

# Bioinformatics III

## Sixth Assignment

Thibault Schowing (2571837)

Wiebke Schmitt (2543675)

May 30, 2018

### Exercise 6.1: Boolean Networks

All the listings are at the end of the exercise.

#### (a) Weighted Interactions

Table 1: Propagation Matrix

	F	E	D	C	B	A
F	0	0	1	0	0	0
E	1	0	0	0	0	0
D	-3	-3	0	0	-3	0
C	0	1	0	0	1	0
B	0	0	0	1	0	0
A	0	0	0	0	1	1

#### (b) Implementation

*When does it make sense to stop the propagation and why?*

We stop the propagation once we meet an already visited state. If we continue, we will just loop over and over again.

*Which sequences do you get when you start from states 1, 4, 21, and 33?*

Sequence with starting state 1: [1, 3, 7, 23, 55, 63, 13, 1]

Sequence with starting state 4: [4, 18, 36, 26, 4]

Sequence with starting state 21: [21, 51, 47, 13, 1, 3, 7, 23, 55, 63, 13]

Sequence with starting state 33: [33, 11, 5, 19, 39, 31, 5]

(c) **Periodic Orbits**

*(1) List these orbits with their respective lengths and basins of attraction*

To make things clearer let's recall the different definition. "If the attractor has only a single state it is called a point attractor, and if the attractor consists of more than one state it is called a cycle attractor. The set of states that lead to an attractor is called the basin of the attractor. States which occur only at the beginning of trajectories (no trajectories lead to them), are called garden-of-Eden states" <sup>1</sup>

---

<sup>1</sup>[https://en.wikipedia.org/wiki/Boolean\\_network#Attractors](https://en.wikipedia.org/wiki/Boolean_network#Attractors)

Table 2: List of states, orbith length, Cycle attractor and relative coverage of the basin of attraction. The basin of attraction's coverage includes the steps before the cycle attractor.

Start State	Period	Basin	Attractor	Basin Coverage
0	1	[]	[0]	1.5625%
1	7	[]	[1, 3, 7, 23, 55, 63, 13]	10.9375%
2	4	[2]	[4, 18, 36, 26]	7.8125%
3	7	[]	[3, 7, 23, 55, 63, 13, 1]	10.9375%
4	4	[]	[4, 18, 36, 26]	6.25%
5	4	[]	[5, 19, 39, 31]	6.25%
6	1	[6, 22, 54, 62, 12]	[0]	9.375%
7	7	[]	[7, 23, 55, 63, 13, 1, 3]	10.9375%
8	1	[8]	[0]	3.125%
9	7	[9]	[1, 3, 7, 23, 55, 63, 13]	12.5%
10	4	[10]	[4, 18, 36, 26]	7.8125%
11	4	[11]	[5, 19, 39, 31]	7.8125%
12	1	[12]	[0]	3.125%
13	7	[]	[13, 1, 3, 7, 23, 55, 63]	10.9375%
14	4	[14]	[4, 18, 36, 26]	7.8125%
15	4	[15]	[5, 19, 39, 31]	7.8125%
16	1	[16, 32, 8]	[0]	6.25%
17	4	[17, 35, 15]	[5, 19, 39, 31]	10.9375%
18	4	[]	[18, 36, 26, 4]	6.25%
19	4	[]	[19, 39, 31, 5]	6.25%
20	1	[20, 50, 44, 8]	[0]	7.8125%
21	7	[21, 51, 47]	[13, 1, 3, 7, 23, 55, 63]	15.625%
22	1	[22, 54, 62, 12]	[0]	7.8125%
23	7	[]	[23, 55, 63, 13, 1, 3, 7]	10.9375%
24	1	[24]	[0]	3.125%
25	7	[25]	[1, 3, 7, 23, 55, 63, 13]	12.5%
26	4	[]	[26, 4, 18, 36]	6.25%
27	4	[27]	[5, 19, 39, 31]	7.8125%
28	1	[28]	[0]	3.125%
29	7	[29]	[1, 3, 7, 23, 55, 63, 13]	12.5%
30	4	[30]	[4, 18, 36, 26]	7.8125%
31	4	[]	[31, 5, 19, 39]	6.25%
32	1	[32, 8]	[0]	4.6875%
33	4	[33, 11]	[5, 19, 39, 31]	9.375%
34	1	[34, 12]	[0]	4.6875%
35	4	[35, 15]	[5, 19, 39, 31]	9.375%
36	4	[]	[36, 26, 4, 18]	6.25%
37	4	[37, 27]	[5, 19, 39, 31]	9.375%
38	4	[38, 30]	[4, 18, 36, 26]	9.375%
39	4	[]	[39, 31, 5, 19]	6.25%
40	1	[40, 8]	[0]	4.6875%

