Fractional Gaussian Noise (FGN)

Parametrization

The (stationary) FGN (Gaussian) process has correlation function at lag k

$$\rho(k) = |k+1|^{2H} - 2|k|^{2H} + |k-1|^{2H}$$

where H is the Hurst parameter or self-similarity parameter, which we assume to be

$$1/2 < H < 1$$
.

so the process has long range properties for H > 1/2. The locations of the process is fixed to $1, 2, \ldots, n$, where n is the dimension of the finite representation of the FGN process.

Hyperparameters

The marginal precision, τ , of the process is represented as

$$\tau = \exp(\theta_1)$$

The Hurst parameter H is represented as

$$H = \frac{1}{2} + \frac{1}{2} \frac{\exp(\theta_2)}{1 + \exp(\theta_2)}$$

and the prior is defined on $\theta = (\theta_1, \theta_2)$.

Specification

The FGN model is specified as

```
f(<whatever>, model="fgn", order=<order>, hyper = <hyper>)
```

The parmeter order gives the order of the Markov approximation. Currently, only order=3 is implemented.

Hyperparameter spesification and default values for model="fgn"

doc Fractional Gaussian noise model

hyper

theta1

theta2

hyperid 13101
name log precision
short.name prec
prior pc.prec
param 3 0.01
initial 1
fixed FALSE
to.theta function(x) log(x)
from.theta function(x) exp(x)

```
hyperid 13102
         name logit H
         short.name H
         prior pcfgnh
         param 0.9 0.1
         initial 2
         fixed FALSE
         to.theta function(x) \log((2*x-1)/(2*(1-x)))
         from.theta function(x) 0.5 + 0.5*exp(x)/(1+exp(x))
constr FALSE
nrow.ncol FALSE
augmented TRUE
aug.factor 5
aug.constr 1
n.div.by
n.required FALSE
set.default.values TRUE
order.default 4
order.defined 34
pdf fgn
Hyperparameter spesification and default values for model="fgn2"
doc Fractional Gaussian noise model (alt 2)
hyper
     theta1
         hyperid 13101
         name log precision
         \mathbf{short.name} \;\; \mathrm{prec}
         prior pc.prec
         param 3 0.01
         initial 1
         fixed FALSE
         to.theta function(x) log(x)
         from.theta function(x) exp(x)
     theta2
         hyperid 13102
         name logit H
         short.name H
```

```
prior pcfgnh
         param 0.9 0.1
         initial 2
         fixed FALSE
         to.theta function(x) \log((2*x-1)/(2*(1-x)))
         from.theta function(x) 0.5 + 0.5*exp(x)/(1+exp(x))
constr FALSE
nrow.ncol FALSE
augmented TRUE
aug.factor 4
aug.constr 1
n.div.by
n.required FALSE
set.default.values TRUE
order.default 4
order.defined 34
pdf fgn
Example
library(FGN)
n = 1000
H = 0.77
y = SimulateFGN(n, H)
y = y - mean(y)
r = inla(y ~-1 + f(idx, model="fgn"),
         data = data.frame(y, idx=1:n),
         control.family =list(hyper = list(prec = list(initial = 12, fixed=TRUE))))
print(c(MLE=FitFGN(y, demean=TRUE)$H,
        Post.mean=r$summary.hyperpar[2,"mean"],
        Post.mode=r$summary.hyperpar[2,"mode"]))
```

Notes

In the example above, then the f(idx,model="fgn") object will expand into a Gaussian of length (order + 1)*n. The first n elements is the FGN model (which is of interrest), then there are order vector of AR1 processes each of length n, and the sum of these AR1 processes is used to represent the FGN.

Another alternative, is f(idx,model="fgn2") object will expand into a Gaussian of length order*n, which are the cumulative sums of the the order vector of AR1 processes each of length n. If order==3, with weighted AR1 processes (and with the given precision), x, xx and xxx, then model="fgn2" return the vector (x + xx + xxx, xx + xxx, xxx) where $\phi_x < \phi_{xx} < \phi_{xxx}$.

The PC-prior for H take two arguments (U, α) where $\text{Prob}(U < H < 1) = \alpha$.