## Bioinformatics III

## Third Assignment

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## Exercise 3.1:

(a) Given the states of the features, you want to infer if two proteins are likely to physically interact. In practice, log-likelihood ratios are used in binary classification:

$$log \frac{P(C|S)}{P(\bar{C}|S)}$$

Derive a term that uses observable probabilities such as  $P(S_i|C)$  to calculate the loglikelihood ratio from training data. How does the actual classification work?

First we have:

$$P(S_i|C) = \frac{P(C|S_i)P(S_i)}{P(C)}$$

And:

$$P(S_i|\bar{C}) = \frac{P(\bar{C}|S_i)P(S_i)}{P(\bar{C})}$$

Then we develop the desired final output

$$\frac{P(S_i|C)}{P(S_i|\bar{C})} \Longleftrightarrow \frac{P(S_i|C)P(C)}{P(S_i|\bar{C})P(\bar{C})} = \frac{P(C|S_i)P(S_i)}{P(\bar{C}|S_i)P(S_i)} = \frac{P(C|S_i)}{P(\bar{C}|S_i)}$$

$$\log \frac{P(C|S)}{P(\bar{C}|S)} = \log \prod_{i}^{n} \frac{P(S_i|C)P(C)}{P(S_i|\bar{C})P(\bar{C})} = \sum_{i}^{n} \log \frac{P(S_i|C)P(C)}{P(S_i|\bar{C})P(\bar{C})} = \Lambda(C|S)$$

The posterior odd is calculated by the odds of an event  $(\frac{p(event)}{1-p(event)})$  multiplied by the likelihood of that event<sup>1</sup>.

 $O(C|S) = \Lambda(C|S)O(C)$ 

To do the classification, we must interate through the data and calculate all the priors and likelihood. The prior P(C) is made from an educated guess

 $<sup>^1</sup>$ Slides V4 - 4

(b) Shortly discuss: What are the practical advantages of the logarithm and the likelihood ratio within this framework? State two reasons why this particular type of classifier may perform poorly on a real world dataset.

The logarithm increase is a monotonically increasing function of x hence, for any positive value the maximum value of a function f(x), the maximum of f(x) is equal to the maximum of log(f(x)). This simplifies the calculation because we don't need the second derivative. A likelihood function is not concave but the log-likelihood is. Also, as seen in part A, with the log-likelihood we can turn a log of products into a sum of logs. The main inconvenient is that this method assume that all the features are independent and do not take in account the eventual correlations between them.

(c) Use the file training1.tsv to build a model. This basically means to determine all necessary priors and likelihoods from part (a). The file layout is explained in README.txt. Report P(C) and P(C

) as well as the ten S<sub>i</sub> (feature number, variant and log-ratio) with the highest absolute log-likelihood ratios. Examine and comment on the results of the training-phase. Which features seem to be the most helpful?

```
Prior probability P(C) = 0.78
Prior probability P(\bar{C}) = 0.22
```

```
Table 1: 10 S_i with the highest absolute log-likelihood ratio
                      -3.7214026458194964
              33
                  0
              11
                  3
                      -2.565631943311438
              87
                  1
                      -2.4686396773241284
              53
                      -2.3351082846996056
                  1
              99
                  1
                      -2.3061207478263537
              59
                  1
                      -2.2779498708596573
                  2
              80
                      -2.2779498708596573
              86
                  3
                      -2.2550655770260692
                  3
              91
                      -2.2173252490432223
              97
                  1
                      -2.2099451417455995
```

