Stochastic Skewness and Index Option Returns

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Abstract

In literature, many researchers focus on information contained in stochastic volatility dynamics, such as CBOE VIX index and its risk premium. However, there are relatively fewer studies on stochastic skewness dynamics. Simple linear regression indicates that stochastic volatility and stochastic skewness have different information about economic fundamentals: regressing CBOE VIX index on CBOE Skewness index, ranging from 1990 to 2013, yields R^2 only 2%. Motivated by such striking yet simple empirical result and advances in reduced-form option pricing literature, we build one discrete-time equilibrium option pricing model with non-normal innovation. In our framework, market return skewness is driven by a stochastic process independent of volatility. Thus volatility and skewness dynamics possess different information about stochastic investment opportunity. Specifically, we use Variance Gamma distribution to generate economic growth shocks. Variance gamma innovation is able to be decomposed into two shocks, one of them is strictly positive, and the other is strictly negative. Our skewness risk factor controls the frequency and magnitude of these two shocks, therefore, controls the switching behavior of economic condition, e.g. from good to bad state. We show in our model that stochastic skewness risk should be priced, provided that investors have recursive utility function. Empirically, using Fama-Macbeth two pass regression and a panel of S&P 500 index option returns, we find that stochastic skewness has positive, statistically significant risk premium, and its sign does not change across our sample period. Moreover, our results show that skewness risk has superior explanation power for index option returns, compared with performance of volatility risk and Merton-type of jump risk. The shock of skewness risk premium is able to predict short-term market excess returns, the R^2 is as high as 7%.

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1 Introduction

Growing evidence suggests that extraordinary average returns could be obtained by trading equity index options. Among various trading strategies, most important and widely studied are delta-hedge straddle returns and out-of-money put option returns. Coval and Shumway (2001) find that on average, at-the-money straddle get returns close to minus 3% per week. Jackwerth (2000) show high risk-adjusted profitability from selling puts.

Many researchers try to explain the risk premium embedded in these two strategies. Generally speaking, two systematic risk factors are involved, stochastic volatility and jumps. For instance, Bakshi and Kapadia (2003) find that the volatility risk premium contributes significantly to high prices for calls and puts. Broadie, Chernov, and Johannes (2009) suggest that a jump factor helps explain the option portfolio returns. However, the effect of stochastic skewness as an independent risk factor on option return is not well explored.

Traditional approaches to generate skewness, according to Heston (1993) and Bate (1996), are to incorporate correlation between return and volatility in the model, or use Merton type of jumps. In order to generate stochastic skewness, it would be tempting to randomizing the mean jump size, and/or the correlation between return and volatility, and/or make jump intensity stochastic (Du (2013) and Seo and Wachter, 2014).

However, randomizing correlation or jump size parameter is not amenable to analytic solution techniques that aid econometric estimation. In addition, empirical results indicate that Merton type of jump is inadequate to explain implied volatility behaviors. Backus, Chernov, and Martin (2011) study option prices in a rare events model similar to that of Rietz (1988) and Barro (2006). They find, however, that the resulting implied volatility level is lower and implied volatility smile is flatter than data, meanwhile, the gap is large. Similarly, Constantinides Jackwerth and Savov (2013) and Chambers, Liebner and Lu (2014) show that volatility and Merton-type jump risk premium are not adequate to explain option returns, especially for out-of-money put option returns.

Incorporating Merton-type of jumps is an indirect way to model skewness, in literature, there is another method directly considering return skewness, that is, using lévy innovation. Unlike normal distribution, most lévy innovations have parameters controlling high moments of its distribution. Earlier literature, such as Carr, Geman, Madan, and Yor (2002), Carr and Wu (2003, 2004) show that lévy innovation is able to match implied volatility smile better than traditional models including stochastic volatility and Merton-type of jumps. The intuition is straightforward, Merton type jumps can only occur finite times in fixed time interval, and its jump size is large. However, most lévy innovations have infinite jump intensity, such processes could have small and moderate size jumps. Suppose investors care about 10% drop of stock market, it is reasonable to assume that they also concerned about 6% drop. Since put option is also able to hedge such kind of jump risk, its price in lévy models is higher. Hence, implied volatility slope is steeper.

Under such lévy framework, there are two methods to make skewness stochastic. One way is provided by Carr and Wu (2007). The authors separate up and down side jumps, and the random proportion of these two type of jumps generates stochastic skewness. Christoffersen, Heston, and Jacobs (2006) and Ornthanalai (2014) propose another method. Instead of using constant parameter to control skewness, they use GARCH process to randomize such parameter, therefore, the conditional skewness become stochastic.

In this paper, instead of following Christoffersen, Heston, and Jacobs (2006) and Ornthanalai (2014) to analize option pricing in reduced-form models, we build an equilibrium model with lévy innovation to highlight the role of stochastic skewness. Although models incorporating Merton-type of jump process are able to generate skewness, such skewness is assumed to contain information about rare disasters. For lévy innovation, there is always one parameter which controls skewness, therefore, the formula for stochastic skewness is intuitive, and skewness has information about jumps with various sizes. For investors with Epstein-Zin recursive utility, state variable governing stochastic skewness is priced. Combined with lévy innovation, such result indicates that in our model, investors care not only rare disasters but also small and media size jump risks. All type of jumps have risk premium.

Empirically, we use CBOE risk neutral skewness index as a proxy for our state variable. Using

Fama-Macbeth two-pass regression with a panel of leverage-adjusted S&P 500 index option returns, we examine whether shocks of stochastic skewness is able to explain cross-section option portfolio returns. Consistent with our model implication, we show that shock of skewness index has better explanation power for index option returns than performance of traditional risk factors such as volatility, price jump and volatility jumps. In addition, using rolling-window Fama-Macbeth regression, the time-series of skewness risk premium is obtained, which has interesting pattern. The risk premium keeps increasing before market downturn, and drop dramatically afterwards. We also explore the relation between skewness risk premium and excess market returns.

The contribution of our paper is twofolds. On the one hand, theoretically, we build an equilibrium asset pricing model to highlight the channel through which the stochastic skewness could affect option prices and its returns. In literature, the widely used equilibrium option pricing model is based on Bollerslev, Tauchen and Zhou (2009). The model assumes normal consumption growth innovation and stochastic volatility, which is extended by Drechsler and Yaron (2011) to have Merton type of jumps and by Du (2013) and Seo and Wachter (2014) to have time-varying jump intensity. Generally speaking, these existing models focus on volatility dynamics and its risk premiums. In Drechsler and Yaron (2011) model, even there is jump component, the authors still only pay attention to its effect on volatility risk premium. Some other researchers, such as Kozhan, Neuberger and Schneider (2013), think that skewness and variance premia are manifestations of the same underlying risk factor. Skewness as an systematic risk factor is ignored in literature. However, as shown in later section, a simple regression test could convey that risk-neutral skewness and volatility contain different information. In our paper, we contribute to such type of modeling framework by allowing consumption growth to have non-normal innovations. Thus skewness can be modeled as a separate risk source different from volatility. Such model incorporates latest advances in reduced-form option pricing model (Ornthanalai, 2014) and allow us to gain intuition about the source of skewness risk premium.

On the other hand, empirically, we use a formal cross-section test to link market skewness risk to option returns. Following Bakshi, Kapadia and Madan (2003), there are some researchers studying effect of market and individual skewness risk on cross section stock returns. Chang, Christoffersen, and Jacobs (2013) find there is market skewness risk premium embedded in cross section stock returns. Boyer, Mitton, and Vorkink (2010) show that high idiosyncratic skewness leads to low expected returns. Conrad, Dittmar and Ghysels (2013) show that individual securities risk-neutral volatility, skewness, and kurtosis are strongly related to future returns. Similarly, Bali, Hu and Murray (2015) suggest that unsystematic part of ex ante measure of individual stock's volatility, skewness and kurtosis from option price is able to predict ex-ante measure of expected returns based on analyst price targets.

However, there are relatively fewer papers about skewness risk and option returns. Bali and Murray (2013) and Boyer and Vorkink (2014), based on positive skewness preference, identify a negative relation between individual risk-neutral skewness and individual option returns. Different from behavioral finance argument, we focus on the role of market skewness as one systematic risk factor. To the authors best knowledge, we provide the first direct testing of market skewness risk premium in option returns. Our empirical method is also different from those in the reduced-form option pricing literature. Conventional procedure in option pricing literature is to calibrate reduced-form option pricing models in an optimization algorithm to match market implied volatility smile curve. Based on such method, it is difficult to conduct statistical tests for factor risk premium with sound economic intuition.

The structure for the rest of this paper is as follows. Section 2 reviews the related literature. In Section 3, we builds the model, conduct simulation test and highlight the intuition and model empirical implication. Section 4 outlines the empirical framework and discusses results. Section 5 makes conclusions and provides some possible directions for future research.

2 Literature Review

Our research is closely related to two streams of literature. One branch is about reduced-form option pricing models trying to fit the implied volatility smiles. Earlier works include Merton (1976), Heston (1993), Duan (1995), Heston and Nandi (2000), Bates (2000) and Pan (2002). Apart from volatility and crash risks, Carr, Geman, Madan, and Yor (2002), Carr and Wu (2003, 2004), Christoffersen, Heston, and Jacobs (2006) and Ornthanalai (2014) propose to use lévy process with infinite jump intensity to model return jump risks. Moreover, some researchers, such as Christoffersen, Jacobs, Ornthanalai and Wang (2008) and Christoffersen, Heston and Jacobs (2009), point out that two volatility factors can generate stochastic leverage effect and more flexible implied volatility term structures, hence leads to a better fit of implied volatility surface. Besides, time-varying jump intensity could also improve model performance, as suggested by Santa-Clara and Yan (2010) and Christoffersen, Jacobs and Ornthanalai (2012).

Our paper is also related to equilibrium risk premium and option pricing models. Pioneer literature includes Bailey and Stulz (1989), Naik and Lee (1990) and Liu, Pan and Wang (2005). In equilibrium models, fundamental assumptions about investors utility, risk factor dynamics and beliefs can be extended. Buraschi and Jiltsov (2006) first provides option pricing and volume implications for an economy with heterogeneous agents who face model uncertainty and have different beliefs on expected returns. Market incompleteness makes options non-redundant, while heterogeneity creates a link between differences in beliefs and option volumes. Based on this model, Buraschi, Trojani and Vedolin (2014) produce evidence for an equilibrium link between investors' disagreement, the market price of volatility and correlation, and the differential pricing of index and individual equity options.

Apart from investors' beliefs, utility function also plays an important role. Bates (2008) studies effect of stock market crashes assuming investors have heterogeneous attitudes towards crash risk. The less crash averse insure the more crash averse through options markets that dynamically complete the economy. The resulting equilibrium is able to explain peso problem, pricing kernel puzzle and the stochastic evolution of option prices. Bollerslev, Tauchen and Zhou (2009) and Drechsler and Yaron (2011) illustrate the source of variance premium via equilibrium stochastic volatility model. Having Epstein-Zin recursive utility functions, investors care about vol-of-vol and time-varying jump risks. Similarly, Du (2013) and Seo and Wachter (2014) highlights the essential role of stochastic disaster risk on fitting implied volatility surface, with recursive utility function. Du (2011) use habit-formation framework to incorporate time-varying risk aversion along with crash risk.

The time-varying volatility and jump- risk premiums explain the observed state-dependent smirk patterns. Polkovnichenko and Zhao (2013) find that probability weighting has effect on pricing kernel dynamics extracted from index option prices. Schreindorfer (2014) build a parsimonious consumption-based asset pricing model based on a (generalized) disappointment averse investor and conditionally Gaussian fundamentals. In his model, regime-switches in endowment volatility interact with the investor tail aversion to produce large endogenous return jumps and a realistic implied volatility smirk.

In addition, market friction also matters. In fact, market friction may be important for option price dynamics. Bollen and Whaley (2004) demonstrate that changes in implied volatility are correlated with signed option volume. Garleanu, Pedersen, and Poteshman (2009) argue that option expensiveness is related to exogenous demand pressure in imperfect markets. The authors develop a theoretical model of option intermediation in which risk-averse market makers provide immediacy and are compensated for inventory risk. Investors are assumed to supply/demand options for exogenous reasons: portfolio insurance, covered call writing, agency issues, or behavioral reasons. Market makers trade a portfolio of options on a given security and are allowed to hedge in the underlying at discrete time intervals. Thus, part of the option risk can be hedged away. The key insight in their paper is that demand pressure for a given option increases its price by an amount proportional to the variance which it contributes to the market makers optimally hedged/diversified portfolio.

3 Economic Model

In this section, we first describe an equilibrium model, in which, the market return has stochastic volatility and stochastic skewness. Different from traditional framework, we assume such risks are generated from two separate random sources. We further discuss empirical implications of the model.

