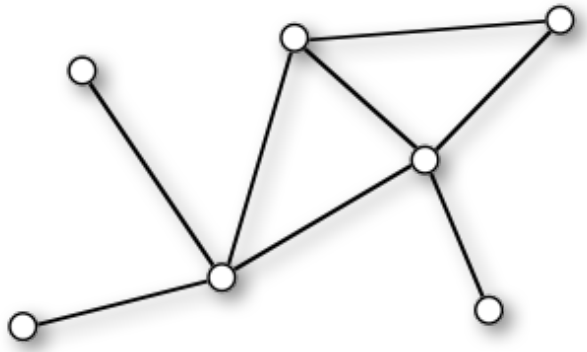
Some Graph Basics

Network <=> Graph

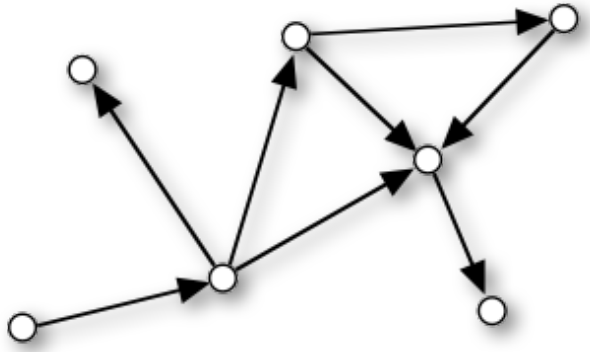
Formal **definition**:

A **graph** *G* is an ordered pair (*V*, *E*) of a set *V* of **vertices** and a set *E* of **edges**.

$$G = (V, E)$$



undirected graph



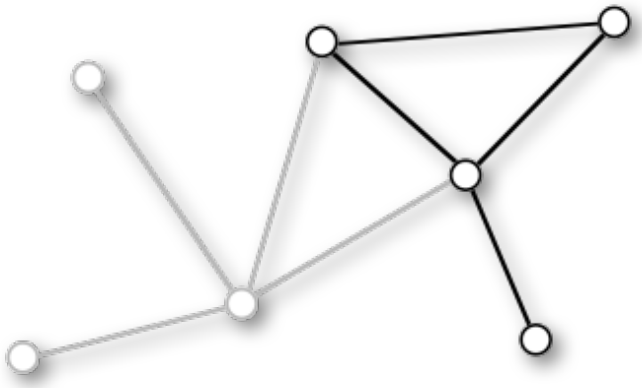
directed graph

If $E = V^{(2)}$ => fully connected graph

Graph Basics II

Subgraph:

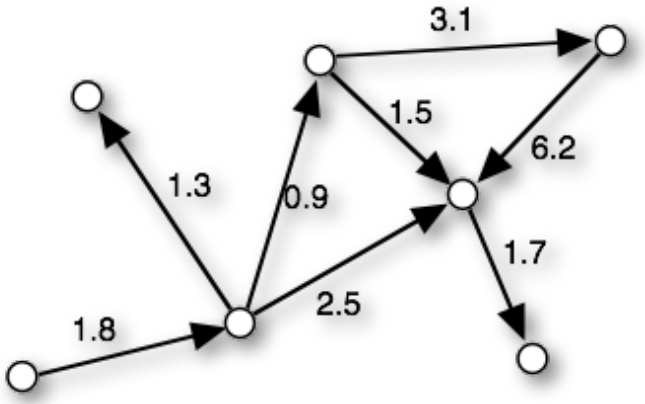
$G' = (V', E')$ is a subset of $G = (V, E)$



Practical question: how to define useful subgraphs?

Weighted graph:

Weights assigned to the edges



Note: no weights for vertices

Walk the Graph

Path = sequence of connected vertices

start vertex => internal vertices => end vertex

Two paths are **independent** (internally vertex-disjoint),

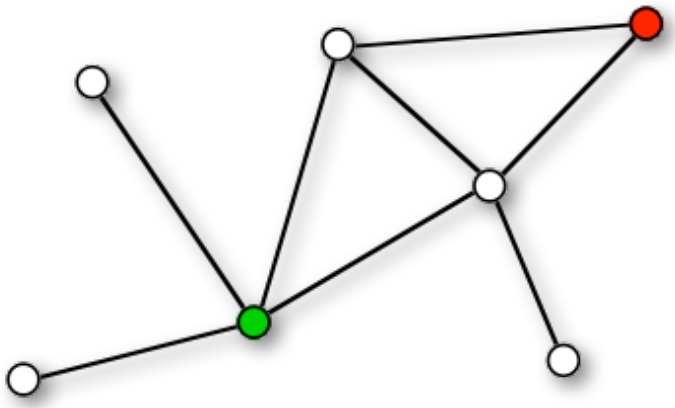
if they have no internal vertices in common.

