Number Theory - Prime numbers

Prime Numbers

- p is prime if 1 and p are its only divisors e.g $3, 5, 7, 11$ …
- p and q are relatively prime (a.k.a. coprime) if $\gcd(p,q) = 1$ e.g $\gcd(4,5) = 1$

➡ There are infinitely many primes

Euler-Fermat Theorem

If $n = p \,.\, q$ and $z = (p-1).(q-1)$

and a such that a and n are relative primes

Then $a^z \equiv 1 \pmod n$

Computational Complexity

Easy problems with prime numbers

- Generating a prime number p
- Addition, multiplication, exponentiation
- Inversion, solving linear equations

Hard problem with prime numbers

- Factoring primes
  e.g. given $\mathbf{n}$ find $\mathbf{p}$ and $\mathbf{q}$ such that $\mathbf{n = p\ .\ q}$