Havel-Hakimi Theorem and Algorithm

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Degree sequence

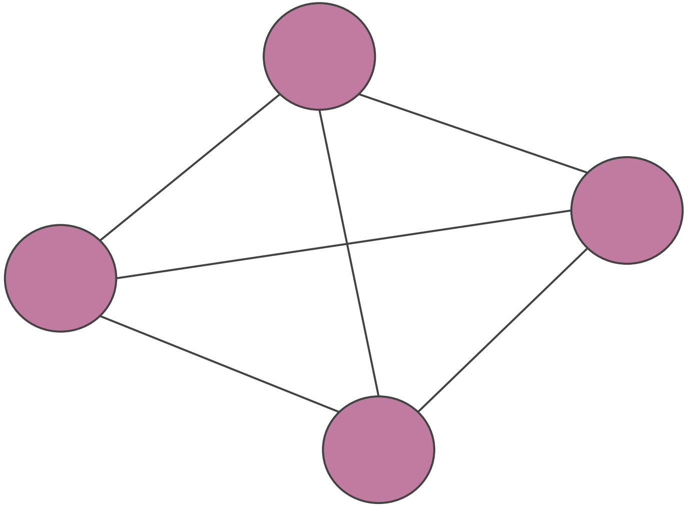
Monotonic non-decreasing sequence of the vertex degrees of given undirected graph.

Graphical sequence

Sequence of numbers which can be degree sequence of some graph

Graph realization

$(d_1, d_2, d_3, \ldots\ldots, d_n)$



- ★ A decision problem in graph theory.
- ★ Given a finite sequence of natural numbers, the problem asks whether there is a labeled simple graph such that the sequence is the degree sequence of this graph.

Determine whether following graphical?

1. 6 6 5 5 4 3 2

No. why?

★ Sum of the degrees is odd.
★ Cannot be odd for any graph
★ Handshake lemma

Determine whether following graphical?

2. 7 6 6 5 4 3 3

- ★ Sum of degrees is even

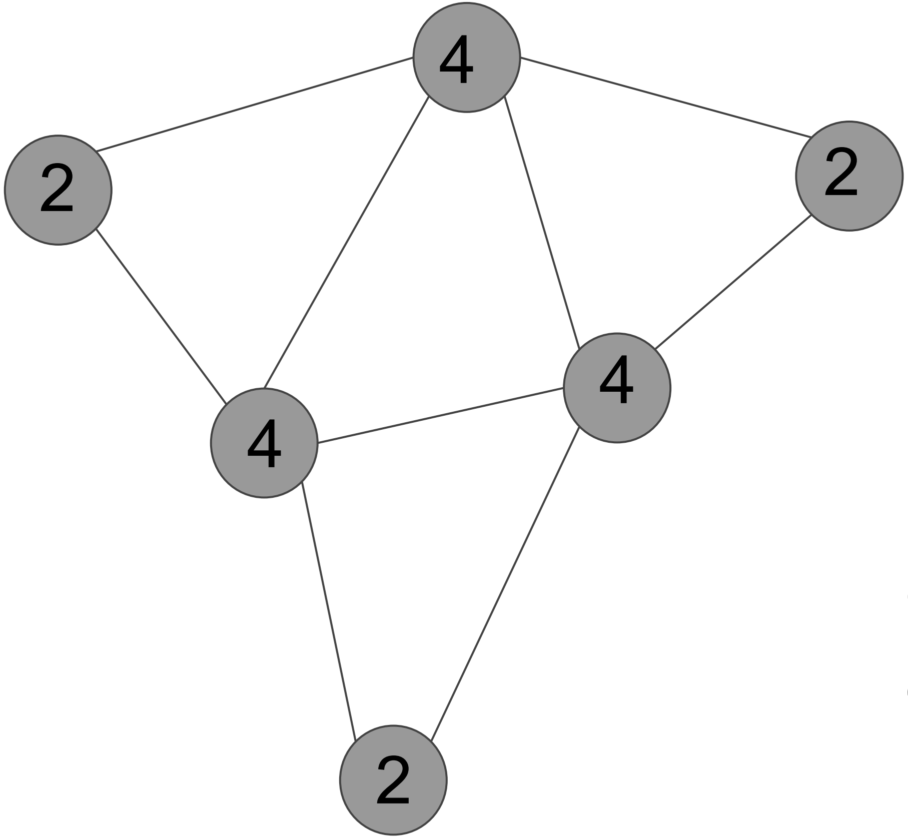
  Still no! Why?

