Series 2 - solutions: epidemic models

Exercise 1. *During the lecture, we have studied the solutions of the Kermack-McKendrick model for initial conditions* $(S(0) > 0, I(0) > 0)$*. Study the solution for the following initial conditions: i)* $(S(0) = 0, I(0) > 0)$*, ii)* $(S(0) = 0, I(0) = 0)$*, and iii)* $(S(0) > 0, I(0) = 0)$*. If possible give the exact analytic solution.*

i) For the initial condition $(S(0) = 0, I(0) > 0)$ *the model reduces to* $dS/dt = 0$*, and* $dI/dt = -\gamma I$*. Consequently the solution is given by* $S(t) = S(0) = 0$ *, and* $I(t) = I(0)e^{-\gamma t}$*.*

ii) For the initial condition $(S(0) = 0, I(0) = 0)$ *the model reduces to* $dS/dt = 0$*, and* $dI/dt = 0$*. Consequently the solution is given by* $S(t) = S(0) = 0$ *, and* $I(t) = I(0) = 0$*.*

iii) For the initial condition $(S(0) > 0, I(0) = 0)$ *the model reduces to* $dS/dt = 0$*, and* $dI/dt = 0$*. Consequently the solution is given by* $S(t) = S(0) > 0$ *, and* $I(t) = I(0) = 0$*.*

Exercise 2. *The following table gives the data from the Eyam's (a village in England) Plague of 1666.*

$Date(1666)$	$Susceptible$	$Infected$
$July3$	235	15
$July19$	201	22
$August3$	154	29
$August19$	121	21
$September3$	108	8
$September19$	97	8
$October20$	83	0

Compute the epidemic threshold.

The epdicemic threshold is given by γ/α*, which in turn can be estimated by* $\gamma/\alpha = (S(0) + I(0) - S^*)/(log(S(0)) - log(S^*))$*. In our case,* $S(0) = 235$*,* $I(0) = 15$*, and* $S^* = 83$*, which gives* $\gamma/\alpha \approx 160$

Exercise 3. *Let assume an epidemic with the following information:* $S(0) = 250$*,* $I(0) = 50$*, and* $S^* = 100$*. What is the number of infected people at the peak of the epidemic (i.e., the maximum number of infected people).*

The peak of the epidemic is reached when the number of susceptible is equal to the epidemic threshold. In our case the epidemic threshold is equal to $\gamma/\alpha \approx 218$ *. Consequently the epidemic peak is reached for* $S(t) = 218$*. Finally the number of infected people at that peak can be evaluated based on the first integral* $S(t) + I(t) - \gamma/\alpha log(S(t)) = S(0) + I(0) - \gamma/\alpha log(S(0))$*, which results in* $I(t) \approx 52$*.*