Stock Recommendations from Stochastic Discounted Cash Flows

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Abstract

We present two stock recommendation systems based on stochastic characterization of firm present value that extends the conventional discounted cash flow analysis. The *single-stock quantile* recommendation system compares the market price of a company's stocks with the estimated distribution of the company's fair value to obtain an individual measure of mispricing, while the *cross-sectional quantile* system builds a relative measure of mispricing using the fair-value distribution of all firms. Both systems use mispricing information to build sell side and buy side portfolios. Statistical tests show that these portfolios consistently deliver significant excess returns even when rebalancing costs are accounted for. Moreover, we show how analysts' indications can be improved by using the first two moments of their recommendations following the same procedure of the cross-sectional quantile system.

Keywords: Stochastic Discounted Cash Flow; Asset Valuation; Valuation Uncertainty; Portfolio Strategy.

JEL codes: G11,G17,G32.

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1. Introduction

Noise and uncertainty play a major role in valuation: all estimates of value are noisy, so we can never know by how much market price has deviated from value , (Black, 1986, p. 533). Noise and uncertainty limit the extent to which an analysis of cash flows can provide useful and accurate information about the value of an asset, and they both affect the determination of target prices and the bottoms-up construction of portfolios. These limitations have been evident since the early history of financial analysis: in their foundational study on security analysis, (Graham and Dodd, 1934, p. 18) state: "The essential point is that security analysis does not seek to determine exactly what is the intrinsic value of a given security. It needs only to establish either that the value is adequate - e.g., to protect a bond or to justify a stock purchase - or else that the value is considerably higher or considerably lower than the market price. For such purposes, an indefinite and approximate measure of the intrinsic value may be sufficient."

In this study, we investigate whether investors can profit from two stock recommendation systems constructed using the information provided by the stochastic discounted cash flow (SDCF) analysis, a new approach for estimating shareholder value proposed the authors of the present work in Bottazzi et al. (2020). The SDCF method provides a valuation measure that takes into account the unavoidable uncertainty of future cash flows by means of a fairly general and theoretically grounded econometric methodology. In the SDCF, the pointwise estimate of the conventional discounted cash flow (DCF) is replaced by a random variable whose empirical distribution, *fair value distribution*, can be used to obtain an estimate of the expected fair value of the company and its degree of uncertainty.

The recommendation systems we introduce, based on the SCDF method, compare the fair value distributions with market prices to detect mispriced assets. Under the hypothesis that some degree of market efficiency is at work[1] , undervalued assets should recover their true value, at least in part, and thus, their prices are expected to increase. As such, they are good candidates for buy portfolios. Conversely, overvalued assets are expected to see a decrease in price and are good candidates for sell portfolios. The misvaluation assessment is run differently in our two recommendation systems. In the single-stock quantile system, the degree and direction of mispricing for a company are derived from the likelihood of obtaining the observed market price from the fair value distribution of that company. In the cross-sectional quantile system, mispricing information is derived

[1] We adopt the point of view of Black (1986): "The noise that noise traders put into stock prices will be cumulative, in the same sense that a drunk tends to wander farther and farther from his starting point. Offsetting this, though, will be the research and actions taken by the information traders. The farther the price of a stock gets from its value, the more aggressive the information traders will become. More of them will come in, and they will take larger positions. They may even initiate mergers, leveraged buyouts, and other restructurings. Thus the price of a stock will tend to move back toward its value over time", (p. 533).

comparatively, jointly using the fair value distributions of all considered companies. In this approach, the mispricing indicator is defined as the difference between the market price of the company and its expected fair value, normalized by the standard deviation of the fair value distribution. The advantage of the former method is that it uses all information available from the fair value distribution, while the latter derives its robustness with respect to possible fair value estimation biases.

The performance analysis of the two systems is carried out using an intercept test of excess portfolio returns, built using the Fama-French three-factor model (Fama and French, 1993) augmented with the momentum factor (Carhart, 1997). We show that equally weighted portfolios composed of the most highly recommended stocks consistently earn positive abnormal gross returns. The comparison with the much weaker results obtained using similar portfolios built from analyst recommendations from the Thomson Reuters Institutional Brokers' Estimate System (I/B/E/S) database further emphasizes the advantage of our methodology. Moreover, our recommendation systems remain profitable even when a high level of turnover costs is included. We believe that this study complements the literature that highlights the necessity for developing probabilistic and statistical tools to extend the conventional DCF approach by including some measure of uncertainty associated with the estimated value, (Bradshaw, 2004; Brown et al., 2015; Baule and Wilke, 2016; Casey, 2001). Finally, as a robustness check of the cross-sectional quantile methodology, we built an analogues system based on analysts recommendations and we find that it improves the the original analysts' indications.

The work is organized as follows. In Section 2 the SDCF methodology is briefly reviewed. In Section 3 the database and variables used in our analysis are described. In Section 4 we introduce the two stocks recommendations systems and in Section 5 we analyse their performance. Finally, we conclude in Section 6.

2. The stochastic discounted cash flow (SDCF) approach

The SDCF approach replaces the usual pointwise estimate of the preset value of a company with a *fair value distribution*, that takes into consideration the intrinsic uncertainty about the future firm performance. We provide a short review of the procedure below. For more details, refer to Bottazzi et al. (2020). Given the difficulty of estimating the debt cash flow using the available data, we adopt an unlevered free cash flow approach and derive the present value of equity V_0 from the present value of the firm $\widetilde{V}_0$, by subtracting the current value of "debt"

$$V_0 = \widetilde{V}_0 - (TD - CsI + MI + PS) \;, \tag{1}$$

where TD represents the total debt, used as a proxy for the market value of debt (Damodaran, 2007; Steiger, 2010; Damodaran, 2012), CsI the cash and short-term investments, MI the minority interest and PS denotes the preferred stocks. The firm present value does not require an estimate of new debt issues or debt repayment and can be obtained directly from an estimate of future unlevered free cash flow CF_t defined at each time t as (Damodaran, 2007, 2012; Chang et al., 2014; Gryglewicz et al., 2019)

$$CF_t := NOPAT_t + D\&A_t - CAPEX_t - \Delta WC_t, \tag{2}$$

where $NOPAT$ denotes the net operating profit after tax, $D\&A$ is depreciation and amortization, $CAPEX$ is capital expenditure and ΔWC is the change in the working capital. In turn, $NOPAT$ can be obtained from the earning before interest, taxes, depreciation, and amortization $EBITDA$ using the relation $NOPAT_t = (EBITDA - D\&A) \cdot (1 - \tau_0)$, where τ_0 represents the marginal tax rate. To derive the firm present value from estimated future cash flow, we make two assumptions[2]. First, a homogeneous cost of capital k is used to discount future cash flows. The cost of capital is computed as the weighted average of the cost of equity, after-tax cost of debt, and cost of preferred stocks. Second, in line with other studies literature (e.g., Damodaran (2007) and Ali et al. (2010)), we consider a two-stage model and assume that there exists a date $T > 0$ and a "perpetual growth rate" g, with $0 < g < k$, such that for any $t \geq T$ it is $CF_{t+1} = CF_t \cdot (1 + g)$. Thus, assuming that all the random quantities are defined on a filtered probability space $(\Omega, \mathcal{F}, (\mathcal{F}_t), \mathcal{P})$, for any possible future realization of cash flows $CF_t(\omega)$ with $\omega \in \Omega$ the associated firm present value reads

$$\widetilde{V}_0(\omega) = \sum_{t=1}^{T} \frac{CF_t(\omega)}{(1+k)^t} + \frac{CF_T(\omega)(1+g)}{(1+k)^T(k-g)} \, . \tag{3}$$

The model heavily depends on reliable estimates of CF_t. To estimate future cash flows, we start by expressing, through a firm specific regression model, all the relevant accounting variables in (2) as *margins* with respect to the revenues. Then we use a battery of econometric models, including stationary models, a local level model and a local linear trend model (Harvey (1990) and Durbin and Koopman (2012)), to describe the dynamics of log-revenues. The models are calibrated at the level of the single firm. For each firm we select the best performing model and we use it in a Monte Carlo exercise to generate future revenues realizations which, substituted in (3), produce a distribution of $\widetilde{V}_0$ values (see Bottazzi et al. (2020) for further details on model selection). The distribution of firm present values is adjusted using (1) and divided by the number of company's outstanding

[2]These assumptions are quite industry standard, e.g., they are similar to those of *Morningstar*® equity research methodology (MorningstarEquityResearchMethodology.pdf), and they are also adopted by our data provider Thomson Reuters Eikon, Datastream database.