- ★ Maximum degree cannot be greater than length of sequence.

Determine whether following graphical?

3. 8 7 6 6 5 3 2 2 2 1

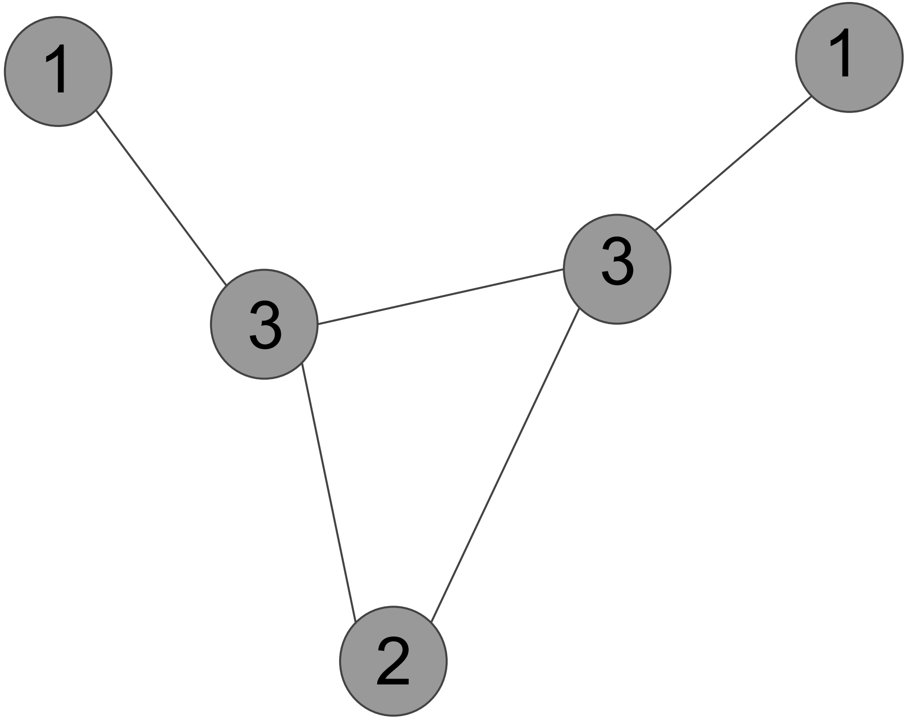
- ★ Sum of sequence is even
- ★ Maximum digit is less than length of sequence.
- ★ Can it be graphical?



Degree sequence :
4 4 4 2 2 2

Remove the highest degree vertex from the graph?

