Information Uncertainty, Volatility Term Structure and Index Option Returns

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Abstract

In this paper, we explore the relation between information uncertainty and S&P 500 index option returns. Since underlying state variable affecting economy is unobservable, investors have to obtain their own estimations based on available information. During such procedure, it is inevitable that their results are contaminated by various kinds of noise signals. Therefore, investors cannot be 100% confident about the their estimations. We model such phenomena through incorporating investors' learning behavior into an equilibrium stochastic volatility model. In the model, we introduce noise signals as a stochastic process independent with economic fundamentals. Such information uncertainty is able to generate time-varying volatility for stock returns, even when volatility of economic fundamental is constant. As a source of risk, for investors with recursive preference, it is priced and is able to explain variance premium and cross-section index option returns. In order to test the model implication, empirically, we construct several proxies for information uncertainty. Consistent with model intuition, we show that information uncertainty as a systematic risk factor is able to explain variance premium term structure and has better performance to explain cross-section index option returns than traditional symmetric risk factors such as volatility and jump.

1 Introduction

Growing evidence suggests that extraordinary average returns may be obtained by trading equity index options. Among various trading strategies, most important and widely studied are delta-hedge straddle returns and out-of-money put option returns. Coval and Shumway (2001) find that on average, at-the-money straddle get returns close to minus 3% per week. Jackwerth (2000) show high risk-adjusted profitability from selling puts.

Many researchers try to explain the risk premium embedded in these two strategies. Generally speaking, two systematic risk factors are involved, stochastic volatility and jump. For instance, Bakshi and Kapadia (2003) find that the volatility risk premium contributes significantly to higher prices for calls and puts. Broadie, Chernov, and Johannes (2009) suggest that a jump factor helps explain the option portfolio returns. However, Constantinides Jackwerth and Savov (2014) and Chambers Foy Liebner and Lu (2014) show that volatility and Merton-type jump risk premium are not adequate to explain option returns, especially for out-of-money put option returns.

We suggest that investors' learning behavior and information uncertainty could help explain index option returns. In the first part of the paper, we use one equilibrium stochastic volatility model to highlight the channel through which information uncertainty could affect option returns. Suppose true dynamics of fundamentals are unobservable, investors have no choice but to learn (estimate) them from various information source. Inevitably, investors will receive noise during estimation, such information uncertainty could generate extra variation for stock prices. If investors is concerned about such information risk, it could be a priced systematic risk factor and affects option returns. Based on such intuition, we build an continuous-time equilibrium asset pricing model. In the models, investors need to learn the unobservable fundamentals and face information uncertainty. Such information risk is able to generate excess variation in stock prices. With Epstein-Zin recursive utility, such risk is priced and contributes to option returns, since options can be used to hedge such risk.

In the second part of the paper, in order to test the model prediction, empirically, we construct three

Electronic copy available at: http://ssrn.com/abstract=2591748

proxies for information uncertainty. Based on these three measures, we use Fama-Macbeth two-pass regression on one panel of leverage-adjusted S&P 500 index returns to conduct formal statistical test on risk premium and model performance. Consistent with our model prediction, we show that risk factors related to information uncertainty has better explanation power for index option returns than traditional ones such as volatility, price jump and volatility jump.

The contribution of our paper is twofolds. On the one hand, theoretically, we build an equilibrium asset pricing model to show the channel through which the investors' learning behavior and information uncertainty could affect option returns. In literature, the widely used equilibrium option pricing model is based on research of Bollerslev, Tauchen and Zhou (2009) and its extensions. Their model assumes normal consumption growth innovations with stochastic volatility and stochastic volatility of volatility. In the model, investors have complete information about economic fundamentals and time-varying volatility of stock return is generated solely from fundamentals.

Our model is different in mechanisms. We assume investors have incomplete information about economic variables, under our framework, due to investors' learning behavior, even the underlying economy totally follows Black-Sholes constant volatility framework, derivatives are still not redundant, because time-varying volatility could be generated when investors attempt to estimate unknown variables and options could be used to hedge such risks. Similar intuition is generated in models proposed by Buraschi and Jiltsov (2006) and Bhamra and Uppal (2009). However, in their models, there should be two investors with different beliefs or risk aversions. The stochastic distribution of wealth contributes to excess return volatility.

Another advantage is that our model could be easily extended to generate two-factor stochastic volatility structure. The importance of volatility term structure on option pricing models is highlighted by Christoffersen, Jacobs, Ornthanalai and Wang (2008), Christoffersen, Heston and Jocobs (2009) and Li and Zhang (2010). Zhou and Zhu (2014) builds equilibrium model with two volatility factors under long-run risk framework. However, existing models always set the two-factor volatility structure in the first place by assumption. In contrast, our model provides a possible economic foundation for such two-factor volatility framework. In our model, one factor could be the timevaring volatility of fundamental variables, while the other volatility factor could be generated from investors' learning behavior and information uncertainty.

On the other hand, empirically, we use Fama-Mecbeth two pass regression to confirm our model prediction. Empirically, there are many research studies linking investors' behavior to cross section of stock returns. However, relatively fewer researchers focus on the relation between option returns and investors' learning behaviors in a formal statistical test. Conventional procedure in option pricing literature is to calibrate reduced-form option pricing models in an optimization algorithm to obtain best estimation of parameters, and then compare model implied volatility skew with data. Suppose the mean square error between model implied volatility skew and real one is small, we call the model a success. One disadvantage of such method lies in the fact that the estimated parameters are not time-consistent, that the reason why in practice the model is needed to be recalibrated frequently. Based on such method, it is difficult to conduct statistical tests for factor risk premium. In the paper, we fill the gap by using a panel of leverage-adjusted index option returns and Fama-Mecbeth two-pass cross section regression to compare explanation power of different risk factors, and draw conclusions from formal statistical test, similar with those for cross section stock returns.

The structure for the rest of this paper is as follows. Section 2 reviews the related literature. Section 3 builds the model. Section 4 outlines the empirical framework. Section 5 presents and discusses results. Section 6 makes conclusions and provides some possible directions for future research.

2 Literature Review

Our research is closely related to three streams of literature. One branch is about reduced-form option pricing models trying to fit the implied volatility smiles. Earlier works include Merton (1976), Heston (1993), Duan (1995), Heston and Nandi (2000), Bates (2000) and Pan (2002). Apart from volatility and crash risks, Carr, Geman, Madan, and Yor (2002), Carr and Wu (2003, 2004, 2007), Christoffersen, Heston, and Jacobs (2006) and Ornthanalai (2014) propose to use levy process with infinite jump intensity to model return jump risks. Moreover, some researchers, such as Christoffersen, Jacobs, Ornthanalai and Wang (2008) and Christoffersen, Heston and Jacobs (2009), point that two volatility factors can generate stochastic leverage effect and more flexible implied volatility term structures, hence leads to a better fit of implied volatility surface. Besides, time-varying jump intensity could also improve model performance, as suggested by Santa-Clara and Yan (2010) and Christoffersen, Jacobs and Ornthanalai (2012).

The second stream of literature is related to equilibrium risk premium and option pricing models. Pioneer literature includes Bailey and Stulz (1989), Naik and Lee (1990) and Liu, Pan and Wang (2005). In equilibrium models, fundamental assumptions about investors utility, risk factor dynamics and beliefs can be extended. Buraschi and Jiltsov (2006) first provides option pricing and volume implications for an economy with heterogeneous agents who face model uncertainty and have different beliefs on expected returns. Market incompleteness makes options non-redundant, while heterogeneity creates a link between differences in beliefs and option volumes. Based on this model, Buraschi, Trojani and Vedolin (2014) produce evidence for an equilibrium link between investors? disagreement, the market price of volatility and correlation, and the differential pricing of index and individual equity options.

Apart from investors' beliefs, utility function also plays an important role. Bates (2008) studies effect of stock market crashes assuming investors have heterogeneous attitudes towards crash risk. The less crash averse insure the more crash averse through options markets that dynamically complete the economy. The resulting equilibrium is able to explain peso problem, pricing kernel puzzle and the stochastic evolution of option prices. Bollerslev, Tauche and Zhou (2009) and Drechsler and Yaron (2011) illustrate the source of variance premium via equilibrium stochastic volatility model. Having Epstein-Zin recursive utility functions, investors care about vol-of-vol and time-varying jump risks. Similarly, Seo and Wachter (2014) highlights the essential role of stochastic disaster risk on fitting implied volatility surface, with recursive utility function. Du (2011) use habit-formation framework to incorporate time-varying risk aversion along with crash risk. The time-varying volatility and jump- risk premiums explain the observed state-dependent smirk patterns. Polkovnichenko and Zhao (2013) find that probability weighting has effect on pricing kernel dynamics extracted from index option prices. Schreindorfer (2014) build a parsimonious consumption-based asset pricing model based on a (generalized) disappointment averse investor and conditionally Gaussian fundamentals. In his model, regime-switches in endowment volatility interact with the investor's tail aversion to produce large endogenous return jumps and a realistic implied volatility smirk.

Discussions above do not involve market friction. In fact, market friction may be important for option price dynamics. Bollen and Whaley (2004) demonstrate that changes in implied volatility are correlated with signed option volume. Gârleanu, Pedersen, and Poteshman (2009) argue that option expensiveness is related to exogenous demand pressure in imperfect markets. The authors develop a theoretical model of option intermediation in which risk-averse market makers provide immediacy and are compensated for inventory risk. Investors are assumed to supply/demand options for exogenous reasons: portfolio insurance, covered call writing, agency issues, or behavioral reasons. Market makers trade a portfolio of options on a given security and are allowed to hedge in the underlying at discrete time intervals. Thus, part of the option?s risk can be hedged away. The key insight in their paper is that demand pressure for a given option increases its price by an amount proportional to the variance which it contributes to the market makers optimally hedged/diversified portfolio.

The third research area closely related to our paper is literature about investor learning behavior. Most previous studies focus on its effect on equity risk premium. Timmermann (1993) provides a simple learning model, in which average dividend growth is unknown. Investors have to get their own estimation from observed dividend process. In such case, the author shows that dividend surprise affects stock price not only through current dividends but also through the effect on expected dividend growth rate, which also changes expected future dividends. Pastor and Veronesi (2003) provide a model with market-to-book ratio as the only state variable. It is observable, yet its long term mean is unknown to investors. Pastor and Veronesi (2006) calibrate the model to value stocks at the time of Nasdaq bubble. They find a positive link between uncertainty about average dividend growth and the level of stock prices. Pastor and Veronesi (2009) study the similar idea in a more detailed model. In the model, future productivity of new technology should be learned by investors. The authors model the process of technology adaption, and use it to explain the different pattern of stock returns for new and old technology firms.

Some papers mentioned above also point out that investors learning behaviors could be an important channel to generate the excess volatility (Grossman and Shiller, 1981). Since the signals about unobservable state variables contain uncertainty (noise), the learning process introduces another source of volatility different from fundamental uncertainty. Therefore, the stock volatility should be larger than the fundamental ones. This intuitive idea is explored by other researchers in different settings. Brennan and Xia (2001) find that non-observability of the expected dividend growth rate introduces an element of learning which increases the volatility of stock price. Bansal and Shaliastovich (2011) show that since information acquiring process is costly, the optimal decision to incur a cost and learn the true economic state can generate sudden change in stock prices, that is, jumps. Benzoni, Collin-Dufresne and Goldstein (2011) build a general equilibrium framework in which expected endowment growth and economic uncertainty are subject to rare jumps. The arrival of a jump triggers investors' learning about the likelihood of future jumps, which produces a permanent shift in option prices and implied volatility smiles. David (2008) shows that investors' learning about the state of future real economic fundamentals from current inflation leads to macroeconomic state dependence of asset valuations and solvency ratios of firms within given rating categories. Such state dependence feature and increasing price of risk generate high credit spreads while maintaining average default losses at historical levels. David and Veronesi (2014) use an equilibrium model in which investors and the central bank learn about composite regimes of economic and policy variables to explain dynamics between implied volatility and interest rate. Shaliastovich (2014) links investor confidence to index option price.

3 Model Specification

In this section, one equilibrium framework about investor learning behavior and information uncertainty is presented. The drift term of cash flow growth is stochastic and unobservable. Investors need to learn for the true state through a signal. The signal contains information about the drift term along with noise, which represents information quality and affects investors learning accuracy. The time-varying learning error will introduce extra variation into return dynamics. For investors with

Epstein-Zin utility function, such risk is priced, therefore, could affect equilibrium option returns.

3.1 General Settings

The basic settings of the model is similar with those in Bansal and Yaron (2004) and Bollerslev, Tauchen and Zhou (2009). In order to sharpen our understandings about the role of information risk, the stochastic volatility channel is shut down. We assume the diffusion term σ_D for dividend growth process D_t is constant, and the drift term μ_t follows a mean-reverting process.

$$\frac{dD_t}{D_t} = \mu_t dt + \sigma_D dW_{D,t} \tag{1}$$

$$d\mu_t = -\alpha \mu_t dt + \sigma_\mu dW_{\mu,t}. \tag{2}$$

Based on the structure above, investor learning behavior is added into the model. It is assumed that investors can not observe μ_t directly, instead, investors can only observe one signal S_t about μ_t, with dynamics given as follows

$$dS_t = \mu_t dt + \sqrt{V_t} dW_{S,t}, \tag{3}$$

where V_t represents the information quality, following the mean-reverting process

$$dV_t = \kappa(\theta - V_t)dt + \sigma_V \sqrt{V_t} dW_{V,t}. \tag{4}$$

The structure for S_t is the standard 'information plus noise' framework. Note that by this setting, we make an abstract assumption about investors' learning behavior. In practice, investors could learn μ_t from the dividend growth data time series D_t along with other information S_t, for instance, analyst issued reports. In our model, we assume all the information is summarized and contained in the signal S_t, including D_t. Such simplification could make model derivation concise without affecting our main intuition.

Under such framework, investors learning process is a standard filtering problem. Using Theorem

10.1 in Lipster and Shiryaev (2001), the filtered dynamics for μ_t is given by

$$d\hat{\mu}_t = -\alpha\hat{\mu}_t dt + \frac{\omega_t}{V_t}(dS_t - \hat{\mu}_t dt) \tag{5}$$

$$\frac{d\omega_t}{dt} = -2\alpha\omega_t - \frac{\omega_t^2}{V_t} + \sigma_\mu^2. \tag{6}$$

where $\hat{\mu}_t = \mathbf{E}[\mu_t|\mathcal{F}_t]$, conditional mean of latent variable at time t. $\omega_t = \mathbf{E}[(\mu_t - \hat{\mu}_t)^2|\mathcal{F}_t]$, filtering error at t. We analyze the model under its steady state. When the filtering system achieves its steady state, that is, $d\omega_t/dt = 0$, ω_t satisfies equation

$$\omega_t^2 + 2\alpha V_t\omega_t - \sigma_\mu^2 V_t = 0. \tag{7}$$

Solving the equation, ω_t is given by

$$\omega_t = -\alpha V_t + \sqrt{\alpha^2 V_t^2 + \sigma_\mu^2 V_t}. \tag{8}$$

Let $\hat{\sigma}_{\mu,t} = \omega_t/S_t$. We use a constant filtering weight in the rest of analysis, based on Equation (8) and long-run mean of V_t and S_t, we get $\hat{\sigma}_\mu$

$$\hat{\sigma}_\mu = -\alpha + \sqrt{\alpha^2 + \frac{\sigma_\mu^2}{\theta}}. \tag{9}$$

Under investors information set, the dynamics of the total system is as follows

$$\frac{dD_t}{D_t} = \hat{\mu}_t dt + \sigma_D(dW_{D,t} + \frac{(\mu_t - \hat{\mu}_t)dt}{\sigma_D}) \tag{10}$$

$$d\hat{\mu}_t = -\alpha\hat{\mu}_t dt + \hat{\sigma}_\mu\sqrt{V_t}(dW_{S,t} + \frac{(\mu_t - \hat{\mu}_t)dt}{\sqrt{V_t}}) \tag{11}$$

$$dV_t = \kappa(\theta - V_t)dt + \sigma_V\sqrt{V_t}dW_{V,t} \tag{12}$$

Equivalently,

$$\frac{dD_t}{D_t} = \hat{\mu}_t dt + \sigma_D dW_{D,t}^* \tag{13}$$

$$d\hat{\mu}_t = -\alpha\hat{\mu}_t dt + \hat{\sigma}_\mu\sqrt{V_t}dW_{S,t}^* \tag{14}$$

$$dV_t = \kappa(\theta - V_t)dt + \sigma_V\sqrt{V_t}dW_{V,t}^*, \tag{15}$$

where $dW^*_{D,t}$, $dW^*_{S,t}$, $dW^*_{V,t}$ are Brownian motions under investors information set. Investor learning brings one new component into the model, which is information uncertainty. To make notation simple, in the rest of the paper, we drop the $*$ and $\hat{\cdot}$, use the following equations

$$\frac{dD_t}{D_t} = \mu_t dt + \sigma_D dW_{D,t} \tag{16}$$

$$d\mu_t = -\alpha\mu_t dt + \sigma_\mu \sqrt{V_t} dW_{S,t} \tag{17}$$

$$dV_t = \kappa(\theta - V_t)dt + \sigma_V \sqrt{V_t} dW_{V,t}, \tag{18}$$

3.2 Risk-Free Rate and Market Price of Risks

To solve the equilibrium, following Bansal and Yaron (2004), we use the continuous time version of Epstein-Zin utility function. Investors consume dividend ($C_t = D_t$), and generate utility from consumption. Based on Duffie and Epstein (1992), the definition is given by

$$J_t = \mathbf{E}_t[\int_t^T f(C_s, J_s)ds]. \tag{19}$$

Thus the representative investor's objective is to choose consumption to optimize the value function J_t

$$J_t = \max_{\{C_s\}} \mathbf{E}_t[\int_t^T f(C_s, J_s)ds], \tag{20}$$

where $f(C, J)$ is a normalized aggregator related to current consumption C_t and continuation value function J_t, and is given by

$$f(C_t, J_t) = \frac{\beta}{1-\frac{1}{\psi}}(1-\gamma)J[(\frac{C}{((1-\gamma)J)^{\frac{1}{1-\gamma}}})^{1-\frac{1}{\psi}} - 1], \tag{21}$$

where β is the rate of time preference, $\gamma > 0$ is the relative risk aversion, and $\psi > 0$ the intertemporal elasticity of substitution (IES). If $\phi = 1/\gamma$, as illustrated by Duffie and Epstein (1992), we obtain the standard additive expected utility of constant relative risk aversion.