Start State	Period	Basin	Attractor	Basin Coverage
41	7	[41, 9]	[1, 3, 7, 23, 55, 63, 13]	14.0625%
42	1	[42, 12]	[0]	4.6875%
43	7	[43]	[13, 1, 3, 7, 23, 55, 63]	12.5%
44	1	[44, 8]	[0]	4.6875%
45	7	[45, 9]	[1, 3, 7, 23, 55, 63, 13]	14.0625%
46	1	[46, 12]	[0]	4.6875%
47	7	[47]	[13, 1, 3, 7, 23, 55, 63]	12.5%
48	1	[48, 40, 8]	[0]	6.25%
49	7	[49, 43]	[13, 1, 3, 7, 23, 55, 63]	14.0625%
50	1	[50, 44, 8]	[0]	6.25%
51	7	[51, 47]	[13, 1, 3, 7, 23, 55, 63]	14.0625%
52	1	[52, 58, 12]	[0]	6.25%
53	7	[53, 59]	[13, 1, 3, 7, 23, 55, 63]	14.0625%
54	1	[54, 62, 12]	[0]	6.25%
55	7	[]	[55, 63, 13, 1, 3, 7, 23]	10.9375%
56	1	[56, 8]	[0]	4.6875%
57	7	[57, 9]	[1, 3, 7, 23, 55, 63, 13]	14.0625%
58	1	[58, 12]	[0]	4.6875%
59	7	[59]	[13, 1, 3, 7, 23, 55, 63]	12.5%
60	1	[60, 8]	[0]	4.6875%
61	7	[61, 9]	[1, 3, 7, 23, 55, 63, 13]	14.0625%
62	1	[62, 12]	[0]	4.6875%
63	7	[]	[63, 13, 1, 3, 7, 23, 55]	10.9375%

(2) Give the relative coverages of the state space by the basins of attraction.

The coverages for each separate basins of attraction + cycle attractor are given in the table 2. In table 3 we give the coverage of each cycle attractor.

Table 3: Relative coverage of the cycle attractors. Details of the basins in table 4.

[0] :	23	35.9375 %
[1, 3, 7, 23, 55, 63, 13] :	21	32.8125 %
[4, 18, 36, 26] :	9	14.0625 %
[19, 39, 31, 5] :	11	17.1875 %

Table 4: State space occupation of the basins of attraction. Details of the basins leading to attractor. Here we included the attractor in the basin.

[0] :	[0, 6, 8, 12, 16, 20, 22, 24, 28, 32, 34, 40, 42, 44, 46, 48, 50, 52, 54, 56, 58, 60, 62]	35.9375 %
[1, 3, 7, 23, 55, 63, 13] :	[[1, 3, 7, 9, 13, 21, 23, 25, 29, 41, 43, 45, 47, 49, 51, 53, 55, 57, 59, 61, 63]]	32.8125 %
[4, 18, 36, 26] :	[2, 4, 36, 38, 10, 14, 18, 26, 30]	14.0625 %
[19, 39, 31, 5] :	[33, 35, 5, 37, 39, 11, 15, 17, 19, 27, 31]	17.1875 %

(d) **Interpretation**

*(1) Give the attractors in terms of active genes and characterize them with a few words*

In the listing 1 the binary transitions are presented for each attractors.

**Attractor [0]** : No gene are activated in this attractor and there is only one period. We can see that the states leading to this attractor are the one when **D** is activated and shuts down genes **B**, **E** and **F** even if **C** is activated, in those case it does not get activated again by **B** as in attractor [4, 18, 36, 26]. Here **A** is not activated and there is no cycle between **B** and **C**.

**Attractor [1, 3, 7, 23, 55, 63, 13]** : Here gene **A** is activated and keeps activating **B**. Thus, whenever **D** is activated and shuts down genes **B**, **E** and **F**, **B** is reactivated again by **A**.

**Attractor [4, 18, 36, 26]** : This cycle happens when gene **B** and **C** are not active at the same time and keep activating each other one step after another.

