NMixNB

Parametrisation

The N-MixtureNB distribution is a negative Binomial mixture of the Binomials, as

$$Prob(y) = \sum_{n=y}^{\infty} {n \choose y} p^n (1-p)^{n-y} \times \frac{\Gamma(n+\delta)}{\Gamma(\delta)n!} q^{\delta} (1-q)^n$$

for responses y = 0, 1, 2, ..., n, where n is Poisson number of trials, and p is probability of success. For δ and q, see below. Replicated reponses $y_1, y_2, ..., y_r$, are iid from the Binomial with the same p, conditioned on the same p from the negative Binomial,

$$\operatorname{Prob}(y_1, \dots, y_r) = \sum_{n=\max\{y_1, \dots, y_r\}}^{\infty} \left\{ \prod_{i=1}^r \binom{n}{y_i} p^n (1-p)^{n-y_i} \right\} \times \frac{\Gamma(n+\delta)}{\Gamma(\delta)n!} q^{\delta} (1-q)^n$$

The negative binomial is parameterisized in terms of the mean λ and overdispersion $1/\delta$, where $q = \delta/(\delta + \mu)$; see the R documentation ?dnbinom for this parameterisation (where $\delta = \text{size}$).

Link-function

The probability p is linked to the linear predictor by

$$p(\eta) = \frac{\exp(\eta)}{1 + \exp(\eta)}$$

for the default logit link, while λ depends on fixed covariates

$$\log(\lambda) = \sum_{j=1}^{m} \beta_j x_j$$

with one vector of covariates for each observation. m can be maximum 10 and minimum 1.

Hyperparameters

The parameters $\theta_1 = \beta_1, \theta_2 = \beta_2, \dots, \theta_m = \beta_m$, and overdispersion $\theta_{11} = \log(1/\delta)$.

Hyperparameter spesification and default values

doc NegBinomial-Poisson mixture

hyper

theta1

hyperid 101121
name beta1
short.name beta1
initial 2.30258509299405
fixed FALSE
prior normal
param 0 0.5
to.theta function(x) x
from.theta function(x) x

```
theta2
    hyperid 101122
    name beta2
    short.name beta2
    initial 0
    fixed FALSE
    prior normal
    param 01
    to.theta function(x) x
    from.theta function(x) x
theta3
    hyperid 101123
    name beta3
    short.name beta3
    initial 0
    fixed FALSE
    prior normal
    param 0 1
    to.theta function(x) x
    from.theta function(x) x
theta4
    hyperid 101124
    name beta4
    short.name beta4
    initial 0
    fixed FALSE
    prior normal
    param 01
    to.theta function(x) x
    from.theta function(x) x
theta5
    hyperid 101125
    name beta5
    short.name beta5
    initial 0
    fixed FALSE
    prior normal
    param 01
    to.theta function(x) x
    from.theta function(x) x
theta6
    hyperid 101126
    name beta6
    short.name beta6
```

```
initial 0
    fixed FALSE
    prior normal
    param 01
    to.theta function(x) x
    from.theta function(x) x
theta7
    hyperid 101127
    name beta7
    short.name beta7
    initial 0
    fixed FALSE
    prior normal
    param 01
    to.theta function(x) x
    from.theta function(x) x
theta8
    hyperid 101128
    name beta8
    short.name beta8
    initial 0
    fixed FALSE
    prior normal
    param 01
    to.theta function(x) x
    from.theta function(x) x
theta9
    hyperid 101129
    name beta9
    short.name beta9
    initial 0
    fixed FALSE
    prior normal
    param 0 1
    to.theta function(x) x
    from.theta function(x) x
theta10
    hyperid 101130
    name beta10
    short.name beta10
    initial 0
    fixed FALSE
    prior normal
    param 0 1
```

```
to.theta function(x) x
         from.theta function(x) x
     theta11
         hyperid 101131
         name overdispersion
         short.name overdispersion
         initial 0
         fixed FALSE
         prior pc.gamma
         param 7
         to.theta function(x) log(x)
         from.theta function(x) exp(x)
status experimental
survival FALSE
discrete TRUE
link default logit probit
pdf nmixnb
```

Specification

- family = nmixnb
- Required arguments: the response Y and covariates X as inla.mdata(Y, X [, additional.covariates])

The response is a matrix where each row are replicates, where responses that are NA's are ignored. The covariates is one or many vectors, matrices or data frames. Each row of the covariates $(x_{i1}, x_{i2}, \ldots, x_{im})$ defines the covariates used for the *i*'th response(s) (the *i*'th row of Y). Note that $\beta_{m+1}, \ldots, \beta_{10}$ are fixed to zero.

Example

In the following example we estimate the parameters in a simulated example with replications.

```
nrep.max = 5
n = 50
y = matrix(NA, n, nrep.max)
x = c()
xx = c()
size = 3
overdispersion = 1/size
intercept = 1

for(i in 1:n) {
   local.x = runif(1) - 0.5
   lambda = exp(2 + local.x)
   N = rnbinom(1, mu=lambda, size=size)
```

```
local.xx = runif(1) - 0.5
    eta = intercept + local.xx
    p = \exp(eta)/(\exp(eta) + 1)
    ## sample the number of replications
    nr = sample(1:nrep.max, 1)
    \#\# and sample these. note that 'y' is initialized with NA's,
    ## so if nr < nrep.max, then
    ## y[i,(nr+1):nrep.max] would be NA.
    y[i, 1:nr] = rbinom(nr, size = N, prob = p)
    x = c(x, local.x)
    xx = c(xx, local.xx)
}
Y = inla.mdata(y, 1, x)
r = inla(Y ~1 + xx,
         data = list(Y=Y, xx=xx),
         family = "nmixnb",
         control.fixed = list(prec.intercept=1, prec=1))
```

Notes