Recurssion Relation

1) $T(2) = ?$

$T(n) = 3T(n-1) + 12n$

Given $T(0) = 5$

~~$T(0) = 3T($~~

If $n = 1$

$T(1) = 3T(1-1) + 12(1)$

$= 3T(0) + 12$

$= 3(5) + 12$ $\quad (\because T(0) = 5)$

$= 15 + 12$

$= 27$

$T(2) = 3T(2-1) + 12(2)$

$= 3T(1) + 24$

$= 3(27) + 24$ $\quad (\because T(1) = 27)$

$= 81 + 24$

$= 105$

2)

a) $T(n) = T(n-1) + C$

$T(n) = T(n-2) + C + C$ $\quad T(n-1) = T(n-1-1) + C$

$T(n) = T(n-3) + C + C + C$

k times

$\boxed{T(n) = T(n-k) + kC}$

$n - k = 1$

$\boxed{k = n-1}$

$T(n) = T(n - n + 1) + (n-1)C$

$= T(1) + (n-1) + C$

$= 0 + (n-1) + C$

$= \boxed{nC - C}$

$\Rightarrow \boxed{O(n)}$

b) $T(n) = 2T(n/2) + n$

$T(n) = 2\left(2T\left(\frac{n}{2^2}\right) + \frac{n}{2}\right) + n$ $\quad T\left(\frac{n}{2}\right) = 2T\left(\frac{n}{2^2}\right) + \frac{n}{2}$

$= 4T\left(\frac{n}{2^2}\right) + n + n$ $\quad T\left(\frac{n}{2^2}\right) = 2T\left(\frac{n}{2^3}\right) + \frac{n}{2^2}$

$T(n) = 4T\left(\frac{n}{2^2}\right) + 2n$

$= 4\left(2T\left(\frac{n}{2^3}\right) + \frac{n}{2^2}\right) + 2n$

$= 8T\left(\frac{n}{2^3}\right) + n + 2n$

$T(n) = 8T\left(\frac{n}{2^3}\right) + 3n$

k time

$$\boxed{T(n) = 2^k T\left(\frac{n}{2^k}\right) + kn}$$

$\frac{n}{2^k} = 1$

$n = 2^k$

$\log n = k \log 2$

$k = \log_2 n$

$T(n) = 2^{\log_2 n} \cdot (0) + \log_2 n \cdot n$

$\Rightarrow \boxed{O(n \log_2 n)}$

c) $T(n) = 2T\left(\frac{n}{2}\right) + C$

$T(n) = 2\left(2T\left(\frac{n}{2^2}\right) + C\right) + C$

$T(n) = 4T\left(\frac{n}{2^2}\right) + 2C + C$

$\boxed{T(n) = 4T\left(\frac{n}{2^2}\right) + 3C} \rightarrow (2^2 - 1)$

$T(n) = 4\left(2T\left(\frac{n}{2^3}\right) + C\right) + 3C$

$= 8T\left(\frac{n}{2^3}\right) + 4C + 3C$

$= 8T\left(\frac{n}{2^3}\right) + 7C$

$\boxed{T(n) = 2^3 T\left(\frac{n}{2^3}\right) + (2^3 - 1)C}$

k time

$\boxed{T(n) = 2^k T\left(\frac{n}{2^k}\right) + (2^k - 1)C}$

$\frac{n}{2^k} = 1$

$\boxed{k = \log_2 n}$

$T(n) = (2^{\log_2 n} - 1) \cdot C$

$= 2^{\log_2 n} C - C$

$\Rightarrow \boxed{O(2^{\log_2 n})}$

d) $T(n) = T\left(\frac{n}{2}\right) + C$

$T(n) = T\left(\frac{n}{2^2}\right) + 2C$

$T(n) = T\left(\frac{n}{2^3}\right) + 3C$

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$T(n) = \cancel{T\left(\frac{n}{2^k}\right)}^{0} + kC$

$\frac{n}{2^k} = 1$

$\boxed{k = \log_2 n}$

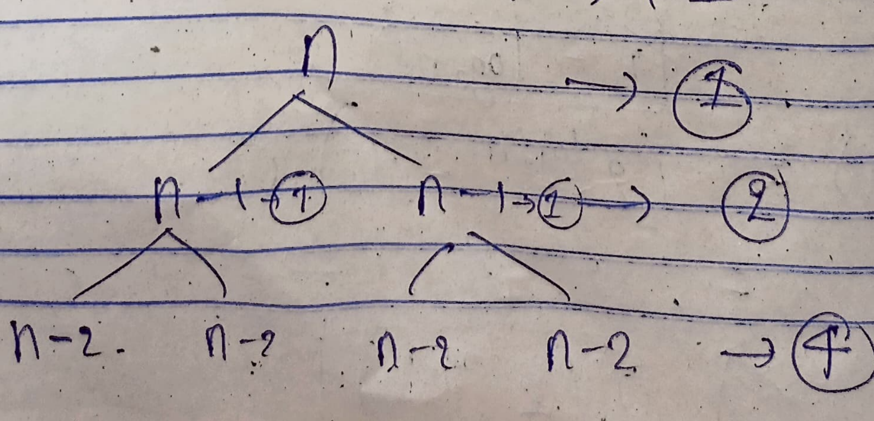
$T(n) = 0 + \log_2 n \, C$

$\boxed{\rightarrow O(\log_2 n)}$

3)

a) $T(n) = 2T(n-1) + 1$

$T(n) = T(n-1) + T(n-1) + 1$



left side Right side

$n-k=0$ $n-k=0$

$n=k$ or $k=n$

$2^0+2^1+2^2+2^3+ \dots +2^k$

$1+2+4+8+ \dots +2^k$

$r = \frac{2}{1} = \frac{4}{2} = \frac{8}{2} = 2$

$a=1$

$$S_n = \frac{a(r^n-1)}{r-1}$$

$$= \frac{1(2^n-1)}{2-1}$$

$$= 2^n-1$$

$$\rightarrow \boxed{O(2^n)}$$

b) $T(n) = 2T\left(\frac{n}{2}\right)+n$

$T(n) = T\left(\frac{n}{2}\right)+T\left(\frac{n}{2}\right)+n$

$n \longrightarrow n$

$\frac{n}{2} \rightarrow \frac{n}{2}$ $\frac{n}{2} \rightarrow \frac{n}{2} \longrightarrow n$

$\frac{n}{4}$ $\frac{n}{4}$ $\frac{n}{4}$ $\frac{n}{4} \rightarrow n$

left sid	Right
$\frac{n}{2^k} = 1$	$\frac{n}{2^k} = 1$
$n = 2^k$	$n = 2^k$
$\boxed{k = \log_2 n}$	$\boxed{k = \log_2 n}$

$n + n + n + \ldots \ldots$ k times

$O(n \cdot k)$

$\Rightarrow \boxed{O(n \cdot \log_2 n)}$