Classical models in biology (exercises) BL.6003
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Series 1: exponential and logistic growth

1. Integrating numerically the exponential model with Euler's method.

Euler's solver is the first method to integrate numerically an ordinary differential equation. It works as follow:

Let $\frac{dN}{dt} = f(N)$ be a differential equation and $N(t = 0) = N_0$ be an initial condition. Then we have to choose a time step δt and recursively we write:

$$N(t + \delta t) = N(t) + f(N(t)) \cdot \delta t \quad \text{with} \quad N(0) = N_0.$$

Use this algorithm to solve the exponential growth model. Test Euler's solver with the following sets of parameters:

a) $r = 0.2$, $\delta t = 0.1$, and $N_0 = 5$

b) $r = -2$, $\delta t = 0.1$, and $N_0 = 5$

c) $r = -2$, $\delta t = 1$, and $N_0 = 5$

What do you remark? How can you explain the behaviour of the solution for the parameters set c) ?

2. Integrating numerically the exponential model with the ode45 solver.

The ode45 solver is a Runge-Kutta method (https://en.wikipedia.org/wiki/Runge–Kutta_methods) that uses an adaptive time step interval δt (we don't aim at reimplementing ode45). We will use the ode45 solver of the library deSolve of R (https://cran.r-project.org/web/packages/deSolve/vignettes/deSolve.pdf).

Using the function ode, solve numerically the exponential model for the same parameters sets as above (without δt that is handled by ode45).

3. Integrating numerically the logistic model with the ode45 solver.

Use the ode45 solver to integrate numerically the logistic growth model. Test the solver with the following set of parameters:

a) $r = 2$, $\alpha = 0.1$ and $N_0 = 1$

b) $r = 2$, $\alpha = 0.1$ and $N_0 = 30$

c) $r = -2$, $\alpha = 0.1$, and $N_0 = 50$