EN4563: ROBOTICS

Mini Project

Kinematic Analysis of a Robot Arm

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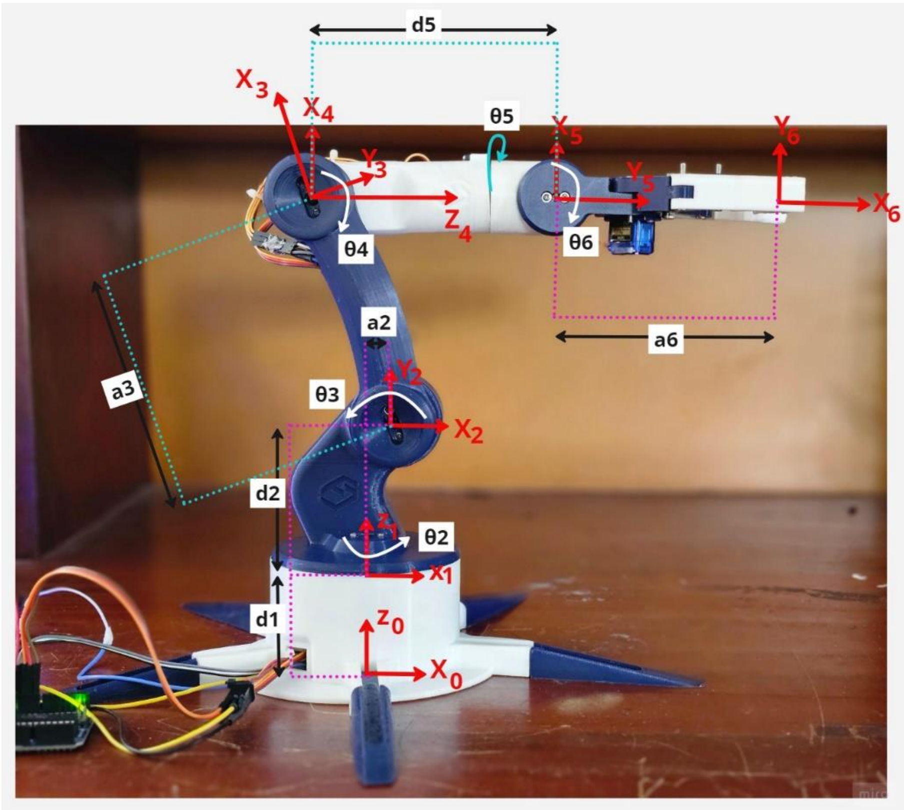
Introduction

This mini project aims to analyze the movement of a 5 degrees of freedom (DoF) robot arm. The robot arm has an RRRRR configuration.

1. Denavit-Hartenberg (DH) Table



Parameter	Length
a2	1.3 cm
a3	12.021 cm
a6	13 cm
d1	6.1 cm
d2	7.001 cm
d5	12.171 cm

Link	a_i	α_i	d_i	θ_i
1	0	0	d1	0
2	a2	$\frac{\pi}{2}$	d2	θ_2^*
3	a3	π	0	$\theta_3^* + k_3$
4	0	$-\frac{\pi}{2}$	0	$\theta_4^* + k_4$
5	0	$\frac{\pi}{2}$	d5	θ_5^*
6	a6	π	0	$\theta_6^* + k_6$

k - Offset angle

```
noname:: 5 axis, RRRRR, stdDH, slowRNE
+---+-----------+-----------+-----------+-----------+-----------+
| j |     theta |         d |         a |     alpha |    offset |
+---+-----------+-----------+-----------+-----------+-----------+
|  1|         q1|     13.101|        1.3|     1.5708|          0|
|  2|         q2|          0|     12.021|    3.14159|   0.785398|
|  3|         q3|          0|          0|    -1.5708|  -0.785398|
|  4|         q4|      12.17|          0|     1.5708|          0|
|  5|         q5|          0|         13|    3.14159|     1.5708|
+---+-----------+-----------+-----------+-----------+-----------+
```

2. Forward Kinematics

Homogeneous transformation matrix from base frame to the end effector frame,

$$R = \begin{bmatrix} R_{11} & R_{12} & R_{13} \\ R_{21} & R_{22} & R_{23} \\ R_{31} & R_{32} & R_{33} \end{bmatrix}, \quad t = \begin{bmatrix} t_{11} \\ t_{21} \\ t_{31} \end{bmatrix}, H = \begin{bmatrix} R_{3\times3} & t_{3\times1} \\ O_{1\times3} & 1_{1\times1} \end{bmatrix}$$