**Attractor [5, 19, 39, 31]** : This attractor is the opposite of attractor [1, 3, 7, 23, 55, 63, 13] for gene **B**, **C** and **D**. Gene **A** is also always activated but the fact that **C** is activated in the first place, shifts the activation of **D** earlier and avoids the total activation before **D** inhibits genes **B**, **E** and **F**.

Listing 1: Output - Binary evolution in the orbits and percentages

```

0 Current orbit: [0]
  Binary evolution:
  [0, 0, 0, 0, 0, 0]
  Average occupancy:
  [0.0, 0.0, 0.0, 0.0, 0.0, 0.0]
5
  Current orbit: [1, 3, 7, 23, 55, 63, 13]
  Binary evolution:
  [0, 0, 0, 0, 0, 1]
  [0, 0, 0, 0, 1, 1]
10 [0, 0, 0, 1, 1, 1]
  [0, 1, 0, 1, 1, 1]
  [1, 1, 0, 1, 1, 1]
  [1, 1, 1, 1, 1, 1]
  [0, 0, 1, 1, 0, 1]
15 Average occupancy:
  [0.2857142857142857, 0.42857142857142855, 0.2857142857142857,
    0.7142857142857143, 0.7142857142857143, 1.0]

  Current orbit: [4, 18, 36, 26]
  Binary evolution:
20 [0, 0, 0, 1, 0, 0]
  [0, 1, 0, 0, 1, 0]
  [1, 0, 0, 1, 0, 0]
  [0, 1, 1, 0, 1, 0]
  Average occupancy:
25 [0.25, 0.5, 0.25, 0.5, 0.5, 0.0]

  Current orbit: [5, 19, 39, 31]
  Binary evolution:
  [0, 0, 0, 1, 0, 1]
30 [0, 1, 0, 0, 1, 1]
  [1, 0, 0, 1, 1, 1]
  [0, 1, 1, 1, 1, 1]
  Average occupancy:
  [0.25, 0.5, 0.25, 0.75, 0.75, 1.0]

```

*(2) Which are the special genes and what are their respective effects on the behavior of the network? For this, explain what is determining the period of the orbits. Further, compare the two shorter orbits with each other. Which gene is responsible for the difference?*

Without genes **A** and **C**, the network shuts down. **A** is particular as it is not inhibited by **D** like the others and because it activates itself. As **A** cannot be activated by any other gene, it leads to odd states, where **A** is active or even states where it is not. It cannot be activated in the middle of a sequence.

**D** is particular because it deactivates **B**, **E** and **F** what has a big effect on the network. **B** and **C** can propagate the activation to the network but they have to be synchronized correctly. If for instance **C** and **D** are activated, the network will be shut down.

