Bym2 model for spatial effects

Parametrization

This model is a reparameterisation of the BYM-model, which is a union of the besag model u^* and a iid model v^*, so that

$$x = \begin{pmatrix} v^* + u^* \\ u^* \end{pmatrix}$$

where both u^* and v^* has a precision (hyper-)parameter. The length of x is $2n$ if the length of u^* (and v^*) is n. The BYM2 model uses a different parameterisation of the hyperparameters where

$$x = \begin{pmatrix} \frac{1}{\sqrt{\tau}}\left(\sqrt{1-\phi}\, v + \sqrt{\phi}\, u\right) \\ u \end{pmatrix}$$

where both u and v are *standardised* to have (generalised) variance equal to one. The *marginal* precision is then τ and the proportion of the marginal variance explained by the spatial effect (u) is ϕ.

Hyperparameters

The hyperparameters are the margainal precision τ and the mixing parameter ϕ. The marginal precision τ is represented as

$$\theta_1 = \log(\tau)$$

and the mixing parameter as

$$\theta_2 = \log\left(\frac{\phi}{1-\phi}\right)$$

and the prior is defined on $\theta = (\theta_1, \theta_2)$.

Specification

The bym2 model is specified inside the `f()` function as

```
f(<whatever>, model="bym2", graph=<graph>,
  hyper=<hyper>, adjust.for.con.comp = TRUE)
```

The neighbourhood structure of `x` is passed to the program through the `graph` argument.

The option `adjust.for.con.comp` adjust the model if the graph has more than one connected compoment, and this adjustment can be disabled setting this option to `FALSE`. This means that `constr=TRUE` is interpreted as a sum-to-zero constraint on *each* connected component and the `rankdef` parameter is set accordingly.

Hyperparameter spesification and default values

doc The BYM-model with the PC priors

hyper

theta1

hyperid 11001
name log precision
short.name prec
prior pc.prec

param 1 0.01

initial 4

fixed FALSE

to.theta `function(x) log(x)`

from.theta `function(x) exp(x)`

theta2

hyperid 11002

name logit phi

short.name phi

prior pc

param 0.5 0.5

initial -3

fixed FALSE

to.theta `function(x) log(x/(1-x))`

from.theta `function(x) exp(x)/(1+exp(x))`

constr TRUE

nrow.ncol FALSE

augmented TRUE

aug.factor 2

aug.constr 2

n.div.by

n.required TRUE

set.default.values TRUE

status experimental

pdf bym2

Example

Details on the implementation

This gives some details of the implementation, which depends on the following variables

nc1 Number of connected components in the graph with size 1. These nodes, *singletons*, have no neighbours.

nc2 Number of connected components in the graph with size ≥ 2.

scale.model The value of the logical flag, if the model should be scaled or not. (Default FALSE)

adjust.for.con.comp The value of the logical flag if the `constr=TRUE` option should be reinterpreted.

The case `(scale.model==FALSE && adjust.for.con.comp == FALSE)`

The option `constr=TRUE` is interpreted as a sum-to-zero constraint over the whole graph. Singletons are given a uniform distribution on $(-\infty, \infty)$ before the constraint.

The case `(scale.model==TRUE && adjust.for.con.comp == FALSE)`

The option `constr=TRUE` is interpreted as a sum-to-zero constraint over the whole graph. Let $Q = \tau R$ be the standard precision matrix from the `besag`-model with precision parameter τ. Then R, except the singletons, are scaled so that the geometric mean of the marginal variances is 1, and R is modified so that singletons have a standard Normal distribution.

The case `(scale.model==FALSE && adjust.for.con.comp == TRUE)`

The option `constr=TRUE` is interpreted as one sum-to-zero constraint over each of the `nc2` connected components of size ≥ 2. Singletons are given a uniform distribution on $(-\infty, \infty)$.

The case `(scale.model==TRUE && adjust.for.con.comp == TRUE)`

The option `constr=TRUE` is interpreted as `nc2` sum-to-zero constraints for each of the connected components of size ≥ 2. Let $Q = \tau R$ be the standard precision matrix from the `besag`-model with precision parameter τ. Then R, are scaled so that the geometric mean of the marginal variances in each connected component of size ≥ 2 is 1, and modified so that singletons have a standard Normal distribution.

Notes

The term $\frac{1}{2}\log(|R|^*)$ of the normalisation constant is not computed, hence you need to add this part to the log marginal likelihood estimate, if you need it. Here R is the precision matrix for the standardised Besag part of the model.

The generic PC-prior for ϕ is available as `prior="pc"` and parameters `param="c(u, alpha)"`, where $\text{Prob}(\phi \leq u) = \alpha$. If $\alpha < 0$ or $\alpha > 1$, then it is set to a value close to the minimum value of α allowed. This prior depends on the graph and its computational cost is $\mathcal{O}(n^3)$.