3.1 Basic Settings of Model

The basic settings of the model are similar with those in Bansal and Yaron (2004) and Bollerslev, Tauchen and Zhou (2009). In the model, we do not distinguish consumption (C_t) and dividend (D_t), therefore, dividend growth is given by $g_{t+1} = \log(D_{t+1}/D_t)$, the general model structure is given by

$$\begin{cases} g_{t+1} = \mu + \sigma_t z_{t+1} + X_{t+1}, \\ \sigma_{t+1}^2 \sim ARG(\delta_\sigma, \beta_\sigma \sigma_t^2, c_\sigma), \\ V_{t+1} \sim ARG(\delta_v, \beta V_t, c_v). \end{cases} \tag{1}$$

Our model uses two contemporaneously independent Levy innovations z_t and X_t to generate shocks in g_t. z_{t+1} follows standard normal distribution with stochastic variance σ_t^2. X_{t+1} follows lévy innovation with infinite jump intensity, whose conditional cumulant exponent is governed by V_{t+1}. Both σ_t^2 and V_t is modeled as autoregressive gamma process with specific parameters.

Take V_t for instance, its conditional moment generating function is

$$\varphi_v(u) = \mathbf{E}_t(\exp(-uV_{t+1})) = \exp(-a(u)\beta_v V_t - b(u)) = \exp(-\frac{c_v u}{1 + c_v u}\beta_v V_t - \delta_v \log(1 + c_v u)). \tag{2}$$

Let $\rho = c_v \beta_v < 1$, conditional moments of V is given by

$$\begin{cases} \mathbf{E}_t(V_{t+1}) = c_v \delta_v + c_v \beta_v V_t \\ \mathbf{Var}_t(V_{t+1}) = c_v^2 \delta_v + 2c_v^2 \beta_v V_t \\ \mathbf{E}_t(V_{t+\tau}) = \frac{c_v \delta_v (1-\rho\tau)}{1-\rho} + \rho^\tau V_t \\ \mathbf{Var}_t(V_{t+\tau}) = \frac{c_v^2 \delta_v (1-\rho\tau)^2}{1-\rho^2} + 2\rho^\tau \frac{c_v(1-\rho^\tau)}{1-\rho} V_t. \end{cases}$$

We further assume X_t follows variance gamma process (Madan and Seneta, 1990) with conditional cumulant exponent function as

$$\Phi_t(u) = -V_t \log(1 - \frac{au + \frac{1}{2}u^2}{b}) = V_t \xi_X(u). \tag{3}$$

where a, b are parameters for the lévy innovation. In principle, as illustrated by Ornthanalai (2014), X_t could be other lévy innovations, such as Normal Inverse Gaussian (Barndorff-Nielsen, 1998), CGMY (Carr, Geman, Madan and Yor, 2002) and Inverse Gaussian (Christoffersen, Heston and Jacobs, 2006), here we just use variance gamma distribution as an example.

Using the property of cumulant exponent, the conditional variance, conditional skewness and conditional kurtosis of dividend growth are given by

$$Var_t(g_{t+1}) = \sigma^2_{g,t+1} = \mathbf{E}_t(\sigma^2_{t+1}) + \mathbf{E}_t(V_{t+1})\xi''_X(0) = (c_\sigma\delta_\sigma + c_\sigma\beta_\sigma\sigma^2_t) + (c_v\delta_v + c_v\beta_v V_t)(\frac{1}{b} + \frac{a^2}{b^2});$$
$$Skew_t(g_{t+1}) = \mathbf{E}(V_{t+1})\xi'''_X(0)/\sigma^{3/2}_{t+1} = (c_v\delta_v + c_v\beta_v V_t)(\frac{3a}{b^2} - \frac{2a^3}{b^3})/\sigma^{3/2}_{g,t+1};$$
$$Kurt_t(g_{t+1}) = \mathbf{E}(V_{t+1})\xi''''_X(0)/\sigma^4_{t+1} = (c_v\delta_v + c_v\beta_v V_t)(\frac{3}{b^2} + \frac{12a^2}{b^3} + \frac{6a^4}{a^4})/\sigma^4_{g,t+1}.$$

where $\xi''_X, \xi'''_X, \xi''''_X$ are second, third and fourth derivatives of the cumulant exponent. Given parameter values calibrated by Ornthanalai (2014), a=-8.19, b=946, $\xi''_X > 0, \xi'''_X < 0, \xi''''_X > 0$.

Such model structure allows us to separate continuous and jump part of g_t dynamics, also allows volatility and skewness risk generating from two different random sources. As will be derived later, the market return dynamics generated from our model shares same conditional moments structure with g_t. Therefore, the variance of market return has two parts, variation from continuous component z_t and variation from jump component X_t. The higher moments are governed solely by X_t (V_t).

Figure 1 plots dynamics of CBOE VIX and Skewness Index Dynamics. The top panel contains VIX dynamics, while the middle panel present risk neutral skewness dynamics. The bottom panel shows residuals extracted from linear regression

$$VIX_t = \alpha + \beta SKEW_t + \epsilon_t.$$

Based on the figure, we know that the dynamics of VIX and Skewness are different, the time series correlation between these two are 0.29. Further linear regression shows that skewness is only able to explain a small portion of VIX variation. The R^2 of regression is around 8%. Therefore, empirical results support our intuition to let variance and skewness controlled independently by two random processes. Furthermore, for reasonable parameters, $\xi'''_X < 0, \xi''''_X > 0$, larger V_t leads to more negative expected skewness and higher kurtosis.

3.2 An Economic View for V_t Process

At anytime t, conditioned on V_t, the g_t innovation X_t follows Variance Gamma distribution, which belongs to the family of generalized hyperbolic distributions. The density function for generalized hyperbolic distributions $f_{GH}(\lambda, \alpha, \beta, \delta, \mu)$ is given by

$$f_{GH}(\lambda, \alpha, \beta, \delta, \mu)(x+\mu) = \frac{\exp(\beta x)}{\sqrt{2\pi}\alpha^{2\lambda-1}\delta^{2\lambda}} \cdot \frac{(\delta\sqrt{\alpha^2-\beta^2})^{\lambda}}{K_{\lambda}(\delta\sqrt{\alpha^2-\beta^2})} \cdot (\alpha\sqrt{\delta^2+x^2})^{\lambda-1/2} K_{\lambda-1/2}(\alpha\sqrt{\delta^2+x^2}).$$

The family of generalized hyperbolic distributions is closely related to time-changed Lévy process (Carr and Wu 2004). Many special cases of time-changed Lévy process, which are widely used in mathematical finance to model stock returns, have distributions belong to generalized hyperbolic family, for instance, Normal Inverse Gaussian (NIG) distributions (Barndorff-Nielsen 1997) and variance gamma (VG) distribution (Madan and Seneta 1990). A general review for applications of generalized hyperbolic distributions is given by Eberlein and Keller (1995). Variance gamma process has distribution as a special case of generalized distribution $f_{GH}(\lambda, \alpha, \beta, 0, 0)$. In our modeling context, at time t, conditioned on V_t, it is $f_{GH}(V_t, \alpha, \beta, 0, 0)$. Suppose at time t, we treat V_t as a constant, then the moment generating function is

$$M_X(u) = (\frac{\alpha^2-\beta^2}{\alpha^2-(\beta+u)^2})^{V_t} = \exp(V_t \xi_X(u)),$$

where

$$\xi_X(u) = -\ln(\frac{\alpha^2-(\beta+u)^2}{\alpha^2-\beta^2}); \quad \beta = a; \quad \alpha^2 = b/2 + a^2.$$

There are mainly 2 parameterizations of the variance gamma distribution, one is based on generalized hyperbolic distribution above, the other one is based on Seneta (2004). In Seneta (2004) paper, the variance gamma distribution is represented by $X(t; \sigma, \nu, \theta)$, where σ, θ, ν controls spread, asymmetry and shape of distribution, respectively.

Based on such parameterization, within our modeling framework, σ, θ, ν are all time-varying,

$$\sigma_t^2 = V_t/b; \quad \theta_t = a\sigma_t^2; \quad \nu_t = V_t^{-1}.$$

It is well-known that variance gamma process can be written as difference of two increasing processes, the first representing the increases in the process and the second representing the decreases. In particular, the two increasing process here are themselves gamma processes. We can then write

$$X(t:\sigma,\nu,\theta)=\gamma_g(t;\mu_g,\nu_g)-\gamma_b(t;\mu_b,\nu_b),$$

where the μ_g,ν_g,μ_b,ν_b are given by

$$\mu_g=\frac{1}{2}\sqrt{\theta^2+\frac{2\sigma^2}{\nu}}+\frac{\theta}{2},\quad \nu_g=(\frac{1}{2}\sqrt{\theta^2+\frac{2\sigma^2}{\nu}}+\frac{\theta}{2})^2\nu$$
$$\mu_b=\frac{1}{2}\sqrt{\theta^2+\frac{2\sigma^2}{\nu}}-\frac{\theta}{2},\quad \nu_b=(\frac{1}{2}\sqrt{\theta^2+\frac{2\sigma^2}{\nu}}-\frac{\theta}{2})^2\nu$$

Note that all the parameters are time-varying, for convenience, here, we omit subscript t. Detailed proof is provided in appendix.

Such framework allows us to interpret γ_g and γ_b as good and bad macroeconomic uncertainties, that is, uncertainties regarding the right and left tail movements in consumption(economic) growth, respectively. Their first and second moments are controlled by V_t. Because the relative importance of these shocks varies through time, there are 'good times' where γ_g dominates, and 'bad times' where γ_b distribution dominates. The stochastic proportion of these two shocks along with their time varying moments generate stochastic volatility and skewness of distribution of economic growth and market returns, which, ultimately, controlled by V_t.

Carr and Wu (2007) apply such techniques to generate stochastic skewness in a reduced-form currency option pricing model. The authors separate positive and negative jumps, and the stochastic proportion of these two types of jump contributes to stochastic skewness. Bekaert and Engstrom (2009) analyze a habit model with good and bad environments in consumption growth. The authors show that economically, their model creates a riskier consumption growth environment, which leads to a large equity premium and substantial precautionary savings demands, while keeping risk free rates low. Segal, Shaliastovich and Yaron (2014), based on the same model developed by Bekaert and Engstrom (2009), find out that good and bad growth components have opposite impact on aggregate growth and asset prices, and the market price of risks for these two components are opposite, both positively contribute to equity premium. In our paper, we summarize these effects using one state variable which controls stochastic skewness and study its effect on option returns.

3.3 Detailed Derivation of Model

To sharpen our intuition about stochastic skewness, we consider a simplified version of the model. The stochastic volatility channels are shut down. The model dynamics is given by

$$\begin{cases} g_{t+1} = \mu + X_{t+1}, \\ V_{t+1} \sim ARG(\delta_v, \beta V_t, c). \end{cases} \tag{4}$$

The derivation given below can be extend to the full version of the model easily.

The representative investor is assumed to have Epstein-Zin recursive utility function,

$$U_t = \{(1-\rho) C_t^{\frac{1-\gamma}{\theta}} + \rho(\mathbf{E}_t(U_{t+1}^{1-\gamma}))^{\frac{1}{\theta}}\}. \tag{5}$$

The parameter $0 < \rho < 1$ is the time discount factor. The parameter $\theta = \frac{1-\gamma}{1-\frac{1}{\psi}}$, with $\gamma \geq 0$ being the risk-aversion parameter and $\psi \geq 0$ the IES parameter. Note that when $\theta = 1$, that is, $\gamma = \frac{1}{\psi}$, the recursive preferences collapse to the standard case of expected power utility, in which case the agent is indifferent to the timing of the resolution of uncertainty of the consumption path. The sign of θ is determined by the magnitudes of risk aversion and elasticity of substitution. When risk aversion exceeds the reciprocal of IES ($\gamma > 1/\psi$), the agent prefers early resolution of uncertainty of consumption path, otherwise, the agent has a preference for late resolution of uncertainty.

For such preference, Epstein and Zin (1989) show that the asset pricing restrictions for gross return $R_{i,t+1}$ satisfy

$$\mathbf{E}_t[\rho^\theta G_{t+1}^{-\frac{\theta}{\psi}} R_{a,t+1}^{-(1-\theta)} R_{i,t+1}] = 1, \tag{6}$$

where G_{t+1} is the aggregate gross growth rate of consumption and $R_{a,t+1}$ is the gross return on an asset that delivers aggregate consumption as its dividends each period. Since in this paper, we do not model consumption and dividend separately, market gross return $R_{M,t+1}$ is also $R_{a,t+1}$. Price-consumption ratio is also price-dividend ratio.

Let $r_{a,t+1} = \log(R_{a,t+1})$, then the logarithm of the intertemporal marginal rate of substitution is

$$m_{t+1} = \theta \log \rho - \frac{\theta}{\psi} g_{t+1} + (\theta - 1) r_{a,t+1}, \tag{7}$$

which means that the innovation in m_{t+1} is driven by the innovations in g_{t+1} and $r_{a,t+1}$. Covariance with the innovation in m_{t+1} determines asset risk premium.

