Batt-Freq-Regulation-Optimization

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Problem Formulation: BESS Operation with Market Participation and Regulation Signals

We consider a single Battery Energy Storage System (BESS) that participates in both the wholesale energy market and a regulation market over a finite time horizon $t = 0, 1, \ldots, T-1$. At each time step, the operator can:

- Charge the battery by buying electricity from the market, $P_{chg}(t) \geq 0$
- Discharge the battery by selling electricity to the market, $P_{dis}(t) \geq 0$
- Provide regulation services by responding to real-time signals $w_{up}(t), w_{down}(t) \in [0, +1]$, which are scaled by the committed regulation capacity $\bar{P}_{up}(t) \geq 0, \bar{P}_{down}(t) \geq 0$
- If we consider buying and selling electricity from the real time market, charge and discharge the battery by buying or selling electricity from the real time market, $R_{chg}(t) \geq 0$, $R_{dis}(t) \geq 0$
- $\bar{P}_{rrs}(t) \geq 0$: Committed Responsive Reserve Service (RRS) capacity at time t (MW).
- $\bar{P}_{ecrs}(t) \geq 0$: Committed ERCOT Contingency Reserve Service (ECRS) capacity at time t (MW).
- $\bar{P}_{ns}(t) \geq 0$: Committed Non-Spinning Reserve (Non-Spin) capacity at time t (MW).
- real-time signals $w_{rrs}(t), w_{ecrs}(t), w_{ns}(t) \in [0, +1]$

Objective

We consider buying and selling electricity from the real time market. Minimize the total cost (or equivalently, maximize profit):

$$\min\ J = J_{DAM} - J_{REG} + J_{RTM} + J_{DIF} \tag{1}$$

$$J_{DAM} = \sum_{t=0}^{T-1} c_{dam}(t) \Big(P_{chg}(t) - P_{dis}(t) \Big) \Delta t \tag{2}$$

$$J_{REG} = \sum_{t=0}^{T-1} \Big(c_{up}(t)\, \bar{P}_{up}(t) + c_{down}(t)\, \bar{P}_{down}(t) + c_{rrs}(t)\, \bar{P}_{rrs}(t) + c_{ecrs}(t)\, \bar{P}_{ecrs}(t) + c_{ns}(t)\, \bar{P}_{ns}(t) \Big) \Delta t \tag{3}$$

$$\begin{aligned} J_{RTM} = \sum_{t=0}^{T-1} c_{rtm}(t) \Big[& \big(R_{\text{chg}}(t) + w_{down}(t)\, \bar{P}_{down}(t) \big) \\ & - \big(R_{\text{dis}}(t) + w_{up}(t)\, \bar{P}_{up}(t) + w_{rrs}(t)\, \bar{P}_{rrs}(t) + w_{ecrs}(t)\, \bar{P}_{ecrs}(t) + w_{ns}(t)\, \bar{P}_{ns}(t) \big) \Big] \Delta t \end{aligned} \tag{4}$$

$$J_{DIF} = p \left(E(0) - E(T) \right) \tag{5}$$

where:

- $c_{dam}(t)$ is the predicted day-ahead market price at time t
- $c_{up}(t)$ and $c_{down}(t)$ are the predicted regulation up/down profit at time t
- $c_{rtm}(t)$ is the predicted real-time market price at time t
- p is the term to evaluate the penalty of the energy state decreases at the end of optimization time horizon
- $c_{rrs}(t)$: Market clearing price for Responsive Reserve Service at time t ($/MW).
- $c_{ecrs}(t)$: Market clearing price for ECRS at time t ($/MW).
- $c_{ns}(t)$: Market clearing price for Non-Spin at time t ($/MW).

Remember, it is possible that $c_{rtm}(t) = c_{dam}(t)$.

Energy Dynamics

The state of charge evolves as:

$$E(t+1) = \gamma_s\, E(t) + (\eta_c\, P_+(t) - \frac{1}{\eta_d}\, P_-(t))\Delta t. \tag{6}$$

$$\begin{aligned} P_{net}(t) = &\big(P_{chg}(t) + w_{\text{down}}(t)\,\bar{P}_{down}(t) + R_{chg}(t)\big) \\ &- \big(P_{dis}(t) + w_{up}(t)\,\bar{P}_{up}(t) + R_{dis}(t) + w_{rrs}(t)\,\bar{P}_{rrs}(t) + w_{ecrs}(t)\,\bar{P}_{ecrs}(t) + w_{ns}(t)\,\bar{P}_{ns}(t)\big) \end{aligned} \tag{7}$$

$$P_+(t) = max(P_{net}(t), 0) \tag{8}$$

$$P_-(t) = max(-P_{net}(t), 0) \tag{9}$$

where:

- γ_s is the storage efficiency over one period, most times $\gamma_s = 1$
- η_c represents the efficiency of converting grid (or other source) power into stored energy inside the battery
- η_d represents how efficiently the stored energy in the battery can be extracted and delivered as usable power

Energy State Constraints

$$E_{\min} \leq E(t) \leq E_{\max}, \quad \forall t \tag{10}$$

where $E_{\min}$ is the minimum allowable energy level, $E_{\max}$ is the maximum allowable energy level.

Charging and Discharging Power Constraints

Since we consider a single time period, we assume that the battery cannot charge and discharge simultaneously in day ahead market. This exclusivity is enforced through the following complementarity constraint (This constraint is breaking convexity, we deal with it by import one more binary variable y and make it MILP):

$$P_{chg}(t) \cdot P_{dis}(t) = 0, \quad \forall t \tag{11}$$

This constraint also need to be applied to real time market behavior:

$$R_{chg}(t) \cdot R_{dis}(t) = 0, \quad \forall t \tag{12}$$

In addition, we impose upper bounds on the total charging and discharging power to reflect the battery's maximum power capabilities:

$$0 \leq P_{chg}(t) + \bar{P}_{down}(t) \cdot w_{down}(t) + R_{chg}(t) \leq P_{chg}^{\max}, \quad \forall t \tag{13}$$

$$0 \leq P_{dis}(t) + w_{up}(t)\,\bar{P}_{up}(t) + R_{dis}(t) + w_{rrs}(t)\,\bar{P}_{rrs}(t) + w_{ecrs}(t)\,\bar{P}_{ecrs}(t) + w_{ns}(t)\,\bar{P}_{ns}(t) \leq P_{dis}^{\max}, \quad \forall t \tag{14}$$

To satisfy the market rules, we also need to impose upper bounds on the day ahead bid behavior:

$$0 \leq P_{\mathrm{chg}}(t) + \bar{P}_{down}(t) \leq P_{\mathrm{chg}}^{\max}, \quad \forall t \tag{15}$$

$$0 \leq P_{dis}(t) + \bar{P}_{up}(t) + \bar{P}_{rrs}(t) + \bar{P}_{ecrs}(t) + \bar{P}_{ns}(t) \leq P_{dis}^{\max}, \quad \forall t \tag{16}$$

Charge Cycle Constraints

To preserve the health and warranty of the battery system, we impose a constraint limiting the battery to at most two full charge cycles per day. This requirement can be expressed as a constraint on the total energy charged during the day:

$$\sum_{t=0}^{T-1} P_{+}(t) \cdot \Delta t \;\leq\; 2\left(E_{\max} - E_{\min}\right) \tag{17}$$