From constraints to classifications: closing the loop in neutron star data analysis

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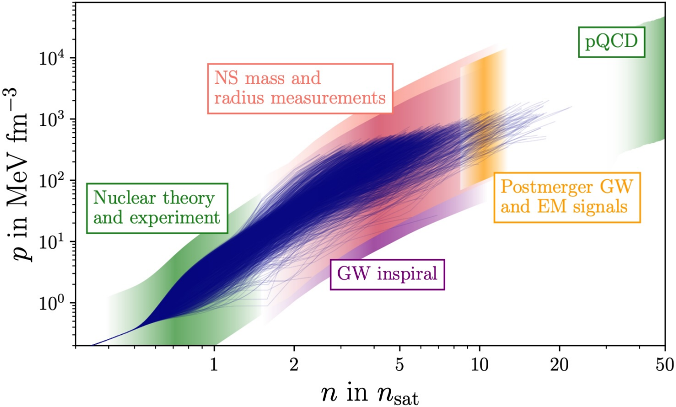
The equation of state (EOS)

- The equation of state (EOS) of dense nuclear matter is still uncertain [1]
- Neutron star (NS) properties depend on the EOS: probe its high density regime (2-8 n_{sat})

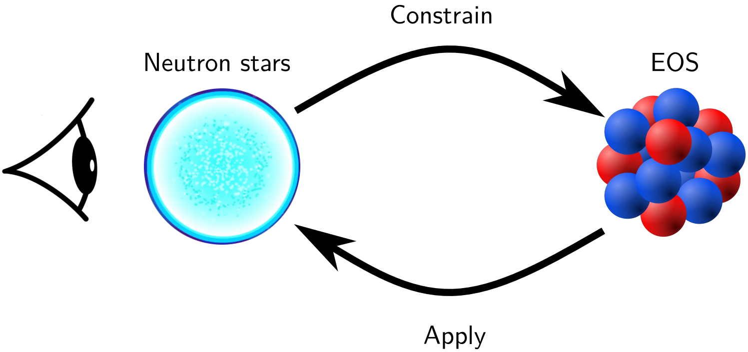


Structure of this talk

Data analysis of neutron star observations is a loop with two stops:

1. Constraining the EOS with neutron star observations
2. Applying EOS knowledge in neutron star data analysis (e.g., GW)

How can we efficiently perform this loop?



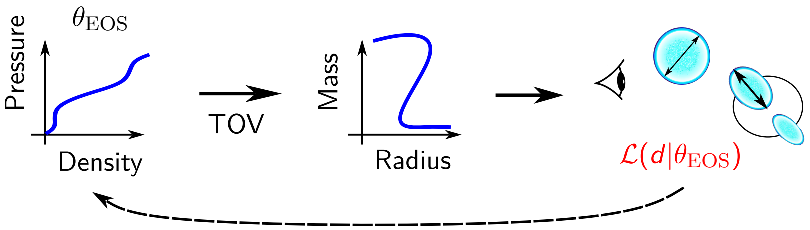
Contents

Constraining the EOS

- EOS determined with Bayesian inference:

$$\mathcal{P}(\theta_{\mathrm{EOS}}|d) \propto \mathcal{L}(d|\theta_{\mathrm{EOS}})\pi(\theta_{\mathrm{EOS}})$$
$$\text{posterior} \propto \text{likelihood} \times \text{prior}$$

- The TOV equations predict NS properties as function of EOS
- Solving the TOV equations is slow: costly likelihood evaluation! Use ML?

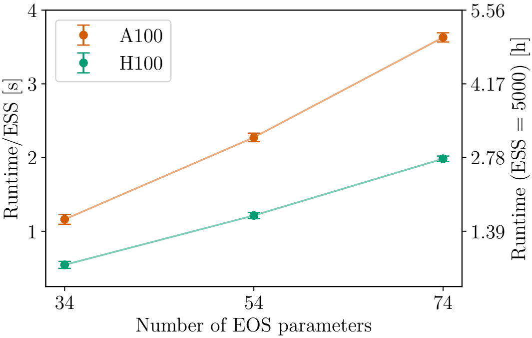


JESTER: accelerated TOV solvers

JESTER [2]: fast EOS code and TOV solver with JAX

- GPU acceleration
- Just-in-time (JIT) compilation

JESTER achieves same speed as machine learning surrogates!



