Space Mission Analysis

# Assignment 3 – due 06.01.2026 23:59

The assignment shall be done in groups of 2-3 students. However, each student must upload the code and report individually via ISIS!

Reports must include the plots and some basic explanations in PDF format as well as your MATLAB code.

Alternatively, you can submit a MATLAB live script (.mlx) showing and commenting on all plots and calculations instead of the PDF.

**Interplanetary Transfers**

1. Solving Lambert's Problem. **(12p.)**

   Implement a function to solve Lambert's problem iteratively using the secant method. The inputs are the two position vectors, $\Delta t$ and the gravitational parameter of the body. Return values should at least include the two velocity vectors and the semi-major axis of the solution. Choose reasonable break conditions. (12p.)

   Tips:

   - The generation of the two necessary initial guesses should be flexible as a diverging solution of the secant method does not generally mean that there is no solution.
   - Implementing the Lambert solver unit-agnostic, e.g., no unit is specified in the function itself and length and time units are provided implicitly by the input values, allows the function to be used for any central body.
   - Including an input parameter that allows the complementary angle of $\Delta\theta$ to be used enables the function to calculate retrograde orbits as well.
   - Error handling for non-converging problems is strongly recommended.
   - Use the provided test cases to test your implementation.

2. Porkchop Plots **(10p.)**

   Create a Porkchop Plot displaying departure C3 and arrival $v_\infty$ for an Earth-Mars transfer in the timeframe of 2030-06-01 to 2031-09-30 for the departure date at Earth and 2030-09-01 to 2033-01-01 for the arrival date at Mars. Include isolines for $\Delta t$ in your Porkchop plot. (10p.)

   Tips:

   - To avoid numerical errors (for this particular example) use astronomical units as length unit and tropical year as time unit. $\mu_{Sun}$ can then be approximated as $4\pi^2$. The latter has no basis in geometrical considerations, it is just pure coincidence.
   - Keep in mind that when computing the angle between the two position vectors you do not necessarily have angles <180°.
   - The position and velocity of Earth and Mars can be obtained from the Jet Propulsion Laboratory (JPL) Horizons system, available here. Using initial values and propagating the planets on their orbits is a valid approximation, but it is recommended to use the ephemerides from the Horizons system.

3. Patched-Conics Trajectory **(12p.)**
   a. Propagate and plot the hyperbolic orbits around Earth and Mars as well as the elliptic heliocentric transfer orbit for the minimum departure C3 solution. Display each orbit in a separate plot and include Earth, Mars and Sun respectively. Assume a periapsis height of 200km for the hyperbolic orbits (6p.)
   b. Propagate/plot the same for the minimum arrival $v_\infty$ solution. (6p.)

   Tips:
   - The Kepler propagation function you have coded needs to be extended to allow hyperbolic orbits to be propagated. The formulas for this extension will be provided in separate slides.
   - Other formulas necessary for the inclusion of hyperbolic orbits will be provided in these slides as well.

Your report should include all graphs, captions and some basic (!) explanations of what you did. The report's form, structure, language etc. will be accounted for with **3p.** as well as the code's structure, appropriate comments etc. will be accounted for with **3p.**

I strongly recommend writing your script as lean as possible and trying to avoid repeating code. Use loops and parameter sets to calculate or plot different orbits.

**All calculations and simulations must be done using MATLAB with own functions (no usage of libraries or proprietary or open-source functions)!**

Exceptions to this are the functions *eci2lla, lla2ecef, eci2ecef* and their reverse versions. Furthermore, functions like *wrapTo360, wrapTo2Pi* and similar as well as *geoplot* are exceptions.

# Test cases for the Lambert Solver:

***Case 1***: Earth-Venus Transfer with simplifications (no inclination between orbital planes, no specific start date)

*Table 1.1: Initial values for the Earth-Venus transfer*

| | |
|---|---|
| $r_1$ | 1 AU |
| $r_2$ | 0.723 AU |
| $\Delta\theta$ | 75 ° |
| $\Delta t$ | 180 days |

3D radius vectors:

$$\boldsymbol{r}_1 = \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix} AU, \qquad \boldsymbol{r}_2 = \begin{bmatrix} 0.723\cos(\Delta\theta) \\ 0.723\sin(\Delta\theta) \\ 0 \end{bmatrix} AU = \begin{bmatrix} 0.187 \\ 0.698 \\ 0 \end{bmatrix} AU$$

As initial guesses:

$$a_1 = a_m, \quad a_2 = 2a_m$$

Converges in five steps with a relative tolerance of $10^{-8}$.

*Table 2.2: Computed values for the Earth-Venus transfer. NOTE: book value of $t_p = 0.1200$ years is likely wrong, my implementation yields $t_p = 0.1100$ years.*

| | |
|---|---|
| $c$ | 1.072 AU |
| $s$ | 1.397 AU |
| $a_m$ | 0.6987 AU |
| $\beta_m$ | 57.73 ° |
| $t_m$ | 0.2769 years |
| $t_p$ | 0.1200 years |

Solution for the velocity vectors (mind the unit conversion from LU = AU, TU = tropical years to LU = km, TU = s):

$$\boldsymbol{v}_1 = \begin{bmatrix} 18.849585 \\ 17.421723 \\ 0 \end{bmatrix} km/s, \qquad \boldsymbol{v}_2 = \begin{bmatrix} -30.405379 \\ -20.372940 \\ 0 \end{bmatrix} km/s$$

***Case 2***: Earth-Mars Transfer of NASA's *Perseverance* Mars rover from 2020-07-30 to 2021-02-18

Initial values for Earth at departure and Mars at arrival are obtained via JPL's Horizon system and converted from AU/day to AU/year for the velocities:

$$\boldsymbol{r}_{Earth} = \begin{bmatrix} 0.605774717586 \\ -0.80374565571 \\ 0.00010684714 \end{bmatrix} AU, \qquad \boldsymbol{v}_{Earth} = \begin{bmatrix} 4.909398453 \\ 3.759466359 \\ 0.0001209199 \end{bmatrix} AU/year$$

$$\boldsymbol{r}_{Mars} = \begin{bmatrix} -0.01300489410 \\ 1.575580535344 \\ 0.033161980867 \end{bmatrix} AU, \qquad \boldsymbol{v}_{Mars} = \begin{bmatrix} -4.920097603 \\ 0.411804813 \\ 0.129389380 \end{bmatrix} AU/year$$

As initial guesses:

$$a_1 = a_m, \quad a_2 = 1.001 a_m$$

Converges in eight steps with a relative tolerance of $10^{-10}$.

*Table 2.1: Computed values for Perseverance's Earth-Mars transfer*

| | |
|---|---|
| $\Delta t$ | 203 days |
| $t_p$ | 109 days |
| $\Delta\theta$ | 143.451 ° |
| $a_m$ | 1.26028 AU |
| $a$ | 1.31751 AU |
| $p$ | 1.24290 AU |
| $e$ | 0.237969 |