

CIRCUIT DESIGN

Part 1:

Design a circuit to compare 2 2-bit unsigned numbers.

Truth table:

a1	a0	b1	b0	$l(a < b)$	$g(a > b)$	$e(a = b)$
0	0	0	0	0	0	1
0	0	0	1	1	0	0
0	0	1	0	1	0	0
0	0	1	1	1	0	0
0	1	0	0	0	1	0
0	1	0	1	0	0	1
0	1	1	0	1	0	0
0	1	1	1	1	0	0
1	0	0	0	0	1	0
1	0	0	1	0	1	0
1	0	1	0	0	0	1
1	0	1	1	1	0	0
1	1	0	0	0	1	0
1	1	0	1	0	1	0
1	1	1	0	0	1	0
1	1	1	1	0	0	1

Sum of Products:

$$l(a < b) = a1'a0'b1'b0 + a1'a0'b1b0' + a1'a0'b1b0 + a1'a0b1b0' + a1'a0b1b0 + a1a0'b1b0$$

$$g(a > b) = a1'a0b1'b0' + a1a0'b1'b0' + a1a0'b1'b0 + a1a0b1'b0' + a1a0b1'b0 + a1a0b1b0'$$

$$e(a = b) = a1'a0'b1'b0' + a1'a0b1'b0 + a1a0'b1b0' + a1a0b1b0$$

Simplification:

$$l(a < b) = a1'b1 + a1'a0'b0 + a0'b1b0 \text{ (K-Map)}$$

a1a0 \ b1b0	00	01	11	10
00				
01	1			
11	1	1		1
10	1	1		

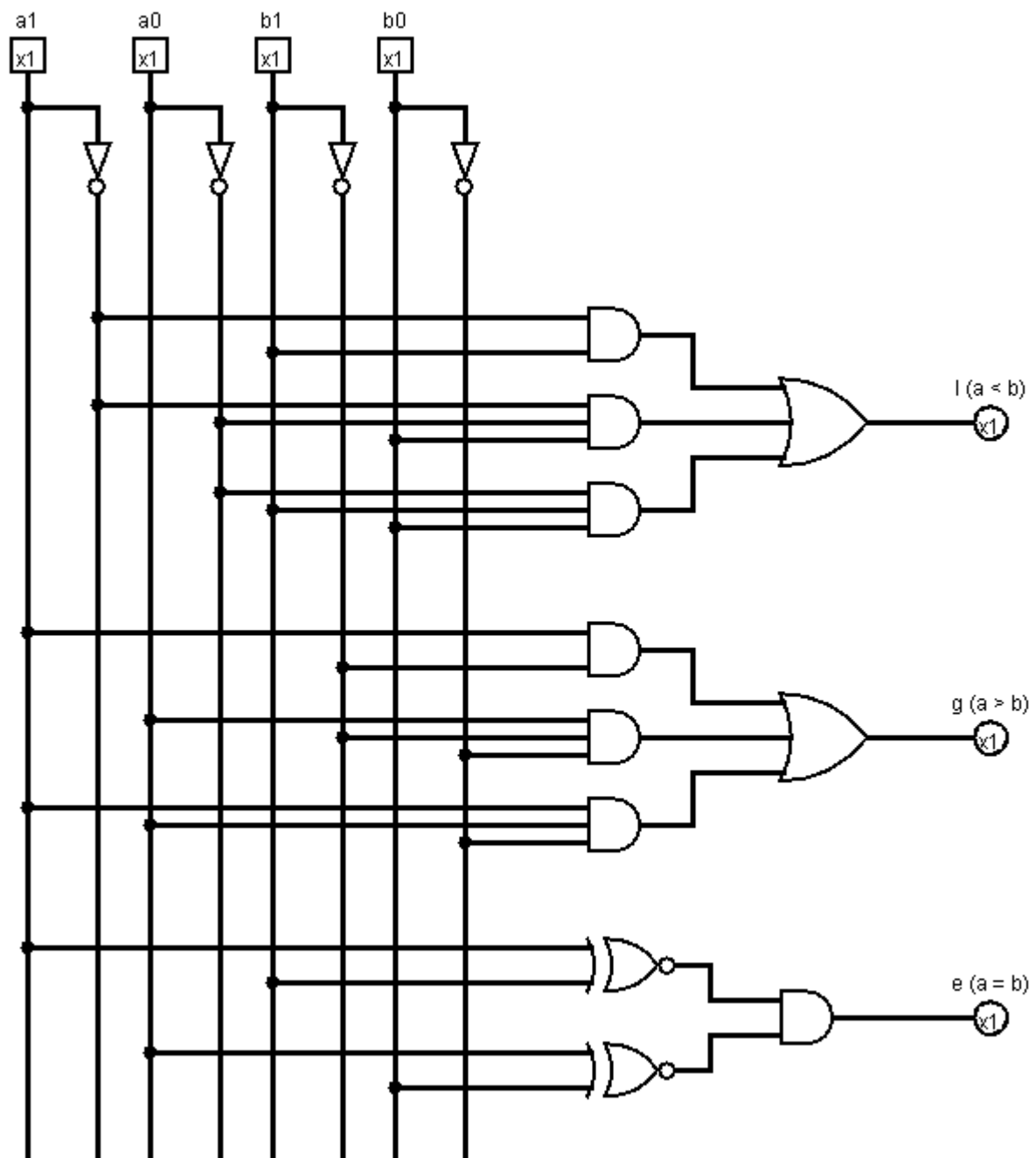
$$g(a > b) = a1b1' + a0b1'b0' + a1a0b0' \text{ (K-Map)}$$

