The quantile log-logistics distribution

Parametrisation

The loglogistic distribution has cdf

$$F(y) = \frac{y^\beta}{\alpha^\beta + y^\beta}, \qquad y \geq 0, \quad \alpha, \beta > 0$$

The quantile function is given by

$$\kappa(q) = F^{-1}(q) = \alpha \left(\frac{q}{1-q} \right)^{1/\beta}, \qquad 0 < q < 1.$$

The median is $\kappa(0.5)$ and equals α. We will use a quantile parameterisation. Define for fixed q,

$$\kappa = \alpha \left(\frac{q}{1-q} \right)^{1/\beta}$$

and

$$s = 1/\beta.$$

Then $0 < \kappa < 1$ is the q-quantile, and $s > 0$ is a scaling parameter. The cdf and the density are now

$$F(y) = \frac{q^* y^{1/s}}{\kappa^{1/s} + q^* y^{1/s}}$$

and

$$f(y) = \frac{q^* \kappa^{1/s} y^{1/s-1}}{s(\kappa^{1/s} + q^* y^{1/s})^2}$$

where $q^* = q/(1-q)$.

Link-function

The quantile κ relates to the linear predictor,

$$\log(\kappa) = \eta$$

using the default log link-function.

Hyperparameters

The hyperparameter is

$$\theta = \log(s)$$

and the prior is given for θ.

Specification

- family $=$ `qloglogistic`

- Required arguments: Observations y and the value of q.

The quantile is given as `control.family=list(quantile=`q`)`.

Hyperparameter spesification and default values

doc A quantile version of the logistic likelihood

hyper

> **theta**
>
>> **hyperid** 60011
>> **name** precision parameter
>> **short.name** prec
>> **initial** 2
>> **fixed** FALSE
>> **prior** loggamma
>> **param** 1 0.001
>> **to.theta** function(x) log(x)
>> **from.theta** function(x) exp(x)

survival FALSE

discrete FALSE

link default log

pdf qloglogistic

Example

```
rqloglogistic = function(eta, s, q=0.5)
{
    qs = q/(1-q)
    u = runif(length(eta))
    x = (u/((1-u)*qs))^s * exp(eta)
    return (x)
}

n = 30
q = .10
s = .1
x = rnorm(n, s=0.2)
eta = 1 + 2*x
y = rqloglogistic(eta=eta, s=s, q=q)
r = inla(y ~ 1 + x,
    data = data.frame(y, x),
    family = "qloglogistic",
    control.family = list(quantile = q))

summary(r)
```

Notes