ATE estimations from real observational data

This notebook examines the use of Bayesian Networks for estimating Average Treatment Effects (ATE) in Observational Studies within the Neyman-Rubin potential outcome framework from real data:

In [23]:
```
import pyAgrum as gum
import pyAgrum.lib.notebook as gnb
import pyAgrum.skbn as skbn
import pyAgrum.causal as csl
import pyAgrum.causal.notebook as cslnb

import numpy as np
import pandas as pd
import matplotlib.pyplot as plt

from scipy.stats import gaussian_kde

pd.set_option('future.no_silent_downcasting', True)
```

Dataset

The dataset under consideration is the Census Adult Income dataset. The objective of this analysis is to determine whether possessing a graduate degree increases the likelihood of earning an income exceeding $50,000 per year.

In [72]:
```
df = pd.read_pickle("../data/df_causal_discovery.p")
df.head()
```

Out[72]:

	age	hours-per-week	hasGraduateDegree	inRelationship	isWhite	isFemale	greaterThan50k
0	39	40	0	0	1	0	0
1	50	13	0	1	1	0	0
2	38	40	0	0	1	0	0
3	53	40	0	1	0	0	0
5	37	40	1	0	1	1	0

In [73]:
```
df.describe()
```

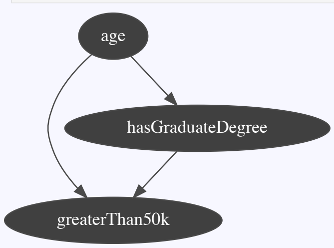
Out[73]:

	age	hours-per-week	hasGraduateDegree	inRelationship	isWhite	isFemale	greaterThan50k
count	29170.000000	29170.000000	29170.000000	29170.000000	29170.000000	29170.000000	29170.000000
mean	38.655674	40.447755	0.052348	0.406616	0.878334	0.331916	0.245835
std	13.722408	12.417203	0.222732	0.491211	0.326905	0.470909	0.430588
min	17.000000	1.000000	0.000000	0.000000	0.000000	0.000000	0.000000
25%	28.000000	40.000000	0.000000	0.000000	1.000000	0.000000	0.000000
50%	37.000000	40.000000	0.000000	0.000000	1.000000	0.000000	0.000000
75%	48.000000	45.000000	0.000000	1.000000	1.000000	1.000000	0.000000
max	90.000000	99.000000	1.000000	1.000000	1.000000	1.000000	1.000000

Bayesian Network Preparation

We begin by focusing exclusively on the age covariate to inform our estimations. We hypothesize that age is a causal factor influencing both the hasGraduateDegree variable and the greaterThan50k outcome.

In [74]:
```
disc = skbn.BNDiscretizer(defaultDiscretizationMethod="NoDiscretization", defaultNumberOfBins=None)
template = disc.discretizedBN(df[["age", "hasGraduateDegree", "greaterThan50k"]])
template.addArcs([("age","hasGraduateDegree"), ("age", "greaterThan50k"), ("hasGraduateDegree", "greaterThan50k")])

gnb.show(template, size="50")
```

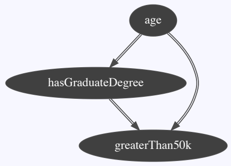


In [75]:
```
p_learner = gum.BNLearner(df, template)
p_learner.useNMLCorrection()
p_learner.useSmoothingPrior(1e-9)

bn = gum.BayesNet(template)
p_learner.fitParameters(bn)

print(p_learner)
```

```
Filename       : /tmp/tmp9oipz0_e.csv
Size           : (29170,3)
Variables      : age[73], hasGraduateDegree[2], greaterThan50k[2]
Induced types  : False
Missing values : False
Algorithm      : MIIC
Score          : BDeu  (Not used for constraint-based algorithms)
Correction     : NML  (Not used for score-based algorithms)
Prior          : Smoothing
Prior weight   : 0.000000
```

In [76]:
```
cbn = csl.CausalModel(bn)

T = "hasGraduateDegree"
Y = "greaterThan50k"
X = "age"
```

In [78]:
```
cslnb.showCausalImpact(cbn, on=Y, doing=T, values={T:1})
```



$$P(greaterThan50k \mid \mathrm{do}(hasGraduateDegree)) = \sum_{age} P\left(greaterThan50k \mid age, hasGraduateDegree\right) \cdot P\left(age\right)$$

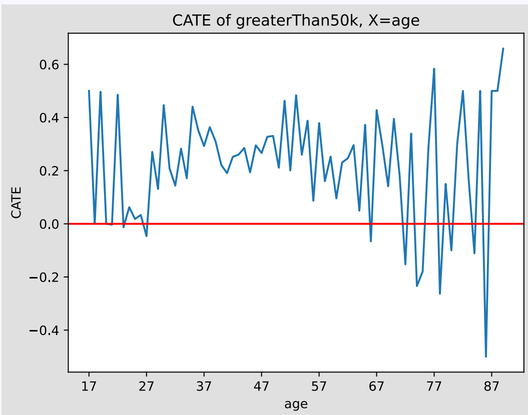
greaterThan50k	
0	1
0.5346	0.4654

Causal Model *Explanation : backdoor ['age'] found.* *Impact*

$$P(greaterThan50k \mid \mathrm{do}(hasGraduateDegree)) = \sum_{age} P\left(greaterThan50k \mid age, hasGraduateDegree\right) \cdot P\left(age\right)$$