a1a0 \ b1b0	00	01	11	10
00		1	1	1
01			1	1
11				
10			1	

$$\begin{aligned}
 e(a = b) &= a1'a0'b1'b0' + a1'a0b1'b0 + a1a0b1b0 + a1a0'b1b0' && \text{(K-Map)} \\
 &= a1'b1' (a0'b0' + a0b0) + a1b1 (a0b0 + a0'b0') && \text{(Distributive x2)} \\
 &= (a1b1 + a1'b1') (a0b0 + a0'b0') && \text{(Distributive)} \\
 &= (a1 \text{ XNOR } b1) (a0 \text{ XNOR } b0)
 \end{aligned}$$

a1a0 \ b1b0	00	01	11	10
00	1			
01		1		
11			1	
10				1

Circuit Diagram:



Part 2:

Design a circuit to subtract 2 2-bit signed numbers.

Truth table:

a1	a0	b1	b0	d1	d0	c (overflow)
0	0	0	0	0	0	0
0	0	0	1	1	1	0
0	0	1	0	1	0	1
0	0	1	1	0	1	0
0	1	0	0	0	1	0
0	1	0	1	0	0	0
0	1	1	0	1	1	1
0	1	1	1	1	0	1
1	0	0	0	1	0	0
1	0	0	1	0	1	1
1	0	1	0	0	0	0
1	0	1	1	1	1	0
1	1	0	0	1	1	0
1	1	0	1	1	0	0
1	1	1	0	0	1	0
1	1	1	1	0	0	0

Sum of Products:

$$d1 = a1'a0'b1'b0 + a1'a0'b1b0' + a1'a0b1b0' + a1'a0b1b0 + a1a0'b1'b0' + a1a0'b1b0 + a1a0b1'b0' + a1a0b1'b0$$

$$d0 = a1'a0'b1'b0 + a1'a0'b1b0 + a1'a0b1'b0' + a1'a0b1b0' + a1a0'b1'b0 + a1a0'b1b0 + a1a0b1'b0' + a1a0b1b0'$$

$$c \text{ (overflow)} = a1'a0'b1b0' + a1'a0b1b0' + a1'b0b1b0 + a1a0'b1'b0$$

Simplification:

$$d1 = a1'b1b0' + a1'a0b1 + a1a0b1' + a1b1'b0' + a1'a0'b1'b0 + a1a0'b1b0 \text{ (K-Map)}$$

a1a0 b1b0	00	01	11	10
00			1	1
01	1		1	
11		1		1
10	1	1		

$$d0 = a0'b0 + a0b0' \text{ (K-Map)}$$

$$= a0 \text{ XOR } b0$$

a1a0 b1b0	00	01	11	10
00		1	1	
01	1			1
11	1			1
10		1	1	

$$c \text{ (overflow)} = a1'b1b0' + a1'a0b1 + a1a0'b1'b0 \text{ (K-Map)}$$

a1a0 b1b0	00	01	11	10
00				
01				1
11		1		
10	1	1		

Circuit Diagram:

