

Towards GPU-accelerated multimessenger inference of neutron star mergers and dense matter physics

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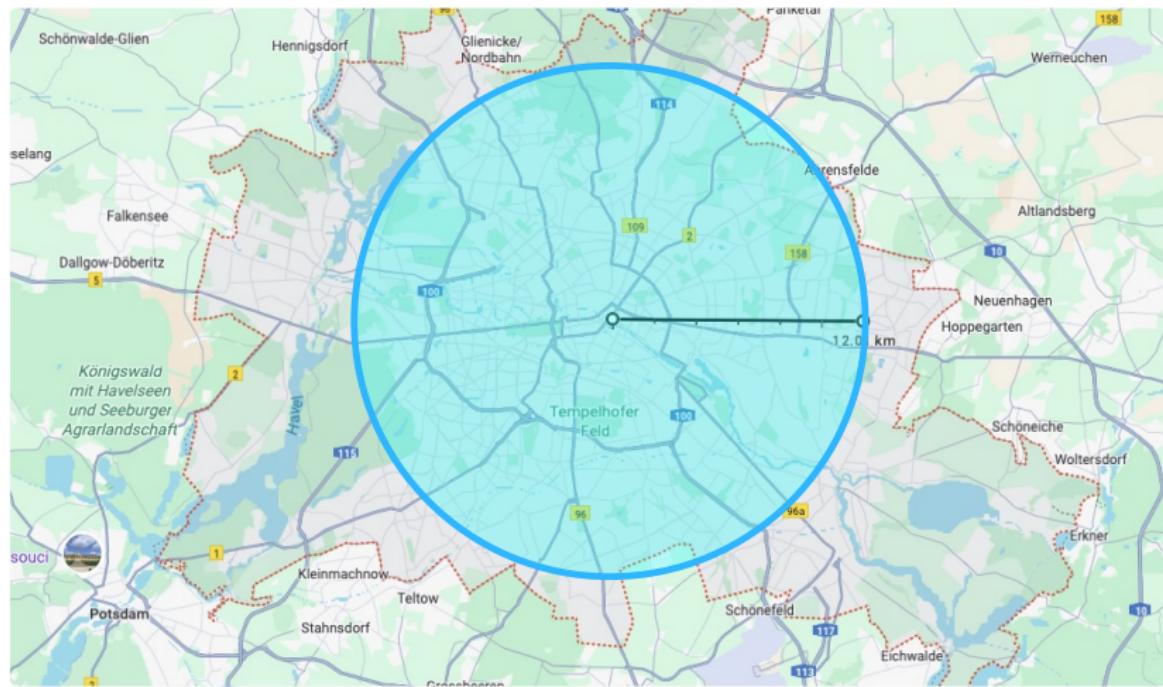
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Neutron stars

- Neutron stars: supernova remnants, densest matter in the universe
- $m \sim 1.2 - 2.3M_{\odot}$, $R \sim 10 - 13$ km

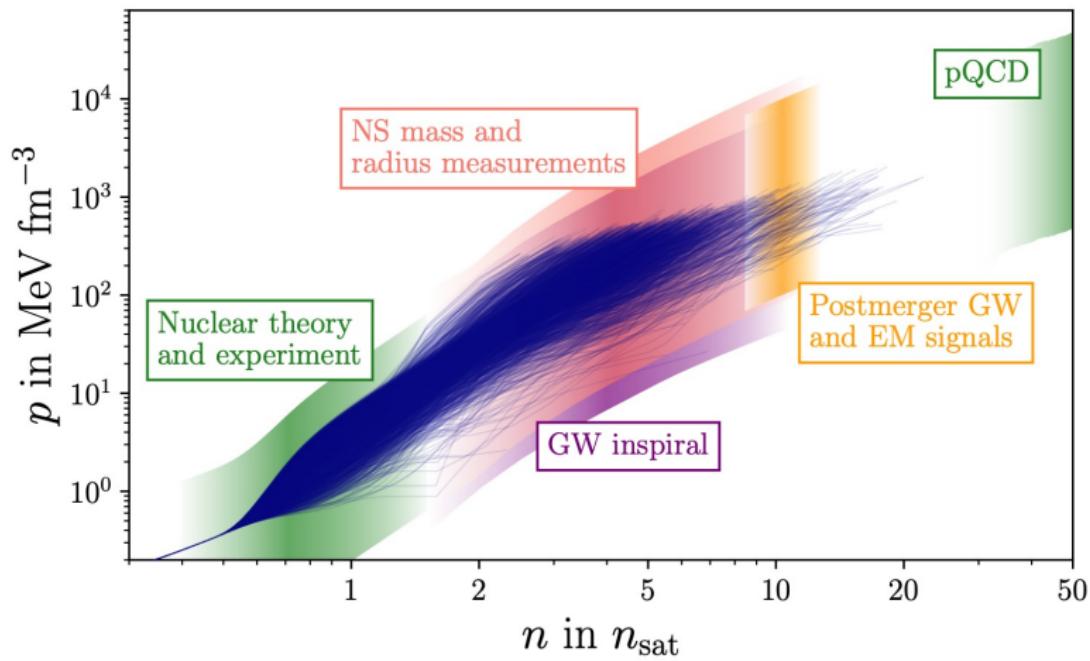
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Equation of state

Neutron stars probe the high-density part of the equation of state (EOS) of dense nuclear matter [1]



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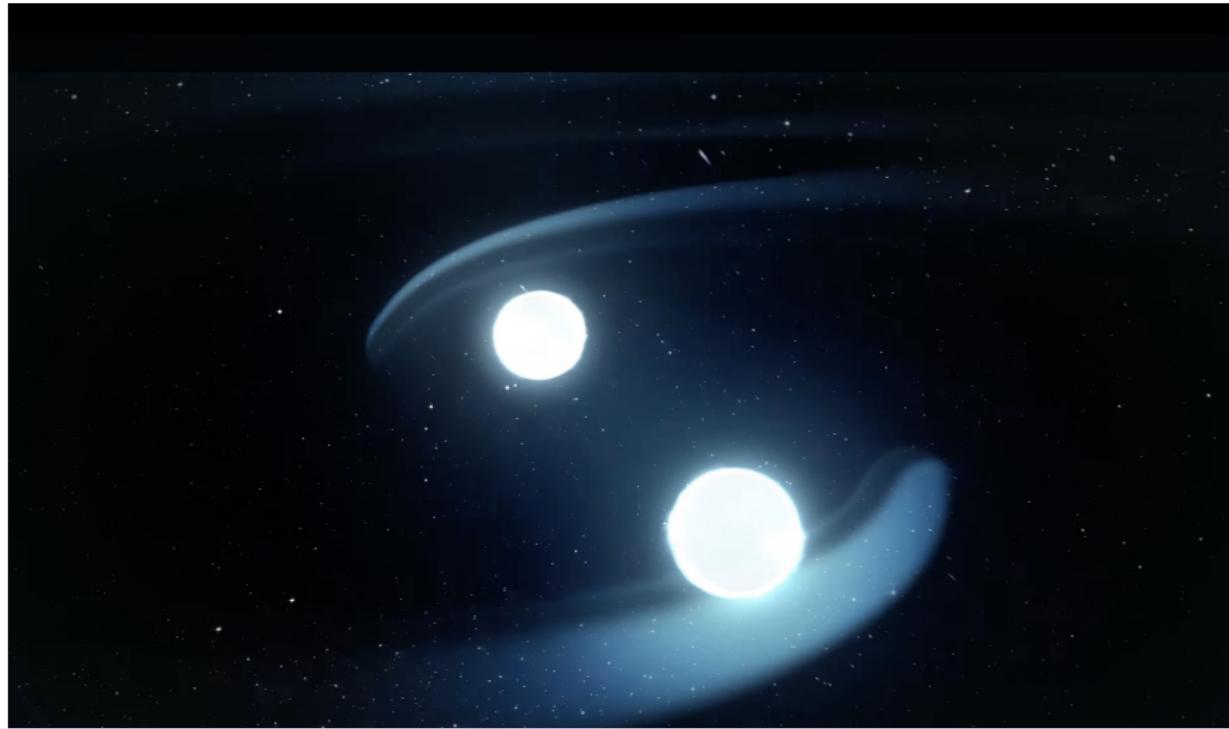
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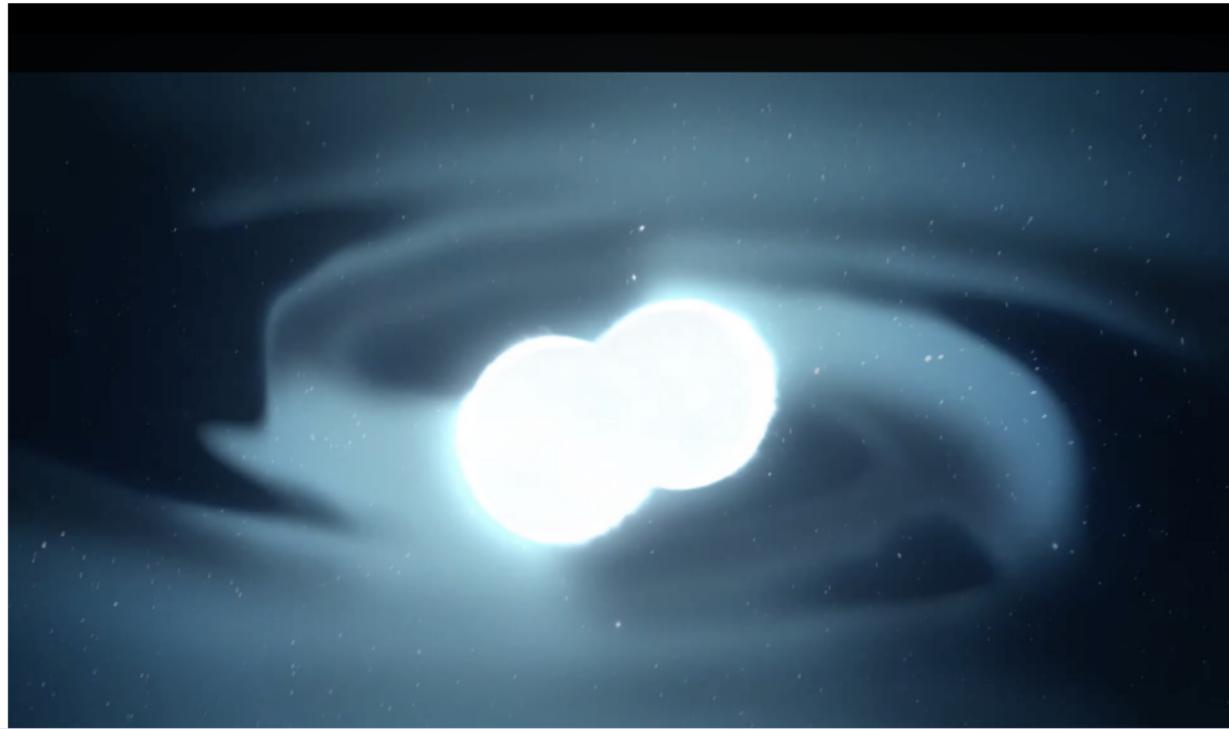
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Neutron star mergers emit gravitational waves and electromagnetic radiation: GW170817 [2, 3]