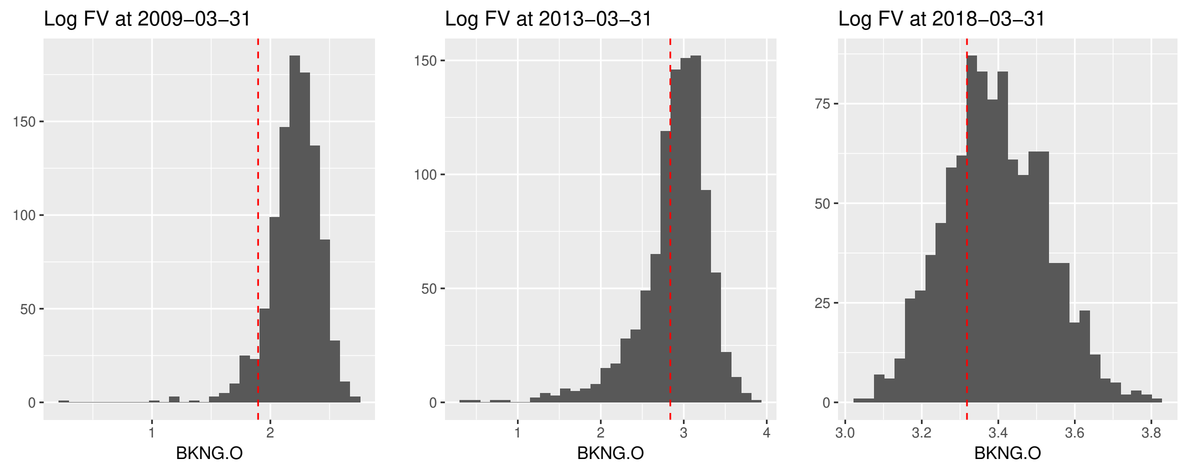

Figure 1: Distribution of the logarithm of the *fair value distribution* for Booking Holdings Inc. (ticker BKNG) computed at different dates. The red dotted lines indicate the market price at the evaluation date.

shares to obtain the *fair value distribution*. Under the hypothesis of the model, $\mathbb{E}[V_0]$ represents the traditional point-wise present value estimate of company's shares. Figure 2 shows the logarithm of the fair value distribution for Booking Holdings Inc. (ticker BKNG) computed at different dates. The red dotted lines indicate the market log-price at the evaluation date. At the end of the first quarter in 2009, the company results heavily undervalued, while it results only mildly undervalued in 2013 Q1 and 2018 Q1.

3. Data and Sample Selection Criteria

While the SDCF model relies upon general considerations, many details of its implementation and validation depend on specific company level data. In this section, we review the different data sources used to develop and test our methodology. All data were taken at a quarterly frequency.

The required equity prices, along with the corresponding fundamental data, are collected from Thomson Reuters Eikon Datastream database. We use the same data sample employed by Bottazzi et al. (2020) of 140 firms[3], which comprises all non-financial firms included in the S&P 500 over the entire period January 2009-December 2017, without missing data, and with sufficient observations of the revenues before January 2009.[4] The sample is rather heterogeneous: 17 firms each in the oil & gas (ICB 1) and basic material (ICB 1000) sectors, 44 industrial firms (ICB 2000), 22 consumer good firms (ICB 3000), 19 healthcare firms (ICB 4000), 12 firms in consumer service sector (ICB 5000), 3 firms

[3] Since we are interested in the comparison of companies within our selected sample of stocks the survivorship bias which may be induced by the sample selection criteria does not compromise our analyses.

[4] See Bottazzi et al. (2020) for further details concerning the sample selection procedures.

Table 1: Summary statistics of analysts' coverage of the 140 firms in our sample, grouped by ICB code. Two "snapshots" are reported: one taken in January 2009, and one in December 2017 (between brackets), which correspond to the beginning and end dates of the period under investigation. For each sector, the average consensus rating level and average number of analysts following the stocks are reported.

	Oil & Gas and Basic Materials	Industrial	Consumer Goods	Healthcare	Consumer Service	Telecommunication and Utilities	Technology	All
ICB codes	1 and 1000	2000	3000	4000	5000	6000 and 7000	9000	
Number of firms	17	44	22	19	12	10	16	140
Rating level								
Mean	2.29 (2.35)	2.50 (2.53)	2.64 (2.68)	2.32 (2.24)	2.5 (2.42)	2.25 (2.80)	2.22 (2.31)	2.42 (2.48)
Number of analysts								
Mean	15.12 (22.35)	12.73 (17.91)	11.77 (18.77)	14.89 (21.05)	17 (24)	13.70 (16.80)	24.12 (26.06)	14.90 (20.39)

in telecommunication sector (ICB 6000), 7 utilities firms (ICB 7000) and 16 technology firms (ICB 9000). We also obtain estimates of firm specific cost of capital k and the companyâs marginal tax rate τ_0 from Datastream. Following industry standard, the discount rate for the cash flow terminal value is computed by considering the fixed corporate tax rate provided by KPMG, instead of the individual tax rate, although the difference is minimal for all firms and all years considered. As we set the terminal year T at 5, the perpetual growth rate g is set equal to the 5-year T-bond rate obtained from the Federal Reserve Economic Data database.

The data used in the analysis of the returns of different portfolios within standard multi-factor models, performed in Section 5, are taken from the Kenneth R. French Data Library.[5] The data on analyst recommendations, used as a benchmarking exercise in Section 5, are obtained from the Datastream. We look at the Summary History-Recommendation file, which compiles a monthly snapshot of each company in the database by a sell-side analyst whose brokerage firm provides data to I/B/E/S. The database tracks the number of analysts following the stock, the average consensus rating level (a number between 1 and 5), along with its standard deviation and the number of analysts upgrading and downgrading their opinions over the previous month. A rating of 1 reflects a strong buy recommendation; 2, a buy; 3, a hold; 4, a sell; and 5, a strong sell. On average, each firm is followed by 20 analysts. Table 1 reports the sector specific summary statistics about analyst coverage.

