

## Relationship between inertial particle divergence and local flow structure in homogeneous isotropic turbulence

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### Abstract:

The divergence and convergence behavior of inertial heavy particles in particle-laden homogeneous isotropic turbulence are investigated to get insight into the relationship between the particle clustering formation and the local flow structure in the turbulent flow. The divergence of particle velocity is quantified by applying a novel tessellation-based technique to direct numerical simulation data for several Stokes numbers. The results show that the particle clustering for small Stokes numbers well obeys the preferential concentration mechanism, whereas for larger Stokes numbers, the preferential concentration is less significant and effects of particle trajectory crossing is observed in cluster regions. Coherent clustering formation behavior will be discussed by filtering out the trajectory crossing effects.

**Keywords:** Inertial particle clustering, Homogeneous isotropic turbulence, Tessellation analysis

### Introduction:

Inertial heavy particles suspended in turbulent flows are frequently observed in environmental and industrial flows. Inertial particles show nonuniform spatial distribution, referred to as clustering, in turbulent flows due to deviation of the inertial particle trajectory from the fluid particle trajectory. To understand the clustering formation process, the divergence of particle velocity is an important quantity. Recently, Oujia et al. [1] proposed a tessellation-based technique to compute the divergence directly based on position and velocity data of discrete particles, and Maurel–Oujia et al. [2] improved the accuracy of the technique. In this work, the novel tessellation-based technique is applied to Lagrangian inertial particle data obtained from direct numerical simulation (DNS) to get insight into the relationship between the particle clustering formation and the local flow structure in the turbulent flow.

### Computational Methods:

Three-dimensional DNS of particle-laden homogeneous isotropic turbulence is performed to obtain the particle position and velocity data. The fluid flow is governed by the incompressible Navier–Stokes equations and solved in the Eulerian framework. The particles are assumed to be small, heavy and spherical Stokes particles and tracked in the Lagrangian framework. Please see Ref. [3] for detail. The number of grid points is  $512^3$ , and the number of particles is  $1.07 \times 10^9$ . The Taylor-microscale Reynolds number of the obtained turbulent flow is  $Re_\lambda = 204$ . The particle inertia is represented by the Stokes number defined as  $St = \tau_p/\tau_\eta$ , where  $\tau_\eta$  is the Kolmogorov time. To calculate the particle velocity divergence  $\nabla \cdot \mathbf{v}$  based on the discrete particle data, we apply the novel tessellation-based technique [1, 2]. When we apply the Voronoi tessellation, each cell segmented by the Voronoi diagram has volume  $V_p$  that approximately corresponds to the inverse of the local number density. Therefore, the divergence  $\nabla \cdot \mathbf{v}$  can be approximately determined by  $\nabla \cdot \mathbf{v} \approx D(\mathbf{x}_p) = (1/V_p)(DV_p/Dt)$ .

### Results and discussion:

We examine the validity of the preferential concentration mechanism based on Maxey’s approximation [4], in which the particle velocity divergence is given by  $\nabla \cdot \mathbf{v} = 2\tau_p Q + \mathcal{O}(\tau_p^2)$ , where  $Q$  is the second invariant

of velocity gradient tensor of the fluid flow field. This approximation is valid for  $St \ll 1$ . Figure 1 shows the spatial distributions of  $Q$  and the particle velocity divergence  $D$ . The distribution of  $D$  for  $St = 0.05$  is similar to that of  $Q$ , indicating the validity of Maxey's approximation. For  $St = 1.0$ , the similarity is weaker than  $St = 0.05$ , indicating less significance of the preferential concentration. We can also observe some cluster regions where both positive and negative divergence coexist. This suggests the crossing of particle trajectories due to the coexistence of particle groups with different particle velocities, which is also known as "caustics". The divergence values due to the caustics would not contribute to the clustering formation. Coherent clustering formation behavior can be extracted by filtering out the trajectory crossing effects.

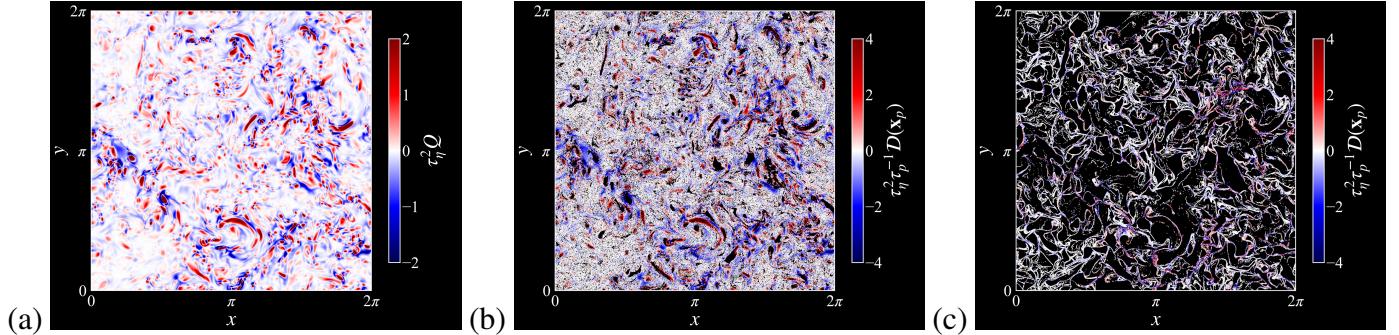


Figure 1: Spatial distribution of (a)  $Q$  and the inertial particles colored by  $D$  for (b)  $St = 0.05$  and (b)  $St = 1.0$ .

## Conclusion:

The divergence of velocity of inertial heavy particles in homogeneous isotropic turbulence has been quantified by applying the novel tessellation-based technique to DNS data to get insight into the particle clustering formation process. The results show that the preferential concentration mechanism becomes less significant as the Stokes number increases. For larger Stokes numbers, coexistence of significant particle divergence and convergence is observed in cluster regions, indicating crossing of particle trajectories. Further analyses on the relationship between the coherent clustering formation and the local flow structure will be presented.

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