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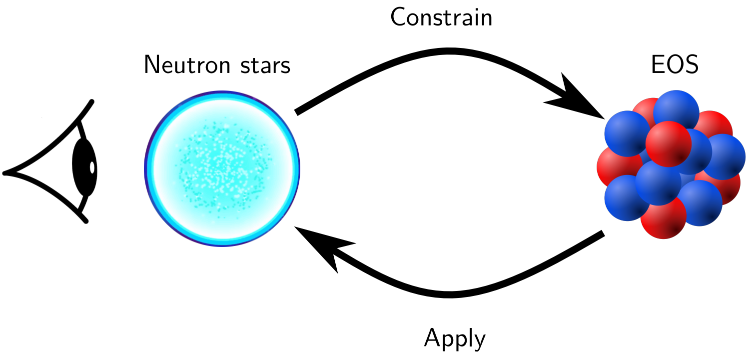
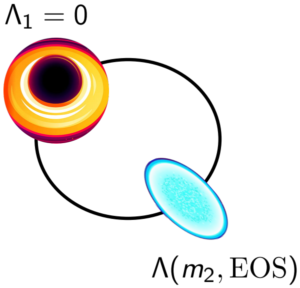
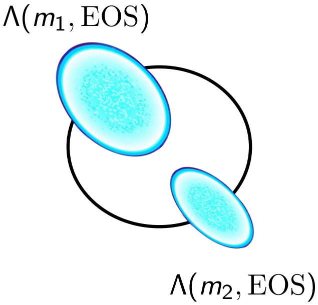


Tidal deformability

- Neutron stars are tidally deformed in a binary
- Quantified by the tidal deformability Λ
- Depends on the EOS: $\Lambda = \Lambda(m, \mathrm{EOS})$ (black holes: $\Lambda = 0$)
- Imprint in the GW phase: $\tilde{\Lambda}(m_i, \Lambda_i)$





Equation of state-informed priors

- GW parameters are determined with Bayesian inference:

$$\mathcal{P}(\theta_{\mathrm{GW}}|d) \propto \mathcal{L}(d|\theta_{\mathrm{GW}})\pi(\theta_{\mathrm{GW}})$$
$$\text{posterior} \propto \text{likelihood} \times \text{prior}$$

- By default, we choose **agnostic priors**: e.g. $\Lambda_i \sim \mathcal{U}(0, 5000)$

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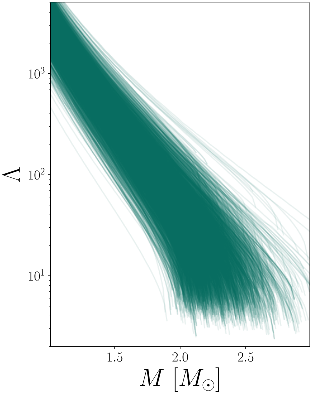
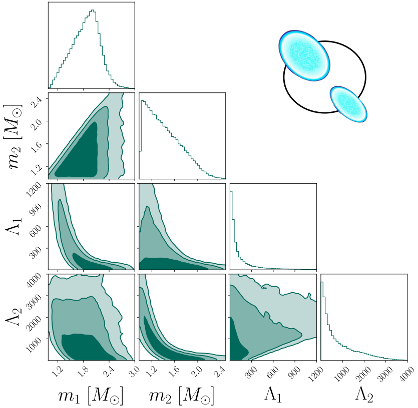
- By default, we choose **agnostic priors**: e.g. $\Lambda_i \sim \mathcal{U}(0, 5000)$
- **But**, we have prior knowledge from the EOS:
  - Masses m_i determined by $M_{\rm max}$
  - Λ_i determined by m_i and EOS
- **Equation of state-informed prior**:

$$\pi(m_1, m_2, \Lambda_1, \Lambda_2) = \int \mathrm{d}\theta_{\rm EOS}\ \pi(m_1, m_2|\theta_{\rm EOS})\pi(\Lambda_1, \Lambda_2|m_1, m_2, \theta_{\rm EOS}) \times \pi(\theta_{\rm EOS})$$

