qLogLogistic likelihood

Parametrisation

The LogLogistic distribution has cumulative distribution function

$$F_0(y) = \frac{1}{1 + \lambda y^{-\alpha}}, \qquad y > 0$$

if variant=0, or

$$F_1(y) = \frac{1}{1 + (\lambda y)^{-\alpha}}, \quad y > 0$$

if variant=1, where

 $\alpha > 0$ is a shape parameter, and

 $\lambda > 0$ is a scale parameter.

The λ is defined implicitely through the quantile, as

$$F_0(y_q) = q$$
, or $F_q(y_q) = q$, $0 < q < 1$

and the linear predictor is defined on y_q .

Link-functions

The parameter λ is linked to the linear predictor, implicitely through

$$y_q = \exp(\eta)$$

Hyperparameters

The α parameter is represented as

$$\theta = \log \alpha$$

and the prior is defined on θ .

Specification

- family equals qloglogistic (regression) or qloglogisticsurv (survival)
- variant=0 (default) or 1, chosing between parameterisation F_0 or F_1 .
- Required arguments: y (regression) or an inla.surv-object using inla.surv() (for survival data), and quantile= q.

Hyperparameter spesification and default values

Regression:

doc A quantile loglogistic likelihood

hyper

theta

hyperid 60011 name log alpha

```
short.name alpha
         initial 1
         fixed FALSE
         prior loggamma
         param 25 25
         to.theta function(x) log(x)
         from.theta function(x) exp(x)
status changed:Oct.25.2017
survival FALSE
discrete FALSE
link default log neglog
pdf qloglogistic
   Survival:
doc A quantile loglogistic likelihood (survival)
hyper
     theta
         hyperid 60021
         name log alpha
         short.name alpha
         initial 1
         fixed FALSE
         prior loggamma
         param 25 25
         to.theta function(x) log(x)
         from.theta function(x) exp(x)
status changed:Oct.25.2017
survival TRUE
discrete FALSE
link default log neglog
pdf qloglogistic
Example
In the following example we estimate the parameters in a simulated case
lam_loglogistic = function(yq, alpha, q, variant = 0)
    if (variant == 0) {
        lambda = yq^alpha * (1/q-1)
    } else if (variant == 1) {
```

```
lambda = 1/yq * (1/(1/q-1))^(1/alpha)
    } else
        stop("ERR")
    return (lambda)
}
rloglogistic = function(n, lambda, alpha, variant=0)
    u = runif(n)
    if (variant == 0) {
        y = (lambda/(1.0/u - 1.0))^(1.0/alpha)
    } else if (variant == 1) {
        y = (1.0/(1.0/u -1.0))^(1.0/alpha) / lambda
    } else {
        stop("ERROR")
    }
}
n = 500
alpha = 2.1
x = c(scale(runif(n)))
eta = 1.1+2.2*x
yq = exp(eta)
for(variant in 0:1) {
    for(q in c(0.2, 0.8)) {
        print(paste("variant=", variant, "quantile=", q))
        lambda = lam_loglogistic(yq, alpha, q, variant=variant)
        y = rloglogistic(n,
                         lambda = lambda,
                         alpha = alpha,
                         variant = variant)
        formula = y \sim 1 + x
        rr=inla(formula,
               family ="qloglogistic",
               data=data.frame(y, x),
               control.family = list(list(variant = variant, quantile = q)))
        print("REGRESSION")
        print(summary(rr))
        event = rep(1,n)
        formula=inla.surv(y,event) ~ 1 + x
        r=inla(formula,
               family ="qloglogisticsurv",
               data = list(y=y, event=event, x=x),
               control.family = list(list(variant = variant, quantile = q)))
        print("SURVIVAL")
        print(summary(r))
```

```
}
```

Notes

• Loglogisticsurv model can be used for right censored, left censored, interval censored data. If the observed times y are large/huge, then this can cause numerical overflow in the likelihood routine. If you encounter this problem, try to scale the observatios, time = time / max(time) or similar.