Listing 1: bayes.py

```
o import math
  import copy
  # For all features, compute the probability (prior) to have 0, 1, 2 or 3
       depending on the output (0 or 1)
  #
  #
  def priors(features, output_indexes):
       priors = \{\}
       # Start to 1 to match the instructions
       feature_nb = 1
10
       for feature in features:
           P_Si_Output = \{\}
           # Values of the feature for a certain output (0 or 1)
S = [feature[i] for i in output_indexes]
           # for all possible feature values \rightarrow [0,1,2,3], set dynamically here
           for value in set (feature):
15
                # Prob of having this 'value' when output is 0 or 1 (depend on
                    output_indexes)
                P_Si_Output [value] = S.count(value) / float(len(S))
            priors [feature_nb] = P_Si_Output
20
           feature_nb += 1
       return priors
  def log_likelihood(Prior_C, Prior_not_C, P_S_C, P_S_notC):
       log_like = [[0.0]*4 \text{ for } lin \text{ range}(len(P_S_C))]
       #For each feature
       # Careful, in P_S_C it's a dict -> start at 1 as "feature 1"
       # in log_like it's a list of list -> feature 1 == [0]
       for feat in P_S_C:
30
           for val in [0,1,2,3]:
                p = math.log((P_S_C[feat][val] * Prior_C)/P_S_notC[feat][val] *
                    Prior_not_C)
                \log_{-1} \text{like} [\text{feat} - 1][\text{val}] = p
```

```
return log_like
  \mathbf{def} \ \ \mathbf{getNMaxLikelihoodRatio} \ (\ \mathbf{likelihoods} \ \ , \ \ \mathbf{N}) :
       \# As we have to loop N times, we'll need to set the max value to zero
       # in order not to pick it more than once.
       likelihoods_copy = copy.deepcopy(likelihoods)
       t\ =\ [\,]
40
       for out in range(N):
           # Will contain (feature number, variant, absolute likelihood ratio)
           info = (0,0,0)
           max = 0
45
           for feat in range(len(likelihoods_copy)):
                for val in range(len(likelihoods_copy[feat])):
                     if abs(likelihoods_copy[feat][val]) > max:
                         max = abs(likelihoods\_copy[feat][val])
                         # Max is calculated with the abs, but the real value is
50
                             stored
                         info = (feat, val, likelihoods_copy[feat][val])
                         likelihoods\_copy[feat][val] = 0.0
           t.append(info)
55
       return t
60 # Read data file
  lines = []
  with open("data/training2.tsv") as f:
       for line in f:
           line = line.split(' \ ')
65
           map(str.strip, line)
lines.append(line)
  # Convert all the elements in float instead of chars
70 # for line in lines:
  # line = list (map(float, line))
lines = [[float(i) for i in line] for line in lines]
  # Number of features
nb_{\text{features}} = len(lines[0]) - 1
  print("Nb_features:_", nb_features)
  \# Get the data by columns: https://stackoverflow.com/questions/44360162/how-to
       -access-a-column-in-a-list-of-lists-in-python
  data_columns = list(zip(*lines))
80 # Problem, columns are now tuples
  data_columns = [list(elem) for elem in data_columns]
  # Features only
  features = data_columns[1:]
  # Output only
  outputs = list (data_columns[0])
  # Indexes according to outputs (1 or 0, first column)
90 interaction_indexes = [i for i,x in enumerate(outputs) if x == 1] #print("interact index, ", interaction_indexes)
   no_interaction_indexes = [i for i,x in enumerate(outputs) if x == 0]
  #print("no-interact index: ", no-interaction-indexes)
  # Prior probabilities
  Prior_C = outputs.count(1) / float(len(outputs))
```

```
print("Prior_probability_of_having_a_connection:_", Prior_C)
100 Prior_not_C = 1 - Prior_C
   print("Prior_probability_of_not_having_a_connection:_", Prior_not_C)
   # For each feature and possible value, calculate the probability according to
       the\ output
105
   \# P_-S_-C = Probability of having S (feature) according to output 1
   P_S_C = priors (features, interaction_indexes)
   \# P_-S_-notC = Probability of having S (feature) according to output 0
110 P_S_notC = priors(features, no_interaction_indexes)
   # Print every probabilities for every feature's values
   # print("Features's values's probabilities if connection: \n")
   # for p in P_-S_-C:
         print ("Feature", p, ":")
115 #
         for val in P_S_C[p]:

print("\tValue: ", val, " prob: ", P_S_C[p][val])
   #
   #
   # print("Features's values's probabilities if no connection: \n")
^{\prime\prime\prime}_{120} # for p in P_S_notC:
# print("Feature", p, ":")
         for val in P_-S_-notC[p]:
   #
              print("\tValue:", val, "prob:", P_S_notC[p][val])
   #
125
   # Now we compute the log likelihood for every features and possible output
   log_like = log_likelihood(Prior_C, Prior_not_C, P_S_C, P_S_notC)
130 #print(log_like)
   \# Get the N (ABSOLUTE) max log-likelihood ratios.
   maxLikelihoods = getNMaxLikelihoodRatio(log_like, 10)
   # Nice printing
   for _ in maxLikelihoods:
       \mathbf{print}(_{-})
```

## Exercise 3.2: Classify real-world network examples

- (a)
- (b)
- (c)