# Circular Normal (von Mises distribution)

### Parametrisation

The circular Normal or von Mises distribution, has density

$$f(y) = \frac{1}{2\pi I_0(\kappa s)} \exp(\kappa s \cos(y - \mu)),$$

for continuously responses y where  $|y - \mu| \le \pi$  and  $|\mu| \le \pi$ . Here,

 $\mu$  is a measure of location, and

 $\kappa$  is a measure of the precision,

s is a fixed scaling, s > 0, and

 $I_0$  is the modified Bessel of first kind and order zero

$$I_0(\tau) = \frac{1}{2\pi} \int_0^{2\pi} e^{\tau \cos \alpha} d\alpha.$$

#### **Link-function**

The "mean" of y is given as  $\mu$  and the mean is linked to the linear predictor as

$$\mu = 2 \arctan(\eta)$$

(Link function "tan")

## Hyperparameters

The "precision"  $\kappa$  is represented as

$$\theta = \log \kappa$$

and the prior is defined on  $\theta$ .

# **Specification**

- family = circularnormal
- Required arguments: y and s (argument scale)

The scalings have default value 1.

#### Hyperparameter spesification and default values

doc The circular Gaussian likelihoood

hyper

theta

hyperid 67001 name log precision parameter short.name prec initial 2 fixed FALSE

```
prior loggamma
param 1 0.01
to.theta function(x) log(x)
from.theta function(x) exp(x)
survival FALSE
discrete FALSE
link default tan
pdf circular-normal
status experimental
```

# Example

In the following example we estimate the parameters in a simulated example with circular Normal responses.

```
ilink = function(x) 2*atan(x)
link = function(x) tan(x/2)
n = 300
z = rnorm(n, sd=0.3)
eta = 1 + z
y.pred = ilink(eta)
## create a simple, almost exact, sampler for the circular normal...
kappa = 5
x = seq(-pi, pi, len = 10000)
d = \exp(kappa*cos(x))
dd = cumsum(d)
dd = dd /max(dd)
cn.icdf.func = splinefun(dd, x, method = "monoH.FC")
rcn = function(n) cn.icdf.func(runif(n))
y = y.pred + rcn(n)
formula = y \sim 1 + z
r=inla(formula, data = data.frame(y, z),
        family = "circularnormal", control.inla = list(cmin = -Inf))
Notes
```

```
Try to use
```

```
control.inla=list(cmin = -Inf)
```

to avoid systematic-errors for low precisions.