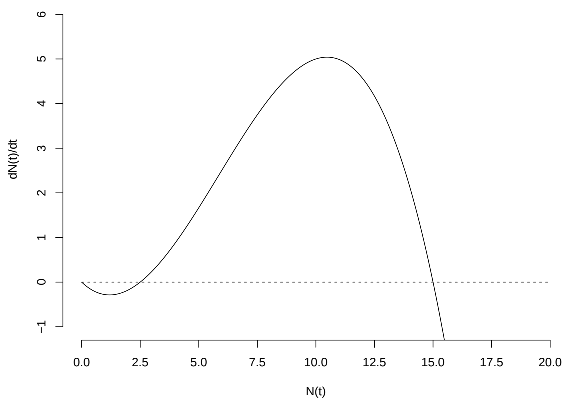
Series 1: population growth models

Exercise 1. *Let us assume that a population $N(t)$ follows the exponential growth model (Matlhusian model) with intrinsic growth rate parameter $r > 0$ and initial population size $N(0) > 0$. We define the doubling time T_2 as the time needed for the population to double in size: $N(\tau + T_2) = 2N(\tau)$.*

Relate the doubling time T_2 to the intrinsic growth rate r. Is this relationship dependent on the time τ?

Could the doubling time T_2 be defined for a population following the logistic growth (Verhulst model)?

Exercise 2. *Let us assume that the differential equation describing the population dynamic $N(t)$ is represented by the following curve:*



1. Write a plausible differential equation $\frac{dN}{dt} = \ldots$.

2. As for the logistic growth model seen during the lecture, describe qualitatively the trajectories of this dynamical system. In particular, describe the trajectories for the initial conditions given by $N(0) = 0, 0.2, 2.5, 5, 15,$ and 18.

3. Find all the equilibrium points and describe their nature (stable or unstable).

Exercise 3. *Let us assume that a population dynamic $N(t)$ is given by the following differential equation:*

$$\frac{dN(t)}{dt} = rN(t)\left(1 - \frac{N(t)}{K}\right) - H,$$

with $H > 0$ a parameter representing the harvesting rate.

Find the equilibrium points and determine their nature (stable or unstable). You will have to discus the equilibrium points under the two conditions $H > rK/4$ and $H \leq rK/4$.

Finally describe the trajectories of this dynamical system under the two conditions.