Leveraging Differentiable Programming in the Inverse Problem of Neutron Stars

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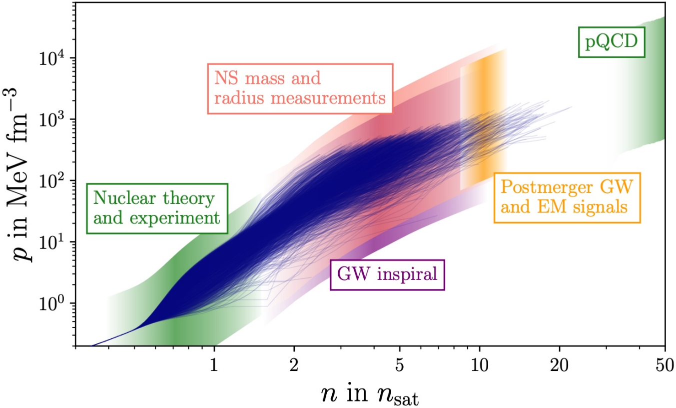
Introduction – Motivation

Neutron stars (NSs) offer unique probes of the high-density regime of the equation of state (EOS) of dense nuclear matter.

Inverse problem of NSs: infer the EOS from observations of NSs (masses, radii, tidal deformabilities, . . .)



Introduction – Differentiable programming with JAX

Main bottleneck: solving Tolman-Oppenheimer-Volkoff equations

Solution: differentiable programming with JAX [1]

- Automatic differentiation to compute gradients of functions
- Use efficient MCMC algorithm and GPU accelerators

Our contributions:

- Fast inference: ~ 0.24 ms per TOV call, ~ 1 h for full MCMC run
- Novel tool to study EOS: gradient descent on NS observables

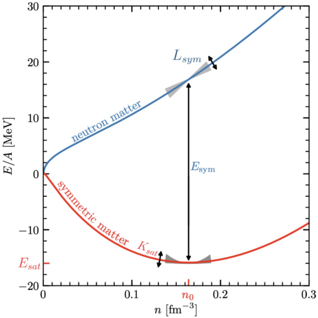
Available open source: JESTER (`tsunhopang/jester`)

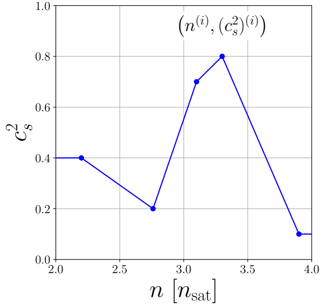
Methods – Equation of state parametrization

Lower density ($< 1 - 2\ n_{\rm sat}$): metamodel [2, 3]

- Taylor expansion of energy per nucleon E/A
- Nuclear empirical parameters ($E_{\rm sym}, L_{\rm sym}, \ldots, E_{\rm sat}, K_{\rm sat}, \ldots$)

Higher density: parametrize $c_s^2(n)$ with grid points and interpolation [4]





Methods – Bayesian inference

Bayesian inference: get posterior of EOS parameters θ_{EOS} with Markov chain Monte Carlo (MCMC) and NS data d

$$p(\theta_{\mathrm{EOS}}|d) \propto p(d|\theta_{\mathrm{EOS}})p(\theta_{\mathrm{EOS}})$$

Computationally expensive: solve TOV equations for many θ_{EOS}!

- `JAX`: compile code, run on GPU
- `flowMC` [5, 6]: MCMC with normalizing flows as proposal distributions

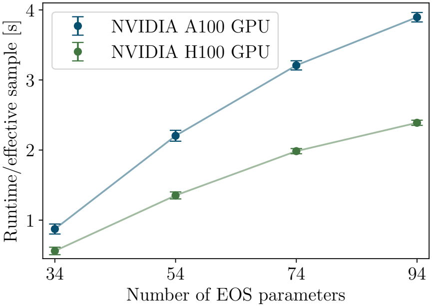
With this, we achieve (on NVIDIA H100 GPU)

- ~ 0.24 ms per TOV call
- Complete MCMC run in ~ 1 h

Results – Validation and scaling

- EOS constraints: nuclear theory (χ_{EFT}), NS observations (heavy PSRs, NICER, GW170817)
- Extend Koehn+ [7] with complete EOS sampling
- Scales well with number of parameters (more $c_s^2(n)$ grid points)

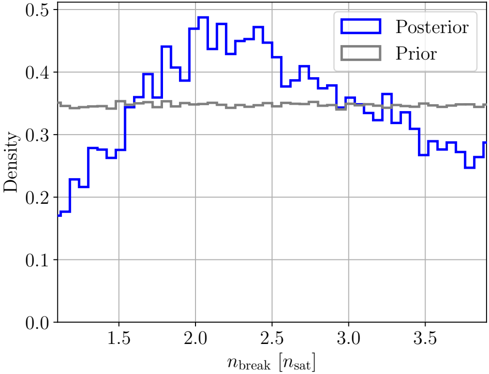
	$R_{1.4}$ [km]	
Constraint	Koehn+	This work
χ_{EFT}	$12.11^{+1.69}_{-3.39}$	$12.59^{+2.24}_{-3.51}$
Radio timing	$13.70^{+1.41}_{-2.17}$	$13.71^{+1.19}_{-1.88}$
PSR J0030+0451	$13.17^{+1.65}_{-2.24}$	$13.48^{+1.42}_{-2.15}$
PSR J0740+6620	$13.39^{+1.57}_{-1.72}$	$13.79^{+1.26}_{-1.73}$
GW170817†	$11.98^{+1.08}_{-1.09}$	$12.40^{+1.33}_{-1.49}$
All	$12.26^{+0.80}_{-0.91}$	$12.62^{+1.04}_{-1.11}$



Results – Measure $\chi_{\rm EFT}$ breakdown

Theory predicts $\chi_{\rm EFT}$ to break down at a density $n_{\rm break}$, around $1 - 2\ n_{\rm sat}$ – can we determine this with NSs?

- Wide, agnostic prior on $n_{\rm break}$: $U(1, 4)\ n_{\rm sat}$
- Only consider heavy PSRs, NICER, GW170817

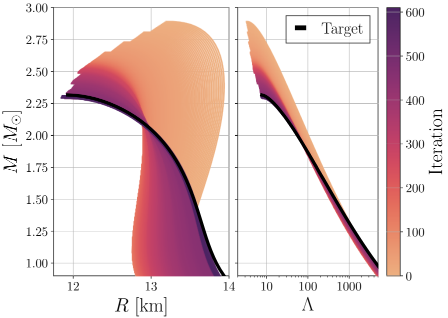


Methods – Variational inference

Alternative to Bayesian inference: optimization with gradients

- $\hat{R}_i, \hat{\Lambda}_i$: "target" tidal deformabilities at masses M_i
- Loss function $L(\boldsymbol{\theta}_{\mathrm{EOS}})$: relative error in tidal deformability Λ
- Gradient descent: $\boldsymbol{\theta}^{(i+1)} \leftarrow \boldsymbol{\theta}^{(i)} - \gamma \nabla L(\boldsymbol{\theta}^{(i)})$
- Efficiently invert a complete NS family to find the EOS

$$L(\boldsymbol{\theta}_{\mathrm{EOS}}) = \frac{1}{N} \sum_{i=1}^{N} \left| \frac{\Lambda_i(\boldsymbol{\theta}_{\mathrm{EOS}}) - \hat{\Lambda}_i}{\hat{\Lambda}_i} \right|$$



Results – degeneracy in metamodel parametrization

Only consider metamodel parameters (no $c_s^2(n)$ extension)