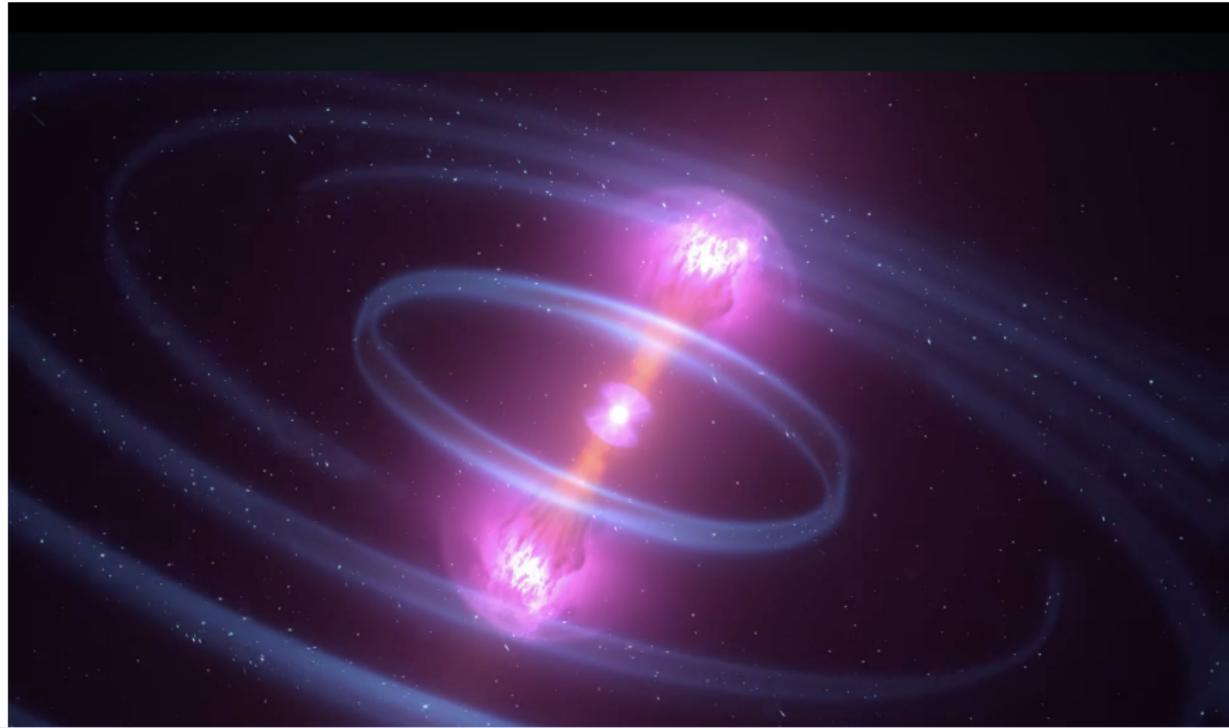
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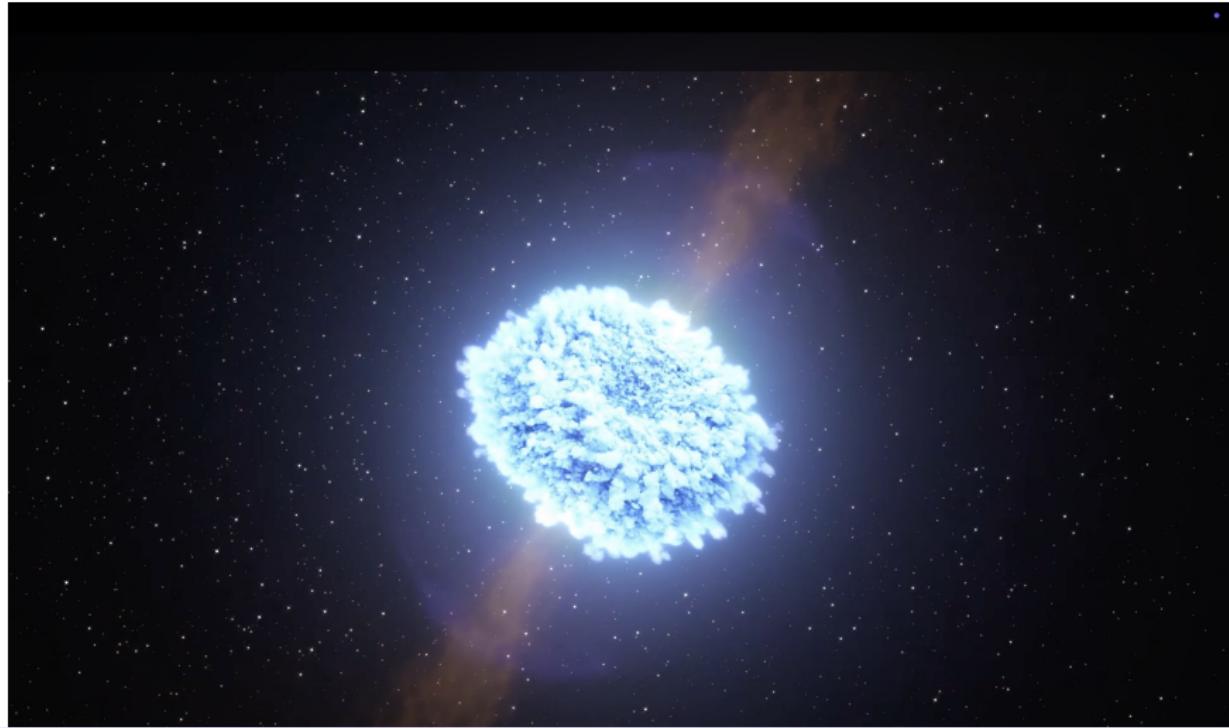
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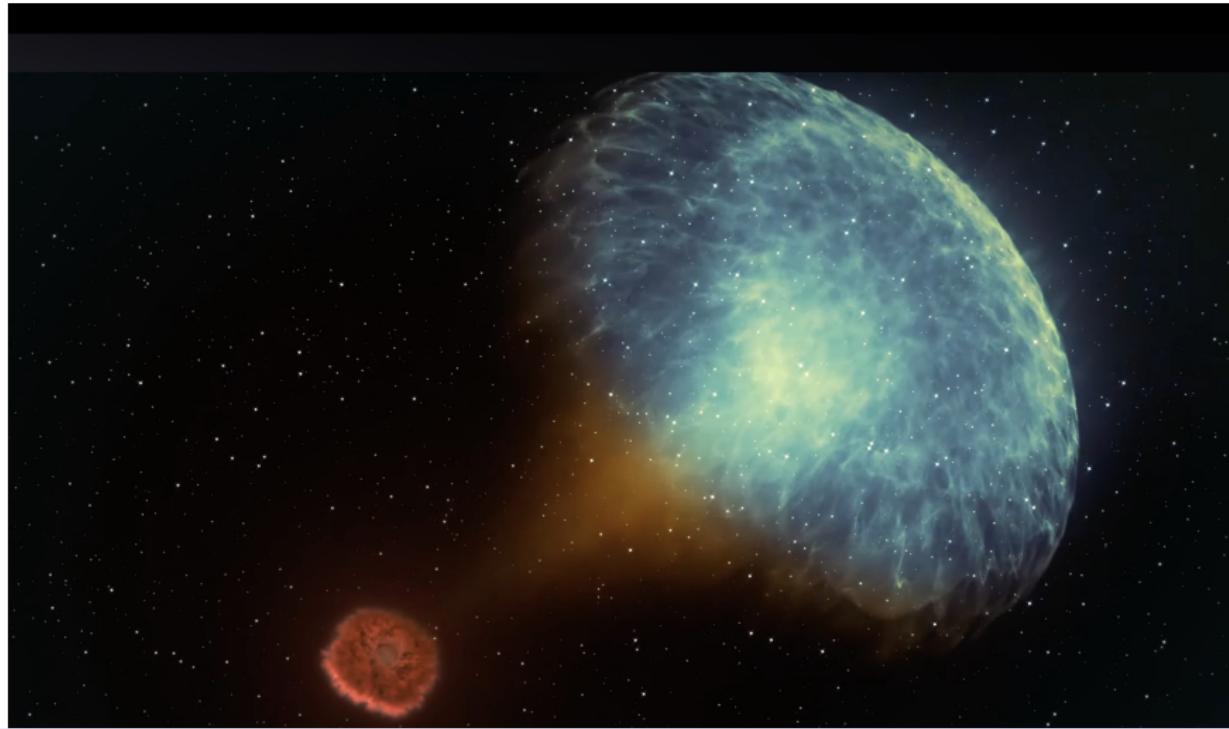
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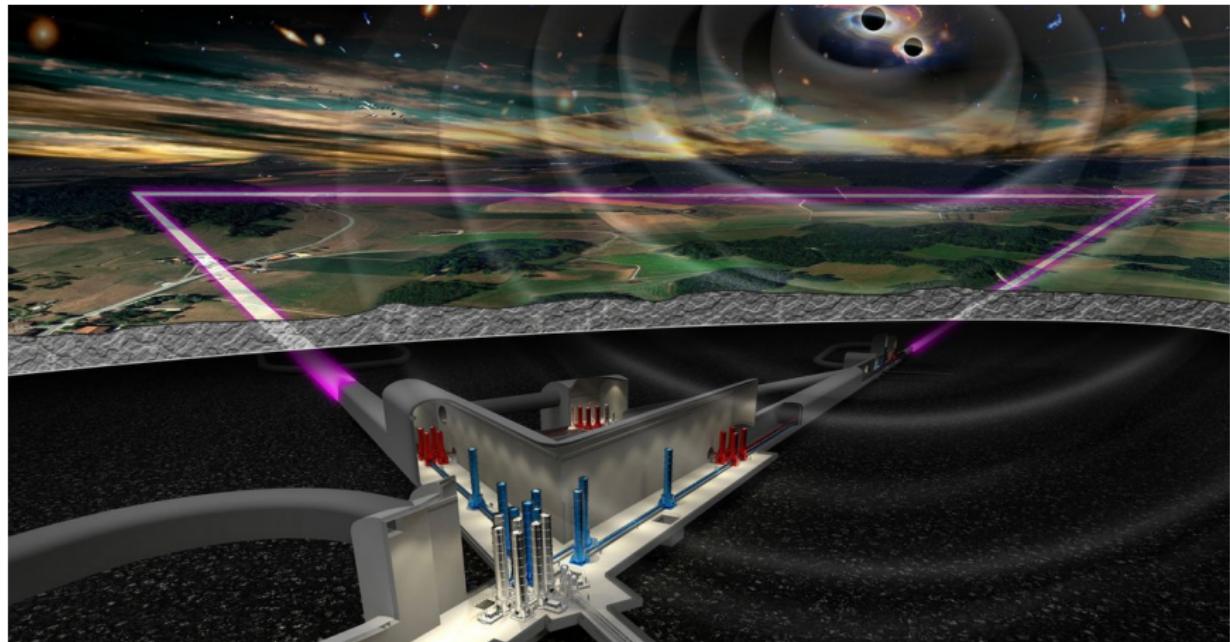
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Future GW detectors: Einstein Telescope

Einstein Telescope: Third-generation ground-based GW detector [4, 5]



(or 2 L-shaped detectors...)

Future GW detectors: Einstein Telescope

Einstein Telescope: Third-generation ground-based GW detector [4, 5]

- Increased sensitivity
 - Louder signals
 - $10^5 - 10^6$ binary black hole mergers/year (now: $\sim 200/10$ years)
 - $10^4 - 10^5$ binary neutron star mergers/year (now: $2/10$ years)
 - $10^2 - 10^3$ multimessenger events/year (now: $1/10$ years)

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 - Longer signals
 - Binary neutron star: 2 hours (vs 2 minutes now)
 - Signals will overlap: joint analysis needed

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Current software cannot handle the ET data analysis problem [6]

My research focus: why – how – what

Why?

To make inference of multimessenger astrophysics scalable

- Prepare for future detectors
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What?

GPU-accelerated Bayesian joint inference framework of neutron star mergers and dense matter physics

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Parameter estimation: Bayesian inference

How do we “measure” source parameters θ for data d ?

