

Towards GPU-accelerated multimessenger inference of neutron star mergers and dense matter physics

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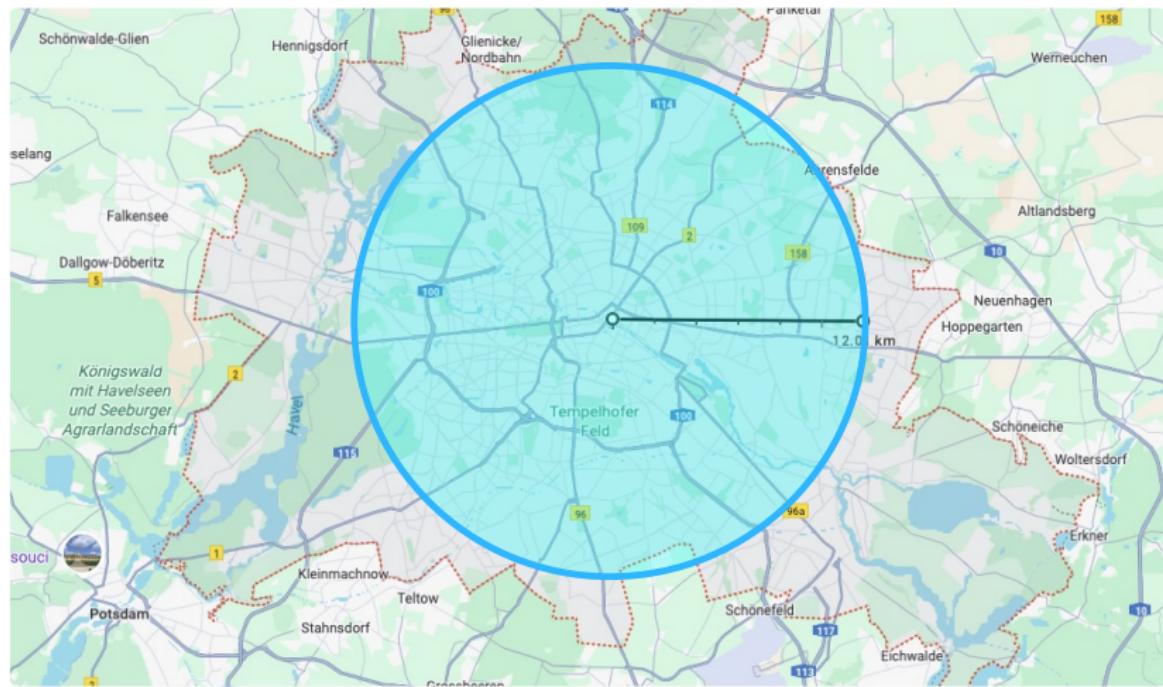
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Neutron stars

- Neutron stars: supernova remnants, densest matter in the universe
- $m \sim 1.2 - 2.3M_{\odot}$, $R \sim 10 - 13$ km

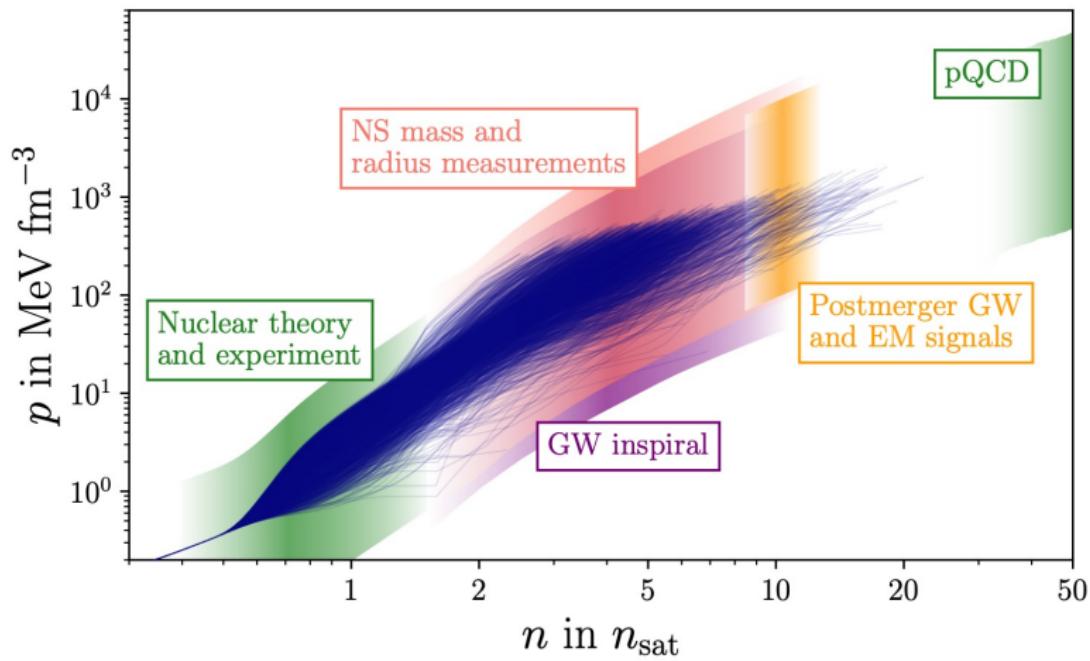
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Equation of state

Neutron stars probe the high-density part of the equation of state (EOS) of dense nuclear matter [1]



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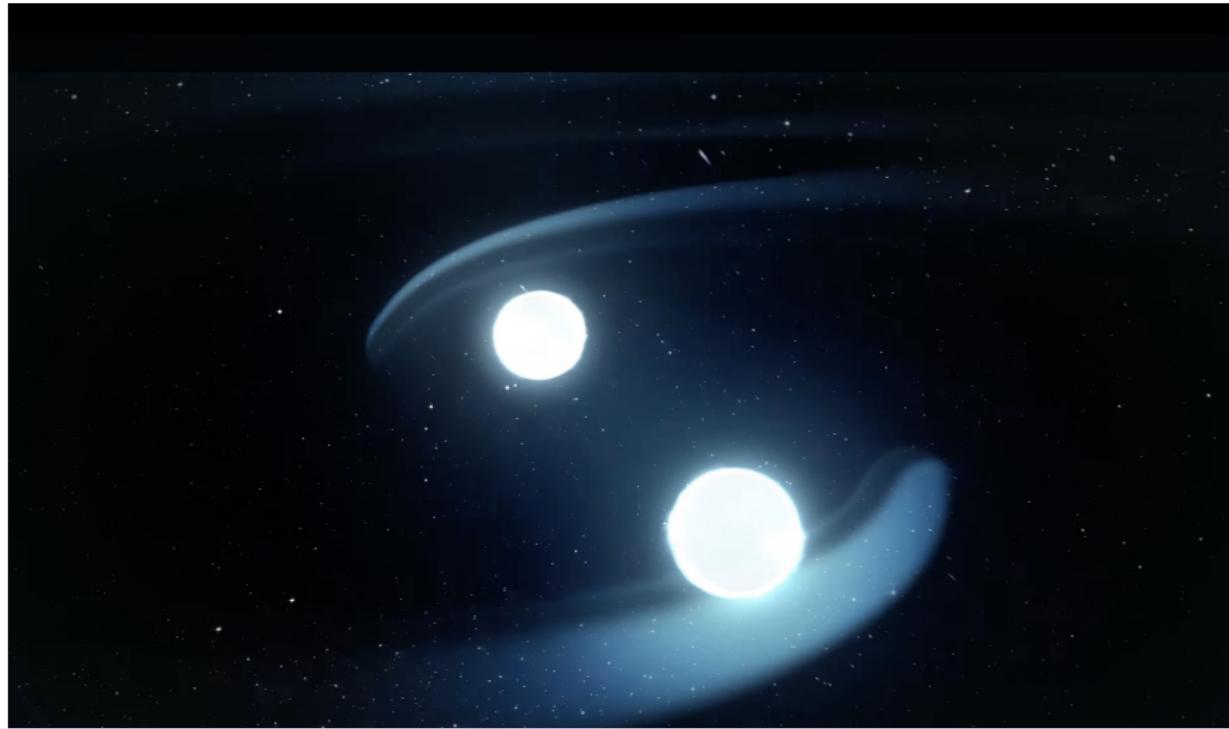
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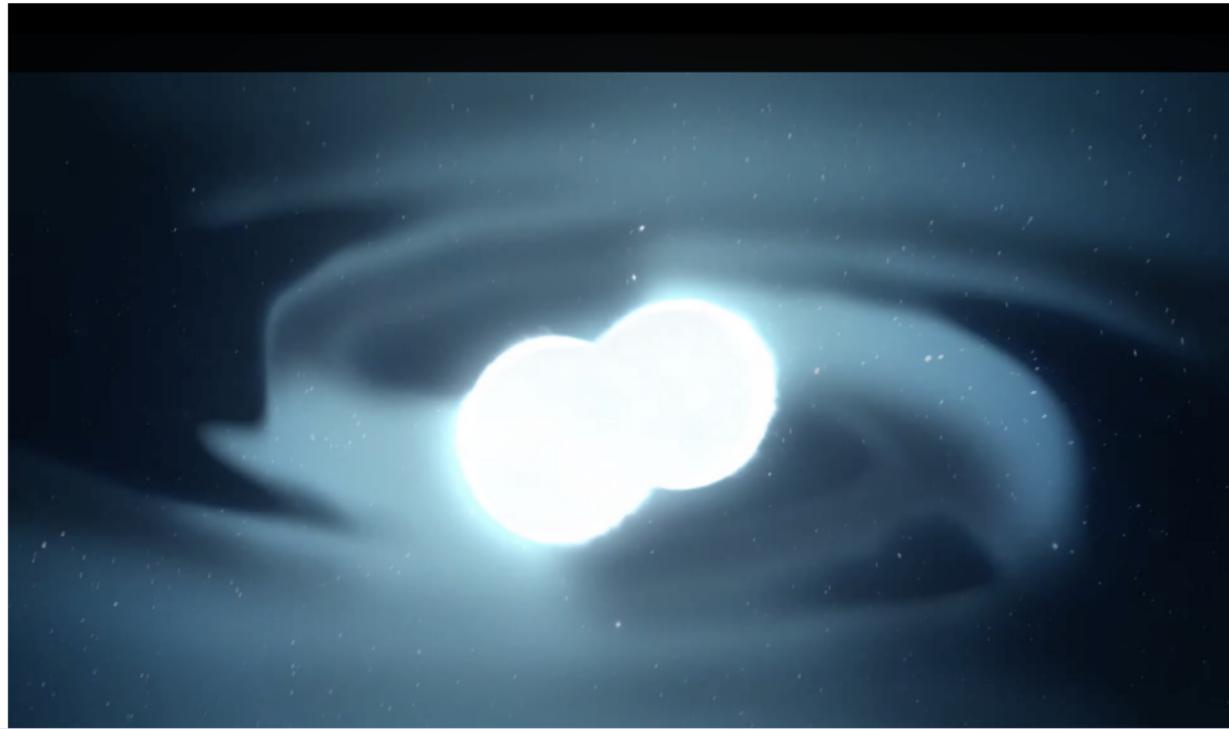
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Neutron star mergers emit gravitational waves and electromagnetic radiation: GW170817 [2, 3]