Vertices *u* and *v* are **connected**, if there exists a path from *u* to *v*.

otherwise: disconnected

Trail = path, in which all edges are distinct

Length of a path = number of vertices || sum of the edge weights



How many paths connect the green to the red vertex?

How long are the shortest paths?

Find the four trails from the green to the red vertex.

How many of them are independent?

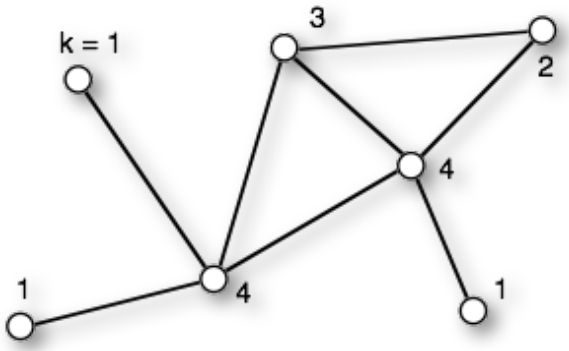
Local Connectivity: Degree/Degree Distribution

Degree *k* of a vertex = number of edges at this vertex

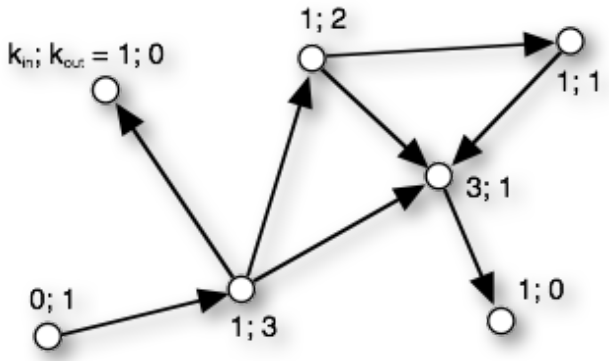
Directed graph => distinguish k_{in} and k_{out}

Degree distribution *P(k)* = fraction of nodes with *k* connections

$$P(k) = \frac{n_k}{N}$$



k	0	1	2	3	4
P(k)	0	3/7	1/7	1/7	2/7



k	0	1	2	3
$P(k_{in})$	1/7	5/7	0	1/7
$P(k_{out})$	2/7	3/7	1/7	1/7

Graph Representation e.g. by adjacency matrix

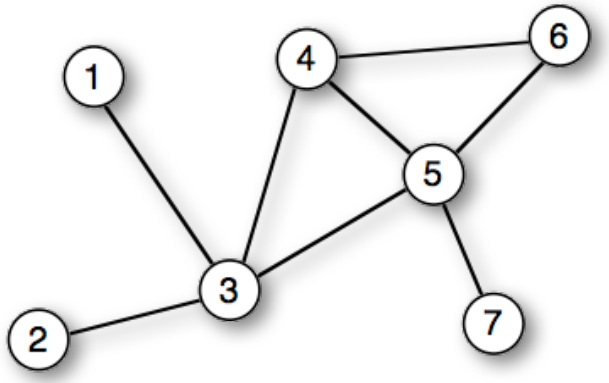
Adjacency matrix is a N x N matrix
with entries M_{uv}

M_{uv} = weight when edge between u and v exists,
0 otherwise

→ symmetric for undirected graphs

+ fast *O(1)* lookup of edges
– large memory requirements
– adding or removing nodes is expensive

Note: very convenient in programming languages that support sparse multi-dimensional arrays
=> Perl



	1	2	3	4	5	6	7
1	–	0	1	0	0	0	0
2	0	–	1	0	0	0	0
3	1	1	–	1	1	0	0
4	0	0	1	–	1	1	0
5	0	0	1	1	–	1	1
6	0	0	0	1	1	–	0
7	0	0	0	0	1	0	–