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Hashiwokakero is NP-complete

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ABSTRACT

In a Hashiwokakero puzzle, one must build bridges to connect a set of islands. We show that deciding the solvability of such puzzles is NP-complete.

1. Introduction

Hashiwokakero ("build bridges!") is a type of puzzle published by Nikoli [3]. Given a set of points in the plane (*islands*), one must connect them using vertical and horizontal line segments (*bridges*). Each island specifies how many bridges that must be connected to it. A bridge must not cross any other bridge or island, at most two bridges may run in parallel, and all islands must be reachable from one another. Fig. 1 shows an example.

Demaine and Hearn [1] survey complexity results on combinatorial games and puzzles, and they list Hashiwokakero as previously unstudied. We show:

Theorem 1. *Deciding the solvability of a Hashiwokakero puzzle is* NP*-complete.*

We construct a reduction from the following NP-hard problem [2]: Given a finite set $P \subset \mathbf{Z}^2$, is there a Hamiltonian circuit in the unit distance graph of P?

2. Reduction

We assume $|P| \geqslant 3$. For each $x \in P$, output an island at $2x$ with bridge requirement $6 - |\{y \in P\colon\ |x-y| = 1\}|$ (*big island*) and an island at $x + y$ with bridge requirement 1 (*small island*) for each $y \in \mathbf{Z}^2 \setminus P$ such that $|x - y| = 1$. Fig. 2 shows an example.

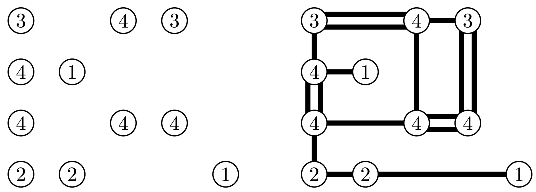

Fig. 1. A Hashiwokakero puzzle and its solution.

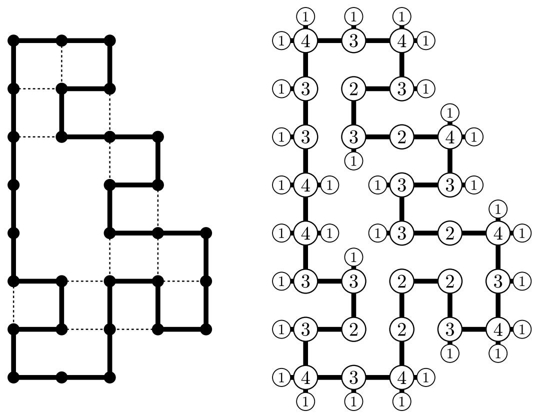

Fig. 2. An example of the reduction and a pair of corresponding solutions.

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Because of the reachability constraint, each small island must be connected to a big island, and by construction, there is only one choice. After all such connections have been made, all big islands have a remaining bridge requirement of 2. Since $|P| \geqslant 3$, the reachability constraint prevents parallel bridges between big islands, and so the connection possibilities now correspond exactly to edges in the unit distance graph of P.

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