Following standard log-linearization procedure, assume price-consumption ratio z_t to be a linear function of state variable, $z_t = A_0 + A_1 V_t$, where A_0 and A_1 are solved by Euler equation (6). Using z_t, $r_{a,t+1}$ is expanded by Taylor' rule around the mean of z_t as in Campbell and Shiller (1988) to obtain

$$r_{a,t+1} = \kappa_0 + \kappa_1 z_{t+1} - z_t + g_{t+1}, \tag{8}$$

where κ_0 and κ_1 are given by

$$\kappa_1 = \frac{\exp(\mathbf{E}_t[z_t])}{1 + \exp(\mathbf{E}_t[z_t])}, \quad \kappa_0 = -\log((1-\kappa_1)^{(1-\kappa_1)}\kappa_1^{\kappa_1})$$

Substitute dynamics of g_{t+1} and $r_{a,t+1}$ into dynamics of m_{t+1}, we get

$$m_{t+1} = f(V_t) - \gamma X_{t+1} - (1-\theta)\kappa_1 A_1 V_{t+1}, \tag{9}$$

where $f(V_t) = \theta \log \rho - \gamma\mu + (\theta - 1)\kappa_0 + (\theta - 1)(\kappa_1 - 1)A_0 - A_1(\theta - 1)V_t$.

The risk free rate is determined endogenously as

$$\begin{aligned} r_t &= -\log(\mathbf{E}_t(\exp(m_{t+1}))) \\ &= -\log(\mathbf{E}_t(\exp(f(V_t) - \gamma X_{t+1} - (1-\theta)\kappa_1 A_1 V_{t+1}))) \\ &= -\log(\exp(f(V_t)))\mathbf{E}_t(\exp(-((1-\theta)\kappa_1 A_1 V_{t+1} - \log(1 - \frac{\gamma^2/2 + a\gamma}{b}))V_{t+1}))) \\ &= -\log(\exp(f(V_t) - a(u_2)V_t - b(u_2))) = a(u_2)V_t + b(u_2) - f(V_t), \end{aligned} \tag{10}$$

where $u_2 = (1-\theta)\kappa_1 A_1 - \xi_X(-\gamma)$.

The Radon-Nikodym derivative for measure transform is

$$\eta_{t,t+1} = \frac{\exp(f(V_t) - \gamma X_{t+1} - (1-\theta)\kappa_1 A_1 V_{t+1})}{\mathbf{E}_t(\exp(f(V_t) - \gamma X_{t+1} - (1-\theta)\kappa_1 A_1 V_{t+1}))} = \frac{\exp(-\gamma X_{t+1} - (1-\theta)\kappa_1 A_1 V_{t+1})}{\varphi_v(u_2)}. \tag{11}$$

One-step ahead joint moment generating function of X_{t+1}, V_{t+1}, under P measure is given by

$$
\begin{aligned}
& \mathbf{E}_t^P(\exp(-\alpha_1 X_{t+1} - \alpha_2 V_{t+1})) \\
= \ & \mathbf{E}_t^P(V_{t+1}\log(1-\frac{\alpha^2/2+a\alpha}{b}) - \alpha_2 V_{t+1}) \\
= \ & \mathbf{E}_t^P(\exp(-(\alpha_2 - \log(1-\frac{\alpha_1^2+a\alpha_1}{b}))V_{t+1})) = \varphi_v(u_1),
\end{aligned}
$$

where $u_1 = \alpha_2 - \log(1-\frac{\alpha_1^2/2+a\alpha_1}{b})$.

Using Radon-Nikodym derivative η_t, we can get one-step joint moment generating function of X_{t+1}, V_{t+1}, under Q measure, it is given by

$$
\begin{aligned}
& \mathbf{E}_t^Q(\exp(-\alpha_1 X_{t+1} - \alpha_2 V_{t+1})) = \mathbf{E}_t^P(\eta_{t,t+1}\exp(-\alpha_1 X_{t+1} - \alpha_2 V_{t+1})) \\
= \ & \mathbf{E}_t^P(\exp(-\gamma X_{t+1} - (1-\theta)\kappa_1 A_1 V_{t+1})\exp(-\alpha_1 X_{t+1} - \alpha_2 V_{t+1}))/\varphi_v(u_2) \\
= \ & \mathbf{E}_t^P(\exp(-(\alpha_1+\gamma)X_{t+1} - ((1-\theta)\kappa_1 A_1 + \alpha_2)V_{t+1}))/\varphi_v(u_2) = \varphi_v(u_3)/\varphi_v(u_2),
\end{aligned}
$$

where $u_3 = (1-\theta)\kappa_1 A_1 + \alpha_2 - \log(1-\frac{(\alpha_1+\gamma)^2/2+a(\alpha_1+\gamma)}{b})$.

Since

$$
\mathbf{E}_t^Q(\exp(-\alpha_1 X_{t+1} - \alpha_2 V_{t+1})) = \varphi_v^*(u_1), \tag{12}
$$

where $*$ means joint moment generating function with parameters under risk neutral measure Q. Using moment-generating functions in P and Q measures, we can get links for parameters between these two measures as follows,

$$
a* = a + \gamma; \quad b^* = b - \gamma^2/2 - a\gamma; \tag{13}
$$

$$
\beta^* = \beta/(1+cu_2) \quad ; c^* = c/(1+cu_2); \quad \delta^* = \delta. \tag{14}
$$

Detailed proofs are in appendix.

Therefore, the model structure is preserved after measure transform, using an equilibrium pricing kernel. Such measure transform make our model different from those reduced-form option pricing models. Based on Equation (8), the risk-neutral dynamics for market return should be

$$
\begin{cases}
r_{M,t+1} = r_{t+1} - \xi_X^*(1)V_{t+1} + X_{t+1}, \\
V_{t+1} \sim ARG(\delta^*, \beta^* V_t, c^*),
\end{cases} \tag{15}
$$

where r_t is equilibrium risk-free rate, which is time varying. $\xi_X^*(1)V_{t+1}$ is the convexity adjustment term.

In order to price options, we need the conditional moment generating function of discounted log stock price $f(\phi)$,

$$f(\phi) = \mathbf{E}_t(\exp(\phi \log S_T)) = S_t^{\phi} \exp(A_t + B_t V_t), \tag{16}$$

where A_t and B_t are calculated recursively by

$$\begin{cases} A_{t+1} - \delta^* \log(1 + c^* u_4) = A_t; \\ -\frac{c^* u_4}{1+c^* u_4}\beta^* = B_t; \\ u_4 = B_{t+1} - \phi\xi_X(1) - \xi_X(\phi); \\ A_T = 0, \quad B_T = 0. \end{cases}$$

The detailed proofs is in appendix. Due to the affine structure of X_t and V_t, the conditional moment generating function is exponential affine in state variable V_t. We can use fast Fourier transform to calculate option prices.

We know that the characteristic function of the log spot price is simply $f(i\phi)$, then

$$\mathbf{E}_t[\max(S_T - K), 0] = f(1)(\frac{1}{2} + \frac{1}{\pi}\int_0^{\infty} \Re[\frac{K^{-i\phi f(i\phi+1)}}{i\phi f(1)} d\phi]) - K(\frac{1}{2} + \frac{1}{\pi}\int_0^{\infty} \Re[\frac{K^{-i\phi} f(i\phi)}{i\phi}]d\phi), \tag{17}$$

where $\Re[\cdot]$ denotes the real part of a complex number. This formula is the same with the transform method proposed by Heston and Nandi (2000). It is different from that of Heston (1993) and others. In particular, it enables us to calculate the expectation once we just have the characteristic function of the logarithm of the spot price, instead of calculating two separate integrals. An option value is simply the discounted expected value of the payoff, $\max(S(T) - K, 0)$, calculated using the risk-neural probabilities, via the characteristic fuction $f(i\phi)$, in particular, the European call option price is given by

$$\begin{aligned} C &= \mathrm{e}^{\int_t^T -r_s ds} \mathbf{E}_t[\max(S_T - K, 0)] \\ &= \frac{1}{2}S_t + \frac{\mathrm{e}^{\int_t^T -r_s ds}}{\pi}\int_0^{\infty} \Re[\frac{K^{-i\phi f(i\phi+1)}}{i\phi f(1)} d\phi] - K\mathrm{e}^{\int_t^T -r_s ds}(\frac{1}{2} + \frac{1}{\pi}\int_0^{\infty} \Re[\frac{K^{-i\phi} f(i\phi)}{i\phi}]d\phi). \end{aligned}$$

As in Black-Scholes formula, the option price can be written as the asset price multiplied by a probability P_1 and the discounted strike price multiplied by a probability P_2. P_2 is the risk-neutral probability of the asset price being greater than K at maturity and the delta of the call value is simply P_1. The other hedge ratios like the vega and the gamma can be calculated by straight differentiation of the option pricing formula. Put option values can be calculated using the put-call parity. In contrast to the Black-Scholes formula, above formula is a function of the current asset price, S_t, and the conditional skewness of return innovation, which is a independent random source.

3.4 Simulation Studies

In this section, we first describe the algorithm used to generate skewness factor and stock return innovations. Then the simulated return sample paths are applied to price put options.

In order to simulate $ARG(1)$ process $\{V_t; t = 1, 2, \ldots, N\}$, according to Gourieroux and Jasiak (2006), it is useful to represent it as

$$V_t = \sum_{i=1}^{Z_t} W_{i,t} + \epsilon_t,$$

where $\epsilon_t, Z_t, W_{i,t}$ are independent conditional on V_{t-1}, and distributed as $Gamma(\delta, c)$, $Poisson(\beta V_{t-1})$, and $Gamma(1, c)$, respectively. Therefore, to simulate V_t process, we first generate N i.i.d. $Gamma(\delta, c)$ which is correspond to ϵ_t. Then, to generate the first simulated element V_1, we need to generate Z_1 according to a $Poisson(\beta V_0)$, where V_0 is initial point of V_t process. After getting Z_1, Z_1 i.i.d. $Gamma(1, c)$ random variable, representing $\{W_{i,1}, i = 1, 2, \ldots, Z_1\}$. With these random variables, the first element V_1 is given by

$$V_1 = \sum_{i=1}^{Z_1} W_{i,1} + \epsilon_1.$$

The second element V_2 is generated in a similar manner. Z_2 is independently drawn from $Poisson(\beta V_1)$. Again, we generate Z_2 i.i.d. $Gamma(1, c)$ and $\{W_{i,2}, i = 1, 2, \ldots, Z_2\}$, following $Gamma(1, c)$ distribution. The second element V_2 is given by

$$V_2 = \sum_{i=1}^{Z_2} W_{i,2} + \epsilon_2.$$

We continue the process to generate $\{V_t; t = 1, 2, \ldots, N\}$.

Once we get the path of V_t, at anytime t, conditioned on V_t, the return dynamics follows Variance Gamma distribution. The simulation method for variance gamma distribution is discussed in Kotz et al (2001). Consistent with our discussion above, it can be seen to be the difference of two i.i.d. gamma variables, whose parameters are governed by V_t. We use such representation to generate observations from the variance gamma distribution.

For option pricing, the model parameter is set to be $\{a, b, c, \delta, \beta\} = \{-8.19, 946, 0.03, 0.48, 1.27\}$. The initial price level is assumed to be 2019.42, which is equal to the close price of S&P 500 index in Jan 16, 2015. Initial value of V_t is assume to be its mean value 0.05, according to the parameters. We consider put options with time-to-maturity 30, 60 and 90 days, and moneyness (strike/price) ranges from 0.9 to 1.1, with step size 0.025. The put option price is computed as

$$\frac{1}{L} \exp(-r\tau) \sum_{i=1}^{L} \max(K - S_T, 0),$$

where $\tau = T - t$ is time to maturity, $L = 50000$ is number of path simulated. Risk free interest rate r is set to be Libor in Jan 16, 2015.

Figure 3 plots return densities and implied volatility smiles for different time-to-maturities. The simulated 30 days returns has skewness -2.20, the sample CBOE 30-day risk neutral skewness has mean value -1.933. As time-to-maturity increases, the return density become less skewed, specifically, 60-day returns has skewness -1.54, and 90-day returns has skewness -1.09. The bottom panel shows dynamics of implied volatility smile. For reasonable parameters, our model is able to generate steep volatility smile. Consistent with theory, skewness affect short-term option more. When strike price moves away from current price level, as time-to-maturity increases, slope of implied volatility fades. For at-the-money options, longer maturity leads to higher implied volatility, because option's time value is larger.

3.5 Empirical Implications of Model

In the equilibrium stochastic skewness model, apart from the return innovation, skewness risk (V_t) is also priced. Therefore, in practice, CBOE skewness index (SKEW) could be applied as a proxy for V_t. The CBOE formula for skewness index is

$$SKEW = 100 - 10 * \mathbf{E}((\frac{R-\mu}{\sigma})^3), \tag{18}$$

where R is stock return, μ is mean of returns, and σ is standard deviation of return. Therefore, CBOE skewness index has negative relation with underlying skewness, just like V_t in our model. As SKEW rises, the left tail of the S&P 500 distribution acquires more weight, and the probabilities of outlier returns become more significant.

Suppose we regard option-implied skewness as a measure of downside risk (Bates, 2000; Pan, 2002; Doran, Peterson, and Tarrant, 2007), then a positive innovation in option-implied market skewness indicates decreasing downside risk in the stock market, which is likely to be related to an improved investment opportunity set. The expected relation between shock of market risk neutral skewness and the market excess return is, therefore, positive, and stocks with low exposures to innovations in market skewness provide a valuable hedge against downside risk of the stock market. Investors require lower returns on these stocks, and we would expect the price of market skewness risk to be positive. Equivalently, if shock of CBOE skewness index SKEW is used as a factor, the risk premium should be negative.