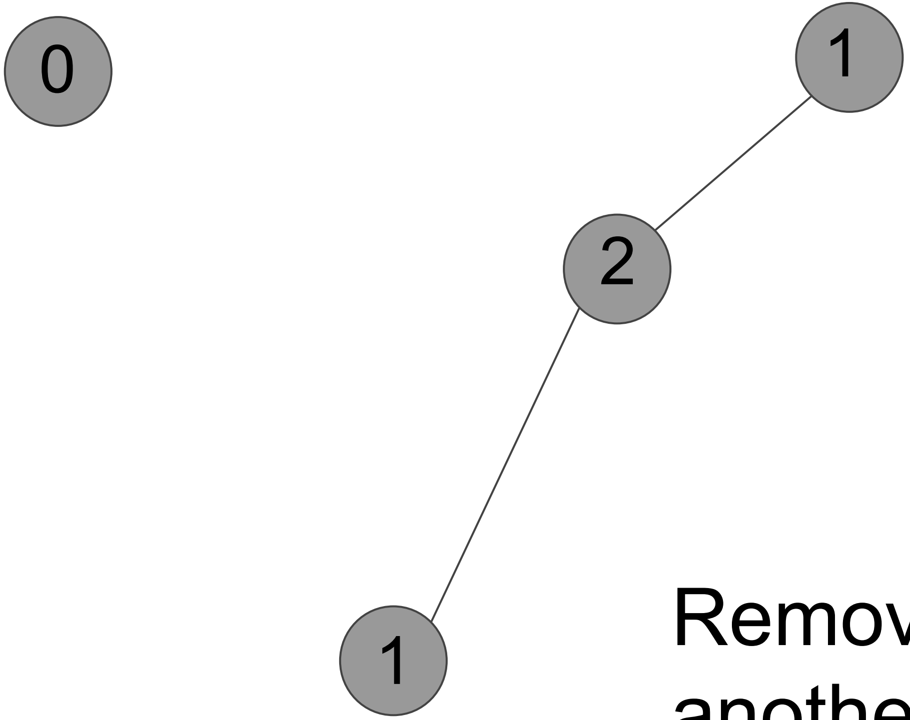
Let’s consider the graph



4 4 4 2 2 2

3 3 2 1 1

Remove another?

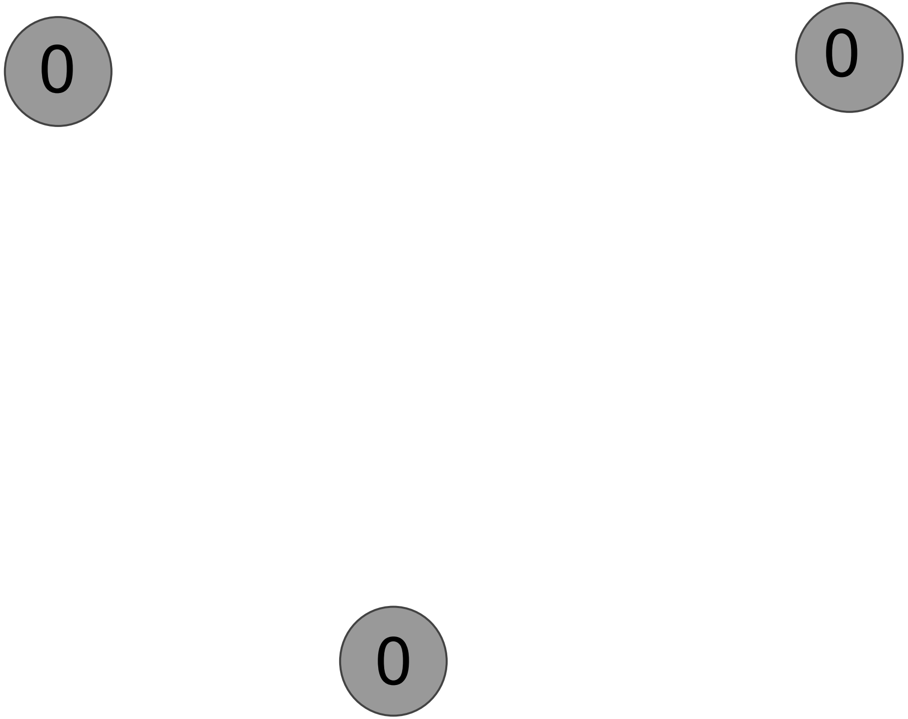


4 4 4 2 2 2

3 3 2 1 1

2 1 1

Remove another?



4 4 4 2 2 2

↓

3 3 2 1 1

↓

2 1 1

↓

0

Consider the non-increasing sequence, $S_1 = (d_1, d_2, d_3, \ldots, d_n)$ of nonnegative integers, where $n \geq 2$ and $d_1 \geq 1$. Then S_1 is graphical if and only if the sequence

$$S_2 = (d_2 - 1, d_3 - 1, \ldots, d_k - 1, d_{k+1} - 1, d_{k+2}, \ldots, d_{n-1}, d_n)$$ is graphical. Where $k = d_1$

Havel - Hakimi theorem

$$(d_2 - 1, d_3 - 1, \ldots, d_k - 1, d_{k+1} - 1, d_{k+2}, \ldots, d_{n-1}, d_n)$$
is graphical