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Bayesian inference:

$$\text{posterior} = p(\theta|d, \mathcal{M}) = \frac{p(d|\theta, \mathcal{M})p(\theta|\mathcal{M})}{p(d|\mathcal{M})} = \frac{\text{likelihood} \times \text{prior}}{\text{evidence}}$$

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Ingredients:

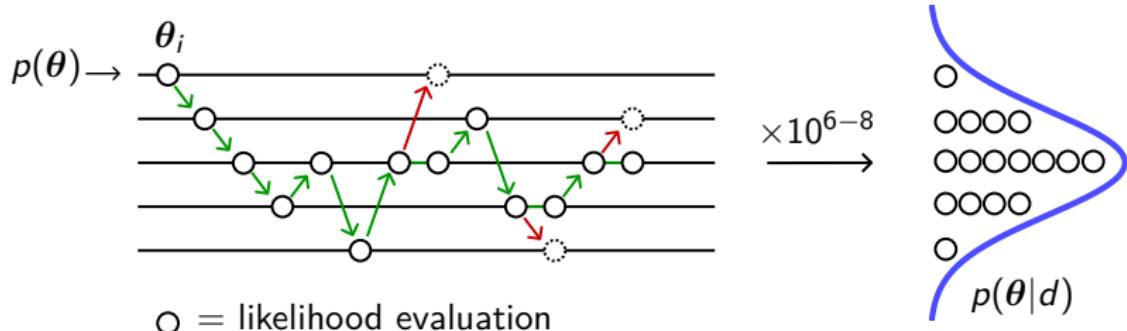
- Posterior: intractable, needs stochastic samplers
- Prior: specified by users, encode beliefs
- Likelihood: computational bottleneck
- Evidence: model selection

Parameter estimation: Markov chain Monte Carlo

How do we sample the posterior? **Markov chain Monte Carlo**

- N chains θ_i explore posterior in parallel
- Evolve chains to new position: proposal
- Compute likelihood \rightarrow accept/reject

Alternatives: nested sampling, sequential Monte Carlo



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- $N_{\text{likelihood}}$: total number of likelihood evaluations
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 - Multimodality/shape posterior
 - Efficiency of sampler (proposals)

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How can we optimize this?

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In particular:

- Python: integrate into existing workflows
- Focus on arrays: widespread applications
- Numpy-like API: (`numpy` → `jax.numpy`)

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In particular:

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- Numpy-like API: (numpy → jax.numpy)

Gamechangers:

- GPU accelerators
- Composable function transformations: `jit`, `grad`, `vmap` & `pmap`

JAX – Function transformations

- `jit`: Just-in-time compilation

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JAX – Function transformations

- `jit`: Just-in-time compilation
- `grad`: Automatic differentiation:
 - Chain rule on computational graph
 - Exact gradients (up to machine precision), fast
- `vmap`, `pmap`: Vectorization, batch processing, parallelization
- These are composable, e.g.:
 - Higher-order derivatives: `grad(grad(f))`
 - Evolve N chains in parallel along gradient of the likelihood

$$\theta \leftarrow \theta + \alpha * \text{vmap}(\text{jit}(\text{grad}(\text{logL}(\theta))))$$

Computational aspects

$$\text{Total runtime} \approx N_{\text{likelihood}} \times \tau_{\text{likelihood}}$$

- $N_{\text{likelihood}}$: total number of likelihood evaluations
 - Dimensionality of θ
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Normalizing flows

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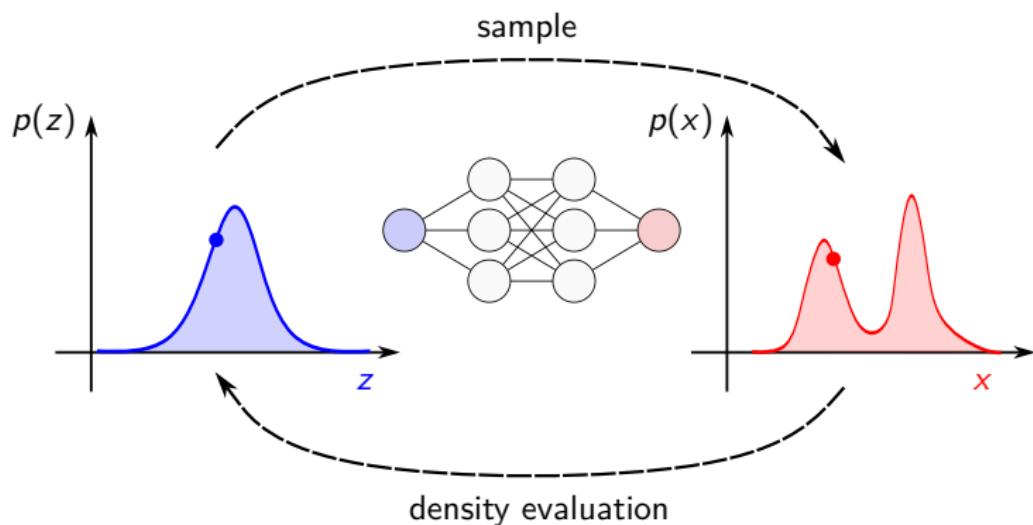
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- Generate new samples

Normalizing flows

Q: Given samples $\{x\} \sim p(x)$, how can we get $p(x)$?

- Evaluate density at any point
- Generate new samples

A: Normalizing flows: generative machine learning model, bijection between **latent** space (Gaussian) and **data** space



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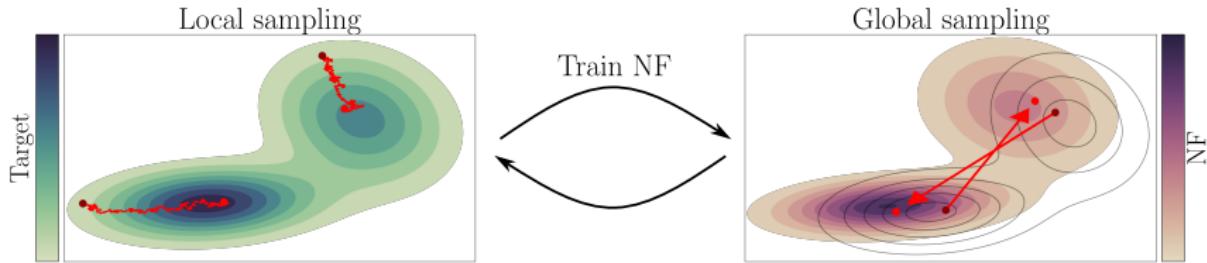
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FLOWMC

FLOWMC  [8, 9]: normalizing flow-enhanced MCMC

- ① Run gradient-based sampler (local sampler)
- ② Train normalizing flow on MCMC chains
- ③ Propose samples from the normalizing flow (global sampler)
- ④ Repeat



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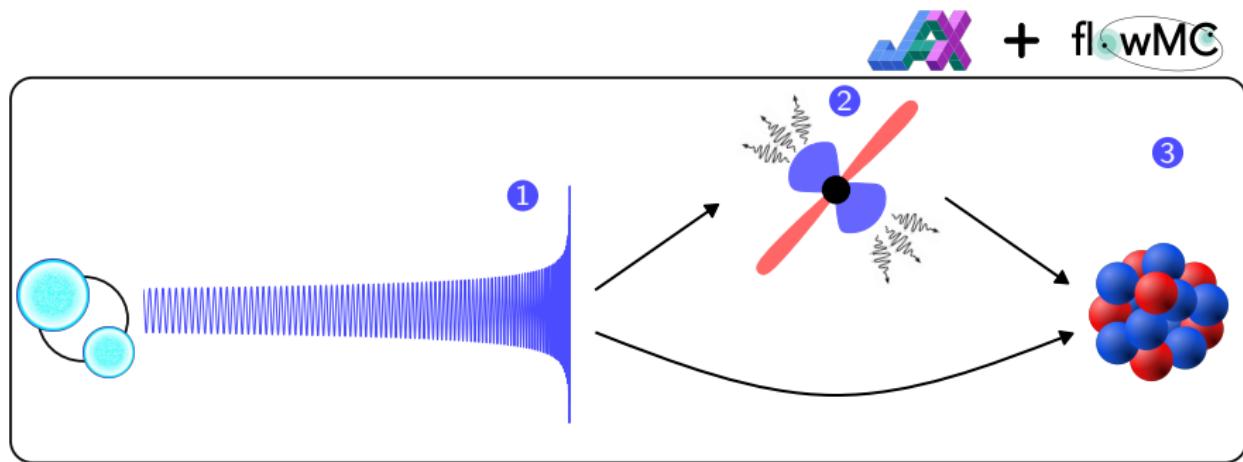
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Overview

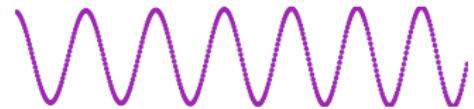
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- ① Gravitational waves
- ② Electromagnetic counterparts
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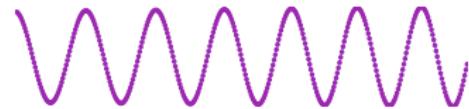
Gravitational waves: RIPPLE

- Waveforms on GPU: $\mathcal{O}(10^3)$ faster
- RIPPLE [10]: from LALSUITE to JAX



Gravitational waves: RIPPLE

- Waveforms on GPU: $\mathcal{O}(10^3)$ faster
- RIPPLE [10]: from LALSUITE to JAX
- Waveforms available in RIPPLE:
 - Binary black holes:
 - IMRPhenomXAS
 - IMRPhenomD
 - IMRPhenomPv2
 - Binary neutron stars:
 - TaylorF2
 - IMRPhenomD_NRTidalv2
 - IMRPhenomPv_NRTidalv2 (by Nihar Gupte)
- Also see GWFEST [11] and SFTS [12]



Gravitational waves: JIM

Parameter estimation: JIM Ω [13, 14]: RIPPLE + FLOWMC

- Verified for binary neutron stars in current generation detectors [14]
- Matches BILBY, passes *pp*-test

Gravitational waves: JIM

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- Matches BILBY, passes *pp*-test
- Runtime: from hours to minutes

Event	Waveform	JIM	PBILBY	RB-BILBY	ROQ-BILBY
		(1 GPU)	(480 cores)	(24 cores)	(24 cores)
GW170817	TF2	15.33 min	9.64 h	3.8 h	–
	NRTv2	25.59 min	10.99 h	4.11 h	1.65 h
GW190425	TF2	18.30 min	8.18 h	2.81 h	–
	NRTv2	21.20 min	4.91 h	2.42 h	0.97 h
Injection	TF2	24.76 min	–	–	–
	NRTv2	18.02 min	–	–	–

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- Up to $5 - 10 \times$ more effective samples (normalizing flow)

Effective sample size	
JIM	4.1×10^4
PBILBY	6.5×10^3
RB-BILBY	5.8×10^3
ROQ-BILBY	7.4×10^3

Gravitational waves: JIM

Parameter estimation: JIM  [13, 14]: RIPPLE + FLOWMC

- Verified for binary neutron stars in current generation detectors [14]
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- More cost-effective/energy-efficient

	kWh	CO ₂ [10 ³ kg]	Trees
JIM	34	11	0.55
PBILBY	4127	1354	67.68
RB-BILBY	80	26	1.32
ROQ-BILBY	sampling	32	0.52
	precompute	27	0.44

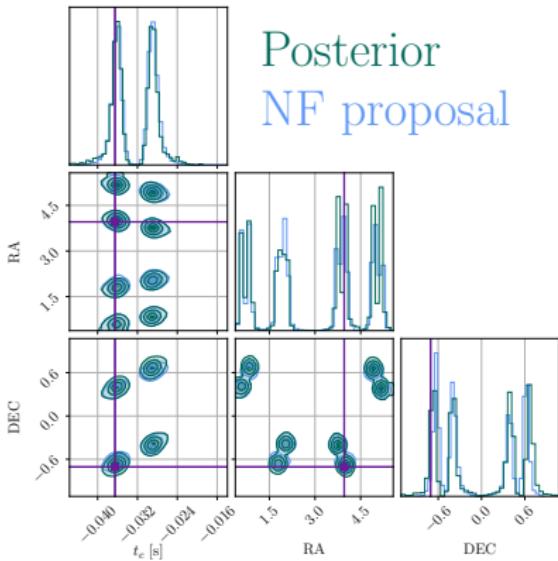
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Work in progress: showing proof-of-concepts!

