

Classical Cryptography

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Example and definitions of a cryptosystem

Caesar Cipher - the oldest cryptosystem

A *shift* cipher – attributed to Julius Caesar (100-44 BC)

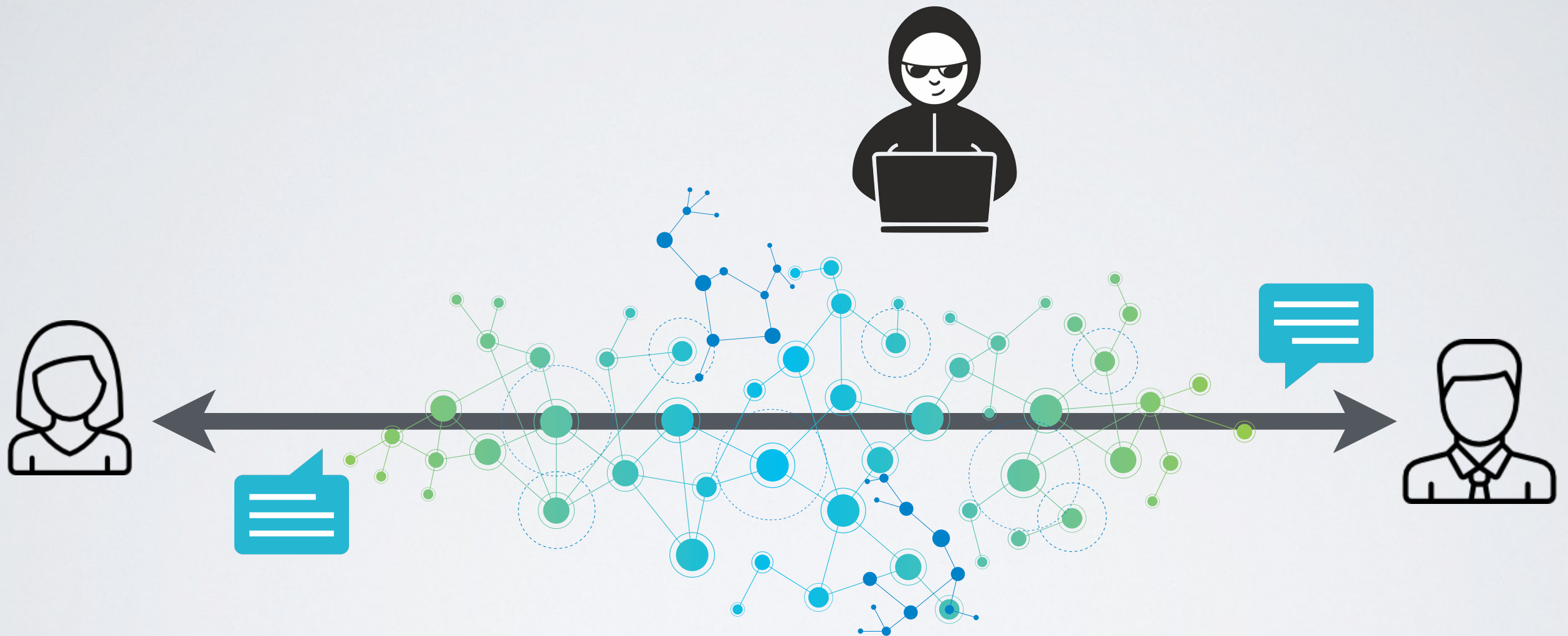