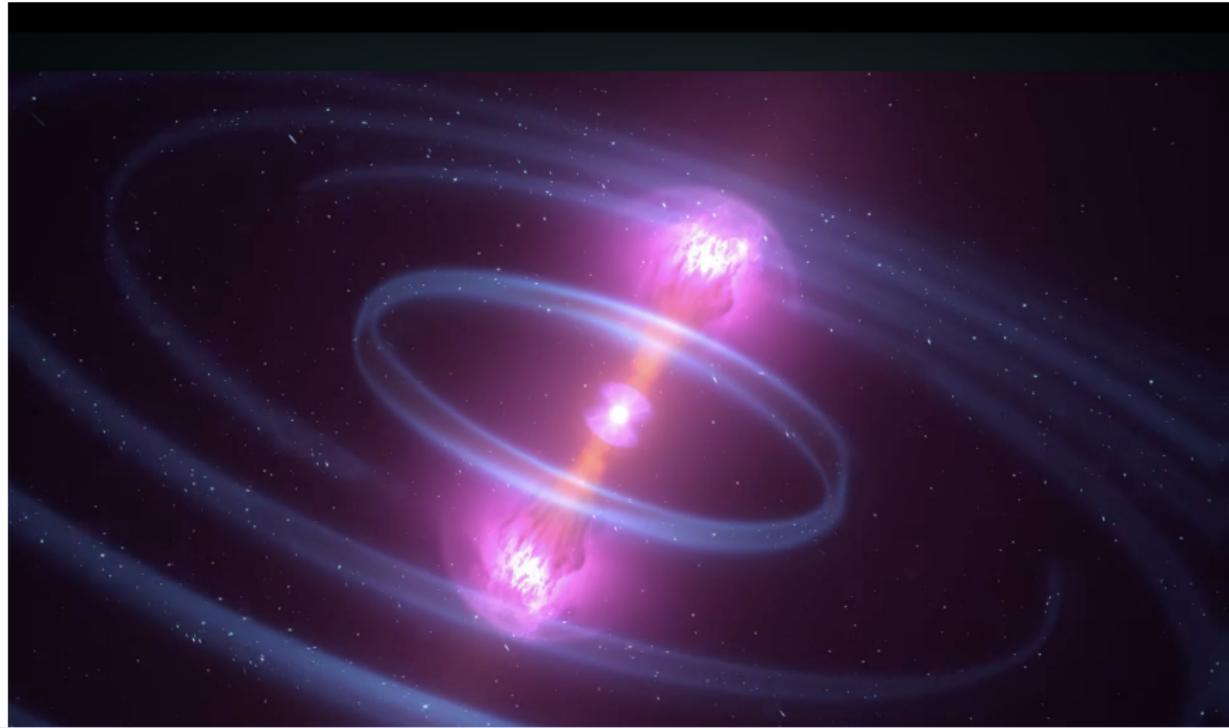
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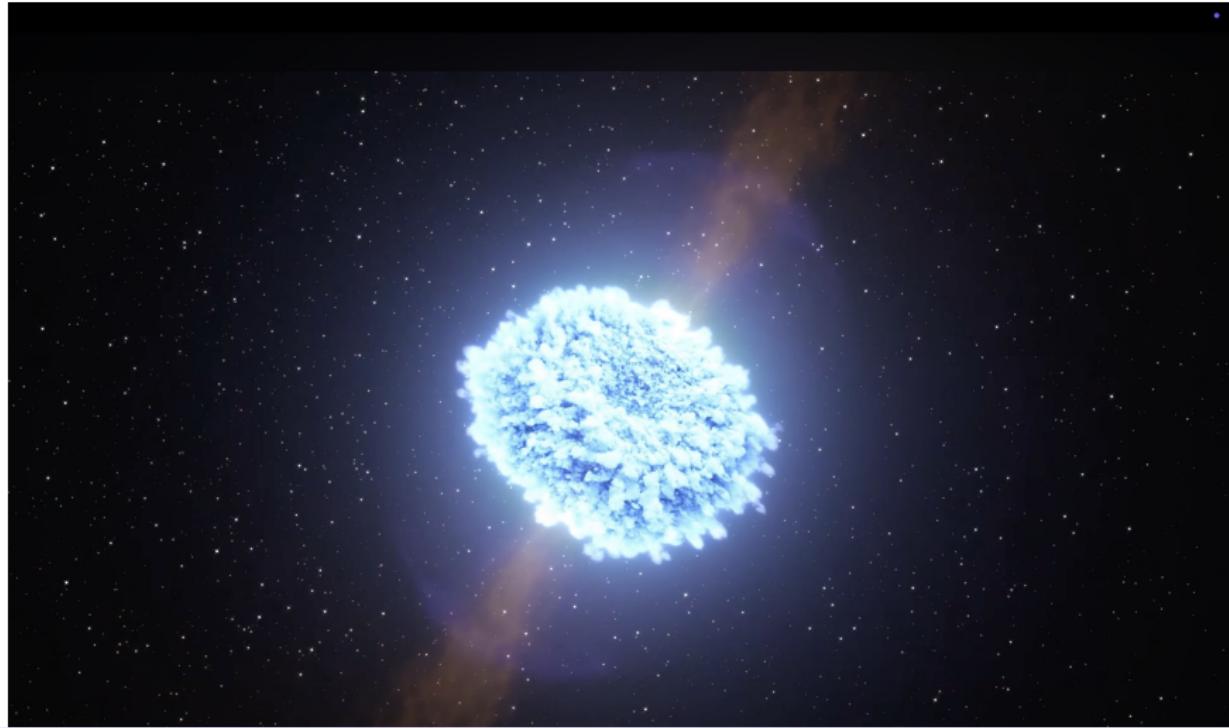
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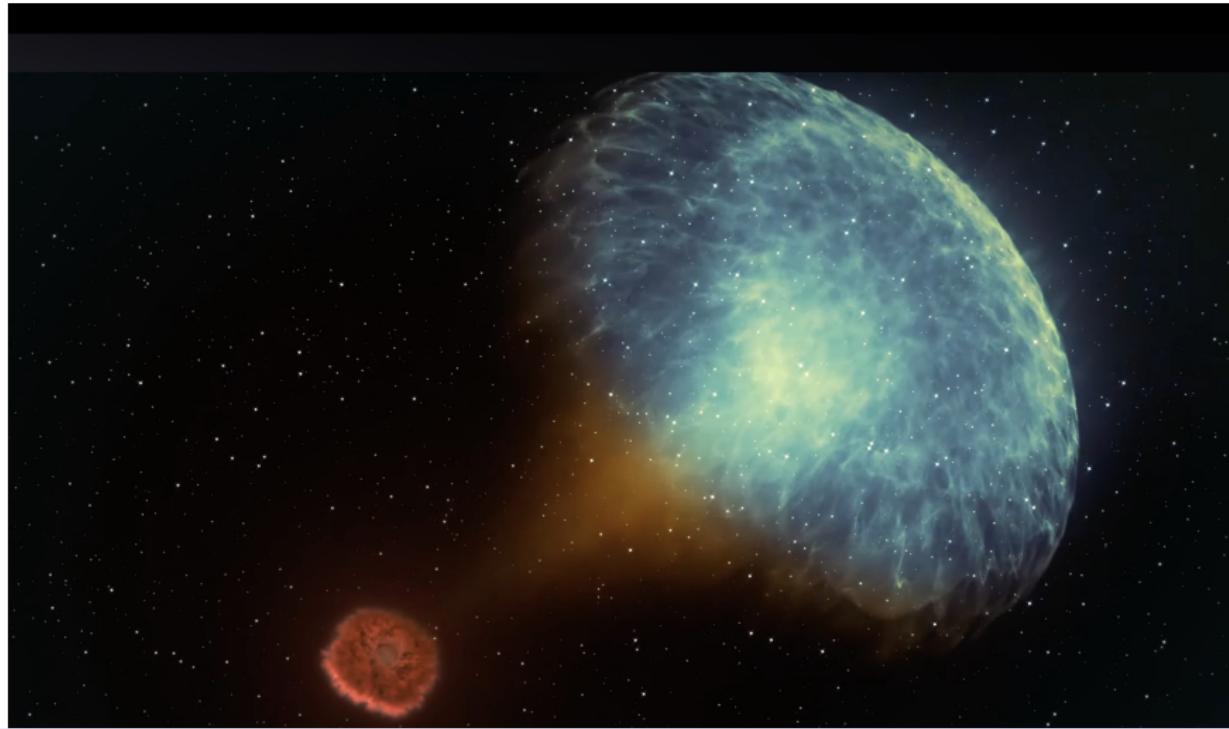
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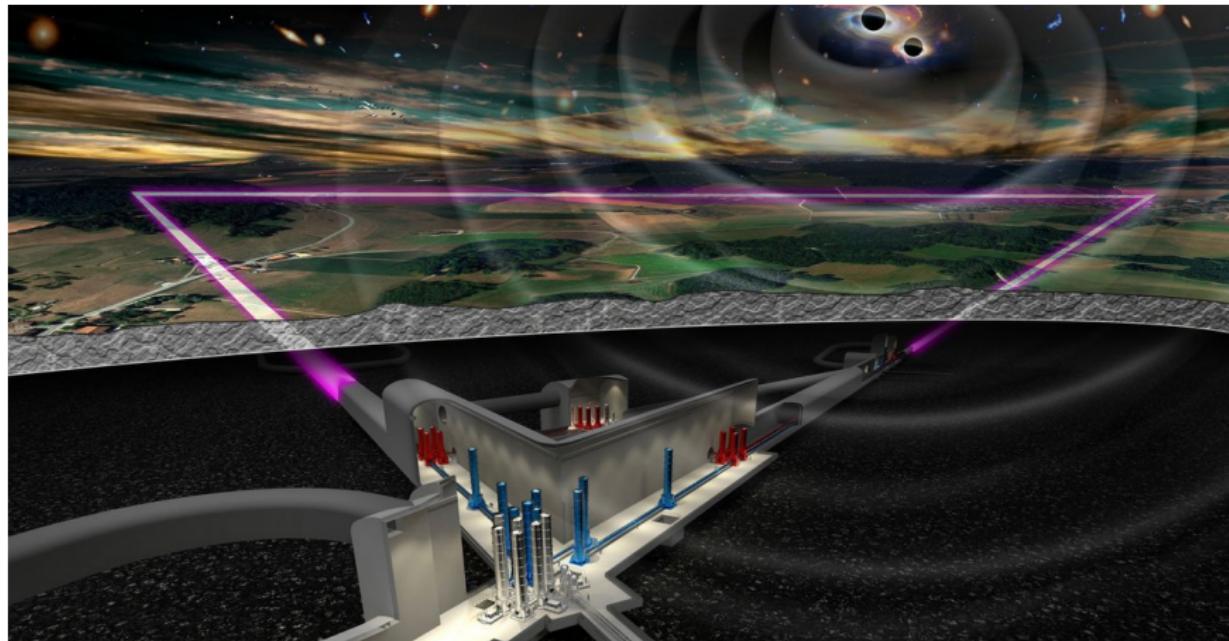
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Future GW detectors: Einstein Telescope

Einstein Telescope: Third-generation ground-based GW detector [4, 5]



(or 2 L-shaped detectors...)

Future GW detectors: Einstein Telescope

Einstein Telescope: Third-generation ground-based GW detector [4, 5]

- Increased sensitivity
 - Louder signals
 - $10^5 - 10^6$ binary black hole mergers/year (now: $\sim 200/10$ years)
 - $10^4 - 10^5$ binary neutron star mergers/year (now: $2/10$ years)
 - $10^2 - 10^3$ multimessenger events/year (now: $1/10$ years)

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 - Binary neutron star: 2 hours (vs 2 minutes now)
 - Signals will overlap: joint analysis needed

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Current software cannot handle the ET data analysis problem [6]

My research focus: why – how – what

Why?

To make inference of multimessenger astrophysics scalable

- Prepare for future detectors
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What?

GPU-accelerated Bayesian joint inference framework of neutron star mergers and dense matter physics

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Parameter estimation: Bayesian inference

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Bayesian inference:

$$\text{posterior} = p(\theta|d, \mathcal{M}) = \frac{p(d|\theta, \mathcal{M})p(\theta|\mathcal{M})}{p(d|\mathcal{M})} = \frac{\text{likelihood} \times \text{prior}}{\text{evidence}}$$

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Ingredients:

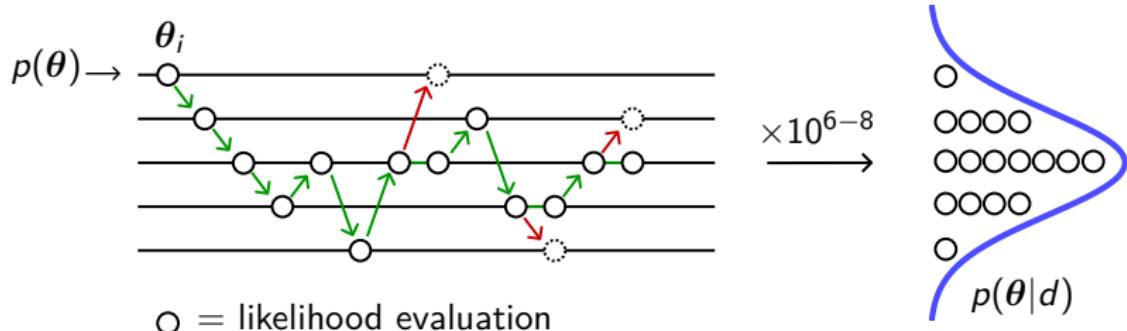
- Posterior: intractable, needs stochastic samplers
- Prior: specified by users, encode beliefs
- Likelihood: computational bottleneck
- Evidence: model selection

Parameter estimation: Markov chain Monte Carlo

How do we sample the posterior? **Markov chain Monte Carlo**

- N chains θ_i explore posterior in parallel
- Evolve chains to new position: proposal
- Compute likelihood \rightarrow accept/reject

Alternatives: nested sampling, sequential Monte Carlo



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 - Multimodality/shape posterior
 - Efficiency of sampler (proposals)

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How can we optimize this?