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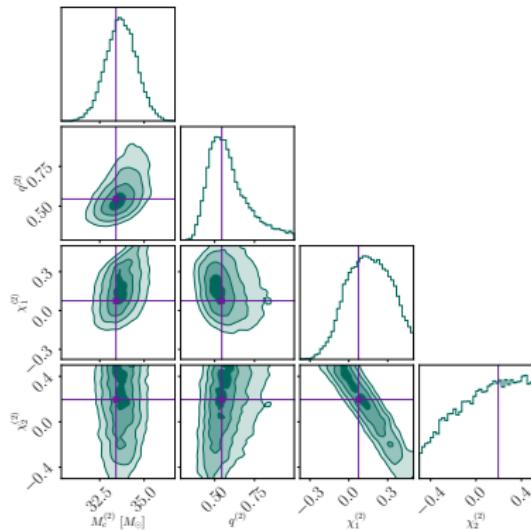
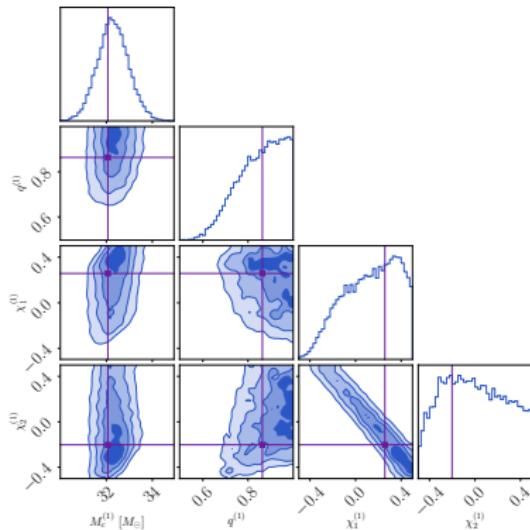
Work in progress: showing proof-of-concepts!

- ET posteriors are multimodal
- Normalizing flows help to jump between modes



Overlapping signals

- Assess scaling of JIM: BBH+BBH with LIGO-Virgo
 - 2 binary black hole mergers: 22 parameters
 - $M_c^{(1)} = 32M_\odot$, $M_c^{(2)} = 33M_\odot$, $\Delta t = 70$ ms
 - $\text{SNR}^{(1)} = 25.76$, $\text{SNR}^{(2)} = 25.24$
 - 1h28m on H100 GPU (vs several days on 16 CPUs [15])



Open call

- Recently: nested samplers in JAX for GW [16]
 - Using RIPPLE waveforms
 - Momentum towards waveforms on GPU?

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- Bridge between LALSUITE and RIPPLE?

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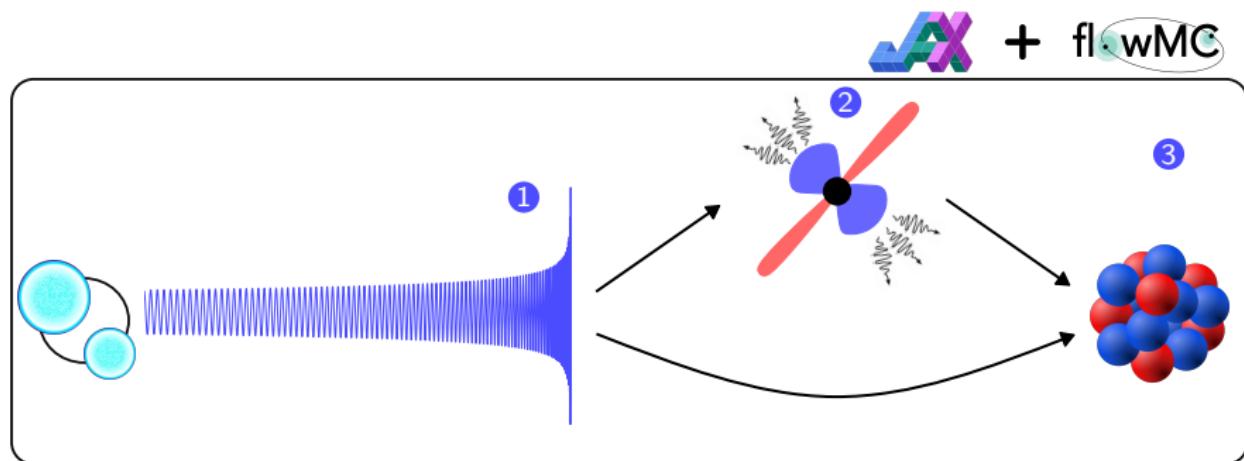
Get in touch if you are interested! We need you!

JIM developers \cap waveform developers = **you**?

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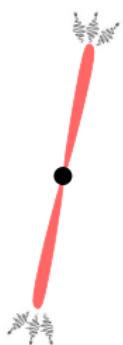
Analyzing a multi-messenger **binary neutron star** signal:

- ① Gravitational waves
- ② **Electromagnetic counterparts**
- ③ Nuclear equation of state



Electromagnetic counterparts (Hauke Koehn)

- Predicting a **GRB afterglow** is slow
- Code libraries too large to 'jaxify': neural network emulators for inference: FIESTA  [17]

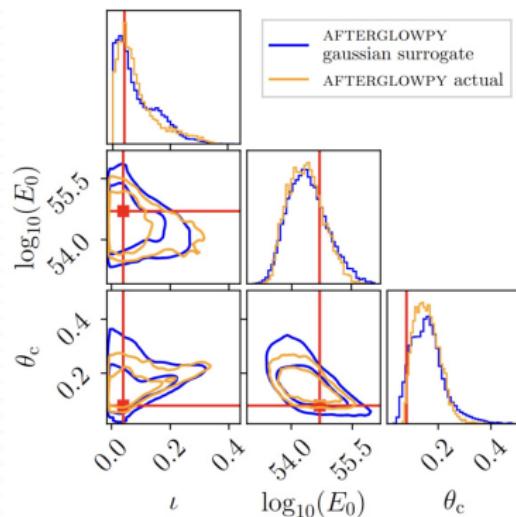


FIESTA

- 1m36s
- 1 H100 GPU

AFTERGLOWPY

- 4 hours
- 30 CPUs



Electromagnetic counterparts (Hauke Koehn)

- Predicting a **GRB afterglow** is slow
- Code libraries too large to ‘jaxify’: neural network emulators for inference: FIESTA  [17]
- Scales well for systematics ‘nuisance parameters’

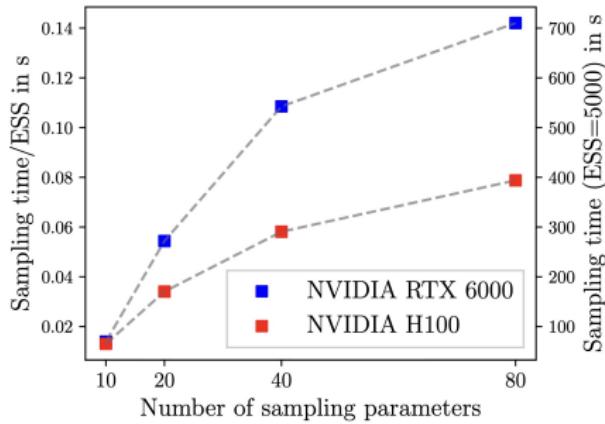


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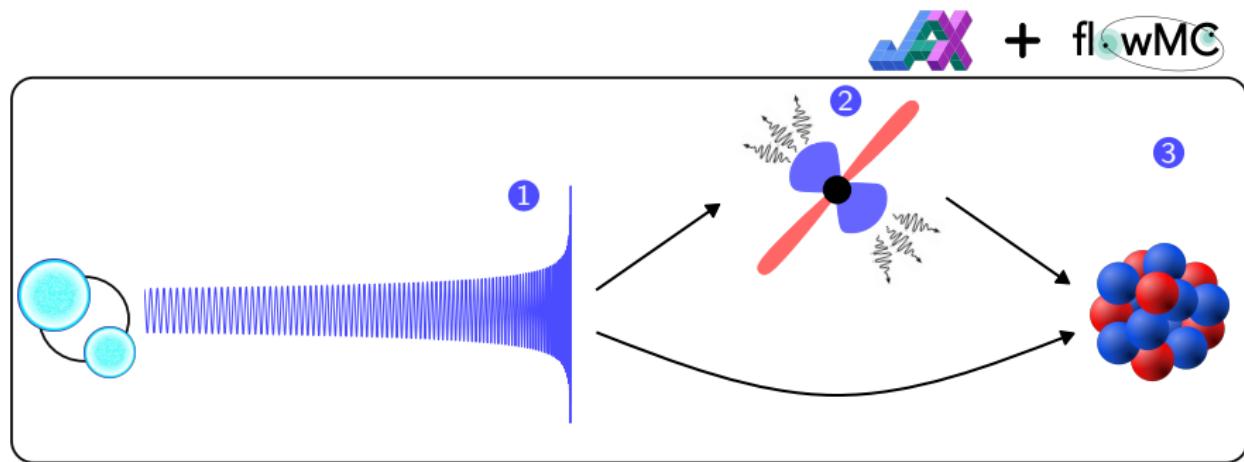
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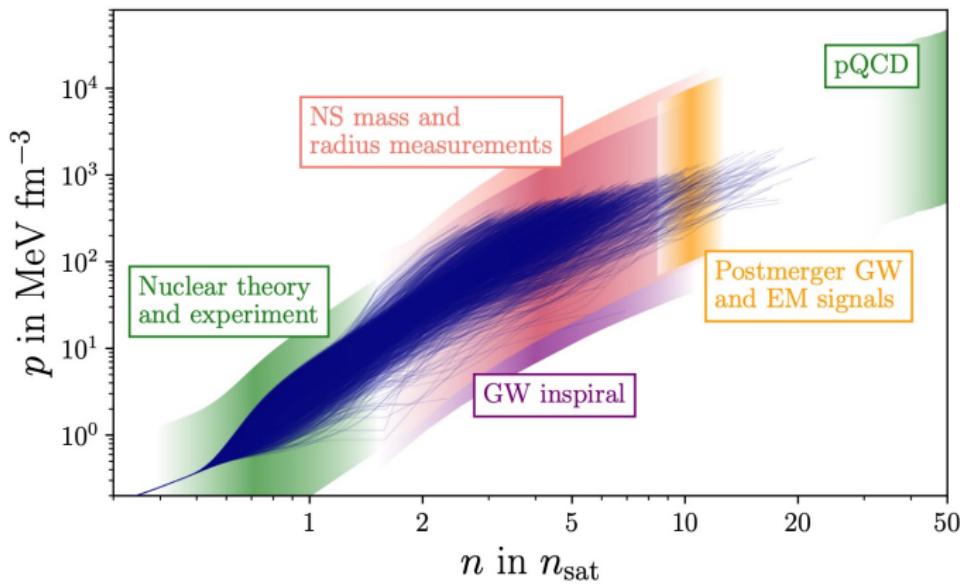
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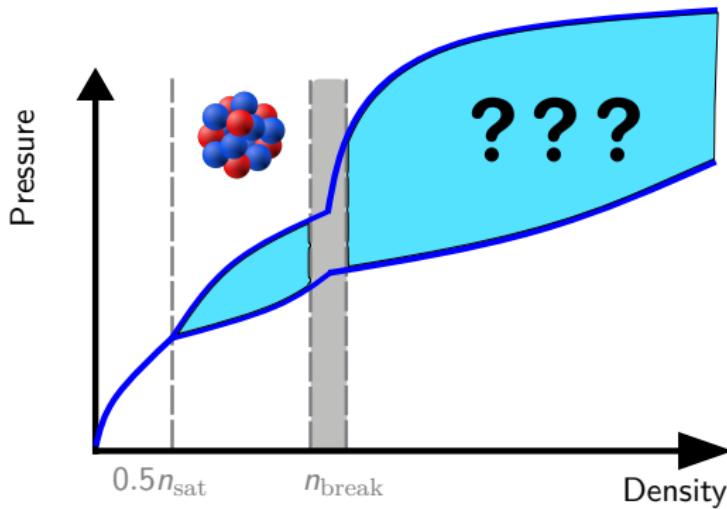
Equation of state inference – warmup

- How do we constrain the EOS from neutron star observations?
- Define parametrization and likelihood