The value function $J = J(W_t, \mu_t, V_t)$ is a function of wealth W_t and two state variables μ_t and V_t. H-J-B equation for J is

$$\max_{\{C\}}\{f(C, J) + \mathcal{A}^c J\} = 0, \tag{22}$$

where $\mathcal{A}^c$ is the infinitesimal generator associate with vector process $\{C_t, \mu_t, V_t\}$. Conjecturing a solution for J of the following form,

$$J(W_t, \mu_t, V_t) = \exp(A_0 + A_1\mu_t + A_2V_t)\frac{W_t^{1-\gamma}}{1-\gamma}. \tag{23}$$

Using standard log-linear approximation, which Campbell (1993) develops in discrete-time, and Chacko and Viceira (2005) use first in continuous-time. Specifically, let g_1 be the long-term mean of the consumption-wealth ratio,

$$g_1 = \exp(E[c_t - \omega_t]), \tag{24}$$

where the lowercase variables are the log variables. With the standard log-linear approximation,

$$\frac{C_t}{W_t} = \exp(c_t - \omega_t) \approx g_1 - g_1 \log g_1 + g_1 \log(\frac{C_t}{W_t}). \tag{25}$$

With the log-linear approximation, the first order condition

$$f_C = J_W \tag{26}$$

leads to the consumption-wealth ratio as

$$\frac{C_t}{W_t} = \beta^{\psi} \exp\{(A_0 + A_1\mu_t + A_2V_t)\frac{1-\psi}{1-\gamma}\}. \tag{27}$$

Therefore, J can be written as

$$J(C_t, \mu_t, V_t) = \beta^{-\psi(1-\gamma)} \exp[\psi(A_0 + A_1\mu_t + A_2V_t)]\frac{C_t^{1-\gamma}}{1-\gamma}. \tag{28}$$

Moreover,

$$f \approx \theta J[g_1\frac{1-\psi}{1-\gamma}(A_0 + A_1\mu_t + A_2V_t + A_3S_t) + \xi], \tag{29}$$

where $\theta = (1-\gamma)/(1-1/\psi)$ and $\xi = g_1 - g_1 \log g_1 + g_1\psi \log \beta - \beta$. Substitute Equation (138) and (139) into H-J-B equation, we can get

$$A_0 = \frac{\psi}{g_1}[\theta\xi + \frac{1}{2}\gamma(1-\gamma)\sigma_D^2 + \kappa\theta\psi A_2]. \tag{30}$$

$$A_1 = \frac{1-\gamma}{\psi g_1 + \alpha}. \tag{31}$$

$$A_2 = \frac{(g_1+\kappa) - \sqrt{(g_1+\kappa)^2 - \psi^2 A_1^2 \sigma_\mu^2 \sigma_V^2}}{\sigma_V^2 \psi}. \tag{32}$$

In order to derive the risk-free rate and market prices of risks. The pricing kernel is given by

$$\pi_t = \exp[\int_0^t f_J(C_s, J_s)ds] f_C(C_t, J_t). \tag{33}$$

$$\frac{d\pi}{\pi} = f_J dt + \frac{df_C}{f_C}. \tag{34}$$

Based on the structure of $f(C, J)$ and value function J, we get the following relation

$$f_J = \xi_1 - g_1(A_1\mu_t + A_2V_t)\frac{1-\gamma\psi}{1-\gamma} \tag{35}$$

$$f_C = \beta^{\psi\gamma} \exp[(\xi_1 + A_1\mu_t + A_2V_t)\frac{1-\gamma\psi}{1-\gamma}] C_t^{-\gamma}, \tag{36}$$

where $\xi_1 = (\theta - 1)\xi - \beta - g_1\frac{1-\gamma\psi}{1-\gamma}A_0$. Applying Ito's lemma to pricing kernel dynamics based on dynamics of f_J and f_C, we have

$$\frac{d\pi_t}{\pi_t} = -r_f dt - \lambda_1 dW_{D,t} - \lambda_2 dW_{S,t} - \lambda_3 dW_{V,t}, \tag{37}$$

where the risk-free interest rate r_t is a linear function of state variables (μ_t, V_t), with constant coefficients

$$r_f = r_0 + r_1\mu_t + r_2V_t, \tag{38}$$

where

$$r_0 = -(\frac{1-\gamma\psi}{1-\gamma})A_2\kappa\theta - \frac{1}{2}\gamma(1+\gamma)\sigma_D^2 - \xi_1. \tag{39}$$

$$r_1 = \gamma + (\alpha + g_1)A_1(\frac{1-\gamma\psi}{1-\gamma}). \tag{40}$$

$$r_2 = (g_1 + \kappa)(\frac{1-\gamma\psi}{1-\gamma})A_2 - \frac{1}{2}(\frac{1-\gamma\psi}{1-\gamma})^2(A_1^2\sigma_\mu^2 + \sigma_V^2A_2^2). \tag{41}$$

Moreover, the market price of risks are

$$\lambda_1 = \gamma. \tag{42}$$

$$\lambda_2 = -(\frac{1-\gamma\psi}{1-\gamma})A_1\sigma_\mu\sqrt{V_t}. \tag{43}$$

$$\lambda_3 = -(\frac{1-\gamma\psi}{1-\gamma})A_2\sigma_V\sqrt{V_t}. \tag{44}$$

3.3 Price Dynamics

The price of the asset having claim on the dividend D_t satisfies the fundamental asset pricing equation,

$$\mathbf{E}_t(\frac{dP_t}{P_t}) + \frac{D_t}{P_t}dt = r_f dt - \mathbf{E}_t[\frac{d\pi_t}{\pi_t}\frac{dP_t}{P_t}]. \tag{45}$$

In the model, the dividend-price ratio is equivalent to the consumption-wealth ratio,

$$\frac{D_t}{P_t} = \frac{C_t}{W_t} = \beta^{\psi}\exp\{(A_0 + A_1\mu_t + A_2 V_t)\frac{1-\psi}{1-\gamma}\}. \tag{46}$$

Applying Ito's lamma to Equation (46), we get the dynamics for stock price

$$\frac{dP_t}{P_t} = \mathbf{E}_t(dP_t/P_t)dt + \sigma_D dW_{D,t} - \frac{1-\phi}{1-\gamma}A_1\sigma_\mu\sqrt{V_t}dW_{S,t} - \frac{1-\phi}{1-\gamma}A_2\sigma_V\sqrt{V_t}dW_{V,t}. \tag{47}$$

Since $\{W_{D,t}, W_{S,t}, W_{V,t}\}$ are independent with each other, the dynamics for p_t can be further written as

$$\frac{dP_t}{P_t} = \mu_{p,t}(\mu_t)dt + \sigma_{p,t}(V_t)dW_{p,t} \tag{48}$$

$$d\mu_t = -\alpha\mu_t dt + \sigma_\mu\sqrt{V_t}dW_{S,t} \tag{49}$$

$$dV_t = \kappa(\theta - V_t)dt + \sigma_V\sqrt{V_t}dW_{V,t}, \tag{50}$$

where $\sigma_{p,t}^2(V_t) = \sigma_D^2 + (1-\psi)^2/(1-\gamma)^2(A_1^2\sigma_\mu^2 + A_2^2\sigma_V^2)V_t$.

Correlation between $dW_{V,t}$ and $dW_{p,t}$ is

$$\rho_V = -\frac{\sqrt{(1-\psi)^2/(1-\gamma)^2 A_2^2\sigma_V^2 V_t}}{\sqrt{\sigma_D^2 + (1-\psi)^2/(1-\gamma)^2(A_1^2\sigma_\mu^2 + A_2^2\sigma_V^2)V_t}}. \tag{51}$$

Similarly, correlation between $dW_{S,t}^*$ and $dW_{p,t}^*$ is

$$\rho_\mu = \frac{\sqrt{(1-\psi)^2/(1-\gamma)^2 A_1^2\sigma_\mu^2 V_t}}{\sqrt{\sigma_D^2 + (1-\psi)^2/(1-\gamma)^2(A_1^2\sigma_\mu^2 + A_2^2\sigma_V^2)V_t}}. \tag{52}$$

The market price of risks for μ_t and V_t are λ_2 and λ_3 of Equation (148). Following Heston (1993) and Duffie, Pan and Singleton (2000), we can perform measure transform from P to Q using pricing kernel.

$$dW_{D,t} = d\tilde{W}_{D,t} - \lambda_1 dt; \tag{53}$$

$$dW_{\mu,t} = d\tilde{W}_{\mu,t} - \lambda_2 dt; \tag{54}$$

$$dW_{V,t} = d\tilde{W}_{V,t} - \lambda_3 dt; \tag{55}$$

where $d\tilde{W}_{\cdot,t}$ are Brownian motions under risk neutral measure Q. Therefore, the risk-neutral dynamics for stock price p_t is

$$\frac{dP_t}{P_t} = -r_f dt + \sigma_{p,t}(V_t) d\tilde{W}_{p,t} \tag{56}$$

$$d\mu_t = -(\alpha\mu_t + \lambda_2 \sigma_\mu \sqrt{V_t}) dt + \sigma_\mu \sqrt{V_t} d\tilde{W}_{S,t} \tag{57}$$

$$dV_t = \tilde{\kappa}(\tilde{\theta} - V_t) dt + \sigma_V \sqrt{V_t} d\tilde{W}_{V,t}, \tag{58}$$

where $\tilde{\kappa} = \kappa - \sigma_V^2 A_2 (1 - \gamma\psi)/(1-\gamma)$, and $\tilde{\theta} = \kappa\theta/\tilde{\kappa}$.

In the model, due to investors' learning activity and noise of information, the stock price p_t has a stochastic mean and stochastic volatility. The stochastic mean is generated from the mean-reverting dividend growth rate μ_t. The stochastic volatility is generated from the time varying information quality. The instantaneous variance $\sigma_{p,t}^2$ is given by

$$\sigma_{p,t}^2(V_t) = \sigma_D^2 + (1-\psi)^2/(1-\gamma)^2 (A_1^2 \sigma_\mu^2 + A_2^2 \sigma_V^2) V_t, \tag{59}$$

which is an linear and increasing function of V_t. Because the dynamics for V_t is widely used square root process, the VIX_τ^2 is

$$VIX_\tau^2 = \sigma_D^2 + (1-\psi)^2/(1-\gamma)^2 (A_1^2 \sigma_\mu^2 + A_2^2 \sigma_V^2)(\underline{A} + \underline{B} V_t), \tag{60}$$

where $\underline{A}$ and $\underline{B}$ are given by

$$\underline{A} = \tilde{\theta}[1 - \frac{1 - \exp(-\tilde{\kappa}\tau)}{\tilde{\kappa}\tau}], \quad \underline{B} = \frac{1 - \exp(-\tilde{\kappa}\tau)}{\tilde{\kappa}\tau}. \tag{61}$$

In a more general model, with stochastic diffusion parameter σ_D for dividend growth process, instantaneous variance $\sigma_{p,t}^2$ could have a richer dynamics, with two volatility factors, consistent with empirical findings provided by Christoffersen, Heston and Jacobs (2009) and Li and Zhang (2010). Detailed derivation for this extended model is provided in appendix.

3.4 Empirical Implication of the Model

Our model highlights one intuition: due to investors' learning activity and noise of information, the stock price p_t has a stochastic mean and stochastic volatility. The stochastic mean is generated from the mean-reverting dividend growth rate μ_t. The stochastic volatility is generated from the time varying information quality. The realized variance $\sigma_{p,t}^2$ is given by

$$\sigma_{p,t}^2(V_t) = \sigma_D^2 + (1-\psi)^2/(1-\gamma)^2(A_1^2\sigma_\mu^2 + A_2^2\sigma_V^2)V_t, \tag{62}$$

which is an linear and increasing function of V_t. Therefore, when V_t is larger, the realized variance should be larger. Similarly, the implied variance is given by

$$VIX_\tau^2 = \sigma_D^2 + (1-\psi)^2/(1-\gamma)^2(A_1^2\sigma_\mu^2 + A_2^2\sigma_V^2)(\underline{A} + \underline{B}V_t), \tag{63}$$

which is also an linear and increasing function of V_t. Hence, when V_t is larger, the implied variance should be larger. Besides, variance premium, the gap between implied and realized volatility is also linear function of V_t.

What the proxy for V_t? From the definition of V_t,

$$dS_t = \mu_t dt + \sqrt{V_t}dW_{S,t}, \tag{64}$$

and investors' learning error

$$\omega_t = -\alpha V_t + \sqrt{\alpha^2 V_t^2 + \sigma_\mu^2 V_t}. \tag{65}$$

We can see that worse information quality, that is, larger V_t will generate larger investors learning error.

Therefore, based on above analysis, the main prediction of our model is as follows

Hypothesis 3.1. *Information uncertainty leads to excess volatility of stock returns. It is a priced risk factor, and the risk premium should contribute to option returns.*

4 Empirical Studies

In this section, the definitions for variables are given first. Then the empirical methodology and results are discussed.

4.1 Data Description

For S&P 500 index option returns, we use the same data in Constantinides, Jackwerth and Savov (2013), downloaded from the authors' website. The dataset is a panel of leverage-adjusted (that is, with a targeted market beta of one) monthly returns of 54 option portfolios split across type (27 call and 27 put portfolios), each with targeted time to maturity (30, 60, or 90 days), and targeted moneyness, that is, strike/price ratio (0.90, 0.925, 0.95, 0.975, 1.00, 1.025, 1.05, 1.075, or 1.10). Each portfolio's weights are adjusted daily to maintain its targeted beta, maturity, and moneyness. The major advantage of this construction is to lower the variance and skewness of the monthly portfolio returns and render the returns close to normal (about as close to normal as the index return), thereby making applicable the standard linear factor pricing methodology.

Table 1 and 2 show the summary statistics for the call and put option returns. Consistent with the arguments in Constantinides, Jackwerth and Savov (2013), the construction method reduces skewness and kurtosis of option returns, making its distribution close to normal.

For the call option returns, we see the same patterns documented by Coval and Shumway (2001). Out-of-money call options have negative returns, and the magnitudes increase as the strike price increases, which means out-of-money call options are over priced. Such phenomena is studied by some researchers, such as Boyer and Vorkink (2014), based on prospect theory proposed by Barbaris, Huang and Santos (2001), and Frazzini and Pedersen (2012), based on investors' leverage preference. As time-to-maturity increases, the out-of-money call option returns become less negative.

When it comes to put option returns, selling short-term out-of-money option earns large positive returns. The returns for 30-day out-of-money put options is more than doubled compared to those with 90-day time-to-maturity. The overprice of out-of-money put option is always the important research topic for option pricing. The leverage effect (negative relation between return and volatility shocks) and Merton type jumps are usually used to explain it. However, although leverage effect can generate skewness for underlying returns over long-horizon, it is unable to affect the volatility skew in short time-to-maturity. Merton jumps could help, but as is shown later, such jump is inadequate.

For proxies of information risk, we consider three measures: analyst earning forecast dispersion (EPSIR), index option trading volume dispersion across strikes (OPIR) and economic policy uncertainty index. EPSIR is calculated using bottom-up method described in Yu (2011). Analyst forecasts for individual stock earnings-per-share (EPS) long-term growth rate (LTG) is aggregated as market-wide measure of information uncertainty. The analyst forecast data is obtained from the unadjusted I/B/E/S summary database. Each month t, standard deviation of firm i analyst forecasts is denoted as $\sigma_{i,t}$. Monthly stock closing prices are shares outstanding are obtained from CRSP. Only common stocks with CRSP item SHRCD=10 or 11 are included. Let $MKTCAP_{i,t}$ denote market value of stock i at the end of month t. The value-weighted average of standard deviation is given by

$$EPSIR_t = \sum_i MKTCAP_{i,t} \cdot \sigma_{i,t} / \sum_i MKTCAP_{i,t}.$$

Another measure for information risk is extracted from S&P 500 index option trading volumes, suggested by Andreou, Kagkadis, Maio and Philip (2014). The authors consider range of strike prices with traded index option as the set of possible future index levels. Investors, based on their information at hand, will form expectation about future index dynamics and trade options with different strikes. Thus, the dispersion of trading volume represents ambiguity about macroeconomic fundamentals. The variable definition is as follows

$$OPIR_t = \sum_{j=1}^{K} \omega_j |X_j - \sum_{j=1}^{K} \omega_j X_j|,$$

where ω_j is the proportion of total trading volume attributed to strike price $X_j, j = 1, \ldots, K$.

The third and last measure for information risk is economic uncertainty index (EPUIR) constructed by Baker, Bloom and Davis (2013). The index is built on three components: the frequency of newspaper references to economic policy uncertainty, the number of federal tax code provisions set to expire, and the extent of forecaster disagreement over future inflation and government purchases.