4. Recommendations from Fair Value Distributions

The fair value distribution defined in the Section 2 can be straightforwardly used to obtain portfolio recommendations for company stocks. The basic idea is to use the valuation model to identify mispriced companies. Under the hypothesis that mispriced

[5]Freely available at https://mba.tuck.dartmouth.edu/pages/faculty/ken.french/data_library.html.

companies will revert to their correct price, undervalued firms represent prospective buys and overvalued firms represent prospective sells. Following standard practice, stocks are classified as strong buy (SB), buy (B), hold (H), sell (S) and strong sell (SS).

4.1. Single-Stock Quantile Recommendations System

Let FV_t^i be the distribution function of the fair value of company i at time t and P_t^i be its market price. The quantity $q_t^i = FV_{t_h}^i(P_{t_h}^i)$ represents the probability that the company's fair value is less than or equal to the observed price. In general, if q_t^i is near 0.5, the market price is near the median of the fair value distribution and we can conclude that, the company is fairly priced. However, if the value assigned by our valuation model to the company is higher (lower) than the market price, then the company is undervalued (overvalued) and $q_{t_h}^i$ is close to zero (one). Based on this consideration, the classification of stocks is performed in the following way[6]:

- if $q_{t_h}^i < 0.125$, the company i is classified SB;
- if $0.125 \leq q_{t_h}^i < 0.25$ it is classified B;
- if $0.25 \leq q_{t_h}^i < 0.75$ it is classified H;
- if $0.75 \leq q_{t_h}^i < 1$ it is classified S;
- and SS if $1 \leq q_{t_h}^i$.

This classification system, denoted as *Single-Stock Quantile* (SSQ), has the advantage of using all the information provided by the distribution of the company's fair value. The recommendation for each firm is obtained using only its own fair value distribution, without referring to the valuation of other firms. Thus, it is possible that some of the recommendation buckets remain empty; for example, that no stock is labeled SB or SS.

4.2. Cross-Sectional Quantile Recommendations System

A second approach is to use the fair value distribution of all firms at the same time. For this, we introduce a second recommendation system based on the definition of a company specific mispricing indicator. Let μ_t^i and σ_t^i be the empirical mean and standard deviation of the distribution of the logarithm of the fair value of stock i at time t, computed using the SDCF method, and let p_t^i be the closing log-price on day t of the same company. The mispricing indicator z_t^i of company i at time t is defined as the difference between the

[6]The class assignment is broadly in line with the values adopted by the *Morningstar*® equity research methodology, see MorningstarEquityResearchMethodology.pdf for further details.

company's expected log fair value and its log price divided by the standard deviation of the log fair value distribution,

$$z_t^i := \frac{p_t^i - \mu_t^i}{\sigma_t^i} \quad i \in \{1, \dots, N\} \ .$$

The uncertainty of the evaluation procedure, captured by the standard deviation σ_t^i, is used to modify the observed difference (in log) between the market price and the expected present value. If uncertainty is high, the indicator is reduced, as the observed difference is considered less significant. If uncertainty is low, the observed difference becomes more relevant and the indicator takes a higher value. Now, consider the empirical distribution function of all mispricing indicators z_t^i, $\forall i = 1, \dots, N$ and let $\rho_t(\alpha)$ be its α-quantile.

- The sock of company i is classified SB if $z_t^i < \rho_t(0.1)$;
- B, if $\rho_t(0.1) \leq z_t^i < \rho_t(0.4)$;
- H, if $\rho_t(0.4) \leq z_t^i < \rho_t(0.6)$
- S, if $\rho_t(0.6) \leq z_t^i < \rho_t(0.9)$;
- and SS if $\rho_t(0.9) \leq z_t^i$.

Firms with a misvaluation indicator near the median of the empirical distribution of all indicators are assigned to the hold class. Firms with a high mispricing with respect to the median are assigned to the sell class and become a strong sell if they are in the top decile. Conversely, firms with a low misvaluation with respect to the median are a buy and a strong buy if they are in the bottom decile. We term this system *Cross-Sectional Quantile* (CSQ). The advantage of this system is that an overall shift in market prices that has no effect on the relative rankings of different companies has no effect on their classification. This system is also insensitive to the presence of a common bias affecting the valuation procedure of different companies.

To test the performance of the SSQ and CSQ systems, we consider 19 non-overlapping periods of six months, from FQ1 2009 to FQ1 2018[7]. At the beginning of each period, we classify the firms using both systems. We use the first available closing price for computing the mispricing indicator. In the case of SSQ, both the number of firms in each class and the associated market capitalization, with respect to our universe of stocks, can vary from period to period, while for CSQ, the number of stocks in each class is constant in all periods. On average, in the SSQ system, the SB class has 31 stocks (28% market capitalization), B has 25 (18%), H has 62 (40%), S has 18 (11%) and SS has 3 (3%);

[7]The period of six months was chosen because it is long enough for the calibration of the cash flow model to be reliable but short enough to give us a sufficient number of data points to analyze. In any case, it is broadly consistent with several portfolio strategies discussed in (Li et al., 2019).