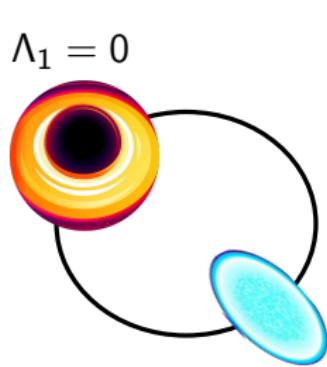
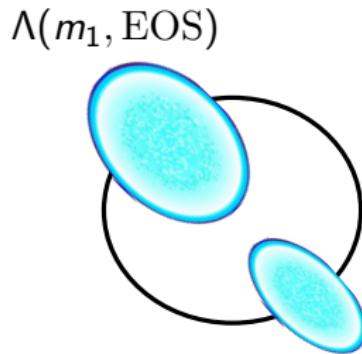
Equation of state inference – parametrization

- $n \leq \frac{1}{2}n_{\text{sat}}$: fixed crust
- $\frac{1}{2}n_{\text{sat}} \leq n \leq n_{\text{break}}$: metamodel EOS, nuclear physics inspired
- $n \leq n_{\text{break}}$: agnostic extension
- > 26 (!) parameters θ_{EOS}



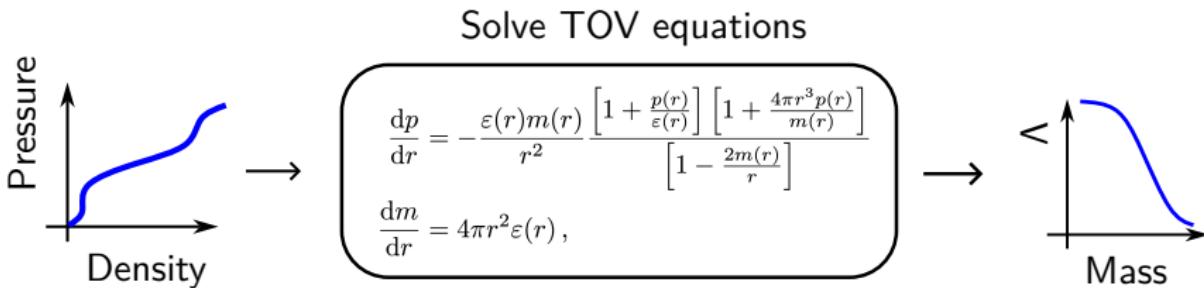
Tidal deformability

- Neutron stars are tidally deformed in a binary, quantified by tidal deformability Λ : affect phase of GWs
- Neutron stars: $\Lambda = \Lambda(m, \text{EOS})$, black holes: $\Lambda = 0$
- GWs give us $p(m_1, m_2, \Lambda_1, \Lambda_2 | d)$ for EOS inference


$$\Lambda(m_2, \text{EOS})$$

$$\Lambda(m_2, \text{EOS})$$

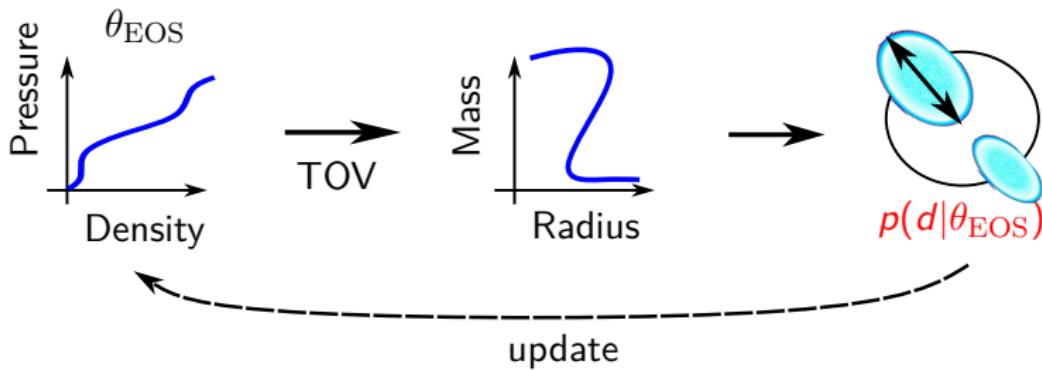
Equation of state

- To predict neutron star properties, we solve the TOV equations: ordinary differential equations (ODEs)



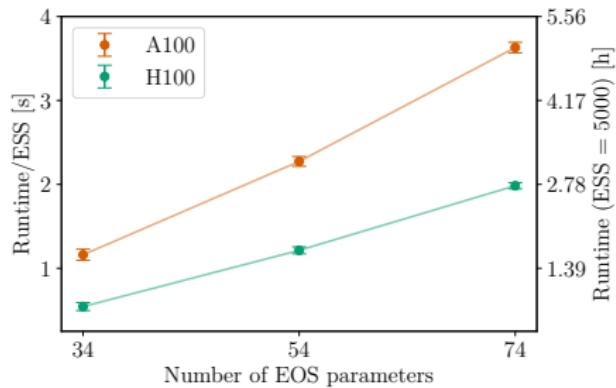
Equation of state

- To predict neutron star properties, we solve the TOV equations: ordinary differential equations (ODEs)
- Done for each sample θ_{EOS} : **costly likelihood**
- How to make this scalable without compromises?

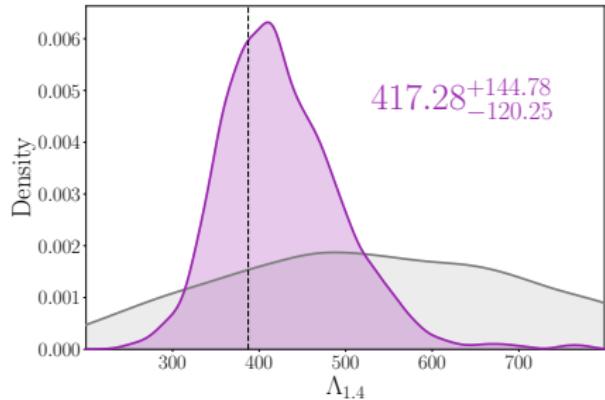
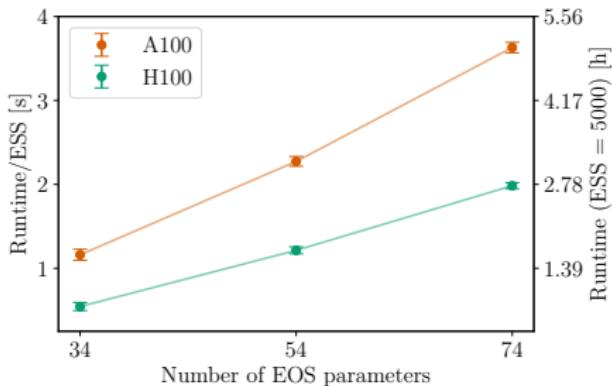


JESTER

- JESTER  [18]: JAX-based TOV solver
 - $1000\times$ faster, without compromises
 - Full inference in \sim hours

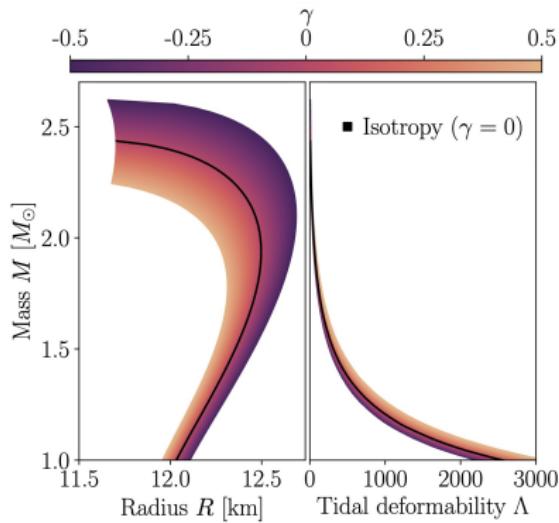


- JESTER  [18]: JAX-based TOV solver
 - $1000\times$ faster, without compromises
 - Full inference in \sim hours
- JIM+JESTER: from GWs to EOS in a few hours
 - Example: 20 binary neutron stars in O5
- Enable systematics studies in EOS inference



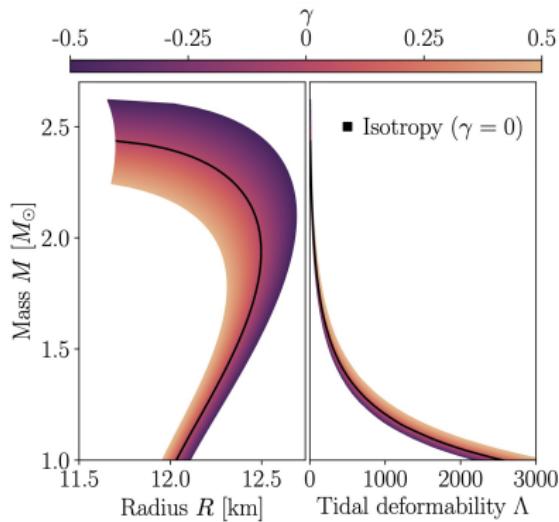
Anisotropy in neutron stars (Peter T. H. Pang)

- Anisotropic pressure: $\gamma \propto p - p_t$



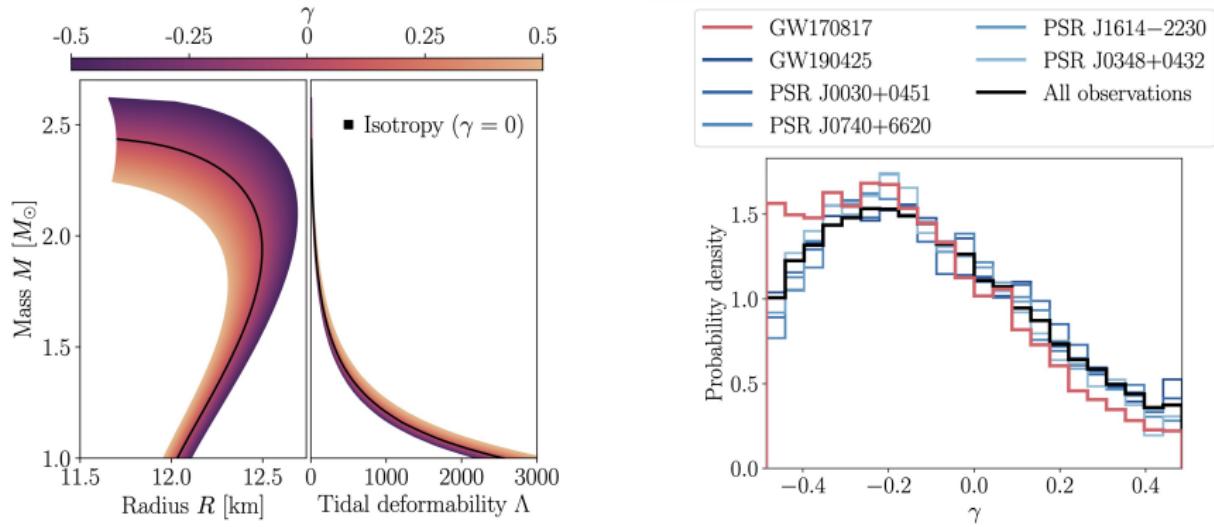
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- Magnetic fields, dark matter clusters, superfluids...