Equation of state-informed priors

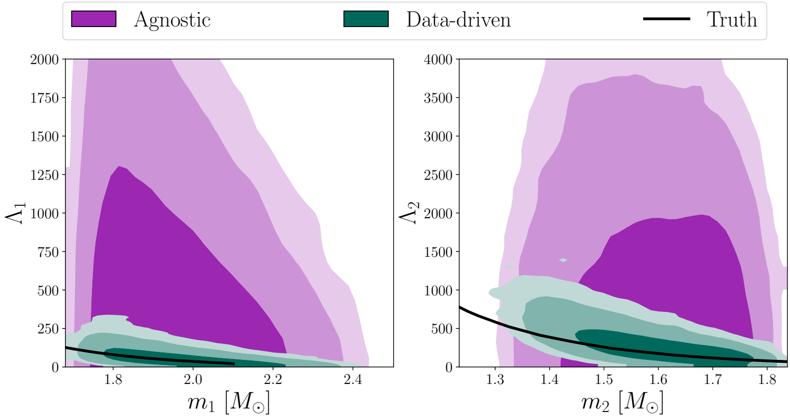
- Example: EOSs with $M_{\max} > 2.0 M_{\odot}$
- Emulate $\pi(m_1, m_2, \Lambda_1, \Lambda_2)$ with a **normalizing flow**





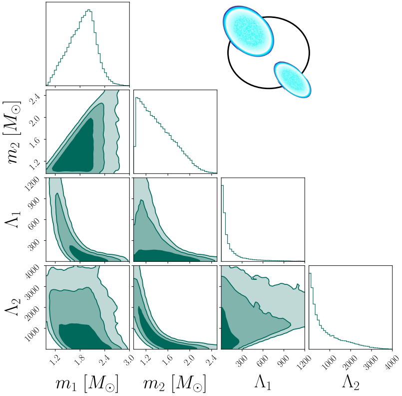
Posterior for single injection

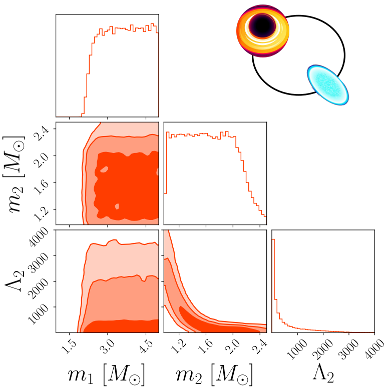
- **Agnostic**: uniform prior for chirp mass, mass ratio, Λ_1, Λ_2
- **Data-driven**: normalizing flow prior $\pi(m_1, m_2, \Lambda_1, \Lambda_2)$
- Tidal content of source more constrained (SNR $= 13$)



Source classification

- Similar to the **binary neutron star (BNS)** prior, we can also construct a **neutron star-black hole (NSBH)** prior
- Use EOS constraints to classify mergers

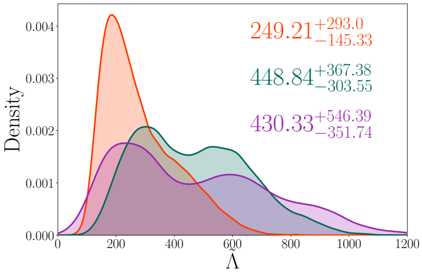




Source classification: proof of concept

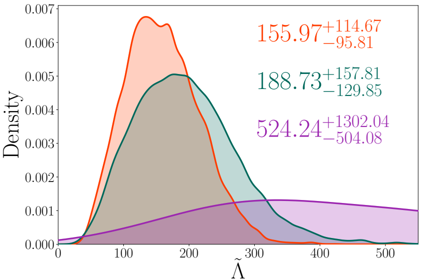
GW170817

$$\ln \mathcal{B}^{\mathrm{BNS}}_{\mathrm{NSBH}} = 44.75$$
$$\ln \mathcal{B}^{\mathrm{BNS}}_{\mathrm{agnostic}} = 2.83$$



GW190425

$$\ln \mathcal{B}^{\mathrm{BNS}}_{\mathrm{NSBH}} = 4.13$$
$$\ln \mathcal{B}^{\mathrm{BNS}}_{\mathrm{agnostic}} = -0.30$$