$R_{11} =$ cos(th6+1.57)*(cos(th5)*(1.0*cos(th3+0.79)*cos(th4-0.79)*cos(th2)+1.0*sin(th3+0.79)*sin(th4-0.79)*cos(th2))+1.0*sin(th2)*sin(th5))-sin(th6+1.57)*(1.0*cos(th3+0.79)*sin(th4-0.79)*cos(th2)-1.0*cos(th4-0.79)*sin(th3+0.79)*cos(th2))

$R_{12} =$ 1.0*cos(th6+1.57)*(1.0*cos(th3+0.79)*sin(th4-0.79)*cos(th2)-1.0*cos(th4-0.79)*sin(th3+0.79)*cos(th2))+sin(th6+1.57)*(cos(th5)*(1.0*cos(th3+0.79)*cos(th4-0.79)*cos(th2)+1.0*sin(th3+0.79)*sin(th4-0.79)*cos(th2))+1.0*sin(th2)*sin(th5))

$R_{13} =$ 1.0*cos(th5)*sin(th2)-1.0*sin(th5)*(1.0*cos(th3+0.79)*cos(th4-0.79)*cos(th2)+1.0*sin(th3+0.79)*sin(th4-0.79)*cos(th2))

$R_{21} =$ -sin(th6+1.57)*(1.0*cos(th3+0.79)*sin(th4-0.79)*sin(th2)-1.0*cos(th4-0.79)*sin(th3+0.79)*sin(th2))-cos(th6+1.57)*(1.0*cos(th2)*sin(th5)-cos(th5)*(1.0*cos(th3+0.79)*cos(th4-0.79)*sin(th2)+1.0*sin(th3+0.79)*sin(th4-0.79)*sin(th2)))

$R_{22} =$ 1.0*cos(th6+1.57)*(1.0*cos(th3+0.79)*sin(th4-0.79)*sin(th2)-1.0*cos(th4-0.79)*sin(th3+0.79)*sin(th2))-sin(th6+1.57)*(1.0*cos(th2)*sin(th5)-cos(th5)*(1.0*cos(th3+0.79)*cos(th4-0.79)*sin(th2)+1.0*sin(th3+0.79)*sin(th4-0.79)*sin(th2)))

$R_{23} =$ -1.0*cos(th2)*cos(th5)-1.0*sin(th5)*(1.0*cos(th3+0.79)*cos(th4-0.79)*sin(th2)+1.0*sin(th3+0.79)*sin(th4-0.79)*sin(th2))

$R_{31} =$ cos(th6+1.57)*sin(th3-1.0*th4+1.57)*cos(th5)-1.0*sin(th6+1.57)*cos(th3-1.0*th4+1.57)

$R_{32} =$ cos(th6+1.57)*cos(th3-1.0*th4+1.57)+sin(th6+1.57)*sin(th3-1.0*th4+1.57)*cos(th5)

$R_{33} =$ -1.0*sin(th3-th4+1.57)*sin(th5)

$t_{11} =$ 1.3*cos(th2)+12.021*cos(th3+0.79)*cos(th2)+13.0*cos(th6+1.57)*(cos(th5)*(1.0*cos(th3+0.79)*cos(th4-0.79)*cos(th2)+1.0*sin(th3+0.79)*sin(th4-0.79)*cos(th2))+1.0*sin(th2)*sin(th5))-13.0*sin(th6+1.57)*(1.0*cos(th3+0.79)*sin(th4-0.79)*cos(th2)-1.0*cos(th4-0.79)*sin(th3+0.79)*cos(th2))-12.171*cos(th3+0.79)*sin(th4-0.79)*cos(th2)+12.171*cos(th4-0.79)*sin(th3+0.79)*cos(th2)

$t_{21} =$ 1.3*sin(th2)-13.0*sin(th6+1.57)*(1.0*cos(th3+0.79)*sin(th4-0.79)*sin(th2)-1.0*cos(th4-0.79)*sin(th3+0.79)*sin(th2))+12.021*cos(th3+0.79)*sin(th2)-13.0*cos(th6+1.57)*(1.0*cos(th2)*sin(th5)-cos(th5)*(1.0*cos(th3+0.79)*cos(th4-0.79)*sin(th2)+1.0*sin(th3+0.79)*sin(th4-0.79)*sin(th2)))-12.171*cos(th3+0.79)*sin(th4-0.79)*sin(th2)+12.171*cos(th4-0.79)*sin(th3+0.79)*sin(th2)

$t_{31} =$ 12.021*sin(th3+0.79)-12.171*cos(th3-th4+1.57)+6.5*cos(th6-th5+1.57)*sin(th3-th4+1.57)-13.0*cos(th3-th4+1.57)*sin(th6+1.57)+6.5*cos(th5+th6+1.57)*sin(th3-th4+1.57)+13.101

