}".format(np.rad2deg(theta_4)))
```

## 2.2 Output

```
Matrix T01:
     1.0
         0
               0
2
         1.0
              0
4
          0
              1.0
                     0
          0
             0
                  1.0
8
     Matrix T12:
10
                   0
                                         0
                                                       0
     1.0
11
```

(continues on next page)

```
6.12323399573677e-17
                                       -1.0
                                                      0
13
14
                 1.0
                        6.12323399573677e-17
15
16
                  0
                                        0
                                                    1.0
17
18
     Matrix T23:
19
     0.707106781186548 -0.707106781186547
20
21
     0.707106781186547 0.707106781186548 0
                                                 0
22
23
             Ω
                               \cap
                                          1.0
                                                 0
25
                               0
                                            0
                                                 1.0
26
27
     Matrix T34:
28
     0.707106781186548 -0.707106781186547
                                           0
                                                500.0
29
     0.707106781186547 0.707106781186548 0
31
                                                 0
32
                                           1.0
33
34
             0
                               0
                                   0
                                                1.0
35
36
37
     Matrix T45:
     1.0 0 0 230.0
39
         1.0 0
                    0
40
41
         0 1.0
                    0
42
43
         0 0 1.0
45
     Matrix T05:
46
     2.22044604925031e-16
                          -1.0
                                                                        853.
47
   →553390593274
48
   6.12323399573677e-17 1.23259516440783e-32
                                                       -1.0
                                                               3.57323395960819e-
   →14
50
                          2.22044604925031e-16 6.12323399573677e-17 583.
            1.0
51
   →553390593274
              0
                                   0
                                                         0
                                                                              1.0
53
54
55
     Problem: location x, y, z
57
58
     x05: 853.5533905932738
     v05: 3.573233959608189e-14
59
     z05: 583.5533905932737
60
61
     Matrix T01 for given problem:
                                                                           (continues on next page)
```

```
1.0
                0
63
64
          1.0
               0
                     0
65
66
          0 1.0
                    0
67
68
           0
               0 1.0
69
70
     Matrix T45 for given problem:
71
     1.0
          0 0
                   230.0
72
73
74
      Ω
         1.0
               0
                     0
              1.0
76
77
              0
                     1.0
           0
78
79
     Matrix T15 for given problem:
80
                           -1.0
                                                          0
81
     2.22044604925031e-16
                                                                          853.
   →553390593274
82
     6.12323399573677e-17 1.23259516440783e-32
                                                          -1.0
                                                                        3.57323395960819e-
83
   →14
                           2.22044604925031e-16 6.12323399573677e-17
85
   →553390593274
86
              0
                                     0
                                                           0
                                                                                 1.0
87
88
     Matrix T14 for given problem:
89
     2.22044604925031e-16
                                    -1.0
90
   →553390593274
91
    6.12323399573677e-17 1.23259516440783e-32
                                                         -1.0
                                                                      2.16489014058873e-
   →14
93
                           2.22044604925031e-16 6.12323399573677e-17 353.
            1.0
94
   →553390593274
              0
                                     0
                                                           0
                                                                                 1.0
96
97
     x14: 853.5533905932738
98
     y14: 2.164890140588733e-14
     z14: 353.55339059327366
100
101
     0.707106781186548 - 0.707106781186547 0 662.634559672906
102
103
     0.707106781186547 0.707106781186548 0 162.634559672906
104
105
                                                                             (continues on next page)
```

```
0
               0
                                     0
                                                  1.0
106
107
               0
                                     0
                                                   0
                                                               1.0
108
      x35: 662.6345596729059
110
      y35: 162.6345596729059
111
      z35: 0.0
112
113
114
      theta_2: -9.54166404439055e-15
115
      theta_3: 45.00000000000014
116
      theta_4: 45.00000000000014
```

**CHAPTER** 

**THREE** 

#### **INDICES AND TABLES**

At the website you can navigate through the menus below:

- genindex
- modindex
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## 3.1 Running the documentation with Sphinx

To run the documentation for this project run the following commands, at the project folder:

**Install Spinxs:** 

python -m pip install sphinx

Install the "Read the Docs" theme:

pip install sphinx-rtd-theme

make clean

make html

## 3.2 GitHub Repository

Find all the files at the GitHub repository here.

## **PYTHON MODULE INDEX**

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i
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```