Figure 1 plots dynamics for these three information risk proxies. Generally speaking, our three information risk proxies have countercyclical dynamics. EPSIR and OPIR share similar features, the correlation between these two is 0.68. Dynamics for EPUIR is slightly different from the other two. Rougly speaking, during the whole sample period, EPSIR and OPIR experience high value during dot-com bubble, 2008 financial crisis and 2010 debt crisis. While EPUIR only spikes during 2008 financial crisis and afterward. The correlations of EPUIR and the other two proxies are around 0.56. It is interesting that EPSIR calculated from analysts' forecast, whose focusing is stock market, is closely related to investors expectation in option market.

Following Constantinides, Jackwerth and Savov (2013), we construct crisis-related factors, Jump, Volatility Jump, Volatility, and Liquidity. Jump is defined as the sum of all daily returns of the S&P 500 that are lower than 4% within each month, zero if there are none. Volatility is defined as the difference between end-month CBOE VIX index value and begin-month CBOE VIX index value of each month. Volatility Jump is defined as the sum of all daily increases in the CBOE VIX index that are greater than 4%, zero otherwise. Liquidity is defined as the innovation of the market-wide liquidity factor proposed by Pastor and Stambaugh (2003) and provided by the Wharton Research Data Services. We also consider two factors related to overall credit market conditions: credit spread, funding liquidity. Credit spread is defined as the credit spread between Moody Baa and Aaa corporate bond. Funding liquidity is the proxy used in paper of Fontaine and Garcia (2012), calculated from cross-section of Treasury securities and obtained from authors' website.

The basic summary statistics for these systematic risk factors are summarized in Table 3. Table 4 reports correlations among various systematic risk factors. Consistent with Figure 1, the correlation data shows that our proxies of information risk have countercyclical dynamics. EPUIR and OPIR are positively correlated to VIX and VIX jumps, and negatively correlated to price jumps. While EPSIR has weak correlation with VIX, however, it is positive correlated with credit spread and funding liquidity.

One prediction of our basic model is that information uncertainty contributes to volatility dynamics. The extended model indicates that, combined with fundamental volatility, information uncertainty is able to generate volatility term structure. Therefore, we construct VIX term structure using S&P 500 index options downloaded from OptionMetrics. The VIX is an estimate of model-free implied volatility measure motivated by Breeden and Litzenberger (1978). If options are available for every strike price, the VIX is calculated as

$$VIX_{T,t}^2 = \frac{2e^{rT}}{T}\{\int_0^{F_t} \frac{1}{K^2} put_t(K;t+T)dK + \int_{F_t}^{\infty} \frac{1}{K^2} call_t(K;t+T)dK\},$$

where F_t is the time t forward price of the S&P 500 at time $t+T$. $put_t(K;t+T)$ and $call_t(K;t+T)$ are the prices at time t of puts and calls expiring at time $t+T$ with strike price K.

The standard approach to estimating the VIX is described by CBOE. Following the whitepaper provided by CBOE[1], We discretize the integral at the available strike prices and truncates it at the smallest and largest strikes $\underline{K}$ and $\overline{K}$,

$$V\hat{I}X_{T,t}^2 = \frac{2e^{rT}}{T} \sum_{K_i=\underline{K}}^{\overline{K}} \frac{1}{K_i^2} option_t(K_i;t+T)\Delta K_i,$$

where $option_t(K_i;t+T)$ is the price of the out-of-money option for strike K_i at time t with expiration date $t+T$. Based on closing quote for S&P 500 index options, $VIX_{T,t}$ with T equals to 1, 2, 3, 6, 9 and 12 months are calculated at the last trading day of each month, from Jan, 1996 to December 2012.

Figure 5 plots dynamics of VIX term structure. Table 5 provides basic summary statistics for VIX^2 with various maturities. Based on the figure and statistics, we know that in both medians and means, longer-term VIX are higher than short-term VIX, indicating that on average, term structure of VIX is upwards sloping. There is substantial variability in the VIX at all horizons. However, short-term VIX are more volatile than long-term VIX.

Given the strong correlations between VIXs at different horizons, I conduct principle analysis to decompose the term structure into six orthogonal factors, whose dynamics are shown in Figure 6.

[1]Downloadable from www.cboe.com/micro/vix/vixwhite.pdf.

The first component is 'level', which follows the familiar pattern of standard VIX index, being low and stable during normal period, while spiking during market downturns. The second component is 'slope'. In normal time, slope is high, indicating an upward sloping term structure. While during crisis, slope turns down and even become negative, indicating a downward sloping term structure.

Our model also predicts that information risk could contribute to variance risk premium. Ealier paper, such as Bollerslev, Tauchen and Zhou (2009), Carr and Wu (2009), Todorov (2010), Bekaert and Hoerova (2014), use volatility spread as a measure of variance premium. One problem of using such measure is that we cannot examine premia associated with different maturities. Therefore, following Dew-Becker et al. (2014), Eraker and Wu (2014) and Johnson (2015), similar with VIX^2, we construct variance risk premium as S&P 500 straddle returns with maturities 1, 2, 3, 6, 9 and 12 months, using options with these maturities. The daily returns of S&P 500 straddle returns are given by

$$SD_{T,t+1} = \frac{straddle_{t+1}(t+S_1) + \ straddle_{t+1}(t+S_2)}{straddle_t(t+S_1) + \ straddle_t(t+S_2)},$$

where $straddle_t t + S$ is the day t price of a straddle with maturity $t+S$, and S_1 and S_2 are the two closest time-to-maturities to the target T. By this method, we construct constant maturity straddle returns from the mixture of two maturity dates nearest to the target date. Then we aggregate daily returns to get monthly ones.

Figure 7 and Table 5 provide information about these straddle returns. On average, consistent with literature, straddles earn negative returns, and they are subject to extreme volatilities. Interestingly, longer-maturity straddles tend to have less negative returns than shorter-maturity assets.

4.2 Empirical Strategy

The model highlights that information risk is able to affect volatility and its term structure. To investigate this implication, we first regress VIX^2 term structure on our proxies for information risk,

$$VIX^2_{T,t+1} = \alpha + \beta\{EPSIR, EPUIR, OPIR\}_t + \epsilon_t, \tag{66}$$

where $VIX^2_{T,t+1}$ is square of VIX with maturity $T = \{1, 2, 3, 6, 9, 12\}$ at the end of month $t+1$. $\{EPSIR, EPUIR, OPIR\}$ are our measure of information risk at month t.

Since VIX^2_T are strongly correlated, the term structure is further decomposed into six orthogonal components: $PC1, PC2, PC3, PC4, PC5, PC6$. These factors are regressed on proxies of information risk

$$PC_{i,t+1} = \alpha + \beta\{EPSIR, EPUIR, OPIR\}_t + \epsilon_t, \quad i = \{1, 2, 3, 4, 5, 6\}. \tag{67}$$

Besides volatility dynamics, the model indicates that for investors with recursive utility function, information risk is priced. Variance premium is a function of information risk. We first construct index straddle returns with different maturities to represent variance premium. Then we explore the model prediction by regressing straddle returns on our measure of information risk,

$$SDRET^2_{T,t+1} = \alpha + \beta\{EPSIR, EPUIR, OPIR\}_t + \epsilon_t, \tag{68}$$

where $SDRET^2_{T,t+1}$ is index straddle returns with maturity $T = \{1, 2, 3, 6, 9, 12\}$. For all these linear regression, robust standard errors of coefficients are calculated by Newey-West adjustment.

Since our information risk is priced in the model, and our empirical measures have countercyclical dynamics. Information risk should be able to explain option returns. We explore this implication in a linear pricing kernel framework via Fama-Macbeth regression, using 54 S&P 500 index option portfolio returns as testing assets. For each asset $i = 1, \ldots, N$, we estimate the risk exposure from the time-series regression

$$R^e_{i,t} = \alpha_i + \beta'_{i,f} f_t + \epsilon_{i,t}, i = 1, \ldots, N, t = 1, \ldots, T, \tag{69}$$

where f represents a vector of risk factors. To estimate the cross-sectional price of risk associated with the factor f, we run a cross-sectional regression of time-series average excess returns, $\mathbf{E}[R^e_t]$, on risk factor exposure,

$$\mathbf{E}[R^e_{i,t}] = \mu_{R,i} = \beta'\lambda_f + \xi_i, i = 1, \ldots, N. \tag{70}$$

Here we impose the restriction that the intercept, corresponding to the excess return on a zero-beta asset, is equal to zero. This restriction increases the power of our tests and ensures that we do not obtain spurious results whereby small differences in factor loadings across correlated portfolios, together with a large premium, appear to fit the cross-section of option returns.

This cross-section regression yields estimates of the cross-sectional prices of risk λ. A good pricing model features an statistically significant and stable prices of risk λ across different cross-sections of test assets, and individual pricing errors ξ_i that are close to zero. We measure such pricing error in three methods. First of all, we calculate the cross-sectional adjusted R^2, which focuses on whether the sum of squared errors is relatively small $(1-\sigma^2_\xi/\sigma^2_{\mu_R})$. Secondly, we get the mean absolute pricing error MAPE $(\sum|\xi|/N)$, which give us an overview of magnitudes about pricing errors. Moreover, we use a χ^2 statistics to test whether the pricing errors are jointly zero-measured by a weighted sum of squared pricing errors $(\xi' cov(\xi)^{-1}\xi \sim \chi^2_{N-K})$, where N is number of assets, K is number of factors and $cov(\xi)$ includes the estimation error in βs. To correct the standard errors for estimation of betas, we report t-statistics of Shanken (1992) in addition to the t-statistics of Fama and MacBeth (1973). The Shanken (1992) adjustment is calculated as follows

$$\sigma^2(\lambda) = \frac{1}{T}[(\beta'\beta)^{-1}\beta'\Sigma_\epsilon\beta(\beta'\beta)^{-1}(1+\lambda'\Sigma_f^{-1}\lambda)+\Sigma_f], \tag{71}$$

$$cov(\xi) = \frac{1}{T}(I_N - \beta(\beta'\beta)^{-1}\beta')\Sigma_\epsilon(I_N - \beta(\beta'\beta)^{-1}\beta') \times (1+\lambda'\Sigma_f^{-1}\lambda), \tag{72}$$

where Σ_f is variance-covariance matrix of the factors. Σ_ϵ is the variance-covariance matrix of the time-series errors, $\epsilon_{i,t}$.

5 Discussion of Results

For VIX^2 with different maturities, Table 6 reports estimates of βs, which represent the sensitivity of risk-neutral variance to our information risk measures. βs estimated are all positive, meaning that when information uncertainty increase, volatility tend to be larger. Statistical test shows that our information uncertainty measures affect long-term variance more than short-term one. βs are statistically significant for long-term variance, typically with maturity larger than half a year. R^2 monotonically increase across maturities. Such pattern is consistent with our intuition. Investors use their information to learn the underlying state of economic fundamentals, which represent stochastic investment opportunity. Therefore, investors' uncertainty should have long-term effect on asset returns.

Since VIX^2 are strongly correlated, we conduct principle analysis to decompose the term structure into six orthogonal components. Their sensitivities to information risk are in Table 7. The biggest takeaways for Table 7 are as follows. First of all, information risk mainly affect first two components of term structure, namely, level and slope. R^2 of regression using level and slope as regressors are much larger than others. Secondly, information uncertainty affect slope more than level, take $EPSIR$ for instance, analyst forecast dispersion could explain 33% variation in slope while could only explain 5% variation in level. Moreover, Level and slope both have positive loadings on information uncertainty measure. Such pattern indicates that when investors are more uncertain about their information at hand, current short-term volatility tend to increase, and long-term future volatility are expected to increase too.

Table 8 presents sensitivities of variance premium with different maturities to our information uncertainty measures. On the one hand, all βs are negative, consistent with literature and our model. Because volatility spikes during market downturns, it serves as a hedging asset. Therefore, straddles which have positive loading on volatility should earn negative returns on average. Our model predicts that information uncertainty is able to increase return volatility and variance premium. Therefore, larger investors' uncertainty about their information should lead to more negative straddle returns. On the other hand, Table 6 and 7 make it clear that information risk affect long-term volatility more. Therefore, we expect similar results for variance premium. The R^2 of our prediction regressions increase and coefficients β become more statistically significant across maturities.

Above linear time-series regression establish the facts that information uncertainty is able to affect variance and variance premium, especially long-term ones. The next step is to examine whether such risk itself has a statistically significant risk premium. We answer this question using Fama-Mecbeth regression with index option portfolio returns as testing assets.

In order to determine the number of factors used in our cross-section regression, we first conduct a principle analysis for option portfolio returns. The principle component analysis is applied for total 54 option portfolio returns and for call and put option portfolio returns separately. Results in Table 9 indicates that there are only two factors matter for dynamics of option returns, the first two factors are able to explain 98% variation. Based on this analysis, we consider eight models, each model applies two factors: market excess return with volatility factor, market excess return with volatility jump factor, market excess return with price jump factor, market excess return with liquidity factor and market excess return with funding liquidity factor, and market excess return with our three proxies for information uncertainty risk. Due to the option portfolio formation method documented by Constantinides, Jackwerth and Savov (2013), by construction, our option returns have theoretical market excess return beta equal to one. Thus, each combination above includes market excess return as one factor.

Table 10 and Figure 8 contains our Fama-Mecbeth regression results. Starting from Figure 8(a), CAPM is unable to explain the returns across index option portfolios. Market factor risk premiums estimated for various models in row 1 Table 10 are never statistically significant. Once the second factor is added into the regression, performance improves dramatically. Figure 8 plots realized portfolio returns against model expected ones, generally speaking, the points are aligned along 45 degree line, except for out-of-money put option with time-to-maturity 30 days. We will discuss these cases later. The first five columns presents performance of models with traditional crisis related factors, and the last three columns show performance of models with our information uncertainty measures. Roughly speaking, models considering information risks have better performance compared with traditional risk factors, the J-statistics are smaller, especially when EPSIR is involved, model is not rejected. The risk premiums for our three proxies are negative and statistically significant, which is consistent with our model prediction and regression results above. Since large information uncertainty leads to large variance and high variance premium, it tend to occur during market falls, hence could serve as hedging assets and receive negative compensation.

Since model incorporate EPSIR has best performance, we examine the model in details. Table 11 records information risk factor loadings of option portfolios in time-series regression, first step of

Fama-Mecbeth regression,

$$R_{i,t}^{e} = \alpha_i + \beta_{i,f}' f_t + \epsilon_{i,t}, i = 1, \ldots, N, t = 1, \ldots, T. \tag{73}$$

For call option portfolios, the factor loadings are not statistically significant. For put option portfolio, all the factor loadings are statistically significant, indicating that our information uncertainty factor is not a useless factor. Besides, holding time-to-maturity constant, the factor loadings decrease with moneyness, out-of-money put options have larger loadings than in-the-money ones, because out-of-money put option typically is applied to hedging market downturn and spiking volatility.

Table 12 presents statistics for pricing errors in cross section regression, second step in Fama-Mecbeth regression,

$$\mathbf{E}[R_{i,t}^{e}] = \mu_{R,i} = \beta' \lambda_f + \xi_i, i = 1, \ldots, N. \tag{74}$$

We calculate the pricing error as the difference between realized returns and model implied ones and conduct t-test to see whether the pricing error for each option portfolio is statistically equal to zero. For most call option portfolios, the pricing errors are small and not statistically different from zero. However, for put option portfolios, pricing errors are statistically significant. For put options with 30-day maturity, pricing errors decrease with moneyness. Realized returns exceeds model implied ones by 70 bps. Such pattern is consistent with linear regression results for variance and variance premium term structure above. Recall that our information proxies affect long-term volatility more, while out-of-money puts with short maturity commonly are used to hedge sudden change in market price and short-term volatility spikes. Therefore, explanation power of our factor for short-term out-of-money puts could be weak.

6 Conclusion

In this study, we model effect of investors' learning behavior on return dynamics. Through an equilibrium stochastic volatility model, we propose a clear channel through which investors' uncertainty about their information at hand could generate extra return variations, and such excess uncertainty should be priced by an investor with Epstein-Zin utility function. Therefore, information uncertainty (risk) is able to affect variance, variance premium and option returns. In empirical studies, we construct three proxies for our information risk. Using these measures, along with VIX term structure, S&P 500 straddle returns and a panel of leverage-adjusted index option portfolio returns, we conduct linear regression and Fama-Macbeth two-pass regression. Consistent with empirical implications of our model, the results show that our information risk proxies are able to affect long-term part of variance and variance premium term structure. In addition, the risk factor has significant negative risk premium and has better performance in explaining cross-section index option returns.

During the linear regression studies with variance premiums, we find that variance risk premium term structure is down-ward sloping, which indicates that investors care about short-term or sudden change of volatility, while tend to ignore long-term volatility. Meanwhile, our information risk proxies have weaker power to predict short-term volatility change, which leads to poor pricing ability of our factor for short-term out-of-money put options. As a result, for future studies, we need to consider model incorporating crisis related factors, for instance, stochastic skewness.

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A Basic Model Derivation

Based on H-J-B equation, we get the following equation,

$$\mathbf{E}[f(C,J)dt + J_C dC + \frac{1}{2}J_{CC}(dC)^2 + J_\mu d\mu + \frac{1}{2}J_{\mu\mu}(d\mu)^2 + J_V dV + \frac{1}{2}J_{VV}(dV)^2] = 0. \tag{75}$$

We can get the derivatives by direct calculation, therefore

$$\mathbf{E}[J_C dC] = (1-\gamma)J\mu_t dt; \tag{76}$$

$$\mathbf{E}[J_\mu d\mu] = -\alpha\psi A_1 J\sigma_D^2 dt; \tag{77}$$

$$\mathbf{E}[J_V(dV)] = \kappa(\theta - V_t)\psi A_2 J dt; \tag{78}$$

$$\mathbf{E}[\frac{1}{2}J_{CC}(dC)^2] = \frac{1}{2}\gamma(\gamma-1)J\sigma_D^2 dt; \tag{79}$$

$$\mathbf{E}[\frac{1}{2}J_{\mu\mu}(d\mu)^2] = \frac{1}{2}(\psi A_1)^2\sigma_\mu^2 V_t J dt; \tag{80}$$

$$\mathbf{E}[\frac{1}{2}J_{VV}(dV)^2] = \frac{1}{2}(\psi A_2)^2\sigma_V^2 V_t dt; \tag{81}$$

Recall that

$$f = \theta J[g_1\frac{1-\psi}{1-\gamma}(A_0 + A_1\mu_t + A_2 V_t) + \xi] \tag{82}$$

Substitute all these equations into Equation (75), we can get

$$\begin{aligned} 0 \;=\;& \theta g_1\frac{1-\psi}{1-\gamma}A_0 + \theta\xi + \frac{1}{2}\gamma(\gamma-1)\sigma_D^2 + \kappa\theta\psi A_2 \\ &+(\theta g_1(\frac{1-\psi}{1-\gamma})A_1 + (1-\gamma) - \alpha\psi A_1)\mu_t \\ &+(\theta g_1\frac{1-\psi}{1-\gamma} - \kappa\psi A_2 + \frac{1}{2}(\psi A_1)^2\sigma_\mu^2 + \frac{1}{2}(\psi A_2)^2\sigma_V^2)V_t \end{aligned} \tag{83}$$

For the equation to hold, the constant term, and coefficients for μ_t and V_t should be zero. Hence, we can get the value for A_0, A_1 and A_2

$$A_0 = \frac{\psi}{g_1}(\theta\xi + \frac{1}{2}\gamma(\gamma-1)\sigma_D^2 + \kappa\theta\psi A_2), \tag{84}$$

$$A_1 = \frac{1-\gamma}{\alpha + \psi g_1}, \tag{85}$$

$$A_2 = \frac{(g_1+\kappa) - \sqrt{(g_1+\kappa)^2 - \psi^2 A_1^2\sigma_\mu^2\sigma_V^2}}{\sigma_V^2\psi}. \tag{86}$$

From Duffie and Epstein (1992), the price kernel π_t is

$$\pi_t = \exp(\int_t f_J(C_s J_s) ds) f_c(C_t, J_t) \tag{87}$$

Take derivative, we get the dynamics for pricing kernel, in the form of stochastic differential equation

$$\frac{d\pi_t}{\pi_t} = f_J dt + \frac{df_C}{f_C}, \tag{88}$$

with

$$f_J = \xi_1 - g_1(A_1\mu_t + A_2 V_t)\frac{1-\gamma\psi}{1-\gamma} \tag{89}$$

$$f_C = \beta^{\psi\gamma}\exp[(\xi_1 + A_1\mu_t + A_2 V_t)\frac{1-\gamma\psi}{1-\gamma}]C_t^{-\gamma}, \tag{90}$$

$$\xi_1 = (\theta-1)\xi - \beta - g_1\frac{1-\gamma\psi}{1-\gamma}A_0. \tag{91}$$

By Itô's lemma,

$$\begin{aligned} df_C &= f_{C,C}dC + \frac{1}{2}f_{C,CC}(dC)^2 + f_{C,\mu}d\mu + \frac{1}{2}f_{C,\mu\mu}(d\mu)^2 + f_{C,V}(dV) + \frac{1}{2}f_{C,VV}(dV)^2, \\ &= -\gamma(\mu_t dt + \sigma_D dW_{D,t})f_C + \frac{1}{2}\gamma(\gamma+1)\sigma_D^2 f_C dt \\ &\quad +(-\alpha\mu_t + \sigma_\mu\sqrt{V_t}dW_{\mu,t})f_C A_1(\frac{1-\gamma\psi}{1-\gamma}) + \frac{1}{2}\sigma_\mu^2 V_t A_1^2(\frac{1-\gamma\psi}{1-\gamma})^2 f_C dt \\ &\quad +(\kappa(\theta - V_t)dt + \sigma_V\sqrt{V_t}dW_{V,t})f_C A_2(\frac{1-\gamma\psi}{1-\gamma}) + \frac{1}{2}\sigma_V^2 V_t A_2^2(\frac{1-\gamma\psi}{1-\gamma})^2 f_C dt. \end{aligned} \tag{92}$$

Substitue Equation (89), (91) and (92) into Equation (88), we can get the dynamics for interest rate and market price of risks in the paper. Suppose pricing kernel is given by

$$\frac{d\pi_t}{\pi_t} = -r_f dt - \lambda_1 dW_{D,t} - \lambda_2 dW_{S,t} - \lambda_3 dW_{V,t}, \tag{93}$$

Then,

$$r_f = r_0 + r_1\mu_t + r_2 V_t, \tag{94}$$

where

$$r_0 = -(\frac{1-\gamma\psi}{1-\gamma})A_2\kappa\theta - \frac{1}{2}\gamma(1+\gamma)\sigma_D^2 - \xi_1. \tag{95}$$

$$r_1 = \gamma + (\alpha + g_1)A_1(\frac{1-\gamma\psi}{1-\gamma}). \tag{96}$$

$$r_2 = (g_1+\kappa)(\frac{1-\gamma\psi}{1-\gamma})A_2 - \frac{1}{2}(\frac{1-\gamma\psi}{1-\gamma})^2(A_1^2\sigma_\mu^2 + \sigma_V^2 A_2^2). \tag{97}$$

Moreover, the market price of risks are

$$\lambda_1 = \gamma. \tag{98}$$

$$\lambda_2 = -(\frac{1-\gamma\psi}{1-\gamma})A_1\sigma_\mu\sqrt{V_t}. \tag{99}$$

$$\lambda_3 = -(\frac{1-\gamma\psi}{1-\gamma})A_2\sigma_V\sqrt{V_t}. \tag{100}$$

In the model, the dividend-price ratio is equivalent to consumption wealth ratio, assume the dividend-price ratio is η_t,

$$\eta_t = \frac{D_t}{P_t} = \beta^\psi \exp((A_0 + A_1\mu_t + A_2V_t)\frac{1-\psi}{1-\gamma}). \tag{101}$$

By Itô's lemma,

$$dP_t = d(\frac{D_t}{\eta_t}) = dD_t\frac{1}{\eta_t} + D_t d(\frac{1}{\eta_t}) = \frac{D_t}{\eta_t}(\mu_t dt + \sigma_D dW_{D,t}) + \frac{D_t}{\eta_t}(-\frac{d\eta_t}{\eta_t} + \frac{(d\eta_t)^2}{\eta_t^2}). \tag{102}$$

Since η_t is a function of μ_t and V_t, use Itô's lemma to η_t, and substitude the results into Equation (102), we can get the price dynamic,

$$\frac{dP_t}{P_t} = \mathbf{E}_t(dP_t/P_t)dt + \sigma_D dW_{D,t} - \frac{1-\phi}{1-\gamma}A_1\sigma_\mu\sqrt{V_t}dW_{S,t} - \frac{1-\phi}{1-\gamma}A_2\sigma_V\sqrt{V_t}dW_{V,t}. \tag{103}$$

Suppose X_1, X_2, X_3 are three independent normal random variables, then $Y = X_1 + X_2 + X_3$ is also a normal random variable. Since $dW_{D,t}, dW_{S,t}, dW_{V,t}$ are independent with each other by our assumption, we can combine these three brownian motion into one, then the price dynamics is given by

$$\frac{dP_t}{P_t} = \mu_{p,t}(\mu_t)dt + \sigma_{p,t}(V_t)dW_{p,t} \tag{104}$$

$$d\mu_t = -\alpha\mu_t dt + \sigma_\mu\sqrt{V_t}dW_{S,t} \tag{105}$$

$$dV_t = \kappa(\theta - V_t)dt + \sigma_V\sqrt{V_t}dW_{V,t}, \tag{106}$$

where $\sigma_{p,t}^2(V_t) = \sigma_D^2 + (1-\psi)^2/(1-\gamma)^2(A_1^2\sigma_\mu^2 + A_2^2\sigma_V^2)V_t$.

The correlation between $dW_{p,t}, dW_{S,t}$ and $dW_{p,t}, dW_{S,t}$ can be calculated by formula

$$\rho = \frac{Cov(X_1, Y)}{\sigma_{X_1}\sigma_Y} = \frac{Var(X_1)}{\sigma_{X_1}\sigma_Y}. \tag{107}$$

B Extension to Two-Factor Volatility Model

B.1 General Settings

Standard long-run risk model with stochastic volatility under continuous-time settings

$$\frac{dC_t}{C_t} = (\mu_c + x_t)dt + \sqrt{V_t}dW_{c,t} \tag{108}$$

$$\frac{dD_t}{D_t} = (\mu_d + \phi x_t)dt + \varphi_d\sqrt{V_t}dW_{d,t} \tag{109}$$

$$dx_t = -\alpha x_t dt + \varphi_x\sqrt{V_t}dW_{x,t} \tag{110}$$

$$dV_t = \kappa_v(\theta_v - V_t)dt + \varphi_v\sqrt{V_t}dW_{v,t}, \tag{111}$$

where C_t is aggregate consumption. D_t is aggregate dividend. V_t is stochastic volatility for consumption process. x_t is long-run risk component.

We add investor learning into the traditional long-run risk model. Suppose that investors do not observe x_t directly, instead, investors can observe one signal y_t about the long-run risk component, with dynamics given as follows

$$dy_t = -\alpha x_t + \sqrt{S_t}dW_{y,t}, \tag{112}$$

where S_t represents the information quality, following the mean-reverting process

$$dS_t = \kappa_s(\theta_s - S_t)dt + \varphi_s\sqrt{S_t}dW_{s,t}. \tag{113}$$

Under such framework, investors learning process is a standard filtering problem. Using Theorem 10.1 in Lipster and Shiryaev (2001), the filtered dynamics for x_t is given by

$$d\hat{x}_t = -\alpha\hat{x}_t dt - \frac{\alpha\omega_t}{S_t}(dy_t + \alpha\hat{x}_t dt) \tag{114}$$

$$\frac{d\omega_t}{dt} = -2\alpha\omega_t - \frac{\alpha^2\omega_t^2}{S_t} + \varphi_x^2 V_t. \tag{115}$$

where $\hat{x}_t = \mathbf{E}[x_t|\mathcal{F}_t]$, conditional mean of latent variable at time t. $\omega_t = \mathbf{E}[(x_t - \hat{x}_t)^2|\mathcal{F}_t]$, filtering error at t. We analyze the model under its steady state. When the filtering system achieves its steady state, that is, $d\omega_t/dt = 0$, ω_t satisfies equation

$$\alpha^2\omega_t^2 + 2\alpha S_t\omega_t - \varphi_x^2 S_t V_t = 0. \tag{116}$$

Solving the equation, ω_t is given by

$$\omega_t = \frac{-S_t + \sqrt{S_t^2 + \varphi_x^2 V_t S_t}}{\alpha}. \tag{117}$$

Let $F_t = \alpha\omega_t/S_t$. We use a constant filtering weight in the rest of analysis, based on Equation (117) and long-run mean of V_t and S_t, we get F

$$F = (\sqrt{1 + \frac{\varphi_x^2 \theta_v}{\theta_s}} - 1)^{-1}. \tag{118}$$

Under investors information set, the dynamics of the total system is as follows

$$\frac{dC_t}{C_t} = (\mu_c + \hat{x}_t)dt + (\sqrt{V_t}dW_{c,t} - (x_t - \hat{x}_t)) \tag{119}$$

$$\frac{dD_t}{D_t} = (\mu_d + \phi\hat{x}_t)dt + (\varphi_d\sqrt{V_t}dW_{d,t} + \varphi(x_t - \hat{x}_t)dt) \tag{120}$$

$$d\hat{x}_t = -\alpha\hat{x}_t dt - F(\sqrt{S_t}dW_{y,t} - \alpha(x_t - \hat{x}_t)) \tag{121}$$

$$dV_t = \kappa_v(\theta_v - V_t)dt + \varphi_v\sqrt{V_t}dW_{v,t} \tag{122}$$

$$dS_t = \kappa_s(\theta_s - S_t)dt + \varphi_s\sqrt{S_t}dW_{s,t}. \tag{123}$$

Equivalently,

$$\frac{dC_t}{C_t} = (\mu_c + \hat{x}_t)dt + \sqrt{V_t}dW^*_{c,t} \tag{124}$$

$$\frac{dD_t}{D_t} = (\mu_d + \phi\hat{x}_t)dt + \varphi_d\sqrt{V_t}dW^*_{d,t} \tag{125}$$

$$d\hat{x}_t = -\alpha\hat{x}_t dt - F\sqrt{S_t}dW^*_{y,t} \tag{126}$$

$$dV_t = \kappa_v(\theta_v - V_t)dt + \varphi_v\sqrt{V_t}dW^*_{v,t} \tag{127}$$

$$dS_t = \kappa_s(\theta_s - S_t)dt + \varphi_s\sqrt{S_t}dW^*_{s,t}, \tag{128}$$

where $dW^*_{c,t}$, $dW^*_{d,t}$, $dW^*_{y,t}$, $dW^*_{v,t}$, $dW^*_{s,t}$ are Brownian motion under investors information set. Investor learning brings one new component into the traditional long-run risk model, which is information uncertainty. The new model is able to generate more flexible variance term structure, due to the two-factor stochastic variance structure.

B.2 Risk-Free Rate and Market Price of Risks

To solve the equilibrium, following Bansal and Yaron (2004), we use the continuous time version of Epstein-Zin utility function. Based on Duffie and Epstein (1992), the definition is given by

$$J_t = \mathbf{E}_t[\int_t^T f(C_s, J_s)ds]. \tag{129}$$

Thus the representative investor's objective is to choose consumption to optimize the value function J_t

$$J_t = \max_{\{C_s\}} \mathbf{E}_t[\int_t^T f(C_s, J_s)ds], \tag{130}$$

where $f(C, J)$ is a normalized aggregator related to current consumption C_t and continuation value function J_t, and is given by

$$f(C_t, J_t) = \frac{\beta}{1-\frac{1}{\psi}}(1-\gamma)J[(\frac{C}{((1-\gamma)J)^{\frac{1}{1-\gamma}}})^{1-\frac{1}{\psi}} - 1], \tag{131}$$

where β is the rate of time preference, $\gamma > 0$ is the relative risk aversion, and $\psi > 0$ the intertemporal elasticity of substitution (IES). If $\phi = 1/\gamma$, as illustrated by Duffie and Epstein (1992), we obtain the standard additive expected utility of constant relative risk aversion.

The value function $J = J(W_t, \hat{x}_t, V_t, S_t)$ is a function of wealth W_t and three state variables. H-J-B equation for J is

$$\max_{\{C\}}\{f(C, J) + \mathcal{A}^c J\} = 0, \tag{132}$$

where $\mathcal{A}^c$ is the infinitesimal generator associate with vector process $(C_t, \hat{x}_t, V_t, S_t)$. Conjecturing a solution for J of the following form,

$$J(W_t, \hat{x}_t, V_t, S_t) = \exp(A_0 + A_1\hat{x}_t + A_2V_t + A_3S_t)\frac{W_t^{1-\gamma}}{1-\gamma}. \tag{133}$$

Using standard log-linear approximation, which Campbell (1993) develops in discrete-time, and Chacko and Viceira (2005) use first in continuous-time. Specifically, let g_1 be the long-term mean of the consumption-wealth ratio,

$$g_1 = \exp(E[c_t - \omega_t]), \tag{134}$$

where the lowercase variables are the log variables. With the standard log-linear approximation,

$$\frac{C_t}{W_t} = \exp(c_t - \omega_t) \approx g_1 - g_1 \log g_1 + g_1 \log(\frac{C_t}{W_t}). \tag{135}$$

With the log-linear approximation, the first order condition

$$f_C = J_W \tag{136}$$

leads to the consumption-wealth ratio as

$$\frac{C_t}{W_t} = \beta^{\psi} \exp\{(A_0 + A_1\hat{x}_t + A_2V_t + A_3S_t)\frac{1-\psi}{1-\gamma}\}. \tag{137}$$

Therefore, J can be written as

$$J(C_t, \hat{x}_t, V_t, S_t) = \beta^{-\psi(1-\gamma)} \exp[\psi(A_0 + A_1\hat{x}_t + A_2V_t + A_3S_t)]\frac{C_t^{1-\gamma}}{1-\gamma}. \tag{138}$$

Moreover,

$$f \approx \theta J[g_1\frac{1-\psi}{1-\gamma}(A_0 + A_1\hat{x}_t + A_2V_t + A_3S_t) + \xi], \tag{139}$$

where $\theta = (1-\gamma)/(1-1/\psi)$ and $\xi = g_1 - g_1 \log g_1 + g_1\psi \log\beta - \beta$. Substitute Equation (138) and (139) into H-J-B equation, we can get

$$A_0 = \frac{1}{\psi g_1}[\theta\xi + (1-\gamma)\mu_c + \kappa_v\theta_v\psi A_2 + \kappa_s\theta_s\psi A_3]. \tag{140}$$

$$A_1 = \frac{1-\gamma}{\psi(g_1+\alpha)}. \tag{141}$$

$$A_2 = \frac{(g_1+\kappa_v) - \sqrt{(g_1+\kappa_v)^2 + \gamma\varphi_v^2}}{\varphi_v^2\psi}. \tag{142}$$

$$A_3 = \frac{(\kappa_s+g_1) - \sqrt{(\kappa_s+g_1)^2 - F^2\psi A_1^2\varphi_s^2}}{\varphi_v^2\psi}. \tag{143}$$