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In particular:

- Python: integrate into existing workflows
- Focus on arrays: widespread applications
- Numpy-like API: (`numpy` → `jax.numpy`)

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In particular:

- Python: integrate into existing workflows
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- Numpy-like API: (numpy → jax.numpy)

Gamechangers:

- GPU accelerators
- Composable function transformations: `jit`, `grad`, `vmap` & `pmap`

JAX – Function transformations

- `jit`: Just-in-time compilation

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JAX – Function transformations

- `jit`: Just-in-time compilation
- `grad`: Automatic differentiation:
 - Chain rule on computational graph
 - Exact gradients (up to machine precision), fast
- `vmap`, `pmap`: Vectorization, batch processing, parallelization
- These are composable, e.g.:
 - Higher-order derivatives: `grad(grad(f))`
 - Evolve N chains in parallel along gradient of the likelihood

$$\theta \leftarrow \theta + \alpha * \text{vmap}(\text{jit}(\text{grad}(\text{logL}(\theta))))$$

Computational aspects

$$\text{Total runtime} \approx N_{\text{likelihood}} \times \tau_{\text{likelihood}}$$

- $N_{\text{likelihood}}$: total number of likelihood evaluations
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Normalizing flows

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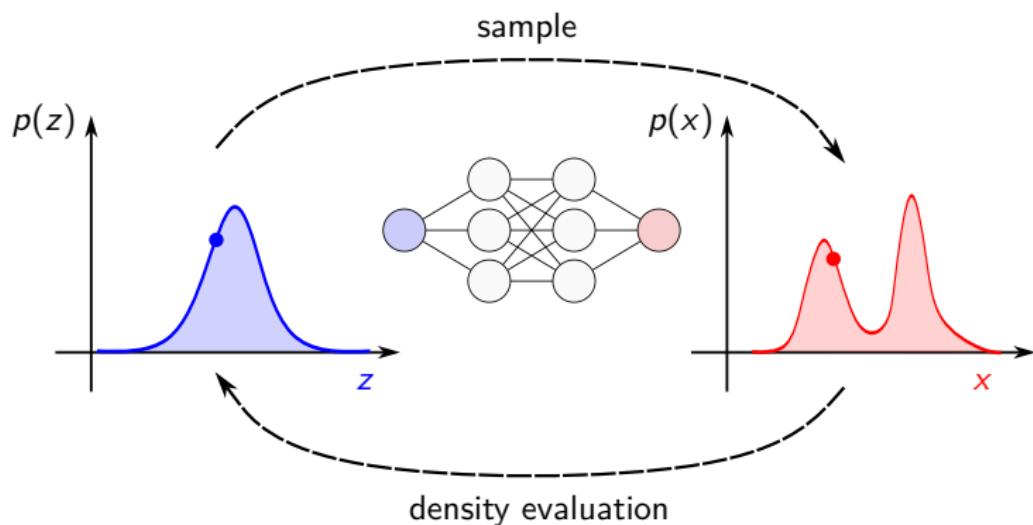
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- Generate new samples

Normalizing flows

Q: Given samples $\{x\} \sim p(x)$, how can we get $p(x)$?

- Evaluate density at any point
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A: Normalizing flows: generative machine learning model, bijection between **latent** space (Gaussian) and **data** space



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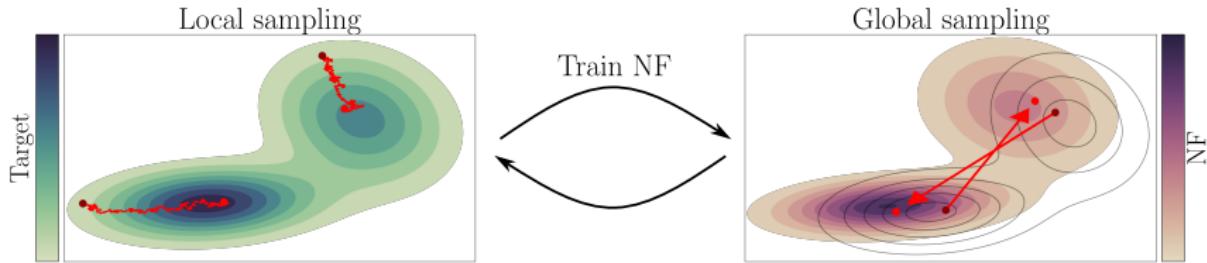
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FLOWMC

FLOWMC  [8, 9]: normalizing flow-enhanced MCMC

- ① Run gradient-based sampler (local sampler)
- ② Train normalizing flow on MCMC chains
- ③ Propose samples from the normalizing flow (global sampler)
- ④ Repeat



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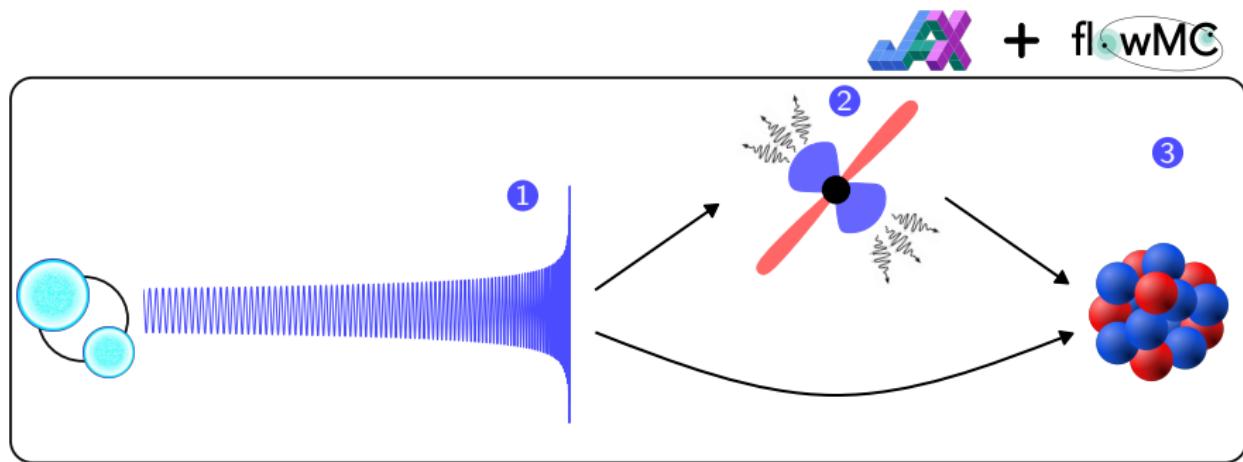
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Overview

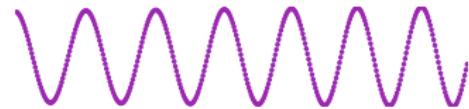
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- ① Gravitational waves
- ② Electromagnetic counterparts
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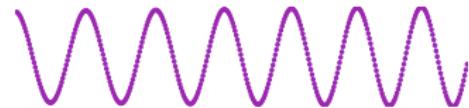
Gravitational waves: RIPPLE

- Waveforms on GPU: $\mathcal{O}(10^3)$ faster
- RIPPLE [10]: from LALSUITE to JAX



Gravitational waves: RIPPLE

- Waveforms on GPU: $\mathcal{O}(10^3)$ faster
- RIPPLE [10]: from LALSUITE to JAX
- Waveforms available in RIPPLE:
 - Binary black holes:
 - IMRPhenomXAS
 - IMRPhenomD
 - IMRPhenomPv2
 - Binary neutron stars:
 - TaylorF2
 - IMRPhenomD_NRTidalv2
 - IMRPhenomPv_NRTidalv2 (by Nihar Gupte)
- Also see GWFEST [11] and SFTS [12]



Gravitational waves: JIM

Parameter estimation: JIM Ω [13, 14]: RIPPLE + FLOWMC

- Verified for binary neutron stars in current generation detectors [14]
- Matches BILBY, passes *pp*-test

Gravitational waves: JIM

Parameter estimation: JIM  [13, 14]: RIPPLE + FLOWMC

- Verified for binary neutron stars in current generation detectors [14]
- Matches BILBY, passes *pp*-test
- Runtime: from hours to minutes

| Event | Waveform | JIM | PBILBY | RB-BILBY | ROQ-BILBY |
|-----------|----------|-----------|-------------|------------|------------|
| | | (1 GPU) | (480 cores) | (24 cores) | (24 cores) |
| GW170817 | TF2 | 15.33 min | 9.64 h | 3.8 h | – |
| | NRTv2 | 25.59 min | 10.99 h | 4.11 h | 1.65 h |
| GW190425 | TF2 | 18.30 min | 8.18 h | 2.81 h | – |
| | NRTv2 | 21.20 min | 4.91 h | 2.42 h | 0.97 h |
| Injection | TF2 | 24.76 min | – | – | – |
| | NRTv2 | 18.02 min | – | – | – |

Gravitational waves: JIM

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- Runtime: from hours to minutes
- Up to $5 - 10 \times$ more effective samples (normalizing flow)

| Effective sample size | |
|-----------------------|-------------------|
| JIM | 4.1×10^4 |
| PBILBY | 6.5×10^3 |
| RB-BILBY | 5.8×10^3 |
| ROQ-BILBY | 7.4×10^3 |

Gravitational waves: JIM

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- More cost-effective/energy-efficient

| | kWh | CO ₂ [10 ³ kg] | Trees |
|-----------|------------|--------------------------------------|-------|
| JIM | 34 | 11 | 0.55 |
| PBILBY | 4127 | 1354 | 67.68 |
| RB-BILBY | 80 | 26 | 1.32 |
| ROQ-BILBY | sampling | 32 | 0.52 |
| | precompute | 27 | 0.44 |

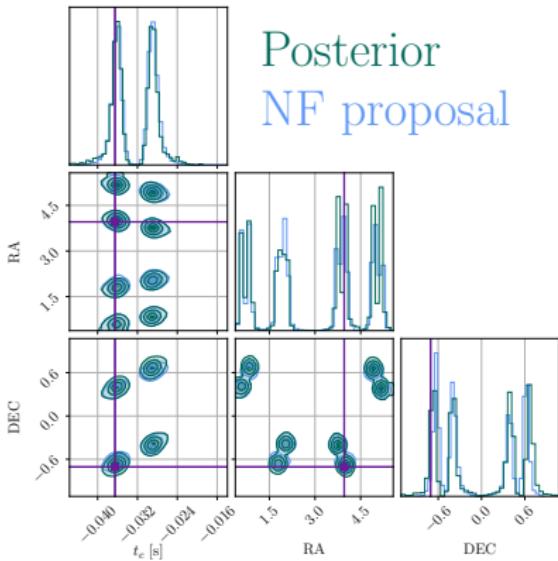
Einstein Telescope

Work in progress: showing proof-of-concepts!