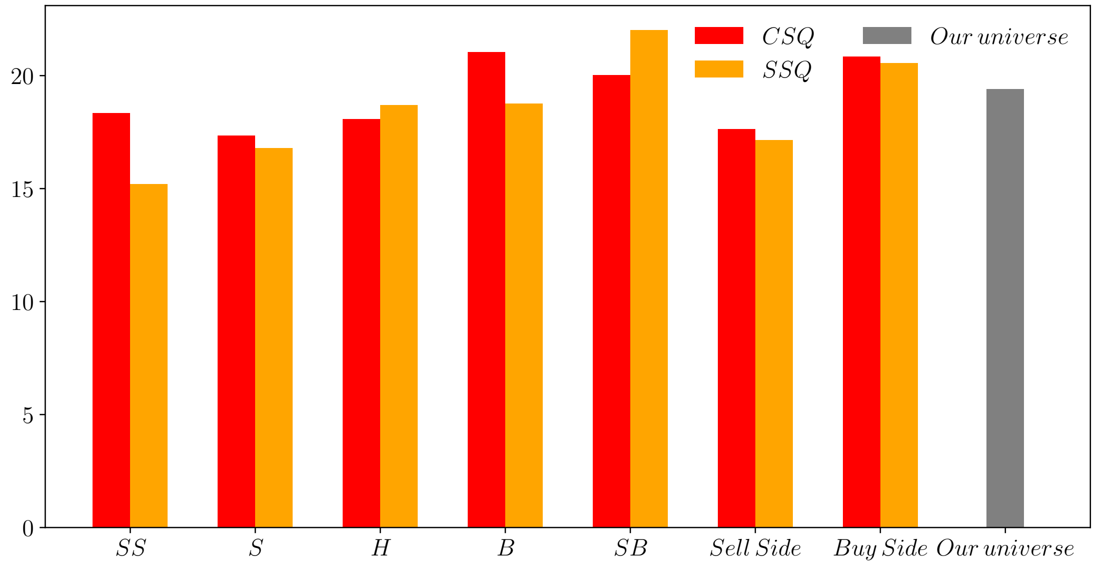

Figure 2: Annualized log-returns in percentage for each of the constructed portfolio according to *CSQ* and to the *SSQ* recommendation. *Our universe* is the return of an equally-weighted portfolio that goes long in all the stocks of our universe. The sample period is April 1, 2009 to September 28, 2018.

while in the *CSQ* system, *SB* class has 14 stocks (13% market capitalization), *B* has 42 (37%), *H* has 28 (16%), *S* has 42 (25%) and *SS* has 14 (9%).

For each recommendation system, we build equally weighted portfolios with all companies in a given rating class at the beginning of each semester and compute the daily returns of these portfolios R_t^p, with p taking values SS, S, H, B, SB, on each day t of the semester[8].

5. Performance Evaluation

We begin with a simple calculation, over the entire period considered, of the annualized log-returns (as percentages) for each of our constructed portfolios. In Figure 2 they are compared with the annualized log-returns of a benchmark equally weighted portfolio that goes long in all the stocks of our universe, labeled *Our universe*. As can be seen, undervalued assets tend to grow significantly faster than overvalued ones. For instance, the annualized log-return of the *SB* portfolio built following the *SSQ* system is 22.00% while that of the *SS* portfolio is 15.20%. This also holds true if we consider a more coarse grained classification, merging portfolios in the buy and sell sides. The use of just two broad classes seems to enhance the performance of the *CSQ* system. The enhanced

[8] In Barber et al. (2001), market-weighted rather than equally weighted portfolios are considered. Their choice is consistent with the use of daily rebalancing and the size of their sample. However, the authors warn about the possibility that using market-weighted returns could bias against finding evidence of abnormal returns, so we opt for a more conservative choice.