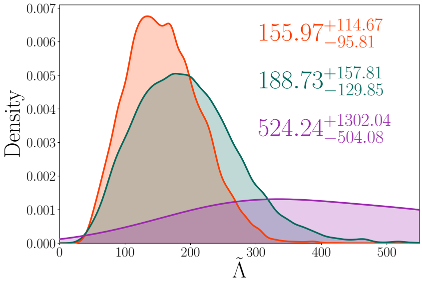


Source classification: proof of concept

GW190425

- $M_{\max} > 2M_\odot$

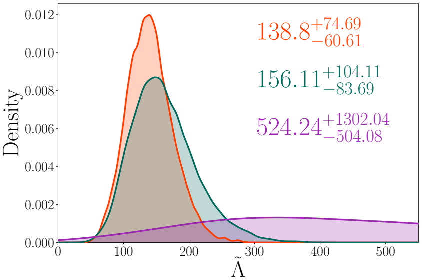
$$\ln \mathcal{B}^{\mathrm{BNS}}_{\mathrm{NSBH}} = 4.13$$
$$\ln \mathcal{B}^{\mathrm{BNS}}_{\mathrm{agnostic}} = -0.30$$



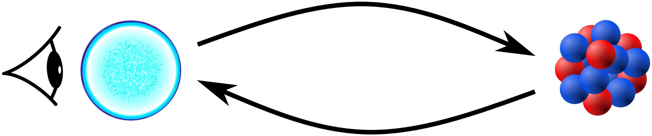
GW190425

+ mass-radius measurements
+ GW170817
+ nuclear theory predictions

$$\ln \mathcal{B}^{\mathrm{BNS}}_{\mathrm{NSBH}} = 8.20$$
$$\ln \mathcal{B}^{\mathrm{BNS}}_{\mathrm{agnostic}} = 0.28$$



Conclusion



- Neutron star observations and EOS constraints form a data analysis loop
- Constraining EOS: TOV solvers can be accelerated with JAX, avoiding the need for machine learning surrogates
- Applying EOS: Normalizing flows can emulate priors on source parameters informed by EOS knowledge
- Equation of state-informed priors enable source classification: confidence depends on EOS constraints

References I

[1] Hauke Koehn et al. "From existing and new nuclear and astrophysical constraints to stringent limits on the equation of state of neutron-rich dense matter". In: (Feb. 2024). arXiv: 2402.04172 [astro-ph.HE].

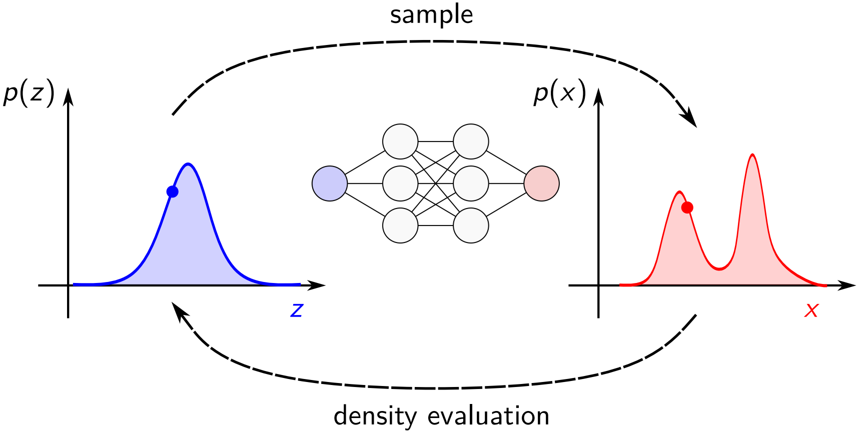
[2] Thibeau Wouters et al. "Leveraging differentiable programming in the inverse problem of neutron stars". In: (Apr. 2025). arXiv: 2504.15893 [astro-ph.HE].

[3] Kaze W. K. Wong, Maximiliano Isi, and Thomas D. P. Edwards. "Fast Gravitational-wave Parameter Estimation without Compromises". In: *Astrophys. J.* 958.2 (2023), p. 129. DOI: 10.3847/1538-4357/acf5cd. arXiv: 2302.05333 [astro-ph.IM].

[4] Thibeau Wouters et al. "Robust parameter estimation within minutes on gravitational wave signals from binary neutron star inspirals". In: *Phys. Rev. D* 110.8 (2024), p. 083033. DOI: 10.1103/PhysRevD.110.083033. arXiv: 2404.11397 [astro-ph.IM].

Normalizing flows

- Trainable, bijective transformation between latent and data space
- Emulate complicated distributions, trained from samples



Projection: 20 BNS in O5

- 20 binary neutron star signals observed with HLV and O5 sensitivity
  - Parameter estimation done with JIM [3, 4]: ∼ 30 mins/event
  - **Agnostic** vs **data-driven** priors
- Use resulting GW posteriors to constrain the EOS
  - ∼ 1 hour with JESTER [2]
- Result: constraints more robust