The Beta-distribution

Parametrisation

The Beta-distribution has the following density

$$\pi(y) = \frac{1}{B(a,b)} y^{a-1} (1-y)^{b-1}, \qquad 0 < y < 1, \quad a > 0, \quad b > 0$$

where B(a, b) is the Beta-function

$$B(a,b) = \frac{\Gamma(a)\Gamma(b)}{\Gamma(a+b)}$$

and $\Gamma(x)$ is the Gamma-function. The (re-)parameterisation used is

$$\mu = \frac{a}{a+b}, \qquad 0 < \mu < 1$$

and

$$\phi = a + b, \qquad \phi > 0,$$

as it makes

$$E(y) = \mu$$
 and $Var(y) = \frac{\mu(1-\mu)}{1+\phi}$.

The parameter ϕ is known as the *precision parameter*, since for fixed μ , the larger ϕ the smaller the variance of y. The parameters $\{a,b\}$ are given as $\{\mu,\phi\}$ as follows,

$$a = \mu \phi$$
 and $b = -\mu \phi + \phi$.

Link-function

The linear predictor η is linked to the mean μ using a default logit-link

$$\mu = \frac{\exp(\eta)}{1 + \exp(\eta)}.$$

Hyperparameter

The hyperparameter is the precision parameter ϕ , which is represented as

$$\phi = s_i \exp(\theta)$$

where $s = (s_i) > 0$ is a fixed scaling, and the prior is defined on θ .

Specification

- family = beta
- Required argument: y
- Optional argument: s (argument scale, default all 1, s > 0)

Hyperparameter spesification and default values

```
doc The Beta likelihood
```

```
hyper
```

```
theta
```

```
hyperid 61001
name precision parameter
short.name phi
initial 2.30258509299405
fixed FALSE
prior loggamma
param 1 0.1
to.theta function(x) log(x)
from.theta function(x) exp(x)
```

survival FALSE

discrete FALSE

link default logit cauchit probit cloglog loglog

pdf beta

Example

In the following example we estimate the parameters in a simulated example.

Notes

None.