Anisotropy in neutron stars (Peter T. H. Pang)

- Anisotropic pressure: $\gamma \propto p - p_t$
- Magnetic fields, dark matter clusters, superfluids...
- Preference for negative anisotropy, but weak evidence [19]



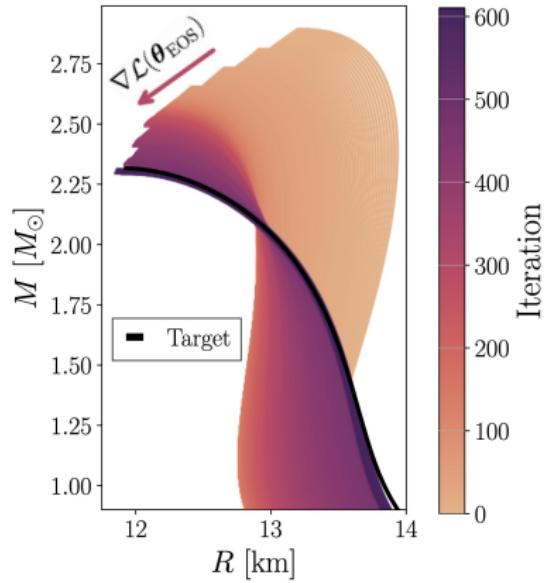
Auto-differentiable ODE solvers

- ODE solvers written in JAX are auto-differentiable
- Frame inference as optimization problem:
 - Gradient descent on loss function $\mathcal{L}(\theta_{\text{EOS}})$

Auto-differentiable ODE solvers

- ODE solvers written in JAX are auto-differentiable
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$$\mathcal{L}(\theta_{\text{EOS}}) = \frac{1}{N} \sum_{i=1}^N \left| \frac{R_i(\theta_{\text{EOS}}) - \hat{R}_i}{\hat{R}_i} \right|$$



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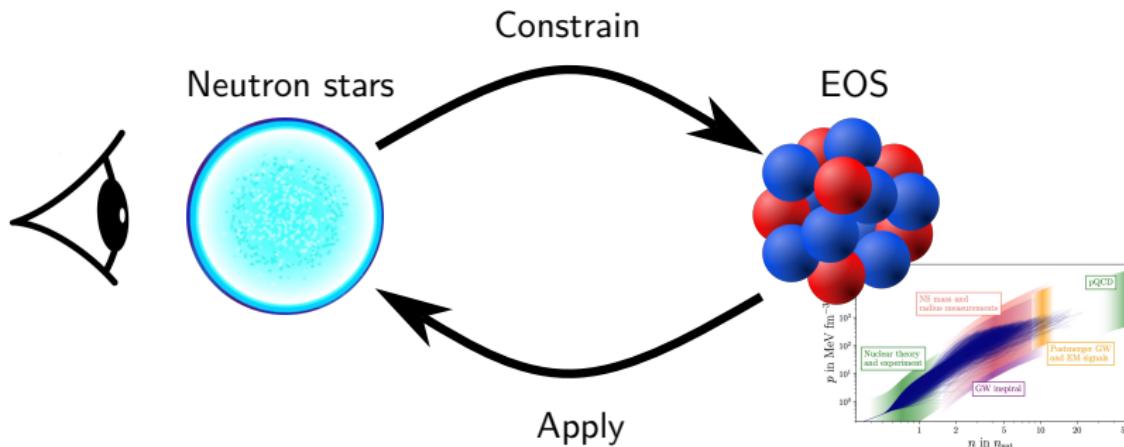
⑤ Conclusion

Neutron star data analysis loop

Data analysis of neutron stars forms a **loop**:

- ① Constraining the EOS with neutron star observations
- ② Applying EOS knowledge in neutron star data analysis (e.g., GW)

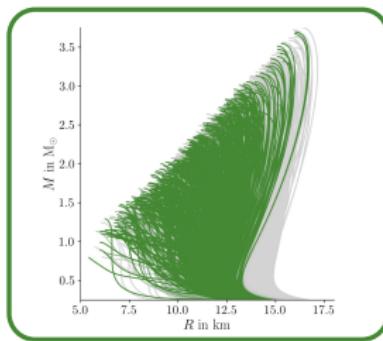
How can we efficiently perform **Step 2**?



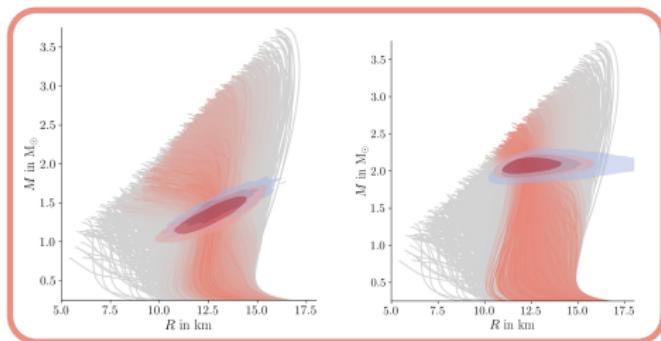
Case study

- Take constraints on EOS from:
 - **Radio timing:** EOS must support $2 M_{\odot}$ neutron stars
 - **Chiral EFT:** nuclear theory predictions (valid $< 2n_{\text{sat}}$)
 - **NICER:** Mass-radius observations of neutron stars

Chiral EFT



NICER



- How can we use this information in GW analyses?

Equation of state-informed priors

- This should enter the prior on Λ_i

$$\mathcal{P}(\theta_{\text{GW}}|d) \propto \mathcal{L}(d|\theta_{\text{GW}})\pi(\theta_{\text{GW}})$$

- By default, we choose **agnostic priors**: e.g. $\Lambda_{1,2} \sim \mathcal{U}(0, 5000)$

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Equation of state-informed priors

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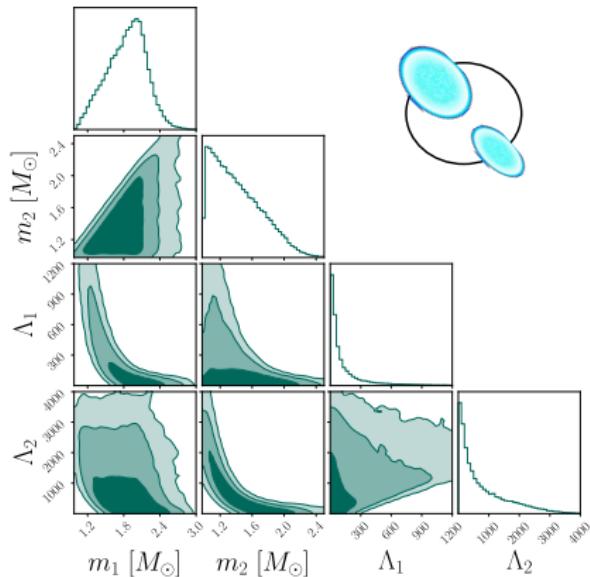
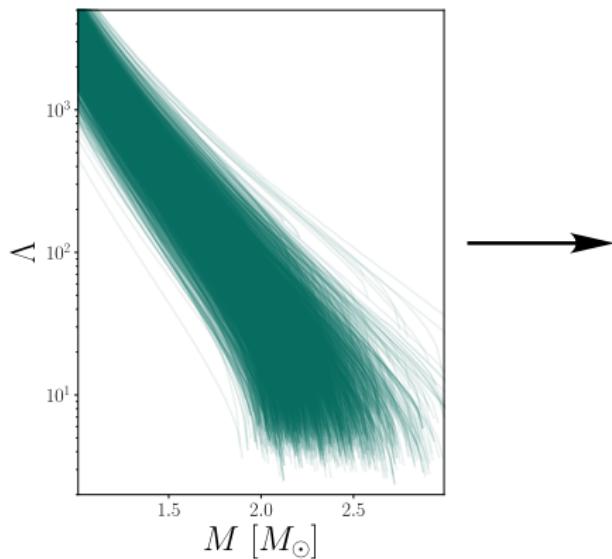
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- **But**, we have prior knowledge from the EOS:
 - Masses m_i determined by M_{\max} (or population)
 - $\Lambda_i = \Lambda_i(m_i, \text{EOS})$
- **Equation of state-informed prior**: take **EOS uncertainty** into account

$$\begin{aligned}\pi(m_1, m_2, \Lambda_1, \Lambda_2) &= \int d\theta_{\text{EOS}} \pi(m_1, m_2 | \theta_{\text{EOS}}) \pi(\Lambda_1, \Lambda_2 | m_1, m_2, \theta_{\text{EOS}}) \\ &\quad \times \pi(\theta_{\text{EOS}})\end{aligned}$$

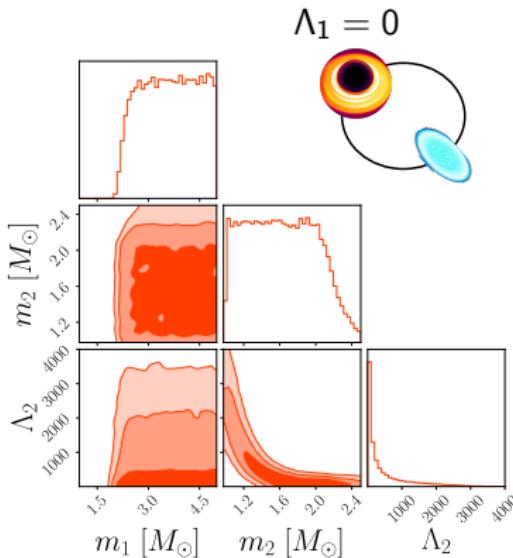
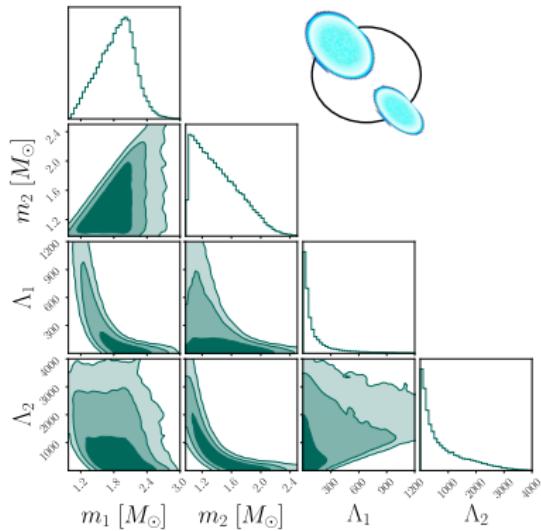
Equation of state-informed priors

- Example: EOSs with $M_{\text{max}} > 2.0M_{\odot}$
- Sample to produce $\pi(m_1, m_2, \Lambda_1, \Lambda_2)$



Source classification

- Similar to the **binary neutron star (BNS)** prior, we can also construct a **neutron star-black hole (NSBH)** prior
 - BH mass in $[M_{\text{max}}(\text{EOS}), 5 M_{\odot}]$
- Classify events with Bayesian model selection

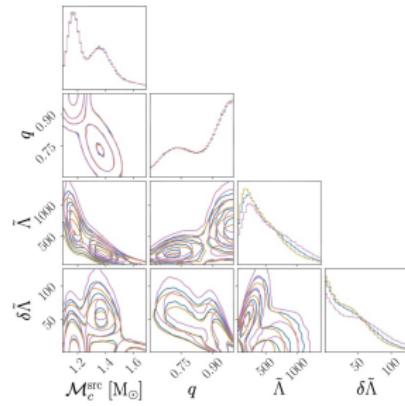
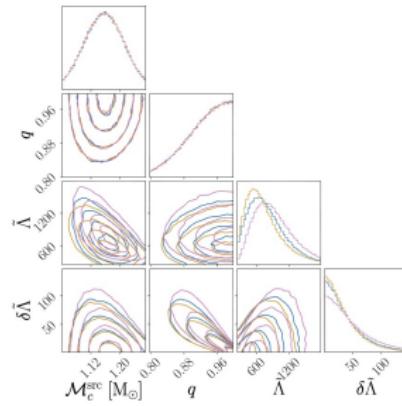
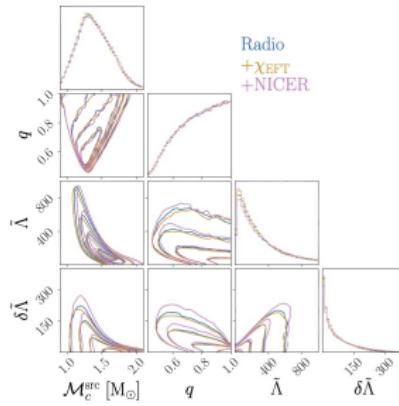