MEET ME AFTER THE TOGA PARTY

PHHW PH DIWHU WKH WRJD SDUWB

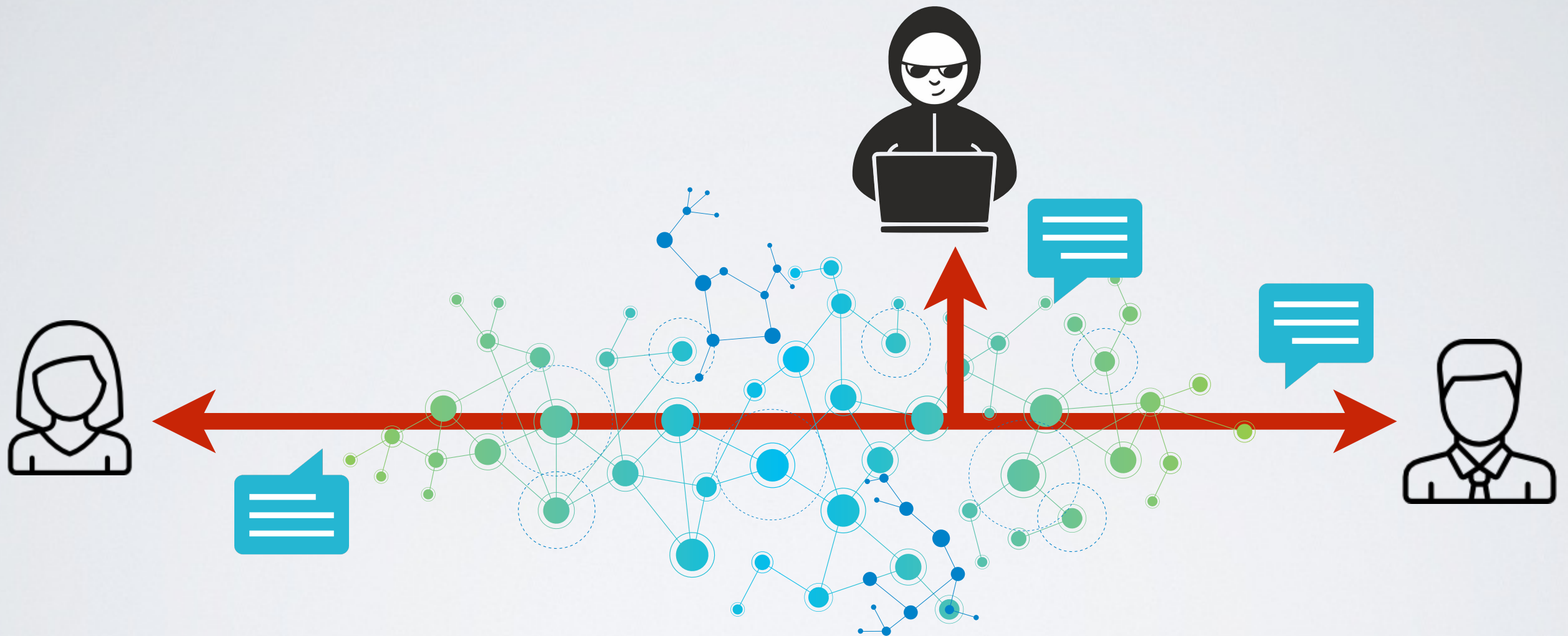
Shift the alphabet 23 places to the right and substitute letters

a	b	c	d	e	f	g	h	i	j	k	l	m	n	o	p	q	r	s	t	u	v	w	x	y	z
D	E	F	G	H	I	J	K	L	M	N	O	P	Q	R	S	T	U	V	W	X	Y	Z	A	B	C