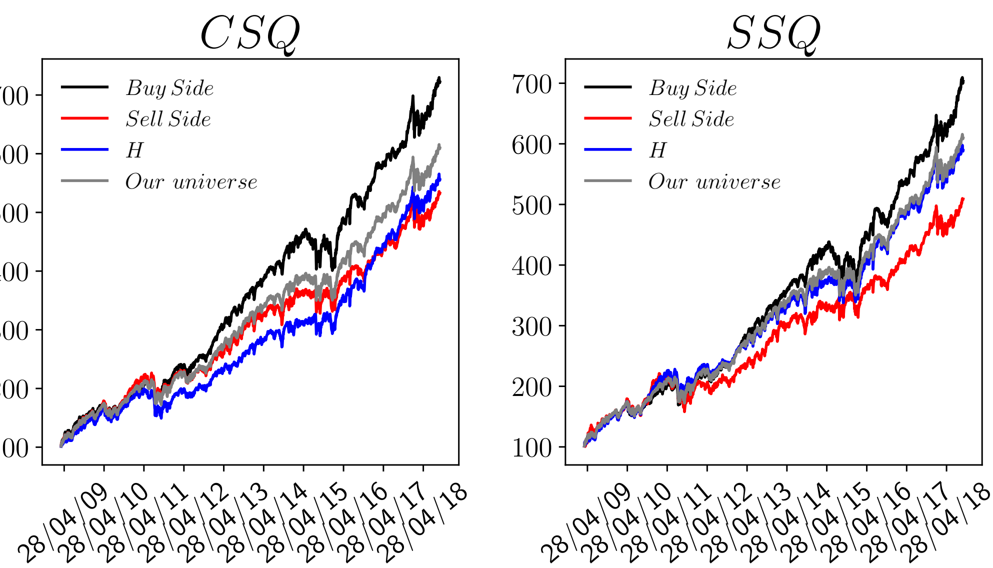

Figure 3: Cumulative sum of daily returns when investing \$100 in the *Buy Side*, *H* and *Sell Side* portfolio constructed with the *CSQ* (left panel) and *SSQ* (right panel) methodologies. The grey line is the cumulative sum of daily returns of *Our universe*. The sample period is from April 1, 2009 to September 28, 2018.

performance of the portfolios obtained with the two recommendation systems is also confirmed when a measure of risk is included. The Sharpe Ratio (Sharpe, 1994) of the *Buy Side* portfolios is 1.43 for the *CSQ* system and 1.40 for the *SSQ* system. They are both significantly higher, according to the Ledoit and Wolf (2008) and Ardia and Boudt (2018) tests (p-value around 0.003), than the Sharpe Ratio of the *Our universe* portfolio, which is 1.24. In turn, the two sell side portfolios have values that are significantly lower than the Our Universe portfolio (1.05 for the *CSQ* system and 0.97 for the *SSQ* system)[9]. Figure 3 represents the cumulative sum of daily returns when investing \$100 in the *Buy Side*, *H* and *Sell Side* portfolios constructed with the *CSQ* (left panel) and *SSQ* (right panel) methodologies, compared with the *Our universe* portfolio.

To obtain a more precise estimate of portfolio performances, we employ an intercept test using the Fama-French three-factor model (Fama and French, 1993), augmented with the momentum factor (Carhart, 1997). We estimate the following daily time-series regression:

$$R_{p,t} - R_{F,t} = \alpha_p + \beta_p(R_{M,t} - R_{F,t}) + s_p, SMB_t + h_p, HML_t + m_p MOM_t + e_{p,t}\,, \quad (4)$$

where $R_{p,t} - R_{F,t}$ denotes the excess return of the selected portfolio over the risk-free

[9] The Sharpe ratio is computed by setting the benchmark return to zero. We compared the portfolios performances using the Sortino ratio (Sortino and Price, 1994) also, obtaining identical results.

Table 2: Estimated gross annual abnormal returns earned by portfolios constructed with our *CSQ* and *SSQ* systems, and using the analysts' recommendation from the I/B/E/S database. The coefficients significant at 10%, 5%, 1% and 0.1% level are marked with '.', '$*$', '$**$' and '$***$' respectively. The *SS* portfolios in the case of *SSQ* system and both the *SS* and *BB* portfolios in the case of analysts are empty for a few rebalancing dates. In such cases, the corresponding returns are set to zero.

α_p	*SS*	*S*	*H*	*B*	*SB*	*Sell Side*	*Buy Side*
CSQ	1.5733	0.8190	2.4977	5.7706 ***	7.0838 ***	1.0076	6.0989 ***
SSQ	<u>1.4665</u>	1.2052	2.0189	3.3869 *	7.9444 ***	1.4182	5.8442 ***
Analysts	<u>11.0480</u> **	5.0222 **	2.4576 *	2.1187	<u>1.9982</u>	5.795 ***	2.0579

rate $R_{F,t}$ for period t, $R_{M,t}$ is the return of the value-weighted market portfolio, SMB_t is the difference between the daily returns of a value-weighted portfolio of small stocks and one of large stocks, HML_t is the difference between the daily returns of a value-weighted portfolio of high book-to-market stocks, MOM_t is the momentum factor and e_{pt} is the error term. The regression yields parameter estimates of α_p, β_p, s_p, h_p and m_p but the relevant parameter here is intercept α_p, as it captures the presence of abnormal returns. The results are reported in Table 2. The most highly recommended stocks (B, SB, and *Buy Side*) earn positive abnormal gross returns, whereas the least recommended ones do not. In addition, the abnormal gross excess returns of these portfolios are greater than that of *Our Universe*, which has a gross annual excess return of 3.34% with a p-value of $3.16e-04$. These results suggest that investors following our SDCF based recommendations and building concentrated portfolios could obtain returns that beat the market.