In order to derive the risk-free rate and market prices of risks. The pricing kernel is given by

$$\pi_t = \exp[\int_0^t f_J(C_s, J_s)ds]f_C(C_t, J_t). \tag{144}$$

$$\frac{d\pi}{\pi} = f_J dt + \frac{df_C}{f_C}. \tag{145}$$

Based on the structure of $f(C, J)$ and value function J, we get the following relation

$$f_J = \xi_1 - g_1(A_1\hat{x}_t + A_2V_t + A_3S_t)\frac{1-\gamma\psi}{1-\gamma} \tag{146}$$

$$f_C = \beta^{\psi\gamma} \exp[(\xi_1 + A_1\hat{x}_t + A_2V_t + A_3S_t)\frac{1-\gamma\psi}{1-\gamma}]C_t^{-\gamma}, \tag{147}$$

where $\xi_1 = (\theta - 1)\xi - \beta - g_1 \frac{1-\gamma\psi}{1-\gamma} A_0$. Applying Ito's lemma to pricing kernel dynamics based on dynamics of f_J and f_C, we have

$$\frac{d\pi_t}{\pi_t} = -r_f dt - \lambda_1 dW^*_{c,t} - \lambda_2 dW^*_{y,t} - \lambda_3 dW^*_{v,t} - \lambda_4 dW^*_{s,t}, \tag{148}$$

where the risk-free interest rate r_t is a linear function of state variables $(\hat{x}_t, V_t, S_t)$, with constant coefficients

$$r_f = r_0 + r_1 \hat{x}_t + r_2 V_t + r_3 S_t, \tag{149}$$

where

$$r_0 = \gamma\mu_c - (\frac{1-\gamma\psi}{1-\gamma}) A_2 \kappa_v - (\frac{1-\gamma\psi}{1-\gamma}) A_3 \kappa_s - \xi_1. \tag{150}$$

$$r_1 = \gamma + \alpha(\frac{1-\gamma\psi}{1-\gamma}) A_1 + g_1 A_1 (\frac{1-\gamma\psi}{1-\gamma}). \tag{151}$$

$$r_2 = -\frac{1}{2}\gamma(\gamma+1) + (\frac{1-\gamma\psi}{1-\gamma}) A_2 - \frac{1}{2}(\frac{1-\gamma\psi}{1-\gamma}) A_2^2 \varphi_v^2 + g_1 A_2 \frac{1-\gamma\psi}{1-\gamma}. \tag{152}$$

$$r_3 = -\frac{1}{2}(\frac{1-\gamma\psi}{1-\gamma})^2 A_1^2 F^2 + (\frac{1-\gamma\psi}{1-\gamma}) A_3 - \frac{1}{2}(\frac{1-\gamma\psi}{1-\gamma})^2 A_3^2 \varphi_s^2 + g_1 A_3 (\frac{1-\gamma\psi}{1-\gamma}). \tag{153}$$

Moreover, the market price of risks are

$$\lambda_1 = \gamma\sqrt{V_t} dW_{c,t}. \tag{154}$$

$$\lambda_2 = (\frac{1-\gamma\psi}{1-\gamma}) A_1 F \sqrt{S_t}. \tag{155}$$

$$\lambda_3 = -(\frac{1-\gamma\psi}{1-\gamma}) A_2 \varphi_v \sqrt{V_t}. \tag{156}$$

$$\lambda_4 = -(\frac{1-\gamma\psi}{1-\gamma}) A_3 \varphi_s \sqrt{S_t}. \tag{157}$$

B.3 Price Dynamics

The market portfolio return defined as

$$r_{m,t} dt = \mathbf{E}_t(\frac{dP_t}{P_t}), \tag{158}$$

$$\mathbf{E}_t(\frac{dP_t}{P_t}) + \frac{D_t}{P_t} dt = r_f dt - \mathbf{E}_t[\frac{d\pi_t}{\pi_t}\frac{dP_t}{P_t}], \tag{159}$$

where $r_{m,t}$ is the continuous compound market portfolio return. The price-dividend ratio is

$$\frac{D_t}{P_t} = \exp(B_0 + B_1\hat{x}_t + B_2V_t + B_3S_t). \tag{160}$$

Using the loglinear approximation, we get

$$\frac{D_t}{P_t} \approx g_{0B} + g_{1B}(B_0 + B_1\hat{x}_t + B_2V_t + B_3S_t). \tag{161}$$

Applying Ito's lamma to Equation (161), we get

$$\frac{dP_t}{P_t} = \frac{dD_t}{D_t} - (B_1d\hat{x}_t + B_2dV_t + B_3dS_t) + \frac{1}{2}[B_1^2(d\hat{x}_t)^2 + B_2^2(dV_t)^2 + B_3^2(dS_t)^2)]. \tag{162}$$

Combined with dynamics of dividend and pricing kernel, based on Equation (159), we can get

$$B_0 = -\frac{1}{g_{1B}}(\mu_d - B_2\kappa_v\theta_v - B_3\kappa_s\theta_s - r_0 + g_{0B}) \tag{163}$$

$$B_1 = -\frac{1}{g_{1B}}(\phi + \alpha B_1 - \gamma_1). \tag{164}$$

$$B_2 = \frac{1}{\varphi_v^2}[-(\kappa_v + g_{1B} - \frac{1-\gamma\psi}{1-\gamma}A_2\varphi_v^2) + \sqrt{(\kappa_v + g_{1B} - \frac{1-\gamma\psi}{1-\gamma}A_2\varphi_v^2)^2 + 2r_2\varphi_v^2}] \tag{165}$$

$$B_3 = \frac{1}{\varphi_s^2}[-(\kappa_s + g_{1B} - \frac{1-\gamma\psi}{1-\gamma}A_3\varphi_s^2) \tag{166}$$

$$+\sqrt{(\kappa_s + g_{1B} - \frac{1-\gamma\psi}{1-\gamma}A_3\varphi_s^2)^2 - (2B_1F^2 - r_3 - (\frac{1-\gamma\psi}{1-\gamma})A_1B_1F^2)}] \tag{167}$$

The dynamics for log stock price is given by

$$\frac{dP_t}{P_t} = \mathbf{E}_t(dP_t/P_t)dt + \varphi_d\sqrt{V_t}dW_{d,t}^* + B_1F\sqrt{S_t}dW_{v,t}^* - B_2\varphi_v\sqrt{V_t}dW_{v,t}^* - B_2\varphi_s\sqrt{S_t}dW_{s,t}^* \tag{168}$$

$$= \mathbf{E}_t(dP_t/P_t)dt + \sqrt{(\varphi_d^2 + B_2^2\varphi_v^2)V_t + (B_1^2F^2 + B_2^2\varphi_s^2)S_t}dW_{r,t}^* \tag{169}$$

where $dW_{r,t}$ is standard Brownian motion. After simplification, we can see that the model generates dynamics of two-factor stochastic volatility model,

$$\frac{dP_t}{P_t} = \mathbf{E}_t(dP_t/P_t)dt + \sqrt{c_vV_t + c_sS_t}dW_{r,t}^* \tag{170}$$

$$dV_t = \kappa_v(\theta_v - V_t)dt + \varphi_v\sqrt{V_t}dW_{v,t}^* \tag{171}$$

$$dS_t = \kappa_s(\theta_s - S_t)dt + \varphi_s\sqrt{S_t}dW_{s,t}^*. \tag{172}$$

Correlation between $dW^*_{v,t}$ and $dW^*_{r,t}$ is

$$\rho_v = \frac{B_2^2\varphi_v^2 V_t}{\sqrt{(\varphi_d^2 + B_2^2\varphi_v^2)V_t + (B_1^2F^2 + B_2^2\varphi_s^2)S_t}\sqrt{B_2^2\varphi_v^2 V_t}}. \tag{173}$$

Similarly, correlation between $dW^*_{s,t}$ and $dW^*_{r,t}$ is

$$\rho_s = \frac{B_2^2\varphi_s^2 S_t}{\sqrt{(\varphi_d^2 + B_2^2\varphi_v^2)V_t + (B_1^2F^2 + B_2^2\varphi_s^2)S_t}\sqrt{B_2^2\varphi_s^2 S_t}}. \tag{174}$$

B.4 Variance Premium

The market price of risks for V_t and S_t are λ_3 and λ_4 of Equation (148), following Heston (1993) and Duffie, Pan and Singleton(2000), the risk premia associated with V_t and S_t are

$$\lambda_3\varphi_v\sqrt{V_t} = -\upsilon_v V_t \ and \ \lambda_4\varphi_s\sqrt{S_t} = -\upsilon_s S_t. \tag{175}$$

Therefore, the risk-neutral process for V_t and S_t are

$$dV_t = \kappa_v^Q(\frac{\kappa_v}{\kappa_v^Q}\theta_v - V_t)dt + \varphi_v\sqrt{V_t}dW^Q_{v,t} \tag{176}$$

$$dS_t = \kappa_s^Q(\frac{\kappa_s}{\kappa_s^Q}\theta_s - S_t)dt + \varphi_s\sqrt{S_t}dW^Q_{s,t}, \tag{177}$$

where $\kappa_v^Q = \kappa_v - \upsilon_v$, and $\kappa_s^Q = \kappa_s - \upsilon_s$. Because the dynamics for V_t and S_t are widely used square root process, the variance swap rate with maturity τ is

$$VS_t = c_v(A_v^Q + B_v^Q V_t) + c_s(A_s^Q + B_s^Q S_t), \tag{178}$$

where A_i^Q and B_i^Q, i = (v, s) are given by

$$A_i^Q = \frac{\kappa_i\theta_i}{\kappa_i^Q}[1 - \frac{1-\exp(-\kappa_i^Q\tau)}{\kappa_i^Q\tau}], \quad B_i^Q = \frac{1-\exp(-\kappa_i^Q\tau)}{\kappa\tau}. \tag{179}$$

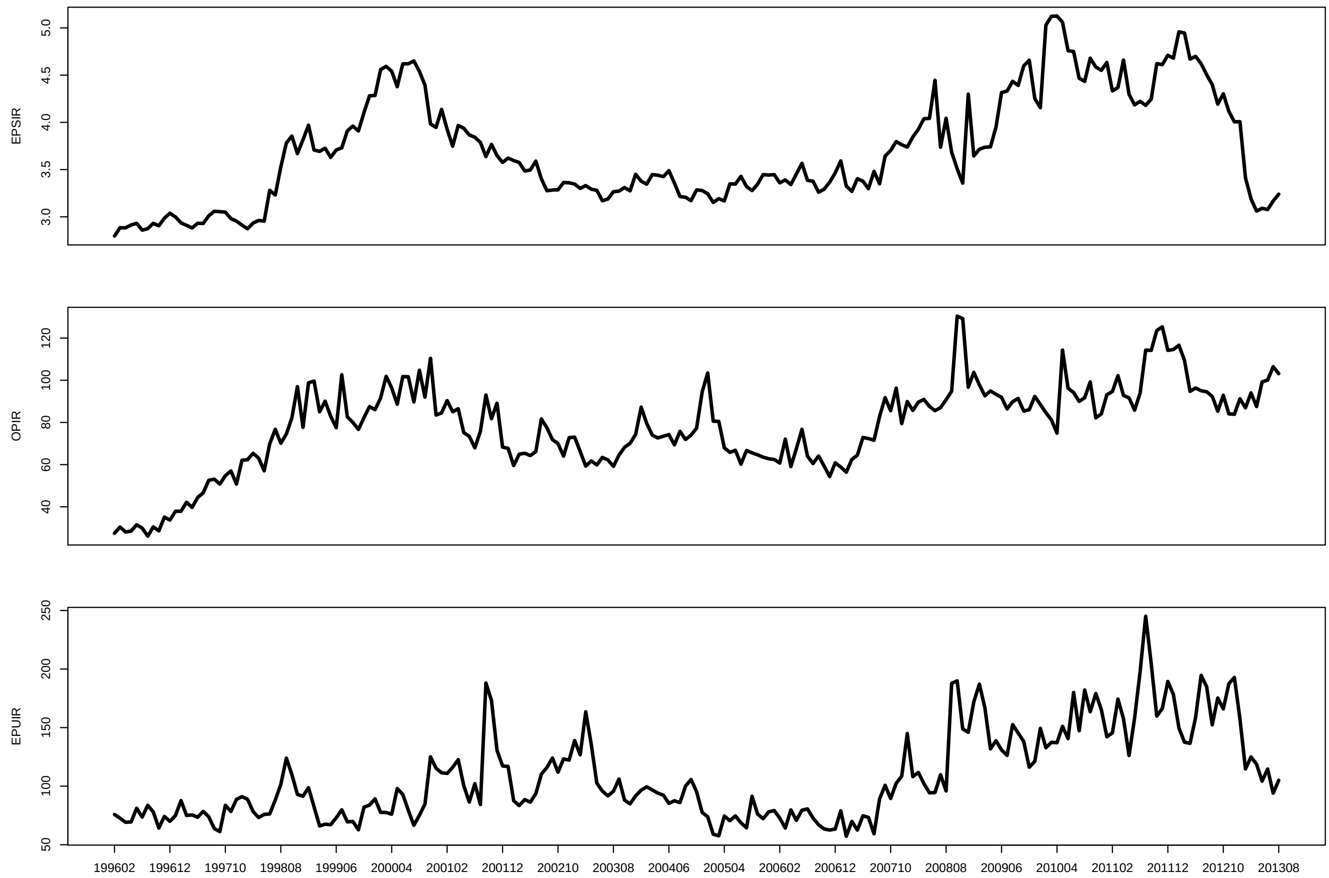
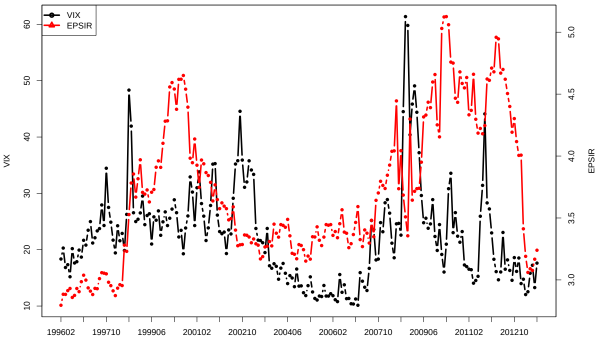
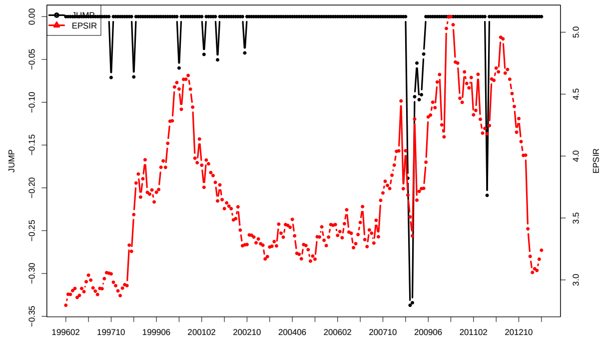


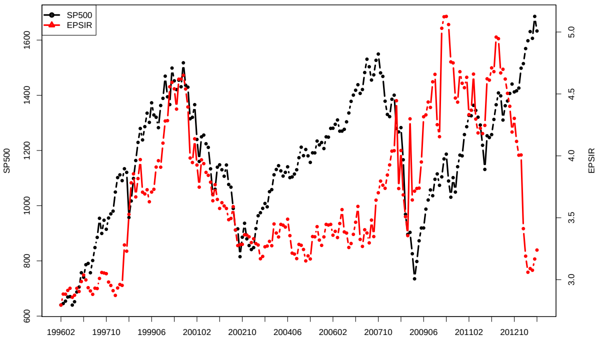
Figure 1: Dynamics of three information risk measures, ranging from Jan, 1996 to Dec, 2012. The top panel contains dynamics for EPSIR. The middle panel show dynamics for OPIR. The bottom panel presents dynamics for EPUIR.