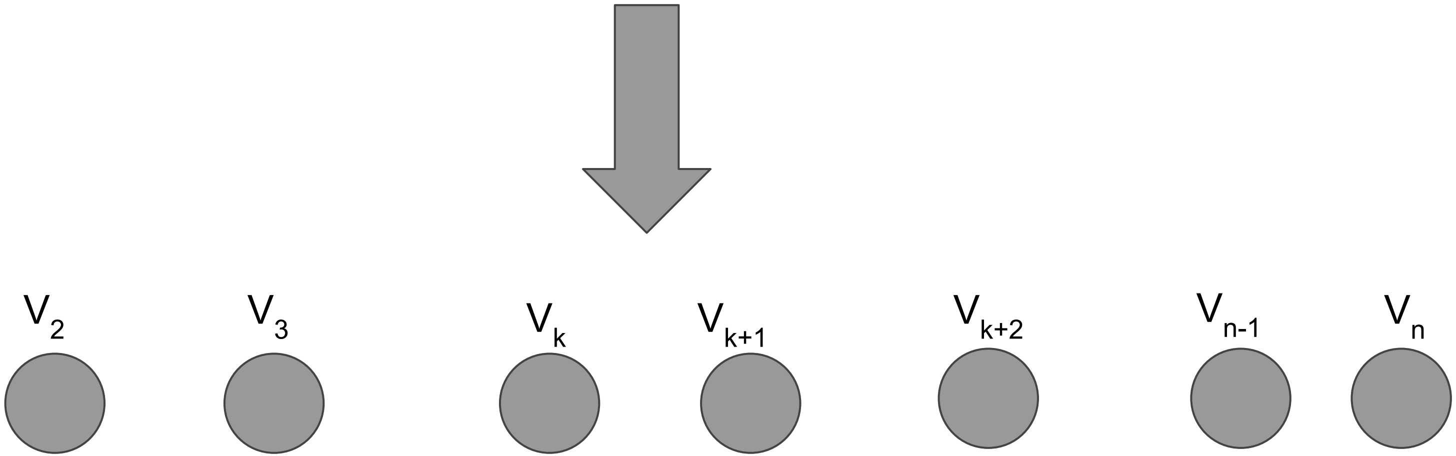
$$\Downarrow$$

$(d_1, d_2, d_3, \ldots, d_n)$ is graphical

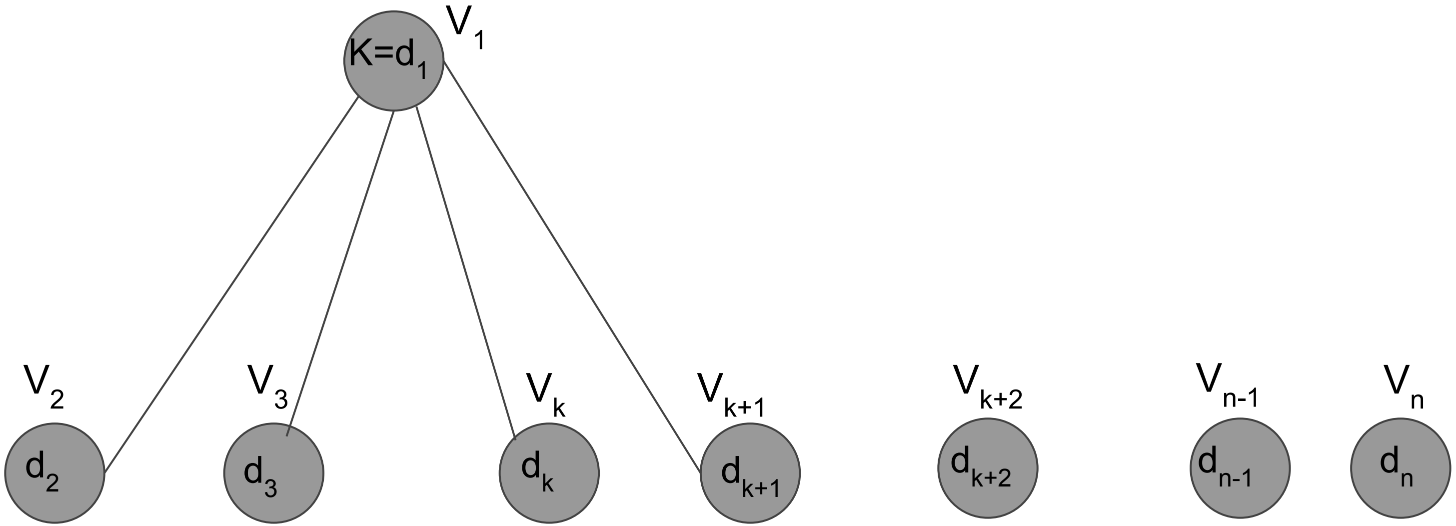
Consider the sufficient condition

$$(d_2 - 1, d_3 - 1, \ldots, d_k - 1, d_{k+1} - 1, d_{k+2}, \ldots, d_{n-1}, d_n)$$