As a further check, we repeat the same analysis using expert recommendations from the I/B/E/S database, as described in Section 3. Let $\bar{A}_t^i$ be the average analysts rating for firm i on date t. We follow Barber et al. (2001) and if $1 \leq \bar{A}_t^i \leq 1.5$ we classify company i as SB; if $1.5 < \bar{A}_t^i \leq 2$ as B; if $2 < \bar{A}_t^i \leq 2.5$ as H; if $2.5 < \bar{A}_t^i \leq 3$ as S; and a SS whenever $\bar{A}_t^i > 3$. The downward shift accounts for the observed over optimistic recommendation scores provided by experts (see Barber et al. (2001) and the references therein). Using the expert recommendation system, we build six-month rebalanced portfolios exactly as we did for our systems and perform the regression in (4). The results in Table 2 show that the experts' *Buy Side* does not provide significant abnormal returns. In fact, abnormal returns are observed for the experts' *Sell Side* portfolio. Moreover, if the Sharpe and Sortino ratio of the *Sell Side*, *Hold* and *Buy Side* are considered, it turns out to be more clear, see Table 3, how Analysts does not provide a consistent recommendation system, i.e., a system where the more profitable portfolios are those with higher recommendations.

The previous calculated returns are the gross of all trading costs. To assess the size

Table 3: This table reports the Sharpe and the Sortino ratios, in parenthesis, for the *H*, *Sell Side* and *Buy Side* portfolios constructed according to *Analysts*, *Cross-Sectional Quantile* and *Single-Stock Quantile*. Sharpe and the Sortino ratios for *Our universe* is also reported.

Sharpe-ratio (Sortino-ratio)	*Analysts*	*CSQ*	*SSQ*	*Our universe*
Sell Side	1.31 (1.72)	1.05 (1.36)	0.97 (1.28)	1.24 (1.59)
H	1.18 (1.52)	1.15 (1.48)	1.11 (1.45)	1.24 (1.59)
Buy Side	1.16 (1.51)	1.43 (1.87)	1.40 (1.83)	1.24 (1.59)

Table 4: Annualized percentage turnover of the *Cross-Sectional Quantile* and *Single-Stock Quantile* portfolios.

	SS	*S*	*H*	*B*	*SB*	*Sell-Side*	*Buy-Side*
CSQ	136.51	158.20	245.24	148.15	142.86	102.38	98.016
SSQ	164.51	236.37	149.70	241.56	167.48	200.81	136.16

of these costs, we calculate a measure of annual turnover. Let t_h with $h = 1, \ldots, 19$ be the rebalancing dates, N_h^p be the number of companies in portfolio p at date t_h and δ_{i,t_h}^p be equal to 1 if company i is in portfolio p at date t_h and zero otherwise. Turnover at date t_{h+1} is calculated as

$$\mathrm{TO}_{t_{h+1}}^p := \sum_i \left| \frac{\delta_{i,t_h}^p}{N_h^p} - \frac{\delta_{i,t_{h+1}}^p}{N_{h+1}^p} \right|$$

where the sum is for all companies composing our universe. Annualized total turnover TO^p is twice the average of the previous quantity across the entire period. The values are reported in Table 4.

The transaction cost of portfolio p is computed as the product of the annualized turnover and the round-trip cost (RTC) (see Baule and Wilke, 2016, and references therein) and the âcritical" round-trip cost, RTC_{crit}^p, which is the rebalancing cost that makes the net abnormal return of the portfolio equal to that of the *Our universe* benchmark:

$$\alpha^p - \mathrm{RTC}_{crit}^p \cdot \mathrm{TO}^p = \alpha^{\text{Our universe}}.$$

The critical round-trip cost is equal to 2.81% for the buy side *CSQ* and 1.84% for the buy side *SSQ* portfolios, the only ones with an abnormal return above that of the benchmark. Table 5 displays the adjusted (for transaction costs) annualized abnormal returns and the adjusted Sharpe ratio (i.e., computed using the adjusted returns, defined as the difference between the actual returns and transaction costs) for different levels of round-trip costs. As can be seen, the considered portfolios remain profitable even when transaction costs are

Table 5: The annualized abnormal returns and adjusted annualized percentage Sharpe ratios for the *Buy Side* portfolios as a function of RTC^p(%) for both *CSQ* and *SSQ* systems. The Sharpe ratios that are significantly different from that of *Our Universe*, according to the Ledoit and Wolf (2008) and Ardia and Boudt (2018) tests, at 10%, 5%, 1% and 0.1% level are marked with '.', '∗', '∗∗' and '∗ ∗ ∗' respectively.