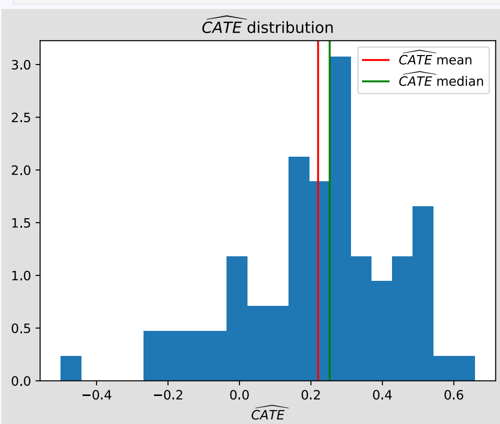
In [10]:
```
_,cpt0,_ = csl.causalImpact(cbn, on=Y, doing=T, knowing={X}, values={T:0})
_,cpt1,_ = csl.causalImpact(cbn, on=Y, doing=T, knowing={X}, values={T:1})

CATE = (cpt1 - cpt0).topandas()
CATE = CATE[Y]["1"]
```

In [11]:
```
plt.plot(CATE)

plt.xticks(np.arange(0, 80, 10))
plt.axhline(y=0, color="r")
plt.title("CATE of greaterThan50k, X=age")
plt.xlabel("age")
plt.ylabel("CATE")
plt.show()
```



In [45]:
```
density = gaussian_kde(pd.cut(CATE.values, bins=10).value_counts())
xs = np.linspace(-0.5,0.7,200)
```

In [63]:
```
plt.hist(x=CATE, density=True, bins=20)
#plt.plot(xs, density(xs))

plt.xlabel("$\widehat{CATE}$")
plt.title("$\widehat{CATE}$ distribution")
plt.axvline(x=CATE.mean(), color="red", label="$\widehat{CATE}$ mean")
plt.axvline(x=CATE.median(), color="green", label="$\widehat{CATE}$ median")
plt.legend()
plt.show()
```



In [64]:
```
def mutilateBN(bn : gum.BayesNet) -> gum.BayesNet:
    """
    Returns a copy of the Bayesian Network with all incoming arcs to the variable T removed.
    """
    res = gum.BayesNet(bn)
    for p_id in bn.parents(T):
        res.eraseArc(p_id, bn.idFromName(T))
    return res

def ATE(bn, X = {}, mutilate = True):
    ie = gum.LazyPropagation(mutilateBN(bn)) if mutilate else gum.LazyPropagation(bn)

    ie.setEvidence({T: 0} | X)
    ie.makeInference()
    p0 = ie.posterior(Y)

    ie.chgEvidence(T,1)
    ie.makeInference()
    p1 = ie.posterior(Y)

    diff = p1 - p0
    return diff.expectedValue(lambda d : diff.variable(0).numerical(d[diff.variable(0).name()]))
```

In [65]:
```python
ATE(bn)
```

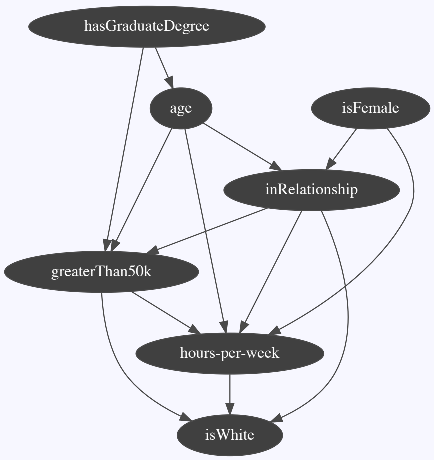
Out[65]:
```
0.2332334047559898
```

Let's examine whether incorporating all available covariates influences the estimation of the ATE. We will employ structure learning techniques to determine the DAG that the algorithm identifies from the data.

In [79]:
```python
disc = skbn.BNDiscretizer(defaultNumberOfBins=5, defaultDiscretizationMethod="uniform")
template = disc.discretizedBN(df)
```

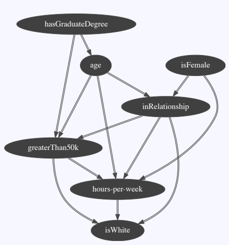
In [81]:
```python
s_learner = gum.BNLearner(df, template)
s_learner.useNMLCorrection()
s_learner.useSmoothingPrior(1e-6)

bn2 = s_learner.learnBN()

print(s_learner)

gnb.showBN(bn2)
```

```
Filename       : /tmp/tmpuny3jv0f.csv
Size           : (29170,7)
Variables      : age[5], hours-per-week[5], hasGraduateDegree[2], inRelationship[2], isWhite[2], isFemale[2], greaterThan50k[2]
Induced types  : False
Missing values : False
Algorithm      : MIIC
Score          : BDeu  (Not used for constraint-based algorithms)
Correction     : NML  (Not used for score-based algorithms)
Prior          : Smoothing
Prior weight   : 0.000001
```



In [82]:
```python
cbn2 = csl.CausalModel(bn2)
```

In [83]:
```python
cslnb.showCausalImpact(cbn2, on=Y, doing=T, values={T:1})
```



Causal Model

$$P(greaterThan50k \mid \text{do}(hasGraduateDegree)) =$$
$$\sum_{age,inRelationship,isFemale} P\left(greaterThan50k \mid age, hasGraduateDegree, inRelationship\right) \cdot P\left(isFemale\right) \cdot P\left(inRelationship \mid age, isFemale\right) \cdot P\left(age \mid hasGraduateDegree\right)$$

Explanation : Do-calculus computations

greaterThan50k	
0	**1**
0.4507	0.5493

Impact

In [71]:
```python
ATE(bn2)
```

Out[71]:
```
0.32517711710620006
```

We observe a higher estimated Average Treatment Effect (ATE) when incorporating all covariates compared to the estimation that accounts only for the `age` covariate.