We could illustrate such point from pricing kernel dynamics. In the pricing kernel equation, we get $(1-\theta)\kappa_1 A_1$ as the market price of risk for V_t. Standard calibration for long-run risk model shows that $\gamma > 1$ and $\phi > 1$. In Bansal and Yaron (2004), $\gamma = 10$ and $\psi = 1.5$. Therefore, $\theta < 0$, the intertemporal substitution effect dominates the wealth effect. Since when V_t increase, the return distribution should have more negative skewness, the price-consumption ratio should be a decreasing function with V_t, meaning that A_1 is less than zero. Therefore, the market price of risk for V_t should be negative. In other words, larger V_t leads to higher volatility and more negative returns, therefore, representing market conditions in recession time. We will test this hypothesis via Fama-Macbeth two pass regression and cross-section S&P 500 index option returns in next section.

4 Empirical Studies

In this section, we first describe data used in our empirical studies. Then the regression framework is introduced. After that, we will discuss our empirical results in details.

4.1 Data Descriptions

For S&P 500 index option returns, we use the same data in Constantinides, Jackwerth and Savov (2013), downloaded from the authors' website. The dataset is a panel of leverage-adjusted (that is, with a targeted market beta of one) monthly returns of 54 option portfolios split across type (27 call and 27 put portfolios), each with targeted time to maturity (30, 60, or 90 days), and targeted moneyness, that is, strike/price ratio (0.90, 0.925, 0.95, 0.975, 1.00, 1.025, 1.05, 1.075, or 1.10). Each portfolio's weights are adjusted daily to maintain its targeted beta, maturity, and moneyness. The major advantage of this construction is to lower the variance and skewness of the monthly portfolio returns and render the returns close to normal (about as close to normal as the index return), thereby making applicable the standard linear factor pricing methodology.

Table 1 and 2 show the summary statistics for the call and put option returns. Consistent with the arguments in Constantinides, Jackwerth and Savov (2013), the construction method reduces skewness and kurtosis of option returns, making its distribution close to normal.

For the call option returns, we see the same patterns documented by Coval and Shumway (2001). Out-of-money call options have negative returns, and the magnitudes increase as the strike price increases, which means out-of-money call options are over priced. Such phenomena is studied by some researchers, such as Boyer and Vorkink (2014), based on prospect theory proposed by Barbaris, Huang and Santos (2001), and Frazzini and Pedersen (2012), based on investors' leverage preference.

As time-to-maturity increases, the out-of-money call option returns become less negative.

When it comes to put option returns, selling short-term out-of-money option earns large positive returns. The returns for 30-day out-of-money put options is more than doubled compared to those with 90-day time-to-maturity. The overprice of out-of-money put option is always the important research topic for option pricing. The leverage effect (negative relation between return and volatility shocks) and Merton type jumps are usually applied to explain such phenomena. However, although leverage effect can generate skewness for underlying returns over long-horizon, it is unable to affect the volatility skew in short time-to-maturity. Merton jumps could help, but as is shown later, such jump risk is inadequate to explain the out-of-money put option returns..

Following Constantinides, Jackwerth and Savov (2013), we construct crisis-related factors, Jump, Volatility Jump, Volatility, and Liquidity. Jump is defined as the sum of all daily returns of the S&P 500 that are lower than 4% within each month, zero if there are none. Volatility is defined as the difference between end-month CBOE VIX index value and begin-month CBOE VIX index value of each month. Volatility Jump is defined as the sum of all daily increases in the CBOE VIX index that are greater than 4%, zero otherwise. Liquidity is defined as the innovation of the market-wide liquidity factor proposed by Pastor and Stambaugh (2003) and provided by the Wharton Research Data Services. We also construct some other proxies for systematic risks, such as credit spread, funding liquidity, economic uncertainty. Credit spread is defined as the credit spread between Moody Baa and Aaa corporate bond. Funding liquidity is the proxy used in paper of Fontaine and Garcia (2012), calculated from cross-section of treasury securities and obtained from authors' website. Economic uncertainty index is constructed by Baker, Bloom and Davis (2013). We define our key systematic risk variable SKEW as the the difference between end-month CBOE skewness index value and begin-month CBOE skewness index value of each month. The sample period ranges from 1990 to 2012.

Table 3 shows that skewness is different from traditional risk factors as volatility and jump. Skewness has positive correlation with market return economic uncertainty and funding liquidity, yet have negative relation with volatility and jump factor.

4.2 Empirical Test Framework

We test a linear pricing kernel framework for our model. For each asset $i = 1, \ldots, N$, we estimate the risk exposure from the time-series regression

$$R^e_{i,t} = \alpha_i + \beta'_{i,f} f_t + \epsilon_{i,t}, i = 1, \ldots, N, t = 1, \ldots, T, \tag{19}$$

where f represents a vector of risk factors. To estimate the cross-sectional price of risk associated with the factor f, we run a cross-sectional regression of time-series average excess returns, $\mathbf{E}[R^e_t]$, on risk factor exposure,

$$\mathbf{E}[R^e_{i,t}] = \mu_{R,i} = \beta' \lambda_f + \xi_i, i = 1, \ldots, N. \tag{20}$$

Here we impose the restriction that the intercept, corresponding to the excess return on a zero-beta asset, is equal to zero. This restriction increases the power of our tests and ensures that we do not obtain spurious results whereby small differences in factor loadings across correlated portfolios, together with a large premium, appear to fit the cross-section of option returns.

This cross-section regression yields estimates of the cross-sectional prices of risk λ. A good pricing model features an statistically significant and stable prices of risk λ across different cross-sections of test assets, and individual pricing errors ξ_i that are close to zero. We measure such pricing error in three methods. First of all, we calculate the cross-sectional adjusted R^2, which focuses on whether the sum of squared errors is relatively small $(1 - \sigma^2_\xi / \sigma^2_{\mu_R})$. Secondly, we get the mean absolute pricing error MAPE $(\sum |\xi|/N)$, which give us an overview of magnitudes about pricing errors. Moreover, we use a χ^2 statistics to test whether the pricing errors are jointly zero-measured by a weighted sum of squared pricing errors $(\xi' cov(\xi)^{-1} \xi \sim \chi^2_{N-K})$, where N is number of assets, K is number of factors and $cov(\xi)$ includes the estimation error in βs. To correct the standard errors for estimation of betas, we report t-statistics of Shanken (1992) in addition to the t-statistics of Fama and MacBeth (1973). The Shanken (1992) adjustment is calculated as follows

$$\sigma^2(\lambda) = \frac{1}{T}[(\beta'\beta)^{-1}\beta'\Sigma_\epsilon\beta(\beta'\beta)^{-1}(1 + \lambda'\Sigma_f^{-1}\lambda) + \Sigma_f], \tag{21}$$

$$cov(\xi) = \frac{1}{T}(I_N - \beta(\beta'\beta)^{-1}\beta')\Sigma_\epsilon(I_N - \beta(\beta'\beta)^{-1}\beta') \times (1 + \lambda'\Sigma_f^{-1}\lambda), \tag{22}$$

where Σ_f is variance-covariance matrix of the factors. Σ_ϵ is the variance-covariance matrix of the time-series errors, $\epsilon_{i,t}$.

4.3 Discussion of Results

In order to determine the number of factors used in our empirical regression, we first conduct a principle analysis using option portfolio returns. The principle component analysis is applied for total 54 option portfolio returns and for call and put option portfolio returns separately. Results in Table 4 illustrate that there are only two factors matter for dynamics of option returns, the first two factors are able to explain 98% variation of option return dynamics.

Based on this analysis, we consider five models, each model has two factors: (1) market excess return with volatility factor; (2) market excess return with price jump factor; (3) market excess return with volatility jump factor; (4) market excess return with liquidity factor; (5) market excess return with skewness factor. The reason for each model to include market excess return is based on the construction method of option returns documented by Constantinides, Jackwerth and Savov (2013). By construction, our option returns should have market beta equal to one.

Table 5 contains our main empirical results. On the one hand, consistent with our hypothesis, shock of CBOE skewness index has a negative risk premium. Recall that suppose we regard option-implied skewness as a measure of jump risk, then under this interpretation of option-implied skewness, a positive innovation in option-implied market skewness indicates decreased jump risk in the stock market, which is likely to be related to an improved investment opportunity set. Thus skewness risk should have positive risk premium. Meanwhile, CBOE skewness index has a linear and negative relation with true return skewness, hence, the risk premium for CBOE skewness index should be negative. The magnitude 8.7346 suggest that one standard deviation change of factor loading will lead to annualized return change of 5.52%.

On the other hand, model involving skewness has best performance among five models: MAPE is smallest, the J-statistics is lowest. Besides, only the model with skewness is not rejected by data. The intuition is clear in Figure 4. Market portfolio alone has very limited explanation power for option returns. As long as one systematic risk factor is added into the system, the performance improves a lot. The points in the figure closely gather along the 45 degree line. However, as mentioned above, traditional systematic risk factors have inadequate explanation power for short dated out-of-money put option returns. Consistent with our argument, for volatility, jump and liquidity factors, 30 days out-of-money put options are underpriced in terms of model implied expected returns. As far as skewness factor is concerned, the underpricing reduces, all the points lie along the 45 degree line.

Table 6 records skewness factor loadings in time-series regression

$$R^e_{i,t} = \alpha_i + \beta'_{i,f} f_t + \epsilon_{i,t}, i = 1, \ldots, N, t = 1, \ldots, T, \tag{23}$$

Similar with results in Constantinides, Jackwerth and Savov (2013), the market beta is close to 1 and always statistically significant, therefore, we do not report market beta in the table. For all call option returns, skewness betas are statistically significant. The magnitudes do not change much across different moneyness. For put option returns, skewness beta is significant for in-the-money options.

α in Table 7 captures time series abnormal returns. When skewness is included in the model, many αs for put option returns are not significant. However, short-term out-of-money put option returns and most call option returns still have significant α. Therefore, although the model has best performance and is not rejected by data, it still has limitation, especially for call option returns, which is difficult for standard risk-based asset pricing theories to handle.

Table 8 shows detailed results for cross section regression

$$\mathbf{E}[R^e_{i,t}] = \mu_{R,i} = \beta' \lambda_f + \xi_i, i = 1, \ldots, N. \tag{24}$$

We calculate the pricing error as the difference between realized returns and model implied ones and conduct t-test to see whether the pricing error for each option portfolio is equal to zero. For most put option portfolios, the pricing errors are not statistically different from zero. However, for call option portfolios, the pricing error is significant different from zero.

Moreover, we run moving window Fama-Mecbeth two-pass regression month-by-month. Since our sample starts from 1990, we begin our regression from Jan, 2000, leaving earlier data as a burn-in time period. This exercise allows us to examine the magnitudes and sign of our skewness risk premium. For convenience, we change the sign of risk premium for CBOE Skewness index. Skewness risk premium time series is presented in the up panel of Figure 5. On the one hand, the sign for skewness risk premium is always positive. On the other hand, skewness risk premium show regime-switching pattern. During normal time, the magnitude of risk premium is larger, and right after crisis, the magnitude of risk premium decreases. One possible explanation is that right after the crisis, investors are less concerned about possible market downturn. Because they tend to think that such kind of crisis won't happen again in the near future.

In order to confirm the pattern of risk premium, we further form a long-short strategy, trying to capture the skewness risk premium. Still, we run moving window Fama-Mecbeth two-pass regression, from the first step time-series regression, the skewness factor loadings for each option portfolios could be obtained. We assume the loadings are β_i. Then, we calculate the 'z-score' for $\beta_{i,t}$,

$$z_{i,t} = \frac{\beta_{i,t} - mean(\beta_{i,t})}{sd(\beta_{i,t})},$$

which is the scaled difference between $\beta_{i,t}$ and its sample mean at month t. $z_{i,t}$ serves as the weight for option portfolio i, suppose the return of option portfolio is r_i, the return for long-short strategy at month $t+1$ is given by

$$r_{t+1,ls} = c_t \sum_i z_{i,t} r_{i,t+1},$$

where c_t is a scale factor, which makes sure that we invest \$1 in each leg. Therefore, we form the dollar-neutral strategy to capture the skewness risk premium. The accumulated return time series is plotted in the middle panel of Figure 5. Consistent with our estimation results for skewness risk premium, we observe big dropdown after market downturn, for instance, 2002 'dot-com' bubble and 2008 financial crisis. The shock for skewness risk premium is presented in the bottom panel.

We further examine whether change of skewness risk premium contains information for future stock market returns. The regression exercise is conducted as follows

$$mktrf_{t+\tau} = \alpha + \beta FactorShock_t + \epsilon_t,$$

where $mktrf$ is market excess return in month $t + \tau$, τ could be $\{1, 2, 3\}$. $FactorShock$ represents shocks of several macro variables, including shock of skewness risk premium, shock of CBOE VIX index, shock of CBOE Skewness Index, shock of credit spread, shock of aggregated analyst forecast dispersion. Liquidity factor excess returns, shock of Economic Policy Uncertainty Index and shock of funding liquidity. The regression results are reported in Table 9. For each panel of the table, first line presents estimation of β, second line contains Newey-West robust standard error, and third line has adjusted R^2. The most important takeaway is that shocks of skewness risk premium has strong predict power for short-term market excess return. The R^2 for 1-month ahead prediction is 7%. The sign of β in the first column of up panel is consistent with our model intuition and time-series dynamics of risk premium, during good time, investors are more concerned about possible market downturn.