is graphical



Consider the sufficient condition



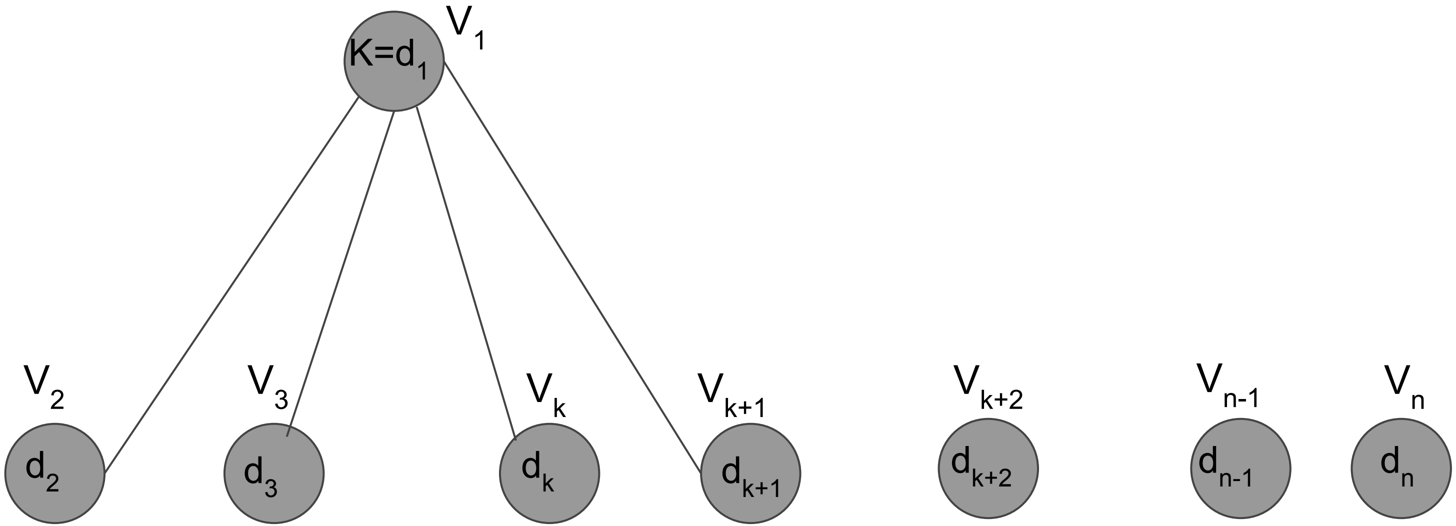
Consider the sufficient condition

$(d_1, d_2, d_3, \ldots, d_n)$ is graphical

$\Downarrow$

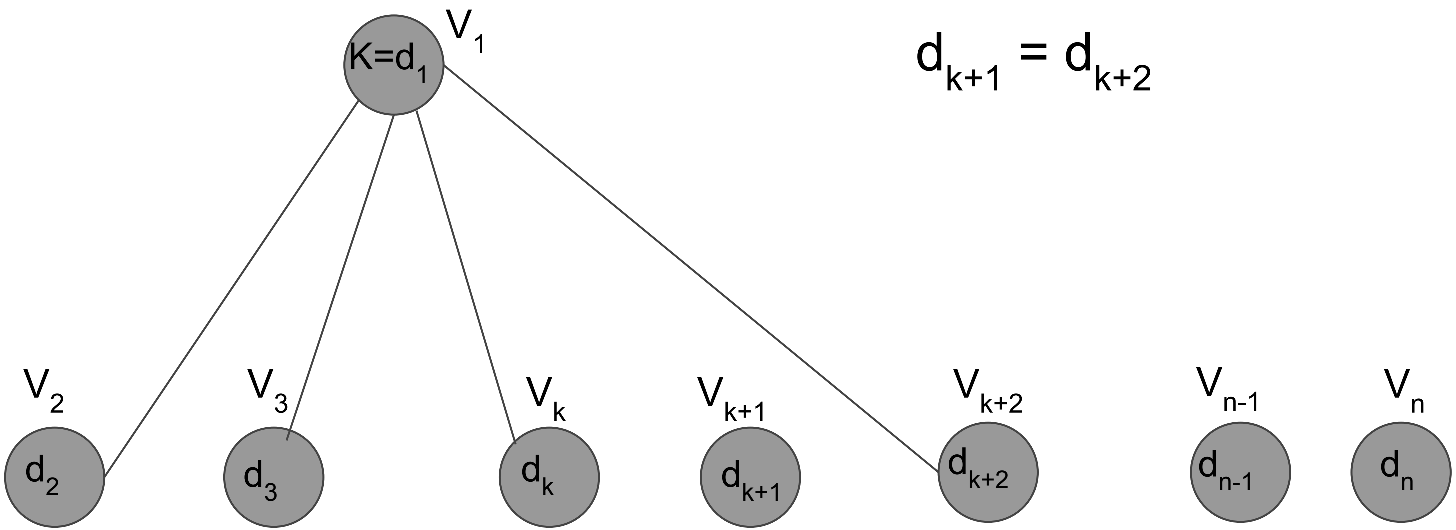
$(d_2 - 1, d_3 - 1, \ldots, d_k - 1, d_{k+1} - 1, d_{k+2}, \ldots, d_{n-1}, d_n)$ is graphical

Consider the necessary condition



Case 1

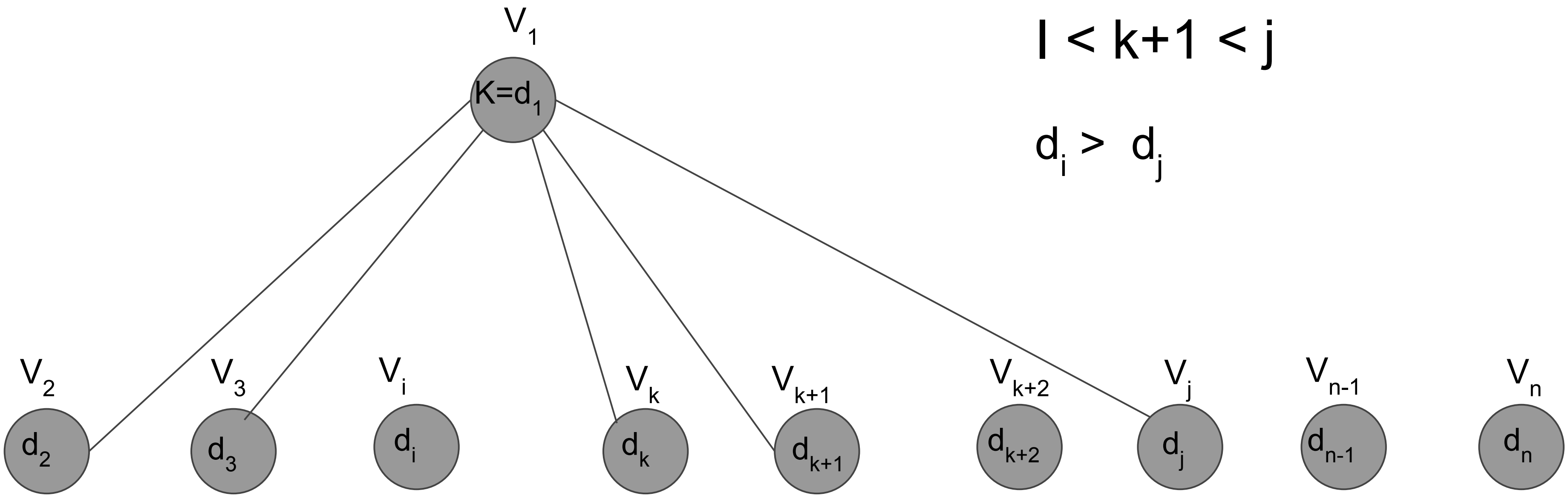
Consider the necessary condition



$$d_{k+1} = d_{k+2}$$

Case 2

Consider the necessary condition

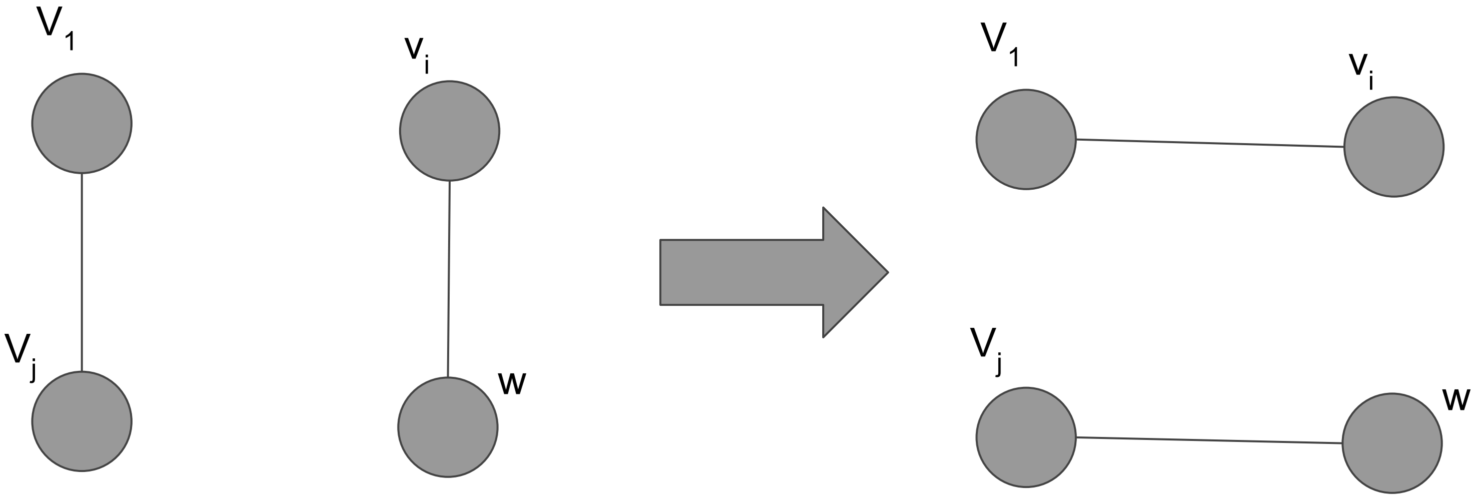


$I < k+1 < j$

$d_i > d_j$

Case 3:

Consider the necessary condition



Case 3:

Consider the necessary condition

Determine whether following is graphical?

8 7 6 6 5 3 2 2 2 1 is graphical iff

6 5 5 4 2 1 1 1 1 is graphical iff

4 4 3 1 1 1 is graphical iff

3 2 1 is graphical iff

1 1 -1

Obviously 1 1 -1 is not graphical. Hence none of the above is graphical.

Application of havel-Hakimi theorem

Determine whether following is graphical?

4 4 4 3 3 2 is graphical iff

3 3 2 2 2 is graphical iff

2 1 1 2 is graphical iff

2 2 1 1 is graphical iff

1 0 1 is graphical iff

Obviously 1 1 0 is graphical. Hence everything above is graphical.

Application of havel-Hakimi theorem

Havel - Hakimi algorithm

Let us define some variables.

Let the sequence as q

S = sum of numbers in given sequence

If S is odd then

Return false

While true do

If min(q) < 0 then

Return false

If max(q) = 0 then

Return true

If max(S) > length(S) -1 then

Return false

$Q \leftarrow (d_2 -1 , d_3 -1, d_{d1+1} -1, d_{d1+2} , \ldots.., d_{length(q)})$

Sort q in non increasing order

1. This condition comes from the handshake lemma which was discussed in RC1. No graph can not contain the sum of degree as an even number.
2. Obviously a graph can not contain a negative number of degree.
3. If the of that sequence is 0 because of the non increasing property and considering above conditions all members of that sequence should be zero. Hence by havel hakim theorem that will be graphical.
4. For a simple graph since it does not have multiple edges max(Q) should be less than or equal to K.

Here k = number of members in given sequence - 1

Havel - Hakimi algorithm

Thank you

References

[1]. http://mathworld.wolfram.com/DegreeSequence.html

[2]. http://mathworld.wolfram.com/GraphicSequence.html

[3]. http://planetmath.org/handshakelemma

[4]. http://coddicted.com/the-havel-hakimi-algorithm/