Quiz Questions - week 4 Applied Biostatistics I Exercises, HS 2019

1) If events A and B are independent, are the following events (a, b, c) dependent or independent
   a) $A, \overline{B}$
   b) $\bar{A}, B$
   c) $\bar{A}, \bar{B}$

   Solution:
   a) From the Low of total probability, we have: $A = A\bar{B} + AB$ and as corollary $P(A\bar{B}) = P(A) - P(AB)$, using the independence of A and B we have $P(AB) = P(A)P(B)$
   As a result, $P(A\bar{B}) = P(A) - P(A)P(B) = P(A)(1 - P(B)) = P(A)P(\bar{B})$
   which by definition means A and $\bar{B}$ are independent.
   b) In the same way can be proved that $\bar{A}$ and B are independent
   c) In similar way from the Low of total probability, we have: $\bar{B} = A\bar{B} + \bar{A}\bar{B}$ and as corollary $P(\bar{A}\bar{B}) = P(\bar{B}) - P(A\bar{B})$
   using the result from a) $P(\bar{A}\bar{B}) = P(\bar{B}) - P(A)P(\bar{B}) = P(\bar{B})(1 - P(A)) = P(\bar{B})P(\bar{A})$
   which again by definition means $\bar{A}$ and $\bar{B}$ are independent.

2) You have a DNA polymer with length L that can break with the same probability at any point.
   A: event of a break where the small part is less than ⅓ ∗ L:
   What is the probability $P(A)$ of event A? consider the continuous case

   Solution:
   Using Geometric probability based on length
   $P(break\ point < position(⅓ * L)) = (⅓ * L) / L = 1/3$
   $P(A) = P(break\ point < position(⅓ * L)) + P(break\ point > position(⅔ * L))$
   $= ⅓ + ⅓ = ⅔$
   For the discrete case where the length of the polymer is given in nucleotide base-pairs (N) and the break can happen only at a phosphodiester bond, the probability has to be corrected:
   $P(A) = P\big(breakpoint < floor(⅓ * N)\big) + P\big(breakpoint > floor(⅔ * N)\big)$
   $P(A) = floor(⅓ * N) / N + floor(⅓ * N) / N$ which for big enough N and considering a certain precision (as always in computational calculations) will be computationally equal to 2/3.

   * Function $floor(x)$ (also in R) rounds to the nearest integer that's smaller than x

3) Freddy and Johnny are friends. Friday evening Freddy wants to meet Johnny (no mobile phones: battery empty). He knows he is with probability ⅔ not at home ($P(Pub) = ⅔$, $P(Home) = ⅓$) and he could be in one of the two famous pubs in the town A and B with equal probability ($P(A|Pub) = P(B|Pub) = 1/2$). He already checked pub B and he was not there.
   What is the probability $P(A|\bar{B})$ he is in pub A?

   Solution: coming soon… will be updated next week