Listing 2: boolean\_network.py

```
0 import copy
  import collections

  class BooleanNetwork:
      def __init__(self, matrix):
9          # Propagation Matrix
            self.matrix = matrix

            # Number of genes/nodes
            self.nb_bits = len(matrix)

10         # Number of possible states (in our example 2**6 = 64)
            self.nb_states = int(2**self.nb_bits)

            # For each gene, we can set a threshold. Here we assume it's 0 for all of
              them
15         self.threshold = [0] * self.nb_bits

      def int_to_binary(self, n):
          """
20         Convert an integer to binary
          /\ Uses the variable: SELF.NB_BITS !

          :Source: https://stackoverflow.com/questions/10411085/converting-integer-to-binary-in-python
          :param n: Integer
25         :return: map object -> use list comprehension to print/loop in etc
          """
            ret = bin(n)[2:].zfill(self.nb_bits)
            return list(map(int, format(ret)))

30     def binary_to_int(self, n):
          """
          Converts binary number (list) to Integer
          :param n: List of bits
          :return: the list converted to int
35         """
            ret = ""
            for bit in n:
                ret += str(bit)
            ret_int = int(ret, 2)
40         return ret_int

      def get_states_sequence(self, start_state):
          """
45         From a passed starting state (INT), calculates all the following states and
            stops when a loop is detected.
          :param start_state: INT - number of the starting state
          :return: The sequence of states,
          """

50         binary_current_state = self.int_to_binary(start_state)

            binary_next_state = [0] * self.nb_bits
            states_sequence = [start_state]

55         state_not_visited = True

            # While the state haven't already been visited...(while not orbit)
            while state_not_visited:

60                 for i in range(0, self.nb_bits):
                    sum_next_state = 0
                    for j in range(0, self.nb_bits):
```

```

        # print("Fucking i: ", i, " and j: ", j)
        # print(binary_current_state[j] * self.matrix[j][i])
65      sum_next_state += binary_current_state[j] * self.matrix[j][i]

        # Now we can set the next state for this gene
        if sum_next_state > self.threshold[i]:
            binary_next_state[i] = 1
70      else:
            binary_next_state[i] = 0

        # If the next state doesn't already exist, we add it to the states list
        # and continue
        # If not, we add it and end the while loop
75      #print("CURRENT STATE: ", binary_next_state)
        if self.binary_to_int(binary_next_state) not in states_sequence:
            states_sequence.append(self.binary_to_int(binary_next_state))
        else:
80      states_sequence.append(self.binary_to_int(binary_next_state))
            state_not_visited = False

        # The state now (starting state) is the next state
85      binary_current_state = copy.copy(binary_next_state)

    return states_sequence

def orbit(self, start_state):
90    """
        :param start_state:
        :return: see comments below
        """
95    # Sequence from start state -> stabilization
        sequence = self.get_states_sequence(start_state)

        # At which state it closes its orbit
        closure = sequence[-1]
100    # Basin = states leading to periodic orbits (cyclic attractor)
        partial_basin = sequence[0:sequence.index(closure)]

        # Periodic orbit (cyclic attractor) of the sequence
105    periodic_orbit = sequence[sequence.index(closure):len(sequence) - 1]

        # Size of the orbit
        orbit_size = len(sequence) - sequence.index(closure) - 1
110    return orbit_size, periodic_orbit, closure, partial_basin

def count_attractors(self):
    """
115    # Returns a counter containing the sets of attractors and their occurrence
    # In our case:
    # Counter({frozenset({0}): 23,
    #          frozenset({1, 3, 7, 13, 55, 23, 63}): 21,
    #          frozenset({39, 19, 5, 31}): 11,
120    #          frozenset({18, 26, 4, 36}): 9})
    #
    # Notice that unfortunately here the order of the attractor is not
    # maintained
    # Source: https://stackoverflow.com/questions/37295981/python-creating-a-
    #          dictionary-with-key-as-a-set-and-value-as-its-count
    #
125    :return:

```



```
"""
all_attractions_list = [sorted(self.orbit(i)[1]) for i in range(self.
    nb_states)]
attractions_count = collections.Counter(frozenset(x) for x in
    all_attractions_list)
130
return attractions_count

def average_occupancies_in_orbit(self, orbit, print_steps = None):
135
    """
    Count occurrence of ABCDEF activated in each states
    :param orbit:
    :return: list of percentages like [0.25, 0.5, 0.25, 0.5, 0.5, 0.0]
    """
140
    total_bits = [0 for _ in range(0, self.nb_bits)]
    percentages = [0.0 for _ in range(0, self.nb_bits)]

    # For every state, convert to binary and increment the occupancy
    for state in orbit:
145
        bin = self.int_to_binary(state)

        # if the optional parameter is true, print the binary states
        # allow to see more clearly the evolution of the different gene
        # activation
        if print_steps:
150
            print(bin)
        #Increment here
        for i in range(0, len(bin)):
            total_bits[i] += bin[i]
        # Calculate percentages
        total_elem = len(orbit)
155
        for i in range(0, self.nb_bits):
            percentages[i] = total_bits[i] / total_elem

    return percentages
160

def get_basin_of_attraction(self):
    """
165
    :return:
    """

    sequence_set = collections.defaultdict(list)
170

    for i in range(0, self.nb_states):
        orbit = self.orbit(i)
        unique_orbit_string = str(sorted(orbit[1]))
        # We have the "previous steps" and the cyclic attractor -> add the
        # previous steps to the set corresponding to the attractor to have
        # the full basin
175
        sequence_set[unique_orbit_string].extend(orbit[3])
        # To have the whole basin, we add also the cycle
        sequence_set[unique_orbit_string].extend(orbit[1])

    for e in sequence_set:
180
        sequence_set[e] = list(set(sequence_set[e]))

    return sequence_set
```

### Exercise 6.2: Differential Expression Analysis

(a) **A**

(b) **B**