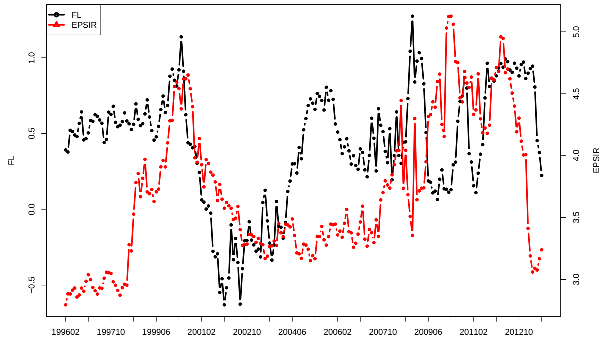
(a) VIX EPSIR Dynamics



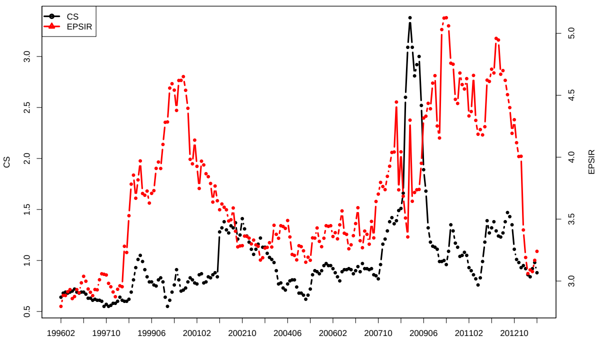
(b) JUMP EPSIR Dynamics



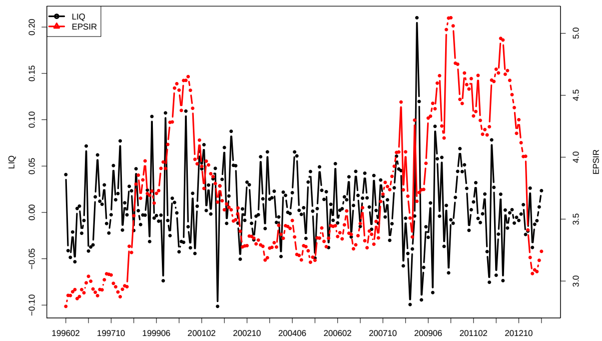
(c) S&P 500 Index EPSIR Dynamics



(d) FL EPSIR Dynamics

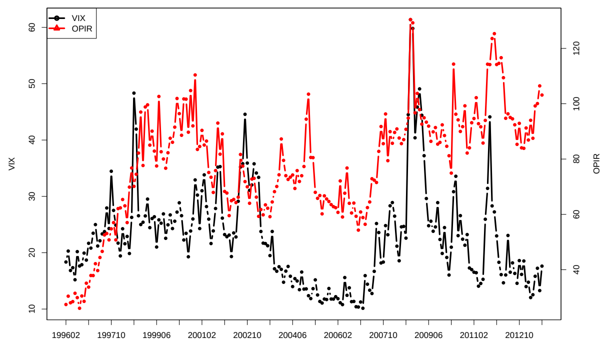


(e) CS EPSIR Dynamics

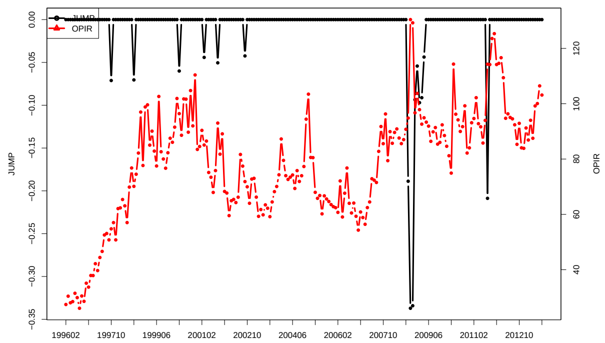


(f) LIQ EPSIR Dynamics

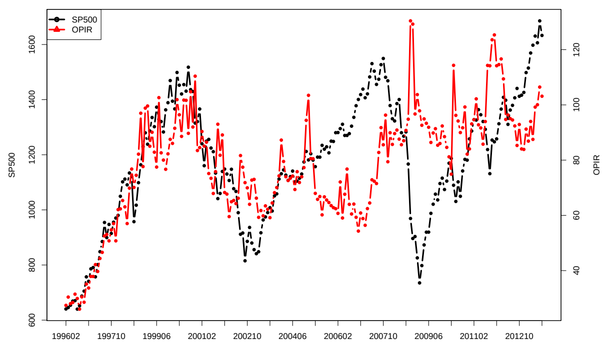
Figure 2: Paired time series dynamics for volatility, index price jump, index price level, funding liquidity, credit spread, market liquity, and analyst forecast dispersion (EPSIR) as a measure for information risks. Time period ranges from Jan, 1996 to Dec, 2012.



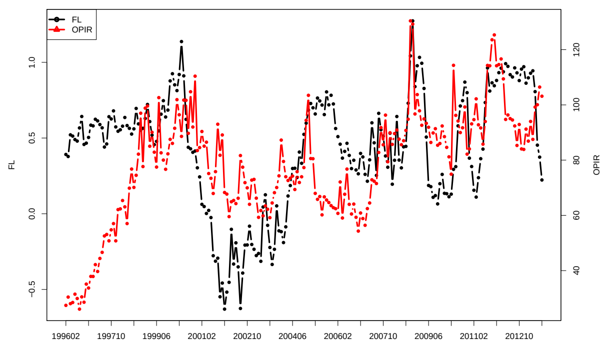
(a) VIX OPIR Dynamics



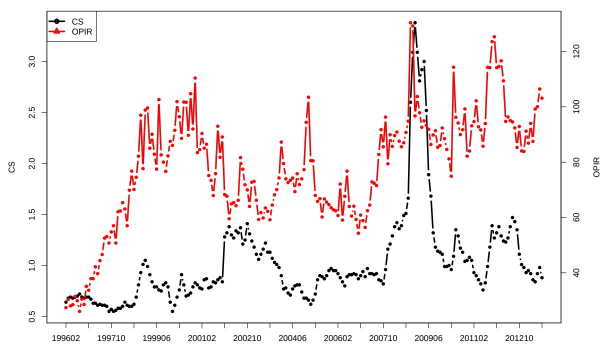
(b) JUMP OPIR Dynamics



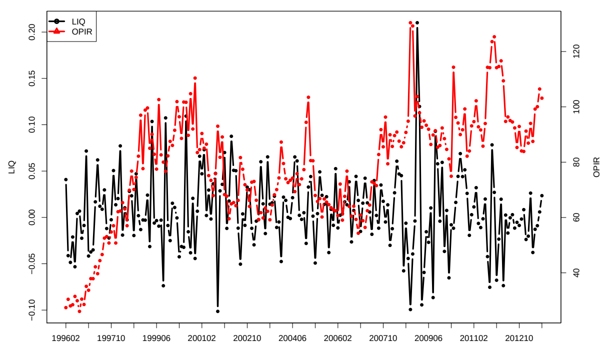
(c) S&P 500 Index OPIR Dynamics



(d) FL OPIR Dynamics

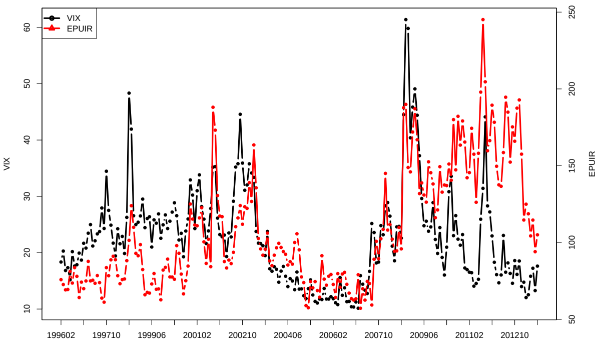


(e) CS OPIR Dynamics

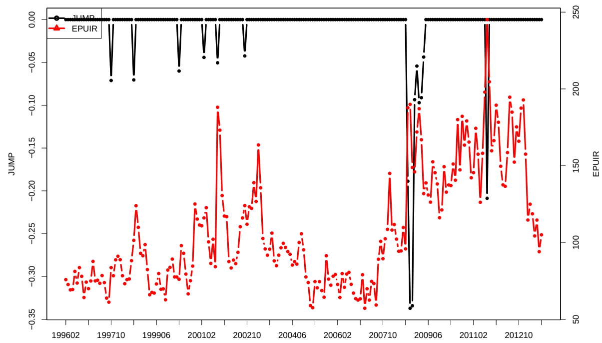


(f) LIQ OPIR Dynamics

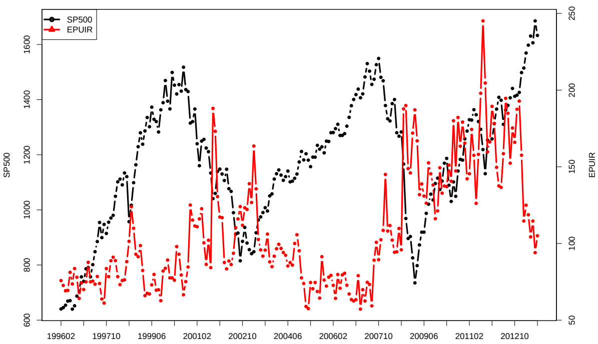
Figure 3: Paired time series dynamics for volatility, index price jump, index price level, funding liquidity, credit spread, market liquity, and option trading volume weighted strike dispersion (OPIR) as a measure for information risks. Time period ranges from Jan, 1996 to Dec, 2012.



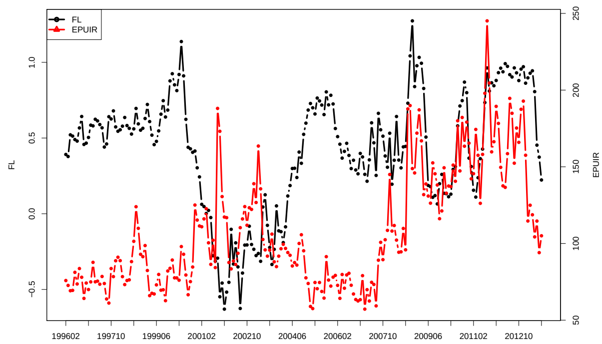
(a) VIX EPUIR Dynamics



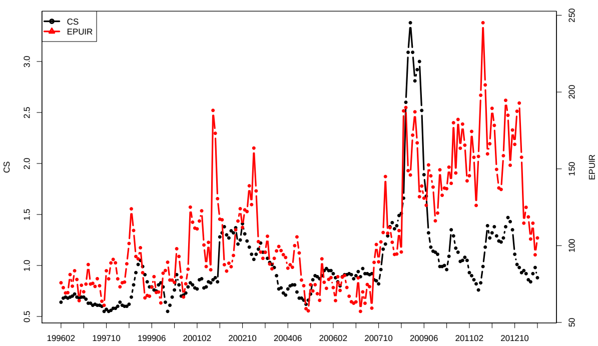
(b) JUMP EPUIR Dynamics



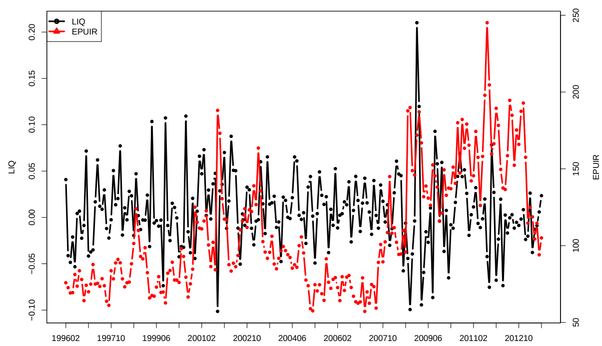
(c) S&P 500 Index EPUIR Dynamics



(d) FL EPUIR Dynamics



(e) CS EPUIR Dynamics



(f) LIQ EPUIR Dynamics

Figure 4: Paired time series dynamics for volatility, index price jump, index price level, funding liquidity, credit spread, market liquidity, and economic policy uncertainty index (EPUIR) as a measure for information risks. Time period ranges from Jan, 1996 to Dec, 2012.

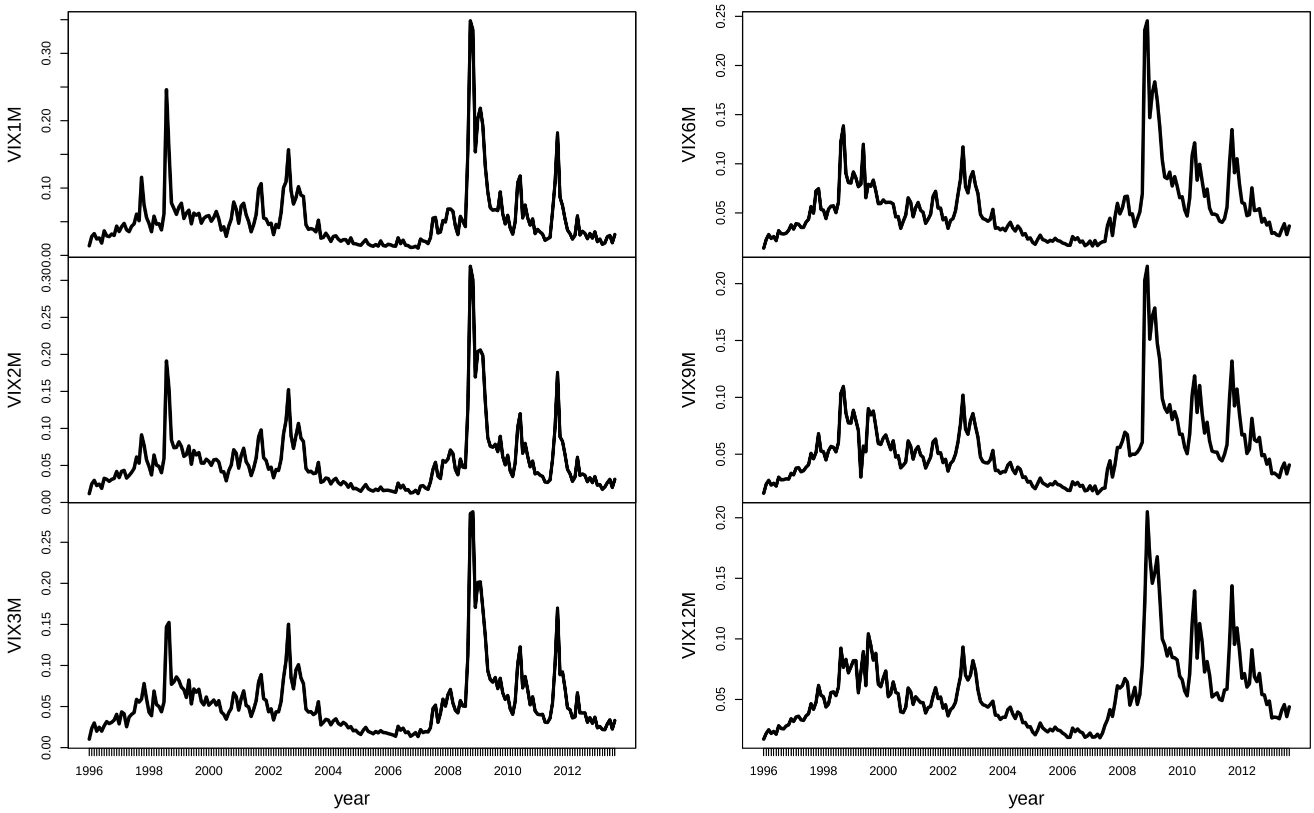


Figure 5: This figure presents dynamics of the VIX^2 term structure, ranging from Jan, 1996 to Dec, 2012. The term structure has six different maturities: 1 month, 2 months, 3 months, 6 months, 9 months and 12 months. VIXiM denotes square of annualized VIX with maturity i month.

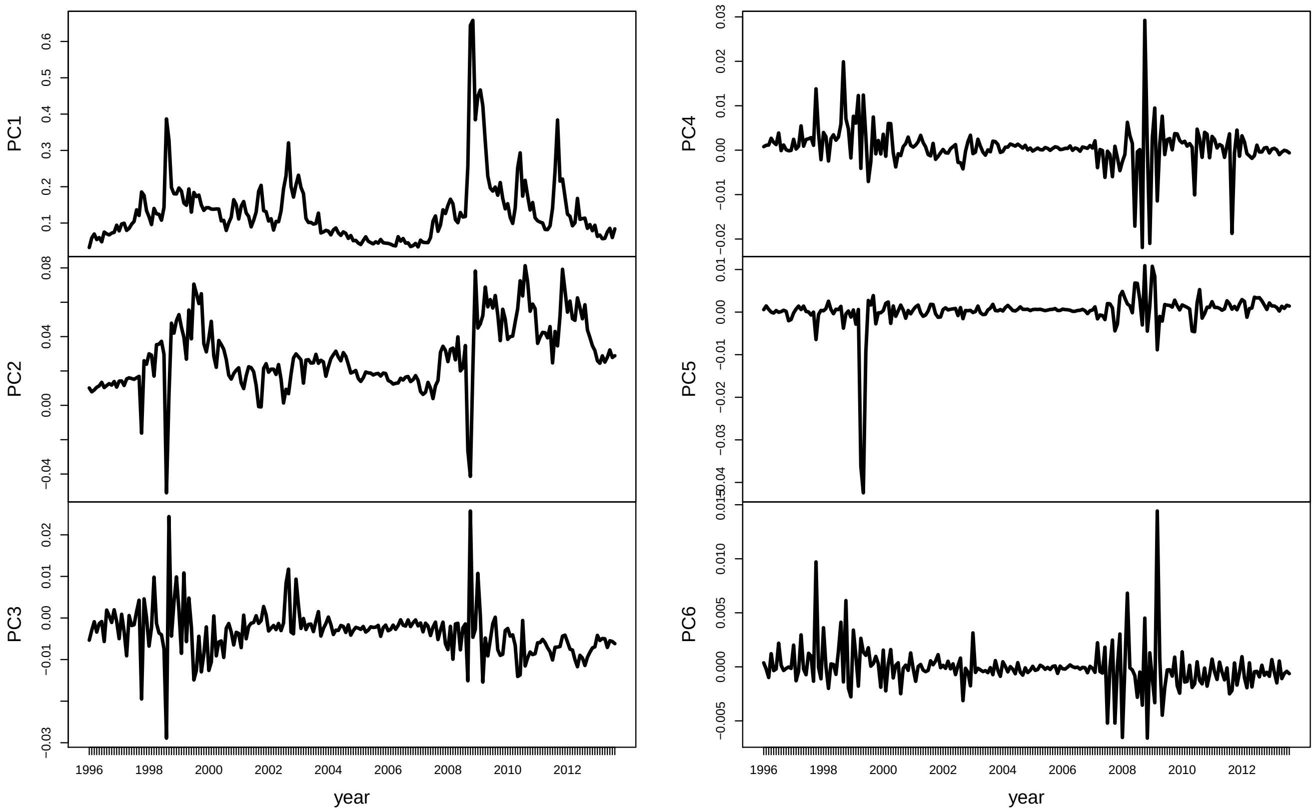


Figure 6: This figure presents dynamics of all principle components of VIX^2 term structure, ranging from Jan, 1996 to Dec, 2012. The term structure has six different maturities: 1 month, 2 months, 3 months, 6 months, 9 months and 12 months. Therefore, we have six principle components. PCi denotes ith principle component.

