Series 2: SI model

1. Phase space and vector field

We aim at using the ode45 solver to integrate numerically the Kermack-McKendrick epidemic model. We will represent the vector field of the system in the phase space.

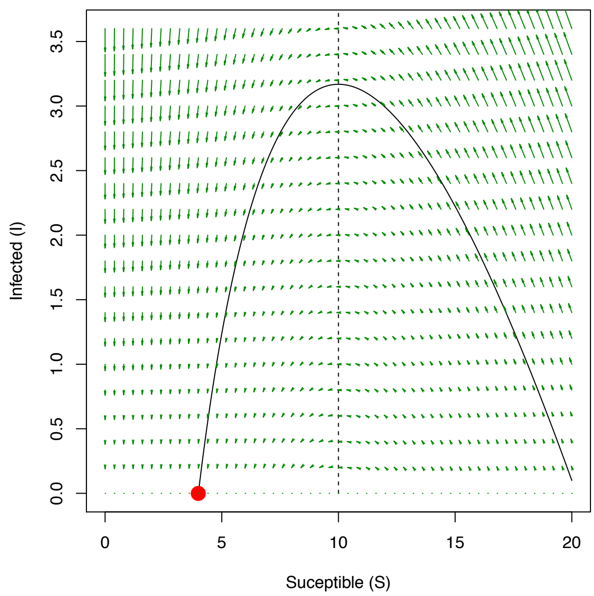
First, you will have to represent the phase space of the model or the parameters value $\alpha = 0.1$ and $\gamma = 1$. On the phase space, draw a vertical line for the epidemic threshold. Also, draw some trajectories with initial condition that you will have chosen.

Second, we will add the vector field of the system on the trajectory. The vector field represents the *speed* of change in the sated variables, i.e., it is given by the differential equation. Each point of the phase space (S, I) is associated the following vector

$$\overrightarrow{v}(S, I) = \begin{bmatrix} \dfrac{dS}{dt} \\ \dfrac{dI}{dt} \end{bmatrix} = \begin{bmatrix} \alpha \cdot S \cdot I \\ \alpha \cdot S \cdot I - \gamma \cdot I \end{bmatrix}.$$

For this, you will need to use the function *quiver* and *meshgrid* of the library *pracma.* The function *meshgrid* helps to generate a grid of *S* and *I* coordinate at which to draw the vectors, while the function *quiver* draws vectors.

Your output should look like



2. SI model without immunity and without death.

We consider a modification of the original Kermack-McKendrick model by assuming that the infected people, once the leave the infected class, become again susceptible (no death). Then, the new model is given by

$$\begin{cases} \dfrac{dS}{dt} = -\alpha \cdot S \cdot I + \gamma \cdot I \\ \dfrac{dI}{dt} = +\alpha \cdot S \cdot I - \gamma \cdot I \end{cases}$$

First, study qualitatively the models; study the sign in the phase space, the equilibrium points, a first integral, etc.

Second, integrate the model numerically and draw some trajectories and the vector field in the phase space. Use the parameters $\alpha = 0.1$ and $\gamma = 1$.