5 Conclusion

In this paper, we build an discrete time equilibrium option pricing model with lévy innovations. Such modeling method allows us to highlight the role of return stochastic skewness, as an independent systematic risk source. For investors with Epstein-Zin utility, such infinite jump risk is priced. Empirically, using Fama-Macbeth two pass regression and a panel of S&P 500 index option returns, we show that stochastic skewness risk is different from traditional volatility and Merton-type jump risks. Stochastic Skewness has superior explanation power for option returns.

We also find that the magnitude of risk premium has regime-switching pattern. During normal time and before crisis, the risk premium is large, however, right after the crisis, the magnitude of risk premium shrink. Shock of skewness risk premium serves as a strong predictor for short-term market excess returns. In our model, the risk premium for skewness is constant, therefore, as a future research direction, models with skewness and time-varying risk premium could be helpful to explain option returns, as suggested by Bekaert and Engstrom (2009), and Du (2011).

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A Derivation

Variance Gamma process as difference of two gamma processes. We can write

$$X(t;\sigma,\nu,\theta)=\gamma_g(t;\mu_g,\nu_g)-\gamma_b(t;\mu_b,\nu_b). \tag{25}$$

Their characteristic functions are given by

$$\phi_{\gamma_g}(u)=(\frac{1}{1-iu\frac{\nu_g}{\mu_g}})^{\frac{\mu_g^2 t}{\nu_g}};$$
$$\phi_{-\gamma_b}(u)=(\frac{1}{1+iu\frac{\nu_b}{\mu_b}})^{\frac{\mu_b^2 t}{\nu_b}}.$$

Suppose μ_g,μ_b,ν_g,ν_b satisfy the following conditions

$$\frac{\mu_g^2}{\nu_g}=\frac{\mu_b^2}{\nu_b}=\frac{1}{\nu}; \tag{26}$$
$$\frac{\nu_g\nu_b}{\mu_g\mu_b}=\frac{\sigma^2\nu}{2}; \tag{27}$$
$$\frac{\nu_g}{\mu_b}-\frac{\nu_b}{\mu_b}=\theta\nu; \tag{28}$$

The product of $\phi_{\gamma_g}(u)$ and $\phi_{-\gamma_b}(u)$ gives

$$(\frac{1}{1-iu(\frac{\nu_g}{\mu_g}-\frac{\nu_b}{\mu_b})-i^2u^2\frac{\mu_g\mu_b}{\nu_g\nu_b}})^{\frac{t}{\nu}},$$

which is exactly characteristic function for $X(t;\sigma,\nu,\theta)$.

Therefore, we can solve for μ_g,ν_g,μ_b,ν_b from conditions (26) - (28). First of all, from Equations (27) and (28), we have

$$(\frac{\nu_g}{\mu_g})^2-\theta\nu(\frac{\nu_g}{\mu_g})-\frac{\sigma^2\nu}{2}=0.$$

Solve it we get

$$\frac{\nu_g}{\mu_g}=\frac{\theta\nu+\sqrt{\theta^2\nu^2+2\sigma^2\mu}}{2}.$$

We choose the positive sign of the square root because the parameters of gamma process are required to be positive. Similarly, we can obtain

$$\frac{\nu_b}{\mu_b}=\frac{-\theta\nu+\sqrt{\theta^2\nu^2+2\sigma^2\mu}}{2}.$$

Then from condition (26), we have

$$\mu_g = \frac{\nu_g}{mu_g} \cdot \frac{1}{\nu},$$

therefore,

$$\mu_g = \frac{1}{2}\sqrt{\theta^2 + \frac{2\sigma^2}{\nu}} + \frac{\theta}{2},$$
$$\nu_g = (\frac{1}{2}\sqrt{\theta^2 + \frac{2\sigma^2}{\nu}} + \frac{\theta}{2})^2 \nu.$$

Similarly, we have

$$\mu_b = \frac{1}{2}\sqrt{\theta^2 + \frac{2\sigma^2}{\nu}} - \frac{\theta}{2},$$
$$\nu_b = (\frac{1}{2}\sqrt{\theta^2 + \frac{2\sigma^2}{\nu}} - \frac{\theta}{2})^2 \nu.$$

Coefficient for state variable in log price-consumption ratio. Recall that

$$m_{t+1} = \theta \log \rho - \frac{\theta}{\psi} g_{t+1} + (\theta - 1) Rra, t + 1, \tag{29}$$

$$z_t = A_0 + A_1 V_t, \tag{30}$$

$$r_{a,t+1} = \kappa_0 + \kappa_1 z_{t+1} - z_t + g_{t+1}. \tag{31}$$

Then,

$$\begin{aligned}
& m_{t+1} + r_{a,t+1} = \theta \log \rho - \frac{\theta}{\psi} g_{t+1} + (\theta - 1) r_{a,t+1} + r_{a,t+1} \\
& = \theta \log \rho - \frac{\theta}{\psi}\mu - \frac{\theta}{\psi} X_{t+1} + \theta r_{a,t+1} \\
& = \theta \log \rho - \frac{\theta}{\psi}\mu - \frac{\theta}{\psi} X_{t+1} + \theta(\tilde{\mu} + \kappa_1 A_1 V_{t+1} - A_1 V_t + X_{t+1}) \\
& = \theta \log \rho - \frac{\theta}{\psi}\mu - \frac{\theta}{\psi} X_{t+1} + \theta\tilde{\mu} + \theta\kappa_1 A_1 V_{t+1} - \theta A_1 V_t + \theta X_{t+1} \\
& = \theta \log \rho - \frac{\theta}{\psi}\mu + \theta\tilde{\mu} - \theta A_1 V_t + \theta\kappa_1 A_1 V_{t+1} + (1 - \gamma) X_{t+1},
\end{aligned}$$

where $\tilde{\mu} = \mu + \kappa_0 + (\kappa_1 - 1) A_0$.

Based on equations above, we calculate Euler equation

$$\begin{aligned}
&\mathbf{E}_t[\exp(m_{t+1}+r_{a,t+1})] \\
&= \exp(\theta\log\rho - \frac{\theta}{\psi}\mu + \theta\tilde{\mu} - \theta A_1 V_t)\mathbf{E}_t[\exp(\theta\kappa_1 A_1 V_{t+1} + (1-\gamma)X_{t+1})] \\
&= \exp(\theta\log\rho - \frac{\theta}{\psi}\mu + \theta\tilde{\mu} - \theta A_1 V_t)\mathbf{E}_t[\mathbf{E}_{t+1}[\exp(\theta\kappa_1 A_1 V_{t+1} + (1-\gamma)X_{t+1})]] \\
&= \exp(\theta\log\rho - \frac{\theta}{\psi}\mu + \theta\tilde{\mu} - \theta A_1 V_t)\mathbf{E}_t[\exp(\theta\kappa_1 A_1 V_{t+1})\mathbf{E}_{t+1}[\exp((1-\gamma)X_{t+1})]] \\
&= \exp(\theta\log\rho - \frac{\theta}{\psi}\mu + \theta\tilde{\mu} - \theta A_1 V_t)\mathbf{E}_t[\exp((\theta\kappa_1 A_1 + \xi_X(1-\gamma))V_{t+1})] \\
&= \exp(-\frac{c\tilde{u}}{1+c\tilde{u}}\beta V_t - \delta\log(1+c\tilde{u}) + \theta\log\rho - \frac{\theta}{\psi}\mu + \theta\tilde{\mu} - \theta A_1 V_t),
\end{aligned}$$

where $\tilde{u} = -\theta\kappa_1 A_1 - \xi_X(1-\gamma)$.

Basic asset pricing theory indicates that $\mathbf{E}_t[\exp(m_{t+1}+r_{a,t+1})] = 1$. Therefore, we get two equations for A_0 and A_1,

$$\begin{aligned}
&-\delta\log(1+c\tilde{u}) + \theta\log\rho - \frac{\theta}{\psi}\mu + \theta\tilde{\mu} = 0, \\
&-\frac{c\tilde{u}}{1+c\tilde{u}}\beta V_t - \theta A_1 V_t = 0.
\end{aligned}$$

The second equation yields

$$\theta^2 c\kappa_1 A_1^2 + (\beta c\theta\kappa_1 - \theta + \theta c\xi_X(1-\gamma))A_1 + \beta c\xi_X(1-\gamma) = 0,$$

through which we get A_1, then use first equation, we get A_0.

Measure transform. The model dynamics is preserved after measure transform. The procedure provides linkage for parameters between P and Q measures.

$$\begin{cases}
u_1 = \alpha_2 - \log(1 - \frac{\alpha_1^2/2 + a\alpha_1}{b}) \\
u_2 = (1-\theta)\kappa_1 A_1 - \log(1 - \frac{\gamma^2/2 + a\gamma}{b}) \\
u_3 = (1-\theta)\kappa_1 A_1 + \alpha_2 - \log(1 - \frac{(\alpha_1+\gamma)^2/2 + a(\alpha_1+\gamma)}{b})
\end{cases} \tag{32}$$

Then we have

$$u_3 - u_2 = \alpha_2 - \log(1 - \frac{\alpha_1^2/2 + \alpha_1(r+a)}{b - \gamma^2/2 - a\gamma}), \tag{33}$$

where

$$\log(1-\frac{\alpha_1^2/2+\alpha_1(r+a)}{b-\gamma^2/2-a\gamma})=\log(1-\frac{\alpha_1^2+a^*\alpha_1}{b^*}). \tag{34}$$

Then, the risk neutral parameters for variance gamma process is

$$a* = a+\gamma; \quad b^* = b-\gamma^2/2-a\gamma. \tag{35}$$

To get the risk-neutral parameters for autoregressive gamma process, note that

$$\begin{aligned}
& a^*(u_1)\beta^* = a(u_3)\beta - a(u_2)\beta \\
\Rightarrow \quad & \frac{c^*u_1}{1+c^*u_1}\beta^* = (\frac{cu_3}{1+cu_3}-\frac{cu_2}{1+cu_2})\beta \\
\Rightarrow \quad & \beta^* = \beta/(1+cu_2) \quad ; c^* = c/(1+cu_2).
\end{aligned}$$

Similarly, from

$$b^*(u_1) = b(u_3) - b(u_2),$$

we can get $\delta^* = \delta$.

Conditional moment generating function. Conditional moment generating function for discounted log stock price is used for option pricing.

$$\begin{aligned}
& \mathbf{E}_t(\exp(\phi \log S_T)) \\
= \quad & S_t^\phi \exp(A_t + B_t V_t) = \mathbf{E}_t(\mathbf{E}_{t+1}(\exp(\phi S_T))) \\
= \quad & \mathbf{E}_t(S_{t+1}^\phi \exp(A_{t+1} + B_{t+1}V_{t+1})) \\
= \quad & \mathbf{E}_t(\exp(\phi(\log S_t - \xi_X(1)V_{t+1} + X_{t+1}) + A_{t+1} + B_{t+1}V_{t+1})) \\
= \quad & S_t^\phi \mathbf{E}_t(\exp(A_{t+1} + (B_{t+1} - \phi\xi_X(1)V_{t+1} + \phi X_{t+1}))) \\
= \quad & S_t^\phi \mathbf{E}_t(\exp(A_{t+1} + (B_{t+1} - \phi\xi_X(1) + \xi_X(\phi))V_{t+1})) \\
= \quad & S_t^\phi \exp(A_{t+1} - \frac{c^*u_4}{1+c^*u_4}\beta^* V_t - \delta^* \log(1+c^*u_4)),
\end{aligned}$$

where $u_4 = B_{t+1} - \phi\xi_X(1) - \xi_X(\phi)$. Then compare coefficients, we get recursive formulas

$$\begin{cases}
A_{t+1} - \delta^* \log(1+c^*u_4) = A_t; \\
-\frac{c^*u_4}{1+c^*u_4}\beta^* = B_t; \\
A_T = 0, \quad B_T = 0.
\end{cases}$$