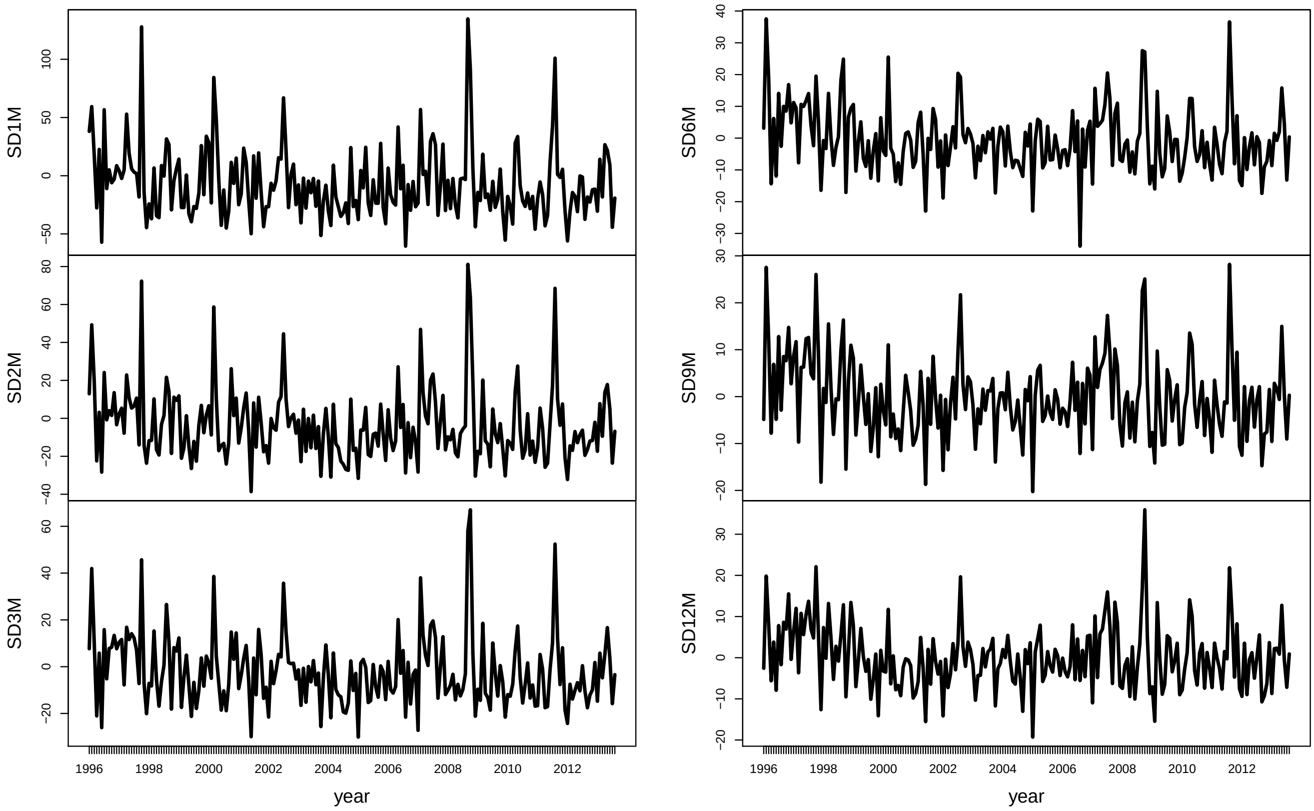
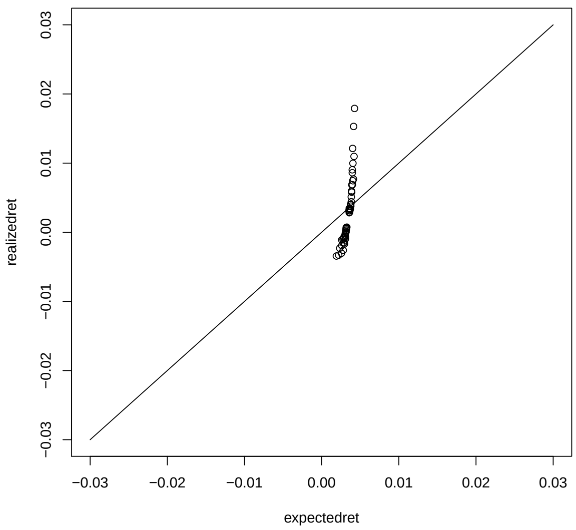
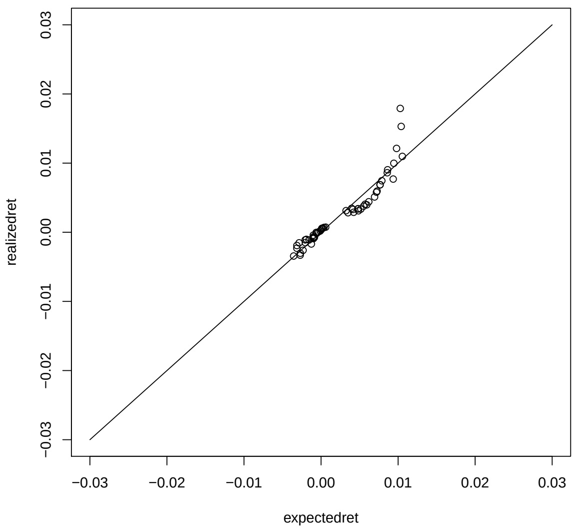


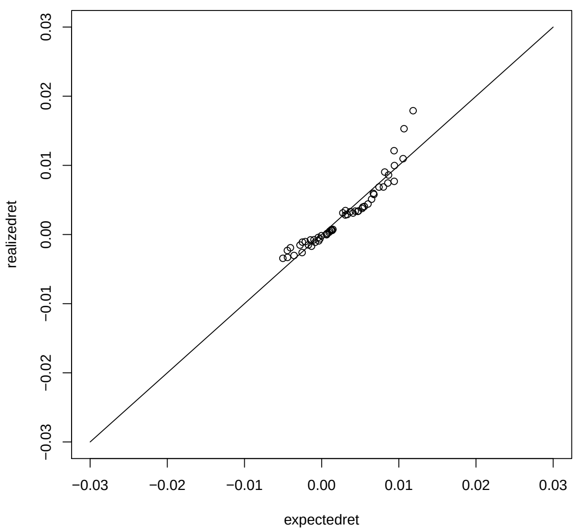
Figure 7: This figure presents dynamics of S&P 500 straddle returns with different maturities, ranging from Jan, 1996 to Dec, 2012. The term structure has six different maturities: 1 month, 2 months, 3 months, 6 months, 9 months and 12 months. SDiM denotes straddle returns with maturity i month.



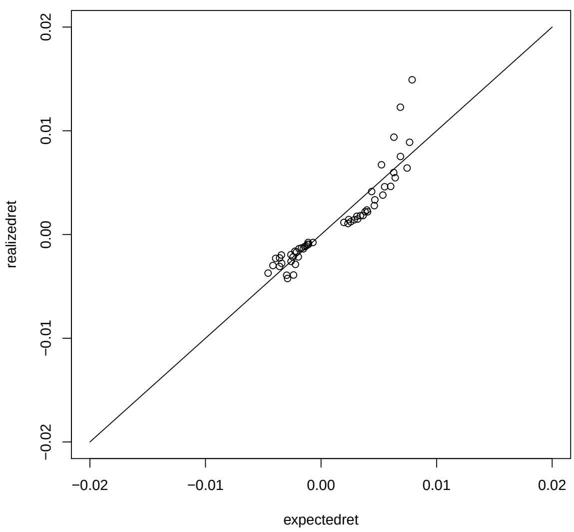
(a) MKT



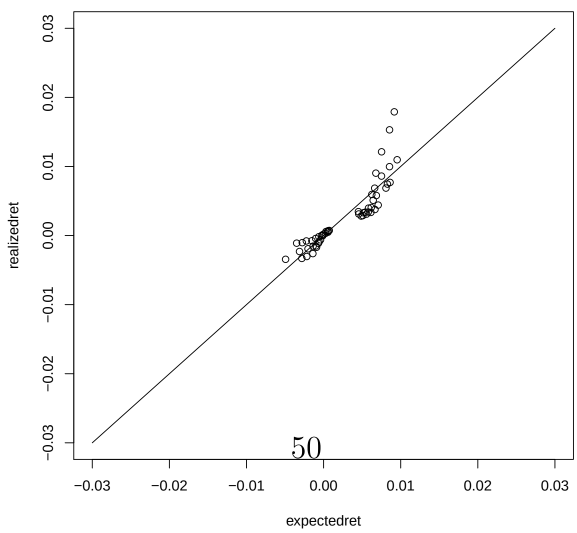
(b) EPSIR



(c) EPUIR

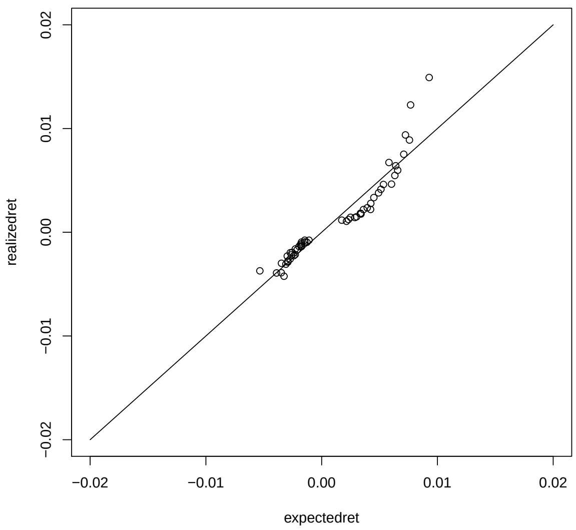


(d) OPIR

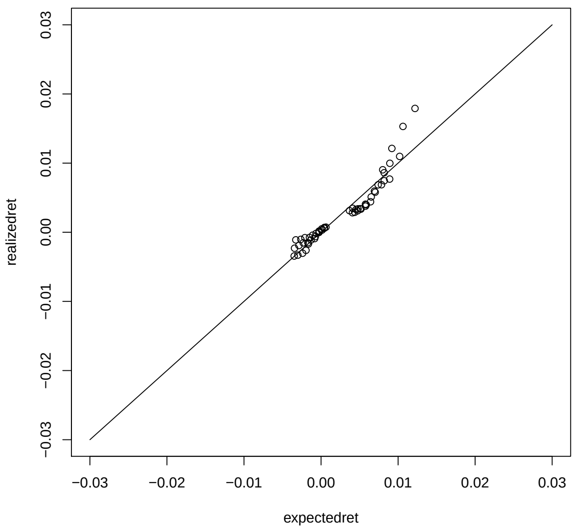


(e) LIQ

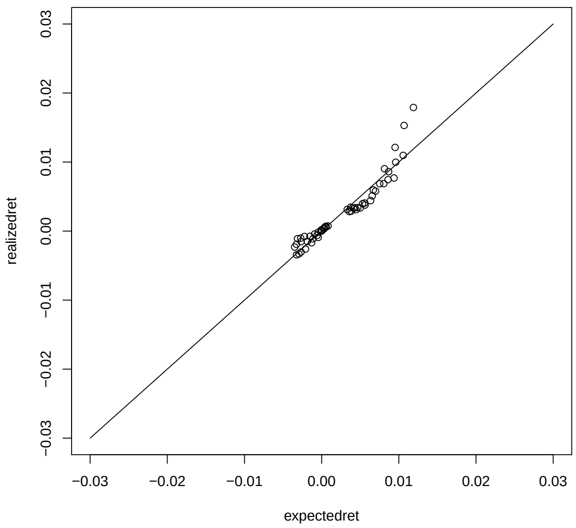
Figure 8: Realized Returns against Expected Returns (first part).



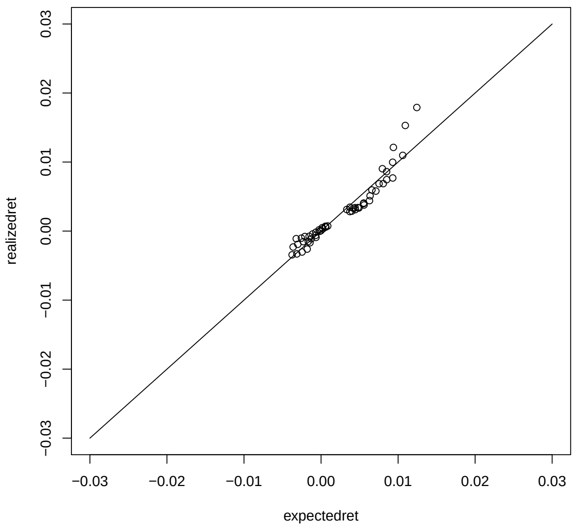
(a) FL



(b) JUMP



(c) VOL



(d) VOLJUMP

Figure 8: Realized Returns against Expected Returns (second part). This figure plots the realized mean excess returns of 54 S&P 500 index option portfolios (27 call and 27 put portfolios) against the mean excess returns predicted by our two-factor models. All the models contain market excess return as one factor. The other factor could be: VIX, CBOE risk neutral volatility index level; VOLJUMP, jumps of VIX; JUMP, S&P 500 index jumps; EPUIR, economic policy uncertainty index; EPSIR, aggregate standard deviation of analyst forecasts about firm long-term earning growth; OPIR, option-volume weighted strike dispersion; FL, funding liquidity; LIQ, market liquidity. The sample period is Jan, 1996 to Dec 2012, via monthly data.

Table 1: **Summary Statistics for S&P 500 Index Call Option Returns**

This table reports summary statistics for percentage monthly excess returns of index call options, ranging from April, 1986 to Jan, 2012. Column names represent option characteristics, for example, C30900 means call option with time-to-maturity 30 days, Strike/PriceLevel 0.9.

	C30900	C0925	C30950	C30975	C301000	C301025	C301050	C301075	C301100
mean	0.17	0.14	0.09	-0.01	-0.11	-0.23	-0.29	-0.29	-0.34
sd	4.28	4.24	4.19	4.17	4.14	4.13	4.06	3.89	3.92
skew	-0.25	-0.23	-0.18	-0.13	0.02	0.28	0.58	0.91	1.39
kurt	0.63	0.63	0.66	0.71	0.78	1.18	2.20	3.61	5.84
	C60900	C60925	C60950	C60975	C601000	C601025	C601050	C601075	C601100
mean	0.17	0.13	0.08	0.01	-0.05	-0.11	-0.12	-0.20	-0.28
sd	4.23	4.20	4.18	4.16	4.17	4.16	4.21	4.16	4.03
skew	-0.23	-0.18	-0.14	-0.07	0.02	0.18	0.36	0.59	0.84
kurt	0.62	0.64	0.72	0.73	0.83	0.95	1.34	1.91	2.91
	C90900	C90925	C90950	C90975	C901000	C901025	C901050	C901075	C901100
mean	0.19	0.16	0.12	0.07	0.05	0.01	-0.01	-0.06	-0.11
sd	4.19	4.17	4.14	4.14	4.15	4.13	4.16	4.21	4.16
skew	-0.20	-0.14	-0.10	-0.05	0.03	0.14	0.25	0.28	0.47
kurt	0.60	0.59	0.62	0.65	0.74	0.85	1.07	1.09	1.41

Table 2: **Summary Statistics for S&P 500 Index Put Option Returns**

This table reports summary statistics for percentage monthly excess returns of index put options, ranging from April, 1986 to Jan, 2012. Column names represent option characteristics, P30900 means put option with time-to-maturity 30 days, Strike/StockLevel 0.9.

	P30900	P30925	P30950	P30975	P301000	P301025	P301050	P301075	P301100
mean	1.86	1.61	1.33	1.06	0.75	0.55	0.48	0.47	0.43
sd	6.33	5.98	5.65	5.38	5.10	4.87	4.72	4.65	4.56
skew	-1.54	-1.47	-1.21	-1.00	-0.86	-0.77	-0.61	-0.48	-0.49
kurt	6.82	6.11	4.98	4.42	4.04	3.49	2.71	2.06	1.84
	P60900	P60925	P60950	P60975	P601000	P601025	P601050	P601075	P601100
mean	1.23	1.13	1.01	0.85	0.66	0.56	0.48	0.45	0.40
sd	6.02	5.69	5.47	5.24	5.05	4.88	4.75	4.70	4.63
skew	-1.27	-1.12	-1.02	-0.94	-0.85	-0.77	-0.67	-0.57	-0.55
kurt	5.58	4.97	4.53	4.09	3.70	3.38	2.87	2.42	2.12
	P90900	P90925	P90950	P90975	P901000	P901025	P901050	P901075	P901100
mean	0.82	0.83	0.78	0.69	0.59	0.54	0.49	0.43	0.42
sd	5.84	5.59	5.41	5.19	5.06	4.91	4.81	4.75	4.67
skew	-1.12	-1.05	-0.98	-0.87	-0.82	-0.75	-0.67	-0.62	-0.58
kurt	5.04	4.79	4.52	3.92	3.58	3.25	3.00	2.69	2.41

Table 3: **Summary Statistics for Systematic Risk Factors**

This table reports summary statistics of systematic risk proxies, ranging from Jan, 1996 to Dec, 2012. SP500 is S&P 500 index level. VIX is CBOE risk neutral volatility index level. VOLJUMP is jumps of VIX. JUMP is S&P 500 index jumps. CS is credit spread between Baa and Aaa bonds. EPUIR is economic policy uncertainty index. EPSIR is aggregate standard deviation of analyst forecasts about firm long-term earning growth. OPIR is option-volume weighted strike dispersion. FL is funding liquidity. LIQ is market liquidity.

