Series 4: Lotka-Volterra model for competition

In this series, we will study the classical Lotka-Volterra model for competition between two species. The abundances of the two species are given by N_1 and N_2. The model is given by the following set of differential equation.

$$\begin{cases} \dfrac{dN_1}{dt} = N_1 \cdot (r_1 - \alpha_{11} \cdot N_1 - \alpha_{12} \cdot N_2) \\ \dfrac{dN_2}{dt} = N_2 \cdot (r_2 - \alpha_{21} \cdot N_1 - \alpha_{22} \cdot N_2) \end{cases}$$

The parameters of the model are $r_i > 0$ the intrinsic growth rates and $\alpha_{ij} > 0$ the competition strength of species *j* on species *i* coefficients.

For the following five sets of parameters, draw the phase space, the non-trivial zero growth isoclines, and the vector field. You will have to determine the equations for the non-trivial zero growth isoclines (hint: they are lines).

1) $\vec{r} = \begin{bmatrix} 2 \\ 2.5 \end{bmatrix}, \quad \alpha = \begin{pmatrix} 1 & 0.5 \\ 0.5 & 1 \end{pmatrix}, \quad \overrightarrow{N}_0 = \begin{bmatrix} 2.5 \\ 0.5 \end{bmatrix}$

2) $\vec{r} = \begin{bmatrix} 3.5 \\ 1.5 \end{bmatrix}, \quad \alpha = \begin{pmatrix} 1 & 0.5 \\ 0.5 & 1 \end{pmatrix}, \quad \overrightarrow{N}_0 = \begin{bmatrix} 0.2 \\ 3 \end{bmatrix}$

3) $\vec{r} = \begin{bmatrix} 1 \\ 3 \end{bmatrix}, \quad \alpha = \begin{pmatrix} 1 & 0.5 \\ 0.5 & 1 \end{pmatrix}, \quad \overrightarrow{N}_0 = \begin{bmatrix} 1.5 \\ 0.1 \end{bmatrix}$

4) $\vec{r} = \begin{bmatrix} 2 \\ 2 \end{bmatrix}, \quad \alpha = \begin{pmatrix} 0.5 & 1 \\ 1 & 0.5 \end{pmatrix}, \quad \overrightarrow{N}_0 = \begin{bmatrix} 3.5 \\ 3 \end{bmatrix}$

5) $\vec{r} = \begin{bmatrix} 2 \\ 2 \end{bmatrix}, \quad \alpha = \begin{pmatrix} 0.5 & 1 \\ 1 & 0.5 \end{pmatrix}, \quad \overrightarrow{N}_0 = \begin{bmatrix} 3 \\ 3.5 \end{bmatrix}$

Explain the differences that you can see between these five sets of parameters and initial condition. In particular, how can we understand the difference between the set 1) and the sets 4) and 5)? What happens with sets 2) and 3) ?