Communication over an **insecure** medium

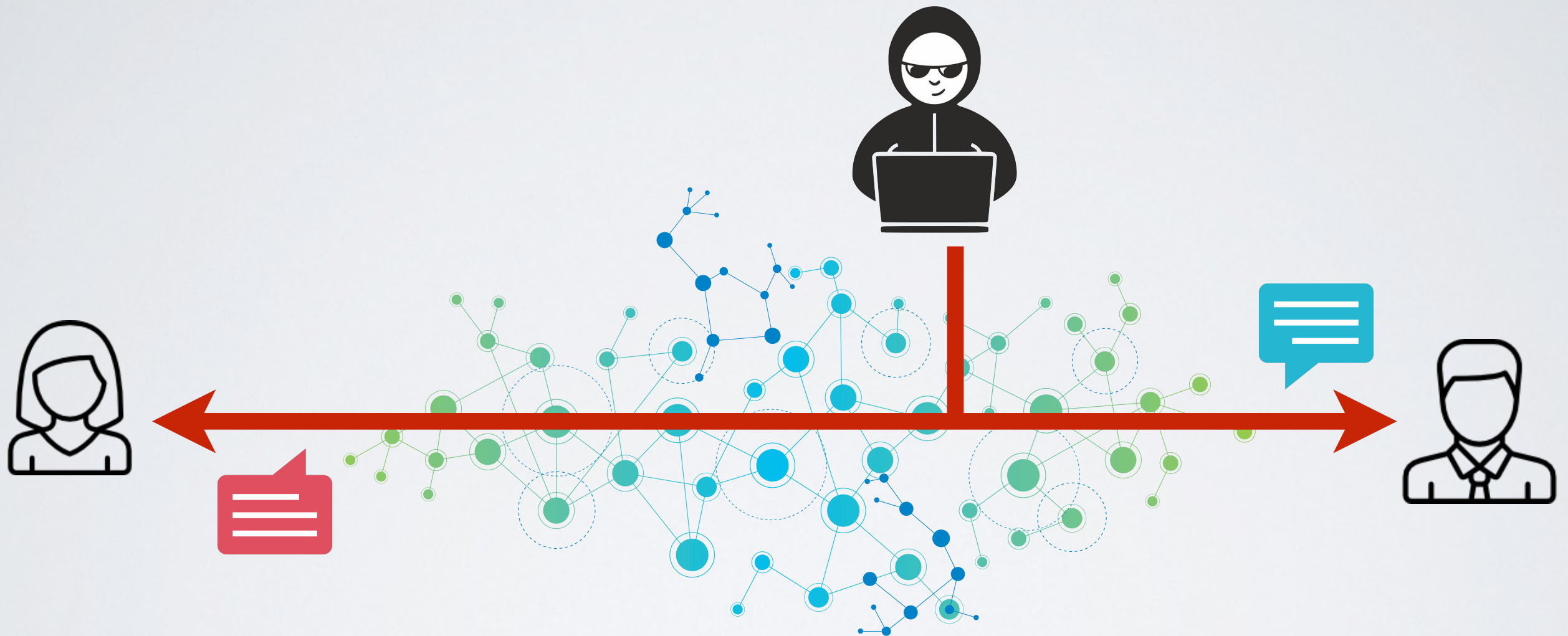


Threat I - **Interception**



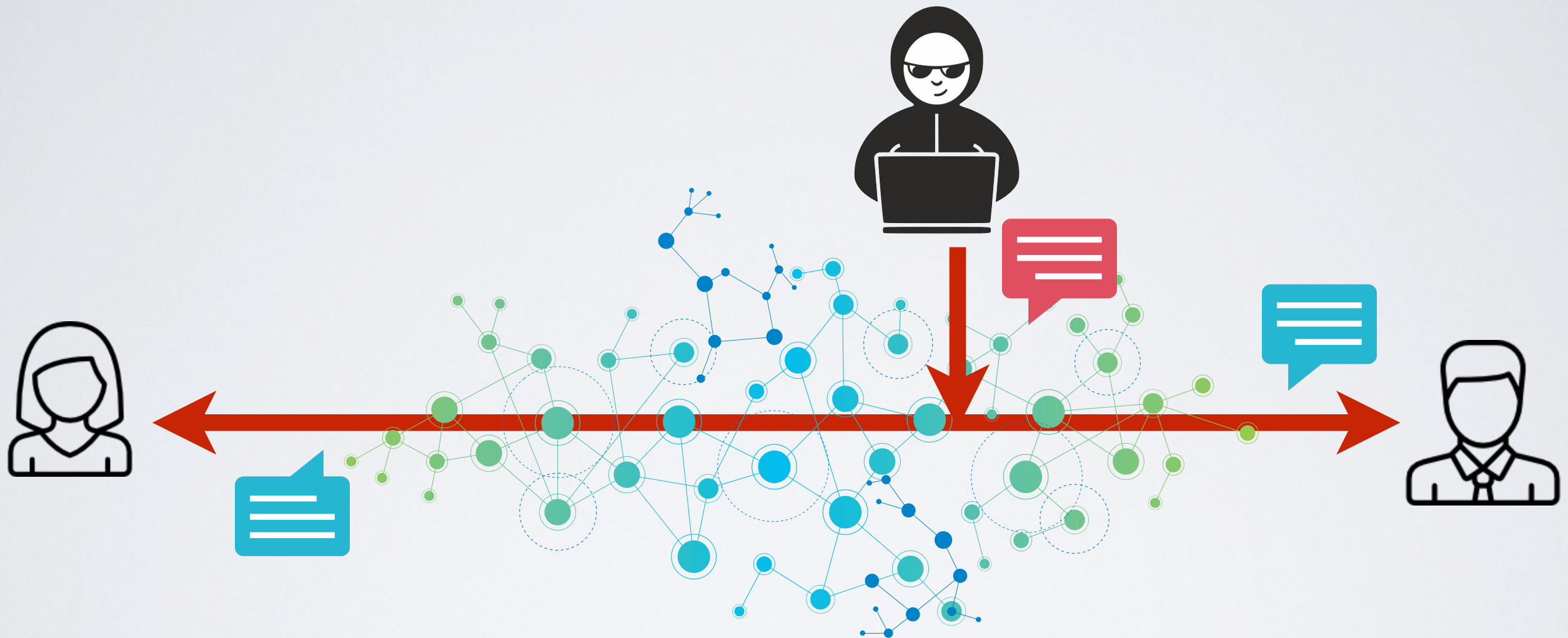
- **Interception** : an attacker can read messages

Threat 2 - **Modification**



- **Modification** : an attacker can modify messages

Threat 3 - **Fabrication**



- **Fabrication** : an attacker can inject messages

Threat 4 - **Interruption**



- **Interruption** : an attacker can block messages

Confidentiality and Integrity of communications



➡ Implement a **virtual trusted channel**
over an insecure medium

Definitions

Plaintext

The message in its clear form (the original message)

Ciphertext

The message in its ciphered form (the encrypted message)

Encryption

Transform a plaintext into ciphertext

Decryption

Transform a ciphertext into a plaintext

Definitions

Cryptographic algorithm

The method to do encryption and decryption

Cryptographic key

An input variable used by the algorithm for the transformation

N-bit security entropy (a.k.a. the key space)

The number of bits necessary to encode the number of possible keys (could be different than the key length)

Representing data as numbers

Cryptographic algorithms are mathematical operations

- ➡ messages and keys must be represented as numbers
for instance : ASCII encoding

Back to Caesar Cipher

Algorithm : shift the alphabet of a certain number of positions

Key : the number of positions to shift

Key space : 25 possible rotations (\sim 5 bits security)

Encoding :

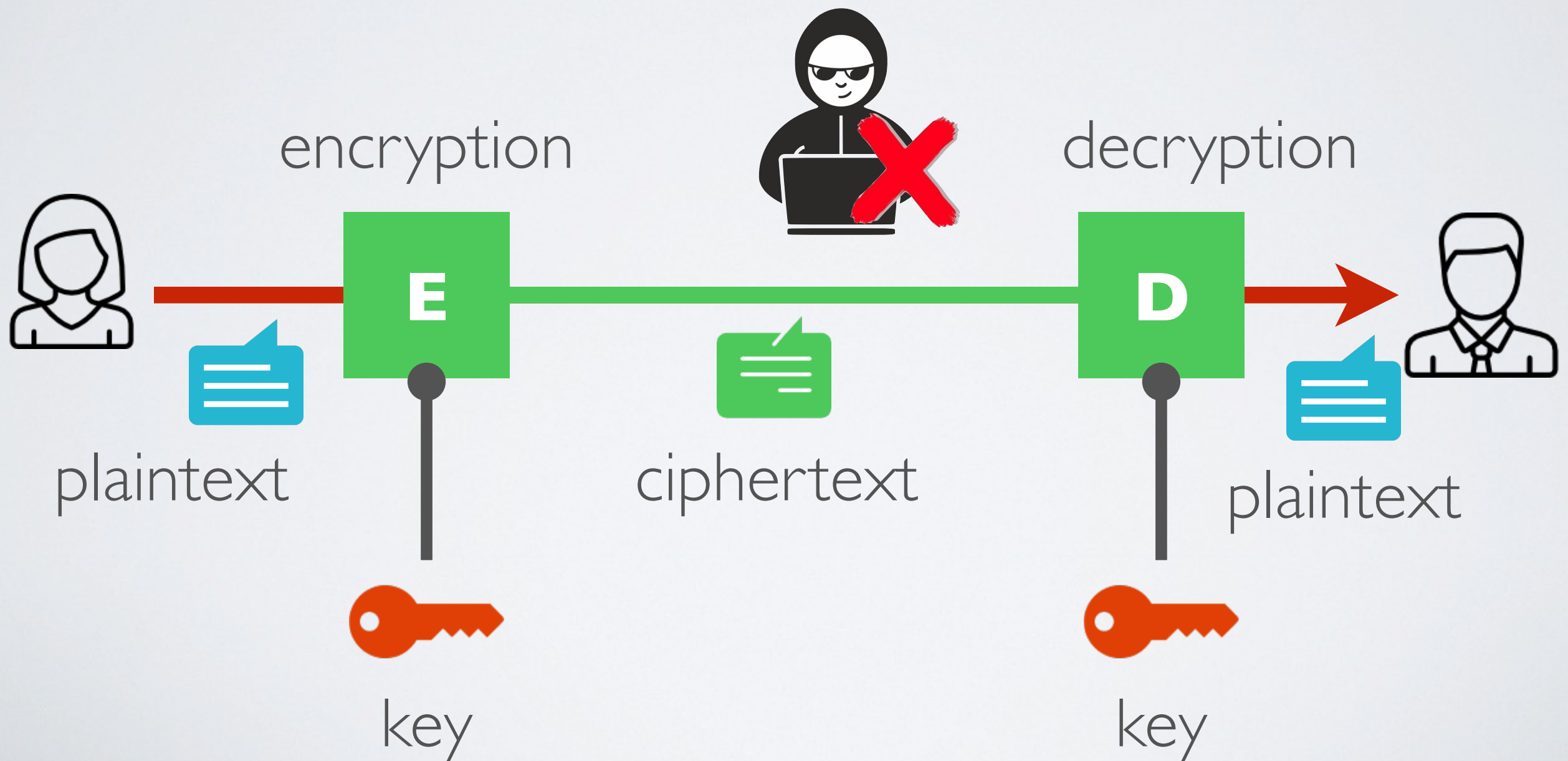
a	b	c	d	e	f	g	h	i	j	k	l	m	n	o	p	q	r	s	t	u	v	w	x	y	z
0	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20	21	22	23	24	25

Encrypting and decrypting one character is obtained as follows:

$$c = E(k, p) = (p + k) \bmod 26$$

$$p = D(k, c) = (c - k) \bmod 26$$

The big picture



Breaking the cipher

The Kerckhoffs' principle (1883)

“The enemy knows the system” - the security of a communication should not rely on the fact that the algorithms are secrets

- ➡ A cryptosystem should be secure even if everything about the system, except the key, is public knowledge

No security by obscurity

Breaking the cipher - the attacker's model

- **Exhaustive Search** (a.k.a brute force)
Try all possible n keys (in average it takes $n/2$ tries)
 - **Ciphertext only**
You know one or several random ciphertexts
 - **Known plaintext**
You know one or several pairs of random plaintext and their corresponding ciphertexts
 - **Chosen plaintext**
You know one or several pairs of chosen plaintext and their corresponding ciphertexts
 - **Chosen ciphertext**
You know one or several pairs of plaintext and their corresponding chosen ciphertexts
- ➔ **A good crypto system resists all attacks**