	Min	Q25	Mean	Median	SD	Q75	Max	Skewness	Kurtosis
SP500	639.95	1025.67	1173.80	1186.69	231.30	1340.53	1685.73	-0.3372	-0.4656
VIX	10.1400	16.2350	22.5889	22.1200	8.7724	26.2050	61.3800	1.3625	2.9287
VOLJUMP	0.0000	0.0000	0.0346	0.0000	0.0923	0.0438	0.8128	4.9021	30.7109
JUMP	-0.3371	0.0000	-0.0085	0.0000	0.0403	0.0000	0.0000	-6.2835	43.4616
CS	0.5500	0.7700	1.0216	0.9100	0.4595	1.1600	3.3800	2.8258	9.7479
FL	-0.6300	0.2066	0.4252	0.4780	0.3889	0.6976	1.2738	-0.5877	-0.1687
LIQ	-0.1014	-0.0141	0.0065	0.0029	0.0409	0.0268	0.2101	0.6019	2.6748
EPUIR	57.2026	77.5724	107.8132	95.2408	38.2557	132.3278	245.1263	0.9174	0.0400
EPSIR	2.7954	3.2805	3.7090	3.5899	0.5795	4.1675	5.1259	0.5205	-0.7543
OPIR	26.0612	64.5743	78.0884	80.5781	20.5824	91.9499	130.4140	-0.3299	0.1934

Table 4: **Correlation of Systematic Risk Factors**

This table reports the correlation of systematic risk proxies, ranging from Jan, 1996 to Dec, 2012. SP500 is S&P 500 index level. VIX is CBOE risk neutral volatility index level. VOLJUMP is jumps of VIX. JUMP is S&P 500 index jumps. CS is credit spread between Baa and Aaa bonds. EPUIR is economic policy uncertainty index. EPSIR is aggregate standard deviation of analyst forecasts about firm long-term earning growth. OPIR is option-volume weighted strike dispersion. FL is funding liquidity. LIQ is market liquidity.

	SP500	VIX	VOLJUMP	JUMP	CS	FL	LIQ	EPUIR	EPSIR	OPIR
SP500	1.0000	-0.3757	-0.1483	0.1674	-0.1647	0.2445	0.0077	0.0116	0.3989	0.5301
VIX	-0.3757	1.0000	0.6412	-0.6028	0.5206	-0.0255	-0.0767	0.3959	0.1218	0.3121
VOLJUMP	-0.1483	0.6412	1.0000	-0.8323	0.4198	0.2022	-0.1774	0.3924	0.0881	0.3563
JUMP	0.1674	-0.6028	-0.8323	1.0000	-0.5202	-0.2531	0.2040	-0.3318	0.0133	-0.3001
CS	-0.1647	0.5206	0.4198	-0.5202	1.0000	0.1069	-0.0180	0.4966	0.2544	0.4196
FL	0.2445	-0.0255	0.2022	-0.2531	0.1069	1.0000	-0.1242	0.1197	0.1719	0.2697
LIQ	0.0077	-0.0767	-0.1774	0.2040	-0.0180	-0.1242	1.0000	-0.1085	-0.0379	-0.0690
EPUIR	0.0116	0.3959	0.3924	-0.3318	0.4966	0.1197	-0.1085	1.0000	0.5616	0.5876
EPSIR	0.3989	0.1218	0.0881	0.0133	0.2544	0.1719	-0.0379	0.5616	1.0000	0.6900
OPIR	0.5301	0.3121	0.3563	-0.3001	0.4196	0.2697	-0.0690	0.5876	0.6900	1.0000

Table 5: **Summary Statistics for VIX^2 Term Structure and Straddle Returns**

This table reports summary statistics for VIX^2 term structure, principle components of VIX^2 term structure and S&P 500 index straddle returns, ranging from Jan, 1996 to Dec, 2012. VIXiM denotes square of annualized VIX with maturity i month. PCi denotes ith principle component. SDi denotes straddle returns constructed vis options with time-to-maturity i month.

	Min	Q25	Mean	Median	SD	Q75	Max	Skewness	Kurtosis
VIX1M	0.011	0.026	0.054	0.043	0.047	0.062	0.348	3.206	13.886
VIX2M	0.012	0.028	0.054	0.044	0.043	0.064	0.319	2.950	11.957
VIX3M	0.010	0.029	0.054	0.044	0.040	0.067	0.287	2.758	10.709
VIX6M	0.014	0.033	0.055	0.048	0.035	0.067	0.245	2.221	7.393
VIX9M	0.015	0.033	0.055	0.049	0.032	0.067	0.215	1.969	5.706
VIX12M	0.017	0.035	0.055	0.049	0.031	0.069	0.205	1.659	3.911
PC1	0.033	0.074	0.131	0.110	0.092	0.157	0.658	2.652	10.043
PC2	-0.051	0.016	0.029	0.026	0.020	0.040	0.081	0.031	1.569
PC3	-0.029	-0.006	-0.003	-0.003	0.006	-0.001	0.026	0.790	6.985
PC4	-0.022	-0.000	0.001	0.001	0.005	0.002	0.029	-0.056	12.575
PC5	-0.042	-0.000	0.000	0.001	0.004	0.001	0.011	-6.575	57.634
PC6	-0.007	-0.001	-0.000	-0.000	0.002	0.000	0.014	2.218	15.401
SD1M	-60.508	-26.817	-6.614	-12.351	30.473	6.628	134.741	1.498	3.810
SD2M	-38.617	-16.112	-3.562	-6.264	18.871	5.962	81.157	1.538	3.892
SD3M	-30.111	-12.025	-1.905	-4.116	14.957	5.405	67.058	1.359	3.473
SD6M	-34.002	-7.851	-0.591	-0.988	10.376	4.666	37.535	0.615	1.363
SD9M	-20.254	-6.061	-0.177	-0.645	8.561	4.228	28.191	0.600	0.806
SD12M	-19.303	-5.280	0.252	-0.266	7.782	4.703	35.935	0.742	1.663

Table 6: **Prediction Regression for VIX Term Structure**

This table reports monthly linear regression results for

$$VIXiM_{t+1} = \alpha + \beta\{EPSIR, OPIR, EPUIR\}_t + \epsilon_{t+1},$$

where $VIXiM_{t+1}$ is VIX^2 with maturity i month at the end of month $t+1$. $\{EPSIR, OPIR, EPUIR\}$ are three proxies for information risk at month t. Estimation and robust standard error for β and adjusted R^2 are presented. The sample period is from January, 1996 to December, 2012. Newey-West adjusted standard errors are used. Signif.codes: 0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 '' 1.

	VIX1M	VIX2M	VIX3M	VIX6M	VIX9M	VIX12M
EPSIR	0.0095	0.0111·	0.0136*	0.0175***	0.0203***	0.023***
	(0.0064)	(0.0061)	(0.0057)	(0.0051)	(0.0046)	(0.0045)
R^2	0.0135	0.0219	0.038	0.0837	0.135	0.1909
OPIR	0.0734·	0.0756*	0.0791*	0.0794**	0.0815***	0.0877***
	(0.0379)	(0.0355)	(0.0330)	(0.0268)	(0.0241)	(0.0224)
R^2	0.1022	0.1279	0.1625	0.2163	0.2742	0.3472
EPUIR	0.0481·	0.0483*	0.0487*	0.0462*	0.0485**	0.0478***
	(0.0250)	(0.0235)	(0.0215)	(0.0183)	(0.0156)	(0.0138)
R^2	0.1527	0.1814	0.2138	0.2549	0.3371	0.3592

Table 7: **Prediction Regression for Principle Components of VIX Term Structure**

This table reports monthly linear regression results for

$$PCi_{t+1} = \alpha + \beta\{EPSIR, OPIR, EPUIR\}_t + \epsilon_{t+1},$$

where PCi_{t+1} is principle components of VIX^2 term structure. $\{EPSIR, OPIR, EPUIR\}$ are three proxies for information risk at month t. Estimation and robust standard error for β and adjusted R^2 are presented. The sample period is from January, 1996 to December, 2012. Newey-West adjusted standard errors are used. Signif.codes: 0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 '' 1.

	PC1	PC2	PC3	PC4	PC5	PC6
EPSIR	0.0355*	0.0194*	-0.0033*	-0.0003	0.0006	-0.0005
	(0.0107)	(0.0019)	(0.0006)	(0.0006)	(0.0005)	(0.0004)
R^2	0.05	0.3309	0.1111	0.0013	0.0054	0.0221
OPIR	0.1883*	0.0498*	-0.0065*	-0.0029	0.0006	-0.0014
	(0.0282)	(0.0056)	(0.0019)	(0.0016)	(0.0015)	(0.001)
R^2	0.1768	0.2755	0.0545	0.016	0.0007	0.0195
EPUIR	0.115*	0.0235*	-0.0026*	-0.0007	0.0022	-0.0005
	(0.0146)	(0.0031)	(0.001)	(0.0009)	(0.0020)	(0.0004)
R^2	0.2293	0.2128	0.0304	0.0035	0.0365	0.0073

Table 8: **Prediction Regression for S&P 500 Straddle Returns**

This table reports monthly linear regression results for

$$SDiM_{t+1} = \alpha + \beta\{EPSIR, OPIR, EPUIR\}_t + \epsilon_{t+1},$$

where $SDiM_{t+1}$ is S&P 500 straddle returns with maturity i. $\{EPSIR, OPIR, EPUIR\}$ are three proxies for information risk at month t. Estimation and robust standard error for β and adjusted R^2 are presented. The sample period is from January, 1996 to December, 2012. Newey-West adjusted standard errors are used. Signif.codes: 0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 '' 1.

	SD1M	SD2M	SD3M	SD6M	SD9M	SD12M
EPSIR	-0.035	-0.0238	-0.0285·	-0.0278*	-0.0278*	-0.024*
	(0.0349)	(0.0210)	(0.0165)	(0.0124)	(0.0115)	(0.0111)
R^2	0.0045	0.0055	0.0127	0.0256	0.037	0.0326
OPIR	-0.1567·	-0.0995·	-0.0924·	-0.0927**	-0.0902***	-0.0716*
	(0.0866)	(0.0547)	(0.0491)	(0.0315)	(0.0265)	(0.0285)
R^2	0.0115	0.0121	0.0167	0.0357	0.0489	0.0365
EPUIR	-0.0726	-0.0281	-0.0241	-0.022	-0.0239	-0.0118
	(0.0542)	(0.0335)	(0.0265)	(0.0182)	(0.0151)	(0.0139)
R^2	0.0086	0.0034	0.0039	0.007	0.0119	0.0034

Table 9: **Principle Component Analysis for S&P 500 Index Option Returns**

This table report principle component analysis for 54 S&P 500 index option return time series. PCA analysis is conducted to all option returns and call, put option returns separately. We report first 10 factors.

	Call& Put		Call		Put	
	eigenvalue	variance(%)	eigenvalue	variance(%)	eigenvalue	variance(%)
comp 1	47.1792	87.3688	25.6783	95.1049	26.1144	96.7200
comp 2	5.7797	10.7031	0.9848	3.6475	0.6658	2.4659
comp 3	0.3885	0.7195	0.1272	0.4710	0.0724	0.2681
comp 4	0.1666	0.3086	0.0644	0.2385	0.0483	0.1787
comp 5	0.0990	0.1833	0.0335	0.1242	0.0212	0.0784
comp 6	0.0724	0.1341	0.0318	0.1178	0.0164	0.0606
comp 7	0.0564	0.1044	0.0198	0.0732	0.0155	0.0574
comp 8	0.0452	0.0838	0.0159	0.0587	0.0100	0.0370
comp 9	0.0340	0.0630	0.0101	0.0373	0.0072	0.0268
comp 10	0.0244	0.0451	0.0070	0.0259	0.0047	0.0175

Table 10: **Fama-Mecbeth Two-Pass Regression Results for Different Models**

This table reports results of Fama-Macbeth two pass regressions. In all regressions, market excess return is always one factor, the other factor includes: VIX, CBOE risk neutral volatility index level; VOLJUMP, jumps of VIX; JUMP, S&P 500 index jumps; LIQ, market liquidity; FL, funding liquidity; EPUIR, economic policy uncertainty index; EPSIR, aggregate standard deviation of analyst forecasts about firm long-term earning growth; OPIR, option-volume weighted strike dispersion; FL, funding liquidity. λs represent market price of risks for each factors. The robust adjusted standard errors are below each λ. We also report cross-sectional adjusted R^2, mean absolute pricing error (MAPE) and J-statistics for each factor combination. Sample period ranges from Jan 1996 to Jan 2012.

	VIX	VIXJUMP	JUMP	LIQ	FL	EPSIR	EPUIR	OPIR
λ_{mkt}	-0.0011	0.0032	0.0035	0.0051	-0.0014	-0.0023	0.0040	-0.0013
t_{NW}	-0.3589	1.0586	1.1463	1.6624	-0.3486	-0.7235	1.3061	-0.3198
t_{Shaken}	-0.3415	1.0408	1.1222	1.4851	-0.3022	-0.5261	1.2209	-0.2984
λ_{factor}	-0.0168	-0.0451	0.0218	0.0587	-0.1828	-0.2731	-0.2215	-0.0730
t_{NW}	-4.4016	-4.8012	4.8002	4.4669	-3.5902	-4.7646	-4.7654	-3.4408
t_{shaken}	-4.0519	-3.9985	3.8458	2.3261	-1.9656	-2.2004	-2.9134	-2.4235
MAPE	0.0959	0.0968	0.1065	0.1407	0.0978	0.1001	0.1023	0.1156
AdjR2	0.9305	0.9304	0.9272	0.9114	0.9320	0.9244	0.9321	0.9312
$T^2(\chi^2_{N-K})$	197.63	201.92	190.58	87.95	87.37	68.7922	113.14	143.30
p-value	0	0	0	0.0014	0.0015	0.0593	0	0

Table 11: **Information Uncertainty Factor Loadings for S&P 500 Option Portfolios**

This table reports information factor loadings and t-statistics. Based on two-pass Fama-Mecbeth regression. The regression is the first step, time-series regression using market excess return and shock of EPSIR as factors. The table is organized by time-to-maturity and moneyness (strike/price level). Column names contain moneyness, and row names contain option type and time-to-maturity. For each option, first line is β (loading) for information uncertainty factor, and second line is its t-statistics. Sample period ranges from Jan 1996 to Jan 2012.

	90%	92.5%	95%	97.5%	100%	102.5%	105%	107.5%	110.5%
call 30	-0.0094	-0.0075	-0.0058	-0.0033	-0.0020	0.0023	0.0041	0.0049	0.0087
	-1.5048	-1.1638	-0.8459	-0.4527	-0.2440	0.2410	0.3802	0.4309	0.6912
call 60	-0.0080	-0.0070	-0.0047	-0.0034	-0.0011	0.0009	0.0041	0.0056	0.0062
	-1.2741	-1.0866	-0.6867	-0.4770	-0.1367	0.1068	0.4193	0.5241	0.5465
call 90	-0.0085	-0.0073	-0.0065	-0.0047	-0.0031	-0.0034	-0.0029	0.0007	0.0017
	-1.3450	-1.1392	-0.9751	-0.6579	-0.4015	-0.4120	-0.3163	0.0744	0.1556
put 30	-0.0468	-0.0470	-0.0445	-0.0402	-0.0350	-0.0299	-0.0256	-0.0224	-0.0197
	-3.0942	-3.5295	-3.8272	-4.0255	-4.1053	-4.0057	-3.7129	-3.1346	-2.9367
put 60	-0.0477	-0.0433	-0.0400	-0.0365	-0.0338	-0.0292	-0.0256	-0.0230	-0.0207
	-3.6523	-3.8224	-3.8776	-3.9842	-4.1203	-3.9046	-3.6831	-3.2481	-3.0660
put 90	-0.0432	-0.0376	-0.0367	-0.0348	-0.0311	-0.0285	-0.0270	-0.0258	-0.0234
	-3.5374	-3.5571	-3.7611	-3.9617	-3.8393	-3.7811	-3.7709	-3.7068	-3.4901

Table 12: **Cross Section Pricing Error for S&P 500 Option Portfolios**

This table reports **E**[RealizedRET-ExpectedRET] and t-statistics. Based on two-pass Fama-Mecbeth regression. The regression is the second step, cross-section regression using market excess return and shock of EPSIR as factor. The table is organized by time-to-maturity and moneyness (strike/price level). Column names contain moneyness, and row names contain option type and time-to-maturity. For each option, first line is mean value of pricing error, and second line is its t-statistics. Sample period ranges from Jan 1996 to Jan 2012.

	90%	92.5%	95%	97.5%	100%	102.5%	105%	107.5%	110.5%
call 30	0.0001	0.0005	0.0004	0.0076	-0.0004	-0.0003	-0.0003	-0.0006	0.0001
	0.4655	1.5465	1.3753	0.0001	-2.0228	-0.8936	-0.6371	-0.7755	0.1156
call 60	0.0005	0.0003	0.0006	0.0003	0.0004	0.0005	0.0013	0.0012	0.0008
	1.1466	1.1428	2.0648	1.2917	1.9426	2.7995	4.7720	2.7683	1.3272
call 90	0.0003	0.0004	0.0003	0.0004	0.0006	0.0001	0.0002	0.0009	0.0010
	1.1553	1.4728	1.1735	1.8345	2.6922	0.4424	0.4068	1.9943	1.4438
put 30	0.0076	0.0049	0.0023	0.0004	-0.0013	-0.0020	-0.0014	-0.0005	-0.0001
	9.0847	7.9747	5.2659	1.2914	-4.8385	-6.5181	-4.9253	-1.2329	-0.3952
put 60	0.0004	0.0005	0.0000	-0.0008	-0.0018	-0.0017	-0.0014	-0.0008	-0.0007
	0.7492	1.6670	0.0706	-4.0117	-7.4388	-6.4865	-5.4125	-2.1769	-2.1024
put 90	-0.0017	-0.0004	-0.0008	-0.0014	-0.0018	-0.0018	-0.0018	-0.0017	-0.0013
	-2.4591	-1.1105	-2.8973	-5.2032	-6.9199	-6.4130	-6.7537	-5.9522	-4.5188