

# Number Theory - Prime numbers

## Prime Numbers

- $p$  is prime if 1 and  $p$  are its only divisors e.g 3, 5, 7, 11 ...
  - $p$  and  $q$  are relatively prime (a.k.a. coprime) if  $\gcd(p,q) = 1$   
e.g  $\gcd(4,5) = 1$
- ➔ There are infinitely many primes

## Euler-Fermat Theorem

If  $n = p \cdot q$  and  $z = (p-1) \cdot (q-1)$

and  $a$  such that  $a$  and  $n$  are relative primes

Then  $a^z \equiv 1 \pmod{n}$

# Computational Complexity

## **Easy problems** with prime numbers

- Generating a prime number  $p$
- Addition, multiplication, exponentiation
- Inversion, solving linear equations

## **Hard problem** with prime numbers

- Factoring primes  
e.g. given  $n$  find  $p$  and  $q$  such that  $n = p \cdot q$