3. Inverse Kinematics

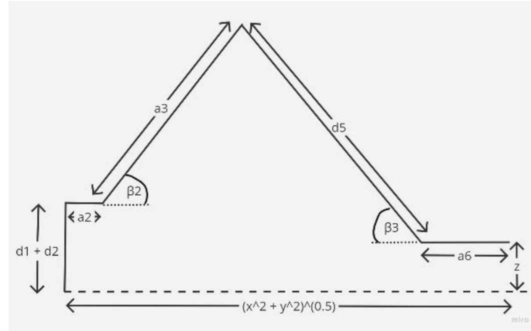
Required equations to solve $\theta_2, \theta_3, \theta_4$;

Eq 01: $\sqrt{x^2 + y^2} = a_2 + a_3 \cos\beta_2 + d_5 \cos\beta_3 + a_6$

Eq 02: $y = x \tan\beta_1$

Eq 03: $Z = d_1 + d_2 + a_3 \sin\beta_2 - d_5 \sin\beta_3$

The given $\beta_1, \beta_2, \beta_3$ has following constraints with our robot arm design.

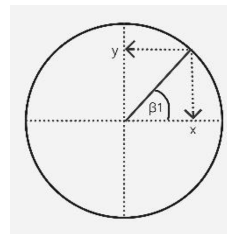


$$0^0 < \beta_1 < 165^0$$

$$0^0 < \beta_2 < 120^0$$

$$-80^0 < \beta_3 < 85^0$$

Finally, the angles required for the servo motors are as follows.



$$\theta_2 = \beta_1$$

$$\theta_3 = \beta_2 - \frac{\pi}{4}$$

$$\theta_4 = \beta_3 - \beta_2 + \frac{\pi}{4}$$

$$\theta_6 = \theta_3 - \theta_4$$

According to our robot arm pick and place orientation, we can keep the wrist angle $\theta_5 = 0^0$.

4. Manipulator Jacobian

$$J = \begin{bmatrix} J_{11} & J_{12} & J_{13} & J_{14} & J_{15} & J_{16} \\ J_{21} & J_{22} & J_{23} & J_{24} & J_{25} & J_{26} \\ 0 & 0 & J_{33} & J_{34} & J_{35} & J_{36} \\ 0 & 0 & \sin(th2) & -\sin(th2) & \sin(th3 - th4 + 1.57)\cdot\cos(th2) & -\sin(th2) \\ 0 & 0 & -\cos(th2) & \cos(th2) & \sin(th3 - th4 + 1.57)\cdot\sin(th2) & \cos(th2) \\ 1 & 1 & 0 & 0 & -\cos(th3 - th4 + 1.57) & 0 \end{bmatrix}$$

J_{11} = -(sin(th2)*(13000.0*cos(th3-th4-th6)+12171.0*sin(th3-th4+1.57)+12021.0*cos(th3+0.79)+1300.0))/1000

J_{12} = -0.001*sin(th2)*(13000.0*cos(th3-th4-th6+)+12171.0*sin(th3-th4+1.57)+12021.0*cos(th3+0.79)+1300.0)

J_{13} = 1.0*cos(th2)*(12.171*cos(th3-th4+1.57)+13.0*sin(th4-th3+th6)-12.021*cos(th3-0.79))

J_{14} = -cos(th2)*(12.171*cos(th3-th4+1.57)+13.0*sin(th4-th3+th6)-12.021*cos(th3-0.79)+12.021*sin(th3+0.79))

J_{15} = 1.0*cos(th3-th4+1.57)*((sin(th2)*(12021.0*cos(th3+0.79)+1300.0))/1000-(sin(th2)*(13000.0*cos(th3-th4-th6)+12171.0*sin(th3-th4+1.57)+12021.0*cos(th3+0.79)+1300.0))/1000)-sin(th3-th4+1.57)*sin(th2)*(12.171*cos(th3-th4+1.57)+13.0*sin(th4-th3+th6)-12.021*cos(th3-0.79)+12.021*sin(th3+0.79))

J_{16} = -cos(th2)*(12.171*cos(th3-th4+1.57)-12.171*cos(th3-th4+1.57)+13.0*sin(th4-th3+th6)-12.021*cos(th3-0.79)+12.021*sin(th3+0.79))

J_{21} = (cos(th2)*(13000.0*cos(th3-th4-th6)+12171.0*sin(th3-th4+1.57)+12021.0*cos(th3+0.79)+1300.0))/1000