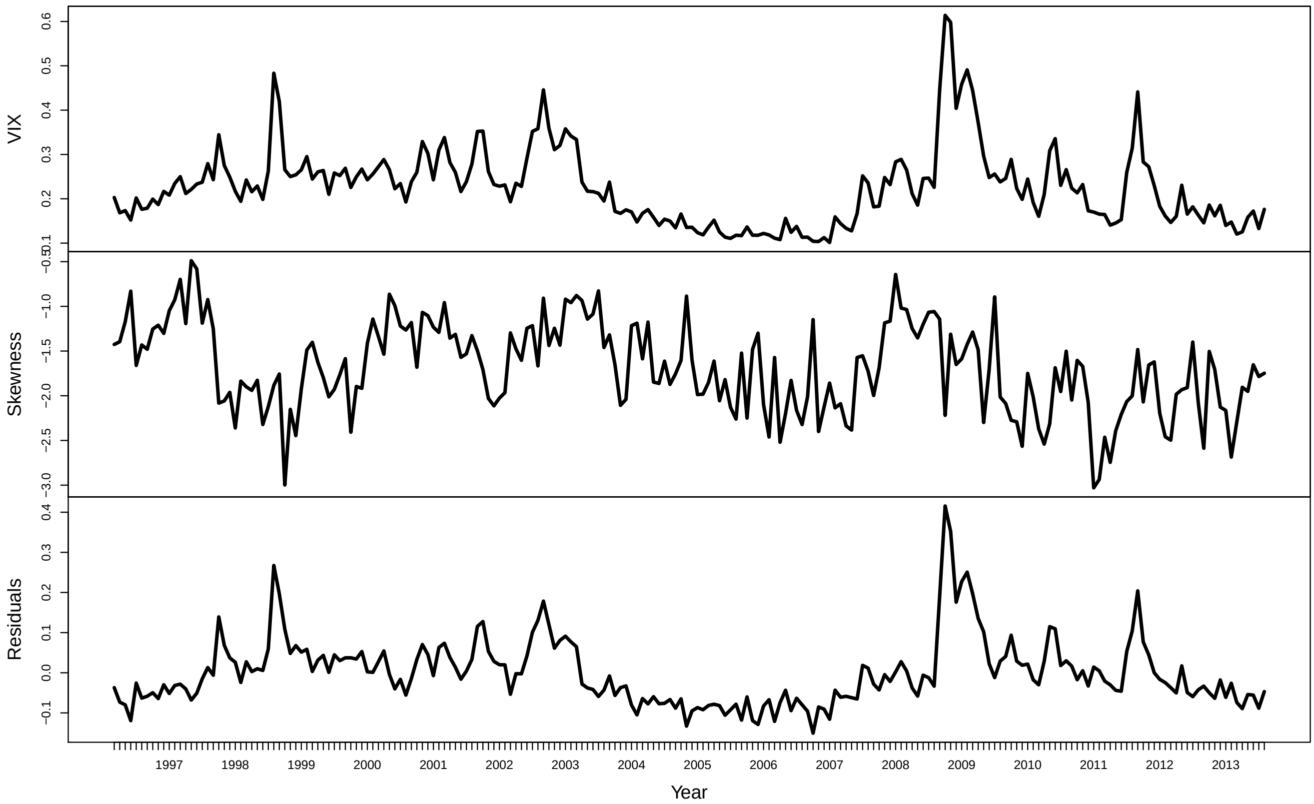
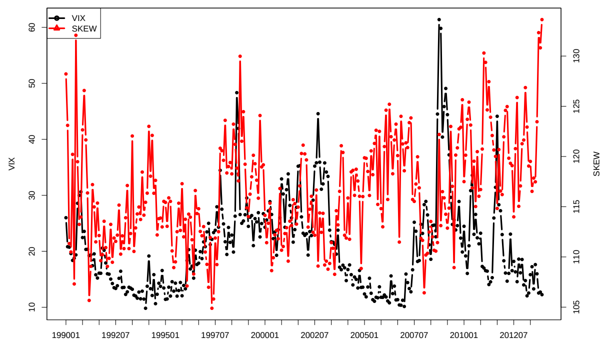
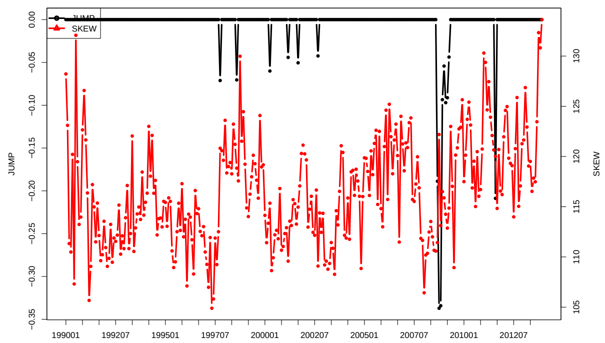


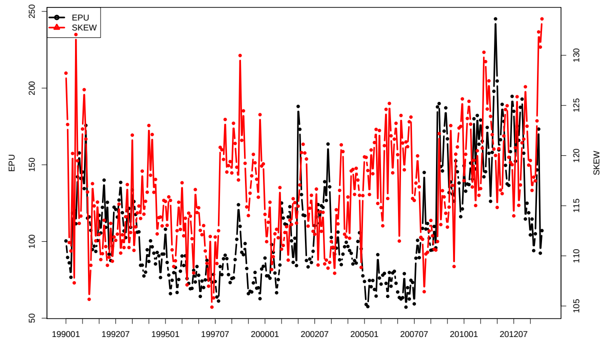
Figure 1: This figure plots dynamics of monthly CBOE VIX Index, CBOE SKEWNESS Index, and residuals in following regression ($VIX_t = \alpha + \beta SKEWNESS_t + \epsilon_t$). The sample period ranges from Jan, 1996 to Aug, 2013. We use VIX and SKEWNESS index values in last trading day of the month t. $\beta = -0.52$, R^2 is around 10%. If we run regression ($SKEWNESS_t = \alpha + \beta VIX_t + \epsilon_t$), $\beta = -0.19$, R^2 is also around 10%.



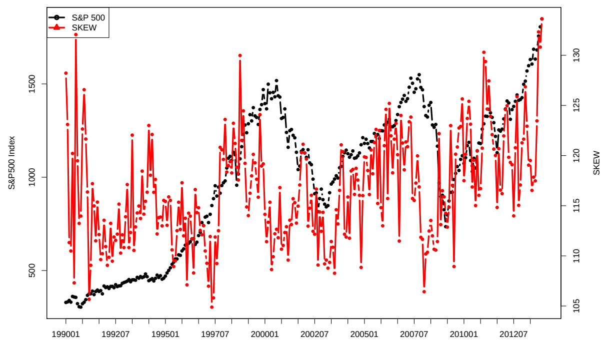
(a) VIX SKEWNESS Dynamics



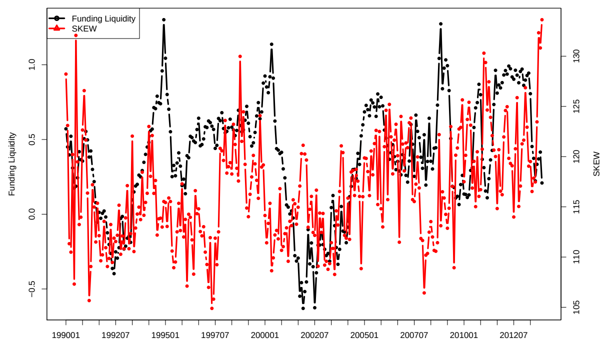
(b) JUMP SKEWNESS Dynamics



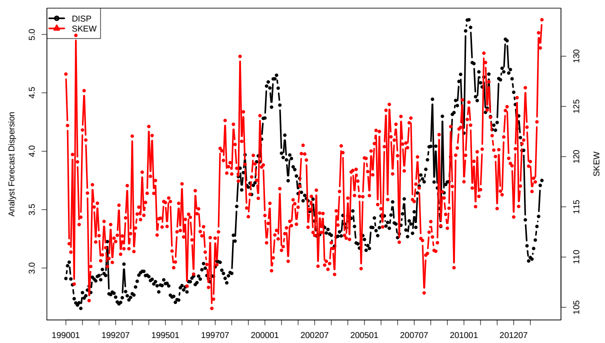
(c) EPU SKEWNESS Dynamics



(d) S&P500 SKEWNESS Dynamics

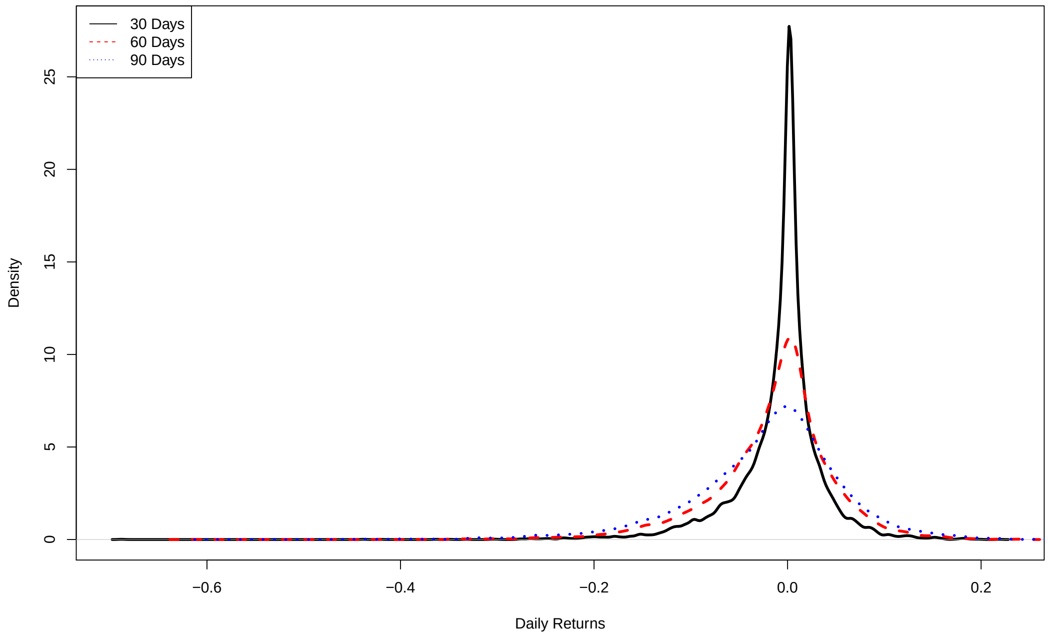


(e) FL SKEWNESS Dynamics

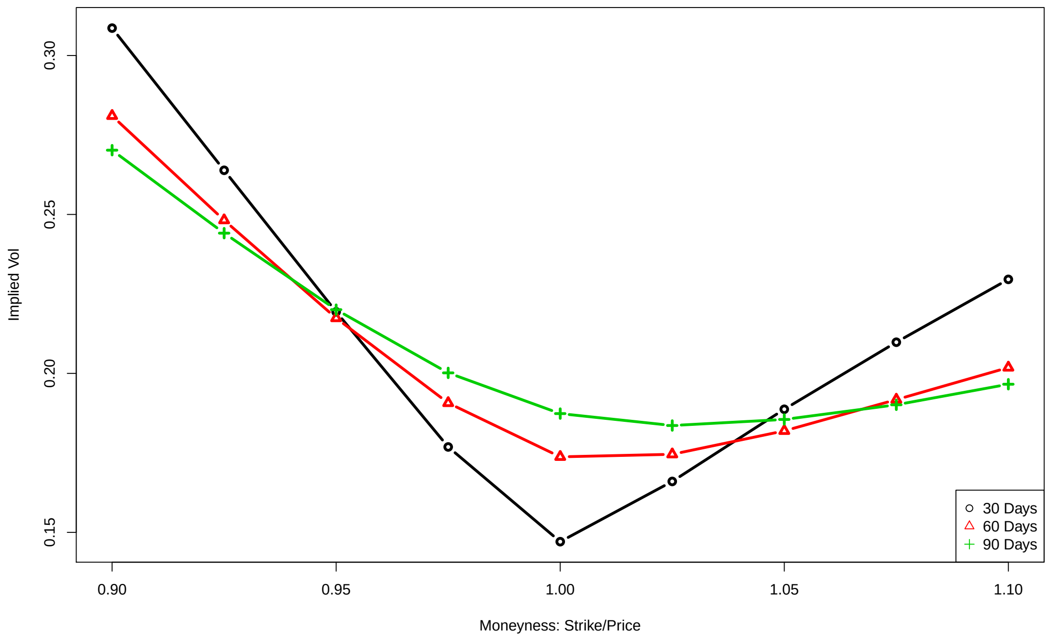


(f) DISP SKEWNESS Dynamics

Figure 2: Monthly time series dynamics for CBOE VIX index, CBOE SKEWNESS index, credit spread, aggregated analyst forecast dispersion, S&P 500 index level, Economic Policy Uncertainty index and funding liquidity, ranging from Jan, 1990 to Dec, 2013.