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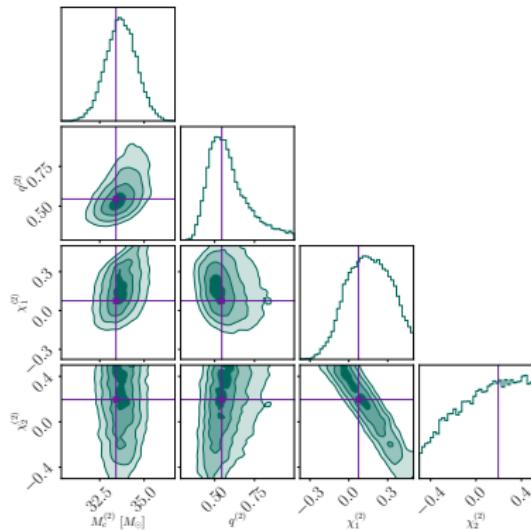
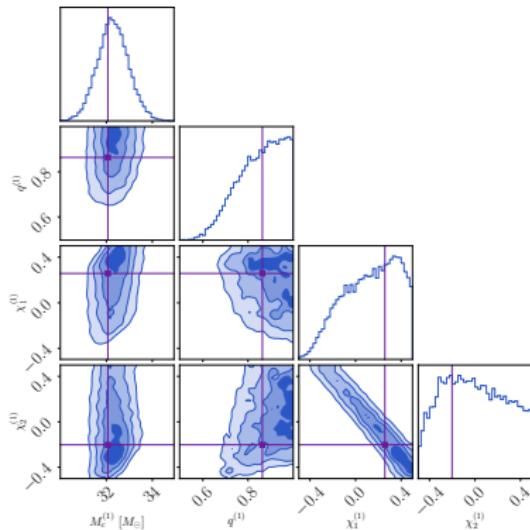
Work in progress: showing proof-of-concepts!

- ET posteriors are multimodal
- Normalizing flows help to jump between modes



Overlapping signals

- Assess scaling of JIM: BBH+BBH with LIGO-Virgo
 - 2 binary black hole mergers: 22 parameters
 - $M_c^{(1)} = 32M_\odot$, $M_c^{(2)} = 33M_\odot$, $\Delta t = 70$ ms
 - $\text{SNR}^{(1)} = 25.76$, $\text{SNR}^{(2)} = 25.24$
 - 1h28m on H100 GPU (vs several days on 16 CPUs [15])



Open call

- Recently: nested samplers in JAX for GW [16]
 - Using RIPPLE waveforms
 - Momentum towards waveforms on GPU?

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- Bridge between LALSUITE and RIPPLE?

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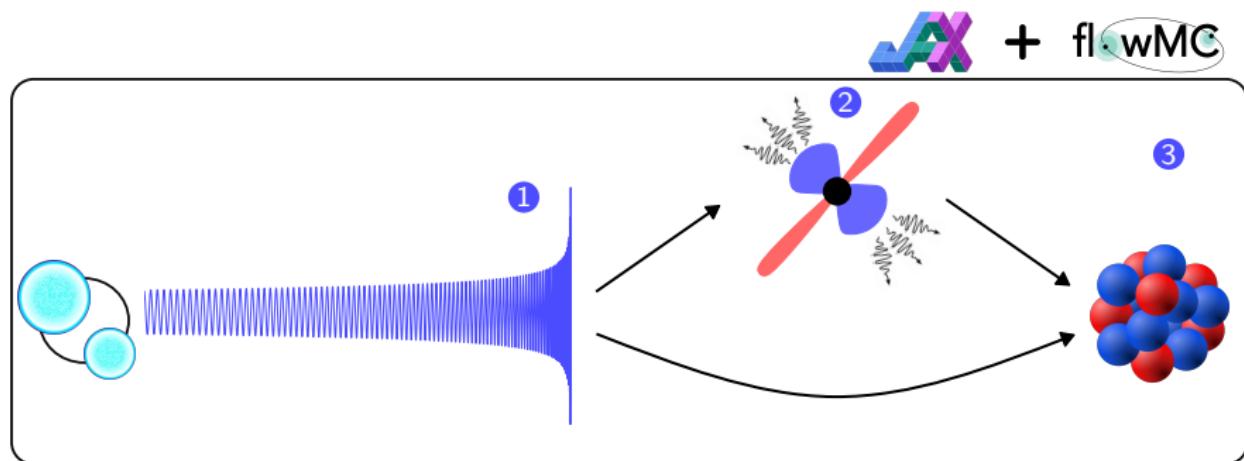
Get in touch if you are interested! We need you!

JIM developers \cap waveform developers = **you**?

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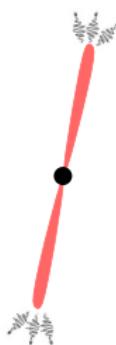
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Electromagnetic counterparts (Hauke Koehn)

- Predicting a **GRB afterglow** is slow
- Code libraries too large to 'jaxify': neural network emulators for inference: FIESTA  [17]

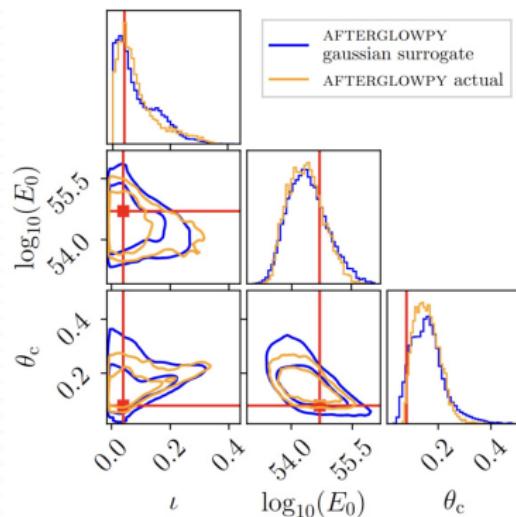


FIESTA

- 1m36s
- 1 H100 GPU

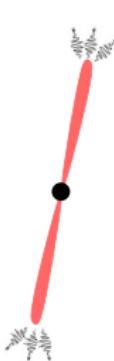
AFTERGLOWPY

- 4 hours
- 30 CPUs



Electromagnetic counterparts (Hauke Koehn)

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- Code libraries too large to ‘jaxify’: neural network emulators for inference: FIESTA  [17]
- Scales well for systematics ‘nuisance parameters’

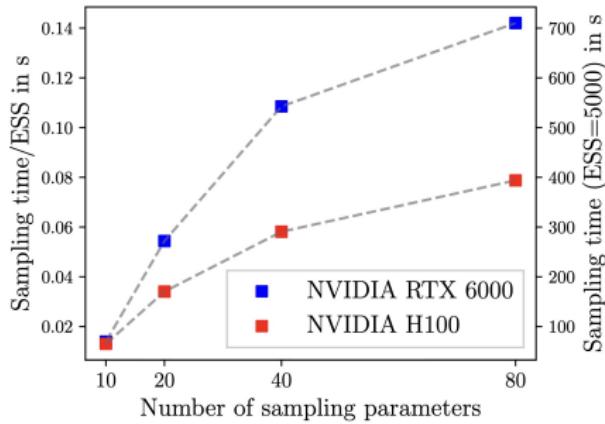


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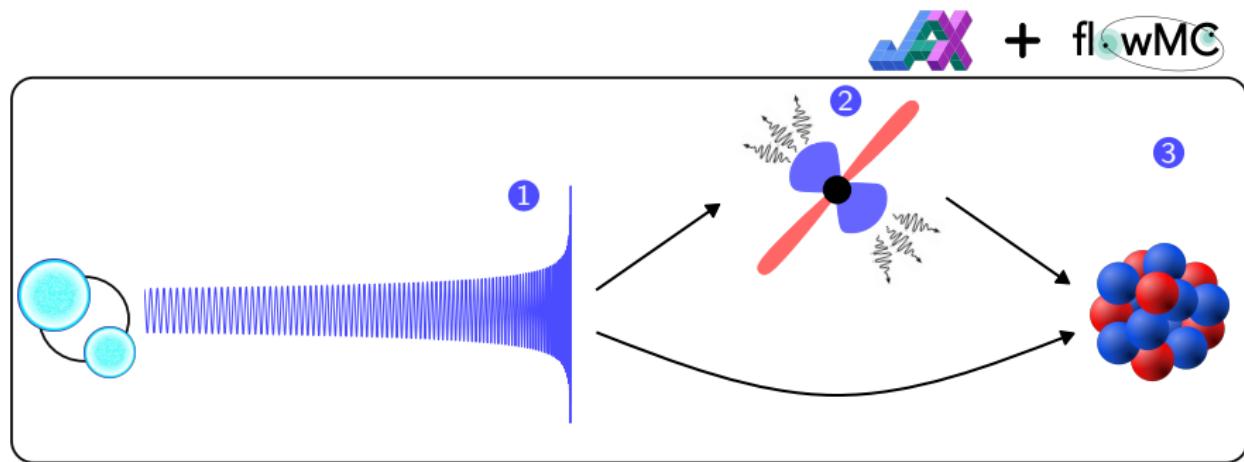
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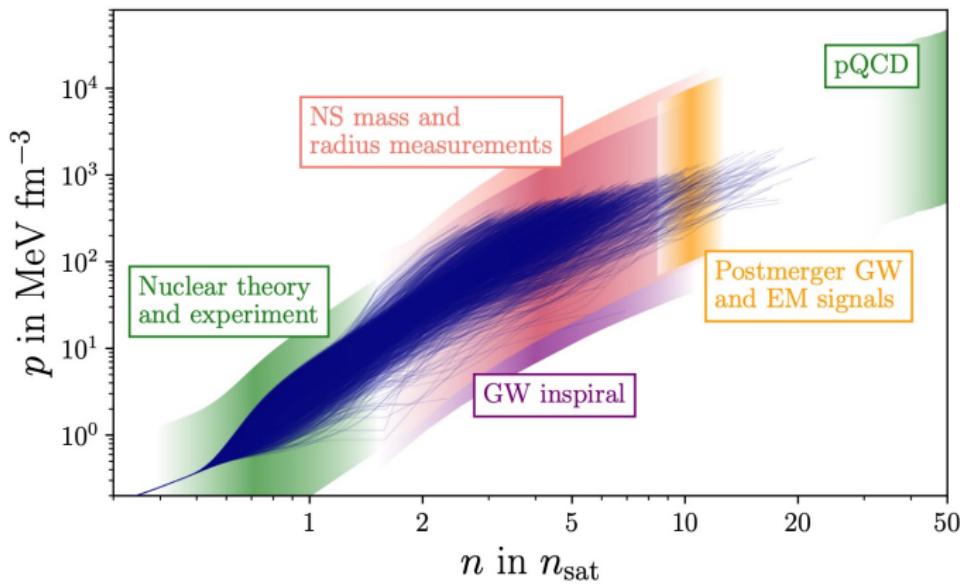
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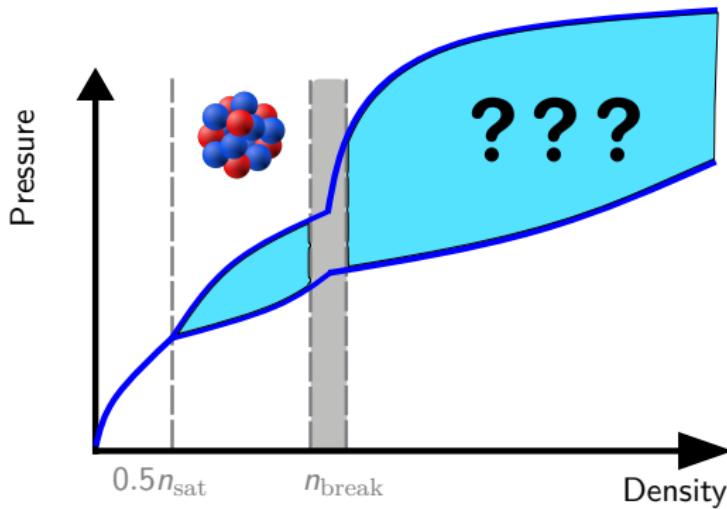
Equation of state inference – warmup

- How do we constrain the EOS from neutron star observations?
- Define parametrization and likelihood



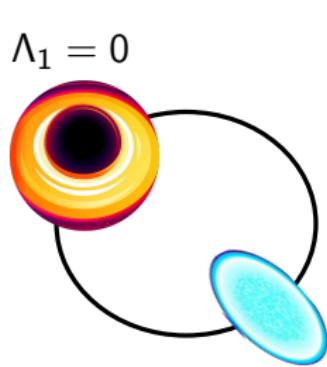
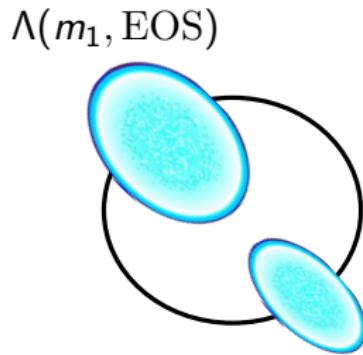
Equation of state inference – parametrization

- $n \leq \frac{1}{2}n_{\text{sat}}$: fixed crust
- $\frac{1}{2}n_{\text{sat}} \leq n \leq n_{\text{break}}$: metamodel EOS, nuclear physics inspired
- $n \leq n_{\text{break}}$: agnostic extension
- > 26 (!) parameters θ_{EOS}



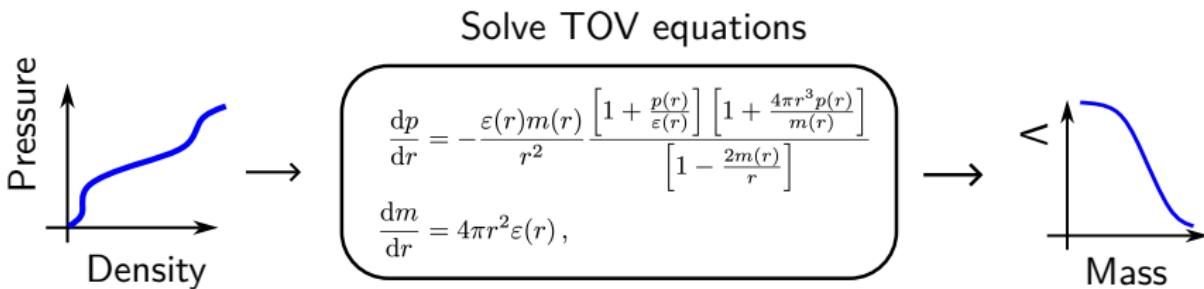
Tidal deformability

- Neutron stars are tidally deformed in a binary, quantified by tidal deformability Λ : affect phase of GWs
- Neutron stars: $\Lambda = \Lambda(m, \text{EOS})$, black holes: $\Lambda = 0$
- GWs give us $p(m_1, m_2, \Lambda_1, \Lambda_2 | d)$ for EOS inference


$$\Lambda(m_2, \text{EOS})$$

$$\Lambda(m_2, \text{EOS})$$

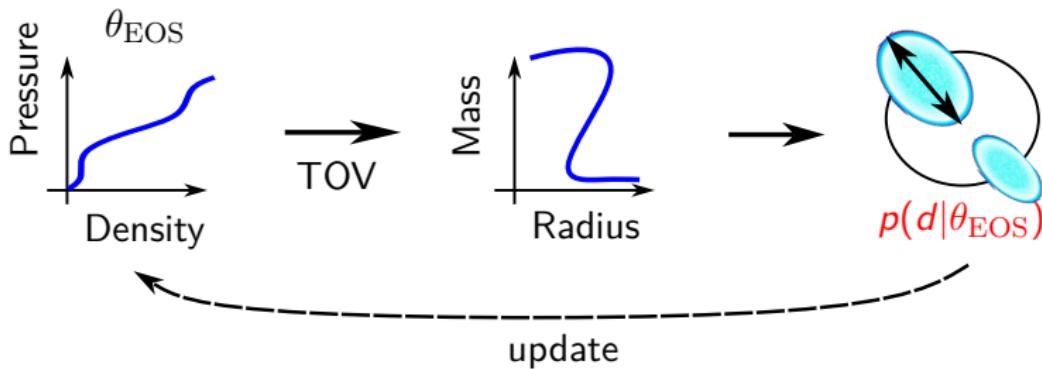
Equation of state

- To predict neutron star properties, we solve the TOV equations: ordinary differential equations (ODEs)



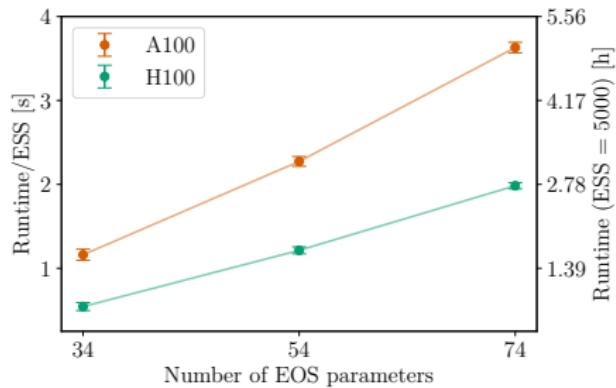
Equation of state

- To predict neutron star properties, we solve the TOV equations: ordinary differential equations (ODEs)
- Done for each sample θ_{EOS} : **costly likelihood**
- How to make this scalable without compromises?

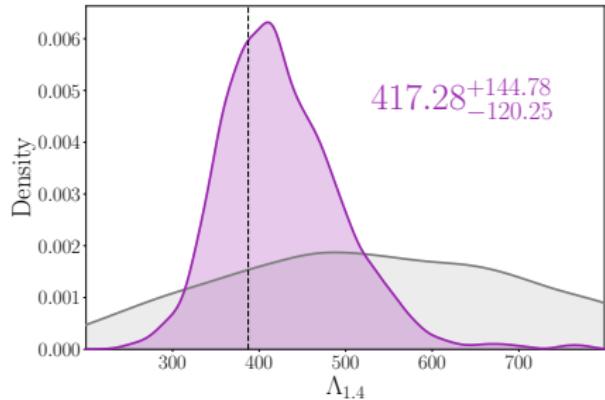
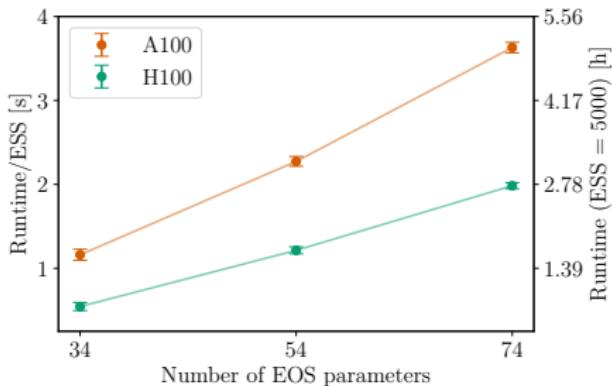


JESTER

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 - $1000\times$ faster, without compromises
 - Full inference in \sim hours

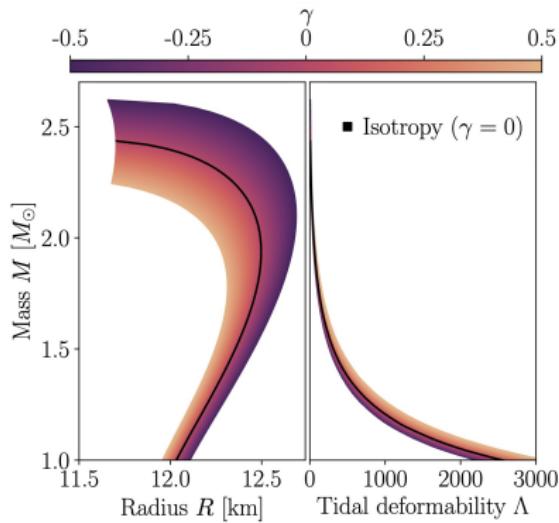


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 - $1000\times$ faster, without compromises
 - Full inference in \sim hours
- JIM+JESTER: from GWs to EOS in a few hours
 - Example: 20 binary neutron stars in O5
- Enable systematics studies in EOS inference



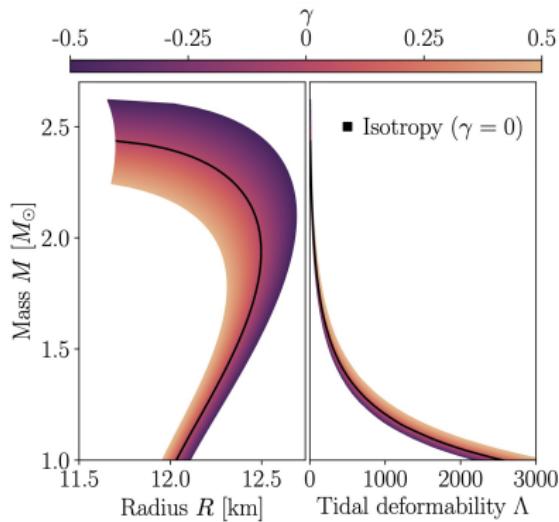
Anisotropy in neutron stars (Peter T. H. Pang)

- Anisotropic pressure: $\gamma \propto p - p_t$



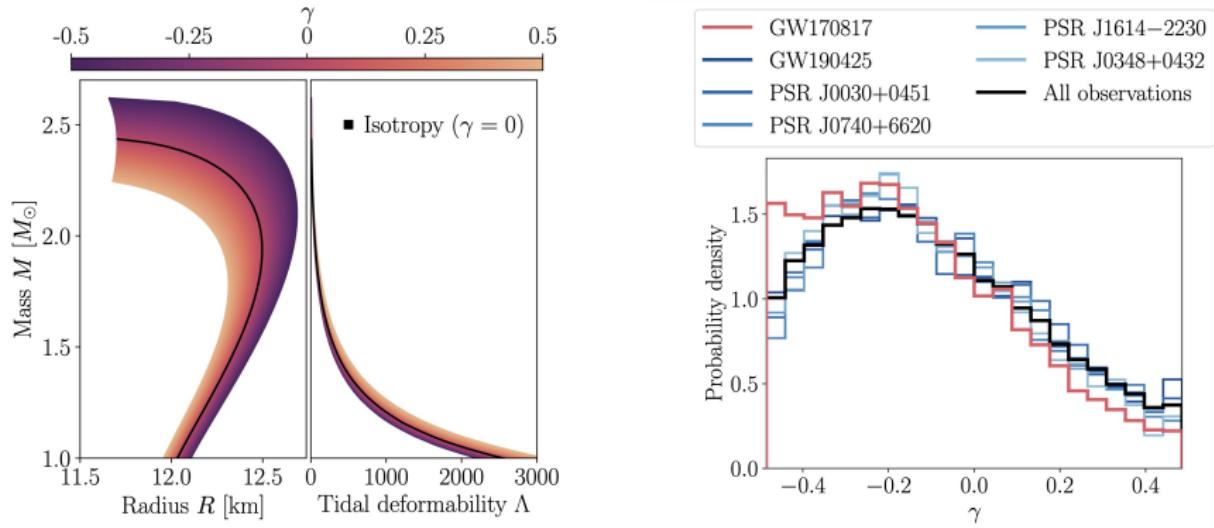
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- Magnetic fields, dark matter clusters, superfluids...