Breaking Caesar cipher

Exhaustive search	Yes
ciphertext only	Statistical Analysis
known plaintext	Look at the first letter and get the shift
chosen plaintext	Choose “A” and get the shift
chosen ciphertext	Choose “A” and get the shift

Statistical Cryptanalysis

- ➔ Monoalphabetic ciphers do not change the relative frequency of letters in a message

Evolution of cryptosystems

Substitution ciphers (a.k.a mono alphabetic ciphers)

➔ Improvement over Caesar cipher

Algorithm : allow an arbitrary permutation of the alphabet

Key : set of substitutions

Key space : $26!$ possible substitutions ($4 \times 10^{26} \sim 89$ bits)

a	b	c	d	e	f	g	h	i	j	k	l	m	n	o	p	q	r	s	t	u	v	w	x	y	z
D	K	V	Q	F	I	B	J	W	P	E	S	C	X	H	T	M	Y	A	U	O	L	R	G	Z	N

if we wish to replace letters

WI RF RWAJ UH YFTSDVF SFUUFYA

Breaking substitution ciphers

Exhaustive search	Doable with a computer
ciphertext only	Statistical analysis
known plaintext	Match letters together
chosen plaintext	Choose ABCDE ... and match letters
chosen ciphertext	Choose ABCDE ... and match letters

Polyalphabetic ciphers (a.k.a Renaissance Cipher)

➔ Vigenere cipher

Algorithm : combine the message and the key

Key : a word

Key space : 26^n (n being the length of the key)

$$\begin{array}{r} \text{wearediscoveredsaveyourself} \\ + \text{deceptivedeceptivedeceptive} \pmod{26} \\ \hline \text{ZICVTWQNGRZGVTWAVZHCQYGLMGJ} \end{array}$$

Advantage : Encryption of a letter is context dependent

Breaking Polyalphabetic Ciphers

exhaustive search	Small key length only
ciphertext only	Statistical analysis for small key length and significant amount of ciphertext
known plaintext	Subtract plaintext from ciphertext
chosen plaintext	Choose AAAAAA ... and match letters
chosen ciphertext	Choose AAAAAA ... and match letters

OTP - One Time Pad

➔ Improvement over Vigenere cipher

Algorithm : combine the message and the key

Key : an infinite random string

Key space : 26^n (n being the length of the plaintext)

	whatanicedaytoday
⊕	yksuftgoarfwfwel

	ZZZJUCLUDTUNNWGQS

Advantage : **this is the perfect cipher !**

Disadvantage : hard to use in practice, how to transmit the key ?

The impossibility of breaking OTP

The ciphertext bears no statistical relationship to the plaintext

➡ No statistical analysis

For any plaintext and ciphertext, there exists a key mapping one to the other, and all keys are equally probable

➡ A ciphertext can be decrypted to any plaintext of the same length

Transposition Cipher

Algorithm : switch letters around a permutation

Key : a set of permutation

Key space : the set of permutations

helloworld

LOLHERDLWO

Breaking Transposition ciphers

brute force	Small key length only
ciphertext only	Hard for large permutations
known plaintext	Match letters together
chosen plaintext	Choose ABCDE ... and match letters
chosen ciphertext	Choose ABCDE ... and match letters

The seeds of modern cryptography

1. **Diffusion**

Mix-up symbols

Transposition Cipher

2. **Confusion**

Replace a symbol with another

Polyalphabetic Cipher

3. **Randomization**

Repeated encryption of the same text are different

OTP

A brief history

~ 2000 years ago	Substitution ciphers (a.k.a mono alphabetic ciphers)
Few Centuries Later	Transposition Ciphers
Renaissance	Polyalphabetic Ciphers
1844	Invention of the Telegraph
1882	One Time Pad
1939	World War II The Enigma Machine
1970	Data Encryption Standard (DES)
1976	Public Key Cryptography (RSA)