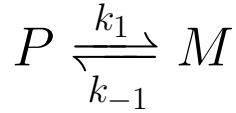


1 Difference in fecundity and mortality

We assume two populations of bacteria, having both logistic growth with different growth rates and carrying capacities. Individuals from any population can switch to the other population.

- $P(t)$ is the population of P (Pathogenic) bacteria at time t
- $M(t)$ is the population of M (Mutualistic) bacteria at time t
- f_P is the fecundity rate of P bacteria
- f_M is the fecundity rate of M bacteria. We have $f_M < f_P$
- μ_P is the mortality rate of P bacteria
- μ_M is the mortality of M bacteria. We have $\mu_M < \mu_P$
- K is the carrying capacity
- k_1 is the switching rate from P to M
- k_{-1} is the switching rate from M to P



The system of differential equation describing the system is:

$$\begin{cases} \frac{dP}{dt} = (f_P(1 - k_1)P + f_M k_{-1}M) \left(1 - \frac{\alpha P + M}{K}\right) - \mu_P P \end{cases} \quad (1a)$$

$$\begin{cases} \frac{dM}{dt} = (f_M(1 - k_{-1})M + f_P k_1 P) \left(1 - \frac{\alpha P + M}{K}\right) - \mu_M M \end{cases} \quad (1b)$$

$$\begin{cases} P(0) = P_0 \end{cases} \quad (1c)$$

$$\begin{cases} M(0) = 0. \end{cases} \quad (1d)$$

The stable state (equilibrium) can be analytically derived and we a simple formula for the ratio P over M at equilibrium (independent of K and α):

$$\frac{M_{eq}}{P_{eq}} = \Theta_2 - \Theta_1 + \sqrt{(\Theta_2 - \Theta_1)^2 + \Delta} \quad (2)$$

Where

$$\Theta_1 = \frac{f_P(1 - k_1)}{2f_M k_{-1}} \quad (3)$$

$$\Theta_2 = \frac{\mu_P(1 - k_{-1})}{2\mu_M k_{-1}} \quad (4)$$

$$\Delta = 4 \frac{k_1 f_P \mu_P}{k_{-1} f_M \mu_M} \quad (5)$$

also we have:

$$\frac{P_{eq}}{M_{eq}} = \frac{\Theta_1 - \Theta_2 + \sqrt{(\Theta_2 - \Theta_1)^2 + \Delta}}{\Delta} \quad (6)$$