Anisotropy in neutron stars (Peter T. H. Pang)

- Anisotropic pressure: $\gamma \propto p - p_t$
- Magnetic fields, dark matter clusters, superfluids...
- Preference for negative anisotropy, but weak evidence [19]



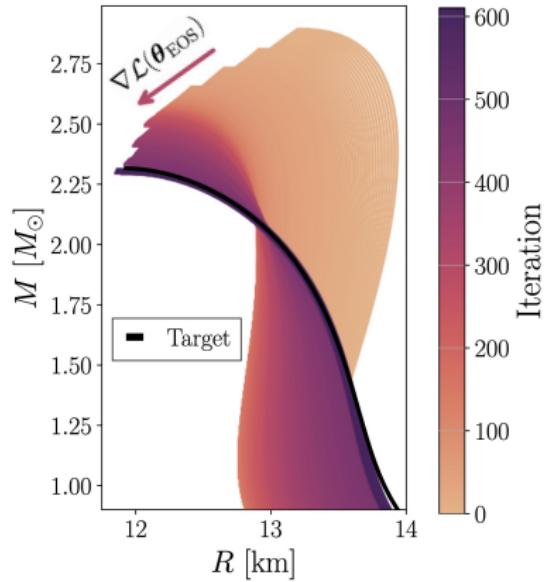
Auto-differentiable ODE solvers

- ODE solvers written in JAX are auto-differentiable
- Frame inference as optimization problem:
 - Gradient descent on loss function $\mathcal{L}(\theta_{\text{EOS}})$

Auto-differentiable ODE solvers

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$$\mathcal{L}(\theta_{\text{EOS}}) = \frac{1}{N} \sum_{i=1}^N \left| \frac{R_i(\theta_{\text{EOS}}) - \hat{R}_i}{\hat{R}_i} \right|$$



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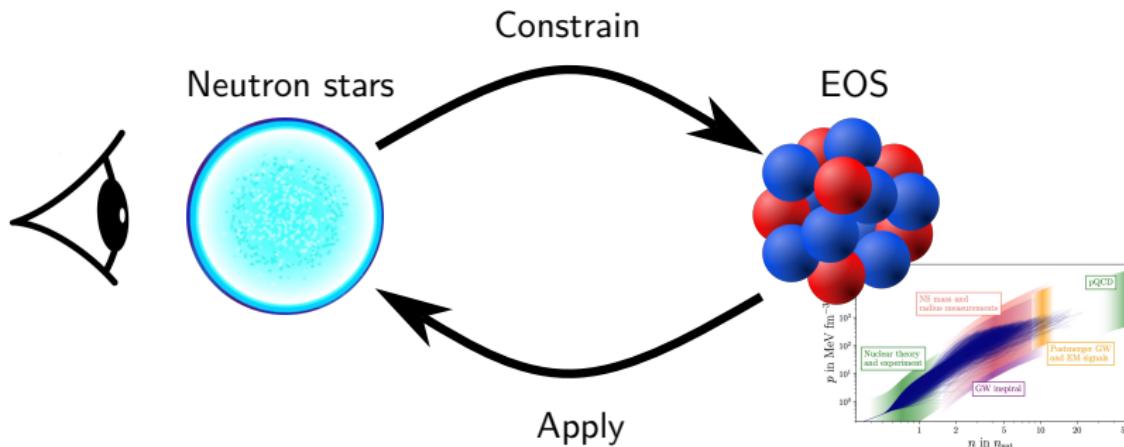
⑤ Conclusion

Neutron star data analysis loop

Data analysis of neutron stars forms a **loop**:

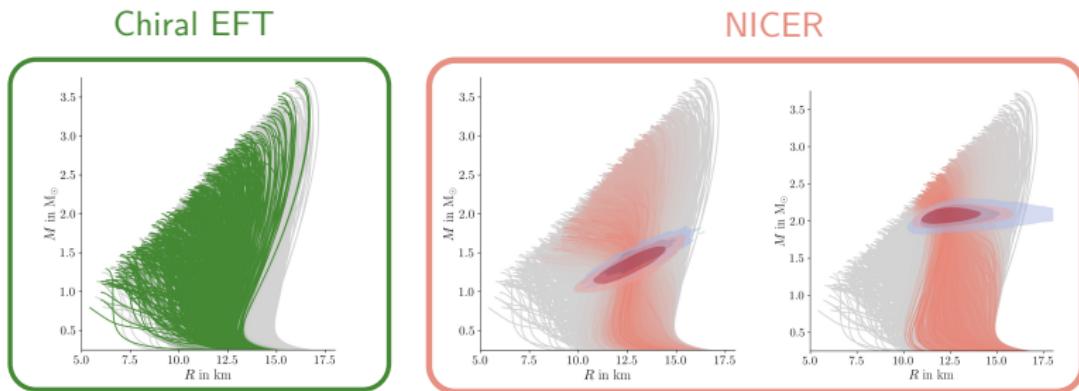
- ① Constraining the EOS with neutron star observations
- ② Applying EOS knowledge in neutron star data analysis (e.g., GW)

How can we efficiently perform **Step 2**?



Case study

- Take constraints on EOS from:
 - **Radio timing:** EOS must support $2 M_{\odot}$ neutron stars
 - **Chiral EFT (χ_{EFT}):** nuclear theory predictions (valid $< 2n_{\text{sat}}$)
 - **NICER:** Mass-radius observations of neutron stars



- How can we use this information in GW analyses?

Equation of state-informed priors

- This should enter the prior on Λ_i

$$\mathcal{P}(\theta_{\text{GW}}|d) \propto \mathcal{L}(d|\theta_{\text{GW}})\pi(\theta_{\text{GW}})$$

- By default, we choose **agnostic priors**: e.g. $\Lambda_{1,2} \sim \mathcal{U}(0, 5000)$

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 - Masses m_i determined by M_{\max} (or population)
 - $\Lambda_i = \Lambda_i(m_i, \text{EOS})$

Equation of state-informed priors

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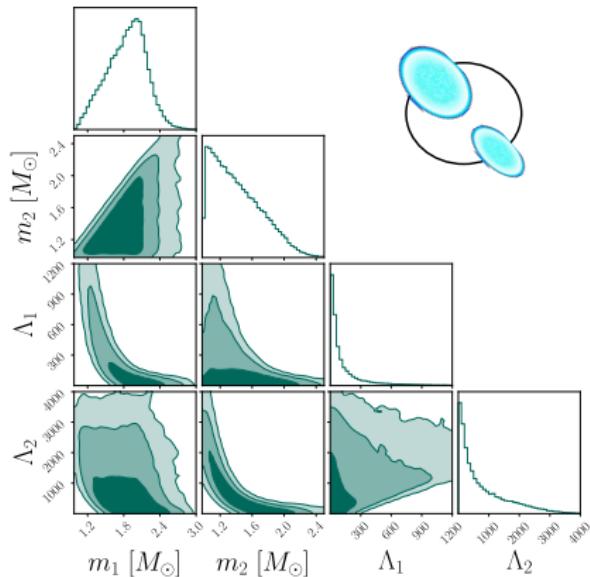
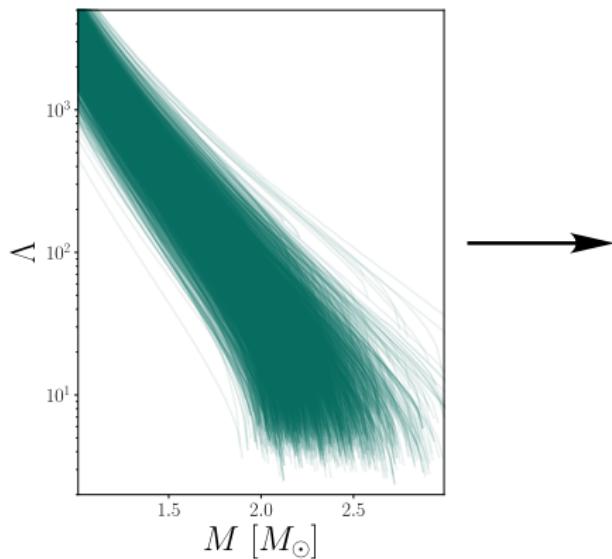
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- **But**, we have prior knowledge from the EOS:
 - Masses m_i determined by M_{\max} (or population)
 - $\Lambda_i = \Lambda_i(m_i, \text{EOS})$
- **Equation of state-informed prior**: take **EOS uncertainty** into account

$$\begin{aligned}\pi(m_1, m_2, \Lambda_1, \Lambda_2) &= \int d\theta_{\text{EOS}} \pi(m_1, m_2 | \theta_{\text{EOS}}) \pi(\Lambda_1, \Lambda_2 | m_1, m_2, \theta_{\text{EOS}}) \\ &\quad \times \pi(\theta_{\text{EOS}})\end{aligned}$$

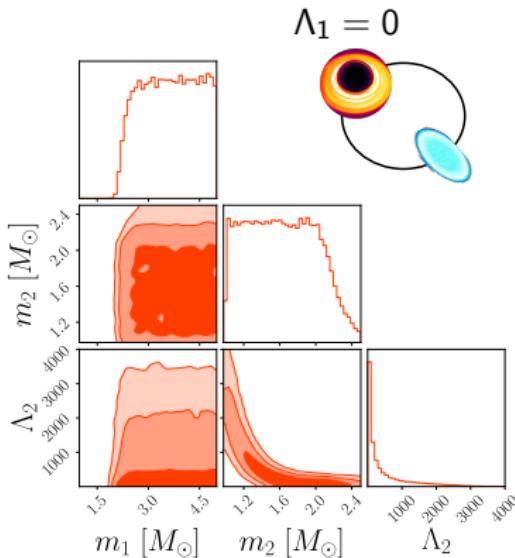
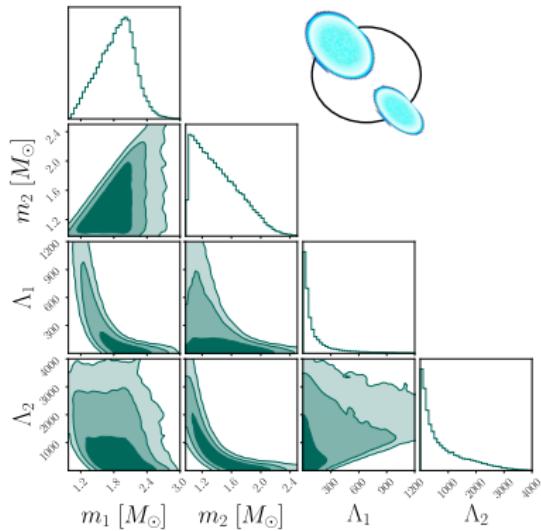
Equation of state-informed priors

- Example: EOSs with $M_{\text{max}} > 2.0M_{\odot}$
- Sample to produce $\pi(m_1, m_2, \Lambda_1, \Lambda_2)$



Source classification

- Similar to the **binary neutron star (BNS)** prior, we can also construct a **neutron star-black hole (NSBH)** prior
 - BH mass in $[M_{\text{max}}(\text{EOS}), 5 M_{\odot}]$
- Classify events with Bayesian model selection

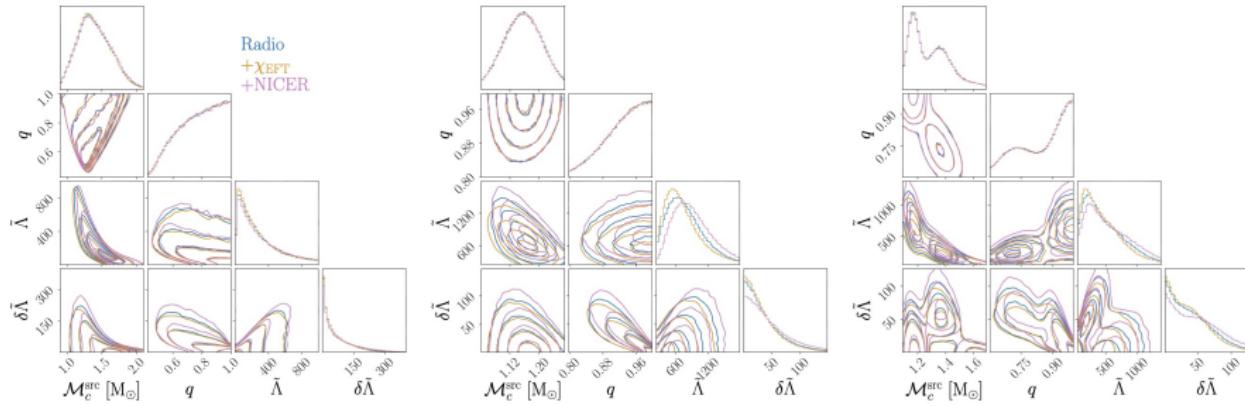


Neural priors

- Masses can also be informed by populations
 - Uniform (agnostic)
 - Gaussian
 - Double Gaussian
- $(\text{BNS/NSBH}) \times (\text{3 populations}) \times (\text{3 EOS constraints}) = 18 \text{ priors}$
- Emulate with normalizing flow: **neural priors**