J_{22} = 0.001*cos(th2)*(13000.0*cos(th3-th4-th6)+12171.0*sin(th3-th4+1.5)+12021.0*cos(th3+0.79)+1300.0)

J_{23} = sin(th2)*(12.171*cos(th3-th4+1.57)+13.0*sin(th4-th3+th6)-12.021*cos(th3-0.79))

J_{24} = -1.0*sin(th2)*(12.171*cos(th3-th4+1.57)+13.0*sin(th4-th3+th6)-12.021*cos(th3-0.79)+12.021*sin(th3+0.79))

J_{25} = 1.0*cos(th3-th4+1.57)*((cos(th2)*(12021.0*cos(th3+0.79)+1300.0))/1000-(cos(th2)*(13000.0*cos(th3-th4-th6)+12171.0*sin(th3-th4+1.57)+12021.0*cos(th3+0.79)+1300.0))/1000)+sin(th3-th4+1.57)*cos(th2)*(12.171*cos(th3-th4+1.57)+13.0*sin(th4-th3+th6)-12.021*cos(th3-0.79)+12.021*sin(th3+0.79))

J_{26} = -1.0*sin(th2)*(12.171*cos(th3-th4+1.57)-12.171*cos(th3-th4+1.57)+13.0*sin(th4-th3+th6)-12.021*cos(th3-0.79)+12.021*sin(th3+0.79))

J_{33} = -1.0*cos(th2)*(1.3*cos(th2)-(cos(th2)*(13000.0*cos(th3-th4-th6)+12171.0*sin(th3-th4+1.57)+12021.0*cos(th3+0.79)+1300.0))/1000)-sin(th2)*(1.3*sin(th2)-(sin(th2)*(13000.0*cos(th3-th4-th6)+12171.0*sin(th3-th4+1.57)+12021.0*cos(th3+0.79)+1300.0))/1000)

J_{34} = cos(th2)*((cos(th2)*(12021.0*cos(th3+0.79)+1300.0))/1000-(cos(th2)*(13000.0*cos(th3-th4-th6)+12171.0*sin(th3-th4+1.57)+12021.0*cos(th3+0.79)+1300.0))/1000)+1.0*sin(th2)*((sin(th2)*(12021.0*cos(th3+0.79)+1300.0))/1000-(sin(th2)*(13000.0*cos(th3-th4-th6)+12171.0*sin(th3-th4+1.57)+12021.0*cos(th3+0.79)+1300.0))/1000)

J_{35} = sin(th3-th4+1.57)*sin(th2)*((cos(th2)*(12021.0*cos(th3+0.79)+1300.0))/1000-(cos(th2)*(13000.0*cos(th3-th4-th6)+12171.0*sin(th3-th4+1.57)+12021.0*cos(th3+0.79)+1300.0))/1000)-sin(th3-th4+1.57)*cos(th2)*((sin(th2)*(12021.0*cos(th3+0.79)+1300.0))/1000-(sin(th2)*(13000.0*cos(th3-th4-th6)+12171.0*sin(th3-th4+1.57)+12021.0*cos(th3+0.79)+1300.0))/1000)

J_{36} = -cos(th2)*((cos(th2)*(13000.0*cos(th3-th4-th6)+12171.0*sin(th3-th4+1.57)+12021.0*cos(th3+0.79)+ 1300.0))/1000-0.001*cos(th2)*(12171.0*sin(th3-th4+1.57)+12021.0*cos(th3+ 0.79)+1300.0))- 1.0*sin(th2)*((sin(th2)*(13000.0*cos(th3-th4-th6)+12171.0*sin(th3-th4+ 1.57)+12021.0*cos(th3+ 0.79)+1300.0))/1000-0.001*sin(th2)*(12171.0*sin(th3-th4+1.57)+12021.0*cos(th3+0.79)+1300.0))

5. Verifications

a. Forward Kinematics Verification

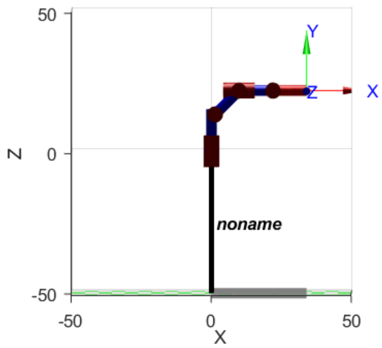
We verified the forward kinematics by giving angles to the homogeneous matrix and checked the x,y,z coordinates. The following is an example showing forward kinematics homogeneous matrix, MATLAB plot and the arm setup when all the variables (th2, th3, th4, th5, th6) equals to zero.

