

Generalized Pareto distribution

Parametrisation

The generalized Pareto (GP) distribution with positive shape parameter has cumulative distribution function

$$F(y; \sigma, \xi) = 1 - \left(1 + \xi \frac{y}{\sigma}\right)^{-1/\xi}, \quad y > 0,$$

for a continuous response y where

ξ : is the shape parameter, $\xi > 0$

σ : is the scale parameter, $\sigma > 0$

The limit for $\xi \downarrow 0$ is $F(y; \sigma, 0) = 1 - \exp(-y/\sigma)$.

Link function

The linear predictor η controls the α quantile of the GP

$$P(y \leq q_\alpha) = \alpha$$

and $q_\alpha = \exp(\eta)$. The scaling σ , is then a function of (q_α, ξ) , as

$$\sigma = \frac{\xi \exp(\eta)}{(1 - \alpha)^{-\xi} - 1}$$

Hyperparameters

The GP model has one hyperparameter. The shape $\xi > 0$ is represented as

$$\theta = \log \xi,$$

and the prior is defined on θ .

Specification

- family=gp
- Required arguments: y and the quantile α .

The quantile is given as `control.family=list(control.link=list(quantile= α)).`

Hyperparameter specification and default values

doc Generalized Pareto likelihood

hyper

theta

hyperid 101201

name shape

short.name xi

initial -2.30258509299405

fixed FALSE

prior loggamma

param 1 15

to.theta function(x) log(x)

from.theta function(x) exp(x)

status experimental

survival FALSE

discrete TRUE

link default quantile

pdf genPareto

Example

```
rgp = function(n, sigma, eta, alpha, xi = 0.001)
{
  if (missing(sigma)) {
    stopifnot(!missing(eta) && !missing(alpha))
    sigma = exp(eta) * xi / ((1.0 - alpha)^(-xi) - 1.0)
  }
  return (sigma / xi * (runif(n)^(-xi) - 1.0))
}
```

```
n = 300
x = runif(n)-0.5
eta = 1+x
alpha = 0.95
xi = 0.3
y = rgp(n, eta = eta, alpha = alpha, xi=xi)

r = inla(y ~ 1+x,
```

```
data = data.frame(y, x),
family = "gp",
control.family = list(control.link = list(quantile = alpha)),
control.predictor = list(compute=TRUE),
verbose=TRUE)

rx = range(c(r$summary.fitted.values$mean, exp(eta)))
plot(r$summary.fitted.values$mean, exp(eta),
     xlim = rx, ylim = rx)
abline(a=0,b=1)
```

Notes