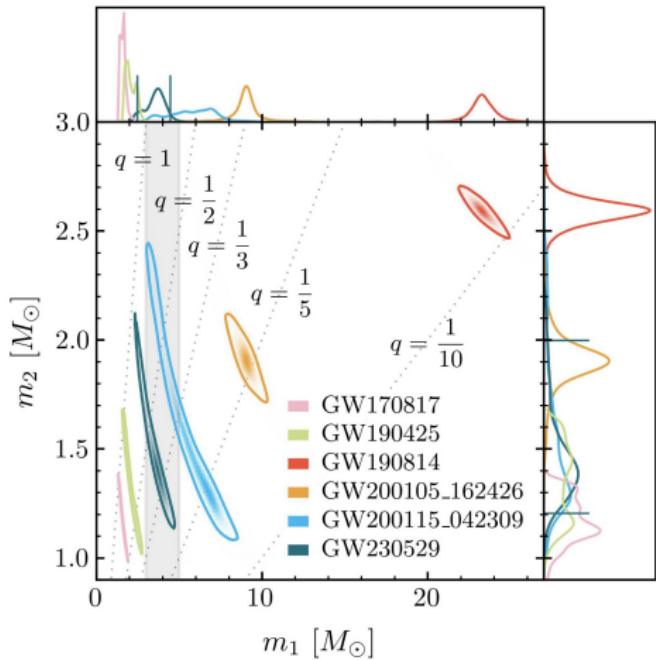
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- (BNS/NSBH) \times (3 populations) \times (3 EOS constraints) = 18 priors
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Application

- Implemented in BILBY
- Consider three “low-mass” GWs
 - GW170817
 - GW190425
 - GW230529



GW170817 – classification

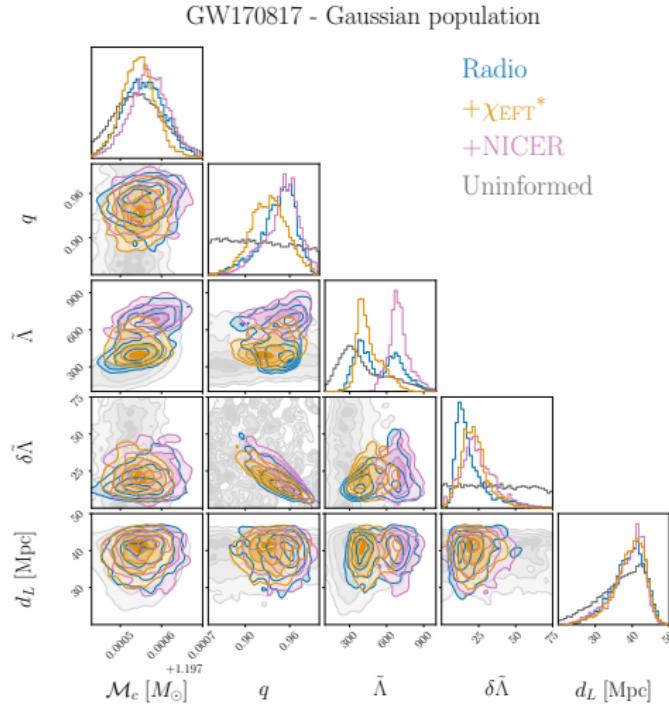
Showing \log_{10} Bayes factors: negative = less preferred

- Strongly prefer BNS over NSBH
- Gaussian population, EOS inconclusive

| Source | Population | EOS Constraints | GW170817 |
|--------|-----------------|-----------------|----------|
| BNS | Uniform | Radio | -0.76 |
| | | + χ EFT | -0.52 |
| | | +NICER | -0.86 |
| | Gaussian | Radio | -0.14 |
| | | + χ EFT | ref. |
| | | +NICER | -0.05 |
| | Double Gaussian | Radio | -0.43 |
| | | + χ EFT | -0.26 |
| | | +NICER | -0.73 |
| NSBH | Uniform | Radio | -224.11 |
| | | + χ EFT | -224.11 |
| | | +NICER | -224.12 |
| | Gaussian | Radio | -224.13 |
| | | + χ EFT | -224.13 |
| | | +NICER | -224.13 |
| | Double Gaussian | Radio | -224.12 |
| | | + χ EFT | -224.13 |
| | | +NICER | -224.12 |

GW170817 – parameter constraints

- More equal mass ratio $q \geq 0.9$
- $\tilde{\Lambda}$ bimodal, resolved by extra EOS information



GW190425 – classification

Showing \log_{10} Bayes factors: negative = less preferred

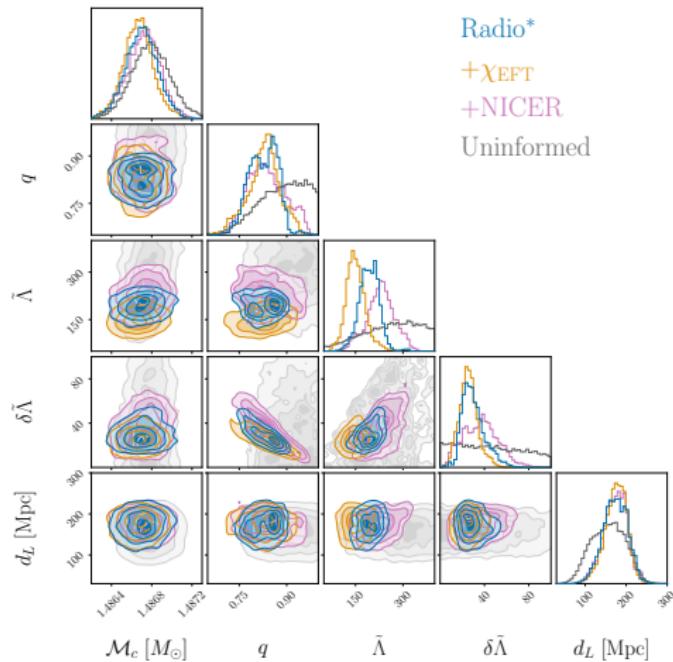
- Prefer BNS over NSBH, but less conclusive
- Most consistent with uniform population

| Source | Population | EOS Constraints | GW190425 |
|--------|-----------------|-----------------|----------|
| BNS | Uniform | Radio | ref. |
| | | + χ EFT | -0.09 |
| | | +NICER | -0.11 |
| | Gaussian | Radio | -8.51 |
| | | + χ EFT | -6.57 |
| | | +NICER | -4.42 |
| | Double Gaussian | Radio | -0.76 |
| | | + χ EFT | -0.56 |
| | | +NICER | -0.89 |
| NSBH | Uniform | Radio | -1.10 |
| | | + χ EFT | -1.10 |
| | | +NICER | -1.19 |
| | Gaussian | Radio | -0.82 |
| | | + χ EFT | -1.01 |
| | | +NICER | -0.98 |
| | Double Gaussian | Radio | -1.65 |
| | | + χ EFT | -3.40 |
| | | +NICER | -2.12 |

GW190425 – parameter constraints

- Less equal masses ($q \leq 0.9$)
- Higher distances

GW190425 - Uniform population



GW230529 – classification

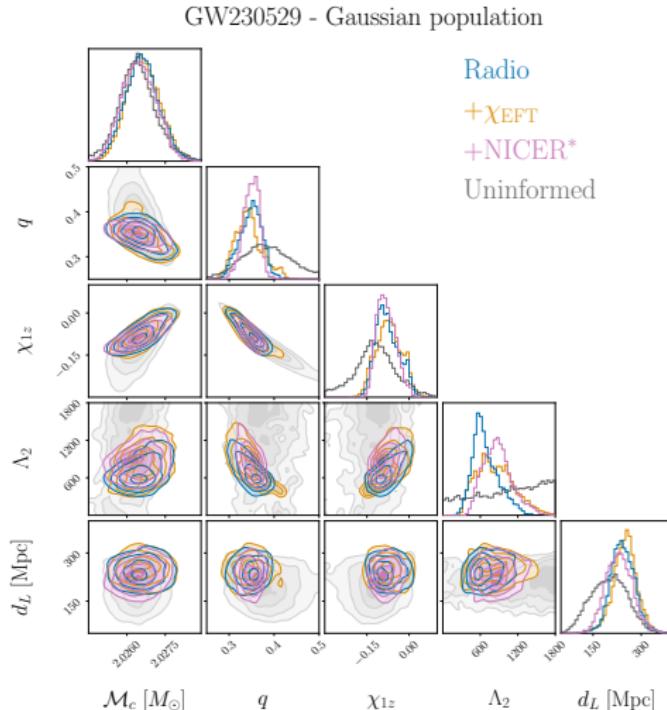
Showing \log_{10} Bayes factors: negative = less preferred

- Decisive evidence for NSBH over BNS
- Weak evidence for population or EOS (low SNR)

| Source | Population | EOS Constraints | GW230529 |
|--------|-----------------|-----------------|----------|
| BNS | Uniform | Radio | -13.23 |
| | | + χ EFT | -13.31 |
| | | +NICER | -13.23 |
| | Gaussian | Radio | -18.90 |
| | | + χ EFT | -18.86 |
| | | +NICER | -18.88 |
| | Double Gaussian | Radio | -13.84 |
| | | + χ EFT | -13.79 |
| | | +NICER | -13.98 |
| NSBH | Uniform | Radio | -0.16 |
| | | + χ EFT | -0.28 |
| | | +NICER | -0.42 |
| | Gaussian | Radio | -0.28 |
| | | + χ EFT | -0.28 |
| | | +NICER | ref. |
| | Double Gaussian | Radio | -0.18 |
| | | + χ EFT | -0.08 |
| | | +NICER | -0.06 |

GW230529 – parameter constraints

- Mass ratio more constrained $\rightarrow \chi_{1z}$ more constrained
- Higher distances



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Conclusion

- Progress on scalable Bayesian inference, with minimal compromises
- Accelerate likelihood-based inference with
 - JAX: GPU for faster likelihoods, automatic differentiation
 - Normalizing flows to aid in inference (sampling, priors)
- GWs: verified for LVK, work in progress for ET
 - Waveform experts more than welcome!
- Kilonova/GRB: emulators for fast inference
- EOS: scalable inference, open up systematics studies

Let's talk!



Thanks for listening!

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Evidence calculation: HARMONIC I

Evidence Z can be computed from posterior samples with HARMONIC [22] with the **harmonic mean estimator**

$$\begin{aligned}\rho &\equiv \mathbb{E}_{P(\theta|d)} \left[\frac{1}{L(\theta)} \right] \\ &= \int d\theta \frac{1}{L(\theta)} P(\theta|d) \\ &= \int d\theta \frac{1}{L(\theta)} \frac{\mathcal{L}(\theta)\pi(\theta)}{Z} = \frac{1}{Z}\end{aligned}$$

Therefore, estimate ρ with posterior samples:

$$\hat{\rho} = \frac{1}{N} \sum_{i=1}^N \frac{1}{L(\theta_i)}, \quad \theta_i \sim P(\theta|d)$$

Evidence calculation: HARMONIC II

Can be interpreted as importance sampling

$$\rho = \int d\theta \frac{1}{Z} \frac{\pi(\theta)}{P(\theta|d)} P(\theta|d),$$

but with target = prior and sampling density = posterior. Therefore, importance sampling is inefficient – how to solve?

New proposal:

$$\begin{aligned}\rho &= \mathbb{E}_{P(\theta|d)} \left[\frac{\varphi(\theta)}{\mathcal{L}(\theta)\pi(\theta)} \right] \\ &= \int d\theta \frac{\varphi(\theta)}{\mathcal{L}(\theta)\pi(\theta)} P(\theta|d) \\ &= \int d\theta \frac{\varphi(\theta)}{\mathcal{L}(\theta)\pi(\theta)} \frac{\mathcal{L}(\theta)\pi(\theta)}{Z} = \frac{1}{Z}\end{aligned}$$

Evidence calculation: HARMONIC III

Use the following estimator:

$$\hat{\rho} = \frac{1}{N} \sum_{i=1}^N \frac{\varphi(\theta_i)}{\mathcal{L}(\theta_i)\pi(\theta_i)}, \quad \theta_i \sim P(\theta|d)$$

Replace the target distribution π with φ : only requirement is that it is normalized

In practice, this can be achieved with a normalizing flow [23].

This has been verified to give accurate evidences (similar values as nested sampling) when GW posteriors are used [24].