	CSQ		*SSQ*	
RTC^p(%)	Ann. Abn. Return (%)	Ann. Sharpe Ratio (%)	Ann. Abn. Return (%)	Ann. Sharpe Ratio (%)
0.00	6.10	1.43 **	5.84	1.40 *
0.31	5.79	1.41 **	5.42	1.37 .
0.63	5.49	1.39 *	4.99	1.34
0.94	5.18	1.37 *	4.57	1.31
1.25	4.87	1.35 .	4.14	1.28
1.31	4.81	1.34	4.06	1.28
1.38	4.75	1.34	3.97	1.27
1.56	4.57	1.32	3.72	1.25
1.88	4.26	1.30	3.29	1.22
2.19	3.95	1.28	2.87	1.20
2.50	3.65	1.26	2.44	1.17
2.81	3.34	1.24	2.01	1.14

fairly high. The buy side *CSQ*, with the lowest turnover, is less sensitive to transaction costs.

5.1. Building z-scores from the analysts' recommendations

How are affected the results in the previous analysis if we replace in the definition of the z-score the mean and the standard deviation of the fair-value distribution with the average and the standard deviation of the price targets given by the IBES database? We name this recommendation approach *Cross-Sectional-IBES* (henceforth *CS-IBES*)[10]. Table 6 summarizes the results. Taking *Our-universe* as baseline, the buy-side of *CSQ* and *SSQ* presents the higher Sharpe Ratio and Sortino gains. We emphasize that also *CS-IBES* provides coherent results for *Buy Side* and *Sell Side* portfolios, in contrast with the methodology employed by Barber et al. (2001). Remarkably, the difference between the Sharpe Ratio of the *CS-IBES* and *CSQ* buy-side is statistically significant at the 5% level, whereas it is not significant the difference between the *CS-IBES* and *Our-universe* buy-side, again according to the test of Ledoit and Wolf (2008) and Ardia and Boudt (2018). The investigation carried out in this section shows that we can improve the

[10]It is worth noting that one of the nice features of this method is that it cannot happen that for some rebalancing date portfolios with the most/least favorably recommended companies are empty, as it may be the case with the rescaling proposed by Barber et al. (2001).

Table 6: This table reports the Sharpe and the Sortino ratios for the H, *Sell Side* and *Buy Side* portfolios constructed according to *Cross-Sectional-IBES*, *Cross-Sectional Quantile* and *Single-Stock Quantile*. Sharpe and the Sortino ratios for *Our universe* is also reported.

Sharpe-ratio (Sortino-ratio)	*CS-IBES*	*CSQ*	*SSQ*	*Our universe*
Sell Side	1.19 (1.53)	1.05 (1.36)	0.97 (1.28)	1.24 (1.59)
H	1.10 (1.43)	1.15 (1.48)	1.11 (1.45)	1.24 (1.59)
Buy Side	1.30 (1.70)	1.43 (1.87)	1.40 (1.83)	1.24 (1.59)

research design of Barber et al. (2001) using the information provided by the average and standard deviation of IBES's price targets with the methodology of the cross-sectional z-score recommendations. In particular, this shows the robustness of the cross-sectional approach, CSQ, to issue consistent recommendation system, thus exhibiting a way to use the uncertainty provided by analysts to construct profitable investment strategies.

6. Conclusions

We propose two recommendation systems based on the comparison of observed market prices with the fair value distributions obtained through the SDCF method introduced by Bottazzi et al. (2020). This approach compares the market price with a probability distribution of fair values, calculated by fitting an econometric model to past revenues, thus attempting to quantify mispricing as well as uncertainty. The *Single-Stock Quantile* system derives recommendations for each company, considering only how far the price of the stock is from the median of the computed fair value distribution. The *Cross-Sectional Quantile* system builds a mispricing indicator for each company and then derives recommendations by comparing the indicators across all companies. While the former method fully uses all information available from the fair value distribution, the latter is more robust with respect to possible fair value estimation biases and also guarantees a constant number of stocks in each recommendation bucket.

For each recommendation system, we build buy and sell side portfolios and estimate the abnormal returns, both gross and net trading costs, earned from diverse investment strategies. The *Buy Sides* provides a significant average annual abnormal gross return of about 6% percent, after controlling for market risk, size, book-to-market, and price momentum effects, which doubles the market abnormal gross return (of Our Universe), which is about 3%. Contrary to the portfolios based on analysts' stock recommendations (i.e., the I/B/E/S recommendation system), our investment strategies (portfolios) are always consistent, as buying stocks with a more favorable recommendation invariably earns a greater annualized log-return than buying stocks with less favorable recommendation. Finally, we show how to use the uncertainty provided by analysts and thus how to improve their recommendations, by relying on the CSQ methodology, which exhibits the robustness of the cross-sectional approach to issue consistent recommendation system.

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