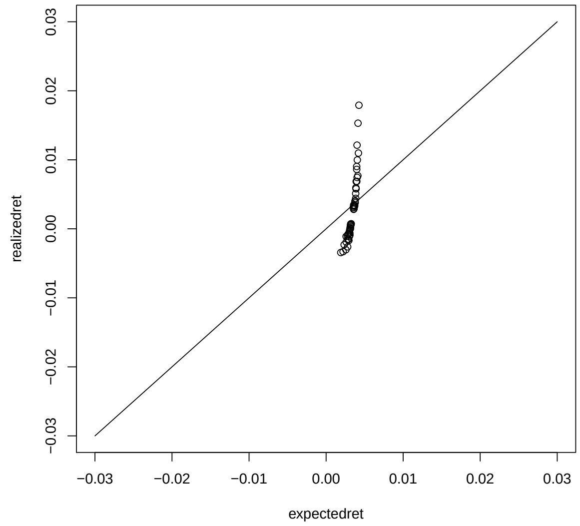


(a) Return Density

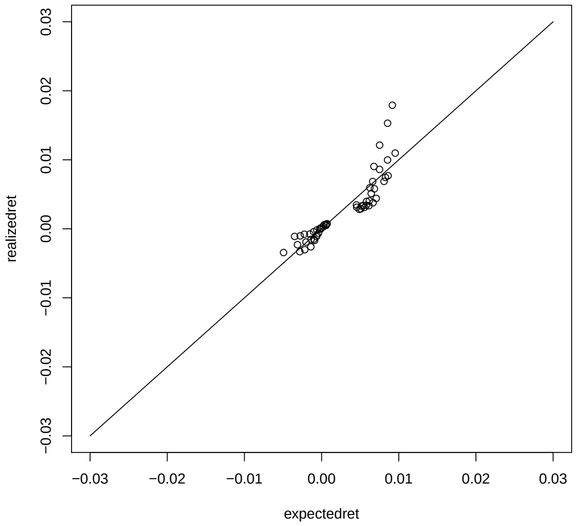


(b) Implied Volatility Smiles

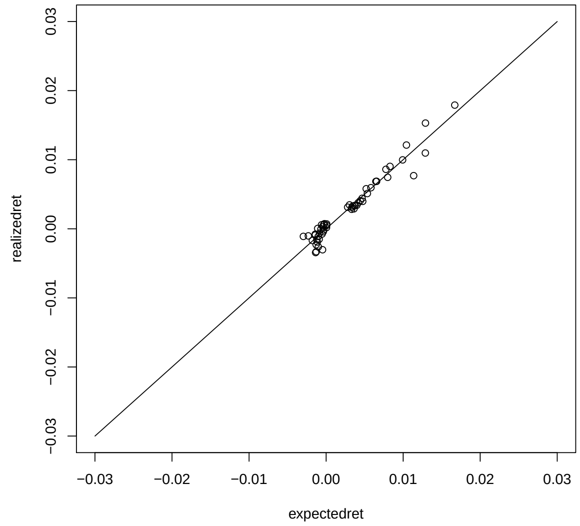
Figure 3: This figure plots simulated return density and implied volatility smiles for put options with different maturities, 30, 60, and 90 days. The model has variance gamma return distribution and stochastic skewness factor following autoregressive gamma process with 1 lag. 50000 paths are simulated.



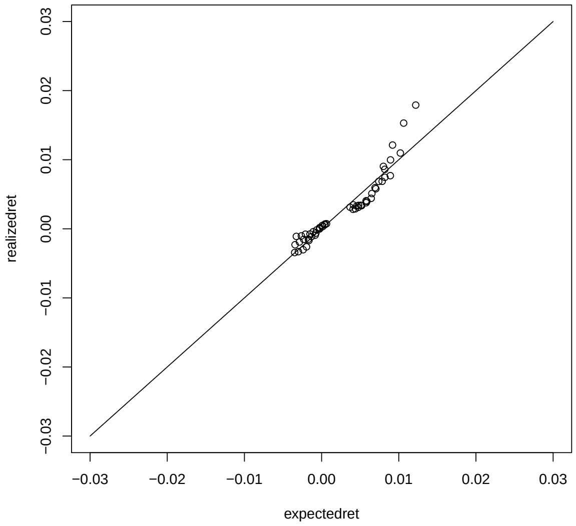
(a) MKT



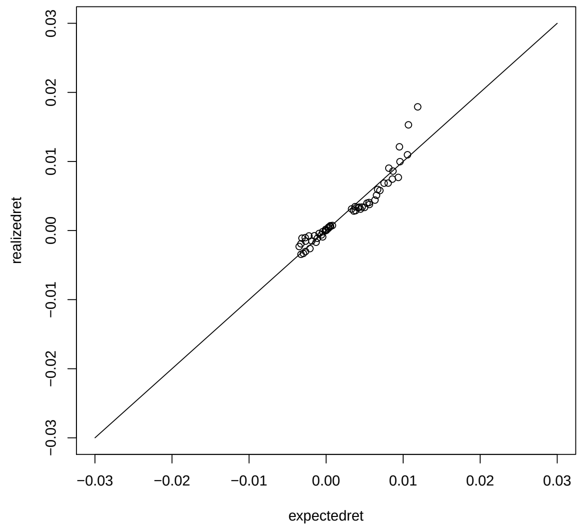
(b) MKT + LIQ



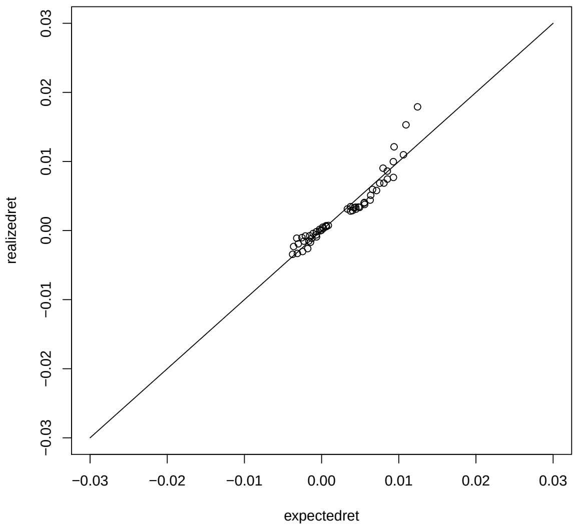
(c) MKT + SKEWNESS



(d) MKT + JUMP



(e) MKT + VOL



(f) MKT + VOLJUMP

Figure 4: Realized Returns against Expected Returns for different models

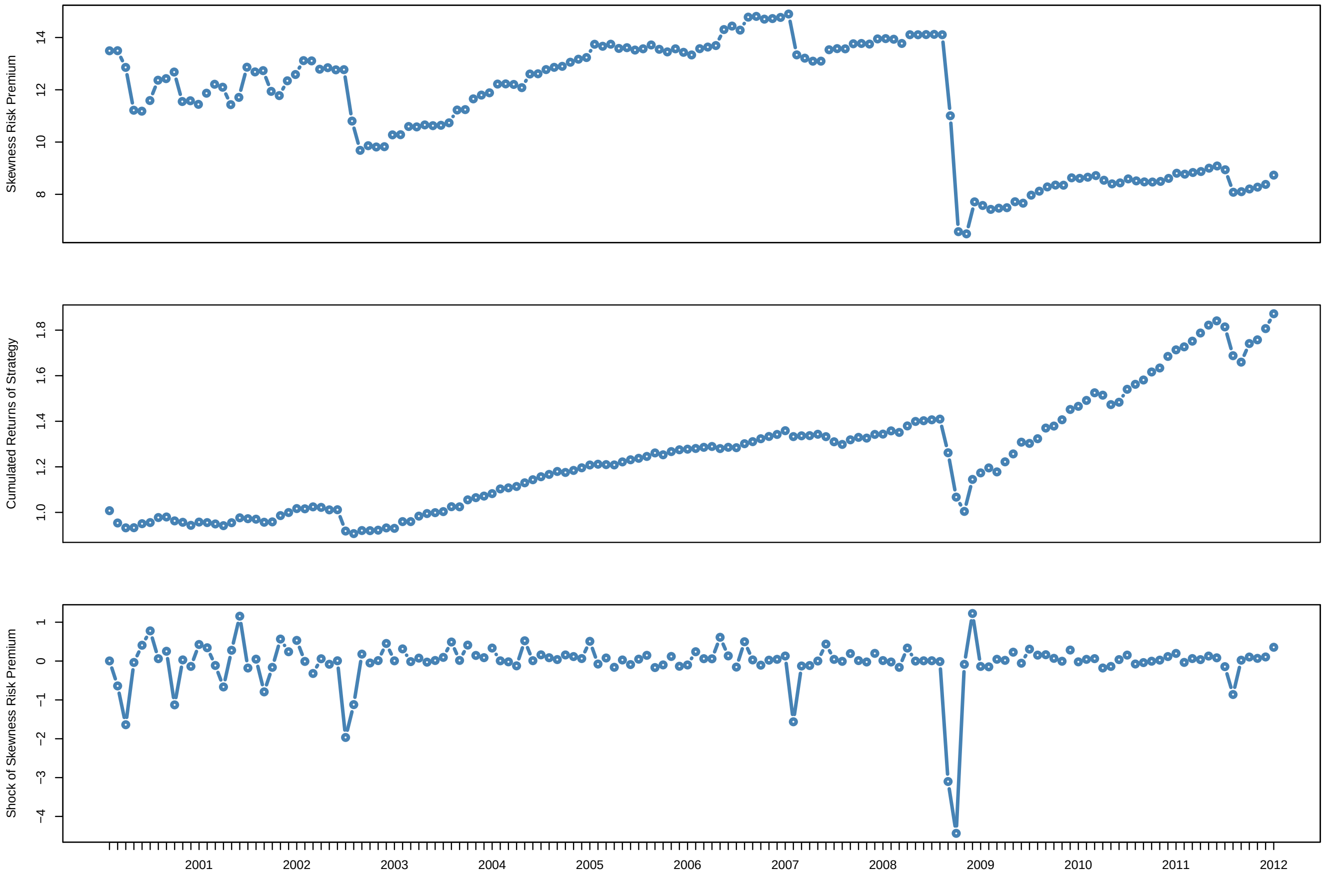


Figure 5: This figure plots time series dynamics for skewness risk premium (up panel), shocks of skewness risk premium (bottom panel), and returns of option trading strategy (middle panel) trying to capture the skewness risk premium. The whole sample ranges from Jan, 1990 to Jan, 2012. The skewness risk premium at month t is generated from Fama-Mecbeth two-pass regression, using information up to month t.The estimation procedure starts from Jan, 2000, the earlier sample period is used as burn-in period. Each month t, we also get skewness factor loadings from Fama-Mecbeth two-pass regression. At month $t+1$, we form long-short strategies using 54 S&P 500 index option portfolios, according to their skewness factor loadings in month t. The cumulative returns are plotted in the middle panel.

Table 1: **Summary Statistics for S&P 500 Index Call Option Returns**

This table reports summary statistics for percentage monthly excess returns of S&P 500 index call options, ranging from April, 1986 to Jan, 2012. Column names represent option characteristics, for example, C30900 means call option with time-to-maturity 30 days, Strike/PriceLevel 0.9.

	C30900	C30925	C30950	C30975	C301000	C301025	C301050	C301075	C301100
mean	0.17	0.14	0.09	-0.01	-0.11	-0.23	-0.29	-0.29	-0.34
sd	4.28	4.24	4.19	4.17	4.14	4.13	4.06	3.89	3.92
skew	-0.25	-0.23	-0.18	-0.13	0.02	0.28	0.58	0.91	1.39
kurt	0.63	0.63	0.66	0.71	0.78	1.18	2.20	3.61	5.84
	C60900	C60925	C60950	C60975	C601000	C601025	C601050	C601075	C601100
mean	0.17	0.13	0.08	0.01	-0.05	-0.11	-0.12	-0.20	-0.28
sd	4.23	4.20	4.18	4.16	4.17	4.16	4.21	4.16	4.03
skew	-0.23	-0.18	-0.14	-0.07	0.02	0.18	0.36	0.59	0.84
kurt	0.62	0.64	0.72	0.73	0.83	0.95	1.34	1.91	2.91
	C90900	C90925	C90950	C90975	C901000	C901025	C901050	C901075	C901100
mean	0.19	0.16	0.12	0.07	0.05	0.01	-0.01	-0.06	-0.11
sd	4.19	4.17	4.14	4.14	4.15	4.13	4.16	4.21	4.16
skew	-0.20	-0.14	-0.10	-0.05	0.03	0.14	0.25	0.28	0.47
kurt	0.60	0.59	0.62	0.65	0.74	0.85	1.07	1.09	1.41

Table 2: **Summary Statistics for S&P 500 Index Put Option Returns**

This table reports summary statistics for percentage monthly excess returns of S&P 500 index put options, ranging from April, 1986 to Jan, 2012. Column names represent option characteristics, P30900 means put option with time-to-maturity 30 days, Strike/StockLevel 0.9.

	P30900	P30925	P30950	P30975	P301000	P301025	P301050	P301075	P301100
mean	1.86	1.61	1.33	1.06	0.75	0.55	0.48	0.47	0.43
sd	6.33	5.98	5.65	5.38	5.10	4.87	4.72	4.65	4.56
skew	-1.54	-1.47	-1.21	-1.00	-0.86	-0.77	-0.61	-0.48	-0.49
kurt	6.82	6.11	4.98	4.42	4.04	3.49	2.71	2.06	1.84
	P60900	P60925	P60950	P60975	P601000	P601025	P601050	P601075	P601100
mean	1.23	1.13	1.01	0.85	0.66	0.56	0.48	0.45	0.40
sd	6.02	5.69	5.47	5.24	5.05	4.88	4.75	4.70	4.63
skew	-1.27	-1.12	-1.02	-0.94	-0.85	-0.77	-0.67	-0.57	-0.55
kurt	5.58	4.97	4.53	4.09	3.70	3.38	2.87	2.42	2.12
	P90900	P90925	P90950	P90975	P901000	P901025	P901050	P901075	P901100
mean	0.82	0.83	0.78	0.69	0.59	0.54	0.49	0.43	0.42
sd	5.84	5.59	5.41	5.19	5.06	4.91	4.81	4.75	4.67
skew	-1.12	-1.05	-0.98	-0.87	-0.82	-0.75	-0.67	-0.62	-0.58
kurt	5.04	4.79	4.52	3.92	3.58	3.25	3.00	2.69	2.41

Table 3: **Correlation of Variables**

This table reports the correlation of variables. Sample period ranges from Jan, 1990 to Dec, 2013. SP500 is S&P 500 index level. VIX is CBOE risk neutral volatility index level. CS is credit spread between Baa and Aaa bonds. EPU is economic policy uncertainty index. FL is funding liquidity. LIQ is market liquidity factor. SKEW is CBOE risk neutral skewness index level.