X out = 34 cm

Y out = 0 cm

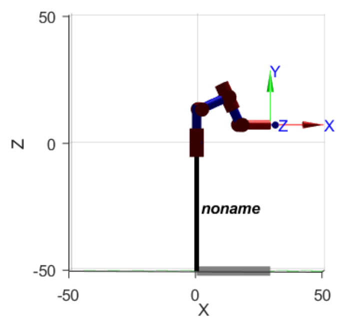
Z out = 22 cm

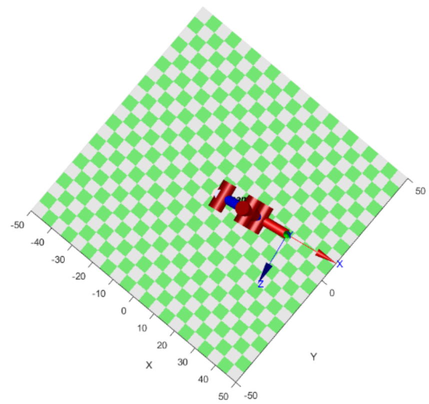
```
fwd =

     1      0      0   33.97
     0      0     -1       0
     0      1      0    21.6
     0      0      0       1
```



b. Inverse Kinematics Verification

We verified the inverse kinematics by giving relevant position(x, y, z) and calculated the joint angles. Then again we calculate the forward kinematics with the calculated joint angles and verify the input coordinates. The results from the MATLAB code and homogeneous matrices are as follows.





```
>> inverse
```

X = 30cm

Y = 5cm

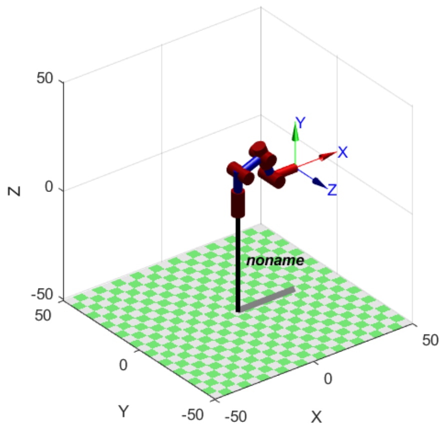
Z = 7cm

```
fwd =
    0.9864         0    0.1644     29.01
    0.1644         0   -0.9864     4.836
         0         1         0     7.001
         0         0         0         1
```

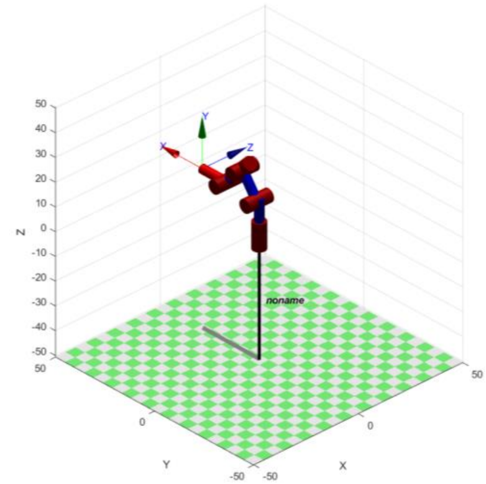
6. Pick and Place Task

<table>
<tr><td colspan="2">Starting Point</td><td>X</td><td colspan="3">30</td><td>Y</td><td colspan="2">0</td><td>Z</td><td colspan="2">6</td></tr>
<tr><td colspan="2">Ending Point</td><td>X</td><td colspan="3">0.2</td><td>Y</td><td colspan="2">30</td><td>Z</td><td colspan="2">14</td></tr>
<tr><td rowspan="2">Joint Angles</td><td>Starting Point</td><td>θ_2</td><td>0^0</td><td>θ_3</td><td>-24^0</td><td>θ_4</td><td>69^0</td><td>θ_5</td><td>0^0</td><td>θ_6</td><td>-69^0</td></tr>
<tr><td>Ending Point</td><td>θ_2</td><td>90^0</td><td>θ_3</td><td>8^0</td><td>θ_4</td><td>46^0</td><td>θ_5</td><td>0^0</td><td>θ_6</td><td>-46^0</td></tr>
<tr><td>Homogeneous Matrix</td><td>Starting Point</td><td colspan="10"><pre>fwd =
     1   0   0   29
     0   0   -1   0
     0   1   0 6.001
     0   0   0   1</pre></td></tr>
</table>

	Ending Point	fwd = $\begin{matrix} 0.0067 & 0 & 1.0000 & 0.1933 \\ 1.0000 & 0 & -0.0067 & 29 \\ 0 & 1 & 0 & 14 \\ 0 & 0 & 0 & 1 \end{matrix}$



Pickup position



Place position