

Autoregressive model of order 1 with covariates (AR1C)

Parametrization

This is an extension of the common autoregressive model (AR1) to include a set of covariates into the conditional mean

$$\begin{aligned}x_1 &\sim \mathcal{N}(0, (\tau(1 - \rho^2))^{-1}) \\x_t &= \rho x_{t-1} + \sum_{j=1}^m \beta_j z_{t-1}^{(j)} + \epsilon_t; \quad \epsilon_i \sim \mathcal{N}(0, \tau^{-1}) \quad t = 2, \dots, n\end{aligned}$$

where $|\rho| < 1$. The latent vector has length $n + m$ and is represented as $(x_1, x_2, \dots, x_n, \beta_1, \dots, \beta_m)$.

Hyperparameters

The precision parameter κ is represented as

$$\theta_1 = \log(\kappa)$$

where κ is the *marginal* precision (when there is no covariates)

$$\kappa = \tau(1 - \rho^2).$$

The parameter ρ is represented as

$$\theta_2 = \log\left(\frac{1 + \rho}{1 - \rho}\right)$$

and the prior is defined on $\theta = (\theta_1, \theta_2)$.

Specification

The AR1C model is specified as

```
f(<whatever>, model="ar1", hyper = <hyper>,  
  control.ar1c = list(Z=Z, Q.beta = Q))
```

The covariates are given in the **matrix** Z and **must have** dimension $n \times m^1$. The prior for $\beta = (\beta_1, \dots, \beta_m)$ is a zero mean Gaussian with a $m \times m$ precision **matrix** Q .

Hyperparameter specification and default values

doc Auto-regressive model of order 1 w/covariates

hyper

theta1

hyperid 14101

name log precision

short.name prec

prior pc.prec

param 1 0.01

initial 4

fixed FALSE

¹Despite the fact that the last row of Z is not used

```

    to.theta function(x) log(x)
    from.theta function(x) exp(x)
  theta2
    hyperid 14102
    name logit lag one correlation
    short.name rho
    prior pc.cor0
    param 0.5 0.5
    initial 2
    fixed FALSE
    to.theta function(x) log((1+x)/(1-x))
    from.theta function(x) 2*exp(x)/(1+exp(x))-1

  constr FALSE

  nrow.ncol FALSE

  augmented FALSE

  aug.factor 1

  aug.constr

  n.div.by

  n.required FALSE

  set.default.values TRUE

  status experimental

  pdf arlc

```

Example

```

n = 500
N = n+1
phi = 0.98
sd.h = 0.4
prec = (1/sd.h)^2
prec.prime = prec / (1-phi^2)
beta = 0.05
h = numeric(n)
y = numeric(n)
z = numeric(n)
s = 0.01
z[n] = NA ## not used
h[1] = rnorm(1, sd = sqrt(1/prec))
y[1] = rnorm(1, sd = sd.h) + rnorm(1, sd = s)
for(i in 2:n) {
  z[i-1] = rnorm(1)
  h[i] = phi * h[i-1] + beta * z[i-1] +
    rnorm(1, sd = sqrt(1/prec.prime))
}

```

```

    y[i] = h[i] + rnorm(1, sd = s)
  }
  idx = 1:n

r = inla(y ~ -1 + f(idx, model="ar1c",
    args.ar1c = list(Z = cbind(z),
                      Q.beta = matrix(1, 1, 1))),
  data = data.frame(y, idx),
  family = "gaussian",
  control.family = list(
    hyper = list(prec = list(
      initial = log(1/s^2),
      fixed=TRUE)))

par(mfrow=c(2, 2))
plot(idx, y, type="l", main = "data")
plot(inla.tmarginal(function(x) sqrt(1/exp(x)),
  r$internal.marginals.hyperpar[[1]]),
  type = "l", lwd=2, main = "sd(h)")
abline(v = sd.h)

plot(inla.smarginal(r$marginals.hyperpar[[2]]),
  type = "l", lwd=2, main = "phi")
abline(v = phi)

## the N'th element is 'beta'
plot(inla.smarginal(r$marginals.random$idx[[N]]), type="l", lwd=2,
  main = "beta")
abline(v = beta)

```

Notes

- If $m = 0$, use `model="ar1"`.