	SP500	VIX	CS	JUMP	EPU	FL	LIQ	SKEW
SP500	1.0000	0.0712	0.0711	0.0020	0.0418	0.2554	0.0489	0.3768
VIX	0.0712	1.0000	0.5678	-0.5745	0.3844	0.0507	-0.0388	-0.1507
CS	0.0711	0.5678	1.0000	-0.5127	0.4971	0.1140	-0.0217	0.0374
JUMP	0.0020	-0.5745	-0.5127	1.0000	-0.3123	-0.2384	0.1723	0.0339
EPU	0.0418	0.3844	0.4971	-0.3123	1.0000	0.0631	-0.1179	0.1394
FL	0.2554	0.0507	0.1140	-0.2384	0.0631	1.0000	-0.0911	0.2578
LIQ	0.0489	-0.0388	-0.0217	0.1723	-0.1179	-0.0911	1.0000	0.0120
SKEW	0.3768	-0.1507	0.0374	0.0339	0.1394	0.2578	0.0120	1.0000

Table 4: **Principle Component Analysis for S&P 500 Index Option Returns**

This table conducts principle component analysis for 54 S&P 500 index option return time series. PCA analysis use all option returns and call, put option returns separately. We report first 10 factors.

	Call& Put		Call		Put	
	eigenvalue	variance(%)	eigenvalue	variance(%)	eigenvalue	variance(%)
comp 1	47.1792	87.3688	25.6783	95.1049	26.1144	96.7200
comp 2	5.7797	10.7031	0.9848	3.6475	0.6658	2.4659
comp 3	0.3885	0.7195	0.1272	0.4710	0.0724	0.2681
comp 4	0.1666	0.3086	0.0644	0.2385	0.0483	0.1787
comp 5	0.0990	0.1833	0.0335	0.1242	0.0212	0.0784
comp 6	0.0724	0.1341	0.0318	0.1178	0.0164	0.0606
comp 7	0.0564	0.1044	0.0198	0.0732	0.0155	0.0574
comp 8	0.0452	0.0838	0.0159	0.0587	0.0100	0.0370
comp 9	0.0340	0.0630	0.0101	0.0373	0.0072	0.0268
comp 10	0.0244	0.0451	0.0070	0.0259	0.0047	0.0175

Table 5: **Cross Section Regression Results for Different Models**

This table reports results of Fama-Macbeth two-pass regressions. In all regressions, market excess return is always one factor, the other factor includes: VIX, shock of CBOE VIX index; VIXJUMP, sum of daily VIX index shocks larger than certain threshold in each month; JUMP, sum of daily S&P500 index returns lower than certain threshold in each month; LIQ, market liquidity factor; SKEW, shock of CBOE skewness index. λs represent estimated risk premiums for each factor. The robust adjusted standard errors are below each λ. We also report cross-sectional adjusted R^2, mean absolute pricing error (MAPE) and J-statistics for each factor combination. Sample period ranges from Jan 1996 to Jan 2012.

	VIX	VIXJUMP	JUMP	LIQ	SKEW
λ_{mkt}	-0.0011	0.0032	0.0035	0.0051	0.0071
t_{NW}	-0.3589	1.0586	1.1463	1.6624	2.2565
t_{Shaken}	-0.3415	1.0408	1.1222	1.4851	1.8179
λ_{factor}	-0.0168	-0.0451	0.0218	0.0587	-8.7346
t_{NW}	-4.4016	-4.8012	4.8002	4.4669	-5.2380
t_{shaken}	-4.0519	-3.9985	3.8458	2.3261	-2.3229
MAPE	0.0959	0.0968	0.1065	0.1407	0.0797
AdjR2	0.9305	0.9304	0.9272	0.9114	0.9138
$T^2(\chi^2_{N-K})$	197.6288	201.9183	190.5805	87.9548	64.6738
p-value	0	0	0	0.0014	0.1115

Table 6: **Skewness Factor Loadings for S&P 500 Option Portfolios**

This table reports asset loadings for skewness factor and t-statistics, based on two-pass Fama-Mecbeth regression. The regression is the first step, time-series regression, using market excess return and shock of CBOE skewness index as factors. The table is organized by time-to-maturity and moneyness (strike/price level). Column names contain moneyness, and row names contain option type and time-to-maturity. For each option, first line is β (loading) for skewness factor, and second line is its t-statistics. Sample period ranges from Jan 1996 to Jan 2012.

	90%	92.5%	95%	97.5%	100%	102.5%	105%	107.5%	110.5%
call 30	0.1078	0.1135	0.1182	0.1206	0.1219	0.1020	0.0988	0.0980	0.1081
	4.1061	4.1744	4.1293	3.9268	3.5486	2.5986	1.9453	2.0296	1.8205
call 60	0.1081	0.1084	0.1109	0.1130	0.1138	0.1084	0.1052	0.1092	0.1115
	4.0902	3.9677	3.8538	3.6898	3.4515	3.0336	2.6232	2.4645	2.3068
call 90	0.1031	0.1041	0.1040	0.1078	0.1057	0.1053	0.1133	0.1235	0.1233
	3.8588	3.8322	3.6649	3.5291	3.2573	3.0154	2.9377	2.9299	2.7480
put 30	-0.1111	-0.0673	-0.0262	0.0034	0.0341	0.0453	0.0569	0.0666	0.0669
	-1.7046	-1.1616	-0.5100	0.0756	0.8800	1.3334	1.8397	2.2509	2.3619
put 60	-0.0472	-0.0034	0.0161	0.0282	0.0409	0.0528	0.0584	0.0625	0.0629
	-0.8454	-0.0682	0.3528	0.6886	1.1072	1.5646	1.8653	2.0708	2.1760
put 90	-0.0170	0.0184	0.0235	0.0405	0.0469	0.0551	0.0621	0.0633	0.0576
	-0.3333	0.3984	0.5390	1.0291	1.2916	1.6247	1.9194	2.0248	1.9454

Table 7: **Time-Series Abnormal Returns for S&P 500 Option Portfolios**

This table reports abnormal returns and t-statistics for option portfolios. Based on two-pass Fama-Mecbeth regression. The regression is the first step, time-series regression using market excess return and shock of CBOE skewness index as factors. The table is organized by time-to-maturity and moneyness (strike/price level). Column names contain moneyness, and row names contain option type and time-to-maturity. For each option, first line is abnormal return α, and second line is its t-statistics. Sample period ranges from Jan 1996 to Dec 2012.

	90%	92.5%	95%	97.5%	100%	102.5%	105%	107.5%	110.5%
call 30	-0.0036	-0.0037	-0.0040	-0.0046	-0.0052	-0.0061	-0.0064	-0.0056	-0.0050
	-2.9482	-2.9277	-3.0105	-3.2730	-3.2587	-3.3793	-3.0405	-2.4840	-2.0175
call 60	-0.0037	-0.0038	-0.0039	-0.0042	-0.0046	-0.0049	-0.0050	-0.0051	-0.0047
	-3.0267	-3.0243	-2.9641	-2.9948	-3.0293	-2.9824	-2.7039	-2.4761	-2.1076
call 90	-0.0035	-0.0037	-0.0038	-0.0039	-0.0041	-0.0044	-0.0042	-0.0043	-0.0042
	-2.8748	-2.9418	-2.9297	-2.7985	-2.7566	-2.6986	-2.3625	-2.2269	-2.0129
put 30	0.0091	0.0069	0.0045	0.0021	-0.0001	-0.0017	-0.0019	-0.0021	-0.0023
	3.0267	2.5800	1.8816	1.0322	-0.0394	-1.1095	-1.3603	-1.5176	-1.7526
put 60	0.0037	0.0028	0.0015	0.0003	-0.0008	-0.0016	-0.0019	-0.0022	-0.0025
	1.4271	1.2121	0.7024	0.1591	-0.4555	-1.0006	-1.3334	-1.5854	-1.8680
put 90	0.0016	0.0010	0.0002	-0.0004	-0.0013	-0.0017	-0.0019	-0.0022	-0.0024
	0.6647	0.4499	0.1178	-0.1949	-0.7613	-1.1000	-1.2934	-1.5064	-1.7780

Table 8: **Cross Section Pricing Error for S&P 500 Option Portfolios**

This table reports **E**[RealizedRET-ExpectedRET] and t-statistics. Based on two-pass Fama-Mecbeth regression. The regression is the second step, cross-section regression using market excess return and shock of CBOE skewness index as factors. The table is organized by time-to-maturity and moneyness (strike/price). Column names contain moneyness, and row names contain option type and time-to-maturity. For each option, first line is mean value of pricing error, and second line is its t-statistics. Sample period ranges from Jan 1996 to Jan 2012.

	90%	92.5%	95%	97.5%	100%	102.5%	105%	107.5%	110.5%
call 30	0.0008	0.0010	0.0010	0.0005	0.0001	-0.0018	-0.0027	-0.0012	-0.0006
	1.8535	2.3827	2.6048	1.6124	0.4542	-2.8827	-2.4135	-1.0563	-0.3733
call 60	0.0007	0.0006	0.0006	0.0005	0.0002	-0.0003	-0.0006	-0.0003	0.0003
	1.7516	1.8027	2.3042	2.1100	0.8889	-0.9971	-0.9164	-0.4096	0.2975
call 90	0.0003	0.0004	0.0003	0.0004	0.0006	0.0001	0.0002	0.0009	0.0010
	1.1553	1.4728	1.1735	1.8345	2.6922	0.4424	0.4068	1.9943	1.4438
put 30	0.0076	0.0049	0.0023	0.0004	-0.0013	-0.0020	-0.0014	-0.0005	-0.0001
	9.0847	7.9747	5.2659	1.2914	-4.8385	-6.5181	-4.9253	-1.2329	-0.3952
put 60	0.0004	0.0005	0.0000	-0.0008	-0.0018	-0.0017	-0.0014	-0.0008	-0.0007
	0.7492	1.6670	0.0706	-4.0117	-7.4388	-6.4865	-5.4125	-2.1769	-2.1024
put 90	0.0006	0.0005	0.0004	0.0005	0.0002	0.0000	0.0007	0.0011	0.0014
	1.7860	1.6284	1.4369	2.0808	0.7599	0.0893	1.4885	2.2033	2.0964

Table 9: **S&P 500 Index Excess Returns Prediction**

This table reports results of market return prediction regressions,

$$mktrf_{t+\tau} = \alpha + \beta FactorShock_t + \epsilon_t,$$

where $mktrf$ is market excess return in month $t+\tau$, τ could be $\{1, 2, 3\}$. $FactorShock$ represents shocks of several macro variables, including shock of skewness risk premium, shock of CBOE VIX index, shock of CBOE Skewness Index, shock of credit spread, shock of aggregated analyst forecast dispersion, liquidity factor excess returns, shock of Economic Policy Uncertainty Index and shock of funding liquidity. For each panel, first line presents estimation of β, second line contains Neway-West robust standard error, and third line shows adjusted R^2. Sample period ranges from Jan 2000 to Jan 2012.

	RPSHOCK	VOLSHOCK	SKEWSHOCK	CSSHOCK	DISPSHOCK	LIQ	EPUSHOCK	FLSHOCK
$\tau = 1$								
β	-0.0222***	0.176*	-0.0018**	-0.1019	0.0222	-0.0985	0.0222	0.176
	(0.0053)	(0.0687)	(0.0007)	(0.0654)	(0.0159)	(0.094)	(0.0053)	(0.0687)
R^2	0.0746	0.0298	0.0272	0.0133	0.0079	0.0076	0.0746	0.0298
$\tau = 2$								
β	0.0023	-0.0208	-0.009	0.087	-0.0199	0.1019	0.0023	-0.0208
	(0.0054)	(0.1240)	(0.0006)	(0.0593)	(0.016)	(0.1349)	(0.0054)	(0.124)
R^2	0.0008	0.0004	0.0073	0.0097	0.0064	0.0081	0.0008	0.0004
$\tau = 3$								
β	-0.0006	0.1251	-0.0002	-0.0737	0.001	0.2243	-0.0006	0.1251
	(0.0093)	(0.042)	(0.0006)	(0.0777)	(0.032)	(0.1032)	(0.0093)	(0.042)
R^2	0.0001	0.0154	0.0002	0.007	0	0.0395	0.0001	0.0154