Neural priors

- Masses can also be informed by populations
 - Uniform (agnostic)
 - Gaussian
 - Double Gaussian
- $(\text{BNS/NSBH}) \times (\text{3 populations}) \times (\text{3 EOS constraints}) = 18 \text{ priors}$
- Emulate with normalizing flow: **neural priors**

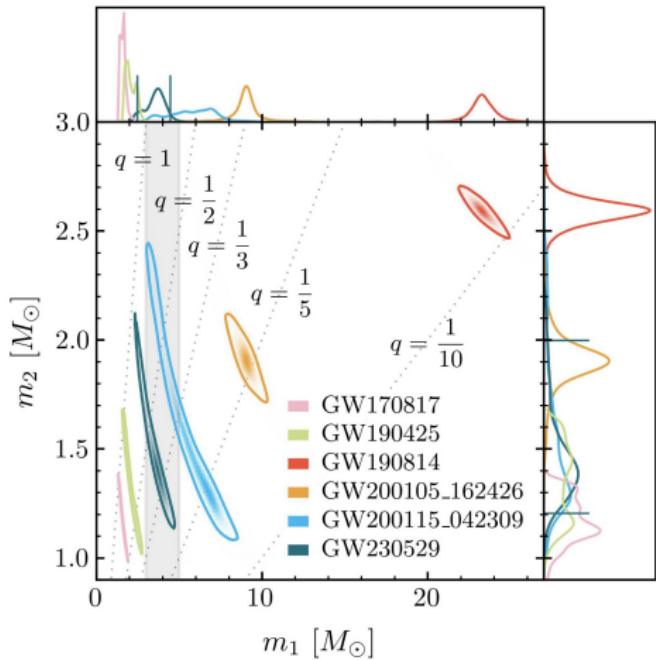
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Application

- Implemented in BILBY
- Consider three “low-mass” GWs
 - GW170817
 - GW190425
 - GW230529



GW170817 – classification

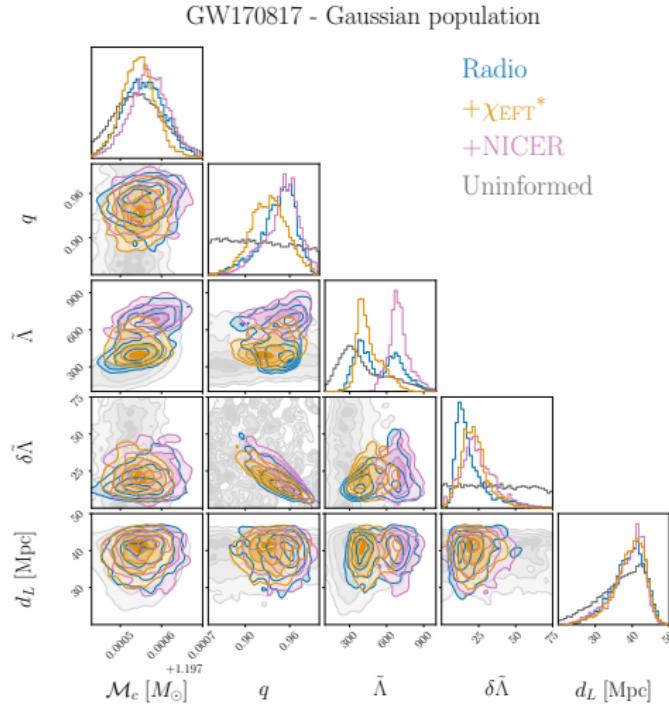
Showing \log_{10} Bayes factors: negative = less preferred

- Strongly prefer BNS over NSBH
- Gaussian population, EOS inconclusive

Source	Population	EOS Constraints	GW170817
BNS	Uniform	Radio	-0.76
		+ χ EFT	-0.52
		+NICER	-0.86
	Gaussian	Radio	-0.14
		+ χ EFT	ref.
		+NICER	-0.05
	Double Gaussian	Radio	-0.43
		+ χ EFT	-0.26
		+NICER	-0.73
NSBH	Uniform	Radio	-224.11
		+ χ EFT	-224.11
		+NICER	-224.12
	Gaussian	Radio	-224.13
		+ χ EFT	-224.13
		+NICER	-224.13
	Double Gaussian	Radio	-224.12
		+ χ EFT	-224.13
		+NICER	-224.12

GW170817 – parameter constraints

- More equal mass ratio $q \geq 0.9$
- $\tilde{\Lambda}$ bimodal, resolved by extra EOS information



GW190425 – classification

Showing \log_{10} Bayes factors: negative = less preferred

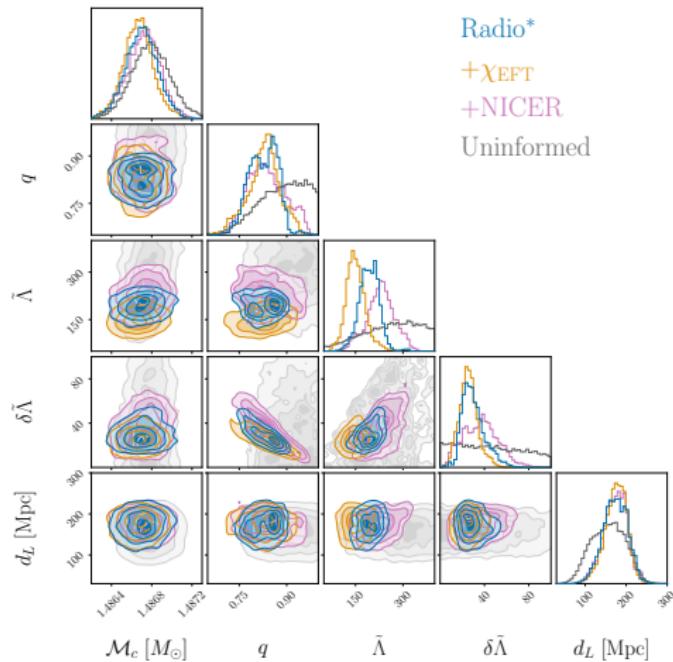
- Prefer BNS over NSBH, but less conclusive
- Most consistent with uniform population

Source	Population	EOS Constraints	GW190425
BNS	Uniform	Radio	ref.
		+ χ EFT	-0.09
		+NICER	-0.11
	Gaussian	Radio	-8.51
		+ χ EFT	-6.57
		+NICER	-4.42
	Double Gaussian	Radio	-0.76
		+ χ EFT	-0.56
		+NICER	-0.89
NSBH	Uniform	Radio	-1.10
		+ χ EFT	-1.10
		+NICER	-1.19
	Gaussian	Radio	-0.82
		+ χ EFT	-1.01
		+NICER	-0.98
	Double Gaussian	Radio	-1.65
		+ χ EFT	-3.40
		+NICER	-2.12

GW190425 – parameter constraints

- Less equal masses ($q \leq 0.9$)
- Higher distances

GW190425 - Uniform population



GW230529 – classification

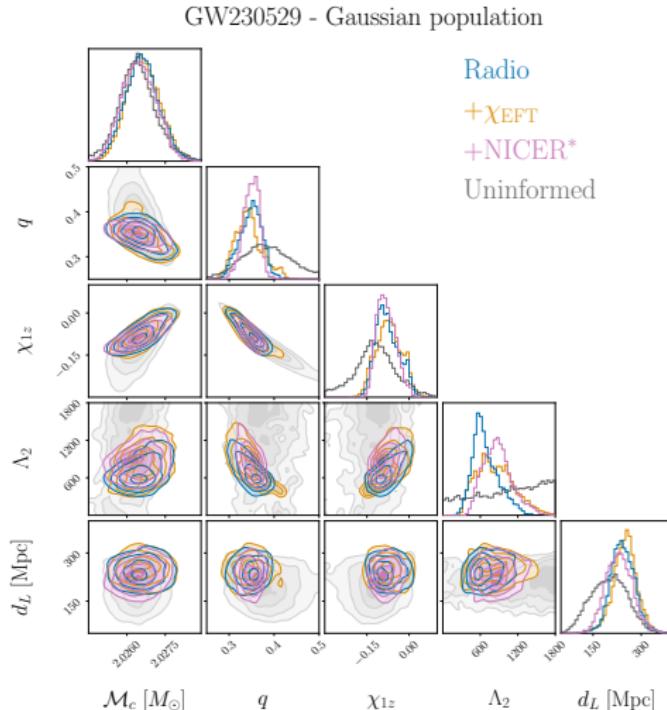
Showing \log_{10} Bayes factors: negative = less preferred

- Decisive evidence for NSBH over BNS
- Weak evidence for population or EOS (low SNR)

Source	Population	EOS Constraints	GW230529
BNS	Uniform	Radio	-13.23
		+ χ EFT	-13.31
		+NICER	-13.23
	Gaussian	Radio	-18.90
		+ χ EFT	-18.86
		+NICER	-18.88
	Double Gaussian	Radio	-13.84
		+ χ EFT	-13.79
		+NICER	-13.98
NSBH	Uniform	Radio	-0.16
		+ χ EFT	-0.28
		+NICER	-0.42
	Gaussian	Radio	-0.28
		+ χ EFT	-0.28
		+NICER	ref.
	Double Gaussian	Radio	-0.18
		+ χ EFT	-0.08
		+NICER	-0.06

GW230529 – parameter constraints

- Mass ratio more constrained $\rightarrow \chi_{1z}$ more constrained
- Higher distances



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Conclusion

- Progress on scalable Bayesian inference, with minimal compromises
- Accelerate likelihood-based inference with
 - JAX: GPU for faster likelihoods, automatic differentiation
 - Normalizing flows to aid in inference (sampling, priors)
- GWs: verified for LVK, work in progress for ET
 - Waveform experts more than welcome!
- Kilonova/GRB: emulators for fast inference
- EOS: scalable inference, open up systematics studies

Let's talk!



Thanks for listening!

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Evidence calculation: HARMONIC I

Evidence Z can be computed from posterior samples with HARMONIC [22] with the **harmonic mean estimator**

$$\begin{aligned}\rho &\equiv \mathbb{E}_{P(\theta|d)} \left[\frac{1}{L(\theta)} \right] \\ &= \int d\theta \frac{1}{L(\theta)} P(\theta|d) \\ &= \int d\theta \frac{1}{L(\theta)} \frac{\mathcal{L}(\theta)\pi(\theta)}{Z} = \frac{1}{Z}\end{aligned}$$

Therefore, estimate ρ with posterior samples:

$$\hat{\rho} = \frac{1}{N} \sum_{i=1}^N \frac{1}{L(\theta_i)}, \quad \theta_i \sim P(\theta|d)$$

Evidence calculation: HARMONIC II

Can be interpreted as importance sampling

$$\rho = \int d\theta \frac{1}{Z} \frac{\pi(\theta)}{P(\theta|d)} P(\theta|d),$$

but with target = prior and sampling density = posterior. Therefore, importance sampling is inefficient – how to solve?

New proposal:

$$\begin{aligned}\rho &= \mathbb{E}_{P(\theta|d)} \left[\frac{\varphi(\theta)}{\mathcal{L}(\theta)\pi(\theta)} \right] \\ &= \int d\theta \frac{\varphi(\theta)}{\mathcal{L}(\theta)\pi(\theta)} P(\theta|d) \\ &= \int d\theta \frac{\varphi(\theta)}{\mathcal{L}(\theta)\pi(\theta)} \frac{\mathcal{L}(\theta)\pi(\theta)}{Z} = \frac{1}{Z}\end{aligned}$$

Evidence calculation: HARMONIC III

Use the following estimator:

$$\hat{\rho} = \frac{1}{N} \sum_{i=1}^N \frac{\varphi(\theta_i)}{\mathcal{L}(\theta_i)\pi(\theta_i)}, \quad \theta_i \sim P(\theta|d)$$

Replace the target distribution π with φ : only requirement is that it is normalized

In practice, this can be achieved with a normalizing flow [23].

This has been verified to give accurate evidences (similar values as nested sampling) when GW posteriors are used [24].