HARMONIC with JIM [24]

Table 1: Total wall times to compute the evidence estimates for the examples discussed in the main text. We run BILBY on 16 CPU cores and JIM + harmonic on 1 GPU.

| Example | Method | $\log(z)$ | Sampling time | Evidence estimation time |
|---------|----------------|-----------------------------|---------------|--------------------------|
| 4D | BILBY | 390.33 ± 0.11 | 31.3 min | — |
| | JIM + harmonic | $390.360^{+0.006}_{-0.006}$ | 3.4 min | 1.9 min |
| 11D | BILBY | 378.29 ± 0.15 | 3.5 h | — |
| | JIM + harmonic | $378.420^{+0.09}_{-0.08}$ | 11.8 min | 2.4 min |

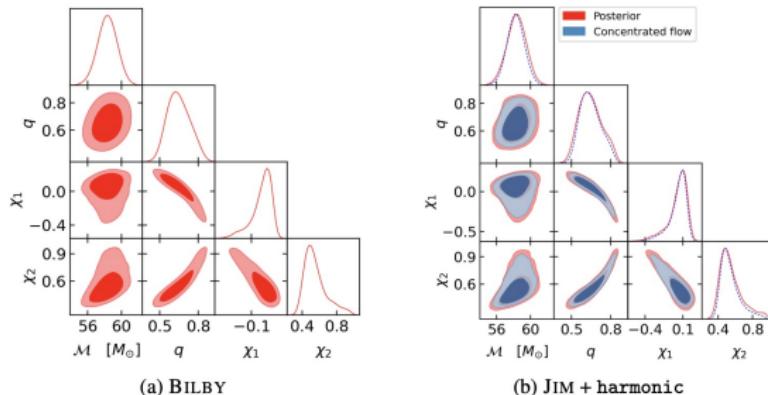
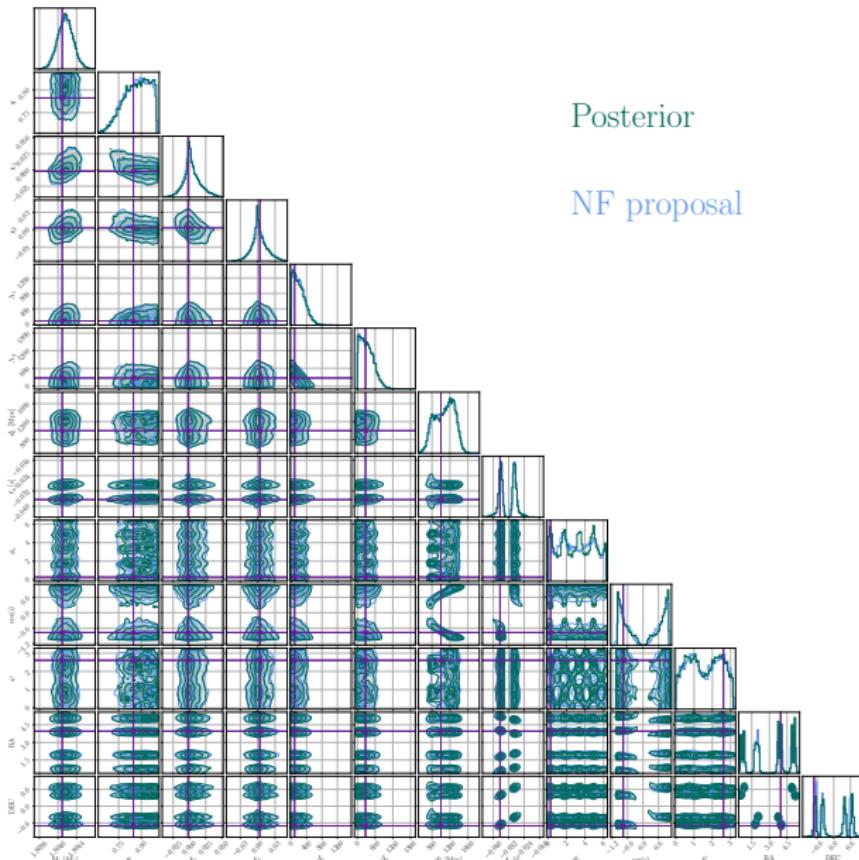
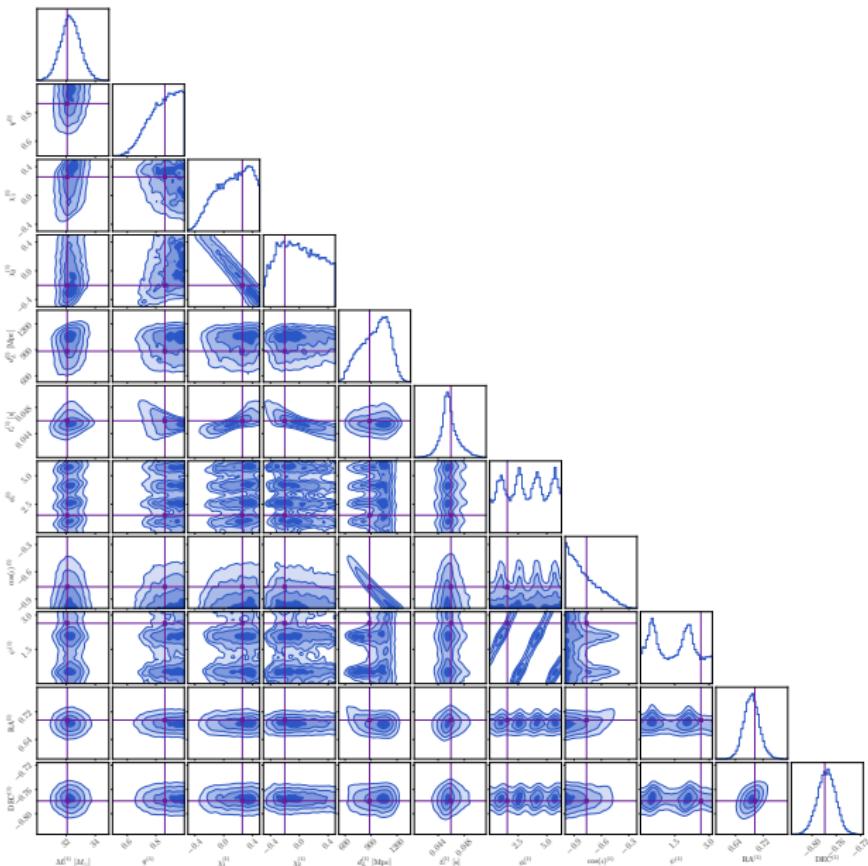


Figure 1: Corner plots for the 4-dimensional posterior samples from (a) BILBY and (b) JIM used for inference (solid red) alongside the concentrated flow at $T = 0.8$ used in the learned harmonic mean (dashed blue).

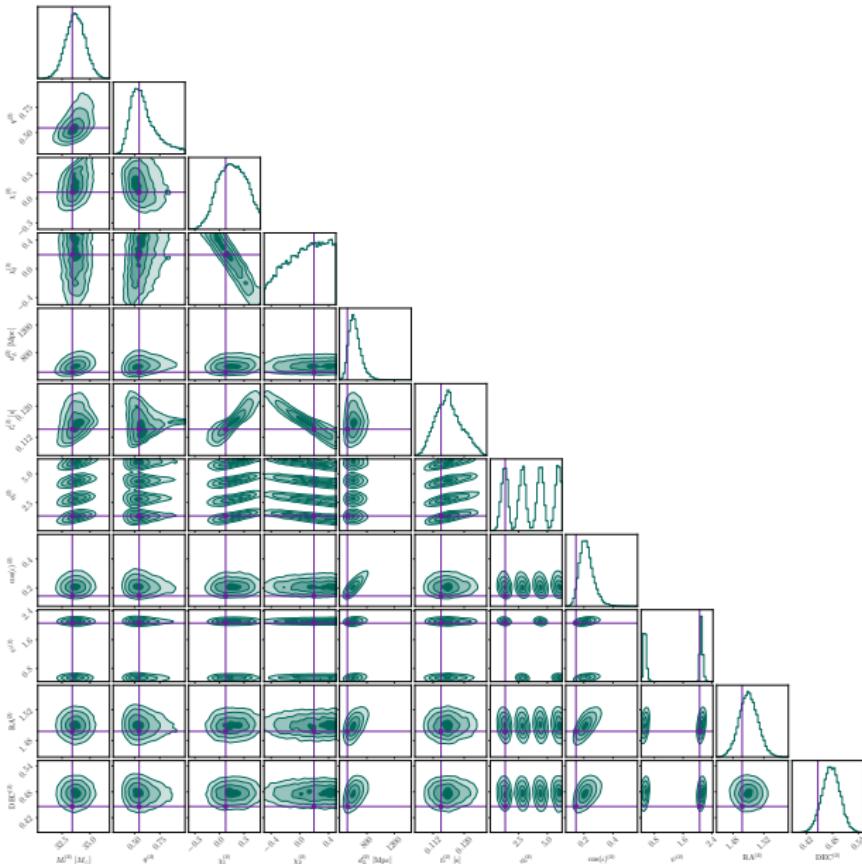
BNS in ET- Δ example: all parameters



Overlapping signals: all parameters signal A

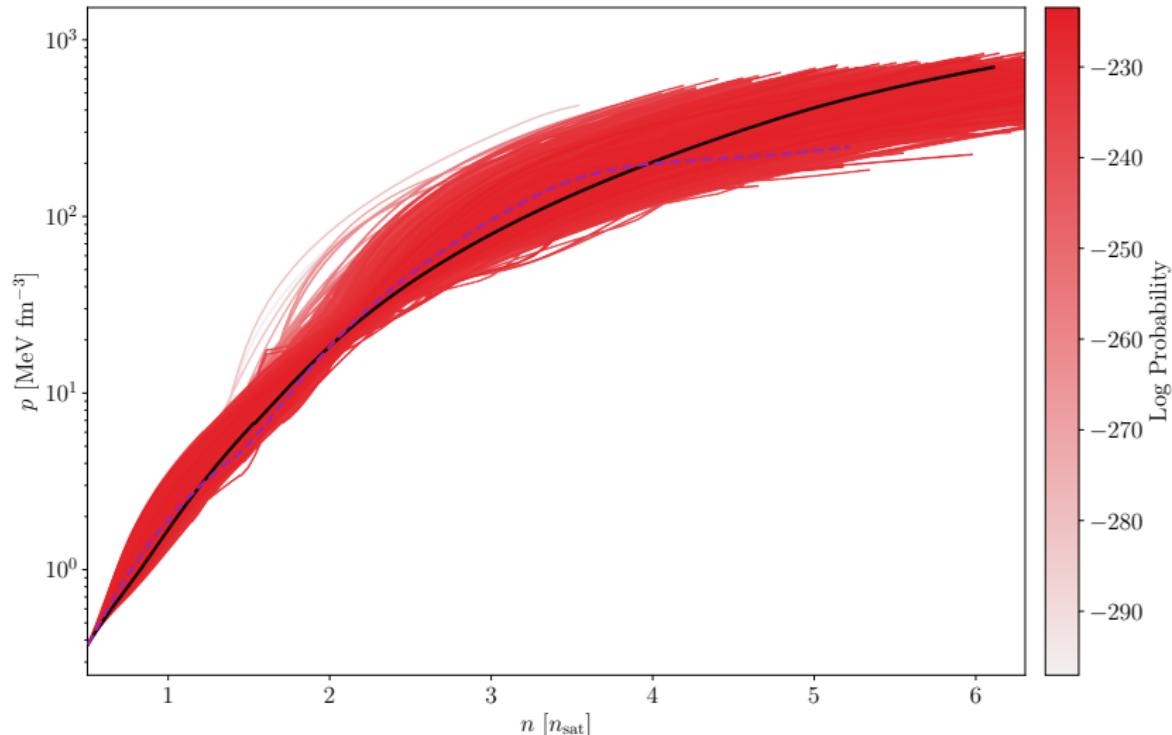


Overlapping signals: all parameters signal B



Equation of state O5 projection with 20 BNS: EOS

- **Purple:** target
- **Red:** posterior EOS samples (**black:** maximum log posterior)



Equation of state O5 projection with 20 BNS: NS