HARMONIC with JIM [24]

Table 1: Total wall times to compute the evidence estimates for the examples discussed in the main text. We run BILBY on 16 CPU cores and JIM + harmonic on 1 GPU.

Example	Method	$\log(z)$	Sampling time	Evidence estimation time
4D	BILBY	390.33 ± 0.11	31.3 min	—
	JIM + harmonic	$390.360^{+0.006}_{-0.006}$	3.4 min	1.9 min
11D	BILBY	378.29 ± 0.15	3.5 h	—
	JIM + harmonic	$378.420^{+0.09}_{-0.08}$	11.8 min	2.4 min

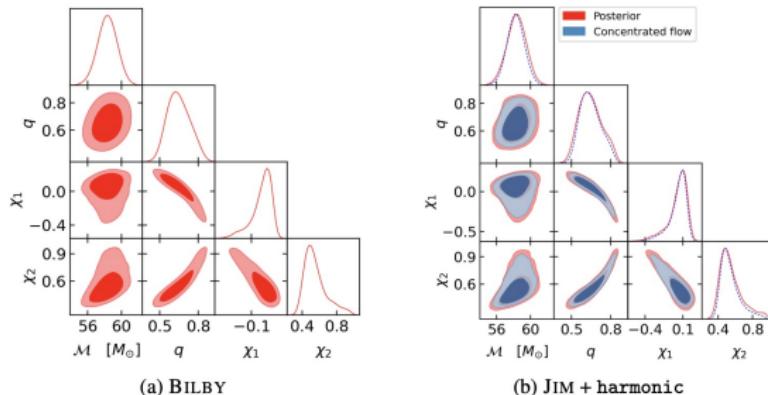
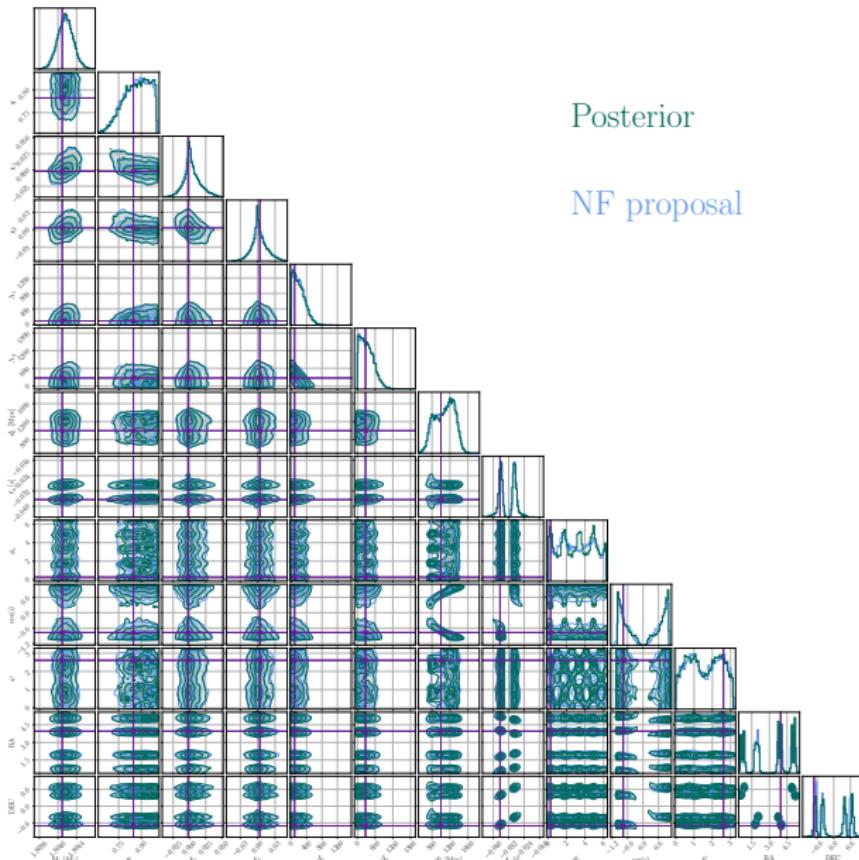
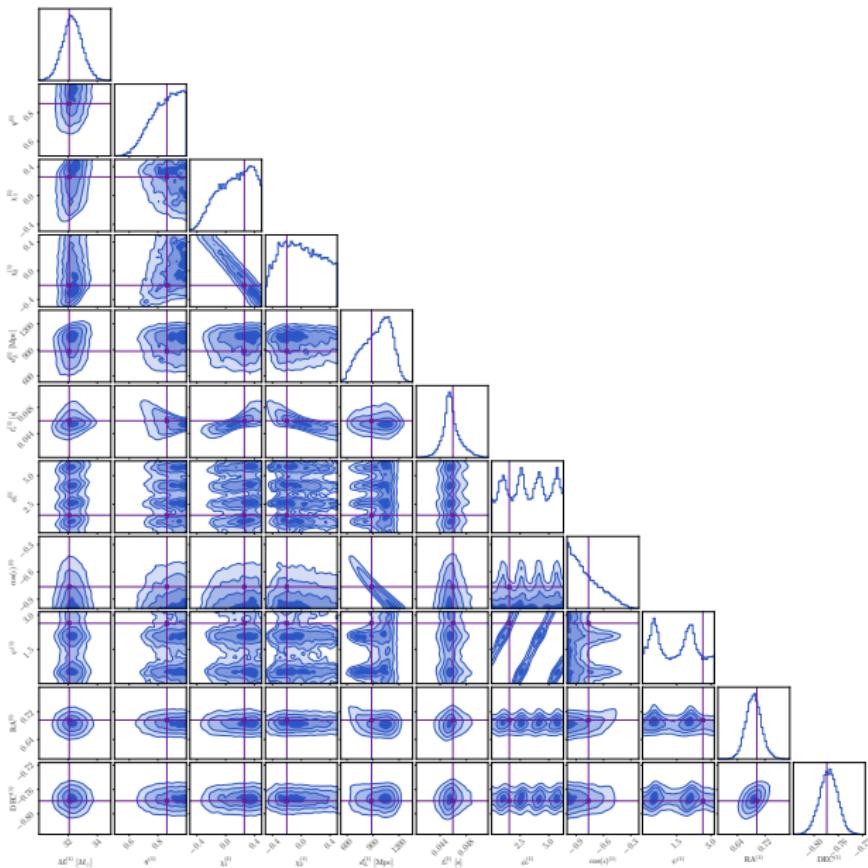


Figure 1: Corner plots for the 4-dimensional posterior samples from (a) BILBY and (b) JIM used for inference (solid red) alongside the concentrated flow at $T = 0.8$ used in the learned harmonic mean (dashed blue).

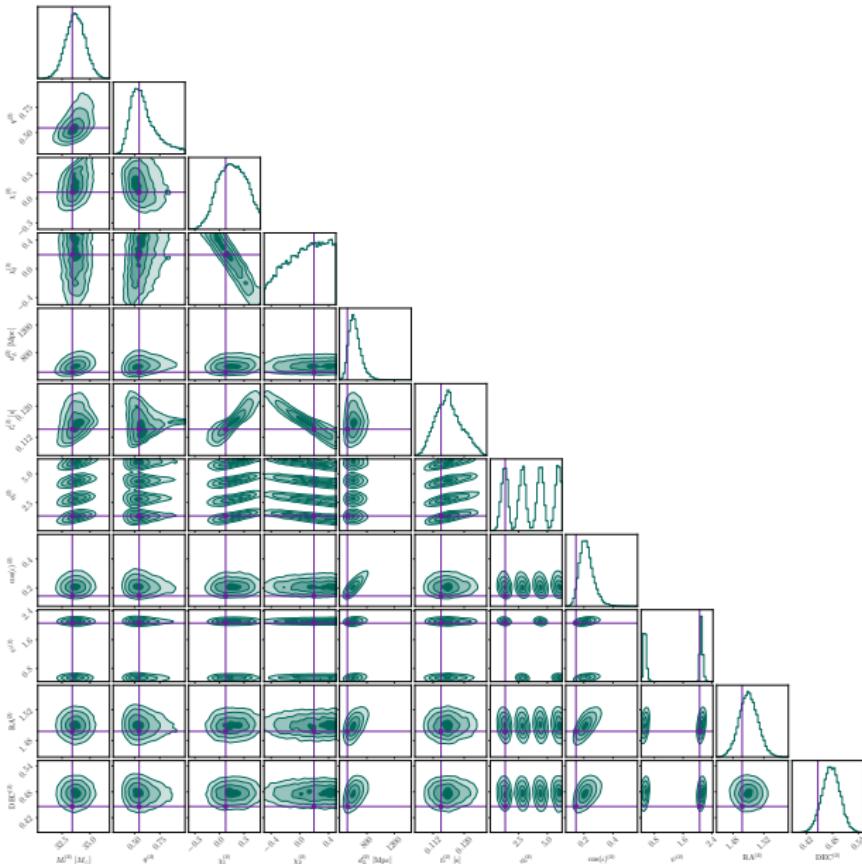
BNS in ET- Δ example: all parameters



Overlapping signals: all parameters signal A

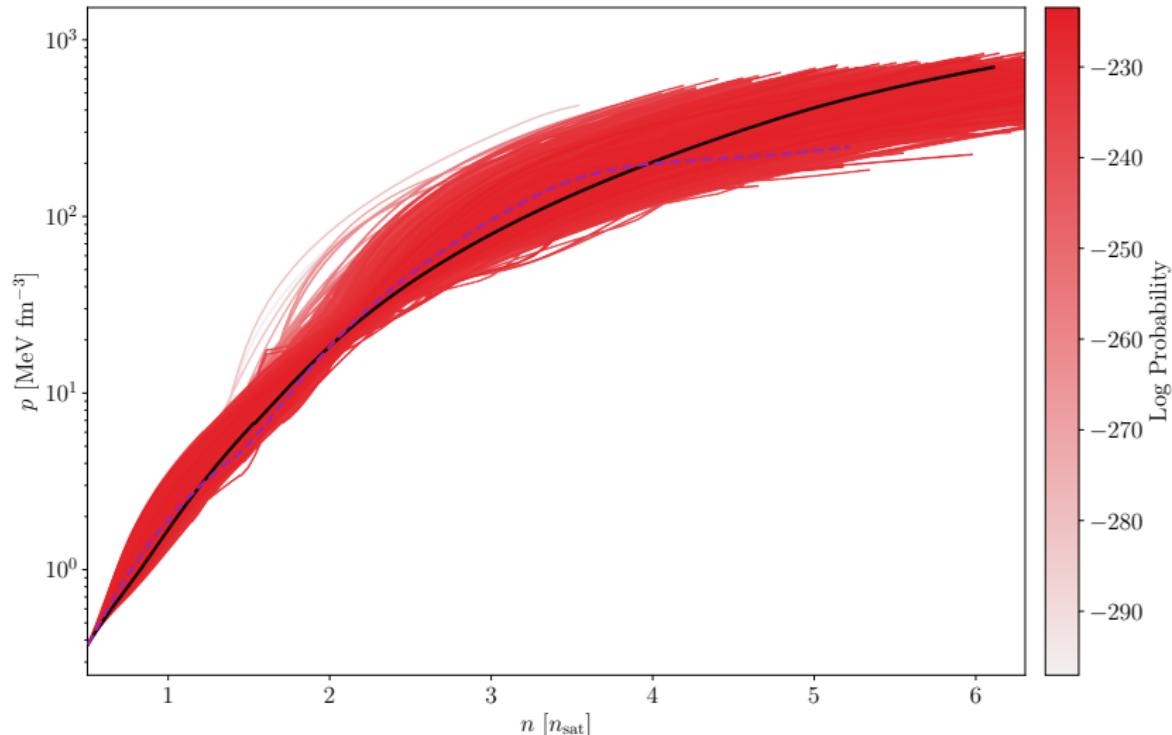


Overlapping signals: all parameters signal B



Equation of state O5 projection with 20 BNS: EOS

- **Purple:** target
- **Red:** posterior EOS samples (**black:** maximum log posterior)



Equation of state O5 projection with 20 BNS: NS

